IR-Safe Observables Extended to Track-Based Measurement

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Based on:

YL, Ian Moult, Solange Schrijnder van Velzen, Wouter Waalewijn, HuaXing Zhu

arXiv:2107.xxxxx

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Motivation

from experiments

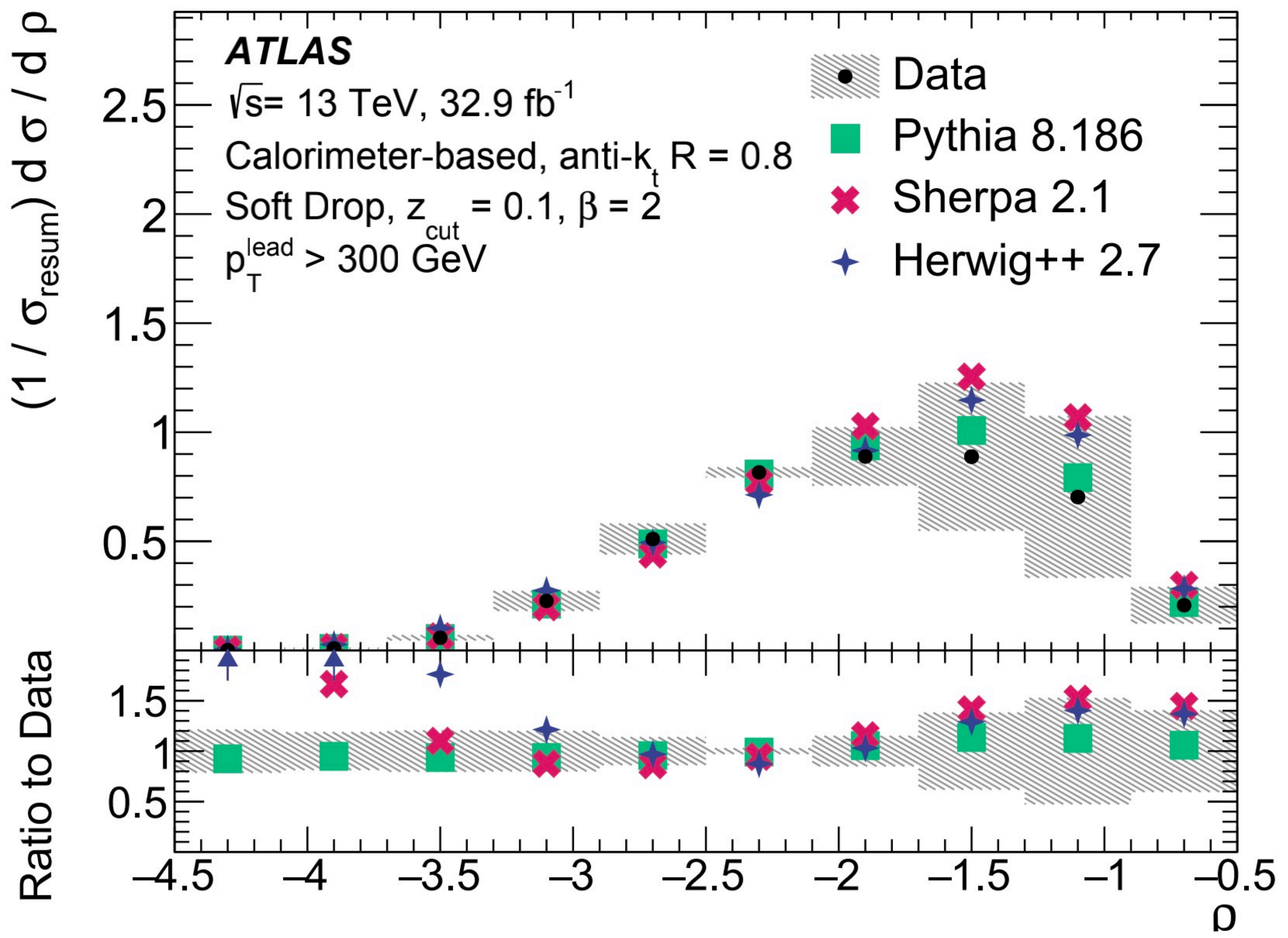
Track-based measurements:

- Superior angular resolution
- To mitigate pileup
- One problem: Track-based calculations are not IR safe in perturbative theory.

