Neutrino masses and **Yukawa interactions**

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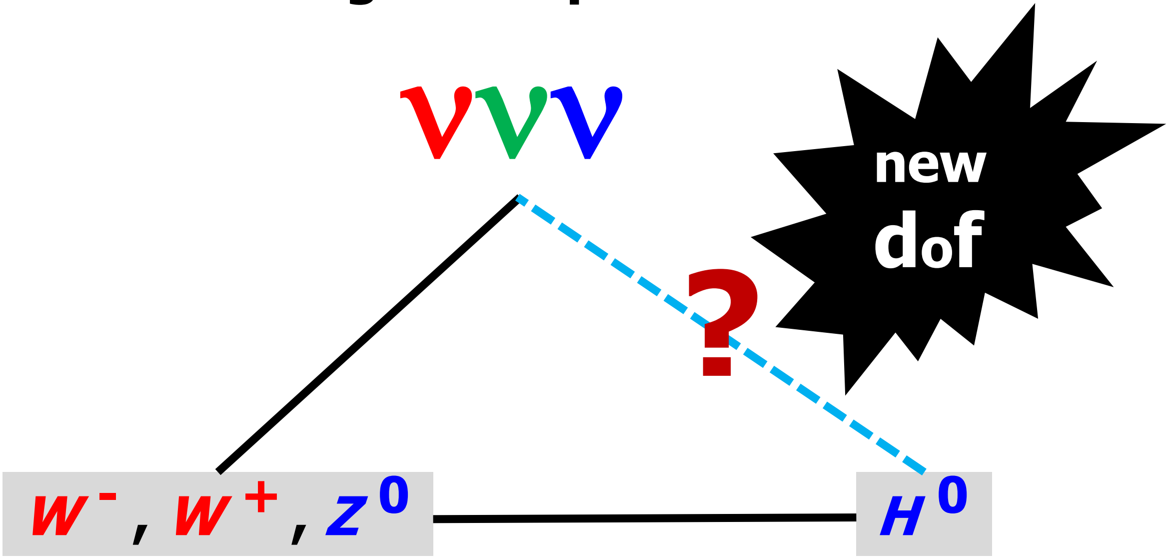


$\nu \nu \nu$

new dof

?

W^-, W^+, Z^0

H^0

第十五届**TeV**物理工作组学术研讨会，2021年7月19日—21日，友谊宾馆

Preliminary statistics: 65 years of ν-physics

1998.06

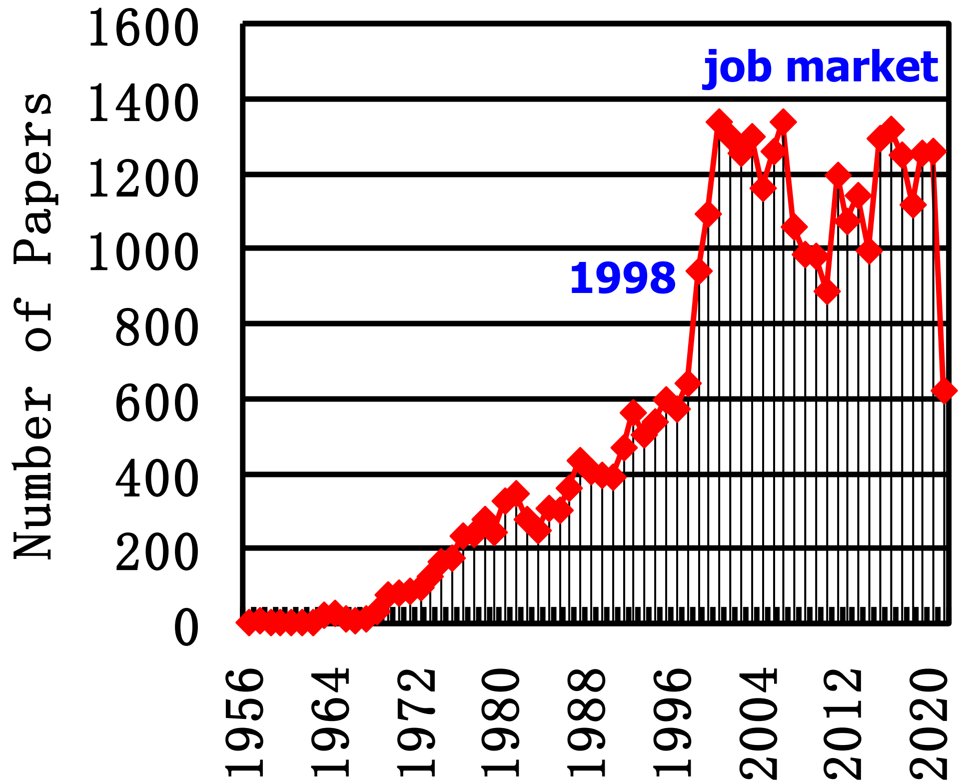
neutrino oscillation

Nobel 2015

1998.12

dark energy

Nobel 2011



NEUTRINOS

1933
β-decay EFT

1957
parity violation

1967
standard model

1973
neutral current

1998
ν-oscillations

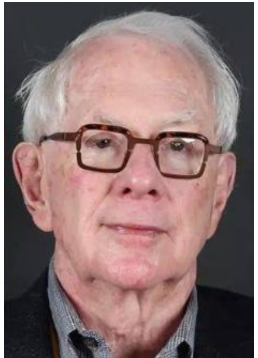
INSPIRE: find title **NEUTRINO** and date **XX**

> 28 000 Papers

Personal observations

★ In 2008, 10 years after 1998, all the ν-theorists and their PhD students were busy with publishing something. Today the situation has changed a lot.

★ But there has been no breakthrough yet, just as predicted by **Sheldon Glashow** on 11 Nov. 2005 at Expert's Restaurant of IHEP. The key issue is that we have no idea about the flavor structures, even though *most* of the flavor parameters have so far been measured in a variety of experiments.



★ In the lack of a powerful *top-down* guiding principle, most of us are following a *bottom-up* way according to our *own* tastes.

> Although nature commences with reason and ends in experience, it is necessary for us to do the opposite, i.e. to commence with experience and from this to proceed to investigate the reason——**Leonard de Vinci**

OUTLINE

Part A —— Some general remarks:
From **Weinberg** 1967 to **Weinberg** 2020 √

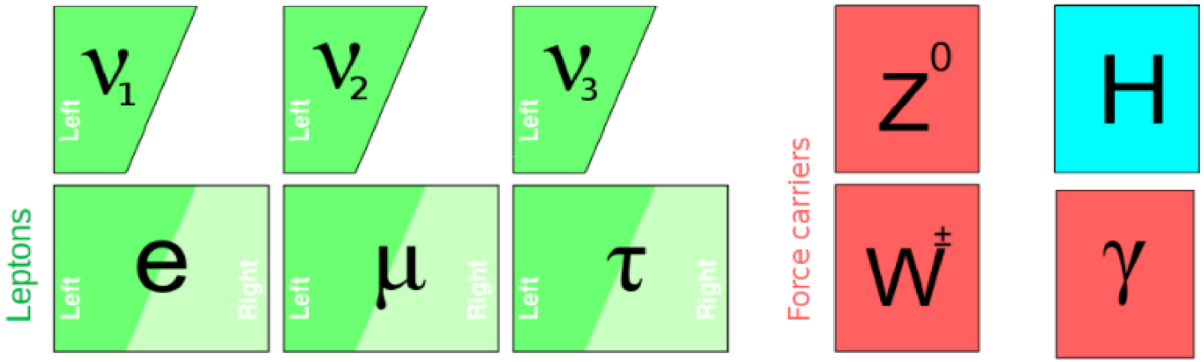
Part B —— Two specific examples:
On **neutrino** EFT and **modular** symmetry

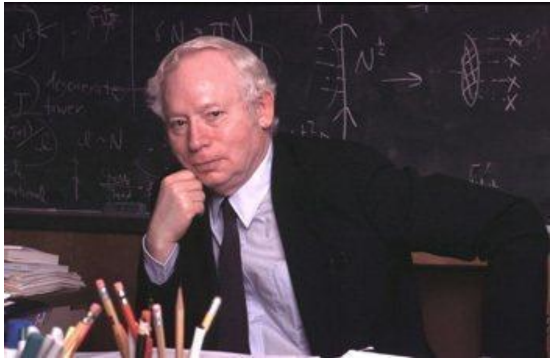
What is wrong with the SM?

In October 1967, Steven Weinberg proposed a model of leptons.

—— its theoretical ingredients are perfect!

—— its particle content looks very strange: it has no quark flavors, no neutrino masses, no flavor mixing and no CP violation.





