Revisit to the b $\to c\tau\nu$ transition: in and beyond the SM

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Outline

- Lepton Flavor Universality
- $R(D^{(*)})$
- Motivation
- Form Factors
- Fit of the Heavy Quark Effective Theory Parameters
- Analyses of New Physics
- Summary and Conclusions

Lepton Flavor Universality

- Leptons of different generations have same couplings in the SM

	$Experiment$	$Theory(SM)$
$\frac{\Gamma_{K^- \to e^- \bar{\nu}_e}}{\Gamma_{K^- \to \mu^- \bar{\nu}_\mu}}$	$(2.488 \pm 0.009) \times 10^{-5}$	$(2.477 \pm 0.001) \times 10^{-5}$
$\frac{\Gamma_{\pi^- \to e^- \bar{\nu}_e}}{\Gamma_{\pi^- \to \mu^- \bar{\nu}_\mu}}$	$(1.230 \pm 0.004) \times 10^{-5}$	$(1.2352 \pm 0.0001) \times 10^{-5}$
$\frac{\Gamma_{D_s^- \to \tau^- \bar{\nu}_\tau}}{\Gamma_{D_s^- \to \mu^- \bar{\nu}_\mu}}$	9.95 ± 0.61	9.76 ± 0.10

Simone Bifani *et al* 2019 J. Phys. G: Nucl. Part. Phys. 46 023001

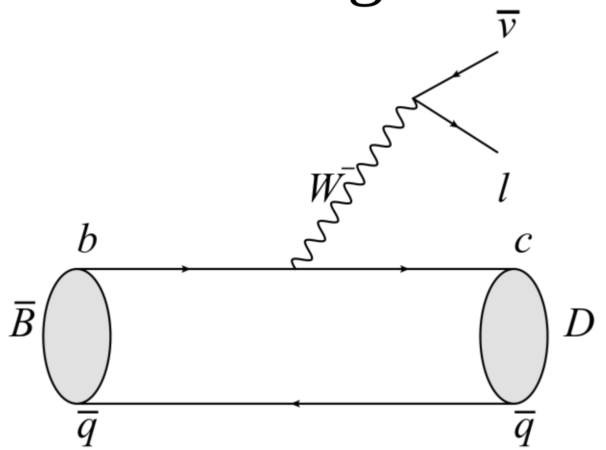
$R(D^{(*)})$ and $R(K^{(*)})$

- $B \to D^{(*)} l \nu$ ($b \to c l \nu$):

  Difference between third generation and the first two generations

  Deviation from the SM: 3 ~ 4σ

$$R(D^{(*)}) = \frac{\mathcal{B}(B \to D^{(*)}\tau\nu)}{\mathcal{B}(B \to D^{(*)}\ell\nu)}, \quad \text{with } \ell = \mu, e$$



- $B \to K^{(*)} l^+ l^-$ ($b \to s l^+ l^-$):

