A high-quality axion from the non-minimal $\mathrm{SU}(6)$ *GUT*

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Background: GUT

Topics	Authors	Publications	Date
Fourth color unification	Pati, Salam	[5]	1974
SU(5) GUT	Georgi, Glashow	[6]	1974
SO(10) GUT	Fritzsch, Minkowski	[7]	1975
Peccei-Quinn mechanism	Peccei, Quinn	[8]	1977
Seesaw mechanism	Yanagida	[9]	1979
	Gell-Mann, Ramond, Slansky	[10]	1979
KSVZ axion	Kim	[11]	1979
	Shifman, Vainshtein, Zakharov	[12]	1980
$\mathrm{SU}(N+4)$ global symmetry	Dimopoulos, Raby, Susskind	[13]	1980
SUSY SU(5) GUT	Dimopoulos, Georgi	[14]	1981
DFSZ axion	Zhitnitsky	[15]	1980
	Dine, Fischler, Srednicki	[16]	1981
Axion in SU(5) GUT	Wise, Georgi, Glashow	[17]	1981
Leptogenesis	Fukugita, Yanagida	[18]	1986
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Table 1: Collections of the major theoretical advances in the GUT and some related BSM physical issues listed in chronological order.

Background: GUT

* In the *minimal* $SU(5)$ GUT, Georgi & Glashow assumed: *``as few leptons (fermions) as possible, no unobserved leptons (fermions) …”.*

* Our assumption: *a successful GUT can address as many BSM issues as possible, with the minimal set of fields unless otherwise necessary*.

* We consider the PQ-quality for the strong CP problem to start with, and we get more than that (see next).

Background: Strong CP

* The strong CP problem, a topological term for the QCD vacuum $\mathscr{L}_\theta = \theta \frac{\alpha_{3c}}{8\pi} G^a_{\mu\nu} \tilde{G}^{\mu\nu\, a}$, and experimentally from the neutron EDM: $|\bar{\theta}| \lesssim 10^{-10}$, with $\bar{\theta} = \theta + \arg\det M_q$, very different from the $\mathcal{O}(1)$ expectation of θ parameter.

* Peccei-Quinn mechanism: to replace θ by a periodic pseudo-scalar field transforming under a global $\mathrm{U}(1)_{\mathrm{PQ}}$ as $a \to a + 2\pi f_a$, with f_a known as the axion decay constant.

* There is a classical window: $10^8\,\mathrm{GeV} \lesssim f_a \lesssim 10^{12}\,\mathrm{GeV}$ [cf. 1803.00993]. The lower bound CANNOT be violated, and the upper bound can be relaxed.

* Axion induced potential: $V_{\mathrm{QCD}} = \Lambda^4_{\mathrm{QCD}}[1 - \cos(a/f_a)]$.

Background: Strong CP

* Invisible axion models such as Kim-Shifman-Vainshtein-Zakharov (KSVZ) and Dine-Fischler-Srednicki-Zhitnisky (DFSZ), a comes as a phase from a complex scalar field $\Phi \supset \frac{1}{\sqrt{2}} v_a \exp(ia/f_a)$.

* PQ quality: $\mathrm{U}(1)_{\mathrm{PQ}}$ symmetry (expressed in terms of Φ) is global and put in by hand in KSVZ/DFSZ models, and the gravity does not respect global symmetries. ['92 Barr, Seckel, Kamionkowski, March-Russell, Holman, Hsu, Kephart, Kolb, Watkins, Widrow.]

Background: Strong CP

* A general operator of $\mathcal{O}_{\not{PQ}}^{d=2m+n} = k\dfrac{|\Phi|^{2m}\Phi^n}{M_{\rm pl}^{2m+n-4}} + H.c.$ ['92 Kamionkowski, March-Russell] $\Delta \mathrm{PQ} = n$ with $\mathrm{PQ}(\Phi) = 1.$

* The $\mathcal{O}_{\not{PQ}}^{d=2m+n}$ shifts the $V_{\rm QCD}$ minima $|\bar{\theta}| = |\langle a \rangle / f_a| \lesssim 10^{-10}$

* if $|k| \sim 10^{-2}$ and $2m + n = 5,\ \Rightarrow f_a \lesssim 10\,\mathrm{GeV}$, ruled out, else if $f_a \sim 10^{12}\,\mathrm{GeV}$ and $2m + n = 5,\ \Rightarrow |k| \lesssim 10^{-55}$, very fine-tuned.

* NB, the renormalizable operators with $2m + n \leq 4$ are even worse in terms of the PQ-quality. This suggests to consider the underlying gauge symmetry of the Φ.

Global Symmetries

* The usual wisdom of a high-quality PQ is to have the $\mathrm{U}(1)_{\mathrm{PQ}}$ as an emergent global symmetry in a gauge theory.

* The QCD has the global symmetries of $\mathcal{G}_{\mathrm{global}} = \mathrm{SU}(3)_L \otimes \mathrm{SU}(3)_R \otimes \mathrm{U}(1)_V \otimes \mathrm{U}(1)_A$, while QCD is vectorial.

* The chiral gauge theory w.o. Unification: to put another confining sector with the SM, e.g. $\mathrm{SU}(5) \otimes \mathcal{G}_{\mathrm{SM}}$ by Gavela, Ibe, Quilez, and Yanagida [1812.08174].