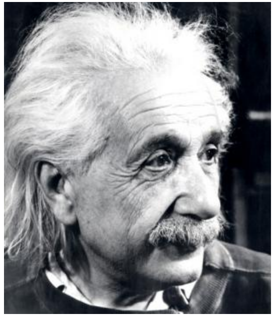
My style is usually not to propose specific models that will lead to specific experimental predictions, but rather to interpret in a broad way what is going on and make very general remarks, like with the development of the point of view associated with effective field theory ---- Weinberg 2021@CERN Courier

The wrong use of Occam's razor!

maximal P violation

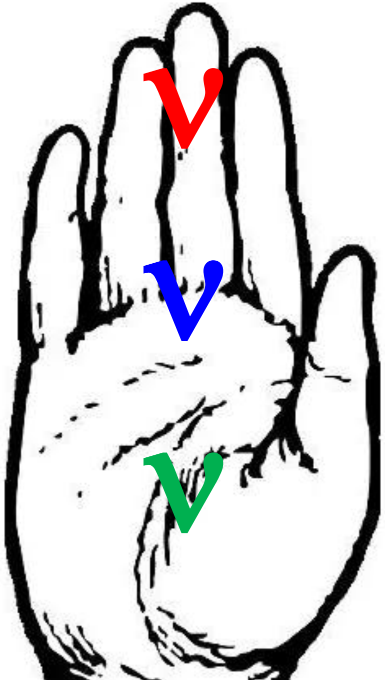
$$\begin{pmatrix} u_{\rm L} \\ d_{\rm L} \end{pmatrix} \begin{matrix} \longleftrightarrow & u_{\rm R} \\ \longleftrightarrow & d_{\rm R} \end{matrix}$$

$$\begin{pmatrix} \nu_{e{\rm L}} \\ e_{\rm L} \end{pmatrix} \begin{matrix} \longleftrightarrow & ? \\ \longleftrightarrow & e_{\rm R} \end{matrix}$$



Albert Einstein:

Everything should be made as simple as possible, but not simpler!



Right-handed neutrinos

A broken flavor democracy?

The right-handed neutrino fields may have big self-interaction couplings (lepton number violation), or a Majorana mass term

$$\frac{1}{2}\overline{(N_{\mathrm{R}})^c}M_{\mathrm{R}}N_{\mathrm{R}}$$

In this case the two neutrino sectors have a huge mass gap, implying a flavor democracy between them.

$$-\mathcal{L}_{\nu+N} = \overline{\nu_{\mathrm{L}}}M_{\mathrm{D}}N_{\mathrm{R}} + \frac{1}{2}\overline{(N_{\mathrm{R}})^c}M_{\mathrm{R}}N_{\mathrm{R}} + \mathrm{h.c.} = \frac{1}{2}\overline{[\nu_{\mathrm{L}} \quad (N_{\mathrm{R}})^c]}\begin{pmatrix} 0 & M_{\mathrm{D}} \\ M_{\mathrm{D}}^T & M_{\mathrm{R}} \end{pmatrix}\begin{bmatrix} (\nu_{\mathrm{L}})^c \\ N_{\mathrm{R}} \end{bmatrix} + \mathrm{h.c.}$$

A new understanding of the *type-I* seesaw mechanism: small neutrino masses originate from the Yukawa interactions which break the flavor democracy.

$$\frac{1}{4}\overline{[\nu_{\mathrm{L}} \quad (N_{\mathrm{R}})^c]}M_{\mathrm{R}}\begin{pmatrix} 1 & 1 \\ 1 & 1 \end{pmatrix}\begin{bmatrix} (\nu_{\mathrm{L}})^c \\ N_{\mathrm{R}} \end{bmatrix}$$

+

$$\frac{1}{4}\overline{[\nu_{\mathrm{L}} \quad (N_{\mathrm{R}})^c]}\left[M_{\mathrm{D}}\begin{pmatrix} 1 & 1 \\ -1 & -1 \end{pmatrix} + \begin{pmatrix} 1 & -1 \\ 1 & -1 \end{pmatrix}M_{\mathrm{D}}^T\right]\begin{bmatrix} (\nu_{\mathrm{L}})^c \\ N_{\mathrm{R}} \end{bmatrix}$$

Yukawa interactions:

$$\overline{\ell_{\mathrm{L}}}Y_\nu\widetilde{H}N_{\mathrm{R}} \longrightarrow M_{\mathrm{D}} = Y_\nu\langle H\rangle$$

The type-I seesaw is in the landscape

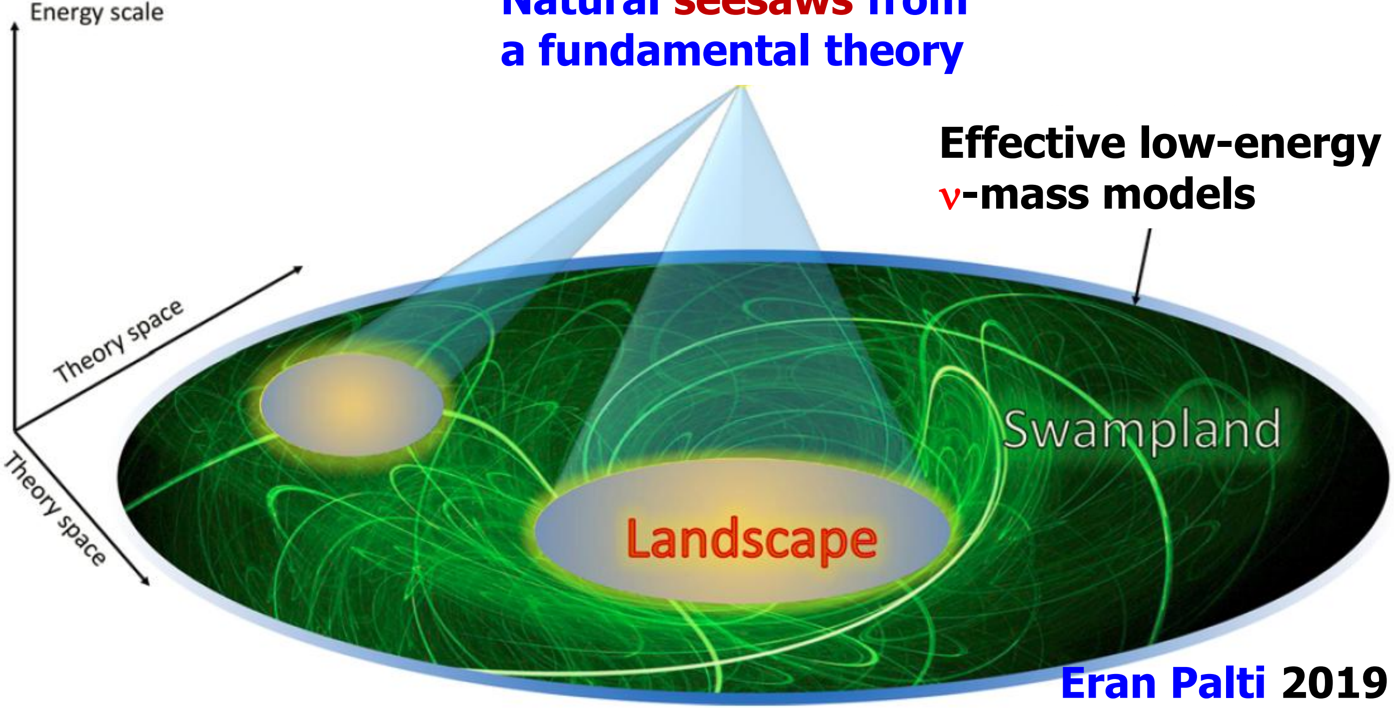
This seesaw picture is well consistent with the spirit of Weinberg's EFT with a unique *d=5* operator (1979).

$$\mathcal{O}_{\rm W} = \frac{\overline{\ell_{\rm L}} \widetilde{H} \widetilde{H}^T \ell^c_{\rm L}}{\Lambda}$$

$$M_\nu \simeq -M_{\rm D} M_{\rm R}^{-1} M_{\rm D}^T = -\langle H \rangle^2 Y_\nu M_{\rm R}^{-1} Y_\nu^T$$

P. Minkowski 1977, T. Yanagida 1979…

Cumrun Vafa 2005



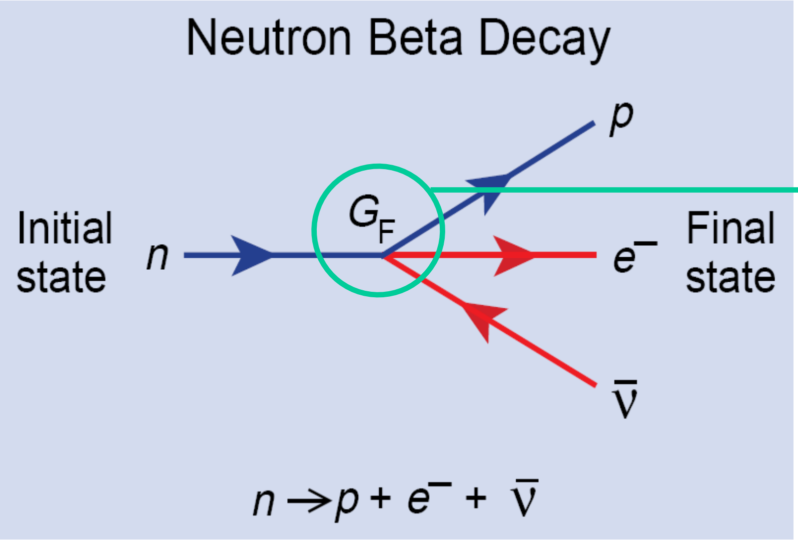
Natural seesaws from a fundamental theory