  Difference between first and second generation

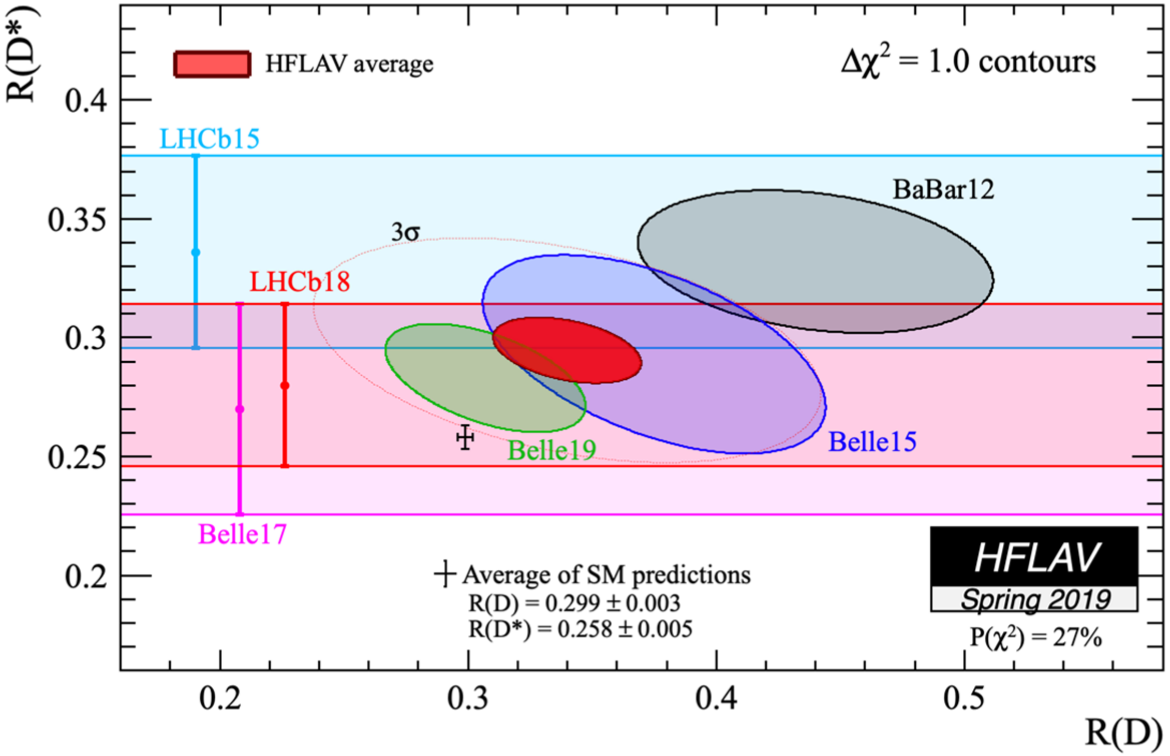
  Deviation from the SM: $R(K)$ 2.6σ

  $R(K^*)$ 2.1 ~ 2.5σ

$$R(K^{(*)}) = \frac{\int_{q^2_{min}}^{q^2_{max}} \frac{d\Gamma(B \to K^{(*)}\mu^+\mu^-)}{dq^2} dq^2}{\int_{q^2_{min}}^{q^2_{max}} \frac{d\Gamma(B \to K^{(*)}e^+e^-)}{dq^2} dq^2}$$

$R(D^{(*)})$ Experimental Status

- The combined results of $R(D^{(*)})$ indicate about 3σ deviation from the SM predictions



$R(D)=0.340\pm0.027\pm0.013$

$R(D^{*})=0.295\pm0.011\pm0.008$

- LHCb reported $R(J/\psi)=\frac{\mathcal{B}(B_c\to J/\psi\tau\nu)}{\mathcal{B}(B_c\to J/\psi\mu\nu)}$ = 0.71±0.17±0.18, which deviate 2σ away from the SM prediction

https://hflav-eos.web.cern.ch/hflav-eos/semi/spring19/main.shtml

Motivation

- $R(D^{(*)})$ anomaly may imply New physics Effect.

- Study of form factors allow us to give more reliable predictions for $R(D^{(*)})$.

- In light of recent data of $R(D^{(*)})$ and the updated form factors, the analyses of New physics can be performed.

Form Factors

Hadronic matrix element:

$$f_-\left(q^2\right)=\frac{m_B^2-m_D^2}{q^2}\left(f_0\left(q^2\right)-f_+\left(q^2\right)\right)$$

$$\left\langle D(p')|\bar{c}\gamma^\mu b|\bar{B}(p)\right\rangle = \boxed{f_+}(q^2)(p+p')^\mu + \boxed{f_-}(q^2)(p-p')^\mu$$

$$q = p - p'$$

In SM:

$$\frac{d\Gamma(B\to D\ell\nu)}{dq^2} = \frac{G_F^2|V_{cb}|^2\eta_{EW}^2}{192\pi^3 m_B^3}\frac{k}{(q^2)^{\frac{5}{2}}}(q^2-m_\ell^2)^2[4k^2q^2(2q^2+m_\ell^2)|\boxed{f_+}|^2+3m_\ell^2|\boxed{f_0}|^2]$$

$$\frac{d\Gamma(B\to D^*\ell\nu)}{dq^2} = \frac{G_F^2|V_{cb}|^2\eta_{EW}^2}{192\pi^3 m_B^3}\frac{k}{(q^2)^{\frac{5}{2}}}(q^2-m_\ell^2)^2\left\{(2q^2+m_\ell^2)[2q^2|\boxed{f}|^2+|\boxed{\mathcal{F}_1}|^2+2k^2(q^2)^2|\boxed{g}|^2]\right.$$
$$\left.+3m_\ell^2k^2q^2|\boxed{\mathcal{F}_2}|^2\right\}$$

Where $k = \sqrt{\frac{[(m_B+m_{D^{(*)}})^2-q^2][(m_B+m_{D^{(*)}})^2-q^2]}{4q^2}}$

$$R(D^{(*)}) = \frac{\mathcal{B}(B\to D^{(*)}\tau\nu)}{\mathcal{B}(B\to D^{(*)}\ell\nu)}, \quad \text{with } \ell=\mu, e$$

C.G. Boyd, B. Grinstein and R.F. Lebed, Precision corrections to dispersive bounds on form-factors, Phys. Rev. D 56 (1997) 6895

Calculation of Form Factors

- Small recoil(Near Max point of q^2):

  Lattice QCD
- Large recoil(Near $q^2 = 0$):

  Light Cone Sum Rule, Perturbative QCD…
- Extrapolation of Form factors:

  Pole model ⊕ z expansion

  Specific Parameterization:

  Boyd-Grinstein-Lebed (BGL)

  Bourrelly-Caprini-Lellouch (BCL)

  Heavy quark effective theory

  Specific Parameterization:

  Caprini-Lellouch-Neubert (CLN)