* Dimopoulos-Raby-Susskind (1980) studied a strongly-interacting theory: an anomaly-free $\mathrm{SU}(N+4)$ chiral gauge theory with N anti-fundamental fermions and one rank-2 anti-symmetric fermion, and it has $\mathcal{G}_{\mathrm{global}} = \mathrm{SU}(N) \otimes \mathrm{U}(1)$, $N \geq 2$.

The SU(6) model

The SU(6) model

* The minimal anomaly-free $\mathrm{SU}(6)$ has fermions of: $2 \times \bar{\mathbf{6}}_{\mathbf{F}} \oplus \mathbf{15}_{\mathbf{F}}$, with the global symmetry of $\mathcal{G}_{\mathrm{global}} = \mathrm{SU}(2)_F \otimes \mathrm{U}(1)_{\mathrm{PQ}}$.

* How to break the $\mathrm{SU}(6)$ to the $\mathcal{G}_{\mathrm{SM}} = \mathrm{SU}(3)_c \otimes \mathrm{SU}(2)_L \otimes \mathrm{U}(1)_Y$? The Georgi-Glashow $\mathrm{SU}(5)$ is a subgroup of the $\mathrm{SU}(6)$, while the direct breaking to the $\mathrm{SU}(5)$ is unwelcome. The sequential breaking of $\mathrm{SU}(5) \to \mathcal{G}_{\mathrm{SM}}$ leads to the proton decays with lower mass scale, hence faster decay rate.

* Alternative pattern is: $\mathrm{SU}(6) \to \mathcal{G}_{331} \to \mathcal{G}_{\mathrm{SM}}$, with $\mathcal{G}_{331} = \mathrm{SU}(3)_c \otimes \mathrm{SU}(3)_L \otimes \mathrm{U}(1)_N$. This is achievable with an adjoint Higgs of $\mathbf{35}_{\mathbf{H}}$ at the GUT scale (1974 Ling-Fong Li).

The SU(6) Higgs sector

- minimal Higgs sector: $\mathbf{6}_{\mathbf{H}}^{\alpha=\mathrm{I},\mathrm{II}}$, $\mathbf{15_H}$, $\mathbf{21_H}$, $\mathbf{35_H}$

- Hierarchies of Higgs VEVs: $\langle \mathbf{35_H} \rangle \sim \Lambda_{\mathrm{GUT}}$,
  $\langle \mathbf{6}_{\mathbf{H}}^{\mathrm{II}} \rangle = v_3 \,, \quad \langle \mathbf{21_H} \rangle = v_6 \,, \quad v_3 \sim v_6 \sim v_{331}$
  $\langle \mathbf{6}_{\mathbf{H}}^{\mathrm{I}} \rangle = v_d = v_{\mathrm{EW}} \sin\beta \,, \quad \langle \mathbf{15_H} \rangle = v_u = v_{\mathrm{EW}} \cos\beta$
  $\Lambda_{\mathrm{GUT}} \gg v_{331} \gg v_{\mathrm{EW}} = (\sqrt{2} G_F)^{-1/2} \simeq 246\,\mathrm{GeV}$

- $\mathbf{6}_{\mathbf{H}}^{\mathrm{II}}$ and $\mathbf{21_H}$ are responsible for the $\mathcal{G}_{331} \to \mathcal{G}_{\mathrm{SM}}$ breaking.

- Two Higgs doublets from $\mathbf{6}_{\mathbf{H}}^{\mathrm{I}}$ (for d^i and ℓ^i) and $\mathbf{15_H}$ (for u^i) are responsible for the EWSB, and they are automatically type-II 2HDM.

- One bonus: free from the μ-problem in the SUSY extension of the $\mathrm{SU}(5)$ GUT $W \supset \mu \mathbf{5}_{\mathbf{H}}^{*} \mathbf{5_H}$, since one cannot form $W \supset \mu \mathbf{6}_{\mathbf{H}}^{\mathrm{I}\,*} \mathbf{15_H}$ term in the $\mathrm{SU}(6)$.