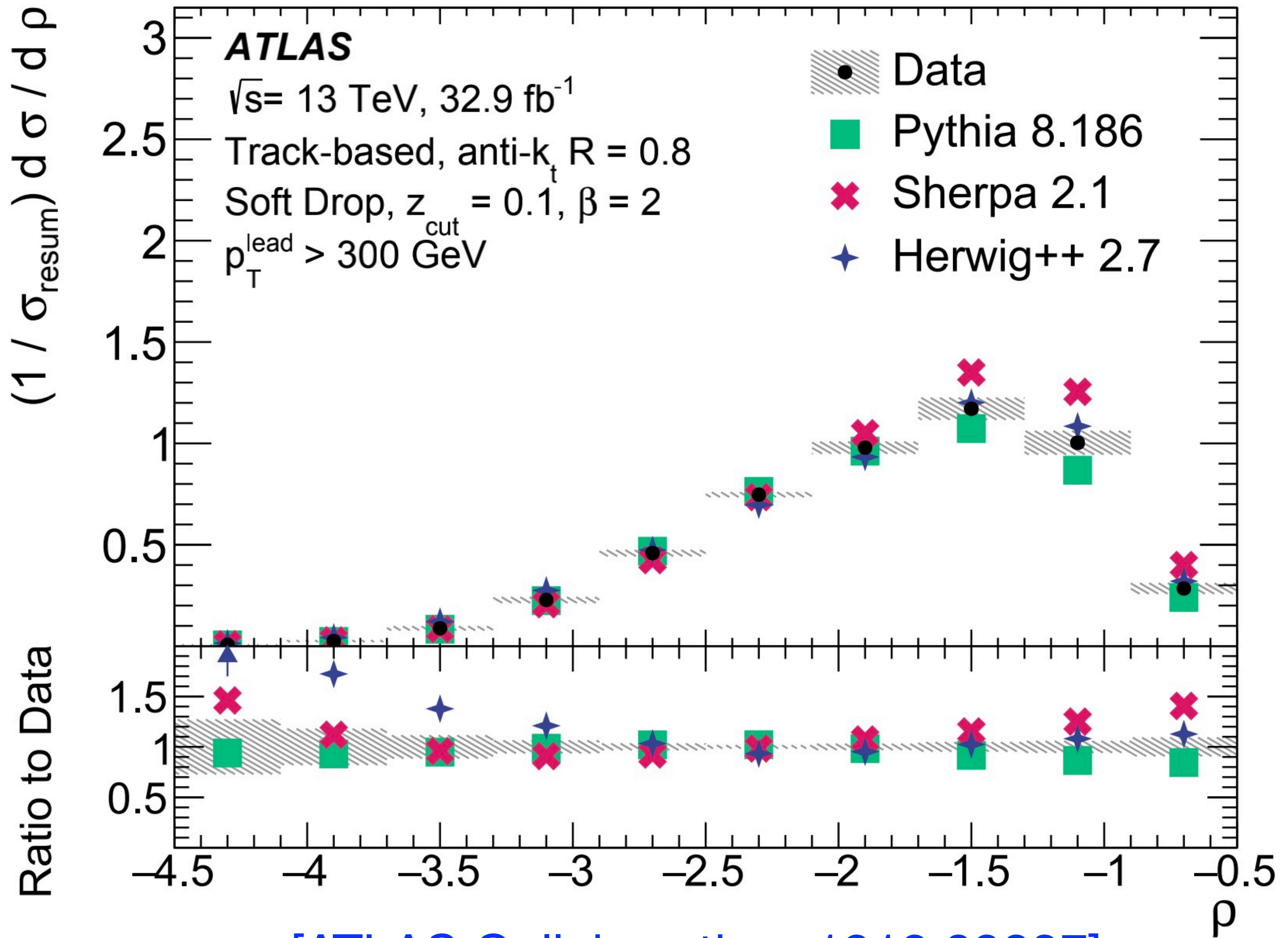
Track Functions [H. Chang, M. Procura, J. Thaler, W. Waalewijn, arXiv: 1303.6637, 1306.6630]

- IR divergences are absorbed into track functions.

calorimeter-based (all-particle)



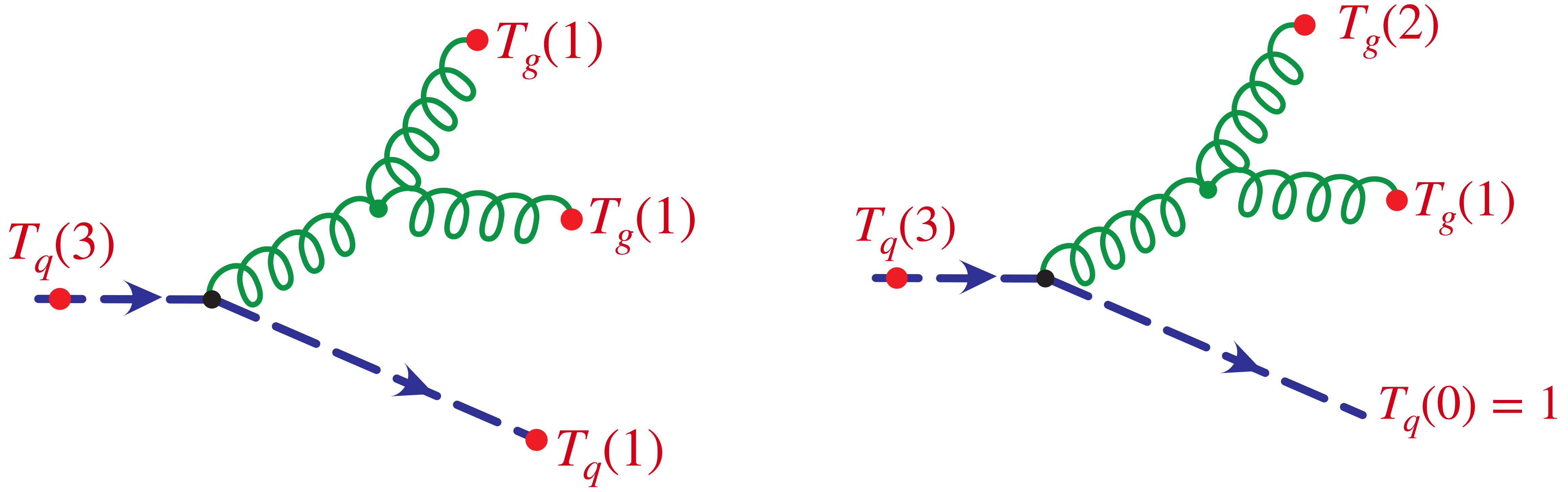
track-based (charged-particle)



[ATLAS Collaboration, 1912.09837]

Outline

- Introduction
  - Definition of track functions
  - Incorporating tracking information
- Track Function Evolution at $\mathcal{O}(\alpha_s^2)$
  - In Mellin Space
- An Example: Prediction for Track EECs
  - Jet Substructure