Form Factors in Heavy Quark Effective Theory

$$h_+ = \xi(w)(1 + \frac{\alpha_s}{\pi}(C_{V_1} + \frac{w+1}{2}(C_{V_2} + C_{V_3})) + (\varepsilon_c + \varepsilon_b)L_1(w) + \varepsilon_c^2\, \delta h_+)$$

$$h_- = \xi(w)(\frac{\alpha_s}{\pi}\frac{w+1}{2}(C_{V_2} - C_{V_3}) + (\varepsilon_c - \varepsilon_b)L_4(w))$$

$$f_+ = \frac{m_B + m_D}{2\sqrt{m_B m_D}}\left(h_+ - \frac{m_B - m_D}{m_B + m_D}h_-\right)$$

$$f_0 = \frac{\sqrt{m_B m_D}}{m_B + m_D}(1+w)\left(h_+ - \frac{m_B + m_D}{m_B - m_D}\frac{w-1}{w+1}h_-\right)$$

where $w = \dfrac{m_B^2 + m_{D^{(*)}}^2 - q^2}{2\, m_B\, m_{D^{(*)}}}$,

$$L_1 = -4(w-1)\chi_2 + 12\chi_3, \quad L_2 = -4\chi_3, \quad L_3 = 4\chi_2, \quad L_4 = 2\eta - 4, \quad L_5 = -1, \quad L_6 = -2\frac{1+\eta}{w+1}$$

Corrections $\mathcal{O}(\alpha_s),\ \mathcal{O}\left(\frac{\Lambda_{QCD}}{m_{b,c}}\right),\ \mathcal{O}\left(\frac{\Lambda_{QCD}^2}{m_c^2}\right)$

$$\xi(w) = 1 - 8\rho^2\frac{\sqrt{1+w} - \sqrt{2}}{\sqrt{1+w} + \sqrt{2}} + (64c - 16\rho^2)\left(\frac{\sqrt{1+w} - \sqrt{2}}{\sqrt{1+w} + \sqrt{2}}\right)^2$$

$$\eta = \eta(1) + \eta'(1)(w-1)$$

$$\chi_2 = \chi_2(1) + \chi_2'(1)(w-1)$$

$$\chi_3 = \chi_3'(1)(w-1)$$

$$\delta h_+, \delta h_{A_1}, \delta h_{T_1}$$

I. Caprini, L. Lellouch and M. Neubert, Dispersive bounds on the shape of $B \to D^{(*)}$ lepton anti-neutrino form-factors, Nucl. Phys. B 530 (1998) 153

Fit of the Heavy Quark Effective Theory Parameters

$$f_i = \frac{1}{P_i(z)\Phi_i(z)} \sum_{n=0}^{+\infty} a_n^i z^n = h_{i,HQET}(z)$$

$$\sum_{i=1}^{7}\sum_{n=0}^{2}(a_{1-,n}^i)^2 \leqslant 1,\ \sum_{i=1}^{7}\sum_{n=0}^{2}(a_{1+,n}^i)^2 \leqslant 1,\ \sum_{i=1}^{3}\sum_{n=0}^{2}(a_{0-,n}^i)^2 \leqslant 1,\ \sum_{i=1}^{3}\sum_{n=0}^{2}(a_{0+,n}^i)^2 \leqslant 1$$

$$a_{0,1,2}^{f_i}(\rho^2, c, \chi_2(1), \chi_2'(1), \chi_3'(1), \eta(1), \eta'(1), \delta h_+, \delta h_{A_1}, \delta h_{T_1})$$

- Lattice QCD results
- Light-cone sum rule results
- Masses of B_c given by experiment, Lattice QCD and model calculation

Fit of the Heavy Quark Effective Theory Parameters

- Results :

$\chi_2(1)$	$\chi_2'(1)$	$\chi_3'(1)$	$\eta(1)$	$\eta'(1)$
0.133(23)	-0.149(19)	0.017(8)	0.365(28)	0.239(114)

ρ^2	c	$\delta_{h_{A_1}}$	δ_{h_+}	$\delta_{h_{T_1}}$
1.120(28)	0.932(212)	-1.304(202)	0.032(133)	-4.888(1975)

$R(D)=0.289\pm0.005$

$R(D^*)=0.237\pm0.008$

Analysis of New Physics

- Model independent
- Detailed models:

NP models	Constrained by
W’	High p_T exprimensts
Charged Higgs	B_c life time, $\mathcal{B}(B_c \to \tau\nu)$
Leptoquark	

Data and χ^2 Fit

	R_D	R_{D^*}	Correlation	$P_\tau(D^*)$	$R_{J/\psi}$	$F_L^{D^*}$
BaBar	$0.440(58)(42)$	$0.332(24)(18)$	-0.27	–	–	–
Belle	$0.375(64)(26)$	$0.293(38)(15)$	-0.49	–	–	–
Belle	–	$0.302(30)(11)$	–	–	–	–
Belle	–	$0.270(35)(^{+0.028}_{-0.025})$	0.33	$-0.38(51)(^{+0.21}_{-0.16})$	–	–
LHCb	–	$0.336(27)(30)$	–	–	–	–
LHCb	–	$0.291(19)(26)(13)$	–	–	–	–
Belle	$0.307(37)(16)$	$0.283(18)(14)$	-0.54	–	–	–
LHCb	–	–	–	–	$0.71(17)(18)$	–
Belle	–	–	–	–	–	$0.60(8)(4)$

