

B+L and PMF from First-order phase transition

一阶相变过程中重子破坏和原初磁场

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Based on work with Yuefeng Di, Jialong Wang, Ruiyu Zhou, Rong-Gen Cai, and Jing Liu, 2012.15625(Phys.Rev.Lett. 126 (2021) 25, 25);
2107.08978

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展望

Gravitational Wave Periods

Milliseconds

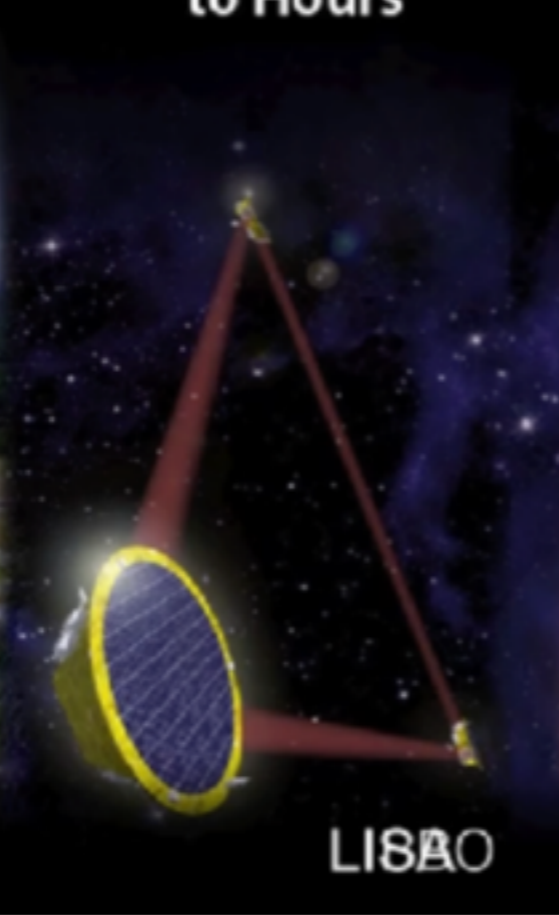
Minutes
to Hours

Years
to Decades

Billions
of Years



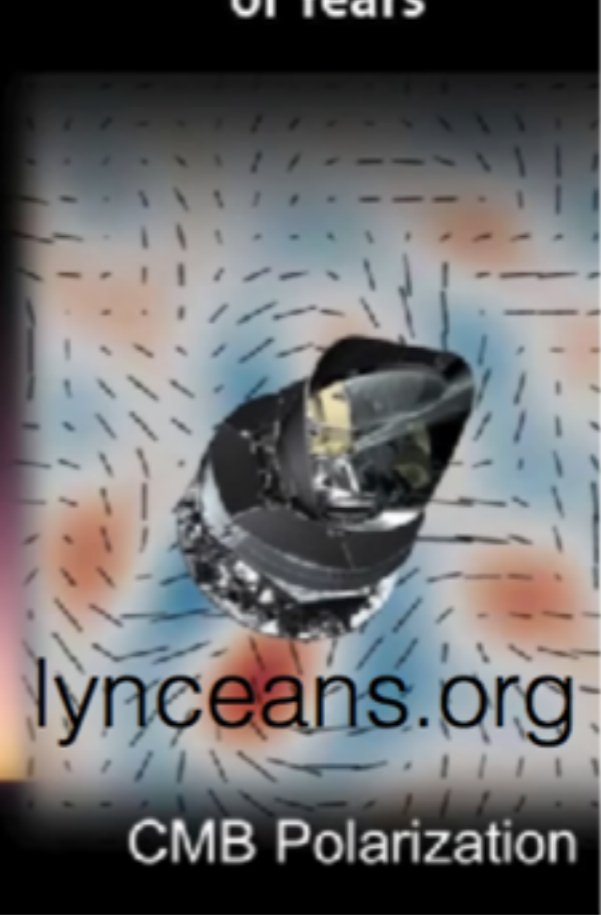
LIGO



LISA



PTA



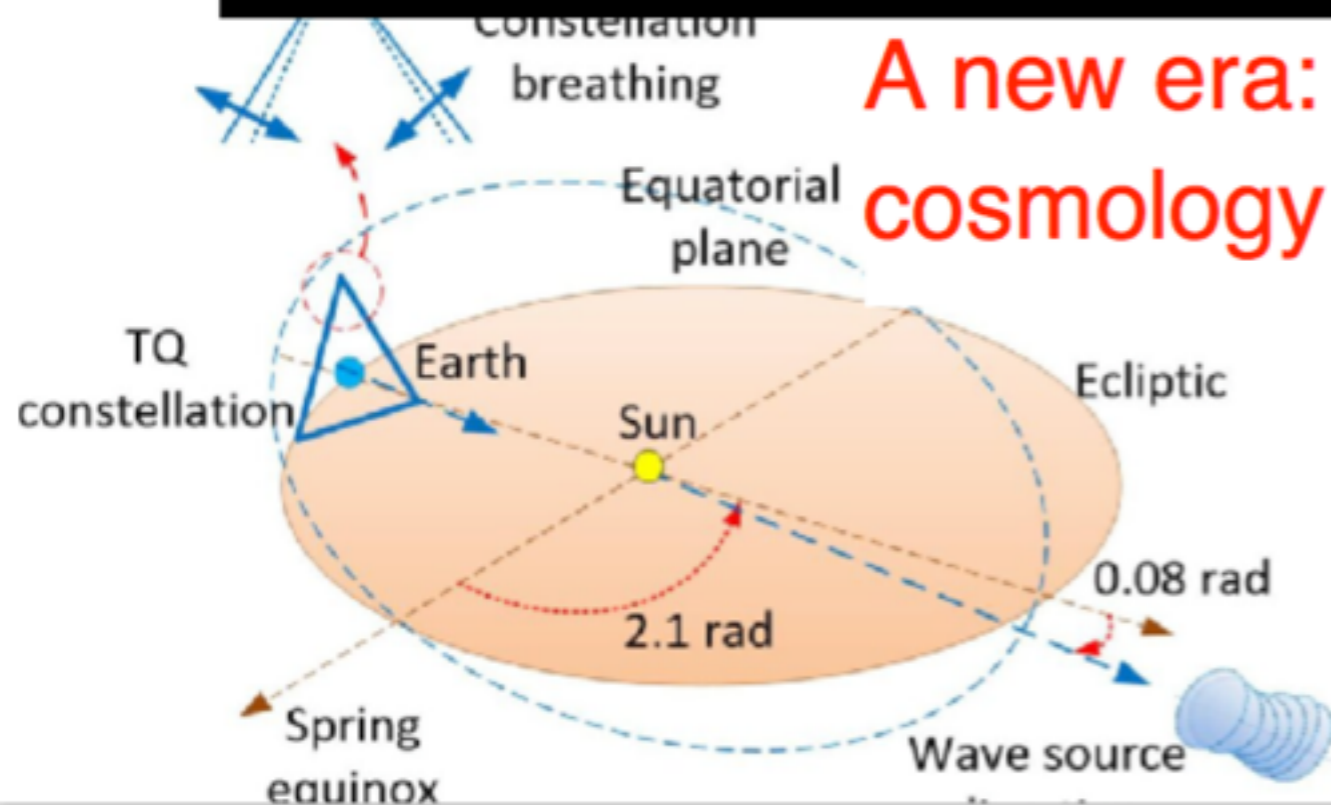
CMB Polarization

lynceans.org

A new era: GWs as a new tool for probing cosmology and high energy physics

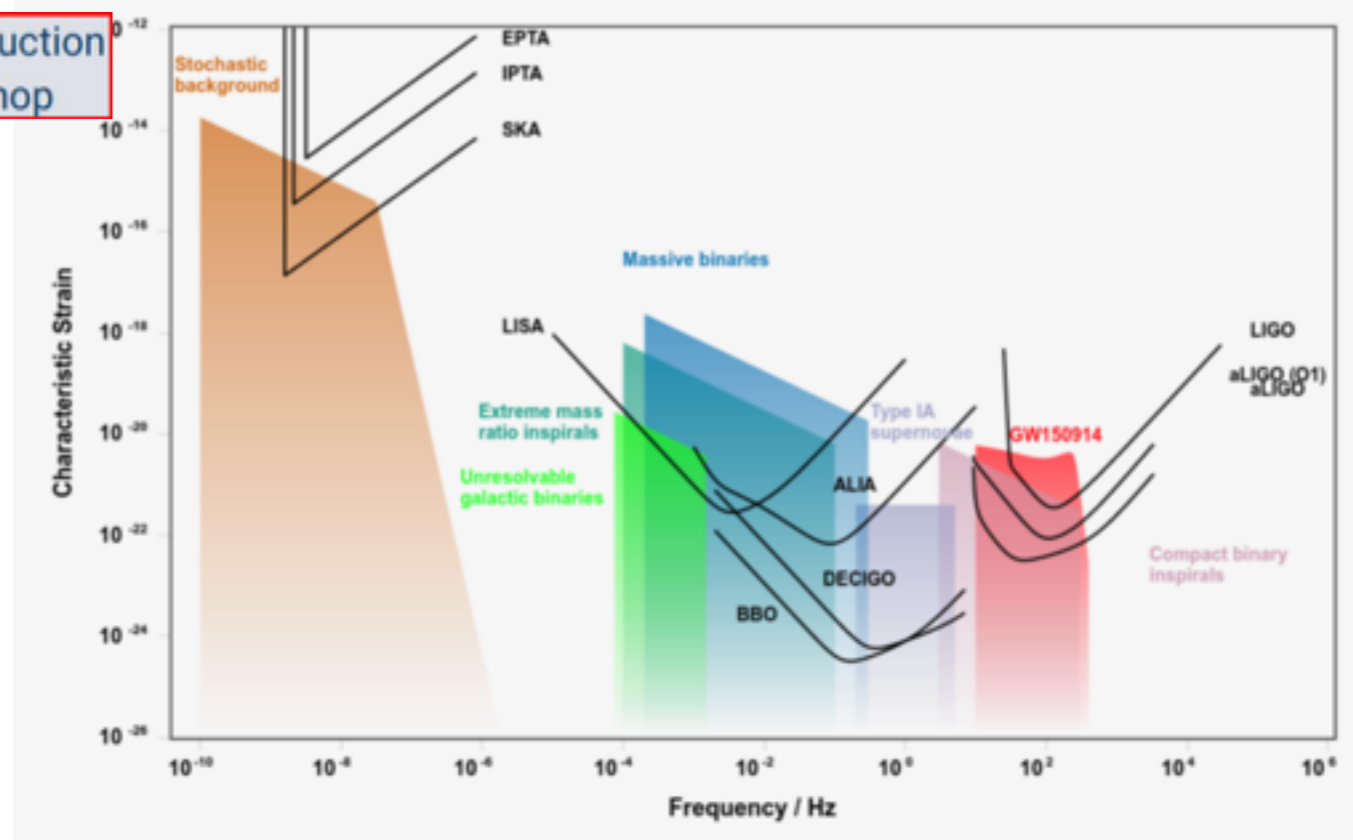
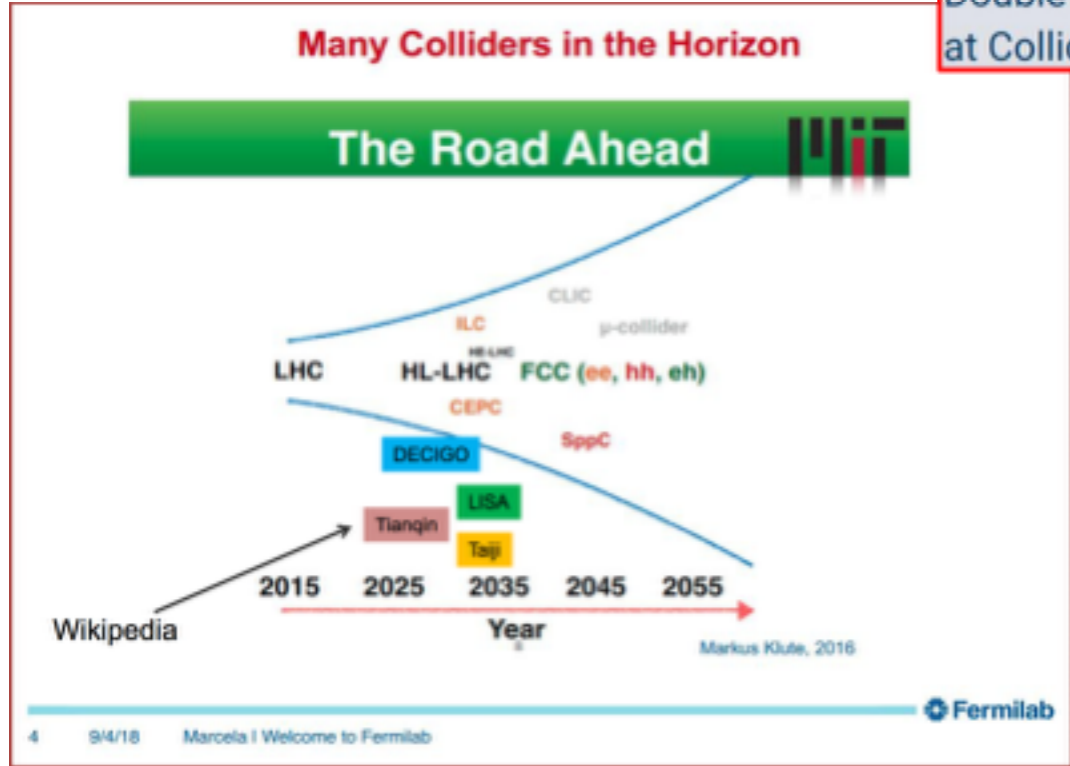
Current: LIGO, PTA, ...

Future GW experiments:
LISA, Taiji, TianQin, BBO,
DECIGO, ET, CE, ...

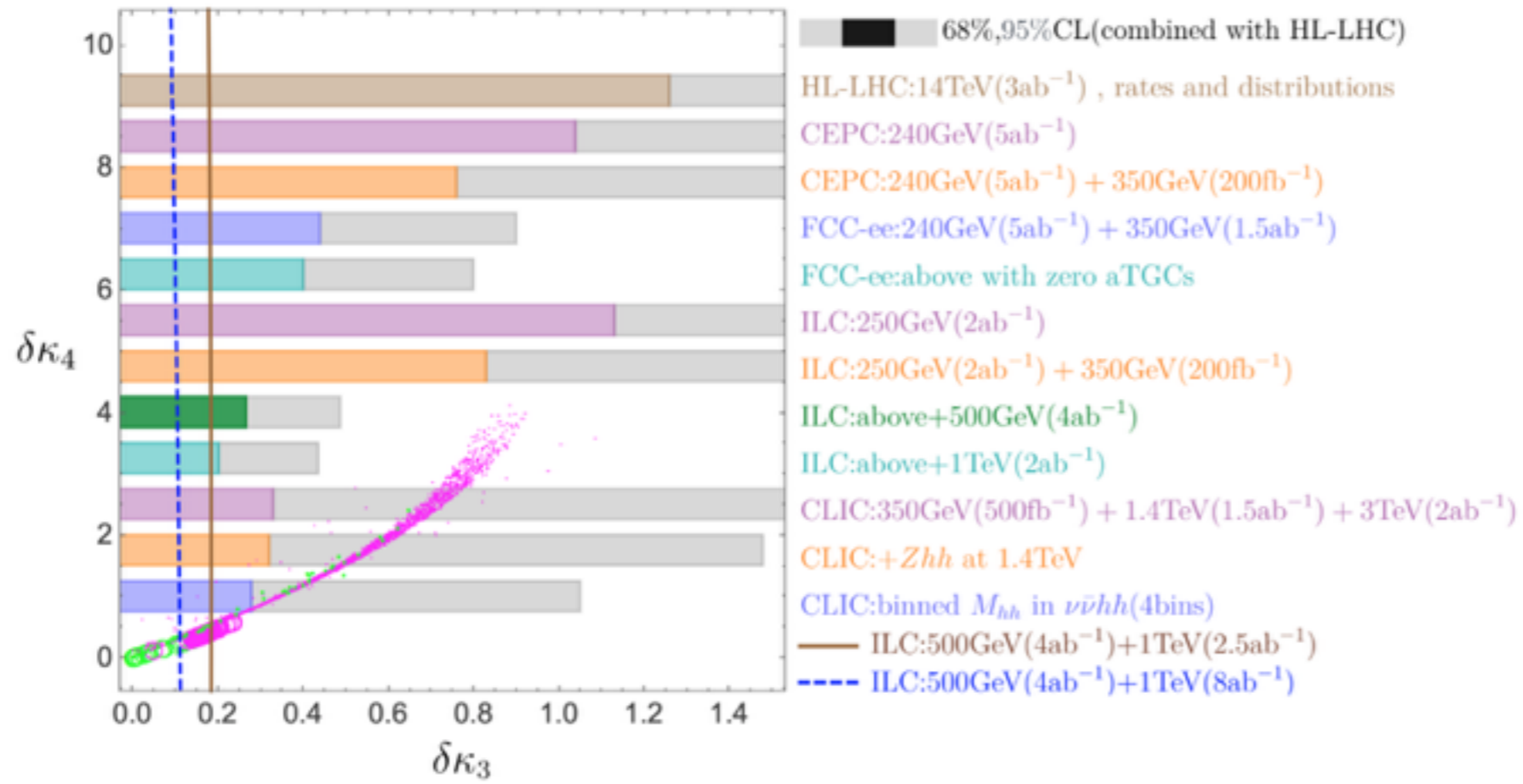


GW and collider search

Double Higgs Production at Colliders Workshop



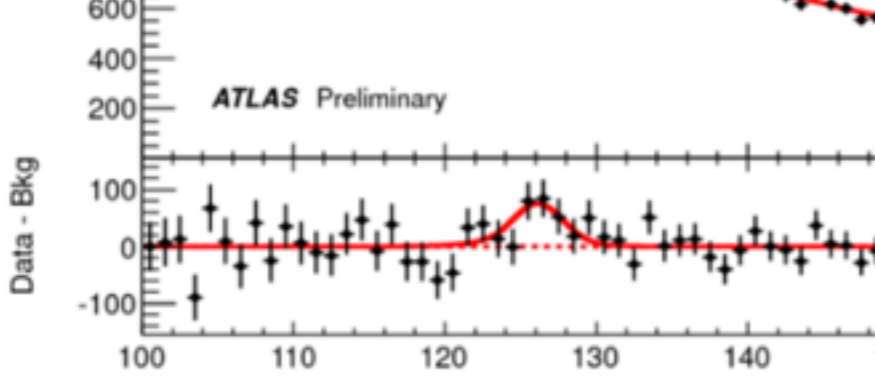
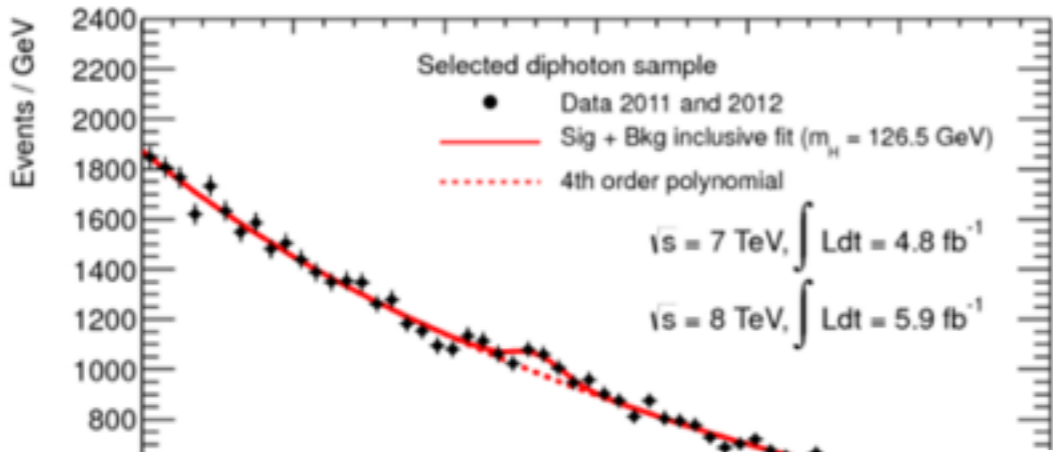
antum Grav. 32 (2015) 015014.



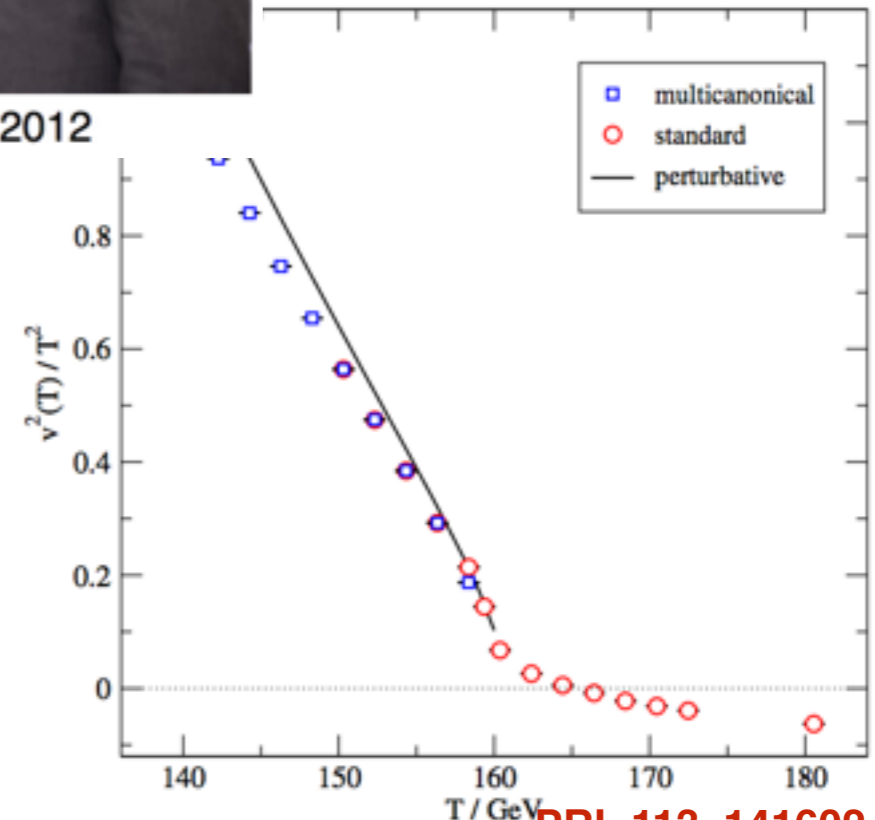
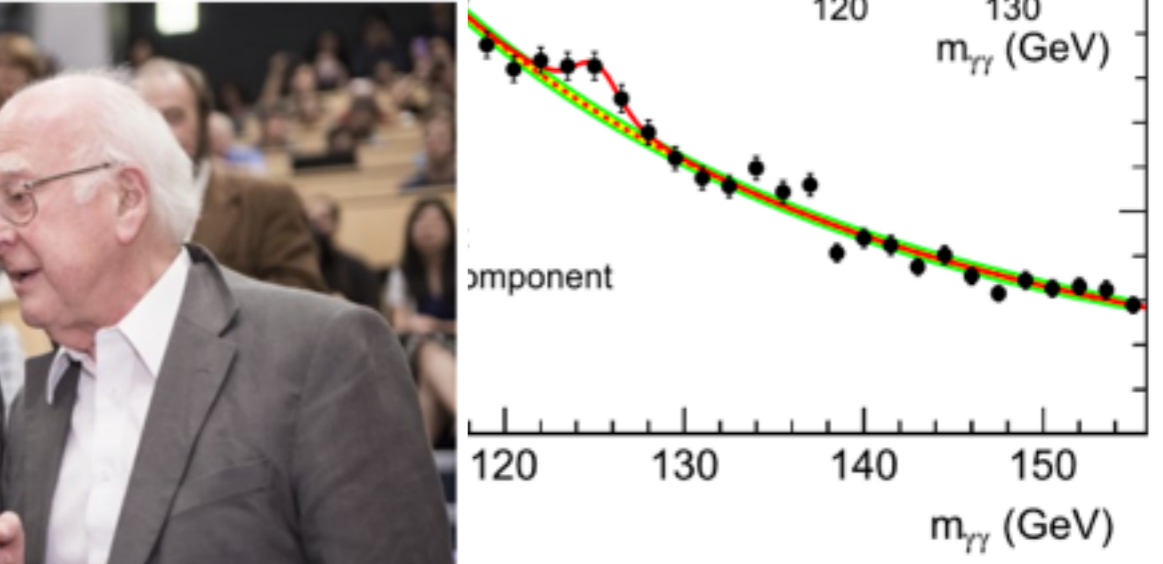
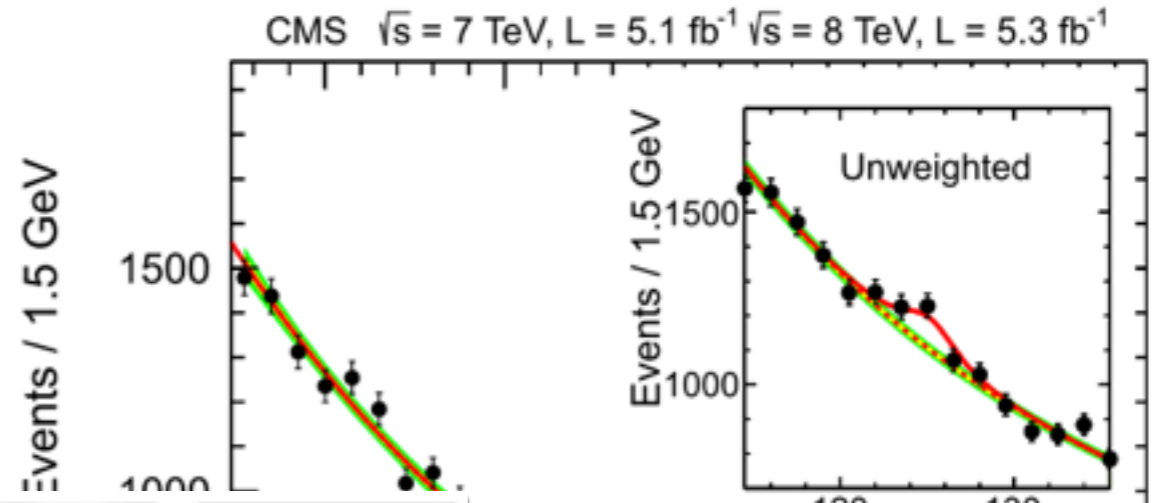
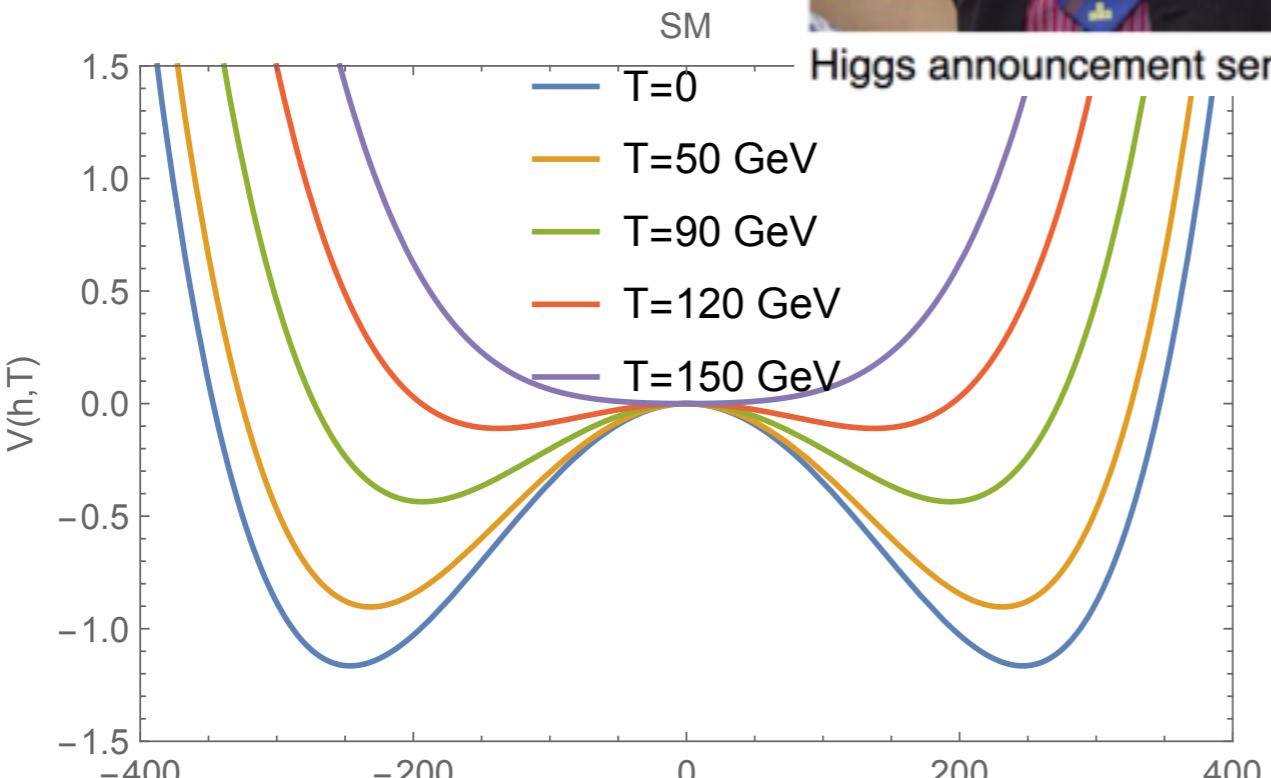
Circles and the dotted points for the GM and xSM scenarios