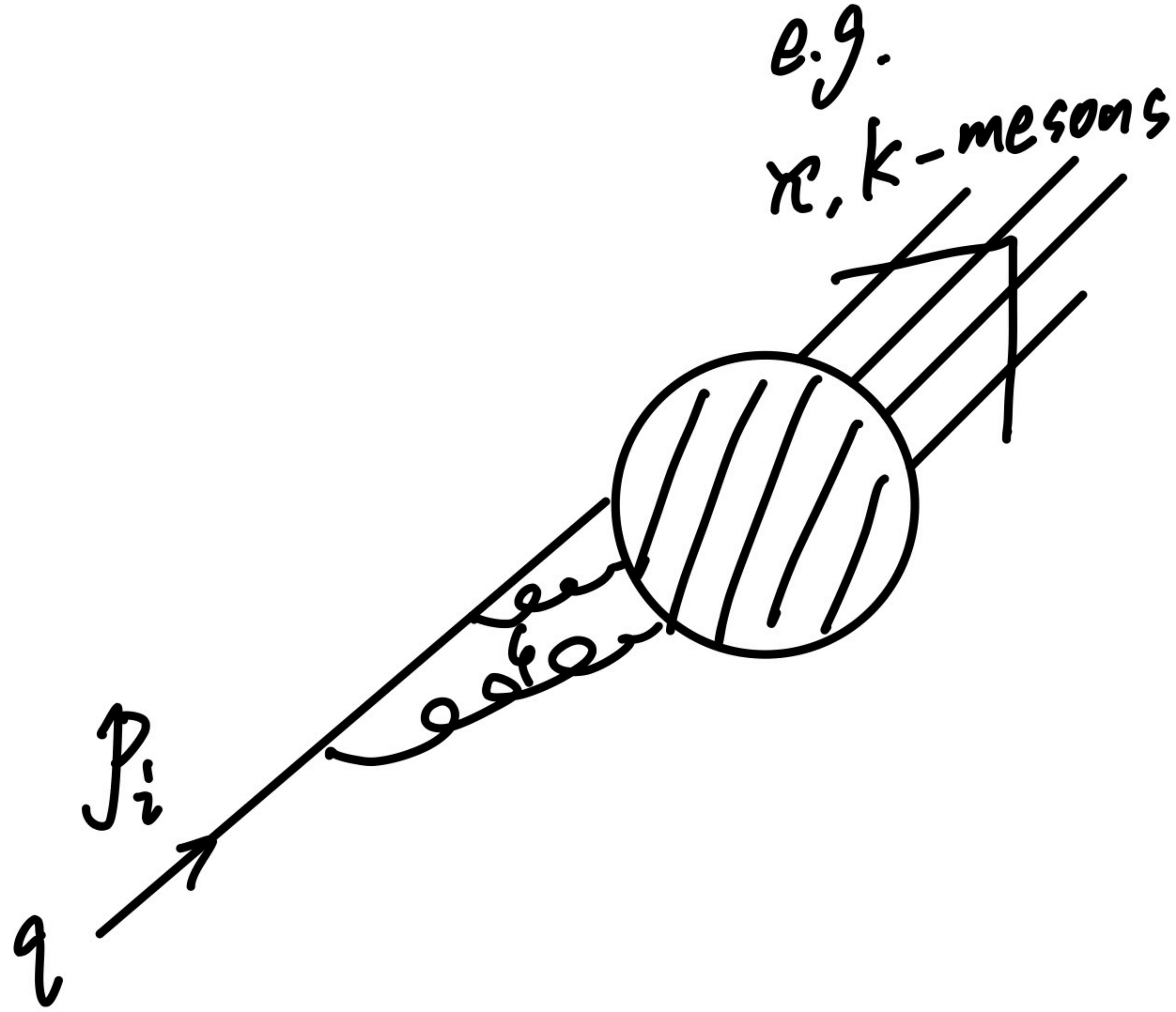
Track Functions $T_i(x,\mu)$

Definition

- The track function $T_i(x,\mu)$ describes the total momentum fraction x of all charged particles (tracks) in a jet initiated by a hard parton i (quark/antiquark or gluon).

$$\bar{p}_i^\mu = x p_i^\mu + O(\Lambda_{\text{QCD}})\ ,(0 \le x \le 1)\ .$$

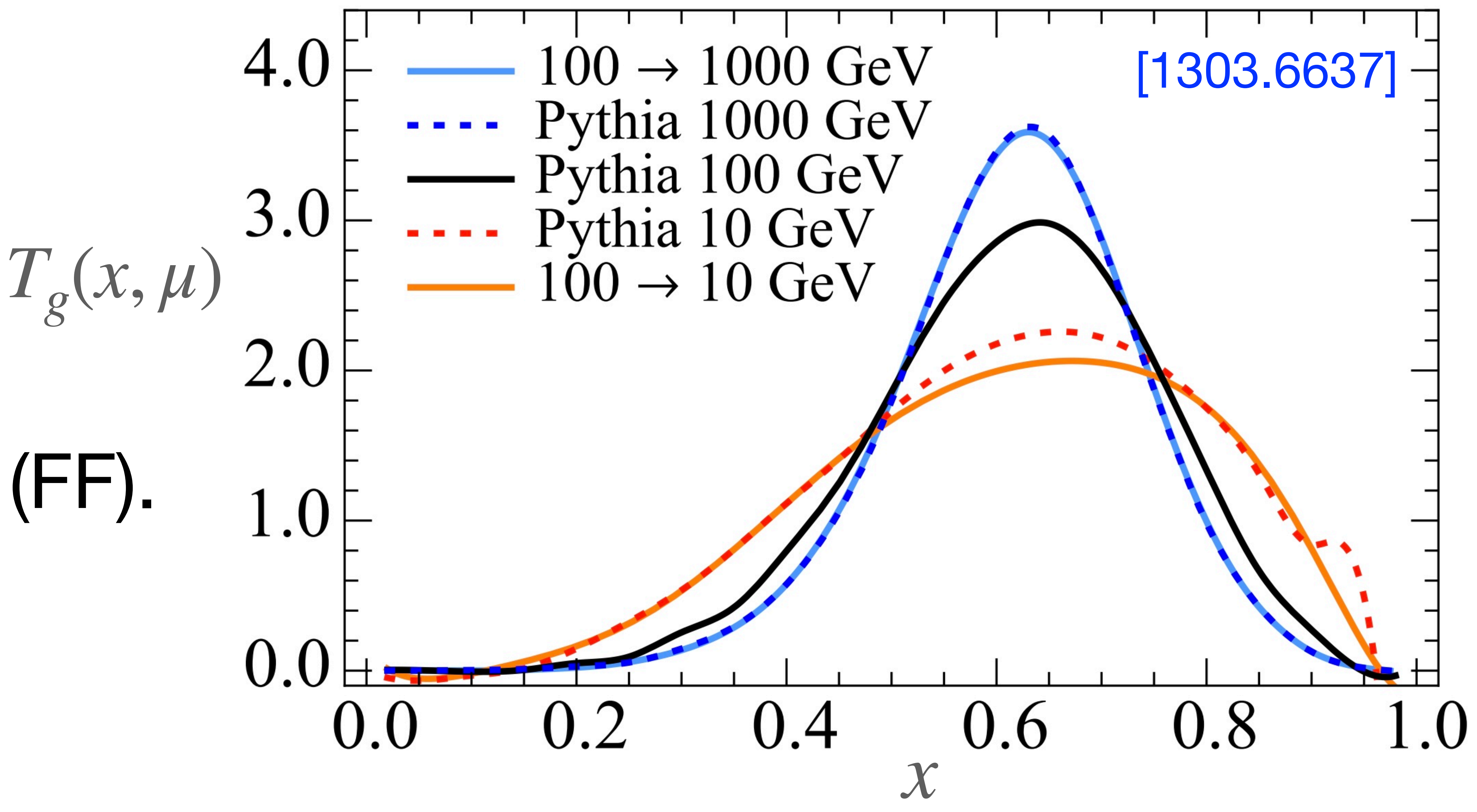
A quark with 4-momentum p_i^μ hadronizes into tracks (charged particles) with total 4-momentum $\bar{p}_i^\mu$:



Track Functions

Features

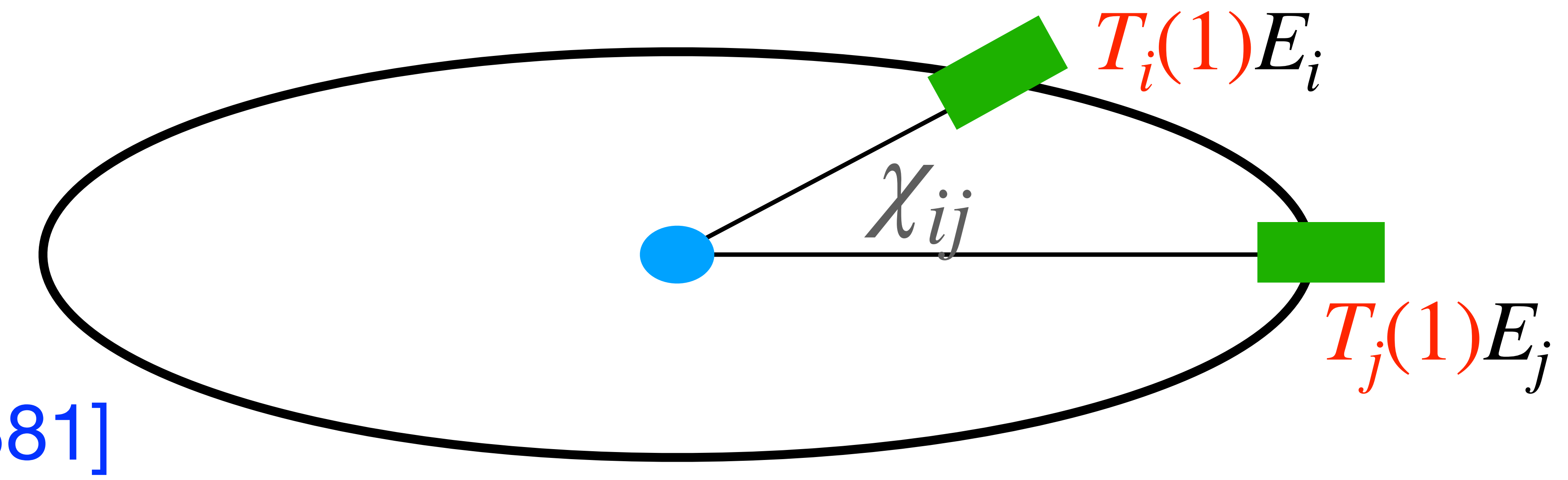
- A generalization of the fragmentation function (FF).
  - Independent of hard process.
  - Fundamentally non-perturbative, with a calculable scale (μ) dependence.
  - Sum rule: $\int_0^1 dx\ T_i(x,\mu) = 1$.



✓Track functions has already been studied at $\mathcal{O}(\alpha_s)$.
[H. Chang, M. Procura, J. Thaler, W. Waalewijn, arXiv: 1303.6637, 1306.6630]

✓New in this talk: The track function formalism applies to IR-safe measurements at $\mathcal{O}(\alpha_s^2)$.

Incorporating Tracks



[1303.6637]

For a δ-function type observable e measured using partons:

$$\frac{d\sigma}{de} = \sum_N \int d\Pi_N \frac{d\sigma_N}{d\Pi_N} \delta\left[e - \hat{e}(p_i^\mu)\right]$$

[Chen, Moult, Zhang, Zhu, 2004.11381]

For an energy correlator at partonic level: e.g. 2-point correlator (EEC)

$$\frac{d\Sigma}{dz} = \sum_{i,j} \int \frac{E_i E_j}{Q^2} \delta\left(z - \frac{1-\cos\chi_{ij}}{2}\right) d\sigma$$

when measured on tracks

Simply

$$E_i^n \to \int dx_i T_i(x_i) x_i^n E_i^n = T_i(n) E_i^n$$

full functional form involved

$$\frac{d\sigma}{d\bar{e}} = \sum_N \int d\Pi_N \frac{d\bar{\sigma}_N}{d\Pi_N} \int \prod_{i=1}^{N} dx_i T_i(x_i) \delta\left[\bar{e} - \hat{e}(x_i p_i^\mu)\right]$$