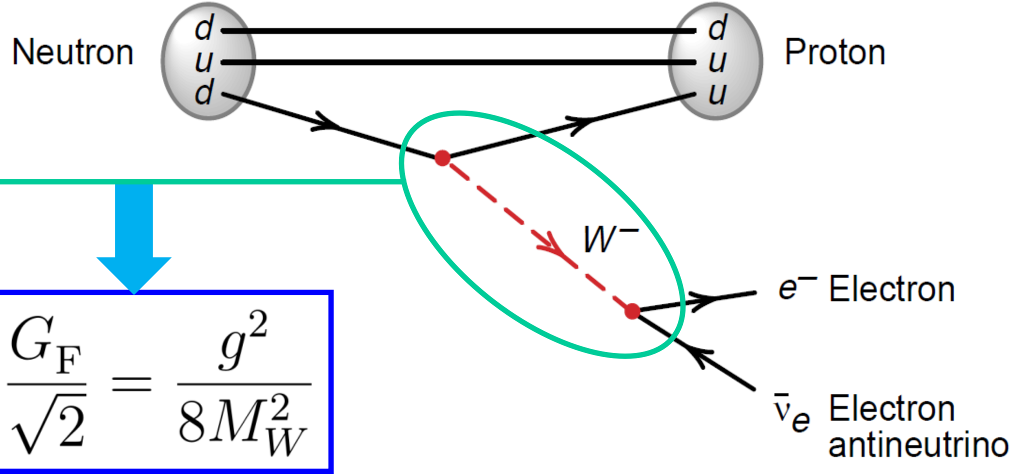
Effective low-energy ν-mass models

Eran Palti 2019

This was historically true

From Fermi's EFT for beta decays to Weinberg's SM, neutrinos did play a role!





$$\boxed{\frac{G_{\mathrm{F}}}{\sqrt{2}} = \frac{g^2}{8M_W^2}}$$

A seesaw-like relation!

The Fermi coupling constant

$G_{\mathrm{F}} \simeq 1.166 \times 10^{-5}\ \mathrm{GeV}^{-2}$

The weak coupling constant

$g \simeq 0.65$ **vs** $M_W \simeq 80.4\ \mathrm{GeV}$

This is the weak charged-current (gauge) interactions in which the neutrinos participate. How about the neutrino Yukawa interactions?

Majorana nature and exact seesaw

Diagonalize the 6×6 Majorana neutrino mass matrix by a 6×6 unitary matrix:

$$\boxed{\begin{pmatrix} U & R \\ S & U' \end{pmatrix}^{\dagger} \begin{pmatrix} 0 & M_{\rm D} \\ M_{\rm D}^{T} & M_{\rm R} \end{pmatrix} \begin{pmatrix} U & R \\ S & U' \end{pmatrix}^{*} = \begin{pmatrix} D_{\nu} & 0 \\ 0 & D_{N} \end{pmatrix}}$$

$$D_{\nu} \equiv {\rm Diag}\{m_1, m_2, m_3\},\ D_{N} \equiv {\rm Diag}\{M_1, M_2, M_3\}$$

$$\boxed{\overline{(N_{\rm R})^c} M_{\rm D}^{T} (\nu_{\rm L})^c = \left[(N_{\rm R})^T \mathcal{C} M_{\rm D}^{T} \mathcal{C} \overline{\nu_{\rm L}}^{T}\right]^{T} = \overline{\nu_{\rm L}} M_{\rm D} N_{\rm R}}$$

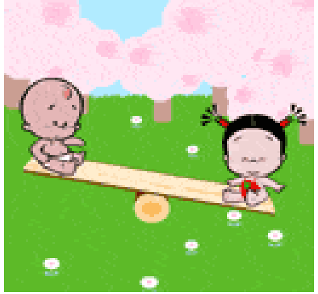
Majorana mass states:

$$\nu' = \begin{bmatrix} \nu'_{\rm L} \\ (N'_{\rm R})^c \end{bmatrix} + \begin{bmatrix} (\nu'_{\rm L})^c \\ N'_{\rm R} \end{bmatrix} = \begin{pmatrix} \nu_1 \\ \nu_2 \\ \nu_3 \\ N_1 \\ N_2 \\ N_3 \end{pmatrix}$$

$$\boxed{(\nu')^c = \nu'}$$

The exact *seesaw* relation between light and heavy Majorana neutrinos

$$\boxed{U D_{\nu} U^{T} = -R D_{N} R^{T}}$$



Three flavor states are linear combinations of six mass states (LFV):

$$\begin{pmatrix} \nu_e \\ \nu_\mu \\ \nu_\tau \end{pmatrix}_{\rm L} = U \begin{pmatrix} \nu_1 \\ \nu_2 \\ \nu_3 \end{pmatrix}_{\rm L} + R \begin{pmatrix} N_1 \\ N_2 \\ N_3 \end{pmatrix}_{\rm L}$$

$$\boxed{UU^{\dagger} + RR^{\dagger} = I}$$

The Euler-like parametrization of everything

Note: the 3×3 **PMNS active** neutrino mixing matrix is not exactly unitary:

$$\boxed{U = AU_0}$$

12 angles
+
12 phases

$$A = f_A \left(O_{36}O_{26}O_{16}O_{35}O_{25}O_{15}O_{34}O_{24}O_{14}\right)$$

$$\simeq I - \frac{1}{2}\begin{pmatrix} s^2_{14}+s^2_{15}+s^2_{16} & 0 & 0 \\ 2\hat{s}_{14}\hat{s}^*_{24}+2\hat{s}_{15}\hat{s}^*_{25}+2\hat{s}_{16}\hat{s}^*_{26} & s^2_{24}+s^2_{25}+s^2_{26} & 0 \\ 2\hat{s}_{14}\hat{s}^*_{34}+2\hat{s}_{15}\hat{s}^*_{35}+2\hat{s}_{16}\hat{s}^*_{36} & 2\hat{s}_{24}\hat{s}^*_{34}+2\hat{s}_{25}\hat{s}^*_{35}+2\hat{s}_{26}\hat{s}^*_{36} & s^2_{34}+s^2_{35}+s^2_{36} \end{pmatrix}$$

unitary:

$$U_0 = O_{23}O_{13}O_{12} = \begin{pmatrix} c_{12}c_{13} & \hat{s}^*_{12}c_{13} & \hat{s}^*_{13} \\ -\hat{s}_{12}c_{23}-c_{12}\hat{s}_{13}\hat{s}^*_{23} & c_{12}c_{23}-\hat{s}^*_{12}\hat{s}_{13}\hat{s}^*_{23} & c_{13}\hat{s}^*_{23} \\ \hat{s}_{12}\hat{s}_{23}-c_{12}\hat{s}_{13}c_{23} & -c_{12}\hat{s}_{23}-\hat{s}^*_{12}\hat{s}_{13}c_{23} & c_{13}c_{23} \end{pmatrix}$$

$$\hat{s}_{ij} \equiv e^{i\delta_{ij}} \sin\theta_{ij}$$

The active-sterile flavor mixing:

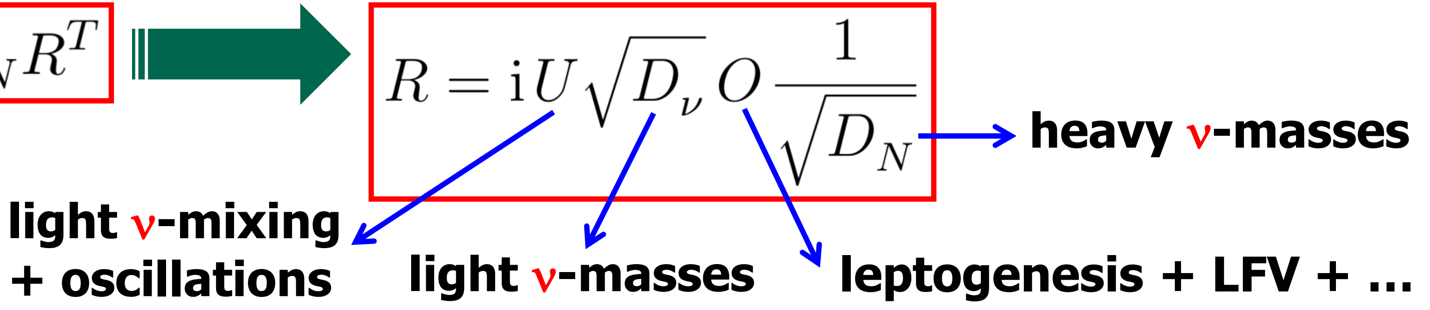
$$R = f_R \left(O_{36}O_{26}O_{16}O_{35}O_{25}O_{15}O_{34}O_{24}O_{14}\right) \simeq \begin{pmatrix} \hat{s}^*_{14} & \hat{s}^*_{15} & \hat{s}^*_{16} \\ \hat{s}^*_{24} & \hat{s}^*_{25} & \hat{s}^*_{26} \\ \hat{s}^*_{34} & \hat{s}^*_{35} & \hat{s}^*_{36} \end{pmatrix}$$

ZZX:
0709.2220
and
1110.0083

An exact factorization of *R*

The active-sterile flavor mixing matrix can be exactly expressed as follows:

$$\boxed{U D_\nu U^T = -R D_N R^T} \Longrightarrow \boxed{R = {\rm i} U \sqrt{D_\nu}\, O \frac{1}{\sqrt{D_N}}}$$



heavy ν-masses

light ν-mixing + oscillations

light ν-masses

leptogenesis + LFV + ...