SNR > 10 points for two-step and one-step SFOEWPT

Implication of 125 GeV Higgs

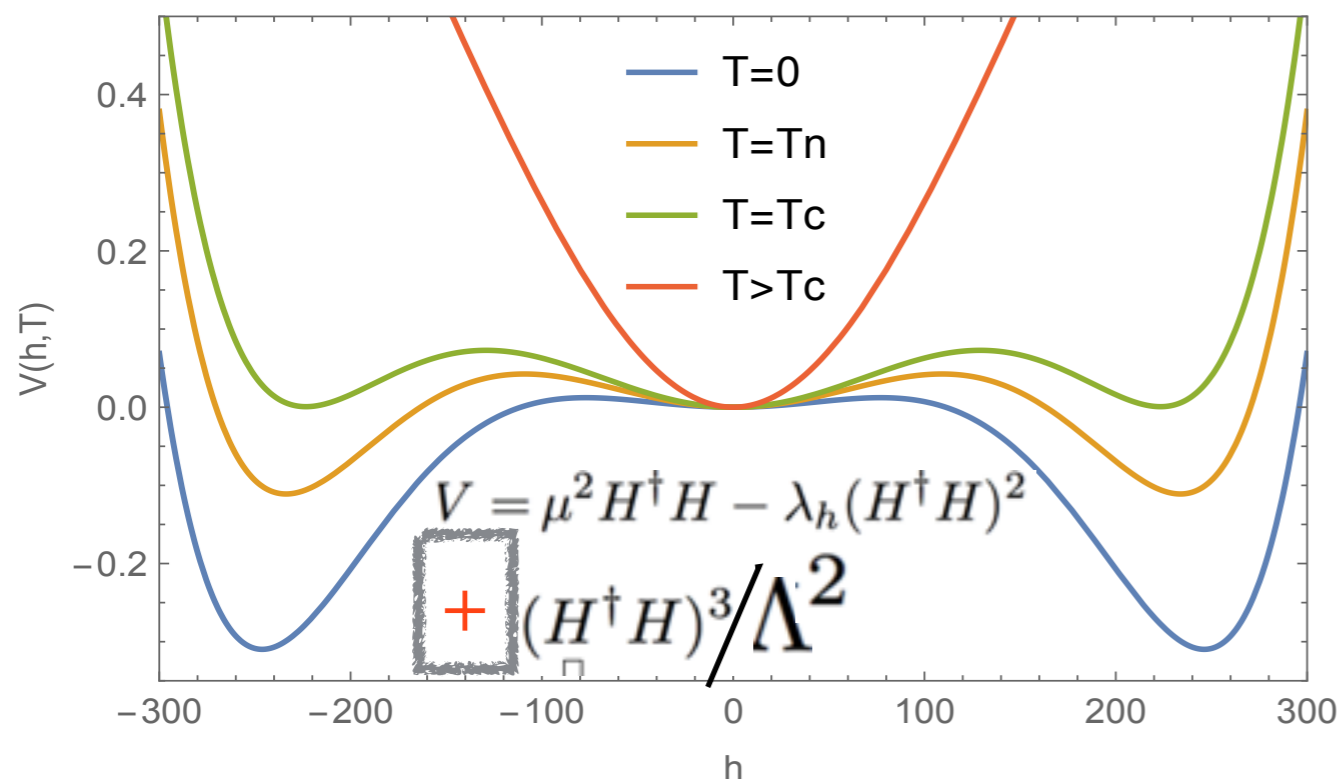
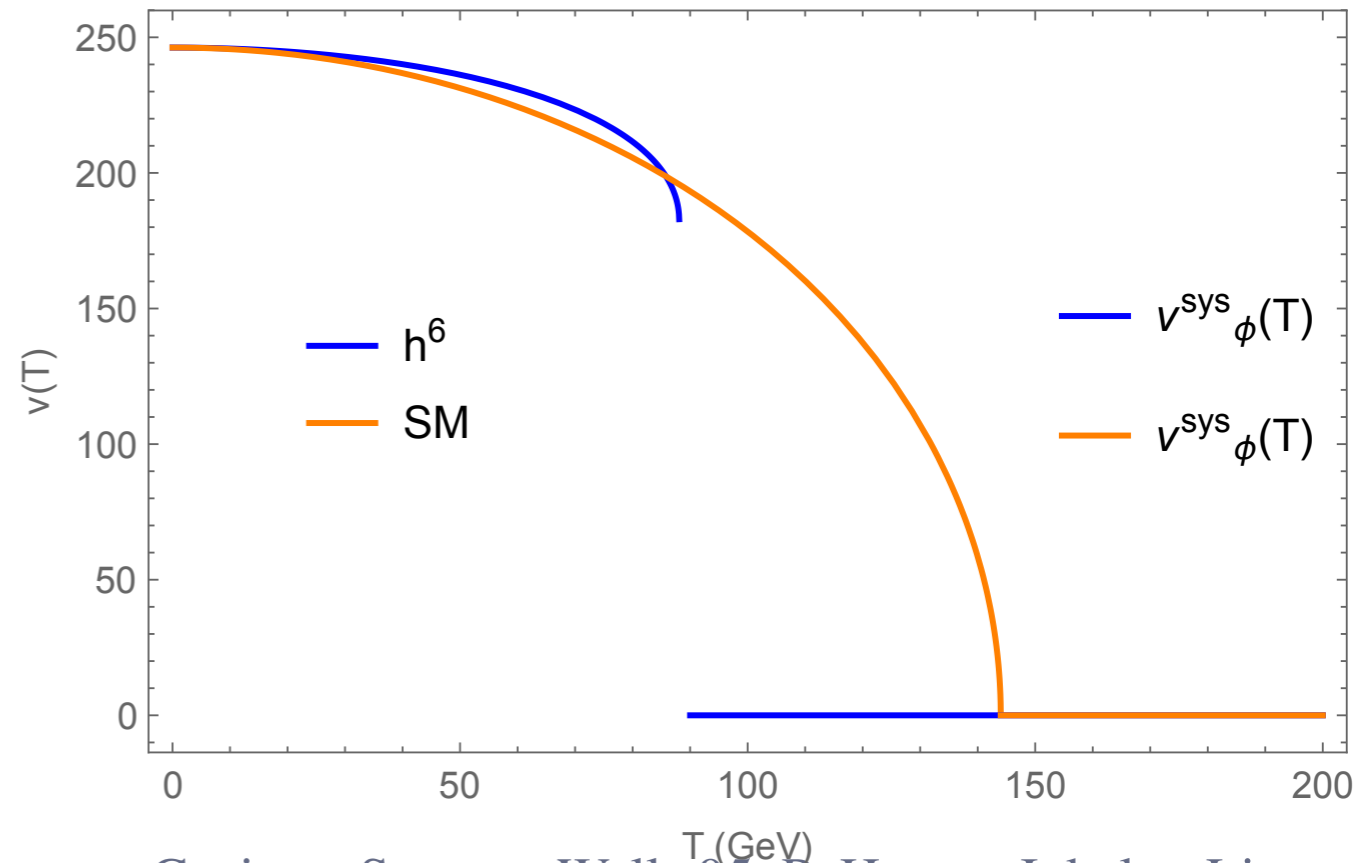
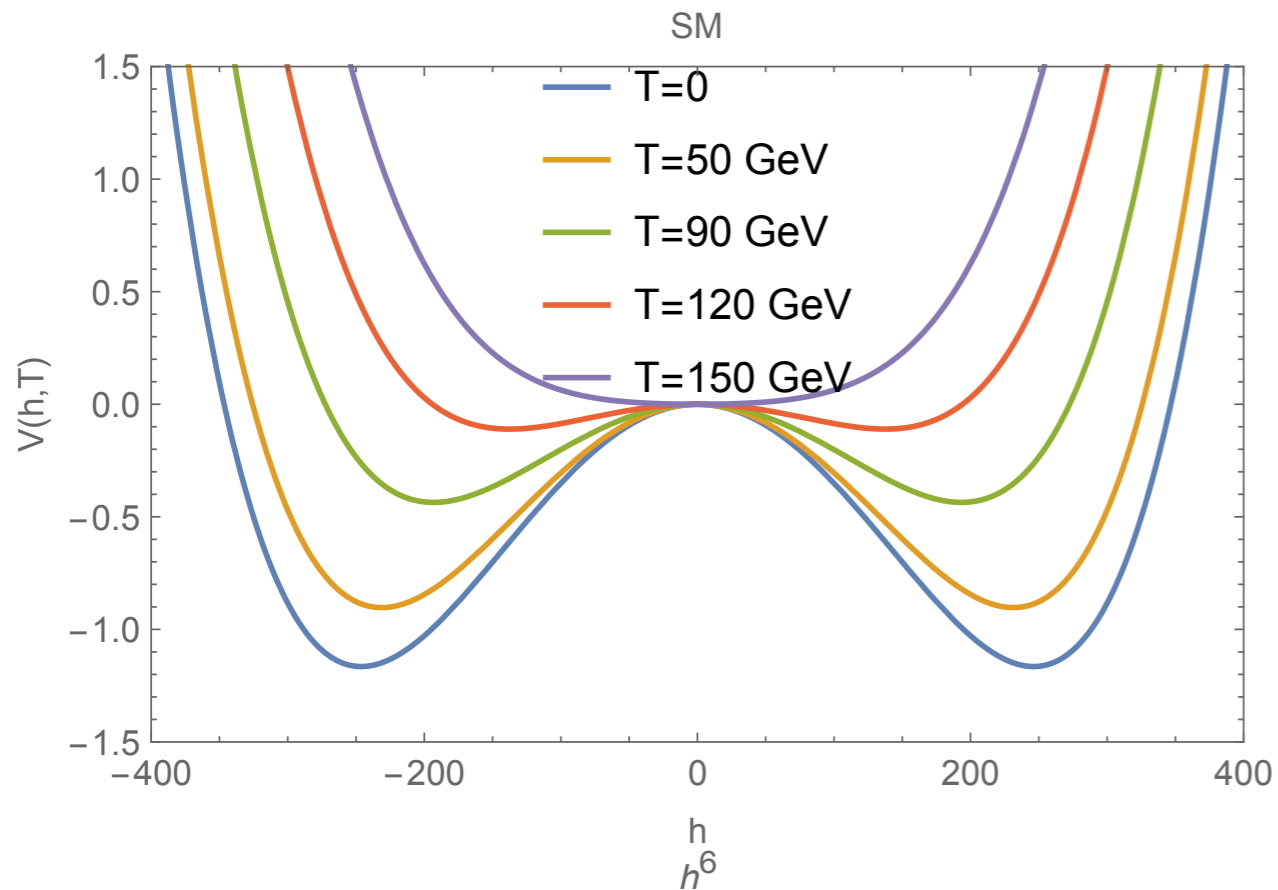


Higgs announcement seminar on 4 July 2012



Higgs Potential Shape??? EFT or ???

First or second order

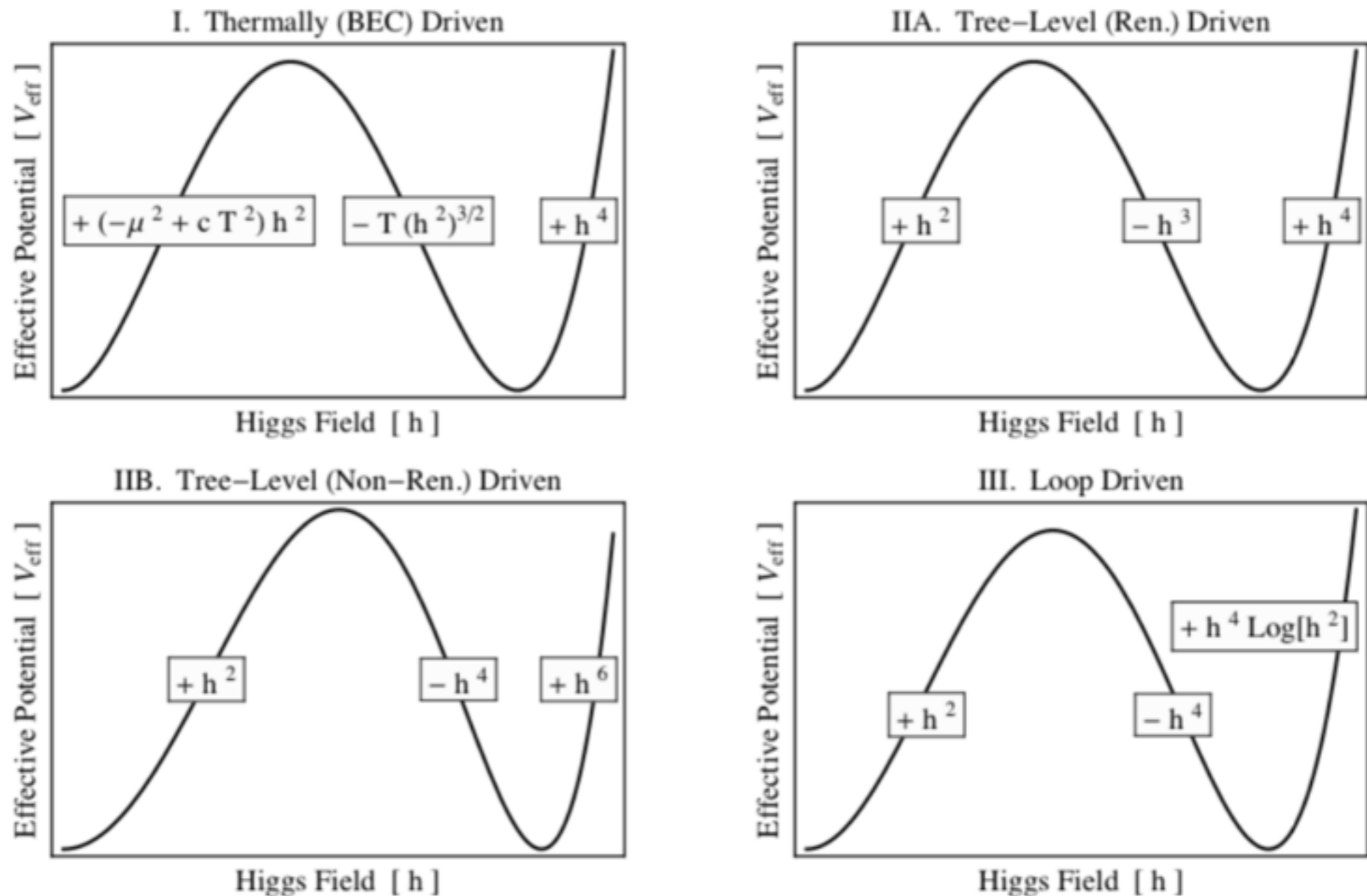


Grojean, Servant, Wells 05, P. Huang, Jokelar, Li, Wagner (2015)

F.P. Huang, Gu, Yin, Yu, Zhang (2015) F.P. Huang, Wan, Wang, Cai, Zhang (2016) Cao, F.P. Huang, Xie, & Zhang (2017)

LHC say the quantum fluctuation (quadratic oscillation) around $h=v$ with $m_h=126$ GeV, not sensitive to the specifically potential shape

Model classes for catalyzing a strongly first order electroweak phase transition



BSM for EWPT

SM+Scalar Singlet

Espinosa, Quiros 93, Benson 93, Choi, Volkas 93, Vergara 96, Branco, Delepine, Emmanuel- Costa, Gonzalez 98, Ham, Jeong, Oh 04, Ahriche 07, Espinosa, Quiros 07, Profumo, Ramsey-Musolf, Shaughnessy 07, Noble, Perelstein 07, Espinosa, Konstandin, No, Quiros 08, Barger, Langacker, McCaskey, Ramsey-Musolf, Shaughnessy 09, Ashoorioon, Konstandin 09, Das, Fox, Kumar, Weiner 09, Espinosa, Konstandin, Riva 11, Chung, Long 11, Barger, Chung, Long, Wang 12, Huang, Shu, Zhang 12, Fairbairn, Hogan 13, Katz, Perelstein 14, Profumo, Ramsey-Musolf, Wainwright, Winslow 14, [Jiang, Bian, Huang, Shu 15](#), Kozaczuk 15, Cline, Kainulainen, Tucker-Smith 17, Kurup, Perelstein 17, Chen, Kozaczuk, Lewis 17, [Cheng, Bian 17, Bian, Tang 18, Chen, Li, Wu, Bian, 19](#)...

SM+Scalar Doublet

Turok, Zadrozny 92, Davies, Froggatt, Jenkins, Moorhouse 94, Cline, Lemieux 97, Huber 06, Froome, Huber, Seniuch 06, Cline, Kainulainen, Trott 11, Dorsch, Huber, No 13, Dorsch, Huber, Mimasu, No 14, Basler, Krause, Muhlleitner, Wittbrodt, Wlotzka 16, Dorsch, Huber, Mimasu, No 17, [Bernon, Bian, Jiang 17, Bian, Liu 18](#),...

SM + Scalar Triplet

Profumo, Ramsey-Musolf 12, Chiang 14, [Zhou, Cheng, Deng, Bian, Wu 18, Zhou, Bian, Guo, Wu 19](#),...

NMSSM

Pietroni 93, Davies, Froggatt, Moorhouse 95, Huber, Schmidt 01, Ham, Oh, Kim, Yoo, Son 04, Menon, Morrissey, Wagner 04, Funakubo, Tao, Yokoda 05, Huber, Konstandin, Prokopec, Schmidt 07, Chung, Long 10, Kozaczuk, Profumo, Stephenson Haskins, Wainwright 15, [Bi, Bian, Huang, Shu, Yin 15, Bian, Guo, Shu 17](#),...

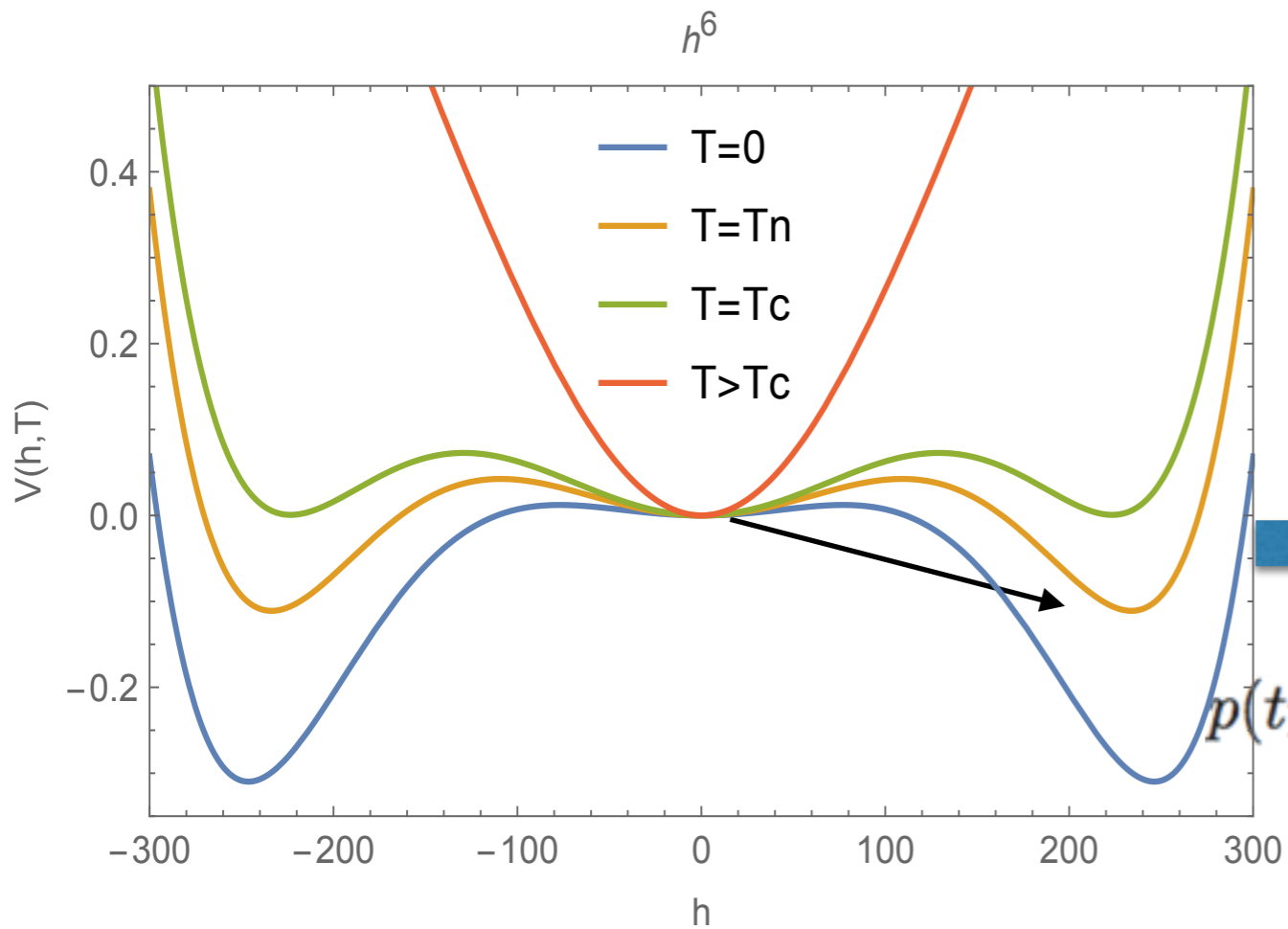
Composite Higgs

Espinosa, Gripaios, Konstandin, Riva 11, Bruggisser, Von Harling, Matsedonskyi, Servant 18, [Bian, Wu, Xie 19](#), De Curtis, Delle Rose, Panico 19, [Bian, Wu, Xie 20](#),...