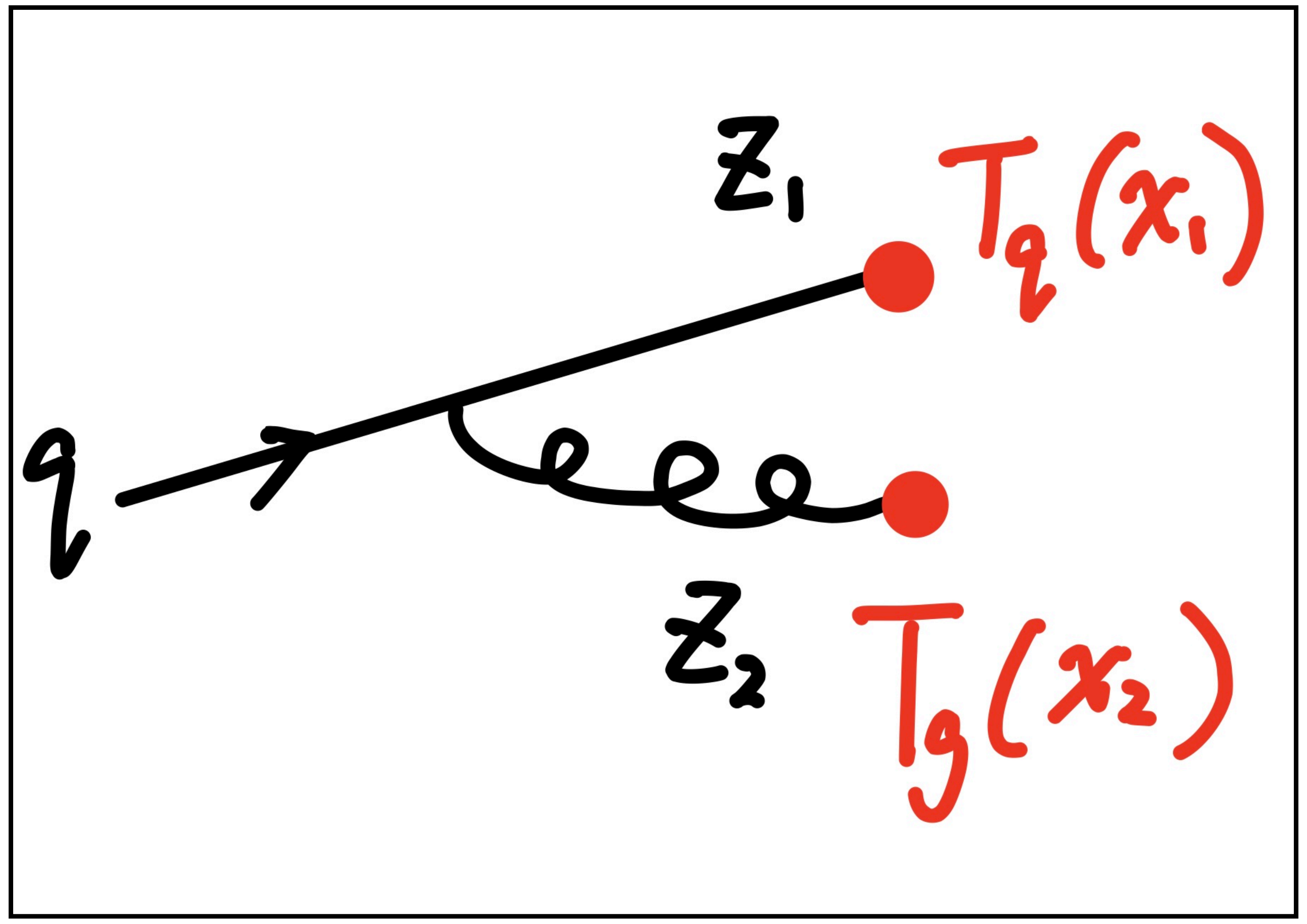
only requiring moments entered as weights

$$\left(\frac{d\Sigma}{dz}\right)_{\text{tr}} = \sum_{i,j} T_i(1) T_j(1) \int \frac{E_i E_j}{Q^2} \delta\left(z - \frac{1-\cos\chi_{ij}}{2}\right) d\bar{\sigma}$$

Track EEC

- The (projected) energy correlator is a natural observable to incorporate tracking information conveniently.

Track Function Evolution



- LO evolution [1303.6637]

$$\frac{d}{d\ln\mu^2}T_i(x,\mu) = a_s(\mu)\sum_{j,k}\int dz dx_1 dx_2 P^{(0)}_{i\to jk}(z_1,z_2)\delta(1-z1-z2)$$
$$\times\, T_j(x_1,\mu)T_k(x_2,\mu)\delta[x-z_1x_1-z_2x_2]\ .$$

Nonlinear, involving contributions from both branches of the splitting.

- Beyond leading order: Involving contributions from multiple branchings.

- Of shift symmetry: $x \to x + a$

While for fragmentation functions:

$$\frac{d}{d\ln\mu^2}d_{h/i}(z,\mu) = \sum_j d_{h/j}\otimes P^T_{ji}(z,\mu)$$

- One of the branches; not depending on correlations between final-state hadrons. → Linearity
- Scale invariant $d(y) \to d(ay)$.

$$\frac{d}{d\ln\mu^2}T_i(x+a) = \sum_X \int\left(\prod_m dx_m dz_m T_{i_m}(x_m+a)\right)P_{i\to i_1\cdots i_m\cdots}(\{z_m\})\ \delta\left(1-\sum_m z_m\right)\delta\left(x-\sum_m x_m z_m\right)$$

Track Function Evolution

- Two independent approaches to extracting the evolution at $\mathcal{O}(\alpha_s^2)$:

◦ Jet Functions ←······Agree······→ ◦ (Projected) Energy Correlators

- Directly calculating track jet functions. [Ritzmann, Waalewijn,1407.3272]
- Remaining IR poles cancel when matching onto track functions.

$$\mathcal{G}_i^{(2)} = T_i^{(2)} + \sum J_{i\to jk}^{(1)} \otimes [T_j^{(1)} T_k^{(0)}] + \sum_{j,k} J_{i\to jk\ell}^{(2)} \otimes [T_j^{(0)} T_k^{(0)} T_\ell^{(0)}]$$

- ‣ Taking moments to extract the evolution for $T(n,\mu)$ (avoiding plus distribution with multiple variables).