The SU(6) fermions

SU(6)	$\mathcal{G}_{331}$	$\mathcal{G}_{\rm SM}$
$\bar{\mathbf{6}}_{\mathbf{F}}^{\rm I}$	$(\bar{\mathbf{3}}, \mathbf{1}, +\frac{1}{3})_{\mathbf{F}}^{\rm I}$	$(\bar{\mathbf{3}}, \mathbf{1}, +\frac{2}{3})_{\mathbf{F}}^{\rm I}$: d_R^c
	$(\mathbf{1}, \bar{\mathbf{3}}, -\frac{1}{3})_{\mathbf{F}}^{\rm I}$	$(\mathbf{1}, \mathbf{2}, -1)_{\mathbf{F}}^{\rm I}$: $(e_L, -\nu_L)$
		$(\mathbf{1}, \mathbf{1}, 0)_{\mathbf{F}}^{\rm I}$: N
$\bar{\mathbf{6}}_{\mathbf{F}}^{\rm II}$	$(\bar{\mathbf{3}}, \mathbf{1}, +\frac{1}{3})_{\mathbf{F}}^{\rm II}$	$(\bar{\mathbf{3}}, \mathbf{1}, +\frac{2}{3})_{\mathbf{F}}^{\rm II}$: D_R^c
	$(\mathbf{1}, \bar{\mathbf{3}}, -\frac{1}{3})_{\mathbf{F}}^{\rm II}$	$(\mathbf{1}, \mathbf{2}, -1)_{\mathbf{F}}^{\rm II}$: $(e_L', -\nu_L')$
		$(\mathbf{1}, \mathbf{1}, 0)_{\mathbf{F}}^{\rm II}$: N'
$\mathbf{15}_{\mathbf{F}}$	$(\bar{\mathbf{3}}, \mathbf{1}, -\frac{2}{3})_{\mathbf{F}}$	$(\bar{\mathbf{3}}, \mathbf{1}, -\frac{4}{3})_{\mathbf{F}}$: u_R^c
	$(\mathbf{1}, \bar{\mathbf{3}}, +\frac{2}{3})_{\mathbf{F}}$	$(\mathbf{1}, \mathbf{2}, +1)_{\mathbf{F}}$: $(\nu_R'^c, e_R'^c)$
		$(\mathbf{1}, \mathbf{1}, +2)_{\mathbf{F}}$: e_R^c
	$(\mathbf{3}, \mathbf{3}, 0)_{\mathbf{F}}$	$(\mathbf{3}, \mathbf{2}, +\frac{1}{3})_{\mathbf{F}}$: (u_L, d_L)
		$(\mathbf{3}, \mathbf{1}, -\frac{2}{3})_{\mathbf{F}}$: D_L

The (D_L, D_R^c) are KSVZ vector-like quarks with masses $\sim f_a$ and $Q = -1/3$.

The SU(6) Yukawa

* The most general Yukawa coupling:
$\mathbf{15_F}\bar{\mathbf{6}}^{\alpha}_{\mathbf{F}}\mathbf{6}^{\alpha\,*}_{\mathbf{H}} + \mathbf{15_F}\mathbf{15_F}\mathbf{15_H} + \bar{\mathbf{6}}^{\alpha}_{\mathbf{F}}(i\sigma_2)_{\alpha\beta}\bar{\mathbf{6}}^{\beta}_{\mathbf{F}}\mathbf{21_H} + H.c.$

* The PQ charge and a discrete $\mathbb{Z}_4$ symmetry (for PQ quality):

	$\bar{\mathbf{6}}^{\alpha}_{\mathbf{F}}$	$\mathbf{15_F}$	$\mathbf{6}^{\alpha}_{\mathbf{H}}$	$\mathbf{15_H}$	$\mathbf{21_H}$	$\mathbf{35_H}$
$\mathrm{U}(1)_{\mathrm{PQ}}$	1	1	2	-2	-2	0
$\mathrm{SU}(2)_{\mathrm{F}}$	$\square$	1	$\square$	1	1	1
$\mathbb{Z}_4$	1	$\frac{1}{2}$	$\frac{3}{2}$	-1	-2	0

At the UV the global $\mathrm{U}(1)_{\mathrm{PQ}}[SU(6)]^2$ anomaly: $N_{\mathrm{SU}(6)} = 9$

The SU(6) Axion

The SU(6) Axion

* The physical axion field comes from: $\mathbf{6}_{\mathbf{H}}^{\mathrm{II}} \supset (\mathbf{1}\,, \mathbf{3}\,, +\frac{1}{3})_{\mathbf{H}}^{\mathrm{II}} \supset \frac{v_3}{\sqrt{2}} \exp(i a_3 / v_3)$ and $\mathbf{21}_{\mathbf{H}} \supset (\mathbf{1}\,, \mathbf{6}\,, +\frac{2}{3})_{\mathbf{H}} \supset \frac{v_6}{\sqrt{2}} \exp(i a_6 / v_6)$
* To impose an orthogonality condition between the $\mathrm{U}(1)_{\mathrm{PQ}}$ $J_{\mathrm{PQ}}^{\mu} = q_3 v_3 (\partial^\mu a_3) + q_6 v_6 (\partial^\mu a_6)$ and the $\mathrm{U}(1)_N$ $J_N^\mu = \frac{1}{3} v_3 (\partial^\mu a_3) + \frac{2}{3} v_6 (\partial^\mu a_6)$ currents. Physical charge: $q \equiv c_1\,\mathrm{PQ} + c_2\,N$.
* 't Hooft global anomaly matching: $N_{\mathrm{SU}(3)_c} = N_{\mathrm{SU}(6)} \Rightarrow c_1 = 1$.
* $a_{\mathrm{phys}} = \cos\phi\, a_3 + \sin\phi\, a_6,\ \tan\phi = \frac{v_3}{2 v_6}$.
* Axion decay const: $9 v_{331}^{-2} = \frac{1}{4} v_6^{-2} + v_3^{-2}$ and $f_a = v_{331}/18$.