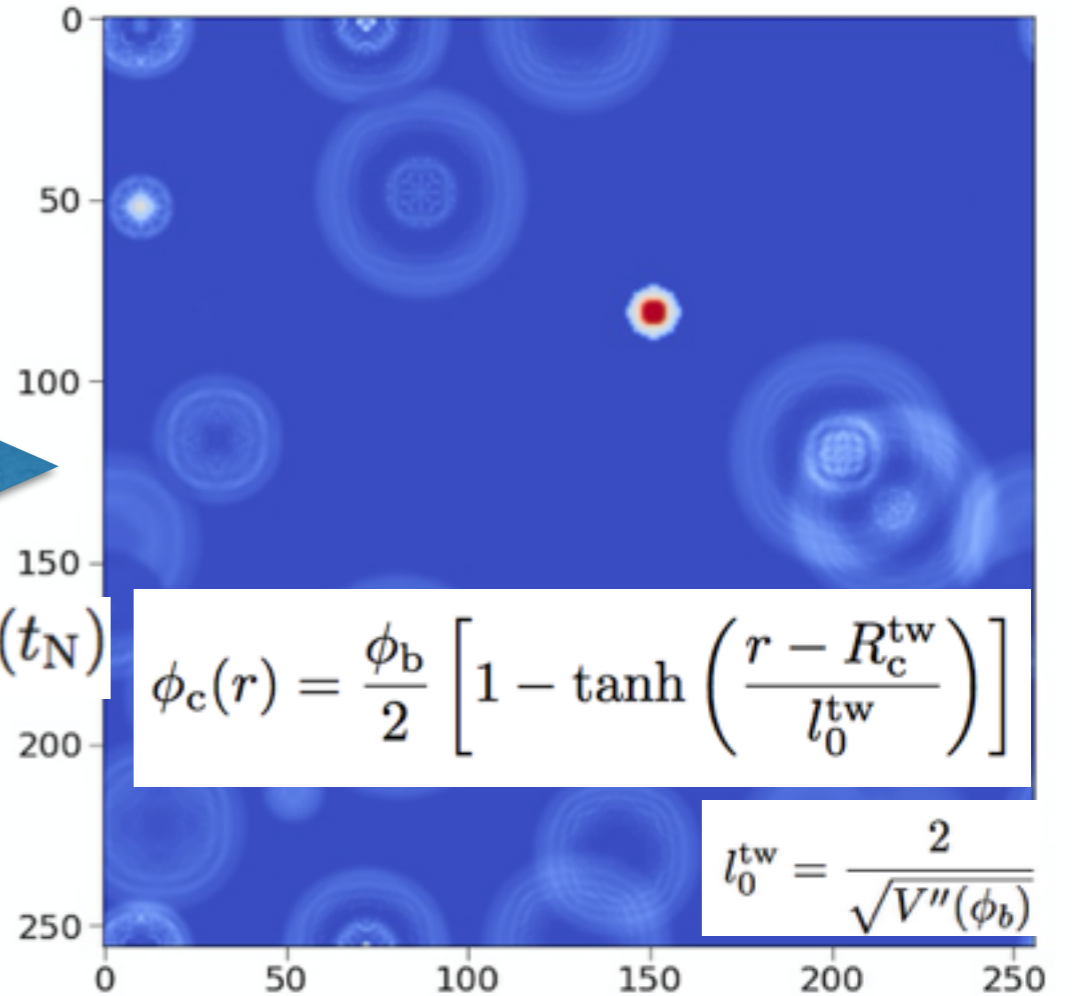
EFT

Grojean, Servant, Wells 05, Bodeker, Froome, Huber, Seniuch 05, Huang, Joglekar, Li, Wagner 15, Cai, Sasaki, Wang 17, [Zhou, Bian, Guo 19](#), ...

Higgs Potential Shape and the Bubble picture



$$p(t_N) = H^4(t_N)$$



$$V = \mu^2 H^\dagger H - \lambda_h (H^\dagger H)^2$$

$$+ (H^\dagger H)^3 / \Lambda^2$$

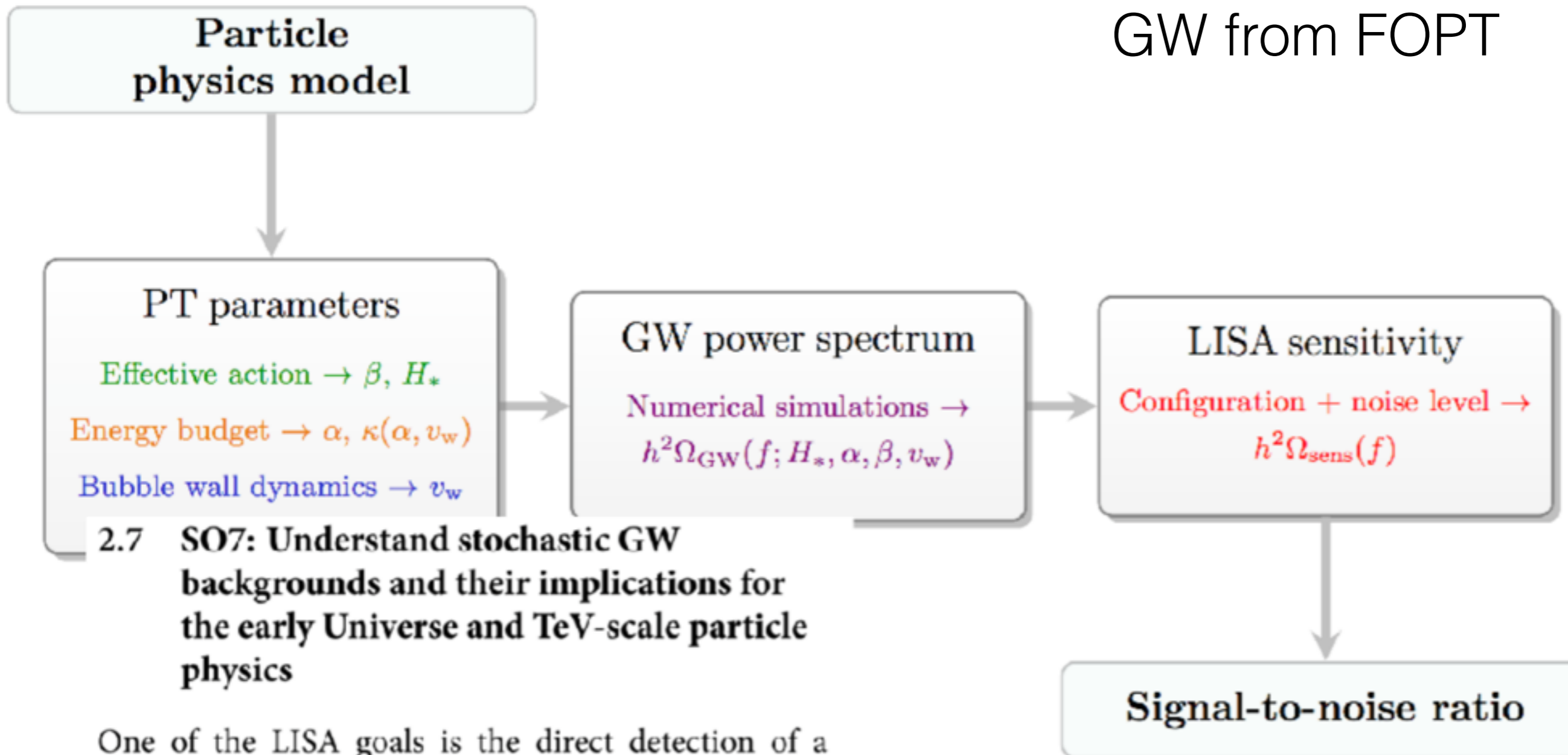
$$\alpha = \frac{\Delta\rho}{\rho_R} = \frac{1}{\rho_R} \left[-V(\vec{\phi}_b, T) + T \frac{\partial V(\vec{\phi}_b, T)}{\partial T} \right] \Big|_{T=T_n}$$

$$p(t) \simeq \Gamma_0 e^{-S(t_N) + \beta(t - t_N)}$$

$$\Gamma_0 \sim \alpha^5 W T_c^4 \quad S(t_N) \sim 4 \ln(m_P / T_N)$$

$$\beta = -d \ln p(t) / dt \Big|_{t_f}$$

$$\frac{\beta}{H_*} \simeq \frac{2S(t_N)}{(1 - T_N / T_c)}$$



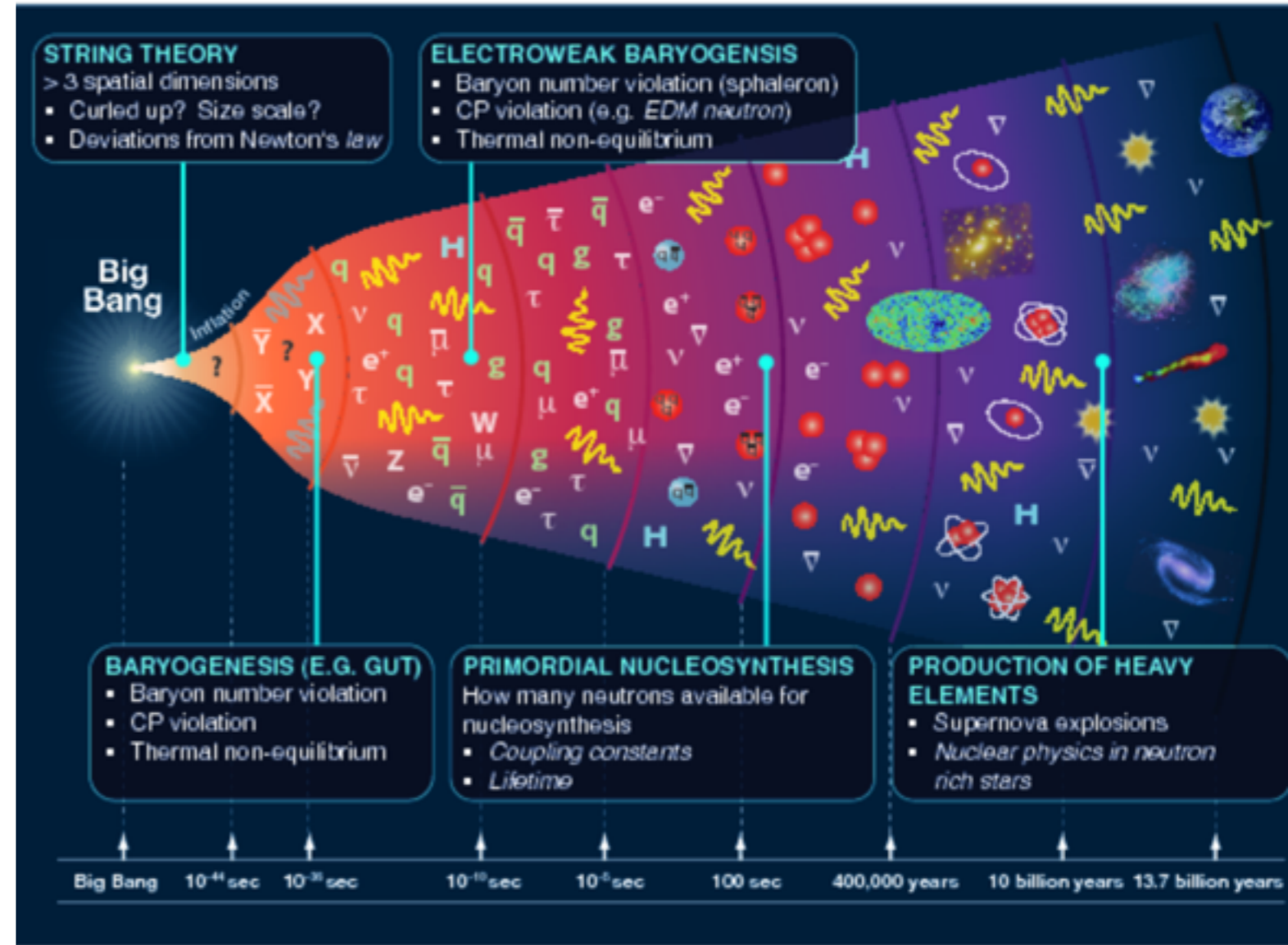
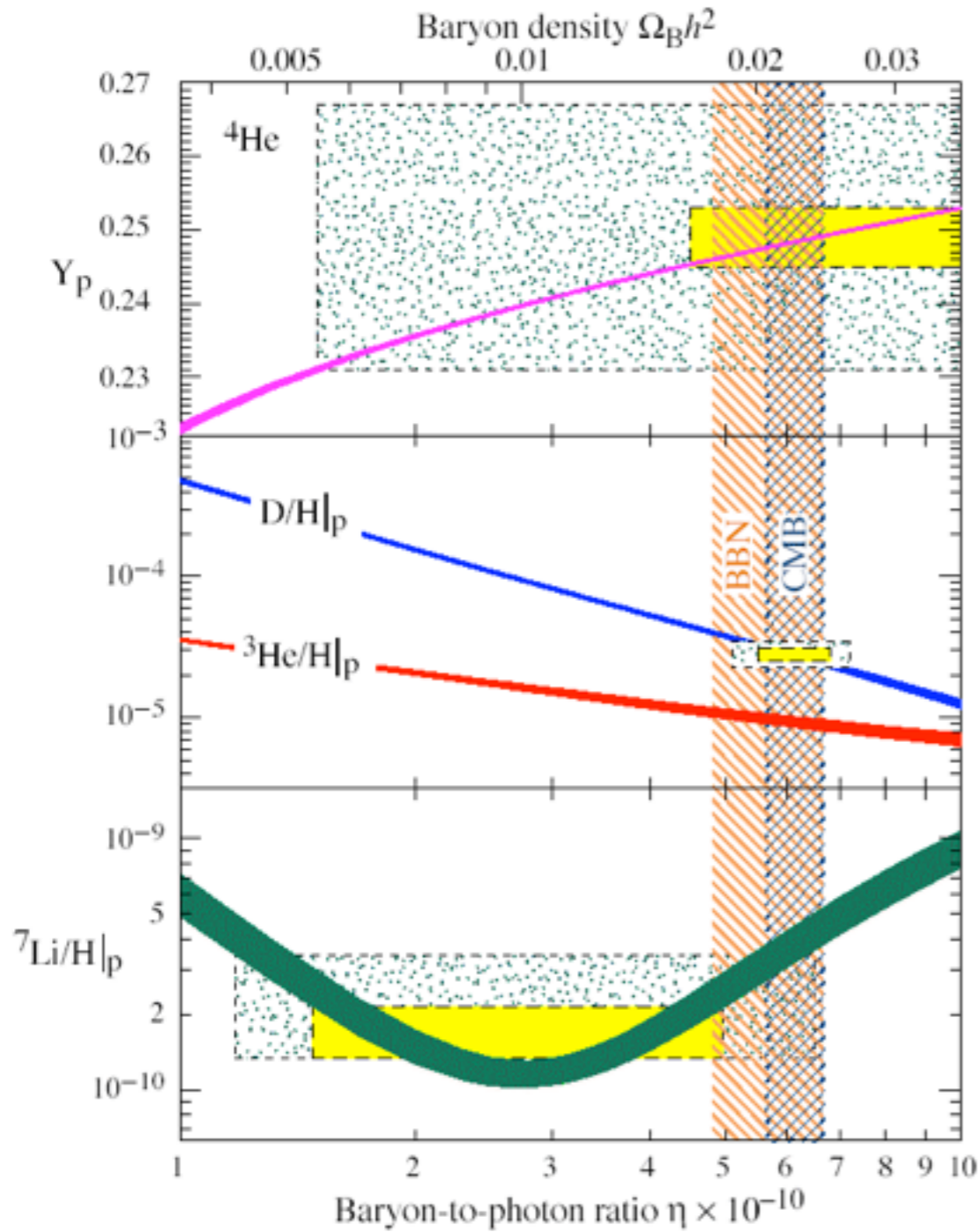
2.7 SO7: Understand stochastic GW backgrounds and their implications for the early Universe and TeV-scale particle physics

One of the LISA goals is the direct detection of a stochastic GW background of cosmological origin (like for example the one produced by a first-order phase transition around the TeV scale) and stochastic foregrounds. Probing a stochastic GW background of cosmological origin provides information on new physics in the early Universe. The shape of the signal gives an indication of its origin, while an upper limit allows to constrain models of the early Universe and particle physics beyond the standard model.

1702.00786

$$\text{SNR} = \sqrt{\mathcal{T} \int_{f_{\min}}^{f_{\max}} df \left[\frac{h^2 \Omega_{\text{GW}}(f)}{h^2 \Omega_{\text{Sens}}(f)} \right]^2}$$

BAU

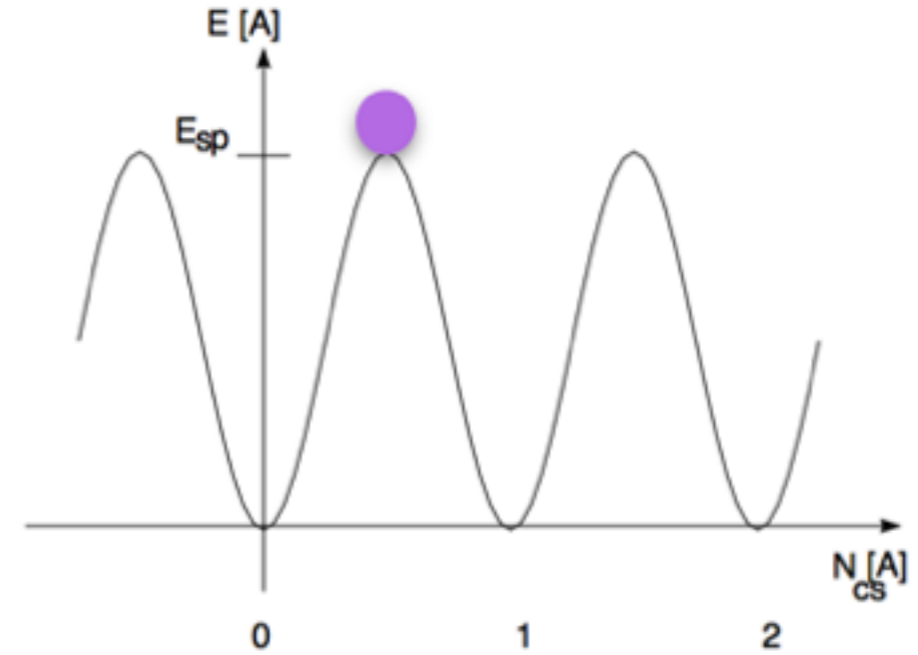
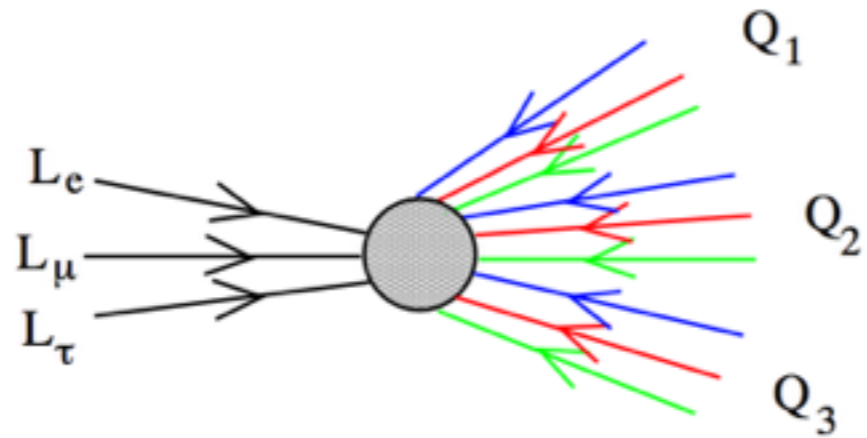


more baryons than anti-baryons (BBN & CMB, etc)

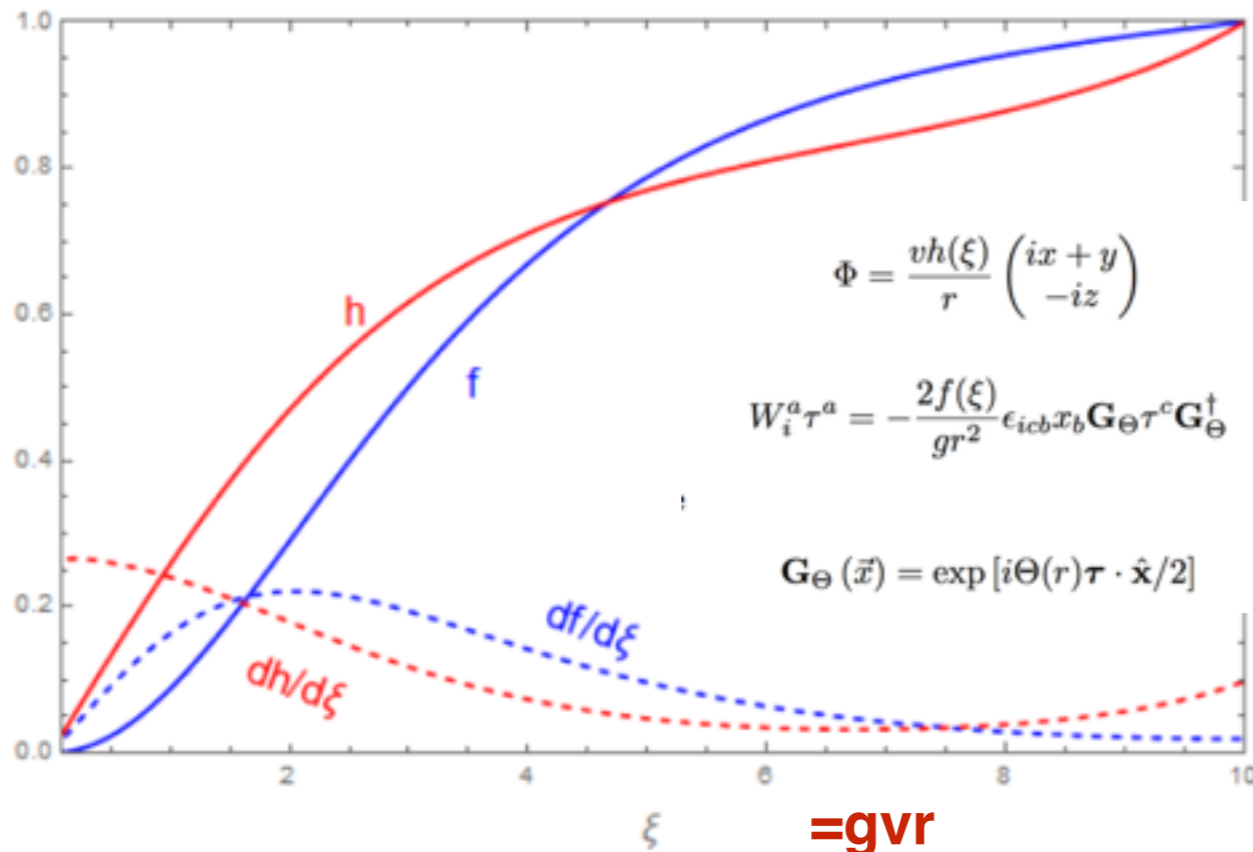
$$\frac{n_b}{s} \approx (0.7 - 0.9) \times 10^{-10} \neq 0$$

B violation and sphaleron

The Standard Model already contains a process that violates B-number. It is known as the electroweak sphaleron ("sphaleron" is Greek for "ready to fall")



Klinkhammer & Manton (1984); Kuzmin, Rubakov, & Shaposhnikov (1985); Harvey & Turner (1990), but also identified earlier by Dashen, Hasslacher, & Neveu (1974) and Boguta (1983)



$$\partial_\mu J_B^\mu = i \frac{N_F}{32\pi^2} \left(-g_2^2 F^{a\mu\nu} \tilde{F}_{\mu\nu}^a + g_1^2 f^{\mu\nu} \tilde{f}_{\mu\nu} \right),$$

$$\Delta B = N_F (\Delta N_{CS} - \Delta n_{CS}),$$

$$N_{CS} = -\frac{g_2^2}{16\pi^2} \int d^3x \, 2\epsilon^{ijk} \text{Tr} \left[\partial_i A_j A_k + i\frac{2}{3} g_2 A_i A_j A_k \right]$$