Inspiration from *hadron physics* :

weak part × strong perturbative part × strong non-perturbative part

The undetermined part is the unknown complex orthogonal matrix: $O O^T = I$

The Casas-Ibarra parametrization (2001):

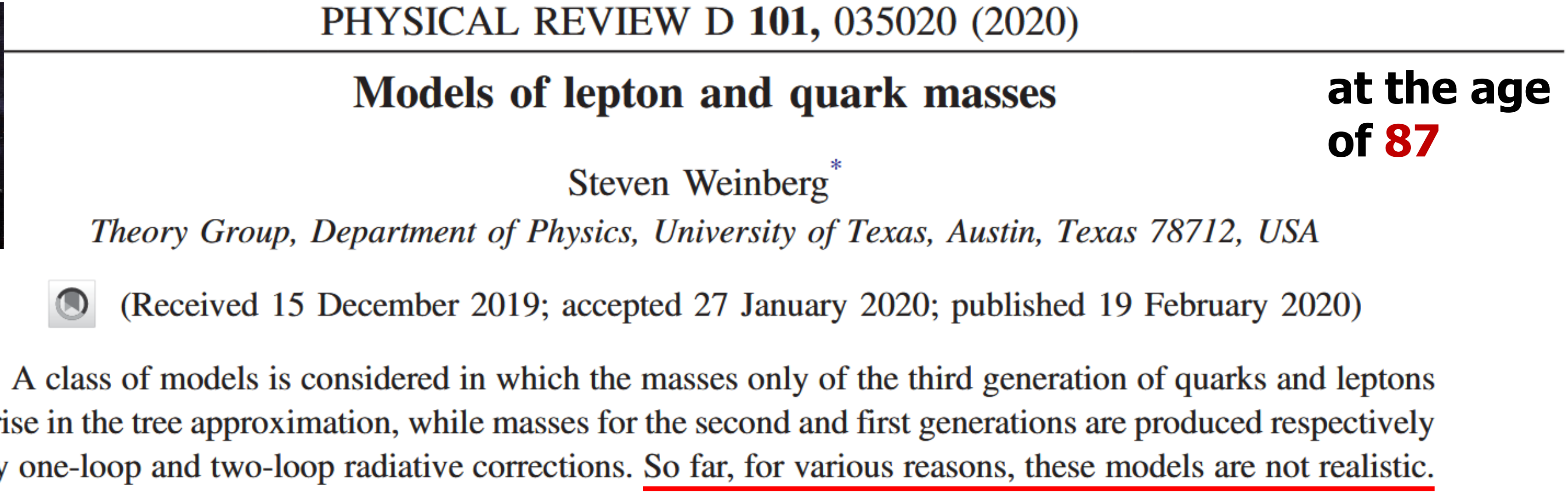
$$\boxed{M_\nu \simeq -M_{\rm D} M_{\rm R}^{-1} M_{\rm D}^T = -\langle H\rangle^2 Y_\nu M_{\rm R}^{-1} Y_\nu^T} \Longrightarrow \boxed{Y_\nu \simeq \frac{\rm i}{\langle H\rangle} U_0 \sqrt{D_\nu}\, O \sqrt{D_N}}$$

★ **This approximate seesaw only contains the dim-5 operator's contributions.**

★ **It is the dim-6 operator that violates the PMNS unitarity (Abada et al, 2007).**

But a factorization has little to do with dynamics

★ **Weinberg**'s conjecture (**2020**): only the **3^{rd} family** of fermions have the **tree level Yukawa** interactions, and the others gain their masses via **loops**.



PHYSICAL REVIEW D **101**, 035020 (2020)

Models of lepton and quark masses

Steven Weinberg*

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(Received 15 December 2019; accepted 27 January 2020; published 19 February 2020)

A class of models is considered in which the masses only of the third generation of quarks and leptons arise in the tree approximation, while masses for the second and first generations are produced respectively by one-loop and two-loop radiative corrections. So far, for various reasons, these models are not realistic.

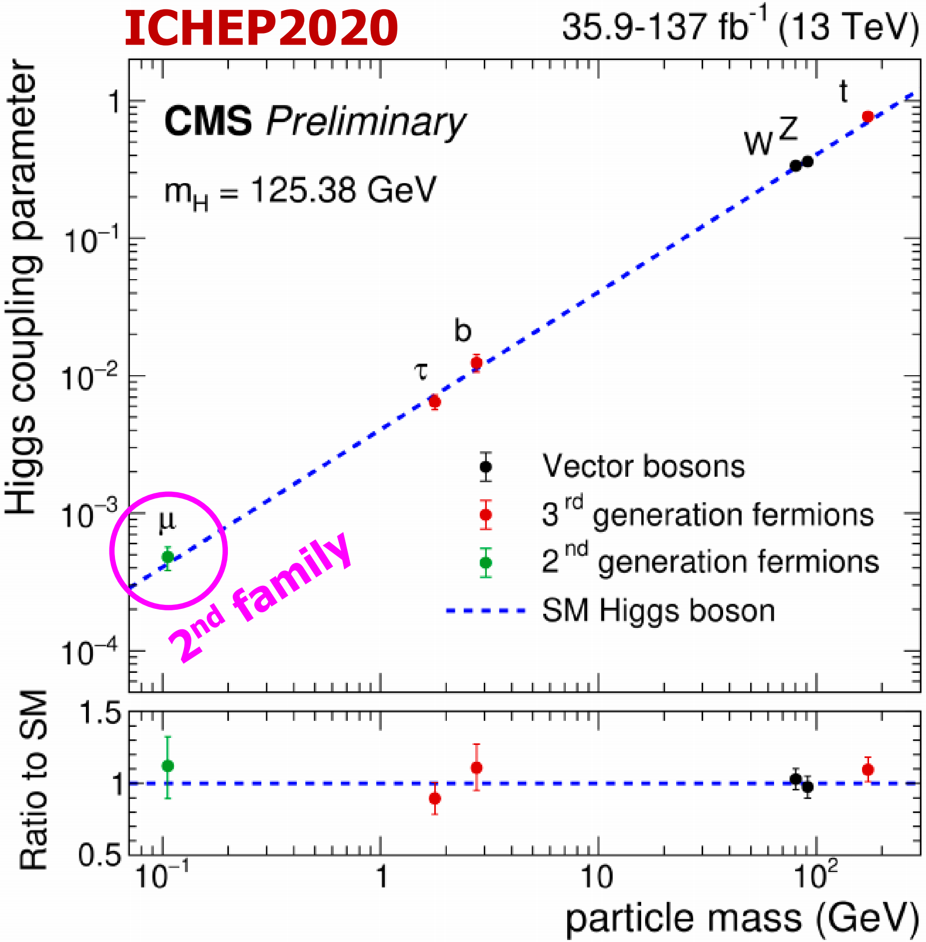
at the age of **87**

Different opinions:

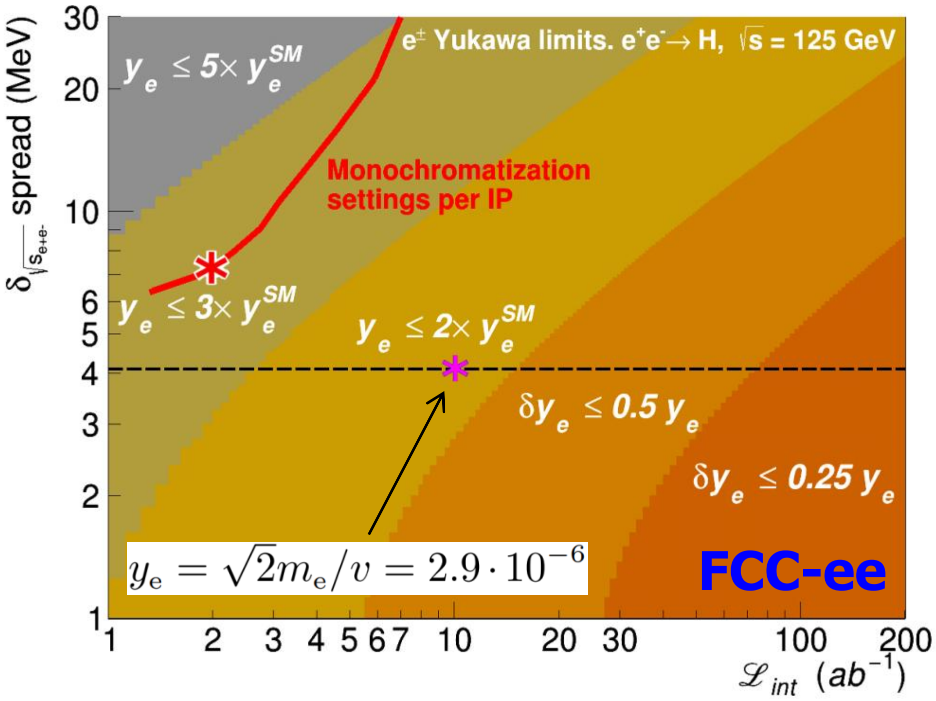
★ **Fermion masses**: primarily stem from **tree-level Yukawa** interactions in SM.