The PQ quality

* The leading PQ-breaking operator respecting the $SU(2)_F$ and $\mathbb{Z}_4$:

  $$\mathcal{O}_{\not{PQ}}^{d=6} = \left[\epsilon_{\alpha\beta}\mathbf{6}_{\mathbf{H}}^{\alpha\,*}\mathbf{6}_{\mathbf{H}}^{\beta\,*}\mathbf{15}_{\mathbf{H}}\right]^2$$

  $$\supset \left[\epsilon_{\alpha\beta}\epsilon_{abc}(\mathbf{1}\,,\mathbf{3}\,,+\frac{1}{3})^{a\,,\alpha}(\mathbf{1}\,,\mathbf{3}\,,+\frac{1}{3})^{b\,,\beta}(\mathbf{1}\,,\mathbf{3}\,,-\frac{2}{3})^{c}\right]^2$$

  if no $\mathbb{Z}_4$: $\mathcal{O}_{\not{PQ}}^{d=3} = \epsilon_{\alpha\beta}\epsilon_{abc}(\mathbf{1}\,,\mathbf{3}\,,+\frac{1}{3})^{a\,,\alpha}(\mathbf{1}\,,\mathbf{3}\,,+\frac{1}{3})^{b\,,\beta}(\mathbf{1}\,,\mathbf{3}\,,-\frac{2}{3})^{c}$ is dangerous in PQ-quality.

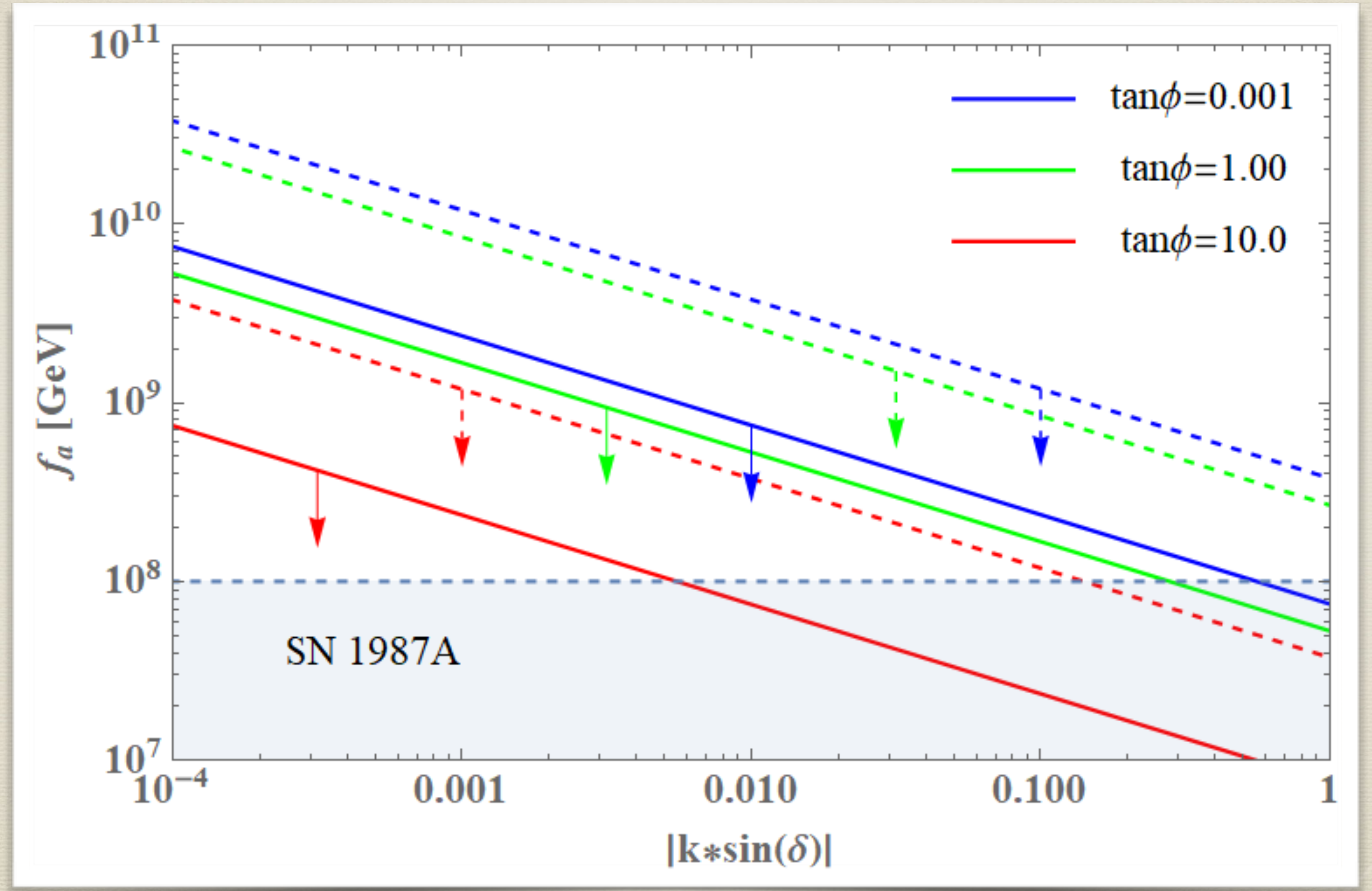
* Axion effective potential:

  $$V = -\Lambda_{\rm QCD}^4 \cos(\frac{a_{\rm phys}}{f_a}) + \frac{|k|(v_u v_d v_3)^2}{4M_{\rm pl}^2}\cos\left(\frac{a_{\rm phys}}{6f_a}+\delta\right)$$

* $$|\bar{\theta}| \equiv \left|\frac{\langle a_{\rm phys}\rangle}{f_a}\right| \lesssim 10^{-10} \Rightarrow f_a \lesssim \frac{3.7\times 10^7\ \cos\phi}{|k\sin(\delta)|^{1/2}}(\tan\beta + \frac{1}{\tan\beta})\,{\rm GeV}$$

  This is solely determined by the symmetry consideration in the $SU(6)$ GUT!