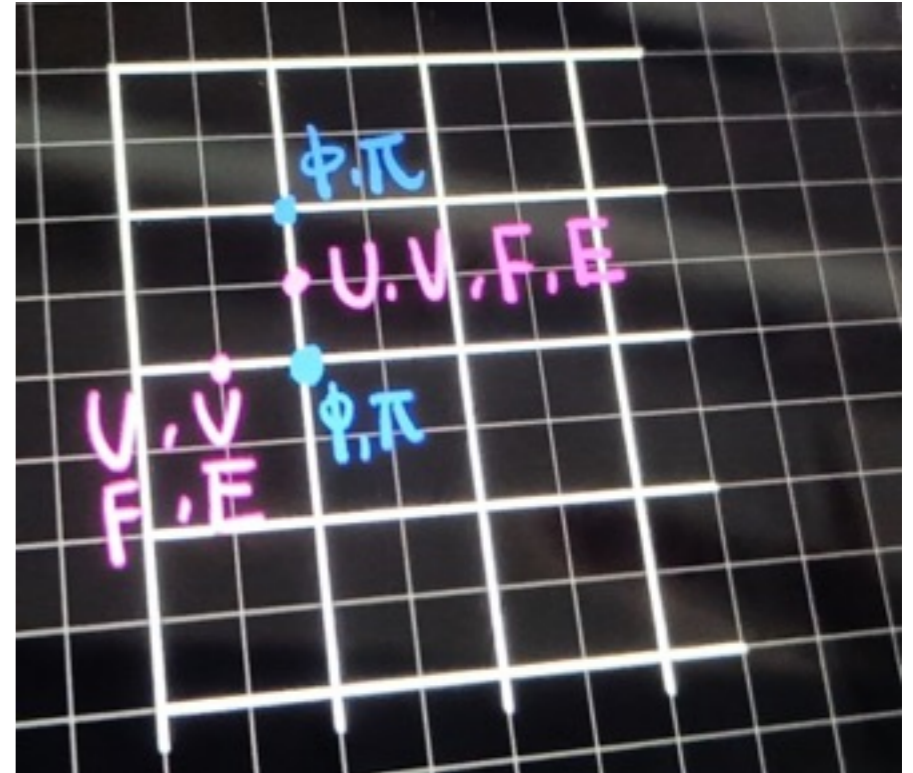
$$n_{CS} = -\frac{g_1^2}{16\pi^2} \int d^3x \, \epsilon^{ijk} \partial_i B_j B_k,$$

Lattice EW field foundation

$\Phi(t, \mathbf{x})$: Higgs field doublet defined on sites; $U_i(t, \mathbf{x})$ and $V_i(t, \mathbf{x})$: SU(2) and U(1) link fields, defined on the link between the neighboring sites \mathbf{x} and $\mathbf{x} + \mathbf{i}$, $\Phi(t, \mathbf{x})$, $U_i(t, \mathbf{x})$ and $V_i(t, \mathbf{x})$ are defined at time steps $t + \Delta t$, $t + 2\Delta t$, . . .; conjugate momentum fields, $\Pi(t + \Delta t/2, \mathbf{x})$, $F(t + \Delta t/2, \mathbf{x})$ and $E(t + \Delta t/2, \mathbf{x})$, are defined at time steps $t + \Delta t/2$, $t + 3\Delta t/2$.

leapfrog

$$\begin{aligned}
 U_i(t, \mathbf{x}) &= \exp\left(-\frac{i}{2}g\Delta x\sigma^a W_i^a\right) & D_i\Phi &= \frac{1}{\Delta x}[U_i(t, \mathbf{x})V_i(t, \mathbf{x})\Phi(t, \mathbf{x} + \mathbf{i}) - \Phi(t, \mathbf{x})] \\
 U_0(t, \mathbf{x}) &= \exp\left(-\frac{i}{2}g\Delta t\sigma^a W_0^a\right) & D_0\Phi &= \frac{1}{\Delta t}[U_0(t, \mathbf{x})V_0(t, \mathbf{x})\Phi(t + \Delta t, \mathbf{x}) - \Phi(t, \mathbf{x})]. \\
 V_i(t, \mathbf{x}) &= \exp\left(-\frac{i}{2}g\Delta x B_i\right) & \Phi(t + \Delta t, \mathbf{x}) &= \Phi(t, \mathbf{x}) + \Delta t\Pi(t + \Delta t/2, \mathbf{x}) \\
 V_0(t, \mathbf{x}) &= \exp\left(-\frac{i}{2}g\Delta t B_0\right). & V_i(t + \Delta t, \mathbf{x}) &= \frac{1}{2}g'\Delta x\Delta t E_i(t + \Delta t/2, \mathbf{x})V_i(t, \mathbf{x}) \\
 U_0(t, \mathbf{x}) &= I_2 \text{ and } V_0(t, \mathbf{x}) = 1 & U_i(t + \Delta t, \mathbf{x}) &= g\Delta x\Delta t F_i(t + \Delta t/2, \mathbf{x})U_i(t, \mathbf{x}),
 \end{aligned}$$



Temporal gauge

$$\begin{aligned}
 \Pi(t + \Delta t/2, \mathbf{x}) &= \Pi(t - \Delta t/2, \mathbf{x}) + \Delta t \left\{ \frac{1}{\Delta x^2} \sum_i [U_i(t, \mathbf{x})V_i(t, \mathbf{x})\Phi(t, \mathbf{x} + \mathbf{i}) \right. \\
 &\quad \left. - 2\Phi(t, \mathbf{x}) + U_i^\dagger(t, \mathbf{x} - \mathbf{i})V_i^\dagger(t, \mathbf{x} - \mathbf{i})\Phi(t, \mathbf{x} - \mathbf{i})] - \frac{\partial U}{\partial \Phi^\dagger} \right\}
 \end{aligned}$$

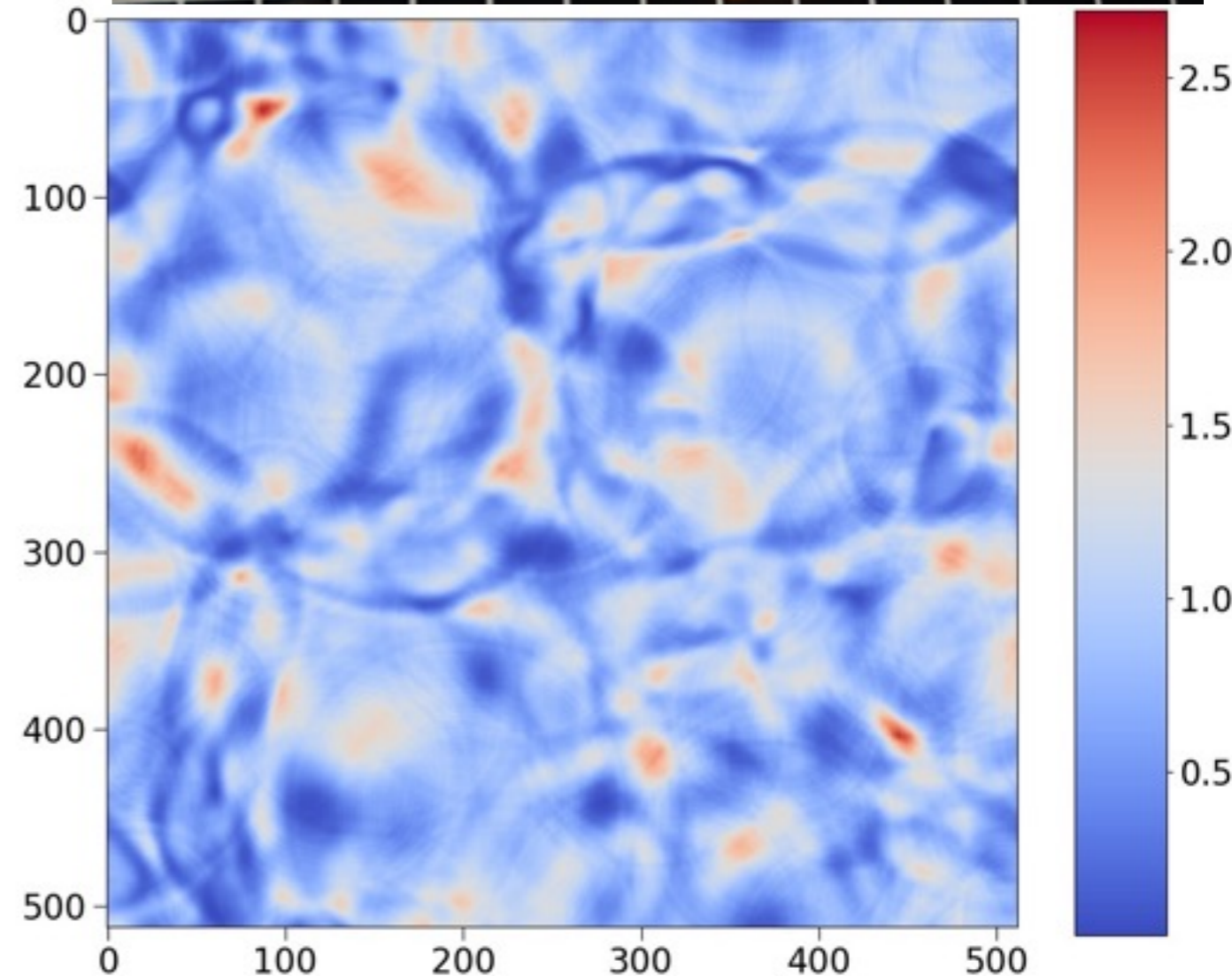
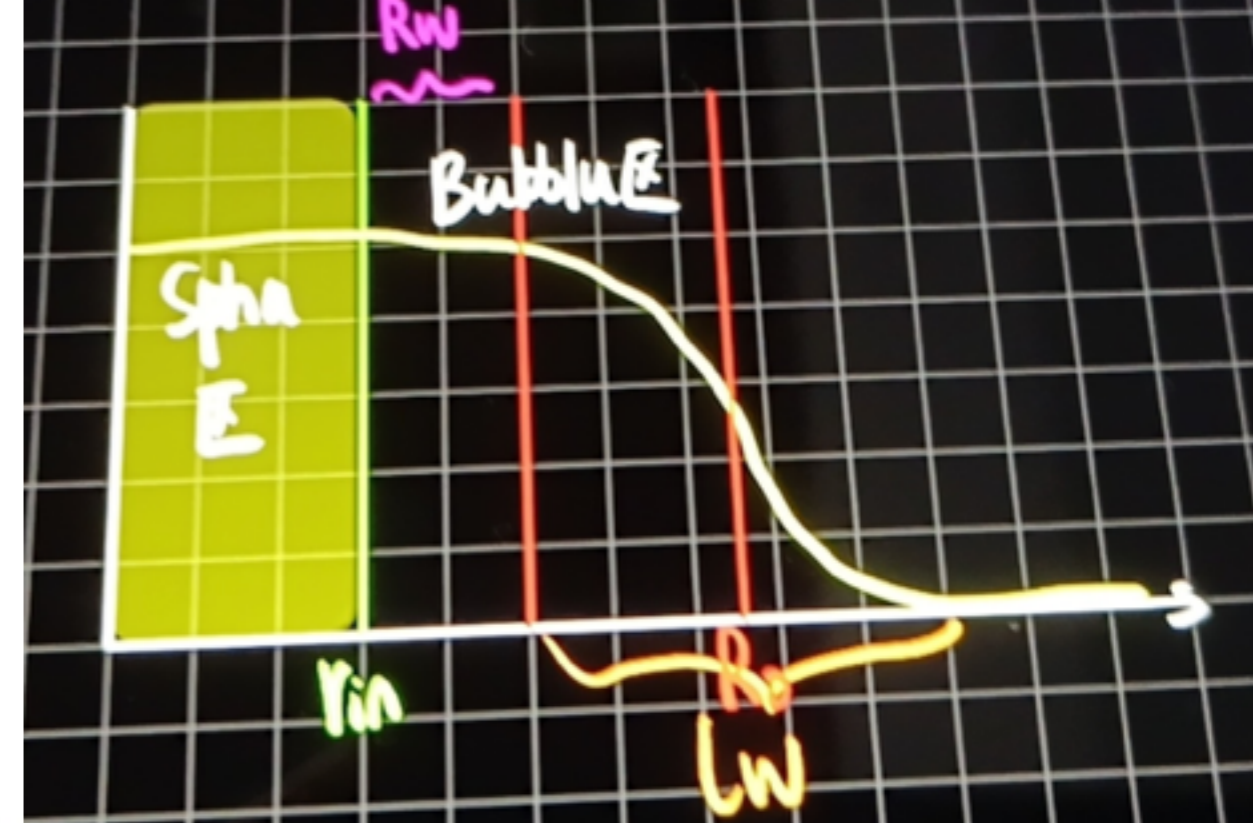
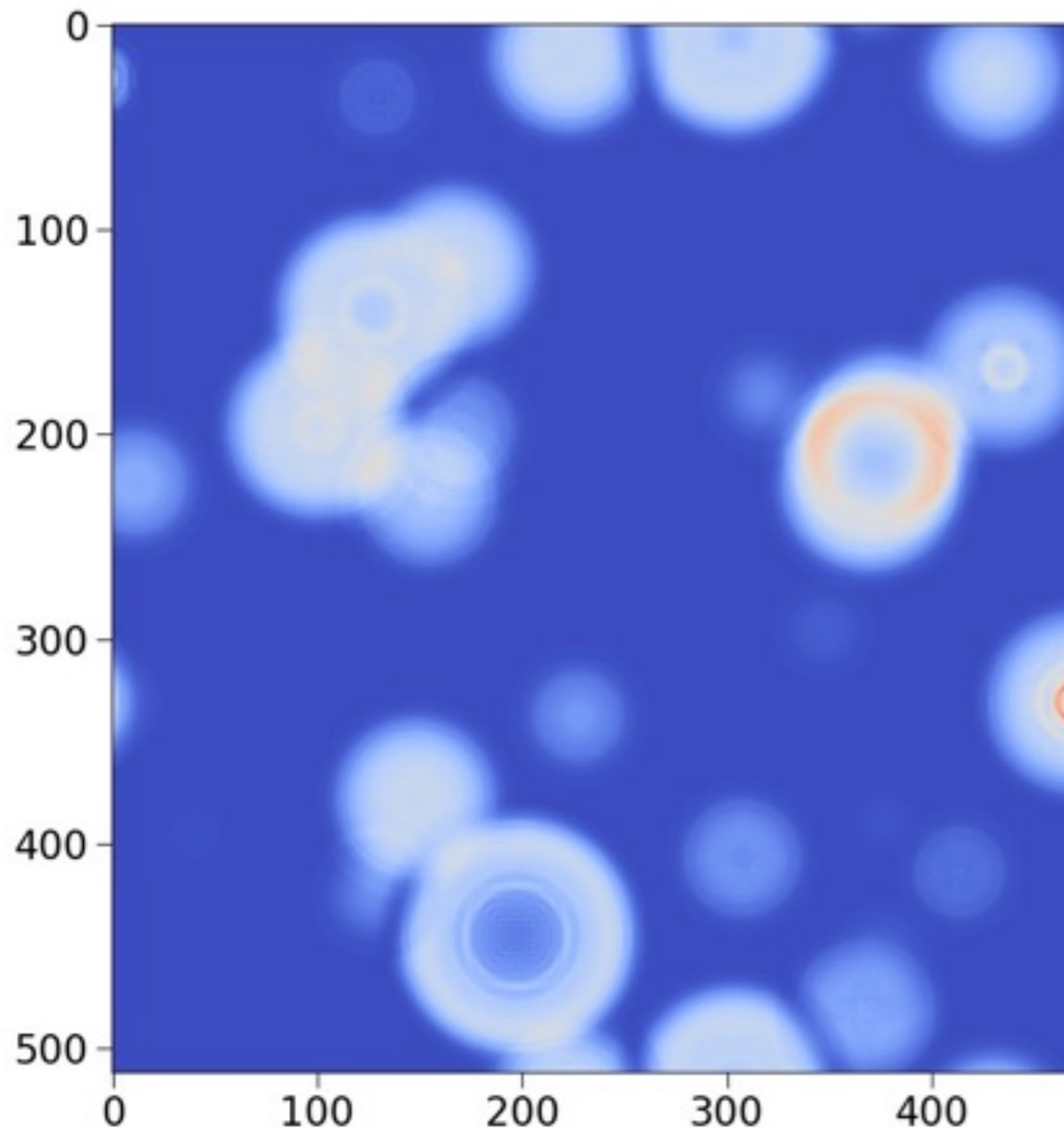
$$\begin{aligned}
 \text{Im}[E_k(t + \Delta t/2, \mathbf{x})] &= \text{Im}[E_k(t - \Delta t/2, \mathbf{x})] + \Delta t \left\{ \frac{g'}{\Delta x} \text{Im}[\Phi^\dagger(t, \mathbf{x} + \mathbf{k})U_k^\dagger(t, \mathbf{x})V_k^\dagger(t, \mathbf{x})\Phi(t, \mathbf{x})] \right. \\
 &\quad \left. - \frac{2}{g'\Delta x^3} \sum_i \text{Im}[V_k(t, \mathbf{x})V_i(t, \mathbf{x} + \mathbf{k})V_k^\dagger(t, \mathbf{x} + \mathbf{i})V_i^\dagger(t, \mathbf{x}) \right. \\
 &\quad \left. + V_i(t, \mathbf{x} - \mathbf{i})V_k(t, \mathbf{x})V_i^\dagger(t, \mathbf{x} + \mathbf{k} - \mathbf{i})V_k^\dagger(t, \mathbf{x} - \mathbf{i})] \right\}
 \end{aligned}$$

$$\begin{aligned}
 \text{Tr}[i\sigma^m F_k(t + \Delta t/2, \mathbf{x})] &= \text{Tr}[i\sigma^m F_k(t - \Delta t/2, \mathbf{x})] + \Delta t \left\{ \frac{g}{\Delta x} \text{Re}[\Phi^\dagger(t, \mathbf{x} + \mathbf{k})U_k^\dagger(t, \mathbf{x})V_k^\dagger(t, \mathbf{x})i\sigma^m \Phi(t, \mathbf{x})] \right. \\
 &\quad \left. - \frac{1}{g\Delta x^3} \sum_i \text{Tr}[i\sigma^m U_k(t, \mathbf{x})U_i(t, \mathbf{x} + \mathbf{k})U_k^\dagger(t, \mathbf{x} + \mathbf{i})U_i^\dagger(t, \mathbf{x}) \right. \\
 &\quad \left. + i\sigma^m U_k(t, \mathbf{x})U_i^\dagger(t, \mathbf{x} + \mathbf{k} - \mathbf{i})U_k^\dagger(t, \mathbf{x} - \mathbf{i})U_i(t, \mathbf{x} - \mathbf{i})] \right\},
 \end{aligned}$$

Bubble with sphaleron

$$\begin{aligned}\partial_0^2 \Phi &= D_i D_i \Phi - 2\lambda(|\Phi|^2 - \eta^2)\Phi - 3(\Phi^\dagger \Phi)^2 \Phi / \Lambda^2, \\ \partial_0^2 B_i &= -\partial_j B_{ij} + g' \text{Im}[\Phi^\dagger D_i \Phi], \\ \partial_0^2 W_i^a &= -\partial_k W_{ik}^a - g \epsilon^{abc} W_k^b W_{ik}^c + g \text{Im}[\Phi^\dagger \sigma^a D_i \Phi].\end{aligned}$$

$$\begin{aligned}\partial_0 \partial_j B_j - g' \text{Im}[\Phi^\dagger \partial_0 \Phi] &= 0, \\ \partial_0 \partial_j W_j^a + g \epsilon^{abc} W_j^b \partial_0 W_j^c - g \text{Im}[\Phi^\dagger \sigma^a \partial_0 \Phi] &= 0.\end{aligned}$$



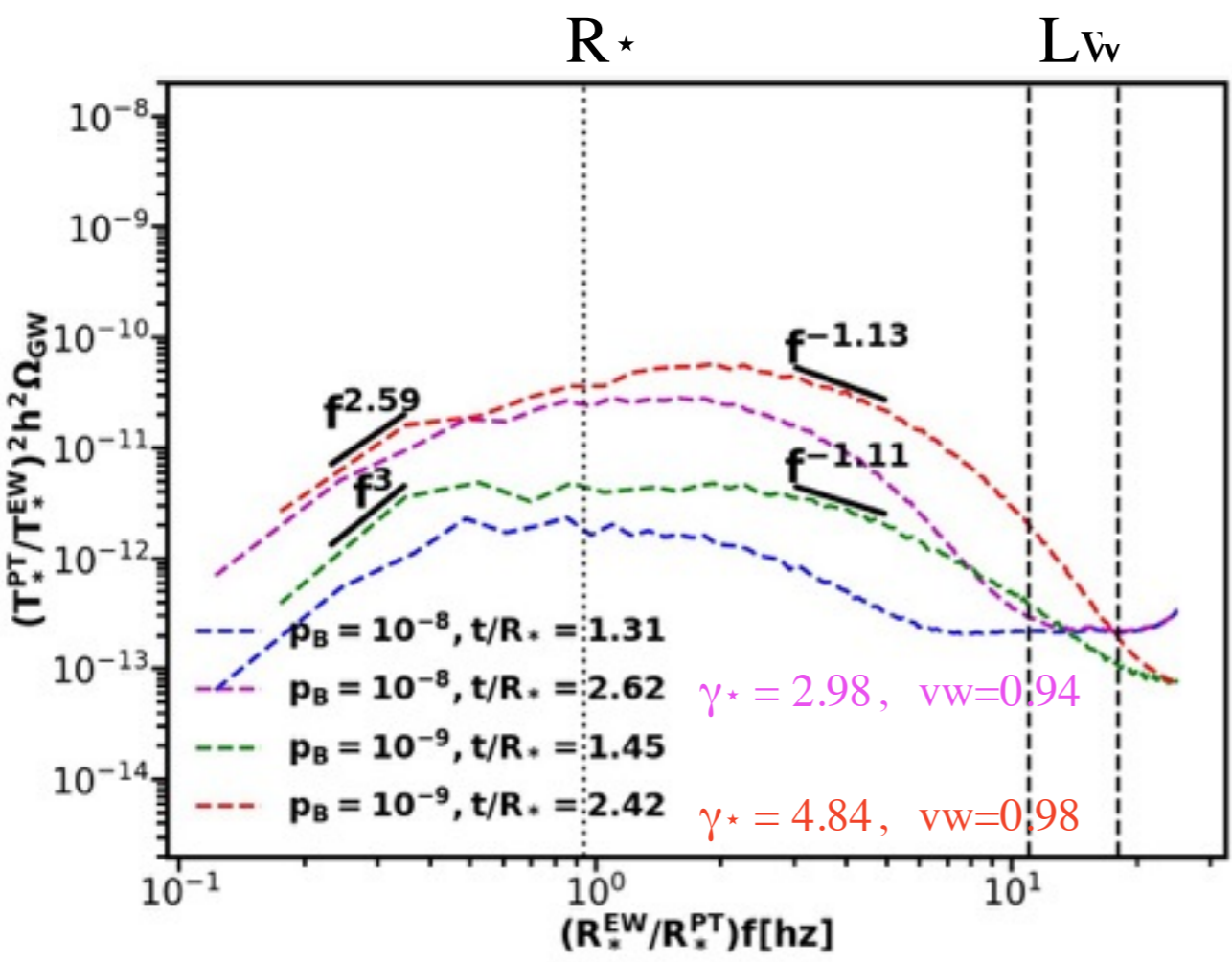
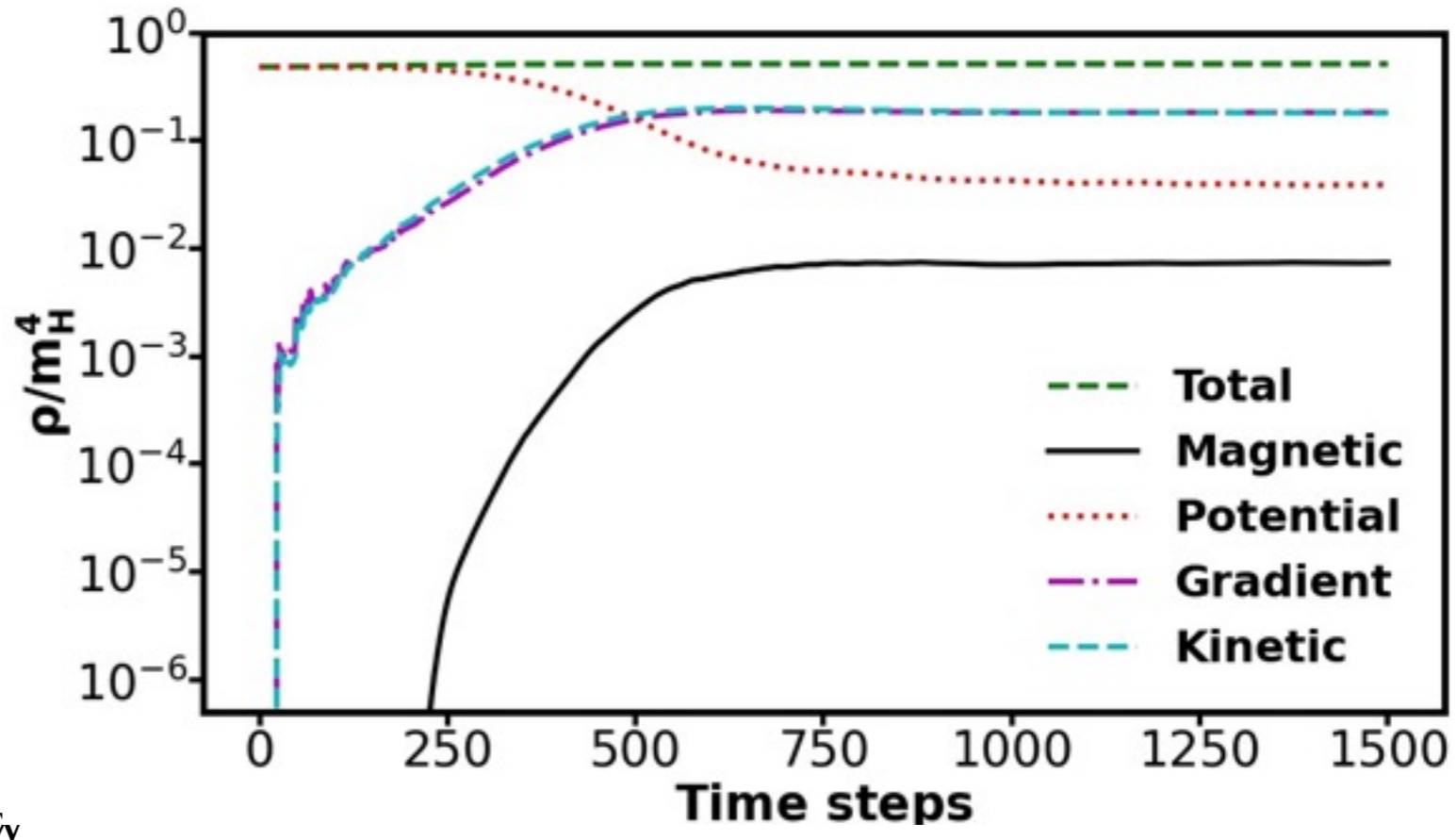
GW from Bubble collisions

$$T_{\mu\nu} = \partial_\mu \Phi^\dagger \partial_\nu \Phi - g_{\mu\nu} \frac{1}{2} \text{Re}[(\partial_i \Phi^\dagger \partial^i \Phi)^2]$$