Experimental data used in the fits

$$\chi^2(C_X) = \sum_{m,n=1}^{\text{data}} (O^{th}(C_X) - O^{exp})_m (V^{exp} + V^{th})^{-1}_{mn} (O^{th}(C_X) - O^{exp})_n$$
$$+ \frac{(R^{th}_{J/\psi}(C_X) - R^{exp}_{J/\psi})^2}{\sigma^2_{R_{J/\psi}}} + \frac{(F_L^{D^*\,th}(C_X) - F_L^{D^*\,exp})^2}{\sigma^2_{F_L^{D^*}}}$$

Effective Hamiltonian with New Physics

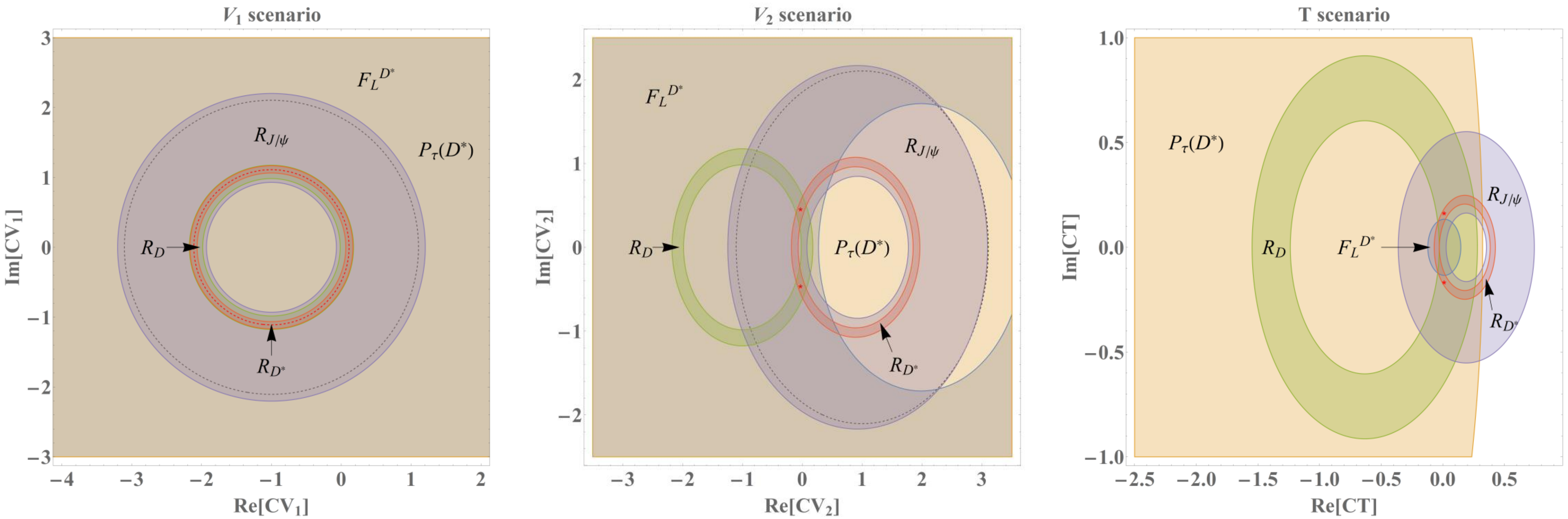
- Weak effective Hamiltonian:

$$\mathcal{H}_{\text{eff}} = \frac{4G_F}{\sqrt{2}} V_{cb} \left[(1 + C_{V_1}) \mathcal{O}_{V_1} + C_{V_2} \mathcal{O}_{V_2} + C_{S_1} \mathcal{O}_{S_1} + C_{S_2} \mathcal{O}_{S_2} + C_T \mathcal{O}_T \right] + \text{H.c.}$$

- The four-fermion operator allowed in b→c$\tau\nu$ transition:

$$\begin{aligned}
\mathcal{O}_{S_1} &= (\bar{c}_L b_R)(\bar{\tau}_R \nu_L)\,, \quad \mathcal{O}_{S_2} = (\bar{c}_R b_L)(\bar{\tau}_R \nu_L) \\
\mathcal{O}_{V_1} &= (\bar{c}_L \gamma^\mu b_L)(\bar{\tau}_L \gamma_\mu \nu_L)\,, \quad \mathcal{O}_{V_2} = (\bar{c}_R \gamma^\mu b_R)(\bar{\tau}_L \gamma_\mu \nu_L) \\
\mathcal{O}_T &= (\bar{c}_R \sigma^{\mu\nu} b_L)(\bar{\tau}_R \sigma_{\mu\nu} \nu_L)
\end{aligned}$$