★ **Flavor mixing**: a mismatch between the **Yukawa** and **CC gauge** interactions, should originate **at the same time** as fermion masses.

Yes, we seem to be on the right track



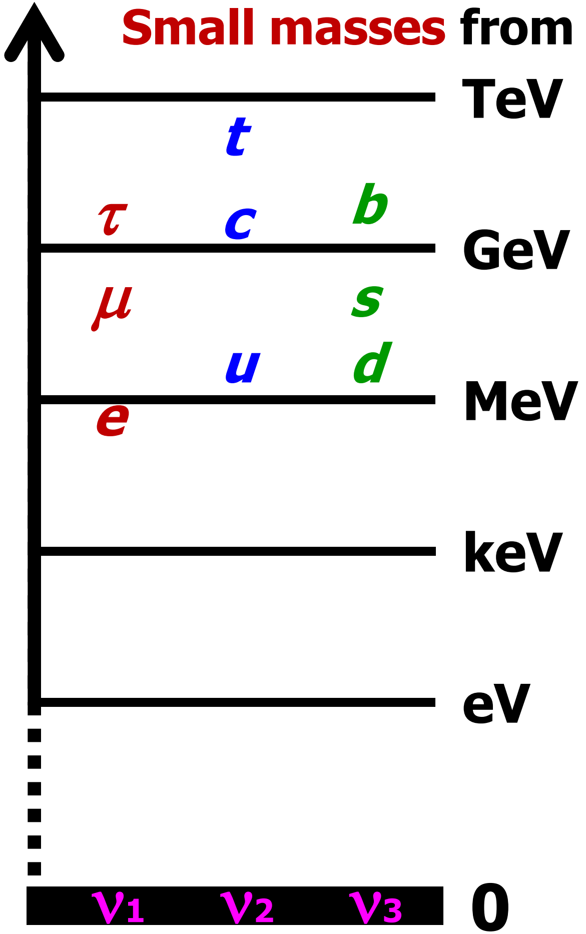
The ambition to measure the electron Yukawa coupling through resonant s-channel Higgs production (D. d'Enterria et al, arXiv: 2107.02686)



$$y_e = \sqrt{2}m_e/v = 2.9\cdot 10^{-6}$$

Hierarchical fermion mass spectrum

Small masses from tree or loop?



Problem 1
flavor
hierarchy

Problem 2
flavor
desert

Problem 3
ν-mass
origin

Example: tree-level nearest-neighbor interactions to generate tiny masses.

S. Weinberg
H. Fritzsch
F. Wilczek + A. Zee
1977

$$M_f = \begin{pmatrix} 0 & B_f \\ B_f^* & A_f \end{pmatrix} \quad \text{with } |B_f| \ll A_f$$

A seesaw mass relation for the lightest fermion

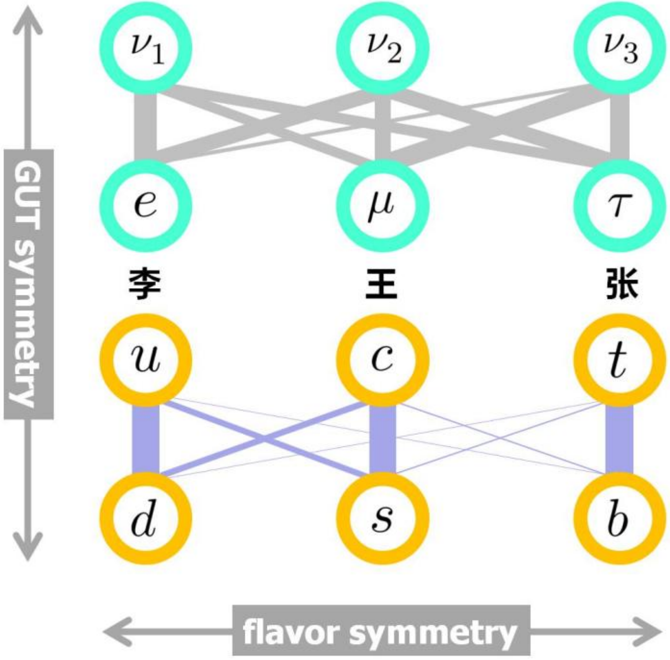
$$\boxed{m_f \sim \frac{|B_f|^2}{A_f}}$$

The Fritzsch texture / double seesaw

$$M_f = \begin{pmatrix} 0 & C_f & 0 \\ C_f^* & 0 & B_f \\ 0 & B_f^* & A_f \end{pmatrix} \Longrightarrow m_f \sim A_f \frac{|C_f|^2}{|B_f|^2}$$

More challenges are from flavor mixing

$$-\mathcal{L}_{\rm cc} = \frac{g}{\sqrt{2}}\left[\overline{\left(u \quad c \quad t\right)_{\rm L}}\,\gamma^{\mu}\,\underset{\textbf{CKM}\uparrow}{V}\begin{pmatrix} d \\ s \\ b \end{pmatrix}_{\rm L} W^{+}_{\mu} + \overline{\left(e \quad \mu \quad \tau\right)_{\rm L}}\,\gamma^{\mu}\,\underset{\textbf{PMNS}\uparrow}{U}\begin{pmatrix} \nu_1 \\ \nu_2 \\ \nu_3 \end{pmatrix}_{\rm L} W^{-}_{\mu}\right] + {\rm h.c.}$$



♣ **I. Esteban et al (2007.14792):**

$$U = \begin{pmatrix} 0.801 \to 0.845 & 0.513 \to 0.579 & 0.143 \to 0.155 \\ 0.234 \to 0.500 & 0.471 \to 0.689 & 0.637 \to 0.776 \\ 0.271 \to 0.525 & 0.477 \to 0.694 & 0.613 \to 0.756 \end{pmatrix}$$

Models: a constant matrix + corrections.

♣ **Particle Data Group (2020):**

$$V = \begin{pmatrix} 0.97401 \pm 0.00011 & 0.22650 \pm 0.00048 & 0.00361^{+0.00011}_{-0.00009} \\ 0.22636 \pm 0.00048 & 0.97320 \pm 0.00011 & 0.04053^{+0.00083}_{-0.00061} \\ 0.00854^{+0.00023}_{-0.00016} & 0.03978^{+0.00082}_{-0.00060} & 0.999172^{+0.000024}_{-0.000035} \end{pmatrix}$$

Models: the identity matrix + corrections.

OUTLINE

Part A —— Some general remarks:

From Weinberg 1967 to Weinberg 2020

Part B —— Two specific examples:

On neutrino EFTs and modular symmetry √

Neutrino masses in the SMEFT (tree level)

The *SMEFT* is built with the SM degrees of freedom and the SM gauge symmetry

$$\mathcal{L} = \mathcal{L}_{\rm SM} + \sum_{d=5}^{\infty} \sum_{i=1}^{n_d} \left(\frac{1}{\Lambda^{d-4}} C_i^{(d)} \mathcal{O}_i^{(d)} + \text{h.c.} \right)$$

Dimension-5: the unique Weinberg operator for ν-masses (S. Weinberg 1979)

Dimension-6: W. Buchmuller, D. Wyler, 1986; B. Grzadkowski et al, 2010; …

Dimension-7: L. Lehman, 2014; Y. Liao, X.D. Ma, 2016; …

Dimension-8: C.W. Murphy, 2020; H.L. Li et al, 2020; …

Dimension-9: Y. Liao, X.D. Ma, 2020; H.L. Li et al, 2020, 2021; …

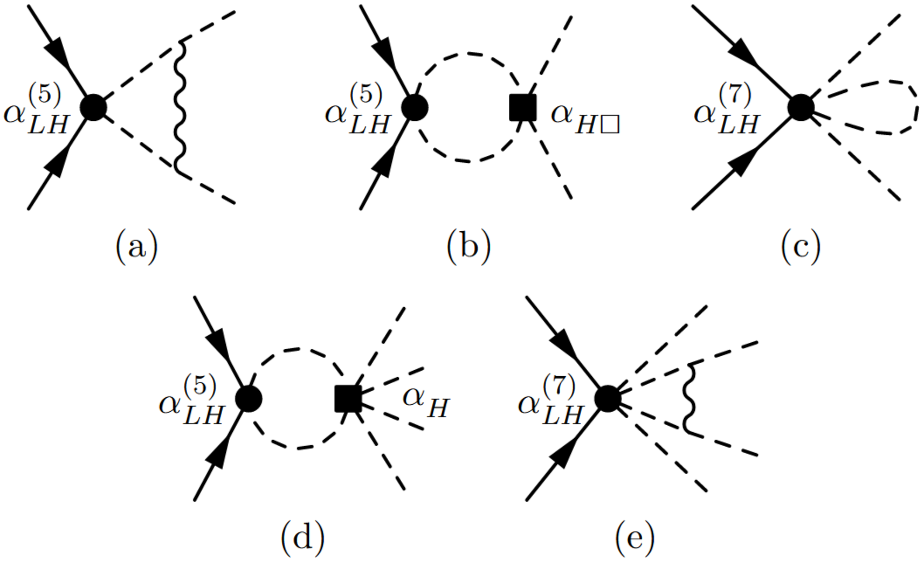
Even (odd) dimensional operators if ($B-L$)/2 is even (odd) (A. Kobach 2016).