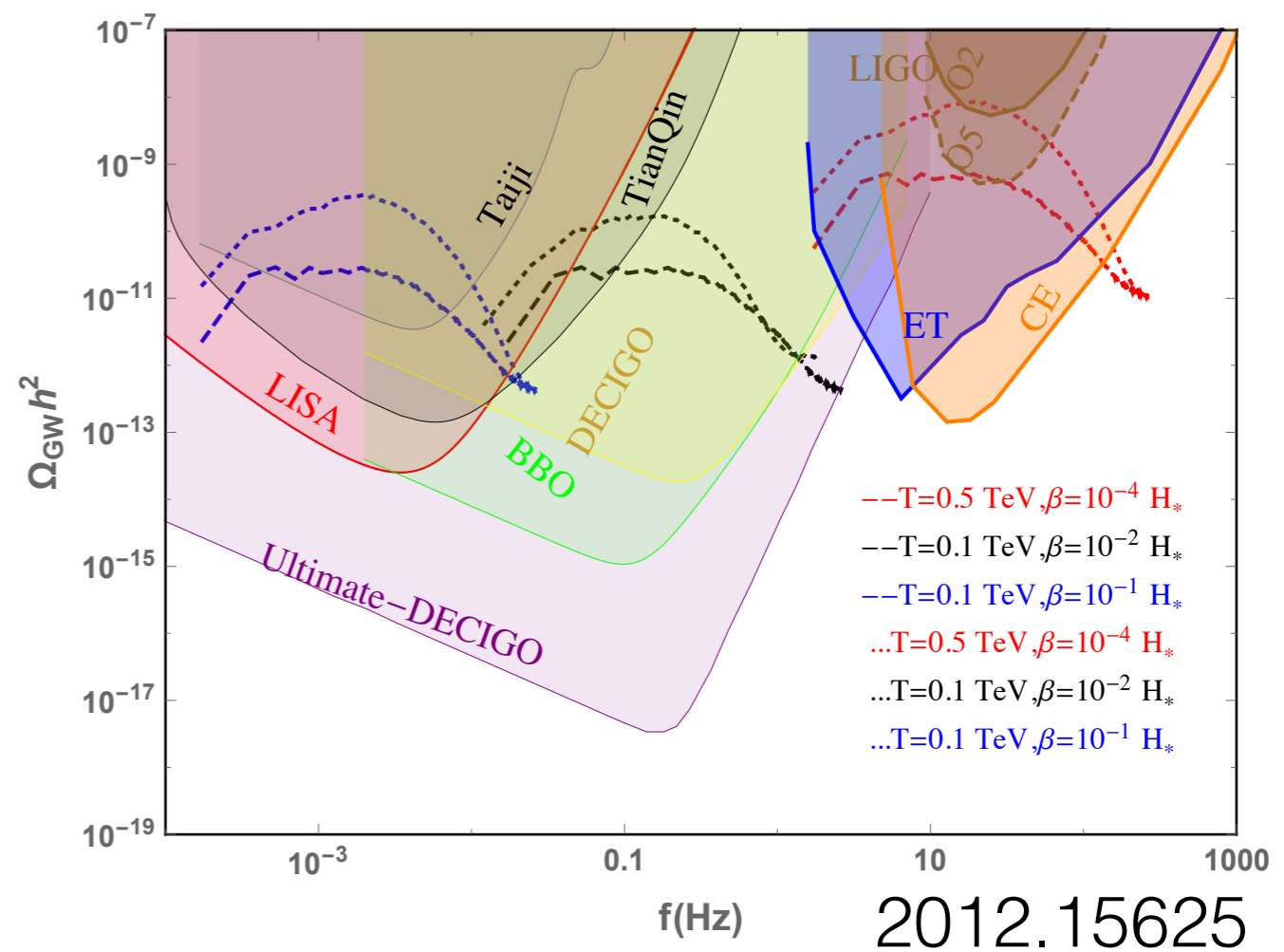
$$\ddot{h}_{ij} - \nabla^2 h_{ij} = 16\pi G T_{ij}^{TT}$$

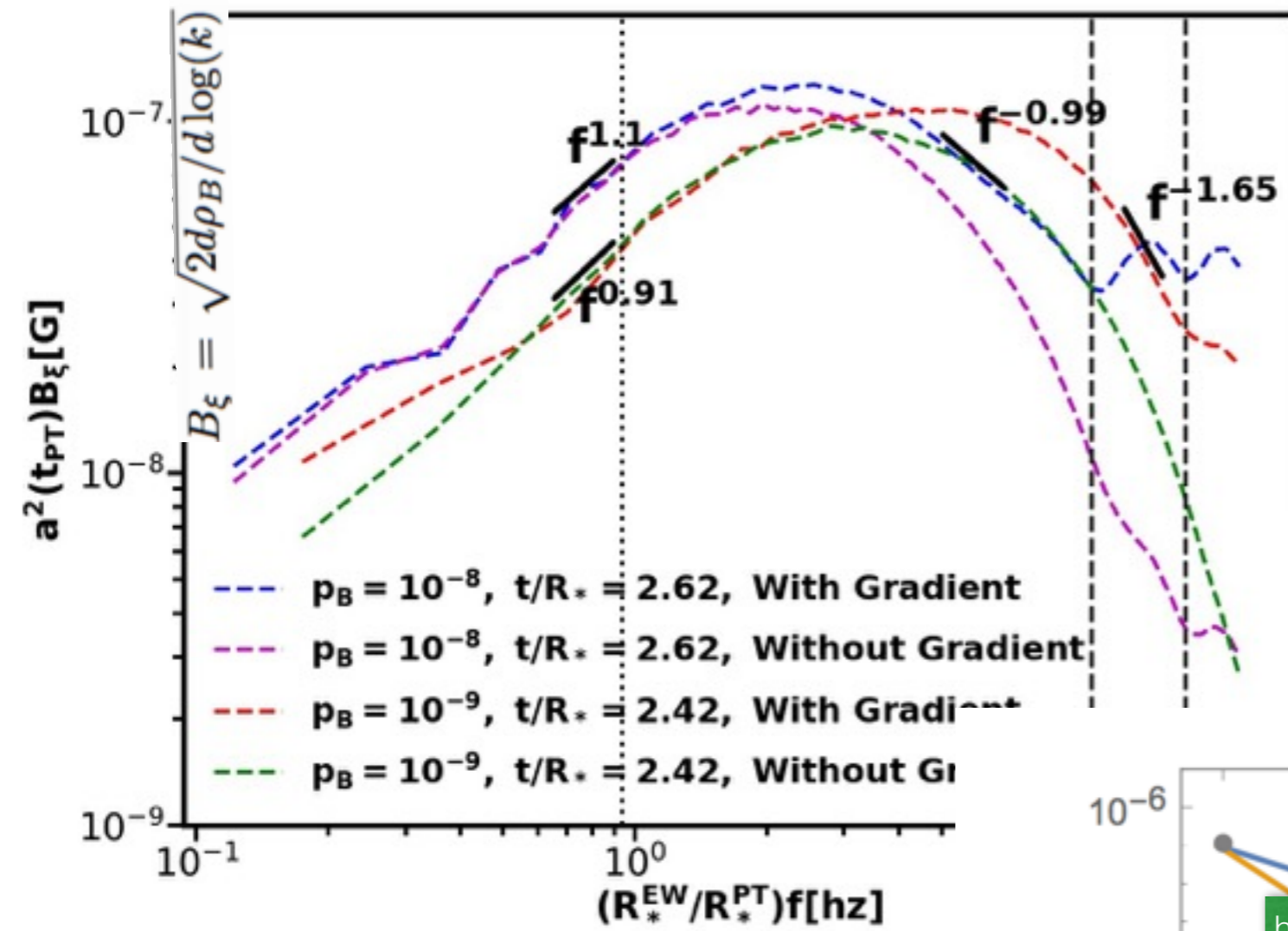
$$\langle \dot{h}_{ij}^{TT}(\mathbf{k}, t) \dot{h}_{ij}^{TT}(\mathbf{k}', t) \rangle = P_h(\mathbf{k}, t) (2\pi)^3 \delta(\mathbf{k} + \mathbf{k}')$$

$$\frac{d\Omega_{\text{gw}}}{d\ln(k)} = \frac{1}{32\pi G \rho_c} \frac{k^3}{2\pi^2} P_h(\mathbf{k}, t)$$



$$\gamma_* = R_* / (2R_c) \quad R_* = \left(\frac{\nu}{N_b} \right)^{1/3}$$





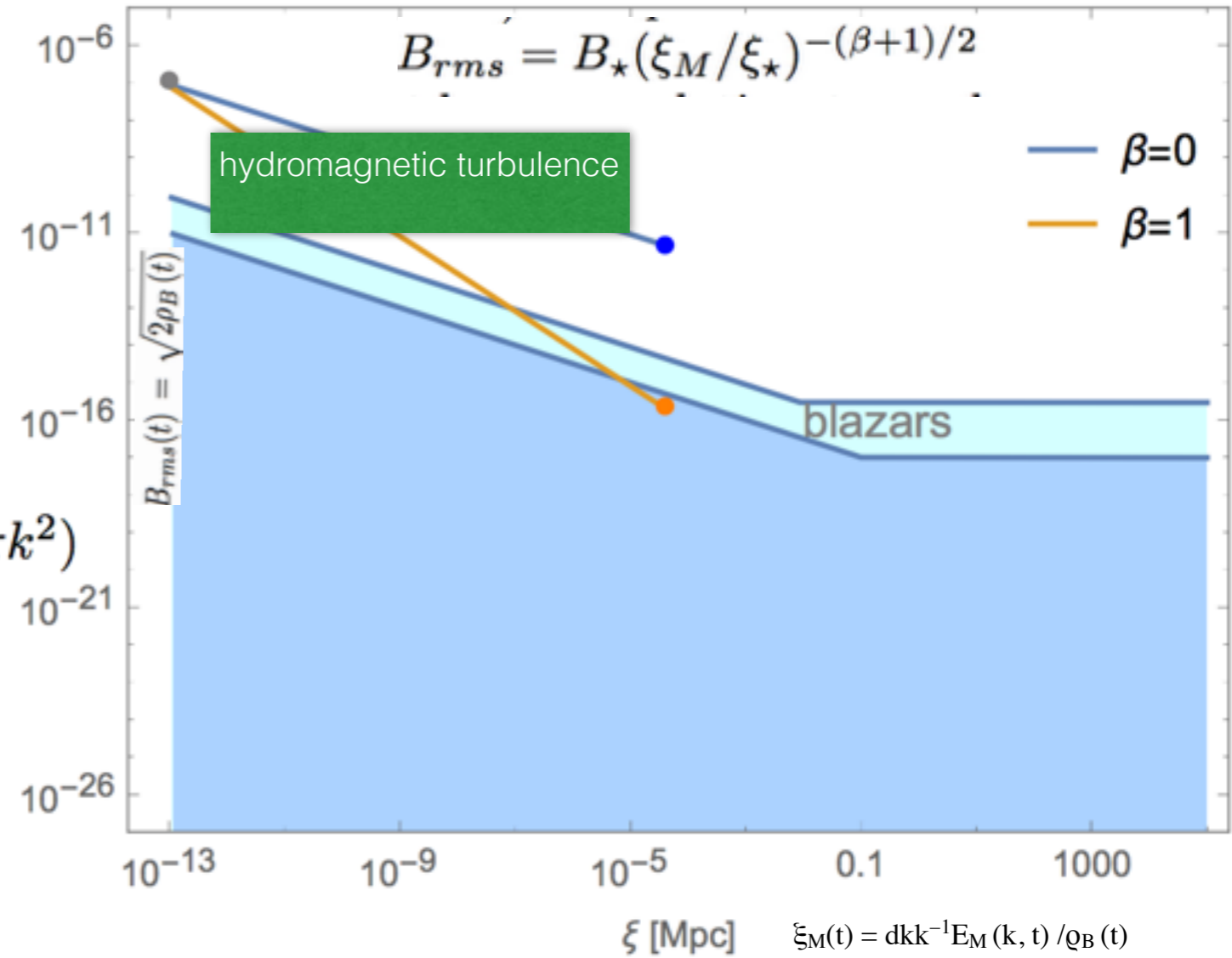
Non-vanishing gradients of the Higgs fields can generate magnetic fields when the bubbles collide.

$$A_{\mu\nu} = \sin \theta_w n^a W_{\mu\nu}^a + \cos \theta_w B_{\mu\nu} - i \frac{2}{g v^2} \sin \theta_w [(D_\mu \Phi)^\dagger (D_\nu \Phi) - (D_\nu \Phi)^\dagger (D_\mu \Phi)]$$

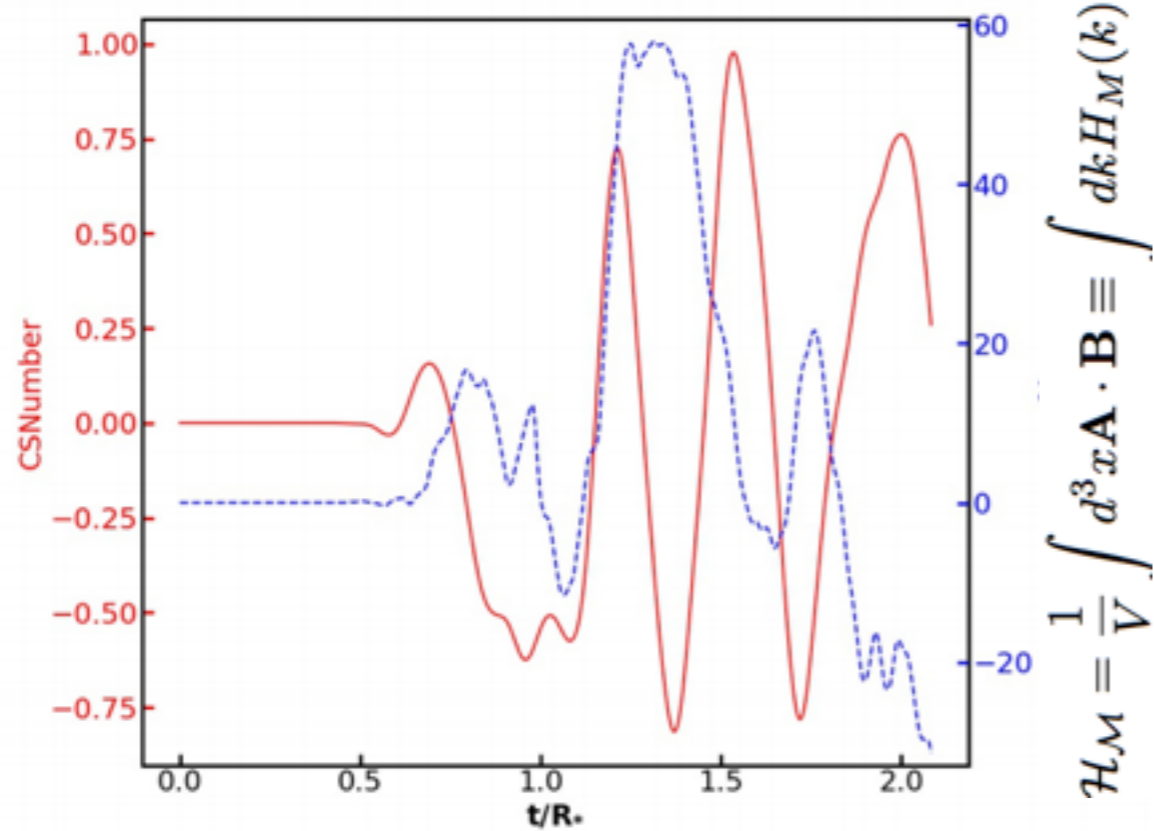
$$\langle B_i^*(\mathbf{k}, t) B_j(\mathbf{k}', t) \rangle = (2\pi)^3 \delta^{(3)}(\mathbf{k} - \mathbf{k}') F_{ij}(\mathbf{k}, t)$$

$$(\delta_{ij} - \hat{k}_i \hat{k}_j) E_M(k, t) / (4\pi k^2) + i \epsilon_{ijl} k_l H_M / (8\pi k^2)$$

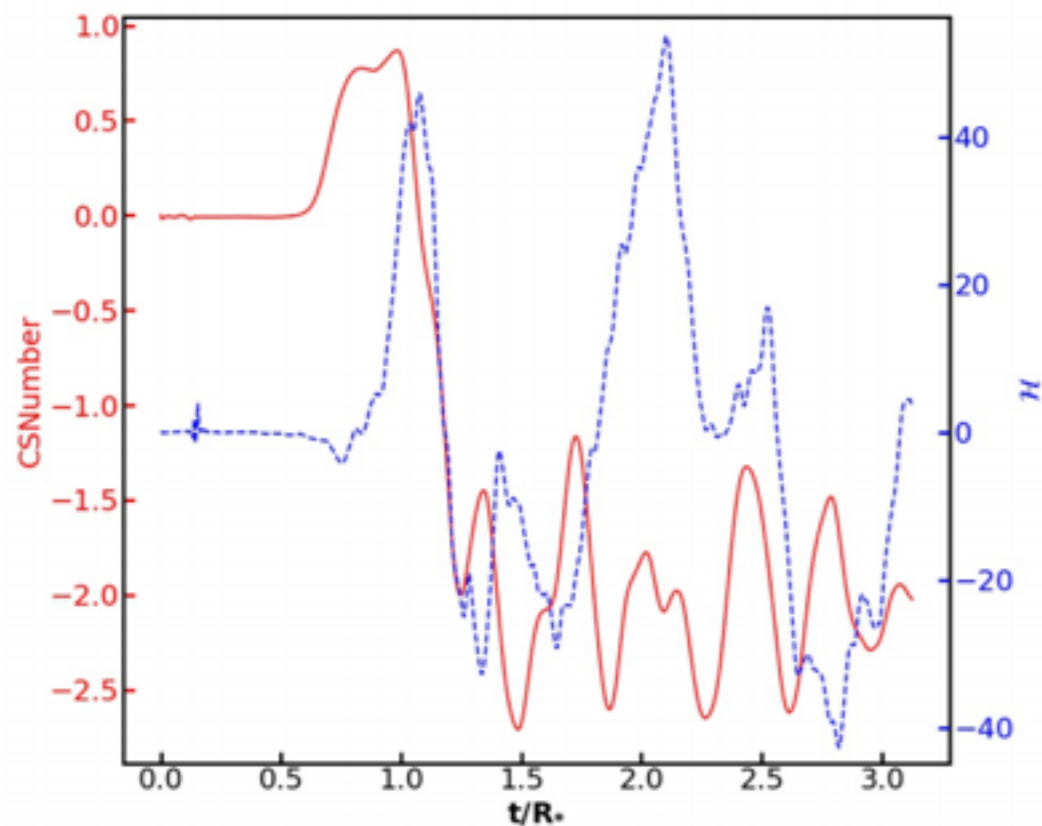
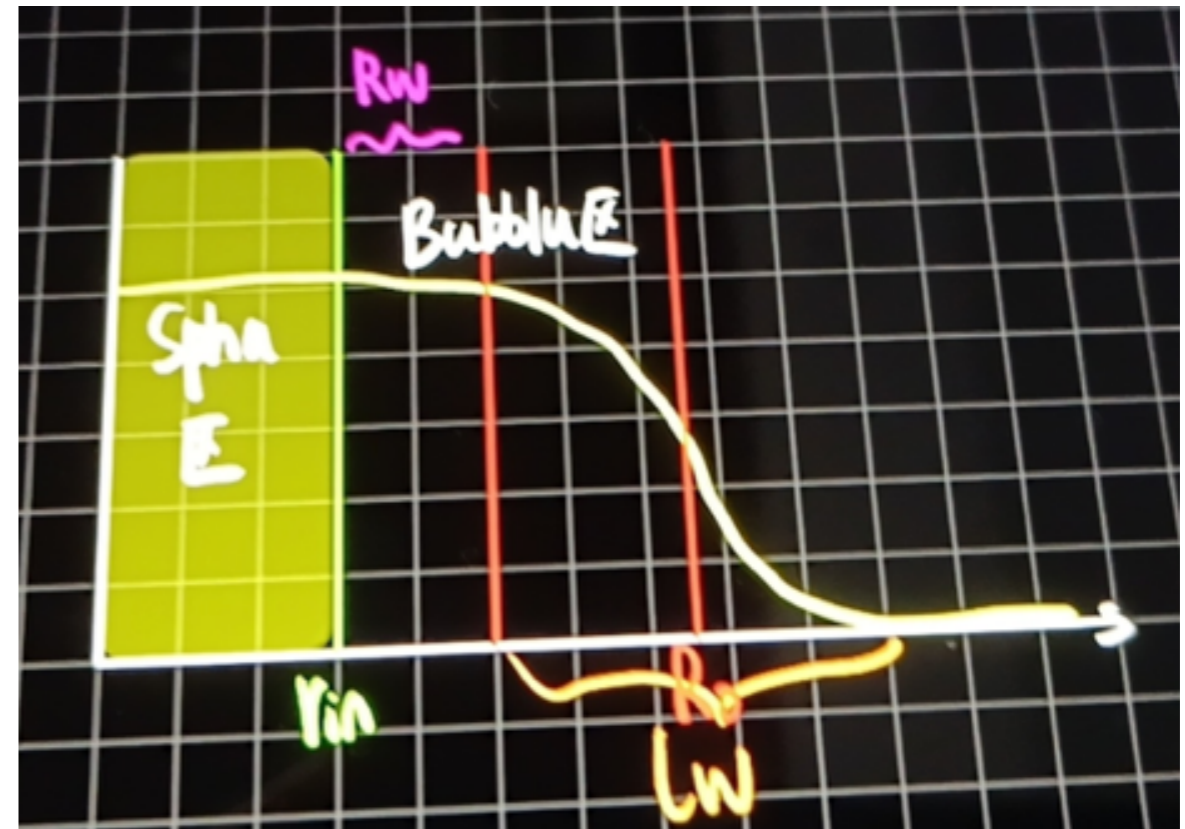
$$\rho_B(t) = \int_0^\infty E_M(k, t) dk$$



CS number and the magnetic helicity



Without sphaleron, the CS number oscillates around zero throughout the PT process