2σ Constraints on the NP Wilson Coefficients



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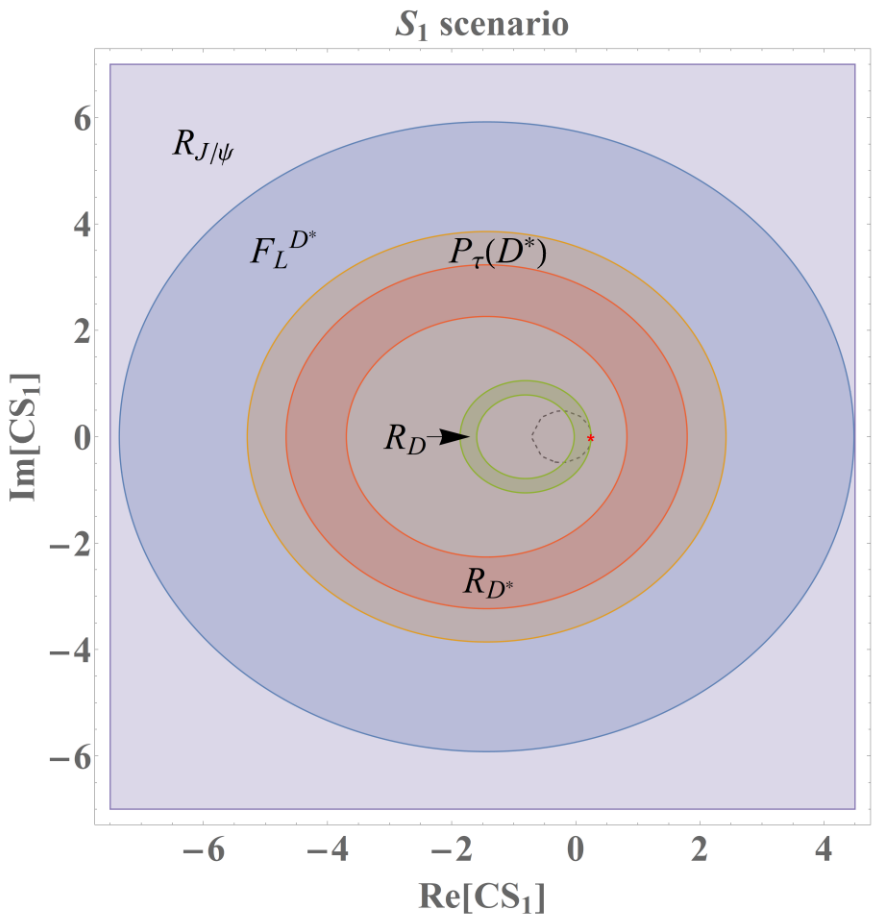