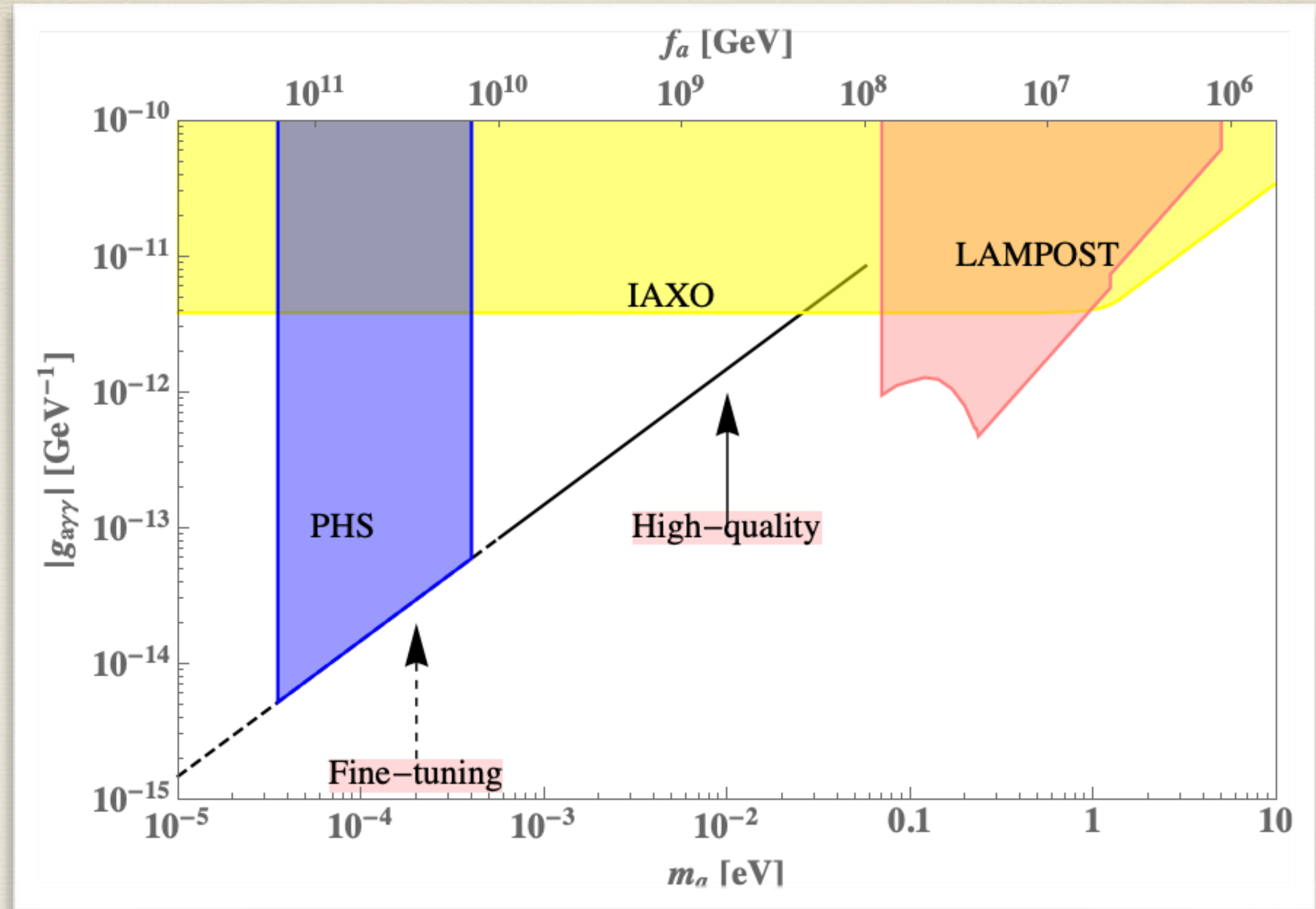
The PQ quality



$10^8\,\text{GeV} \lesssim f_a \lesssim 10^{10}\,\text{GeV}$

$$m_a = 5.70\left(\frac{10^{12}\,\text{GeV}}{f_a}\right)\mu\text{eV} \sim (10^{-4}, 10^{-2})\text{eV}$$

The axion searches



$$g_{a\gamma\gamma} = (\frac{E}{N_{\mathrm{SU(3)}_c}} - 1.92)\left(\frac{1.14 \times 10^{-3}\,\mathrm{GeV}}{f_a}\right)\mathrm{GeV}^{-1}$$

$$\mathrm{U(1)_{PQ}[U(1)_{em}]^2 \ \ anomaly\ \ factor:} \qquad E = \sum_f \mathrm{PQ}_f \dim(\mathscr{C}_f)\mathrm{Tr} q_f^2 = 24$$

$\frac{E}{N_{\mathrm{SU(3)}_c}} = \frac{8}{3} \Rightarrow$ ***A KSVZ axion with the DFSZ coupling***

The Axion domain walls

* Back to the axion effective potential:

$$V = \Lambda_{\rm QCD}^4[1 - \cos(\frac{a_{\rm phys}}{f_a})] + \frac{|k|(v_u v_d v_3)^2}{4M_{\rm pl}^2}\cos\left(\frac{a_{\rm phys}}{6f_a} + \delta\right)$$

* The PQ-breaking term can avoid the cosmologically dangerous domain walls from the first periodic term.