$E_i^n \to T_i(n) E_i^n$

- n-point correlators on tracks involves no higher moments than $T(n)$.
- n-point correlator $\xrightarrow[z\to 0]{\text{Pole Cancellation}}$ Evolution for $T(n,\mu)$
  - ‣ The evolution for the lower moments can be checked at wide-angle $0 < z \leq 1$ region.

checks on the track function formalism

➡ Mellin Space

Why we study moments of track functions?

- For a δ-function type IR-safe observable, [2004.11381]

$$\frac{d\sigma}{de} = \int d^4x e^{iq\cdot x}\langle 0 | O(x)\delta(e-\hat{e})O^\dagger(0)|0\rangle$$

  - Infinite number of correlators

    $\rightarrow$ an infinite number of $T(n)$'s

  - Complicated dependence on $T(x)$
  - Its moments $\rightarrow$ weighted cross sections.

- For (projected) n-point energy correlators, tracking information can be easily incorporated with a finite number of the moments.

non-linear equations for the x-space track functions

- Linear RG equations for $\mathbf{T} =$

$$\{T_i(n), T_{i_1}(k)T_{i_2}(n-k), \cdots, T_{i_1}(1)\cdots T_{i_n}(1)\}^{\mathrm{t}}$$

  - Matrix form: $\frac{d}{d\ln\mu^2}\mathbf{T} = \mathbb{R}\mathbf{T}$
  - $\mathbb{R}$: related to moments of timelike splitting functions.

- Simpler to do resummation
- Give information on full-x evolution equations.

➡ Mellin Space

Evolution for moments of track functions

- For the first moment: the same as that of FFs up to all orders in perturbation theory:

For Moments of Fragmentation Functions:

$$\frac{d}{d\ln\mu^2} d_{h/i}(n) = -\sum_j d_{h/j}(n)\gamma^T_{ji}(n+1)$$

$$\frac{d}{d\ln\mu^2} T_g(1) = -\gamma_{gg}(2) T_g(1) + \sum_q \left(-2\gamma_{qg}(2)\right) T_q(1)$$

$$\frac{d}{d\ln\mu^2} T_q(1) = -\gamma_{gq}(2) T_g(1) + \left(-\gamma_{qq}(2) - \gamma_{\bar{q}q}(2)\right) T_q(1) + \sum_{q'\neq q} \left(-\gamma_{q'q}(2) - \gamma_{\bar{q}'q}(2)\right) T_{q'}(1)$$

- Higher moments involve products of ≥ 2 track functions

$$\frac{d}{d\ln\mu^2} T_i(n) \supset a_s^N T_{i_1}(1) T_{i_2}(1) \cdots T_{i_n}(1) \text{ for } n < N+1$$

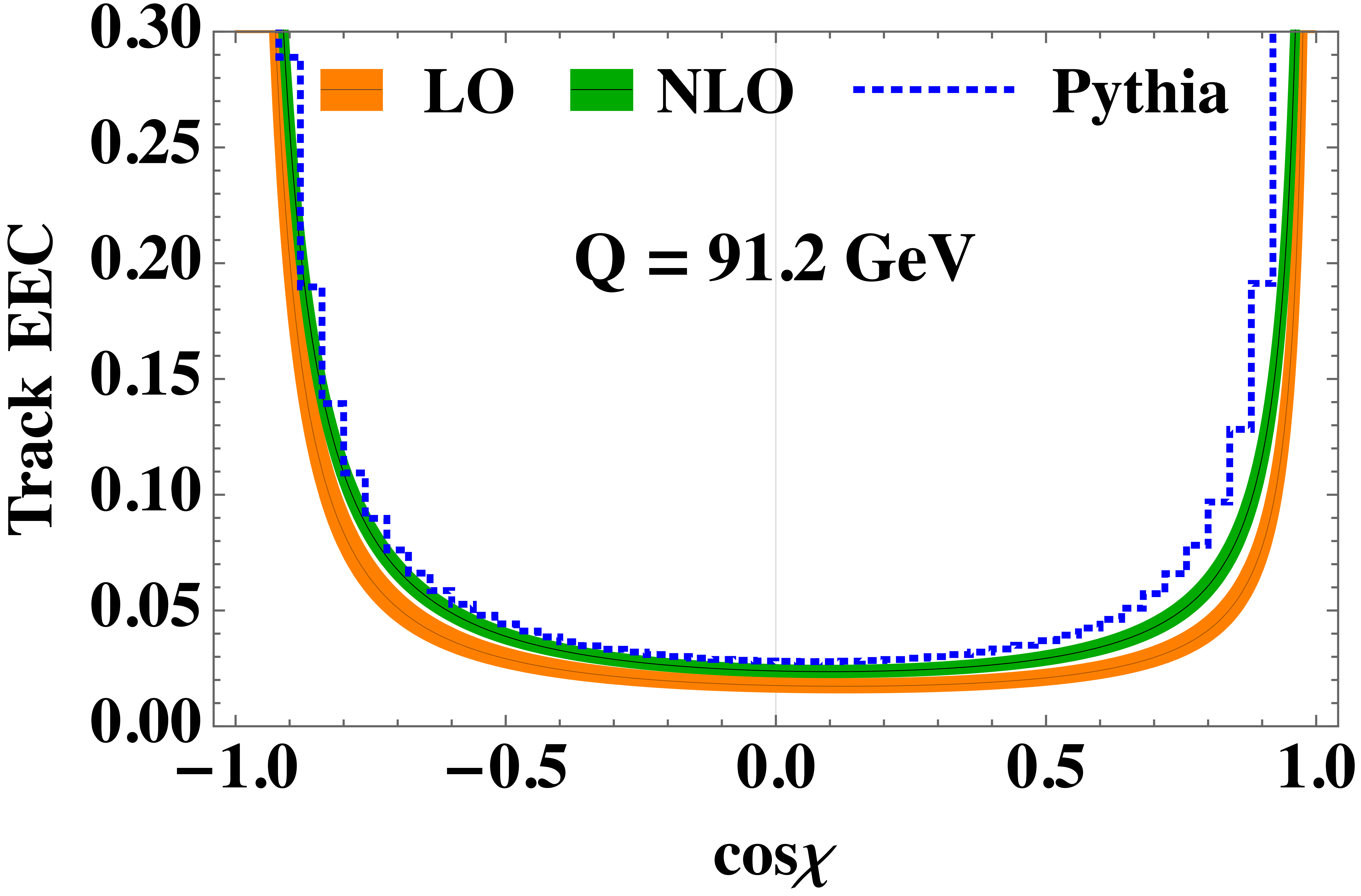
- The expressions for the evolution can be simplified in terms of shift-invariant objects (named central moments).