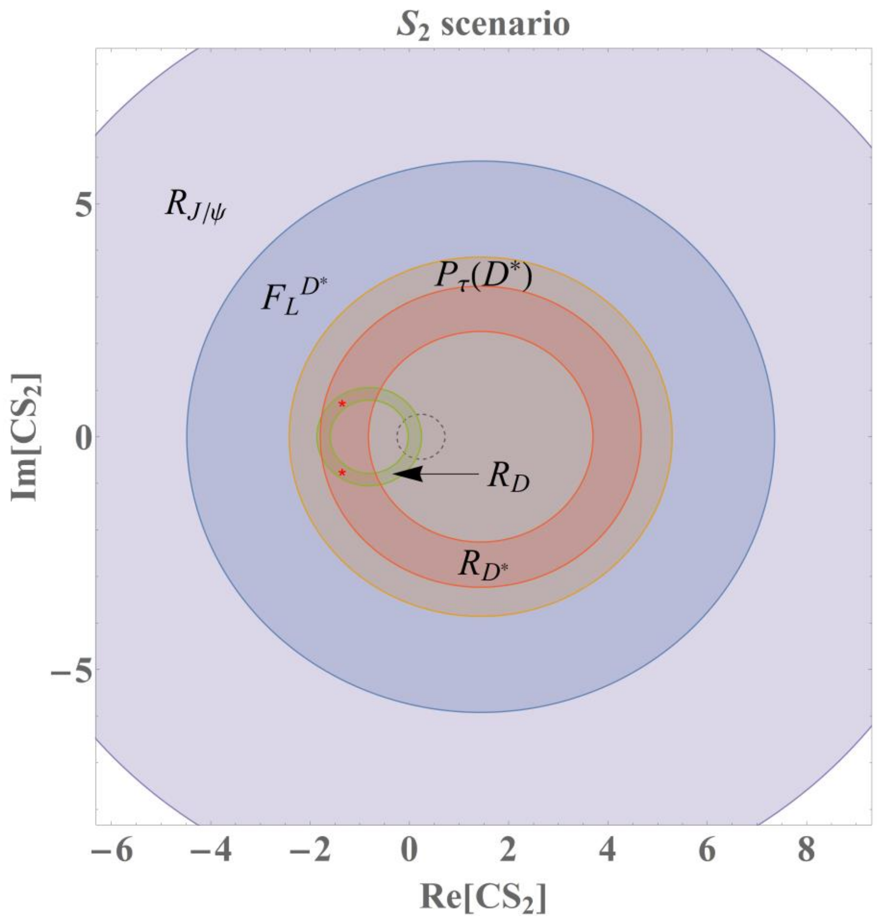

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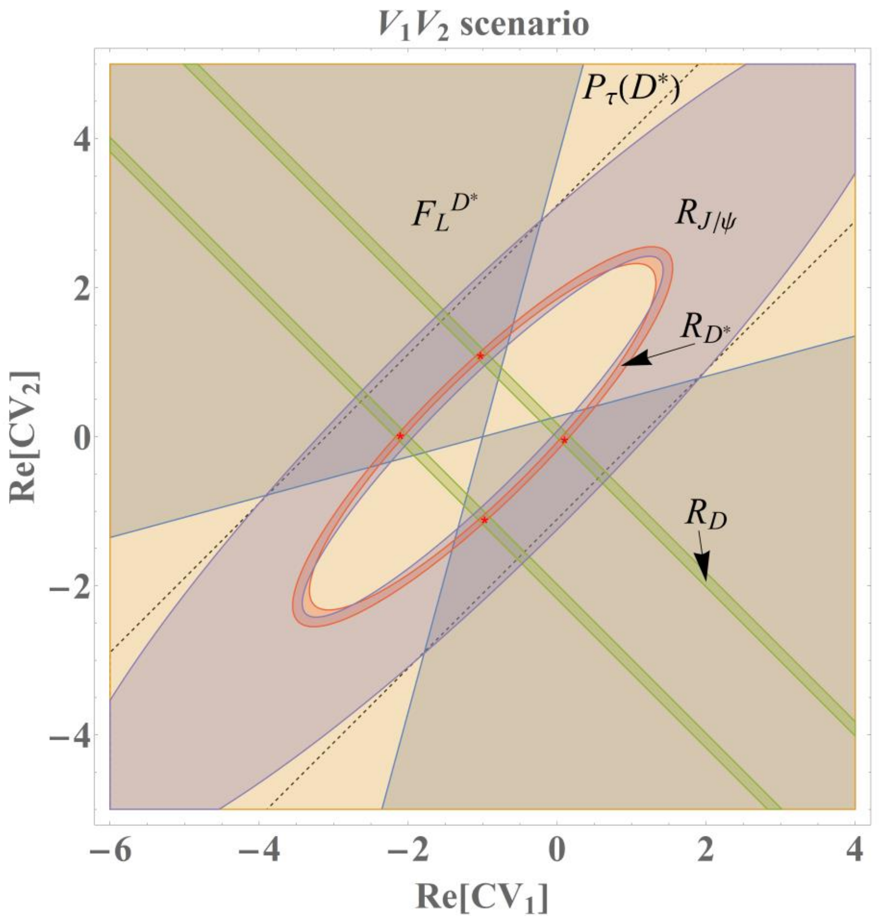
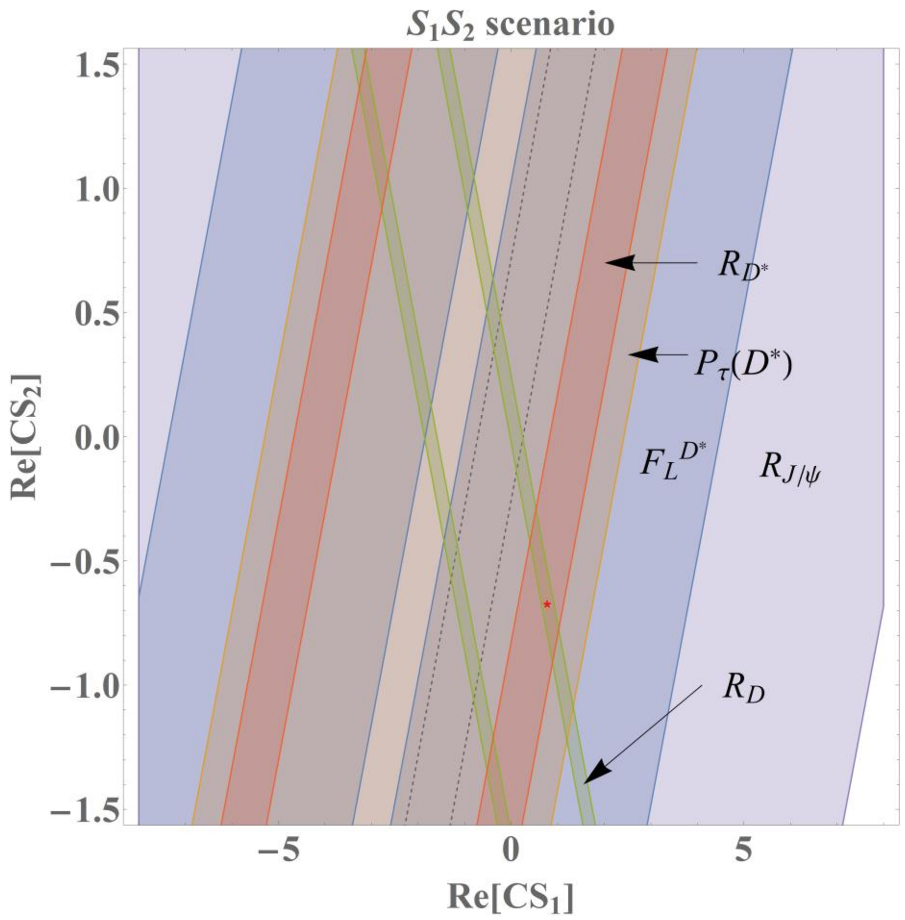