* In our case:

$$t_{\rm form} \sim 10^2\,{\rm sec}\left(\frac{10^{13}\,{\rm GeV}}{v_{331}}\right) \sim \mathcal{O}(10^4) - \mathcal{O}(10^6)\,{\rm sec}$$

$$t_{\rm dec} \approx \frac{\sigma_{\rm DW}}{v_{331}^4} \sim 10^{-66}\,{\rm sec}\left(\frac{M_{\rm pl}v_{331}}{v_u v_d}\right)^2\left(\frac{10^{13}\,{\rm GeV}}{v_{331}}\right)^3$$

$$\sim \mathcal{O}(10^{-8}) - \mathcal{O}(10^{-6})\,{\rm sec}$$

The SU(6) Unification

The SU(6) unification (one-loop)

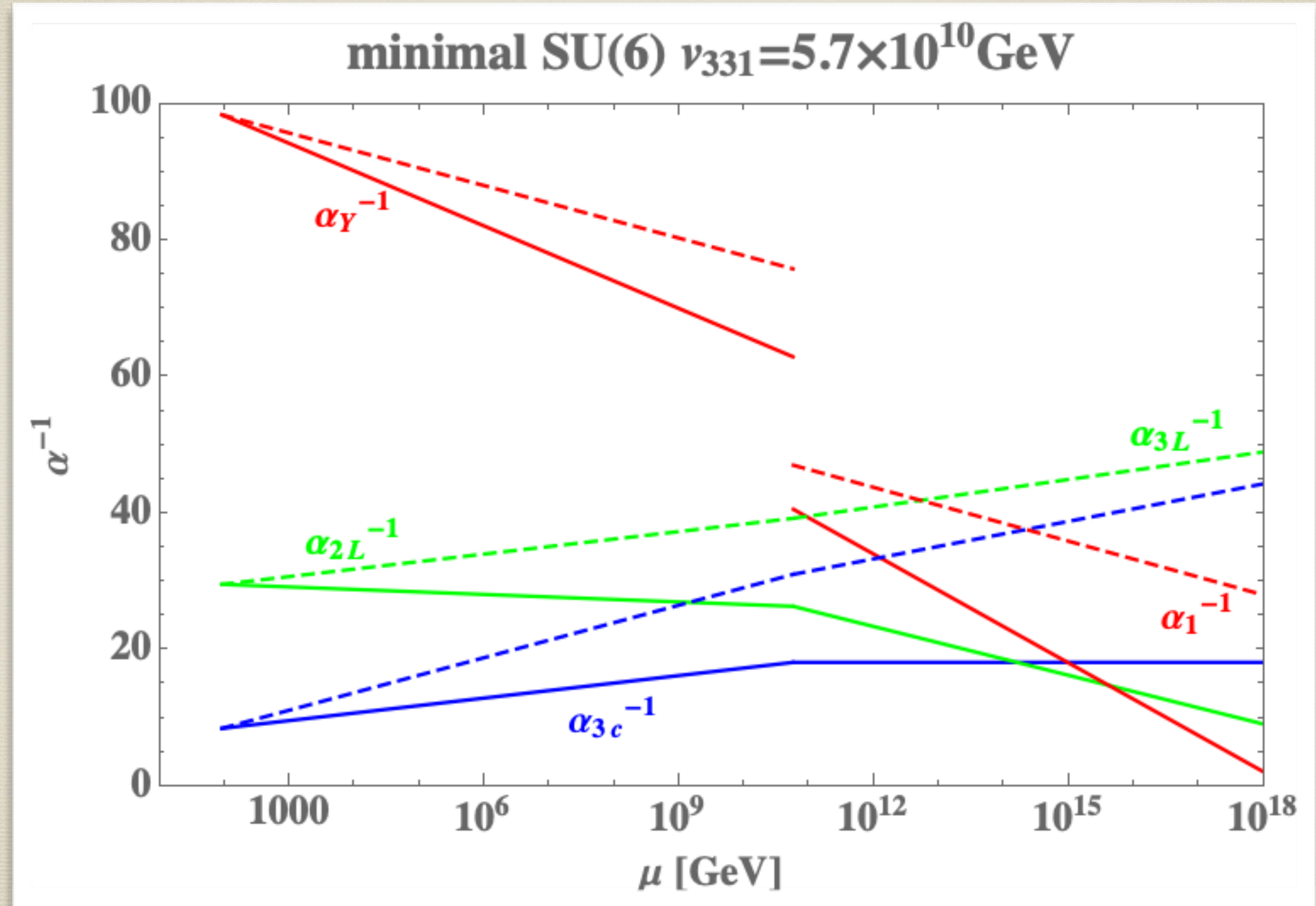
* The gauge couplings: $(\alpha_{3c}, \alpha_{3L}, \alpha_N)$ for the $\mathcal{G}_{331}$, and $(\alpha_{3c}, \alpha_{2L}, \alpha_Y)$ for the $\mathcal{G}_{\rm SM}$. Use $\alpha_1 = \frac{4}{3}\alpha_N$ for the $\mathcal{G}_{331}$ embedding into the $\mathrm{SU}(6)$.