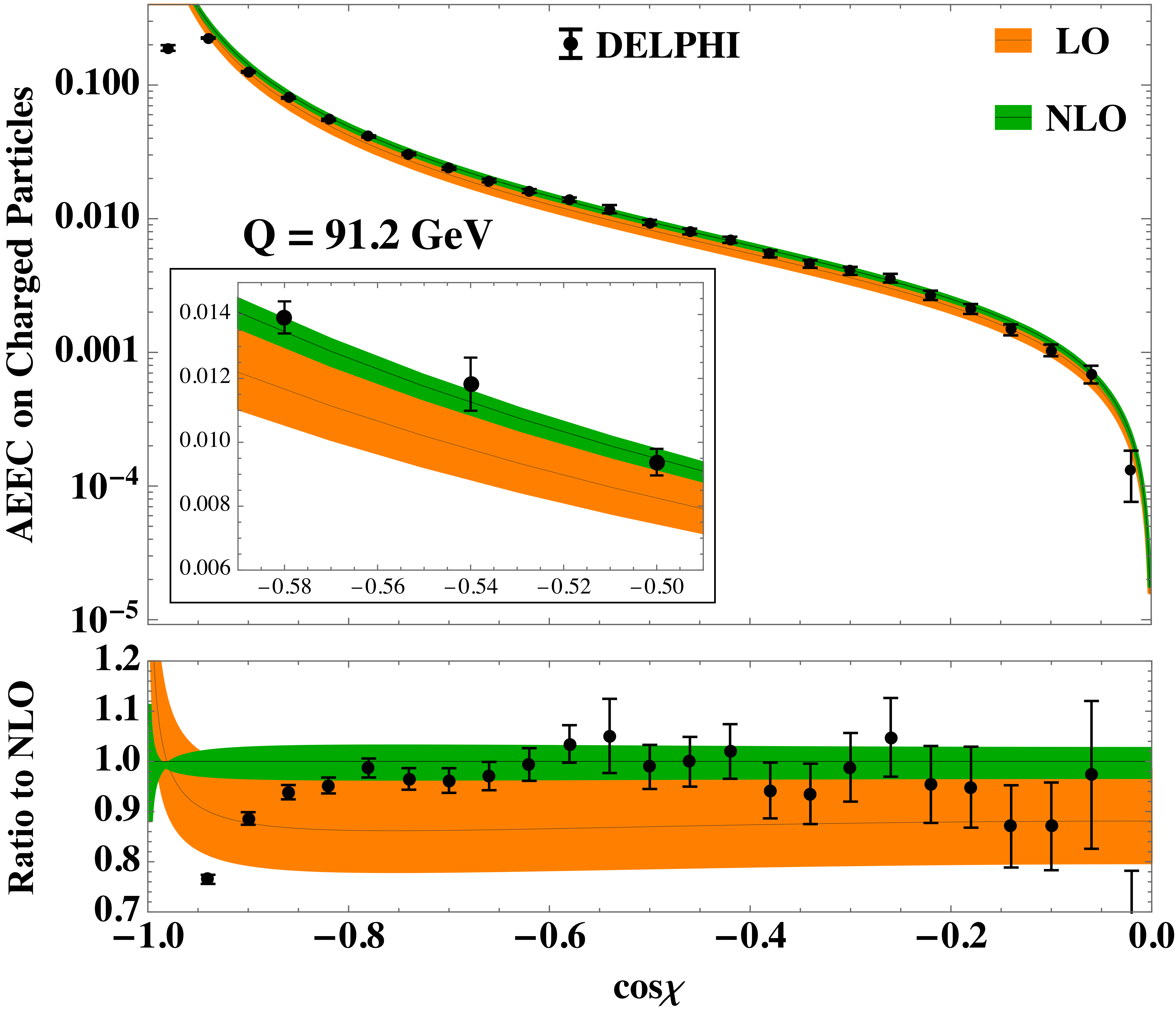
➡ Mellin Space

$\mathcal{O}(\alpha_s^2)$ evolution for moments of track functions

Prediction for Track EECs

- Here, $\text{EEC} = \frac{1}{\sigma}\frac{d\Sigma}{d\cos\chi}$
- $\text{AEEC}(\cos\chi) = \text{EEC}(\cos\chi) - \text{EEC}(-\cos\chi)$, $\cos\chi \le 0$