Exclusion

NP scenario	value (with $\mathcal{B}(B_c \to \tau\nu) < 0.1$)	χ^2/dof	Correlation
V_1	$(1 + Re[C_{V_1}])^2 + (Im[C_{V_1}])^2 = 1.236(38)$	13.70/11	–
V_2	$-0.030(34) \pm 0.460(52)i$	12.92/11	± 0.59
S_1	$0.245 + 0.000i$	32.77/11	–
S_2	$0.072 \pm 0.461i$	39.10/11	–
T	$0.011(62) \pm 0.165(60)i$	16.79/11	± 0.98
(S_1, S_2)	$(-0.785, -1.041)$	32.01/11	–

- S_1 , S_2 and (S_1 , S_2) are excluded
- Those models which only generate scalar operators are excluded

Predictions for the Observables

Scenario	$R(D)$	$R(D^*)$	$P_\tau(D)$	$P_\tau(D^*)$
SM	0.289(5)(0)	0.237(8)(0)	0.328(3)(0)	−0.490(5)(0)
V_1	0.358(6)(11)	0.293(10)(9)	0.328(3)(0)	−0.490(5)(0))
V_2	0.334(6)(30)	0.300(10)(12)	0.328(3)(0)	−0.490(5)(1)
T	0.300(5)(26)	0.303(21)(34)	0.314(3)(48)	−0.357(25)(75)
(V_1, V_2)	0.333(6)(31)	0.300(10)(13)	0.328(3)(0)	-0.490(5)(1)
(V_1, S_1)	0.337(6)(30)	0.298(10)(12)	0.268(3)(87)	-0.502(4)(16)
(V_1, S_2)	0.332(5)(30)	0.300(10)(12)	0.264(3)(74)	-0.478(5)(14)
(V_1, T)	0.336(6)(30)	0.299(10)(15)	0.340(3)(15)	-0.479(4)(17)
(V_2, S_1)	0.318(6)(30)	0.297(10)(13)	0.523(3)(39)	−0.447(7)(10)
(V_2, S_2)	0.333(6)(32)	0.299(10)(12)	0.587(3)(43)	−0.535(3)(9)
(V_2, T)	0.328(6)(28)	0.299(21)(12)	0.396(2)(12)	−0.402(12)(23)
(S_1, T)	0.337(6)(29)	0.299(13)(12)	0.486(3)(41)	−0.428(5)(9)
(S_2, T)	0.333(6)(29)	0.300(15)(12)	0.487(3)(44)	−0.463(7)(13)

Predictions for the Observables

Scenario	$F_L^{D^*}$	$\mathcal{A}_{FB}(D)$	$A_{FB}(D^*)$
SM	0.467(4)(0)	0.360(1)(0)	-0.057(6)(0)
V_1	0.467(4)(0)	0.360(1)(0)	-0.057(6)(0)
V_2	0.470(4)(3)	0.360(1)(0)	0.016(4)(10)
T	0.401(13)(40)	0.357(1)(25)	0.013(15)(20)
(V_1, V_2)	0.470(4)(3)	0.360(1)(0)	0.318(5)(9)/-0.047(4)(11)
(V_1, S_1)	0.463(4)(6)	0.365(1)(7)	-0.063(4)(9)
(V_1, S_2)	0.472(4)(5)	0.365(1)(5)	-0.050(4)(7)

Scenario	$F_L^{D^*}$	$\mathcal{A}_{FB}(D)$	$A_{FB}(D^*)$
(V_1, T)	0.463(4)(7)	0.352(1)(10)	-0.039(4)(25)
(V_2, S_1)	0.491(4)(4)	0.327(2)(9)	0.003(3)(9)
(V_2, S_2)	0.463(4)(3)	0.311(2)(12)	-0.031(4)(8)
(V_2, T)	0.423(7)(12)	0.310(2)(9)	0.011(9)(9)
(S_1, T)	0.459(5)(7)	0.313(2)(8)	0.012(6)(9)
(S_2, T)	0.440(5)(4)	0.309(2)(10)	-0.007(7)(12)

Leptoquark

	SM quantum number $[\mathrm{SU}(3)\times\mathrm{SU}(2)\times\mathrm{U}(1)]$	Spin	Fermions coupled to
R_2	$(3, 2, 7/6)$	0	$\bar{c}_R\nu_L, \bar{b}_L\tau_R$
S_1	$(\bar{3}, 1, 1/3)$	0	$\bar{b}^c_L\nu_L, \bar{c}^c_L\tau_L, \bar{c}^c_R\tau_R$
U_1	$(3, 1, 2/3)$	1	$\bar{c}_L\gamma_\mu\nu_L, \bar{b}_L\gamma_\mu\tau_L, \bar{b}_R\gamma_\mu\tau_R$