For a given mass dimension ($2n + 5$), there is only a unique operator that can give neutrino masses at the tree level (F. Bonnet et al, 2009; Y. Liao, 2011):

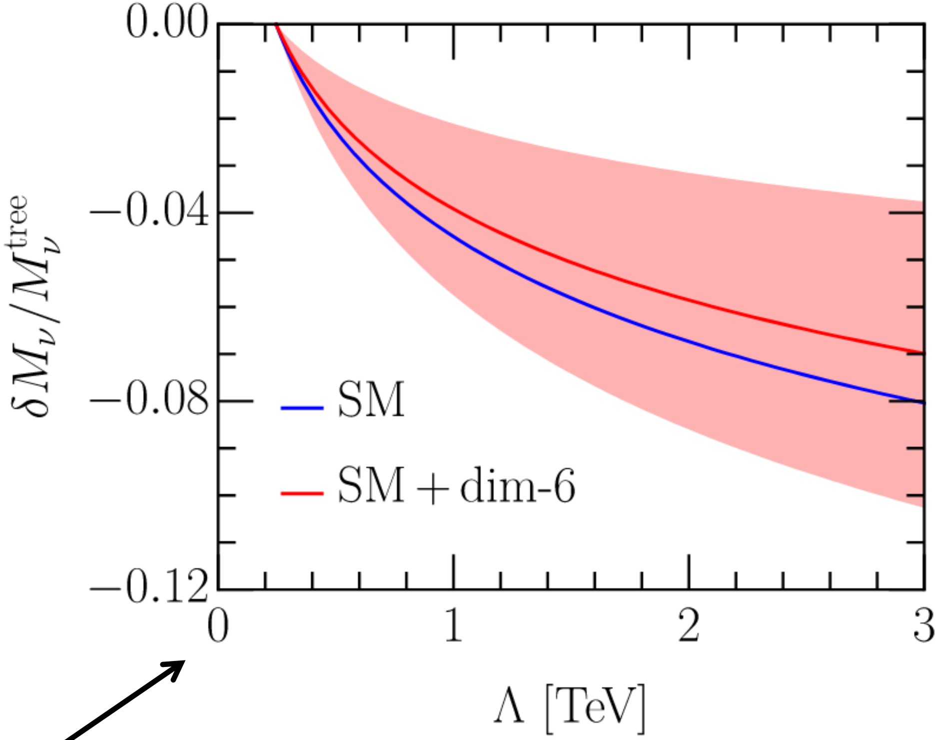
$$\mathcal{O}_{LH}^{(2n+5)} = \overline{\ell_{\rm L}} \widetilde{H} \widetilde{H}^T \ell_{\rm L}^c \left(H^\dagger H \right)^n \Longrightarrow M_\nu^{\rm tree} = -\frac{v^2}{\Lambda} \left(\alpha_{LH}^{(5)} + \alpha_{LH}^{(7)} \frac{v^2}{2\Lambda^2} \right)$$ **(up to dim-7)**

Neutrino masses in the SMEFT (one loop)

Neutrino masses get the one-loop corrections from dim-(5 + 6 + 7) operators



(examples of the one-loop diagrams)



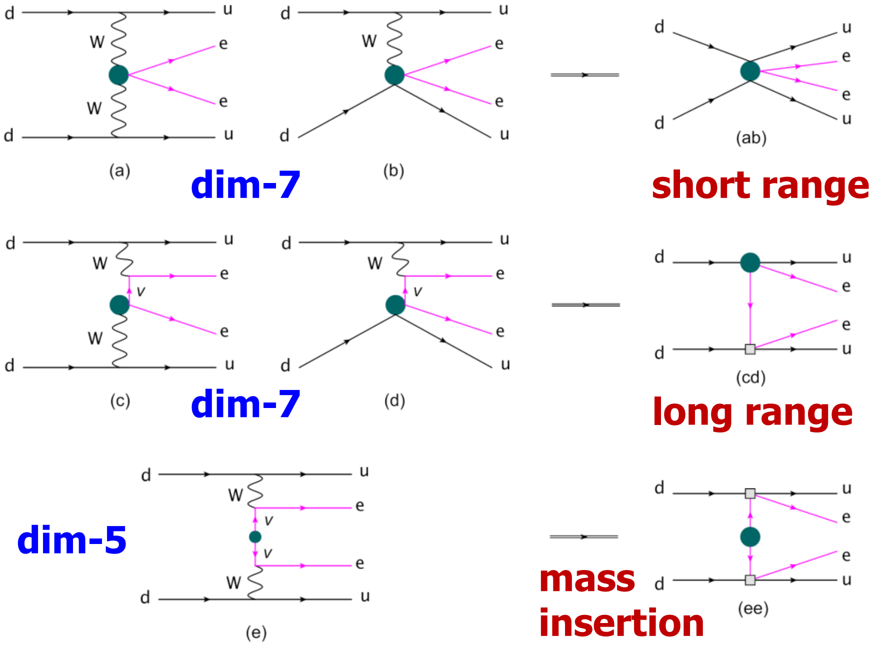
Radiative corrections are in particular triggered by dim-6 terms.

An illustration for a ball-park feeling.

M. Chala, A. Titov, 2104.08248;
J. Ellis et al, 2012.02779.

More on neutrino masses in the SMEFT

Neutrinoless double-beta decay: apart from the dim-5 Weinberg operator, the next contribution is from the dim-7 operators:



dim-7

short range

dim-7

long range

dim-5

mass insertion

V. Cirigliano et al, 2017; 2018;
M. Horoi, A. Neacsu, 1706.05391;
Y. Liao, X.D. Ma, 2019; …

Some other aspects:

♦ Lepton flavor violation in the SMEFT

A. Crivellin, S. Najjari, J. Rosiek, 2014;
S. Davidson, 2016;
A. Crivellin et al, 2017;
S. Davidson, 2021;
M. Ardu, S. Davidson, 2103.07212; …

♦ The SMEFT extended with sterile ν's

F. del Aguila et al, 2009;
S. Bhattacharya, J. Wudka, 2016;
Y. Liao, X.D. Ma, 2017;
A. Datta et al, 2021;
B. H.L. Li et al, 2105.09329; …

Neutrino EFT

ν-EFT is defined as an EFT after integrating out heavy degrees of freedom in a given ultraviolet neutrino model, e.g., the type-I seesaw model.

The type-I seesaw with three right-handed neutrinos:

$$\mathcal{L}_{\rm UV} = \mathcal{L}_{\rm SM} + \overline{N_{\rm R}}\,{\rm i}\not{\partial} N_{\rm R} - \left(\frac{1}{2}\overline{N^c_{\rm R}} M N_{\rm R} + \overline{\ell_{\rm L}} Y_\nu \widetilde{H} N_{\rm R} + {\rm h.c.}\right)$$

Integrate out heavy degrees of freedom at the tree level up to dim-6 operator

$$\mathcal{L}^{\rm tree}_{\rm EFT} = \mathcal{L}_{\rm SM} + \left[\frac{1}{2} C^{(5)}_{\alpha\beta} \mathcal{O}^{(5)}_{\alpha\beta} + {\rm h.c.}\right] + C^{(6)}_{\alpha\beta} \mathcal{O}^{(6)}_{\alpha\beta} \quad \textbf{with} \quad \boxed{C^{(5)}_{\alpha\beta} = \left(Y_\nu M^{-1} Y^T_\nu\right)_{\alpha\beta}, C^{(6)}_{\alpha\beta} = \left(Y_\nu M^{-2} Y^\dagger_\nu\right)_{\alpha\beta}}$$

$$\boxed{\mathcal{O}^{(5)}_{\alpha\beta} = \overline{\ell_{\alpha\rm L}} \widetilde{H} \widetilde{H}^T \ell^c_{\beta\rm L}}$$

Weinberg operator

$$\begin{aligned}\mathcal{O}^{(6)}_{\alpha\beta} &= \left(\overline{\ell_{\alpha\rm L}} \widetilde{H}\right) {\rm i}\not{\partial} \left(\widetilde{H}^\dagger \ell_{\beta\rm L}\right) \\ &= \frac{1}{4}\left[\left(\overline{\ell_{\alpha\rm L}} \gamma^\mu \ell_{\beta\rm L}\right)\left(H^\dagger {\rm i}\overleftrightarrow{D}_\mu H\right) - \left(\overline{\ell_{\alpha\rm L}} \gamma^\mu \tau^I \ell_{\beta\rm L}\right)\left(H^\dagger {\rm i}\overleftrightarrow{D}^I_\mu H\right)\right]\end{aligned}$$

Warsaw basis

violate unitarity of the PMNS matrix.