* non-SUSY: $m_Z \leq \mu \leq v_{331} : (b^{(1)}_{\mathrm{SU}(3)_c}, b^{(1)}_{\mathrm{SU}(2)_L}, b^{(1)}_{\mathrm{U}(1)_Y}) = (-7\,, -3\,, 7)$
  $v_{331} \leq \mu \leq \Lambda_{\rm GUT} : (b^{(1)}_{\mathrm{SU}(3)_c}, b^{(1)}_{\mathrm{SU}(3)_L}, b^{(1)}_{\mathrm{U}(1)_1}) = (-5\,, -\frac{11}{3}\,, \frac{43}{6})$

* SUSY:
  $m_Z \leq \mu \leq v_{331} : (b^{(1)}_{\mathrm{SU}(3)_c}, b^{(1)}_{\mathrm{SU}(2)_L}, b^{(1)}_{\mathrm{U}(1)_Y}) = (-3\,, 1\,, 11)$
  $v_{331} \leq \mu \leq \Lambda_{\rm GUT} : (b^{(1)}_{\mathrm{SU}(3)_c}, b^{(1)}_{\mathrm{SU}(3)_L}, b^{(1)}_{\mathrm{U}(1)_1}) = (0\,, \frac{13}{2}\,, \frac{29}{2})$

* SUSY extension: $\mathbf{21_H}$ super-multiplet is anomalous, we include a $\overline{\mathbf{21_H}}$.

The SU(6) unification (one-loop)



$\alpha_{3c}(m_Z)\,,\alpha_{\rm em}(m_Z)\,,\sin^2\theta_W(m_Z)$ as inputs

The SU(6) unification (one-loop)

- To impose the unification condition at the UV:
  $\alpha_{3c}^{-1}(\Lambda_{\rm GUT}) = \alpha_{3L}^{-1}(\Lambda_{\rm GUT}) = \alpha_{1}^{-1}(\Lambda_{\rm GUT}) = \alpha_{\rm GUT}^{-1}(\Lambda_{\rm GUT})$

- Benchmark $v_{331} = 5.7 \times 10^{10}\,\text{GeV}$, we find:
  $\Lambda_{\rm GUT} \approx 7.8 \times 10^{15}\,\text{GeV}\,, \quad \alpha_{\rm GUT}^{-1}(\Lambda_{\rm GUT}) = 18.14$
  $\sin^2\theta_W(m_Z) = 0.22923$
  PDG: $\sin^2\theta_W(m_Z) = 0.23117 \pm 0.00016$

- Proton lifetime:
  $$\tau[p \to e^+\pi^0] \sim 10^{36}\,\text{yrs}\left(\frac{\alpha_{\rm GUT}^{-1}}{35}\right)^2\left(\frac{\Lambda_{\rm GUT}}{10^{16}\,\text{GeV}}\right)^4$$
  $\approx 9.8 \times 10^{34}\,\text{yrs}$
  SK limit: $\tau_p \gtrsim 2.4 \times 10^{34}\,\text{yrs}$, can be probed in the future HyperK.

Summary

* We show a *non-minimal* $\mathrm{SU}(6)$ GUT model with the minimal setup to achieve a high-quality axion by identifying the $\mathrm{U}(1)_{\mathrm{PQ}}$ as the Abelian component of the emergent global symmetries.

* The axion decay constant: $10^8\,\mathrm{GeV} \lesssim f_a \lesssim 10^{10}\,\mathrm{GeV}$ purely from the symmetry consideration, w.o. much fine-tuning of the EFT parameter.

* The GUT spectrum contains vector-like KSVZ D-quarks, and heavy leptons && singlet neutrinos, type-I seesaw.

* Safe from the cosmological constraints.

* The origin of the discrete symmetries?

* The improved RGEs, with two-loops and the mass threshold effects.

Backups

The SU(6) Yukawa

- At the $\mathcal{G}_{331} \to \mathcal{G}_{\rm SM}$ breaking:
  $\mathbf{15_F} \bar{\mathbf{6}}_{\mathbf{F}}^{\rm II} \mathbf{6}_{\mathbf{H}}^{\rm II\,*} + H.c. \supset$
  $(\mathbf{3}, \mathbf{3}, 0)_{\mathbf{F}} \otimes (\bar{\mathbf{3}}, \mathbf{1}, +\frac{1}{3})_{\mathbf{F}}^{\rm II} \otimes (\mathbf{1}, \bar{\mathbf{3}}, -\frac{1}{3})_{\mathbf{H}}^{\rm II} + H.c.$
  $(\mathbf{1}, \bar{\mathbf{3}}, +\frac{2}{3})_{\mathbf{F}} \otimes (\mathbf{1}, \bar{\mathbf{3}}, -\frac{1}{3})_{\mathbf{F}}^{\rm II} \otimes (\mathbf{1}, \bar{\mathbf{3}}, -\frac{1}{3})_{\mathbf{H}}^{\rm II} + H.c.$
  $\Rightarrow m_D \sim m_{e'} \sim m_{\nu'} \simeq \mathcal{O}(v_{331})$

  D-hadron lifetime: $\tau_D \sim m_D^{-1} \sim \mathcal{O}(10^{-36}) - \mathcal{O}(10^{-34})$ sec, Vs. the BBN constraint of $\tau_Q \lesssim 10^{-2}$ sec.