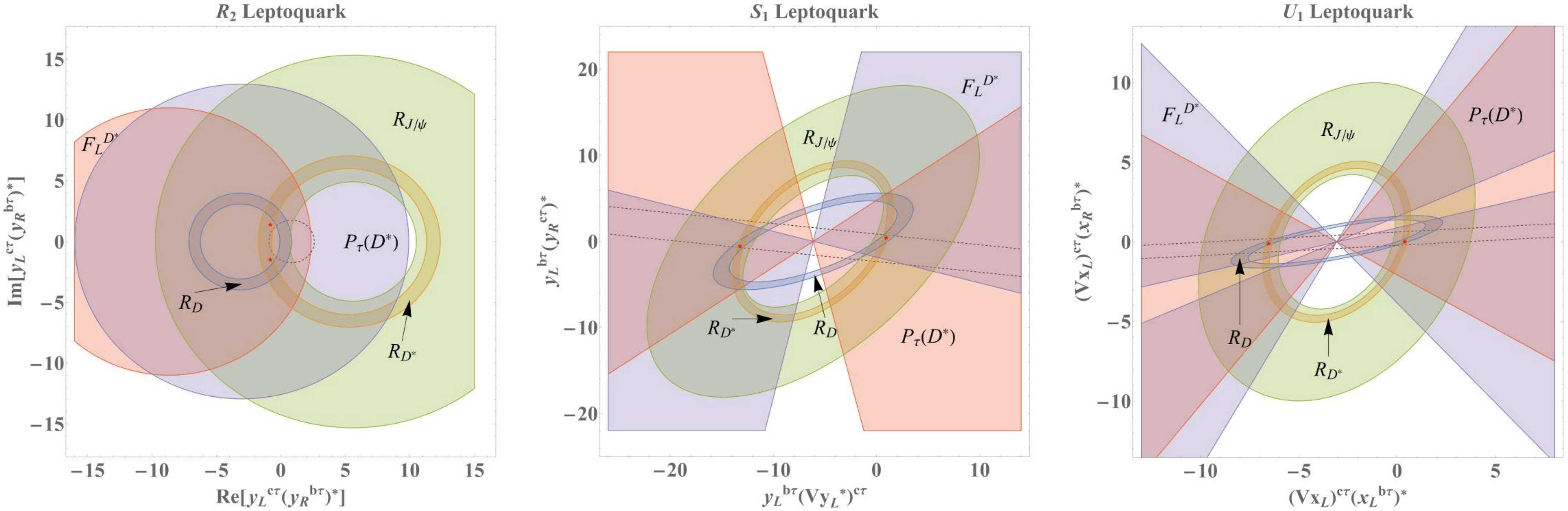
- Lagrangian:

$$\mathcal{L}_{R_2} = \left(y_R^{b\tau}\bar{b}_L\tau_R + y_L^{c\tau}\bar{c}_R\nu_L\right) Y_{2/3} + \text{H.c.}$$

$$\mathcal{L}_{S_1} = \left((V^*_{\text{CKM}}y_L)^{c\tau}\bar{c}^c_L\tau_L - y_L^{b\tau}\bar{b}^c_L\nu_L + y_R^{c\tau}\bar{c}^c_R\tau_R\right) Y_{1/3} + \text{H.c.}$$

$$\mathcal{L}_{U_1} = \left((V_{\text{CKM}}x_L)^{c\tau}\bar{c}_L\gamma_\mu\nu_L + x_L^{b\tau}\bar{b}_L\gamma_\mu\tau_L + x_R^{b\tau}\bar{b}_R\gamma_\mu\tau_R\right) X^\mu_{2/3} + \text{H.c.}$$

2σ Constraints on the Leptoquark Couplings



Predictions for the Observables with LQ Model

LQ Type	value (with $\mathcal{B}(B_c \to \tau\nu) < 0.1$)	χ^2/dof	corr
R_2	$(-0.164(398), \pm 1.446(117))$	22.87/11	± 0.29
S_1	$(0.936(270), 0.476(509))$	12.70/11	0.92
S_1	$(-13.224(270), -0.476(509))$	12.70/11	0.92
U_1	$(0.391(85), 0.061(86))$	13.17/11	0.78
U_1	$(-6.535(85), -0.061(86))$	13.17/11	0.78

Disfavoured

LQ type	$R(D)$	$R(D^*)$	$P_\tau(D)$	$P_\tau(D^*)$	$F_L^{D^*}$	$\mathcal{A}_{FB}(D)$	$\mathcal{A}_{FB}(D^*)$
S_1	0.330(5)(29)	0.301(10)(13)	0.193(5)(145)	−0.474(7)(20)	0.480(4)(13)	0.375(1)(12)	-0.061(5)(5)
U_1	0.338(6)(30)	0.298(10)(12)	0.268(3)(87)	−0.502(4)(16)	0.463(4)(6)	0.365(1)(7)	-0.063(6)(9)

Summary and Conclusions

- Fit the parameters in the HQET parametrization including the $\mathcal{O}(\alpha_s, \Lambda_{\rm QCD}/m_{b,c})$ corrections and part of $\mathcal{O}(\varepsilon_c^2)$ correlations
- Our calculations of $R(D^{(*)})$ in SM are smaller than the predictions of HFLAV and have ~4σ deviation from the experiments
- The NP models that generate only scalar operators are ruled out, such as the charged Higgs models
- The R_2 Leptoquark model is disfavored to explain the $R(D^{(*)})$ anomalies
- Our calculations of $R(D^{(*)})$ in new physics scenario can well explain the experiments

Thank you!