A. Brancano et al, 2003; …

In the minimal unitarity violation scheme which contains the above tree-level operators, *unitarity violation* of the PMNS matrix can be well constrained by current experimental data on precision EW measurements and LFV processes (S. Antusch et al, 2006; S. Antusch, O. Fischer, 2014; M. Blennow et al, 2017; …)

A self-consistent calculation of μ → *e* + γ (1)

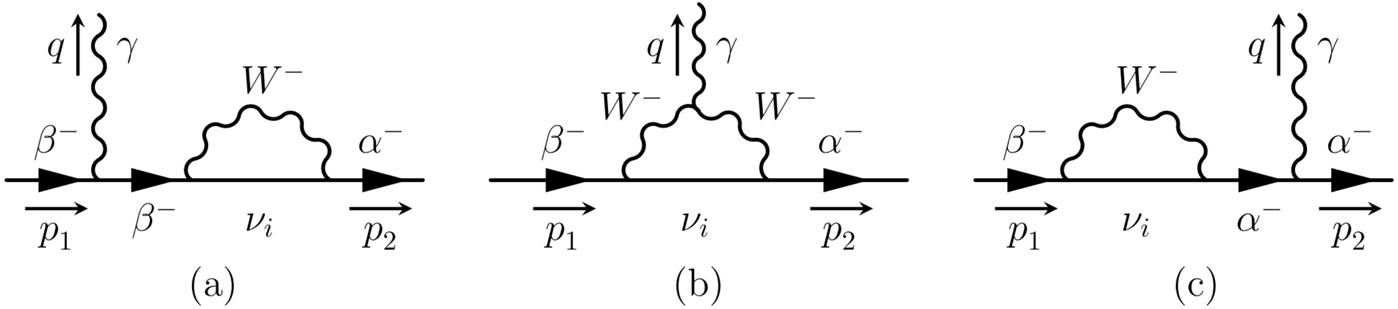
After SSB, the effective Lagrangian with tree-level dim-5 and dim-6 operators

$$\mathcal{L}_{\rm EFT}^{\rm tree} \supset \overline{\nu_{\rm L}}\, {\rm i} \not{\partial} \nu_{\rm L} - \left(\overline{l_{\rm L}} M_l l_{\rm R} + \frac{1}{2}\overline{\nu_{\rm L}} \widehat{M}_\nu \nu_{\rm L}^{\rm c} + {\rm h.c.}\right) + \left(\frac{g_2}{\sqrt{2}} \overline{l_{\rm L}}\gamma^\mu U \nu_{\rm L} W_\mu^- + {\rm h.c.}\right)$$

$$\boxed{U = (1-\frac{1}{2}RR^\dagger)U_0}$$

$$\boxed{R = \frac{v}{\sqrt{2}}Y_\nu M^{-1}}$$

contributing to the LFV decays like μ → *e* + γ via the one-loop diagrams (a)—(c) in the unitary gauge:



(a) (b) (c)

T.P. Cheng, L.F. Li, 1980; A. Ilakovac, A. Pilaftsis, 1995; R. Alonso et al, 2013; Z.Z. Xing, D. Zhang, 2020.

The amplitude of diagrams (a)—(c) is

$${\rm i}\mathcal{M}_{abc} = \frac{-{\rm i}eg_2^2}{2\left(4\pi\right)^2 M_W^2}\sum_{i=1}^{3} U_{\alpha i}U_{\beta i}^* \left(-\frac{5}{6}+\frac{m_i^2}{4M_W^2}\right)\left[\epsilon_\mu^* \overline{u}\left(p_2\right){\rm i}\sigma^{\mu\nu}q_\nu\left(m_\alpha P_{\rm L}+m_\beta P_{\rm R}\right)u\left(p_1\right)\right]$$

a result *inconsistent* with that obtained in the full type-I seesaw model.

A self-consistent calculation of $\mu \to e + \gamma$ (2)

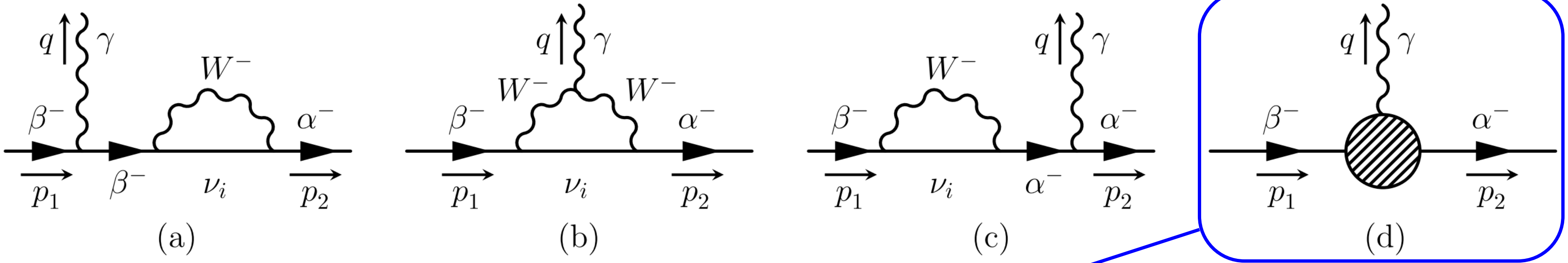
The reason is the missing of contributions from one-loop matching operators:

$$\mathcal{L}^{(6)}_{\rm loop} \supset \frac{\left(Y_\nu M^{-2} Y_\nu^\dagger Y_l\right)_{\alpha\beta}}{24\,(4\pi)^2}\left[g_1\left(\overline{\ell_{\alpha \rm L}}\sigma_{\mu\nu}E_{\beta \rm R}\right)HB^{\mu\nu} + 5g_2\left(\overline{\ell_{\alpha \rm L}}\sigma_{\mu\nu}E_{\beta \rm R}\right)\tau^I H W^{I\mu\nu}\right] + {\rm h.c.}$$

SSB

$$-\frac{eg_2^2}{12\,(4\pi)^2\,M_W^2}\overline{l_{\rm L}}\sigma_{\mu\nu}RR^\dagger M_l l_{\rm R}F^{\mu\nu} + {\rm h.c.}$$

contributing directly via diagram (d):
(D. Zhang, S. Zhou, 2102.04954)



(a) (b) (c) (d)

The total amplitude turns out to be

$$\mathrm{i}\mathcal{M}_{\rm tot} = \frac{-\mathrm{i}eg_2^2}{2\,(4\pi)^2\,M_W^2}\left[\sum_{i=1}^{3}U_{\alpha i}U^*_{\beta i}\left(-\frac{5}{6}+\frac{m_i^2}{4M_W^2}\right)\underline{-\frac{1}{3}\left(RR^\dagger\right)_{\alpha\beta}}\right]\left[\epsilon^*_\mu\overline{u}\left(p_2\right)\mathrm{i}\sigma^{\mu\nu}q_\nu\left(m_\alpha P_{\rm L}+m_\beta P_{\rm R}\right)u\left(p_1\right)\right]$$

This result is *consistent* with the full type-I seesaw picture —— a good thing!

Flavor symmetries

Flavor symmetry is a powerful guiding principle for ***neutrino*** **model building**?

—— in principle, no problem.

—— in practice, a lot to pay.

S_3 , S_4 , A_4 , A_5 , $U(1)_F$, $SU(2)_F$, modular, ….

- ♣ Abelian or non-Abelian
- ♣ continuous or discrete
- ♣ local or global
- ♣ broken spontaneously or explicitly

● flavons; ● assignments; ● how to break?

★ **Froggatt-Nielsen** mechanism (1979)

Example (T. Kobayashi, ZZX, 1997):

$$M'_{\rm u} \sim \langle H_2 \rangle \begin{pmatrix} 0 & \epsilon_{\rm u}^3 & 0 \\ \epsilon_{\rm u}^3 & \epsilon_{\rm u}^2 & \epsilon_{\rm u}^2 \\ 0 & \epsilon_{\rm u}^2 & 1 \end{pmatrix} , \quad M'_{\rm d} \sim \langle H_1 \rangle \begin{pmatrix} 0 & \epsilon_{\rm d}^3 & 0 \\ \epsilon_{\rm d}^3 & \epsilon_{\rm d}^2 & \epsilon_{\rm d}^2 \\ 0 & \epsilon_{\rm d}^2 & 1 \end{pmatrix}$$

CKM = identity matrix + perturbations

Hierarchy of masses and flavor mixing

★ **Discrete** flavor symmetries (1978—)

Example (K.S. Babu, X.G. He, 2005):

$$M_l = \sqrt{3}\, v U_\omega \begin{pmatrix} \lambda_l & 0 & 0 \\ 0 & \lambda'_l & 0 \\ 0 & 0 & \lambda''_l \end{pmatrix} , \quad M_\nu \simeq -\frac{\lambda_\nu^2 v_\phi^2}{M} \begin{pmatrix} \xi & 0 & -\zeta \\ 0 & 1 & 0 \\ -\zeta & 0 & \xi \end{pmatrix}$$