- $\bar{\mathbf{6}}_{\mathbf{F}}^{[\rm I} \bar{\mathbf{6}}_{\mathbf{F}}^{\rm II]} \mathbf{21_H} + H.c. \supset$
  $(\mathbf{1}, \bar{\mathbf{3}}, -\frac{1}{3})_{\mathbf{F}}^{[\rm I} \otimes (\mathbf{1}, \bar{\mathbf{3}}, -\frac{1}{3})_{\mathbf{F}}^{\rm II]} \otimes (\mathbf{1}, \mathbf{6}, +\frac{2}{3})_{\mathbf{H}} + H.c.$
  $\Rightarrow m_{N,N'} \simeq \mathcal{O}(v_{331})$

The SU(6) Yukawa

* $\mathbf{15_F}\bar{\mathbf{6}}^{\mathrm{I}}_{\mathbf{F}}\mathbf{6}^{\mathrm{I}\,*}_{\mathbf{H}} + H.c. \supset$
$(\mathbf{3}, \mathbf{2}, +\frac{1}{3})_{\mathbf{F}} \otimes (\bar{\mathbf{3}}, \mathbf{1}, +\frac{2}{3})^{\mathrm{I}}_{\mathbf{F}} \otimes (\mathbf{1}, \mathbf{2}, -1)^{\mathrm{I}}_{\mathbf{H}} + H.c.$
$(\mathbf{1}, \mathbf{2}, -1)_{\mathbf{F}} \otimes (\mathbf{1}, \mathbf{1}, +2)^{\mathrm{I}}_{\mathbf{F}} \otimes (\mathbf{1}, \mathbf{2}, -1)^{\mathrm{I}}_{\mathbf{H}} + H.c.$
$\Rightarrow m_{d,\ell} \simeq \mathcal{O}(v_{\mathrm{EW}})$
$\mathbf{15_F 15_F 15_H} + H.c. \supset$
$(\mathbf{3}, \mathbf{2}, +\frac{1}{3})_{\mathbf{F}} \otimes (\bar{\mathbf{3}}, \mathbf{1}, -\frac{4}{3})^{\mathrm{I}}_{\mathbf{F}} \otimes (\mathbf{1}, \mathbf{2}, +1)_{\mathbf{H}} + H.c.$
$\Rightarrow m_u \simeq \mathcal{O}(v_{\mathrm{EW}})$
The EW EFT = type-II 2HDM

The SU(6) Yukawa

* $\bar{\mathbf{6}}_{\mathbf{F}}^{[\mathrm{I}}\bar{\mathbf{6}}_{\mathbf{F}}^{\mathrm{II}]}\mathbf{15_H} + H.c. \supset$
  $(\mathbf{1}, \mathbf{2}, -1)_{\mathbf{F}}^{[\mathrm{I}} \otimes (\mathbf{1}, \mathbf{1}, 0)_{\mathbf{F}}^{\mathrm{II}]} \otimes (\mathbf{1}, \mathbf{2}, +1)_{\mathbf{H}} + H.c.$
  $\Rightarrow m_{\nu N'} \sim m_{\nu' N} \simeq \mathcal{O}(v_{\mathrm{EW}})$

* Type-I seesaw: $(\nu, N')\mathcal{M}(\nu, N')^T \Rightarrow m_\nu \sim \dfrac{v_{\mathrm{EW}}^2}{v_{331}}, m_{N'} \sim v_{331}$

The SU(6) fermions

* To obtain the spectrum: $N = \frac{1}{3}\mathrm{diag}(-\mathbb{I}_3\,, +\mathbb{I}_3)$ for the $\mathrm{SU}(6)$ fundamental, and $Y = \mathrm{diag}(\frac{1}{3} + 2N\,, \frac{1}{3} + 2N\,, -\frac{2}{3} + 2N)$ for the $\mathrm{SU}(3)_L$ fundamental.
* In the $\mathrm{SU}(5)$ GUT: $\bar{\mathbf{5}}_{\mathbf{F}} = d_R^c \oplus \ell_L$, $\mathbf{10}_{\mathbf{F}} = q_L \oplus u_R^c \oplus e_R^c$
* In the $\mathrm{SU}(6)$ GUT: $\bar{\mathbf{6}}_{\mathbf{F}}^{\mathrm{I}} \supset d_R^c \oplus \ell_L$, $\mathbf{15}_{\mathbf{F}} \supset q_L \oplus u_R^c \oplus e_R^c$
* Additional $\mathrm{SU}(6)$ fermions are vectorial: $(D_L\,, D_R^c)$, $(N\,, N')$, $(\ell'_L\,, \ell'^c_R)$