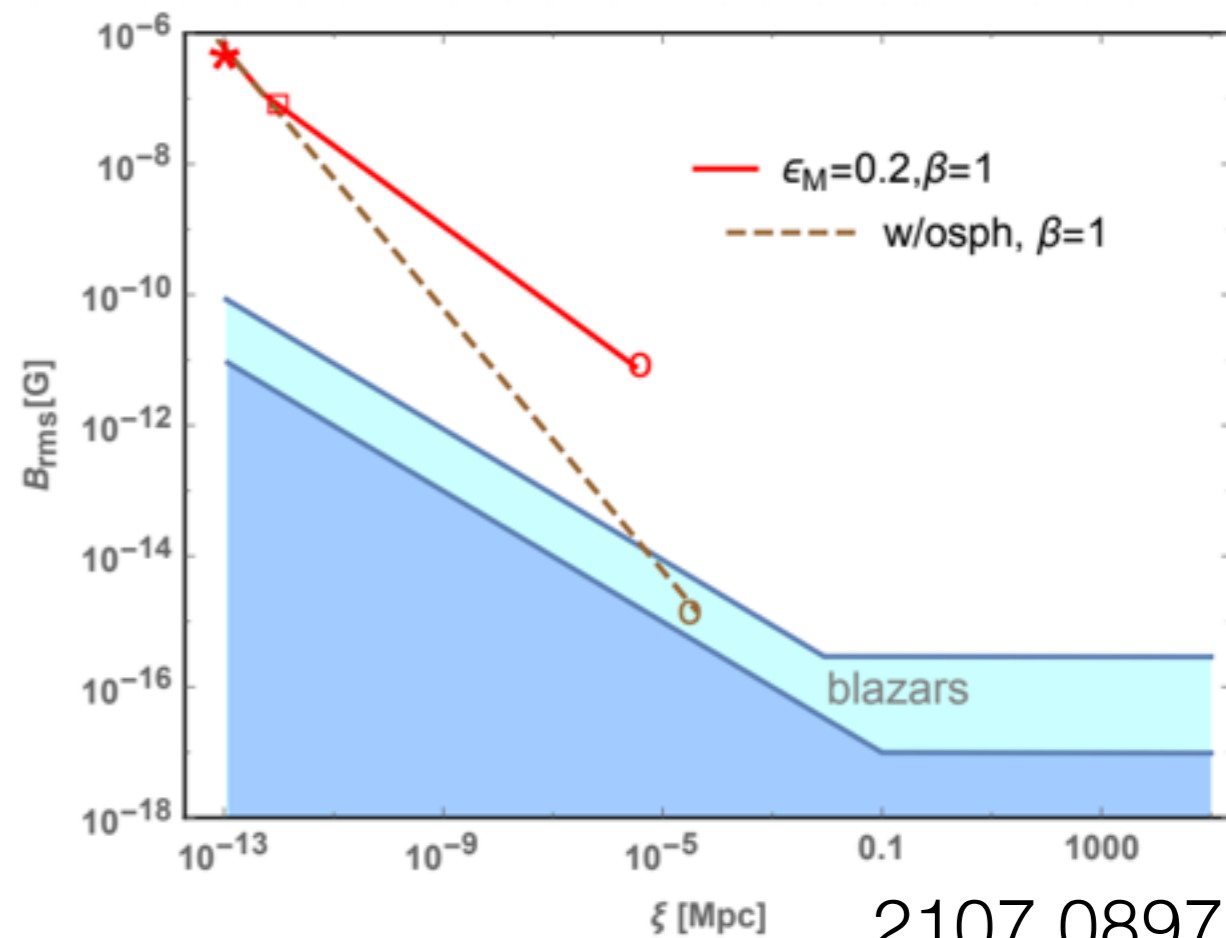
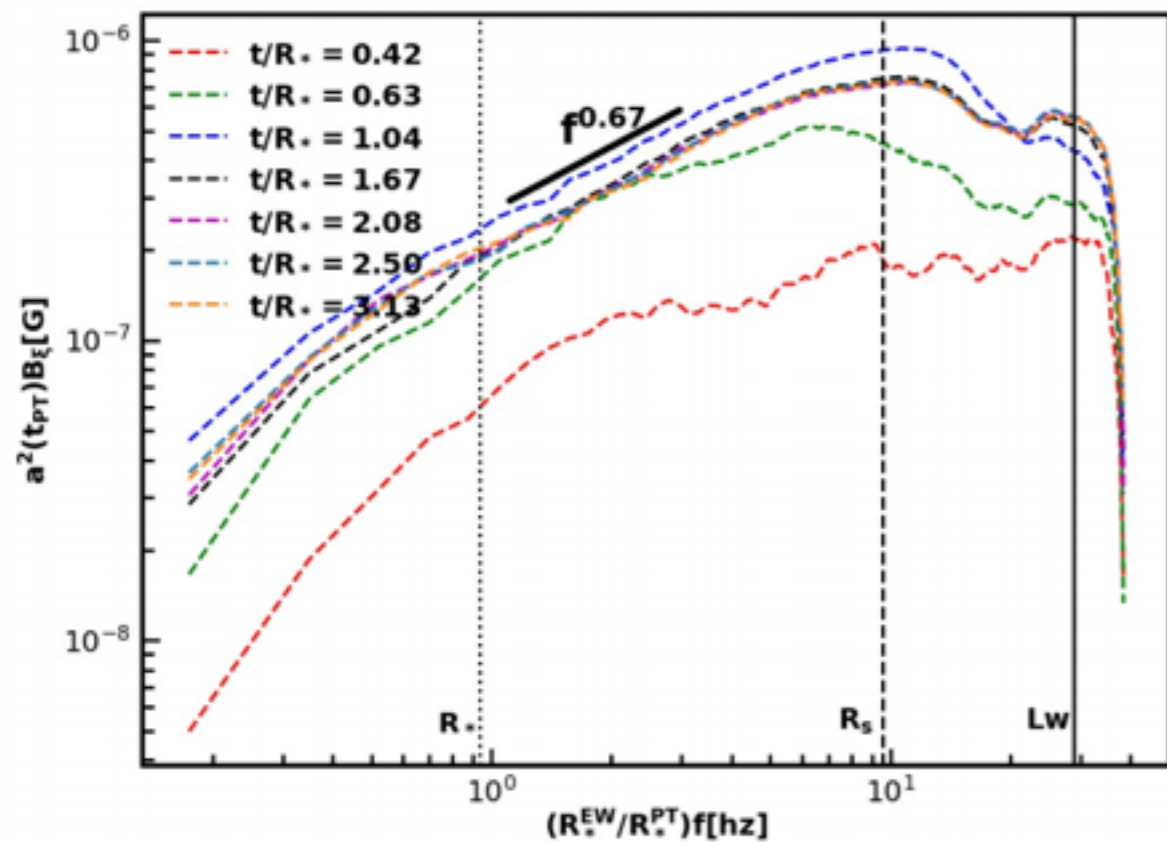
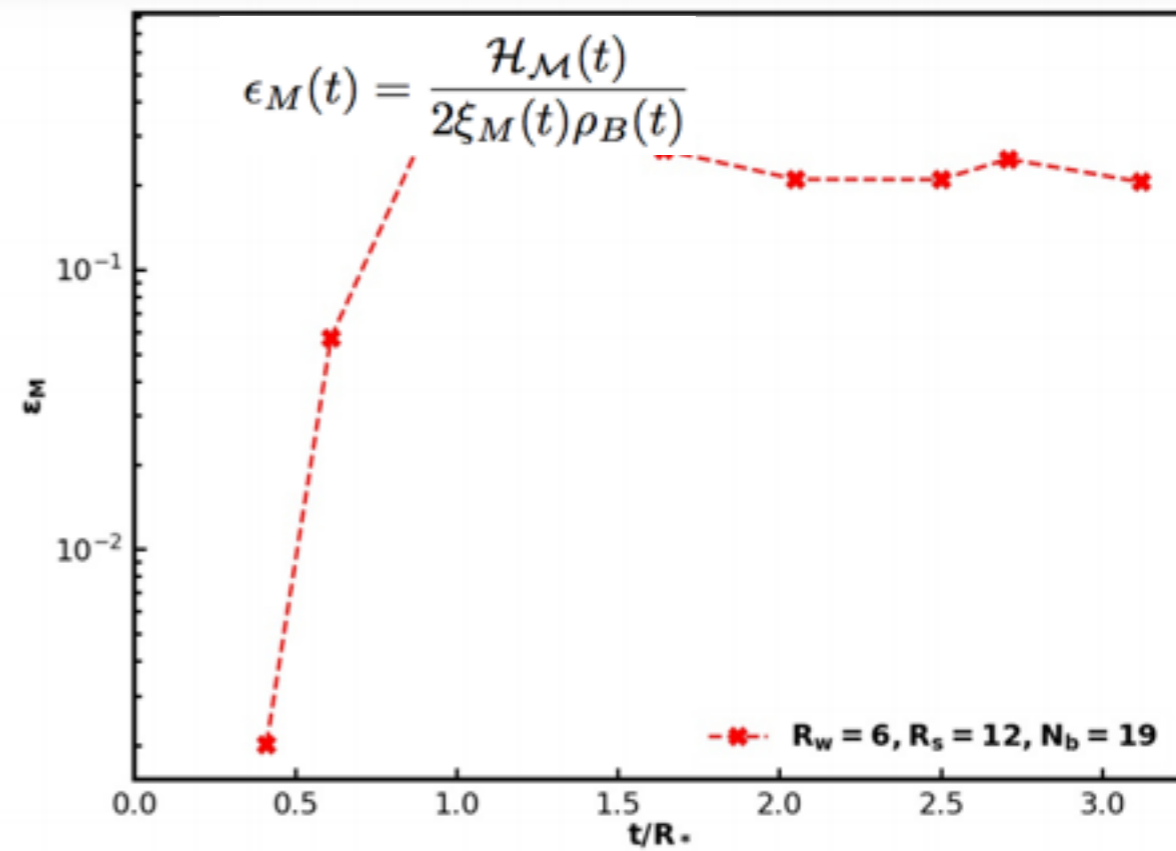
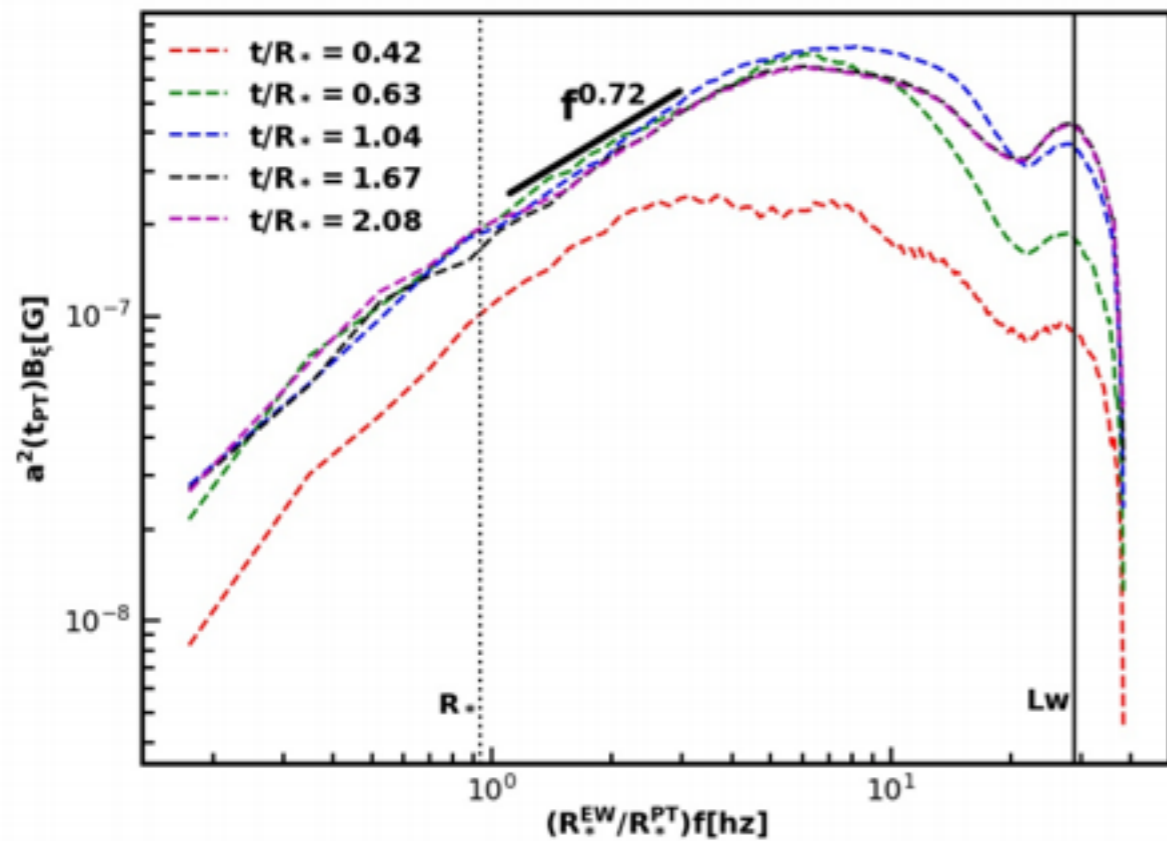


B + L anomaly equation: $\Delta N_B = 3 \Delta N_{CS} \sim 3$;

Magnetic helicity and NCS (and NB):

$|H| \sim 18 \Delta N_{CS} (\sim 6 \Delta N_B)$.

MF versus Sphaleron



Summary and future

Observation of the cosmic Magnetic field seeded by phase transition, with GW production may hint the B+L violation

Bubble dynamics

Interaction between the bubble wall and Plasma, and interaction among different bubbles

- 1) Magnetic field feedback to the phase transition
- 2) Baryogenesis and/or at fast-wall request by the GW

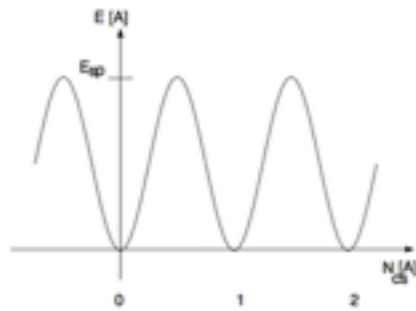
Higgs Potential shape

- 1) The future collider prospect, with diHiggs, Zh and/or Zhh production
- 2) Thin wall or thick wall tell by gravitational wave, wall profile and GW spectrum

谢谢！

Backup

BNPC, v/T and EW sphaleron



$$\partial_\mu J_B^\mu = i \frac{N_F}{32\pi^2} \left(-g_2^2 F^{a\mu\nu} \tilde{F}_{\mu\nu}^a + g_1^2 f^{\mu\nu} \tilde{f}_{\mu\nu} \right),$$

$$\Delta B = N_F (\Delta N_{CS} - \Delta n_{CS}),$$

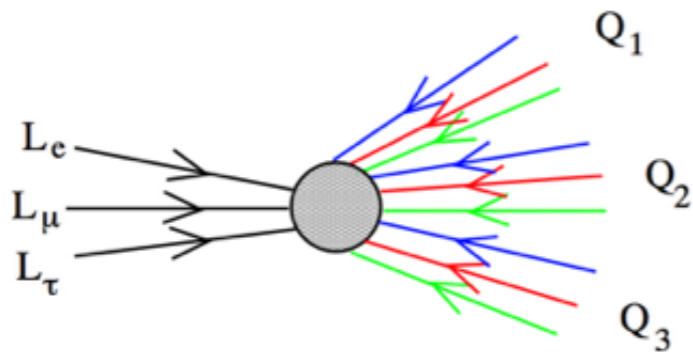
$$N_{CS} = -\frac{g_2^2}{16\pi^2} \int d^3x \, 2\epsilon^{ijk} \text{Tr} \left[\partial_i A_j A_k + i \frac{2}{3} g_2 A_i A_j A_k \right],$$

$$n_{CS} = -\frac{g_1^2}{16\pi^2} \int d^3x \, \epsilon^{ijk} \partial_i B_j B_k,$$

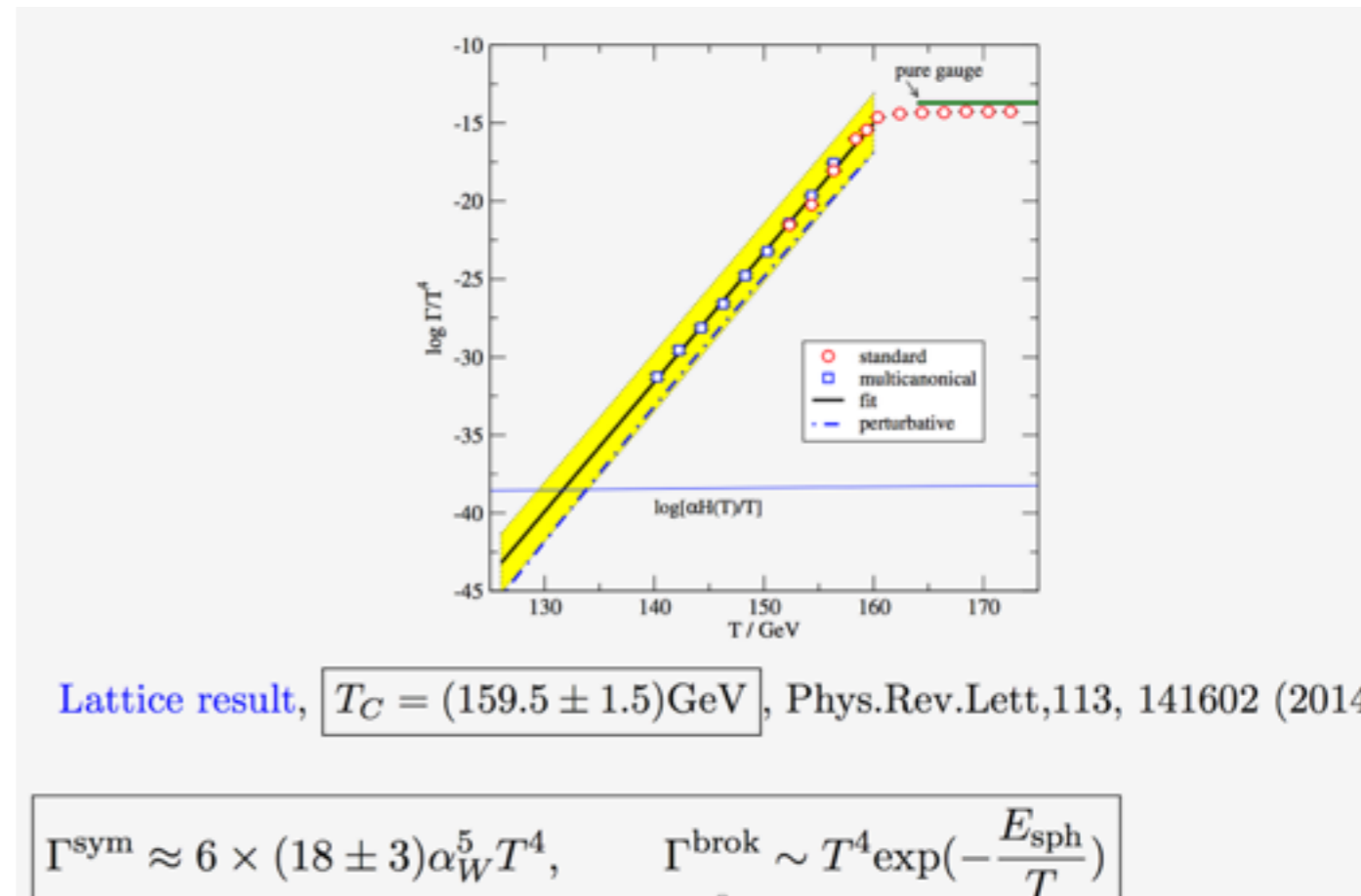
$$A_i \rightarrow U A_i U^{-1} + \frac{i}{g_2} (\partial_i U) U^{-1},$$

$$\delta N_{CS} = \frac{1}{24\pi^2} \int d^3x \, \text{Tr} \left[(\partial_i U) U^{-1} (\partial_j U) U^{-1} (\partial_k U) U^{-1} \right] \epsilon^{ijk}.$$

The Standard Model already contains a process that violates B-number. It is known as the electroweak sphaleron ("sphaleron" is Greek for "ready to fall").



Klinkhammer & Manton (1984); Kuzmin, Rubakov, & Shaposhnikov (1985); Harvey & Turner (1990) but also identified earlier by Dashen, Hasslacher, & Neveu (1974) and Boguta (1983)



Washout avoidance, BNPC

$$\Gamma_{\text{sph}} = A_{\text{sph}}(T) \exp[-E_{\text{sph}}(T)/T] < H(T)$$

$$PT_{\text{sph}} \equiv \frac{E_{\text{sph}}(T)}{T} - 7 \ln \frac{v(T)}{T} + \ln \frac{T}{100 \text{ GeV}} \quad PT_{\text{sph}} > (35.9 - 42.8)$$

$$E_{\text{sph}}(T) \approx E_{\text{sph},0} \frac{v(T)}{v}$$

$$\frac{v(T)}{T} > (0.973 - 1.16) \left(\frac{E_{\text{sph},0}}{1.916 \times 4\pi v/g} \right)^{-1}$$

Sphaleron details

SM, one higgs

$$E_{\text{sph},T} = \frac{4\pi v[T]}{g} \int_0^\infty d\xi \left[4 \left(\frac{df}{d\xi} \right)^2 s_\mu^2 + \frac{8}{\xi^2} f^2 (1-f)^2 s_\mu^4 + \frac{\xi^2}{2} \left(\frac{dh}{d\xi} \right)^2 s_\mu^2 + s_\mu^2 ((1-f)^2 h^2 - 2fh(1-f)(1-h)c_\mu^2 + f^2(1-h)^2 c_\mu^2) + \frac{\xi^2}{g^2 v[T]^4} (V_{\text{eff}}[\mu, h, T]) \right] \quad (1)$$

$$\frac{d^2 f}{d\xi^2} = \frac{2}{\xi^2} f(1-f)(1-2f)s_\mu^2 - \frac{1}{8}(2h^2(1-f) - 2h(1-h)(1-2f)c_\mu^2 + 2f(1-h)^2 c_\mu^2),$$

$$\frac{d}{d\xi} \left(\xi^2 \frac{dh}{d\xi} \right) = 2h(1-f)^2 - 2f(1-f)(1-2h)c_\mu^2 - 2f^2(1-h)c_\mu^2 + \frac{\xi^2}{g^2 v[T]^4} \frac{\partial V_{\text{eff}}}{\partial h},$$

SM+S

$$E_{\text{sph}}[f, h, k] = \frac{4\pi v}{g_2} \int_0^\infty d\xi \left[4 \left(\frac{df}{d\xi} \right)^2 + \frac{8}{\xi^2} (f-f^2)^2 + \frac{\xi^2}{2} \left(\frac{dh}{d\xi} \right)^2 + h^2(1-f)^2 + \frac{\xi^2 v_S^2}{2 v^2} \left(\frac{dk}{d\xi} \right)^2 + \frac{\xi^2}{g_2^2 v^4} V_{\text{eff}}(h, k, T) \right]$$

For the xSM model, we consider the following sphaleron field ansatz [1]:

$$A_i(\mu, r, \theta, \phi) = -\frac{i}{g} f(r) \partial_i U(\mu, \theta, \phi) U^{-1}(\mu, \theta, \phi), \quad (1)$$

$$H(\mu, r, \theta, \phi) = \frac{v(T)}{\sqrt{2}} \left[(1-h(r)) \begin{pmatrix} 0 \\ e^{-i\mu} \cos \mu \end{pmatrix} + h(r) U(\mu, \theta, \phi) \begin{pmatrix} 0 \\ 1 \end{pmatrix} \right], \quad (2)$$

$$S(\mu, r, \theta, \phi) = v_S(T) k(r), \quad (3)$$

where A_i are SU(2) gauge fields, and the matrix U is defined as

$$U(\mu, \theta, \phi) = \begin{pmatrix} e^{i\mu}(c_\mu - i s_\mu c_\theta) & e^{i\phi} s_\mu s_\theta \\ -e^{-i\phi} s_\mu s_\theta & e^{-i\mu}(c_\mu + i s_\mu c_\theta) \end{pmatrix}, \quad (4)$$

where the $s_{\mu(\theta)} = \sin \mu(\theta)$ and $c_{\mu(\theta)} = \cos \mu(\theta)$. The sphaleron energy is obtained for $\mu = \pi/2$ [2]. From the sphaleron energy in the main body of this paper, the equations of motion can be found:

$$\frac{d^2 f}{d\xi^2} = \frac{2}{\xi^2} f(1-f)(1-2f) - \frac{v[T]^2 h^2}{4\Omega[T]^2} (1-f), \quad (5)$$

$$\frac{d}{d\xi} \left(\xi^2 \frac{dh}{d\xi} \right) = 2h(1-f)^2 + \frac{\xi^2}{g^2 v[T]^2 \Omega[T]^2} \frac{\partial V_{\text{eff}}(h, k, T)}{\partial h}, \quad (6)$$