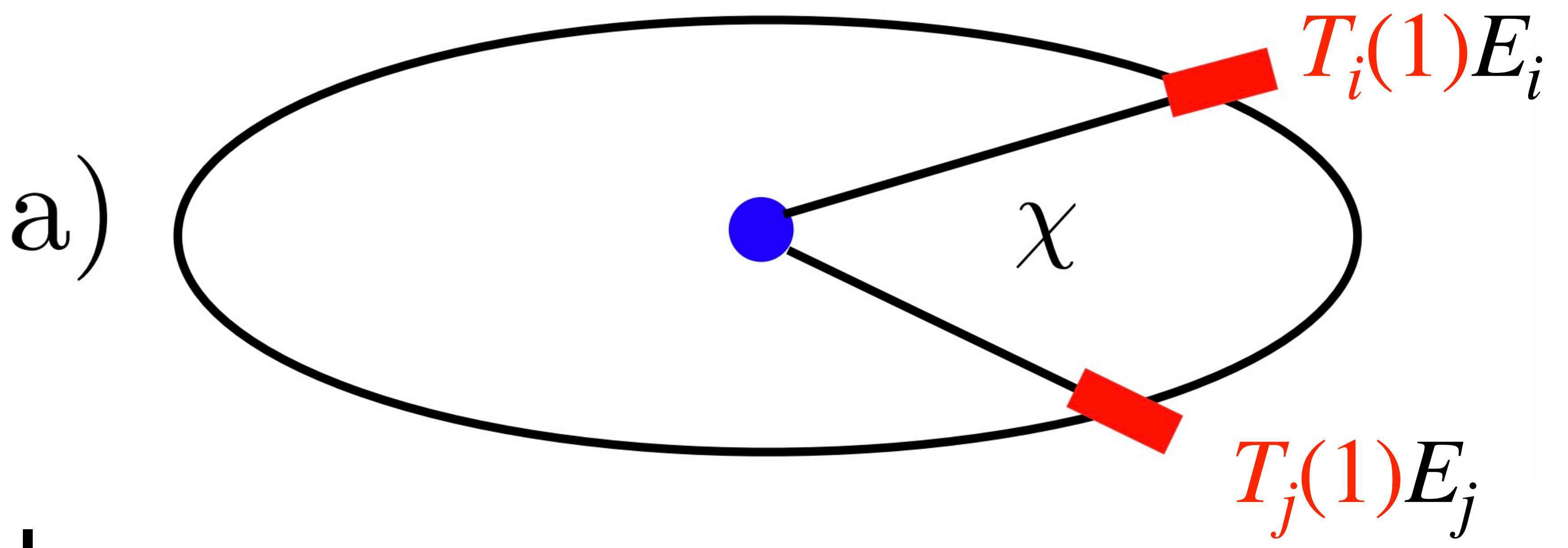




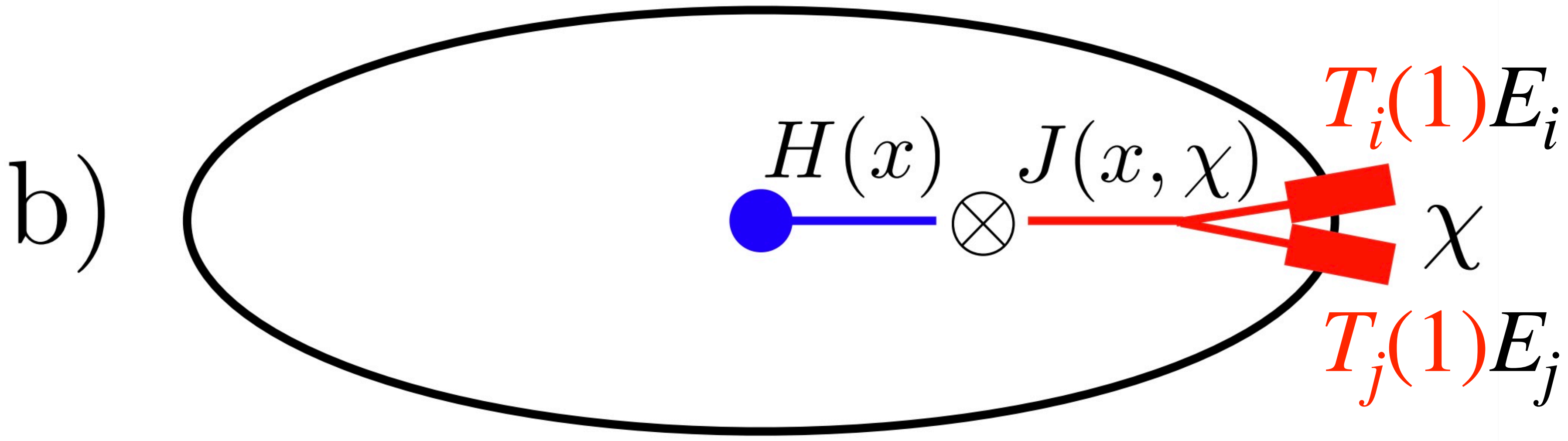
Jet Substructure

In the collinear limit:

[2004.11381]



- The energy correlator is a jet substructure observable.
- Jet function constants (jet functions with the logarithmic dependence excluded):
  - The moments $T_i(n)$ appear as the coefficients.
  - e.g. for EECs, up to $\mathcal{O}(\alpha_s^2)$



$$j_2^g = \frac{1}{4}T_g(2) + a_s\left\{T_g(1)T_g(1)C_A\left(-\frac{449}{150}\right) + \sum_q T_q(1)T_{\bar{q}}(1)T_F\left(-\frac{7}{25}\right)\right\}$$

$$+a_s^2\left\{T_g(1)T_g(1)\left\{C_A^2\left(-\frac{527\zeta(3)}{10} + \frac{133639871}{3240000} - \frac{2159\pi^2}{1800} + \frac{19\pi^4}{90}\right) + C_A n_f T_F \frac{139}{270}\right\} + \sum_q T_q(1)T_{\bar{q}}(1)\cdots\right\}$$

- Resummation

Summary & Outlook

- Track functions offer a QFT approach to calculating track-based observables.
  - Superior angular resolution.
  - Reduced effect of pile-up.
- Track function formalism studied beyond leading order:
  - Evolution for moments of track functions at $\mathcal{O}(\alpha_s^2)$.
  - Two-loop track EEC.
    - The (project) energy correlators interface in a simple manner with tracking information through the moments, allowing for high order calculations.

> - High precision
> - Strong check on formalism

- This formalism allows IR-safe observables to be computed on any subset of final-state hadrons specified by some particular quantum numbers.

➡ More: Generalized fragmentation functions (GFFs)

[B. Elder, M. Procura, J. Thaler, W. Waalewijn, K. Zhou, 1704.05456]

Thanks!