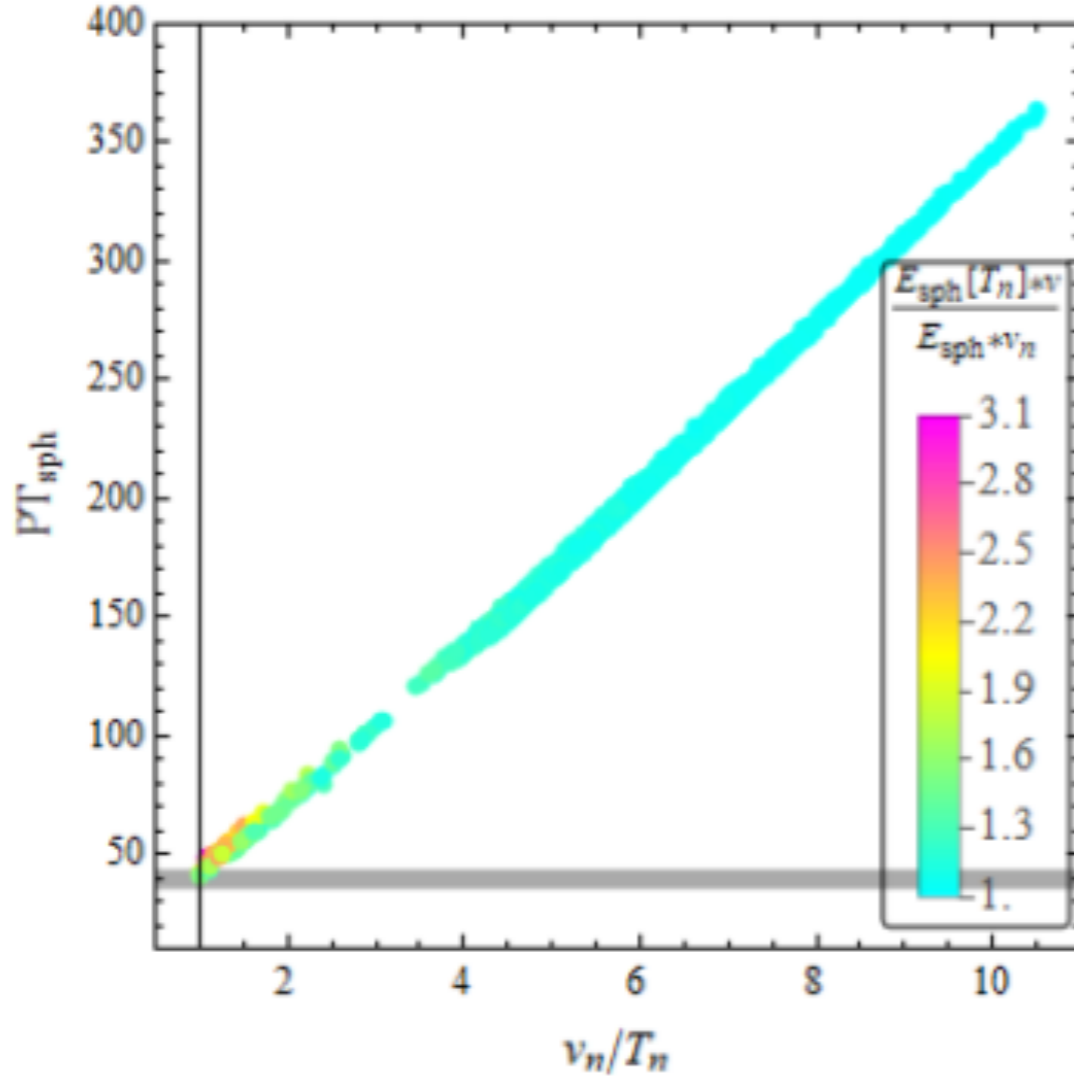
$$\frac{d}{d\xi} \left(\xi^2 \frac{dk}{d\xi} \right) = \frac{\xi^2}{g^2 v_S[T]^2 \Omega[T]^2} \frac{\partial V_{\text{eff}}(h, k, T)}{\partial k}. \quad (7)$$

The sphaleron solutions can be obtained with the following boundary conditions,

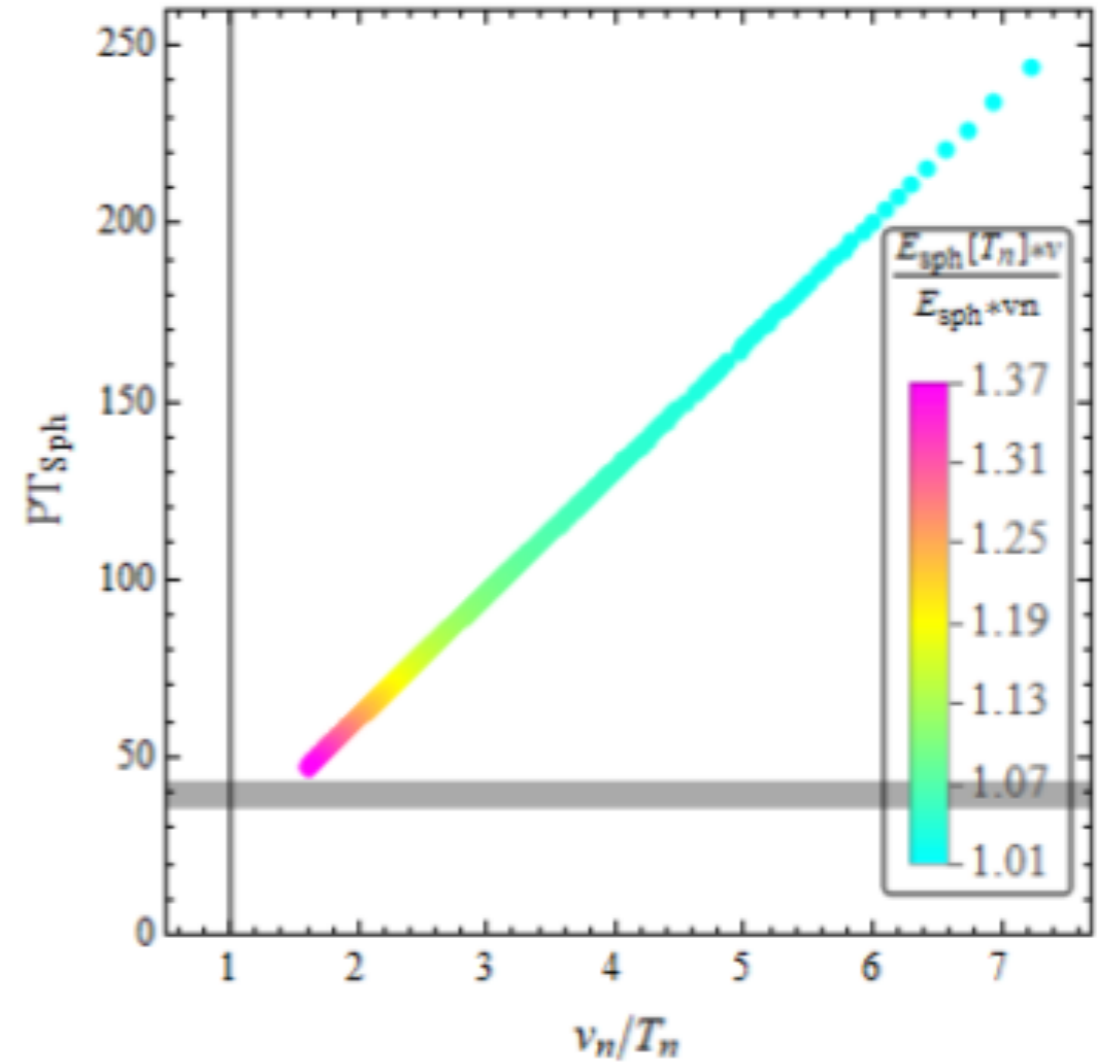
$$\lim_{\xi \rightarrow 0} f(\xi) = 0, \quad \lim_{\xi \rightarrow 0} h(\xi) = 0, \quad \lim_{\xi \rightarrow 0} k'(\xi) = 0,$$

$$\lim_{\xi \rightarrow \infty} f(\xi) = 1, \quad \lim_{\xi \rightarrow \infty} h(\xi) = 1, \quad \lim_{\xi \rightarrow \infty} k(\xi) = 1. \quad (8)$$

SFOPT condition



SM+singlet



SMEFT

For the “xSM” model, the gauge invariant finite temperature effective potential is found to be:

$$V(h, s, T) = -\frac{1}{2}[\mu^2 - \Pi_h(T)]h^2 - \frac{1}{2}[-b_2 - \Pi_s(T)]s^2 + \frac{1}{4}\lambda h^4 + \frac{1}{4}a_1 h^2 s + \frac{1}{4}a_2 h^2 s^2 + \frac{b_3}{3}s^3 + \frac{b_4}{4}s^4, \quad (C1)$$

with the thermal masses given by

$$\begin{aligned} \Pi_h(T) &= \left(\frac{2m_W^2 + m_Z^2 + 2m_t^2}{4v^2} + \frac{\lambda}{2} + \frac{a_2}{24} \right) T^2, \\ \Pi_s(T) &= \left(\frac{a_2}{6} + \frac{b_4}{4} \right) T^2, \end{aligned} \quad (C2)$$

$$V_T(h, T) = V(h) + \frac{1}{2}c_{hT}h^2$$

$$V(H) = -m^2(H^\dagger H) + \lambda(H^\dagger H)^2 + \frac{(H^\dagger H)^3}{\Lambda^2}$$

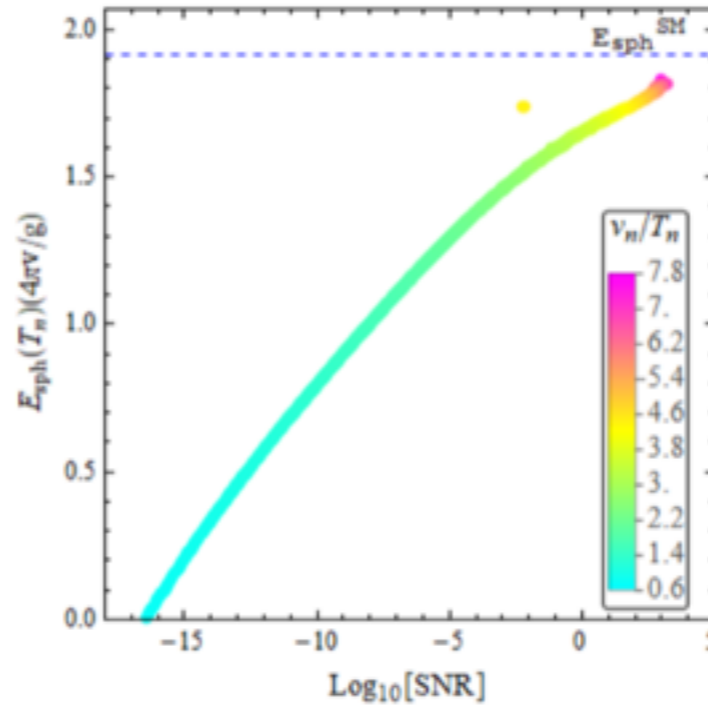
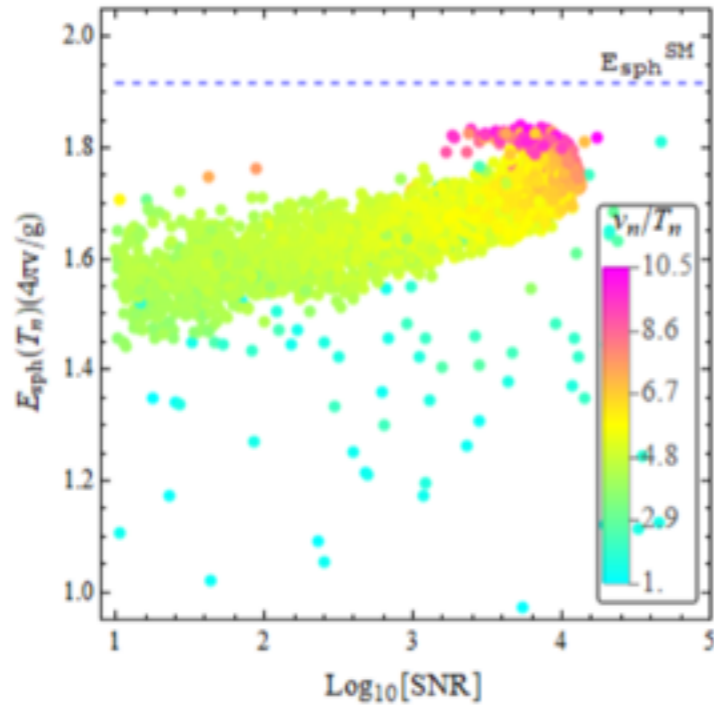
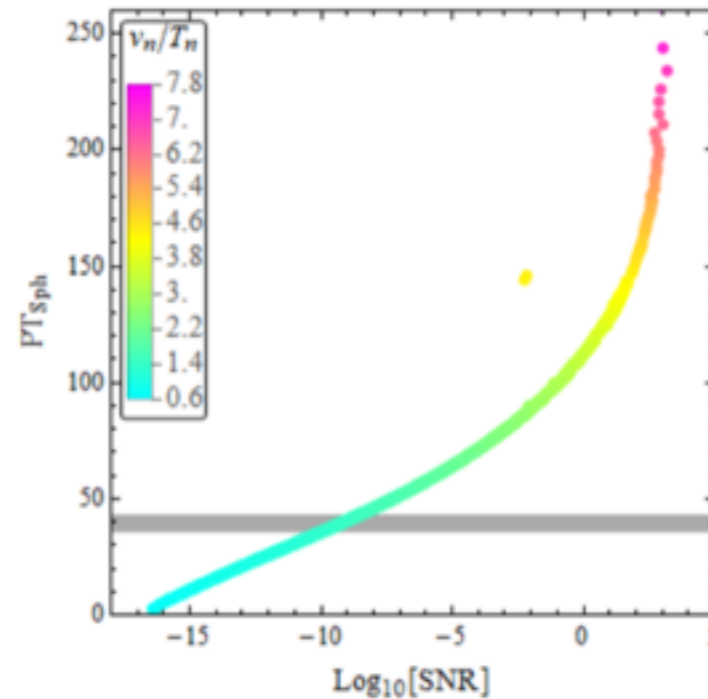
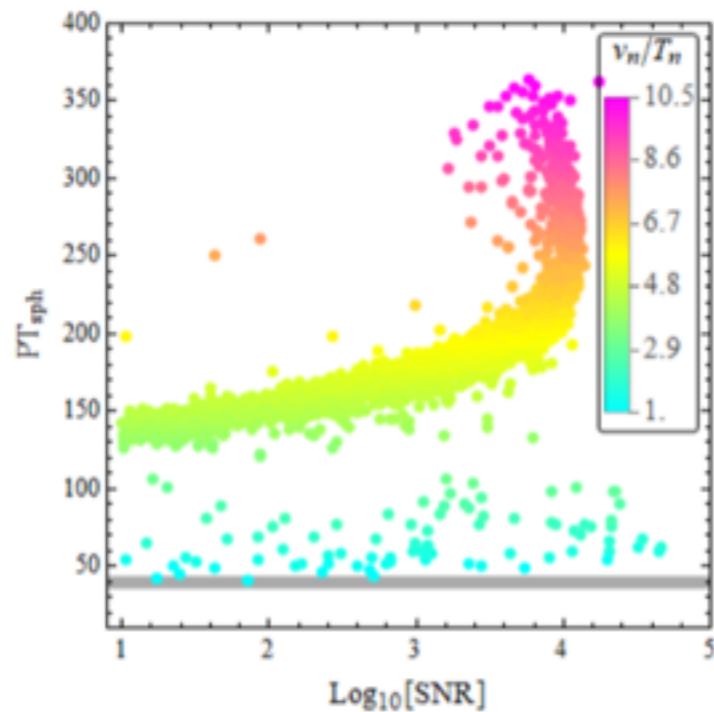
$$c_{hT} = (4y_t^2 + 3g_+^2 + g'^2 + 8\lambda)T^2/16$$

Search for sphaleron with GW

Gravitational waves can be searched for by cross-correlating outputs from two or more detectors, with the resulting signal-to-noise ratio(SNR)

$$\text{SNR} = \sqrt{\mathcal{T} \int df \left[\frac{h^2 \Omega_{\text{GW}}(f)}{h^2 \Omega_{\text{exp}}(f)} \right]^2}$$

where T is the duration of the data in years and Ω_{exp} the power spectral density of the detector.



GW sources

$$\Omega_{\text{GW}}(f) = \begin{cases} \Omega_{\text{GW}*} \left(\frac{f}{f_*}\right)^{n_{\text{GW}1}} & \text{for } f < f_*, \\ \Omega_{\text{GW}*} \left(\frac{f}{f_*}\right)^{n_{\text{GW}2}} & \text{for } f > f_*, \end{cases}$$

Table 1. Cosmological GW sources

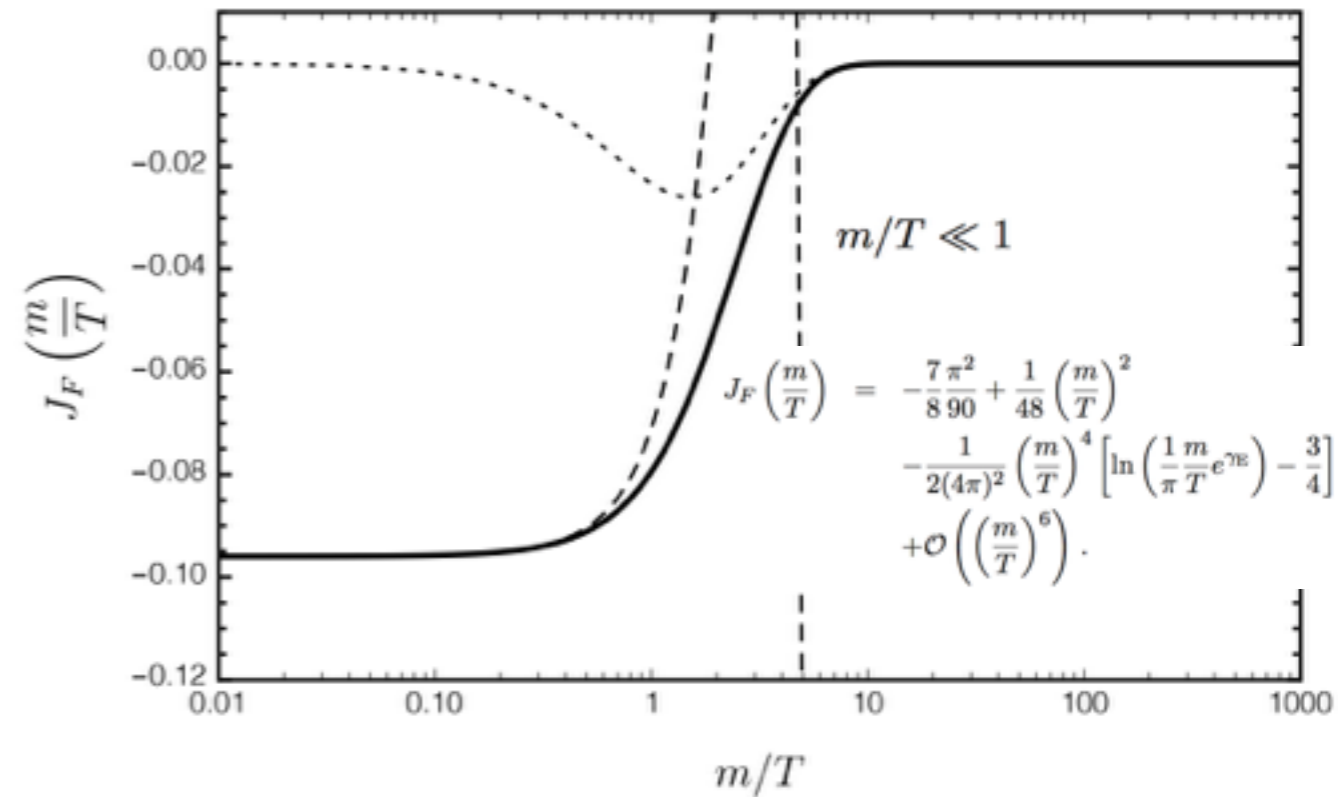
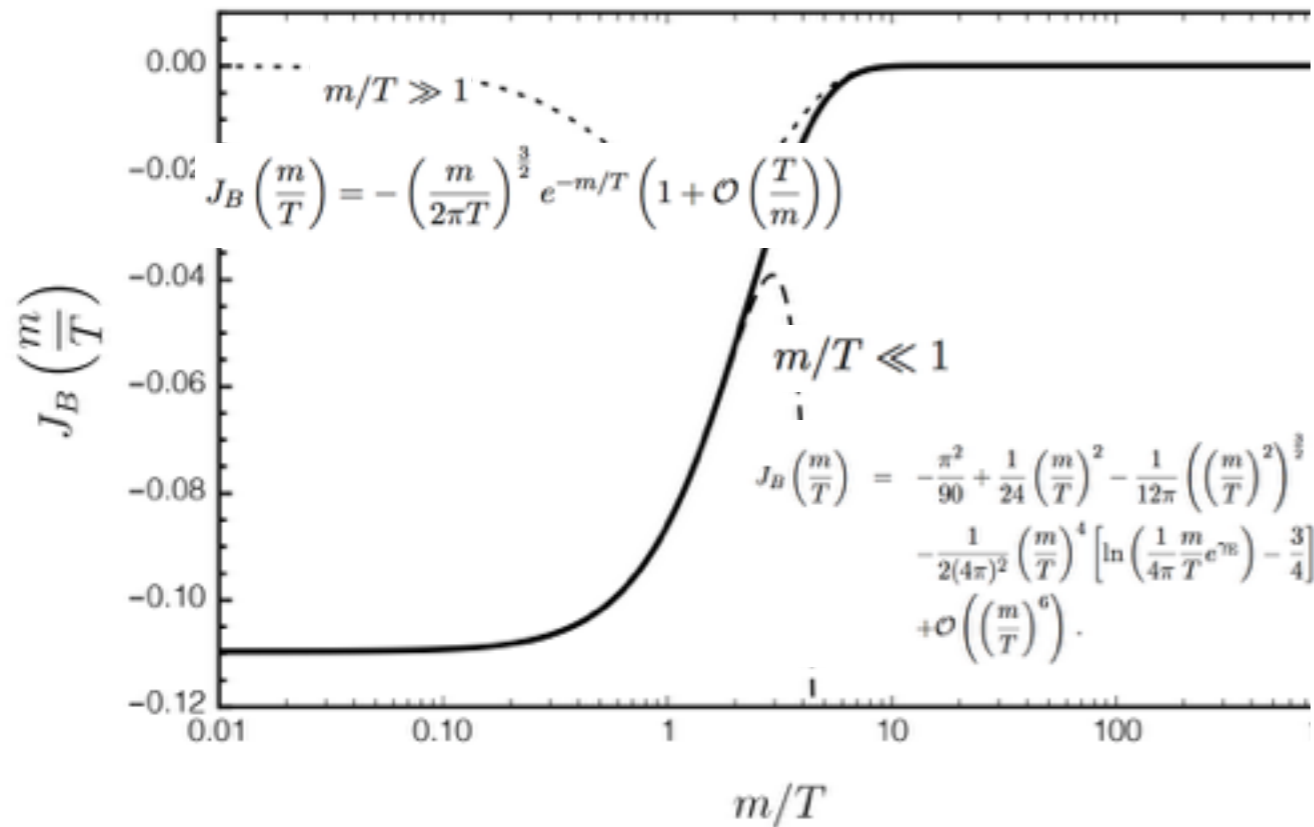
1807.00786

source	$n_{\text{GW}1}$	$n_{\text{GW}2}$	f_* [Hz]	Ω_{GW}
Phase transition (bubble collision)	2.8	-2	$\sim 10^{-5} \left(\frac{f_{\text{PT}}}{\beta}\right) \left(\frac{\beta}{H_{\text{PT}}}\right) \left(\frac{T_{\text{PT}}}{100 \text{ GeV}}\right)$	$\sim 10^{-5} \left(\frac{H_{\text{PT}}}{\beta}\right)^2 \left(\frac{\kappa_\phi \alpha}{1+\alpha}\right)^2 \left(\frac{0.11 v_w^3}{0.42 + v_w^2}\right)$
Phase transition (turbulence)	3	-5/3	$\sim 3 \times 10^{-5} \left(\frac{1}{v_w}\right) \left(\frac{\beta}{H_{\text{PT}}}\right) \left(\frac{T_{\text{PT}}}{100 \text{ GeV}}\right)$	$\sim 3 \times 10^{-4} \left(\frac{H_{\text{PT}}}{\beta}\right) \left(\frac{\kappa_{\text{turb}} \alpha}{1+\alpha}\right)^{3/2} v_w$
Phase transition (sound waves)	3	-4	$\sim 2 \times 10^{-5} \left(\frac{1}{v_w}\right) \left(\frac{\beta}{H_{\text{PT}}}\right) \left(\frac{T_{\text{PT}}}{100 \text{ GeV}}\right)$	$\sim 3 \times 10^{-6} \left(\frac{H_{\text{PT}}}{\beta}\right) \left(\frac{\kappa_\phi \alpha}{1+\alpha}\right)^2 v_w$
Preheating ($\lambda\phi^4$)	3	cutoff	$\sim 10^7$	$\sim 10^{-11} \left(\frac{g^2/\lambda}{100}\right)^{-0.5}$
Preheating (hybrid)	2	cutoff	$\sim \frac{g}{\sqrt{\lambda}} \lambda^{1/4} 10^{10.25}$	$\sim 10^{-5} \left(\frac{\lambda}{g^2}\right)^{1.16} \left(\frac{v}{M_{\text{pl}}}\right)^2$
Cosmic strings (loops 1)	[1, 2]	[-1, -0.1]	$\sim 3 \times 10^{-8} \left(\frac{G\mu}{10^{-11}}\right)^{-1}$	$\sim 10^{-9} \left(\frac{G\mu}{10^{-12}}\right) \left(\frac{\alpha_{\text{loop}}}{10^{-1}}\right)^{-1/2}$ (for $\alpha_{\text{loop}} \gg \Gamma G\mu$)
Cosmic strings (loops 2)	[-1, -0.1]	0	$\sim 3 \times 10^{-8} \left(\frac{G\mu}{10^{-11}}\right)^{-1}$	$\sim 10^{-9.5} \left(\frac{G\mu}{10^{-12}}\right) \left(\frac{\alpha_{\text{loop}}}{10^{-1}}\right)^{-1/2}$ (for $\alpha_{\text{loop}} \gg \Gamma G\mu$)
Cosmic strings (infinite strings)	[0, 0.2]	[0, 0.2]	—	$\sim 10^{-[11,13]} \left(\frac{G\mu}{10^{-8}}\right)$
Domain walls	3	-1	$\sim 10^{-9} \left(\frac{T_{\text{ann}}}{10^{-2} \text{ GeV}}\right)$	$\sim 10^{-17} \left(\frac{\sigma}{1 \text{ TeV}^3}\right)^2 \left(\frac{T_{\text{ann}}}{10^{-2} \text{ GeV}}\right)^{-4}$
Self-ordering scalar fields	0	0	—	$\sim \frac{511}{N} \Omega_{\text{rad}} \left(\frac{v}{M_{\text{pl}}}\right)^4$
Self-ordering scalar + reheating	0	-2	$\sim 0.4 \left(\frac{T_R}{10^7 \text{ GeV}}\right)$	$\sim \frac{511}{N} \Omega_{\text{rad}} \left(\frac{v}{M_{\text{pl}}}\right)^4$
Magnetic fields	3	$\alpha_B + 1$	$\sim 10^{-6} \left(\frac{T_*}{10^2 \text{ GeV}}\right)$	$\sim 10^{-16} \left(\frac{B}{10^{-10} \text{ G}}\right)$
Inflation+reheating	~ 0	-2	$\sim 0.3 \left(\frac{T_R}{10^7 \text{ GeV}}\right)$	$\sim 2 \times 10^{-17} \left(\frac{r}{0.01}\right)$
Inflation+kination	~ 0	1	$\sim 0.3 \left(\frac{T_R}{10^7 \text{ GeV}}\right)$	$\sim 2 \times 10^{-17} \left(\frac{r}{0.01}\right)$
Particle prod. during inf.	-2ϵ	$-4\epsilon(4\pi\xi - 6)(\epsilon - \eta)$	—	$\sim 2 \times 10^{-17} \left(\frac{r}{0.01}\right)$
2nd-order (inflation)	1	drop-off	$\sim 7 \times 10^5 \left(\frac{T_{\text{reh}}}{10^9 \text{ GeV}}\right)^{1/3} \left(\frac{M_{\text{inf}}}{10^{16} \text{ GeV}}\right)^{2/3}$	$\sim 10^{-12} \left(\frac{T_{\text{reh}}}{10^9 \text{ GeV}}\right)^{-4/3} \left(\frac{M_{\text{inf}}}{10^{16} \text{ GeV}}\right)^{4/3}$
2nd-order (PBHs)	2	drop-off	$\sim 4 \times 10^{-2} \left(\frac{M_{\text{PBH}}}{10^{20} \text{ g}}\right)^{-1/2}$	$\sim 7 \times 10^{-9} \left(\frac{\mathcal{A}^2}{10^{-3}}\right)^2$
Pre-Big-Bang	3	$3 - 2\mu$	—	$\sim 1.4 \times 10^{-6} \left(\frac{H_s}{0.15 M_{\text{pl}}}\right)^4$

Thermal effective scalar potential for PT study

$$V_T(\phi, T) = V_0(\phi) + T^4 \left[\sum_B J_B \left(\frac{M_B}{T} \right) + \sum_F J_F \left(\frac{M_F}{T} \right) \right]$$

all fermions **F** and bosons **B** that are relativistic at temperature **T**



High-T expansion

$$V_T(\phi) = V_0(\phi) + \frac{T^2}{24} \left(\sum_S M_S^2(\phi) + 3 \sum_V M_V^2(\phi) + 2 \sum_F M_F^2(\phi) \right) - \frac{T}{12\pi} \left(\sum_S \left(M_S^2(\phi) \right)^{3/2} + \sum_V \left(M_V^2(\phi) \right)^{3/2} \right) + \text{higher order terms.}$$

MS, MV, MF are the masses of the scalar fields **S**, vector fields **V** and fermionic fields **F**

Bubble, Sphaleron and BAU

Instanton

$$\frac{\Gamma}{V} = A(T)e^{-S_3/T}$$

$$\begin{aligned} \frac{S_3(T_N)}{T_N} - \frac{3}{2} \ln \left(\frac{S_3(T_N)}{T_N} \right) \\ = 152.59 - 2 \ln g_*(T_N) - 4 \ln \left(\frac{T_N}{100 \text{ GeV}} \right) \end{aligned}$$

Bubble nucleation

$$S_3(T_N)/T_N \sim 140-150$$

Washout avoid

$$\Gamma_{\text{sph}} = A_{\text{sph}}(T) \exp[-E_{\text{sph}}(T)/T] < H(T)$$

$$E_{\text{sph}}(T) \approx E_{\text{sph},0} \frac{v(T)}{v}$$

$$\frac{v(T)}{T} > (0.973 - 1.16) \left(\frac{E_{\text{sph},0}}{1.916 \times 4\pi v/g} \right)^{-1} \quad 1708.03061$$

GW parameters and FOPT

Bounce solution

$$S_3(T) = \int 4\pi r^2 dr \left[\frac{1}{2} \left(\frac{d\phi_b}{dr} \right)^2 + V(\phi_b, T) \right]$$

$$\lim_{r \rightarrow \infty} \phi_b = 0, \quad \left. \frac{d\phi_b}{dr} \right|_{r=0} = 0$$

Bubble nucleation:

$$\Gamma \approx A(T) e^{-S_3/T} \sim 1$$

Latent heat:

$$\alpha = \frac{1}{\rho_R} \left[-(V_{\text{EW}} - V_f) + T \left(\frac{dV_{\text{EW}}}{dT} - \frac{dV_f}{dT} \right) \right] \Big|_{T=T_*}$$

phase transition inverse duration: $\frac{\beta}{H_n} = T \frac{d(S_3(T)/T)}{dT} \Big|_{T=T_n}$

GW from FOPT

$$\Omega_{\text{GW}}(f)h^2 \approx \Omega_{\text{sw}}(f)h^2 + \Omega_{\text{turb}}(f)h^2$$

Sound Wave: $\Omega h_{\text{sw}}^2(f) = 2.65 \times 10^{-6} (H_* \tau_{\text{sw}}) \left(\frac{\beta}{H}\right)^{-1} v_b \left(\frac{\kappa_\nu \alpha}{1 + \alpha}\right)^2 \left(\frac{g_*}{100}\right)^{-\frac{1}{3}} \left(\frac{f}{f_{\text{sw}}}\right)^3 \left(\frac{7}{4 + 3(f/f_{\text{sw}})^2}\right)^{7/2}$

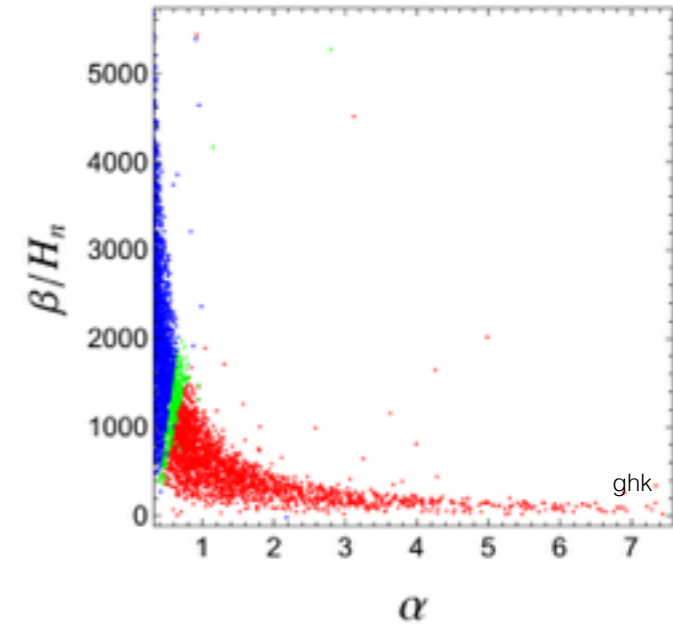
phase transition duration:

$$\tau_{\text{sw}} = \min \left[\frac{1}{H_*}, \frac{R_*}{\bar{U}_f} \right], \quad H_* R_* = v_b (8\pi)^{1/3} (\beta/H)^{-1}$$

Root-mean-square four-velocity of the plasma

$$\bar{U}_f^2 \approx \frac{3}{4} \frac{\kappa_\nu \alpha}{1 + \alpha}$$

$$f_{\text{sw}} = 1.9 \times 10^{-5} \frac{\beta}{H} \frac{1}{v_b} \frac{T_*}{100} \left(\frac{g_*}{100}\right)^{\frac{1}{6}} \text{ Hz}$$



MHD turbulence:

$$\Omega h_{\text{turb}}^2(f) = 3.35 \times 10^{-4} \left(\frac{\beta}{H}\right)^{-1} \left(\frac{\epsilon \kappa_\nu \alpha}{1 + \alpha}\right)^{\frac{3}{2}} \left(\frac{g_*}{100}\right)^{-\frac{1}{3}} v_b \frac{(f/f_{\text{turb}})^3 (1 + f/f_{\text{turb}})^{-\frac{11}{3}}}{[1 + 8\pi f a_0 / (a_* H_*)]}$$

$$f_{\text{turb}} = 2.7 \times 10^{-5} \frac{\beta}{H} \frac{1}{v_b} \frac{T_*}{100} \left(\frac{g_*}{100}\right)^{\frac{1}{6}} \text{ Hz}$$

3.1. Models of time-correlated processes

The principal results of this paper are referred to a fiducial power-law spectrum of characteristic GW strain

$$h_c(f) = A_{\text{GWB}} \left(\frac{f}{f_{\text{yr}}} \right)^\alpha, \quad (1)$$

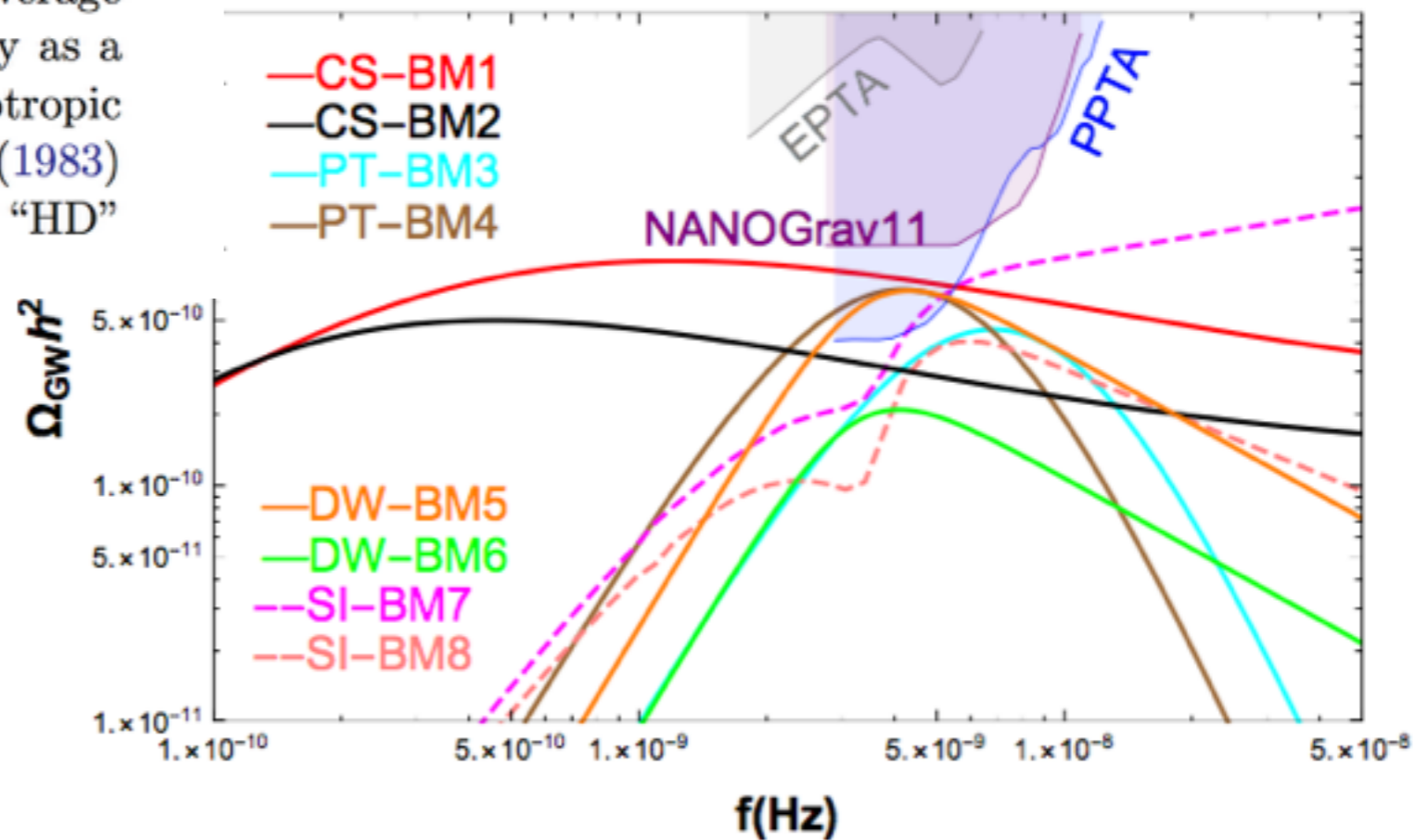
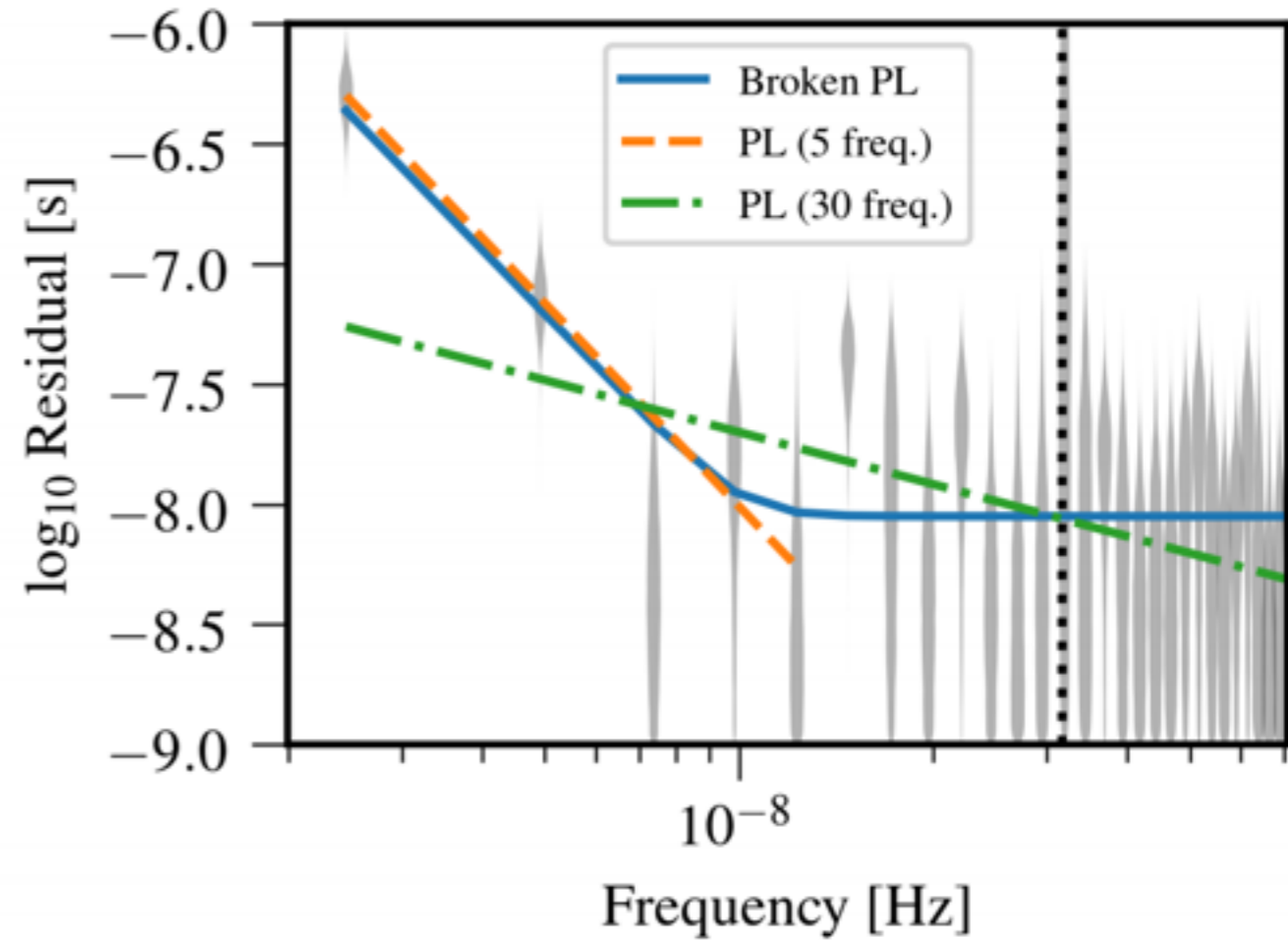
with $\alpha = -2/3$ for a population of inspiraling SMBHBs in circular orbits whose evolution is dominated by GW emission (Phinney 2001). We performed our analysis in terms of the timing-residual cross-power spectral density

$$S_{ab}(f) = \Gamma_{ab} \frac{A_{\text{GWB}}^2}{12\pi^2} \left(\frac{f}{f_{\text{yr}}} \right)^{-\gamma} f_{\text{yr}}^{-3}. \quad (2)$$

where $\gamma = 3 - 2\alpha$ (so the fiducial SMBHB $\alpha = -2/3$ corresponds to $\gamma = 13/3$), and where Γ_{ab} is the overlap reduction function (ORF), which describes average correlations between pulsars a and b in the array as a function of the angle between them. For an isotropic GWB, the ORF is given by Hellings & Downs (1983) and we refer to it casually as “quadrupolar” or “HD” correlations.

2009.04496

GW for NanoGrav ???

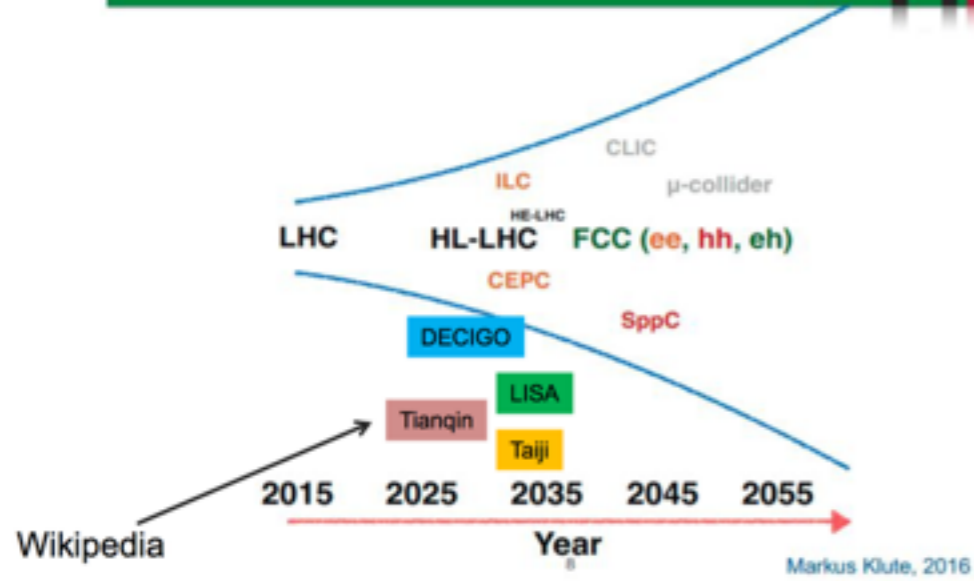


Test of SFOEWPT

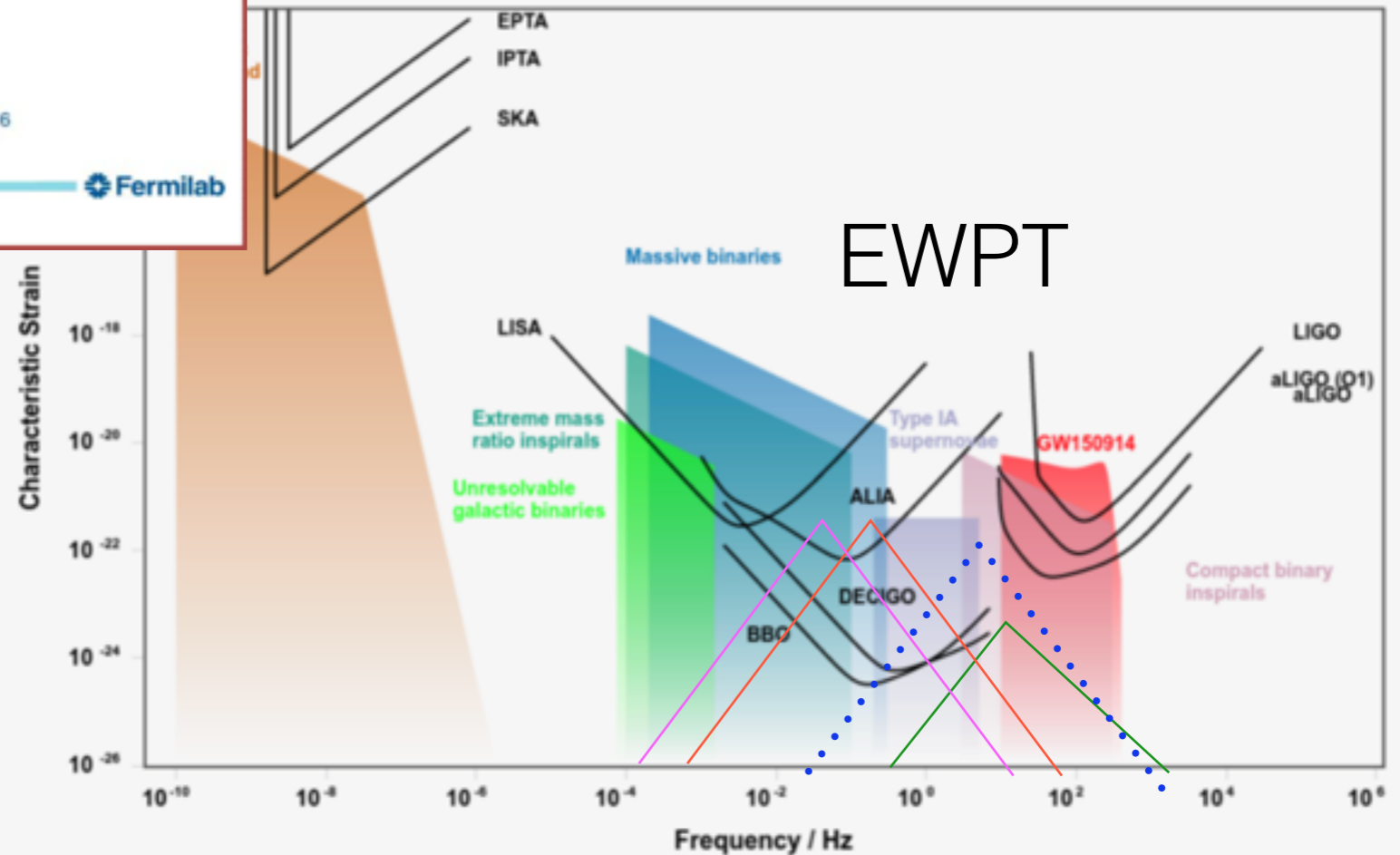
Many Colliders in the Horizon

Double Higgs Production at Colliders Workshop

The Road Ahead



Markus Klute, 2016



EWPT

Why SFOEWPT

$$\frac{n_b}{s} \approx (0.7 - 0.9) \times 10^{-10} \neq 0$$