PMNS = constant matrix + corrections

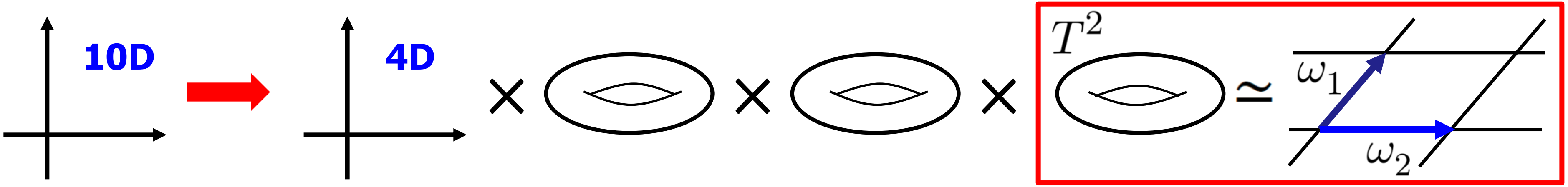
Specific **constant** flavor mixing pattern

New development: modular flavor symmetry with its own advantages + disadvantages

New progress: modular symmetry

The modular symmetry: G. Altarelli, F. Feruglio, 2006; F. Feruglio, 1706.08749

Orbifold compactification: 10D string theory → 4D SM + 3 copies of 2D torus.



$$S = \int \mathrm{d}^4x\, \mathrm{d}^6y \mathcal{L}_{\mathrm{10D}} \longrightarrow \int \mathrm{d}^4x \mathcal{L}_{\mathrm{eff}} \longrightarrow \tau \equiv \frac{\omega_1}{\omega_2}$$

: modular parameter

Modular transformation (*a*, *b*, *c*, *d* are integer, and *ad* − *bc* = 1):

$$\begin{pmatrix} \omega_1 \\ \omega_2 \end{pmatrix} \to \begin{pmatrix} \omega_1' \\ \omega_2' \end{pmatrix} = \begin{pmatrix} a & b \\ c & d \end{pmatrix} \begin{pmatrix} \omega_1 \\ \omega_2 \end{pmatrix} \Rightarrow \boxed{\tau \to \gamma\tau = \frac{\omega_1'}{\omega_2'} = \frac{a\tau + b}{c\tau + d}}$$

$$S: \ \tau \longrightarrow -\frac{1}{\tau}\,, \quad \begin{pmatrix} a & b \\ c & d \end{pmatrix} = \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix}$$

duality

$$T: \ \tau \longrightarrow \tau + 1\,, \quad \begin{pmatrix} a & b \\ c & d \end{pmatrix} = \begin{pmatrix} 1 & 1 \\ 0 & 1 \end{pmatrix}$$

shift

Modular group:

$$\Gamma \simeq \left\{ S, T \mid S^2 = I, (ST)^3 = I \right\}$$

Modular group has finite subgroups.

Modular invariance as flavor symmetry

The quotient group of modular group and its principal congruence subgroups:

$$\boxed{\Gamma_N \simeq \left\{S,T \mid S^2=I,(ST)^3=I,T^N=I\right\}} \longrightarrow \Gamma_2\simeq S_3,\ \Gamma_3\simeq A_4,\ \Gamma_4\simeq S_4,\ \Gamma_5\simeq A_5$$

$$\tau \to \gamma\tau = \frac{\omega_1'}{\omega_2'} = \frac{a\tau+b}{c\tau+d}$$

$$\chi^{(I)} \to (c\tau+d)^{-k_I}\,\rho^{(I)}(\gamma)\,\chi^{(I)}$$

supermultiplet

weight

unitary representation matrix of finite modular group

key point

$$Y_{\mathrm{r}}^{(k)} \equiv \begin{pmatrix} f_1(\tau)\\ f_2(\tau)\\ \vdots \end{pmatrix}$$

Under modular transformation the superpotential is invariant:

$$\boxed{Y_{I_1\cdots I_n}(\tau) \longrightarrow (c\tau+d)^{k_Y}\,\rho_Y(\gamma)\,Y_{I_1\cdots I_n}(\tau)}$$

Modular transformation is the transformation of modulus τ, and a Yukawa coupling depends on τ.

It is always possible to choose a proper basis:

$$\boxed{Y_{\mathrm{r}}^{(2k)}(\gamma\tau) = (c\tau+d)^{2k}\,\rho_{\mathrm{r}}(\gamma)\,Y_{\mathrm{r}}^{(2k)}(\tau),\quad \gamma\in\overline{\Gamma}\equiv\Gamma/Z_2}$$

transform as the irreducible representation of Γ_N .

$$\mathcal{W}(\tau,\chi) \longrightarrow \mathcal{W}(\tau,\chi) = \sum_n \sum_{\{I_1,\cdots,I_n\}} Y_{I_1\cdots I_n}(\tau)\,\chi^{(I_1)}\cdots\chi^{(I_n)}$$

← **The Modular form of Yukawa couplings.**

Example: modular A4 (1)

For the modular group $\Gamma_3 \simeq A_4$ **with weight 2 and Dedekind η-function,**

$$Y_1(\tau) = \frac{\mathrm{i}}{2\pi}\left[\frac{\dot\eta(\tau/3)}{\eta(\tau/3)} + \frac{\dot\eta(\tau/3+1/3)}{\eta(\tau/3+1/3)} + \frac{\dot\eta(\tau/3+2/3)}{\eta(\tau/3+2/3)} - 27\frac{\dot\eta(3\tau)}{\eta(3\tau)}\right] = 1 + 12q + 36q^2 + 12q^3 + \cdots \ ,$$

$$Y_2(\tau) = \frac{-\mathrm{i}}{\pi}\left[\frac{\dot\eta(\tau/3)}{\eta(\tau/3)} + \omega^2\frac{\dot\eta(\tau/3+1/3)}{\eta(\tau/3+1/3)} + \omega\frac{\dot\eta(\tau/3+2/3)}{\eta(\tau/3+2/3)}\right] = -6q^{1/3}\left(1 + 7q + 8q^2 + \cdots\right) \ ,$$

$$Y_3(\tau) = \frac{-\mathrm{i}}{\pi}\left[\frac{\dot\eta(\tau/3)}{\eta(\tau/3)} + \omega\frac{\dot\eta(\tau/3+1/3)}{\eta(\tau/3+1/3)} + \omega^2\frac{\dot\eta(\tau/3+2/3)}{\eta(\tau/3+2/3)}\right] = -18q^{2/3}\left(1 + 2q + 5q^2 + \cdots\right) \ ,$$

where $\eta(-1/\tau) = \eta(\tau)\sqrt{-\mathrm{i}\tau}$ **,** $\eta(\tau+1) = \eta(\tau)\exp(\mathrm{i}\pi/12)$ **,** $q = \exp(\mathrm{i}2\pi\tau)$ **,** $\omega = \exp(\mathrm{i}2\pi/3)$

An explicit model in MSSM (T. Kobayashi et al, 2018):

$$\ell_\mathrm{L} = (\ell_{e\mathrm{L}}, \ell_{\mu\mathrm{L}}, \ell_{\tau\mathrm{L}})^T \sim \underline{\mathbf{3}} \ , \quad N_\mathrm{R} = (N_{e\mathrm{R}}, N_{\mu\mathrm{R}}, N_{\tau\mathrm{R}})^T \sim \underline{\mathbf{3}} \ , \quad E_{e\mathrm{R}} \sim \underline{\mathbf{1}} \ , \ E_{\mu\mathrm{R}} \sim \underline{\mathbf{1}}'' \ , \ E_{\tau\mathrm{R}} \sim \underline{\mathbf{1}}' \ ;$$

$$H_1 \sim \underline{\mathbf{1}} \ , \quad H_2 \sim \underline{\mathbf{1}} \ , \quad Y(\tau) \sim \underline{\mathbf{3}} \ ,$$

$$\begin{aligned}\mathcal{W}_\mathrm{lepton} = {} & \alpha_l\,(E_{e\mathrm{R}})_{\underline{\mathbf{1}}}\,(H_1)_{\underline{\mathbf{1}}}\,[\ell_\mathrm{L}Y(\tau)]_{\underline{\mathbf{1}}} + \beta_l\left(E_{\mu\mathrm{R}}\right)_{\underline{\mathbf{1}}''}(H_1)_{\underline{\mathbf{1}}}\,[\ell_\mathrm{L}Y(\tau)]_{\underline{\mathbf{1}}'} + \gamma_l\,(E_{\tau\mathrm{R}})_{\underline{\mathbf{1}}'}\,(H_1)_{\underline{\mathbf{1}}}\,[\ell_\mathrm{L}Y(\tau)]_{\underline{\mathbf{1}}''} \\ & + g_1\,(N_\mathrm{R})_{\underline{\mathbf{3}}}\,(H_2)_{\underline{\mathbf{1}}}\cdot[\ell_\mathrm{L}Y(\tau)]_{\underline{\mathbf{3}}_\mathrm{S}} + g_2\,(N_\mathrm{R})_{\underline{\mathbf{3}}}\,(H_2)_{\underline{\mathbf{1}}}\cdot[\ell_\mathrm{L}Y(\tau)]_{\underline{\mathbf{3}}_\mathrm{A}} \\ & + M_0\,(N_\mathrm{R}N_\mathrm{R})_{\underline{\mathbf{3}}_\mathrm{S}}\cdot[Y(\tau)]_{\underline{\mathbf{3}}} \ .\end{aligned}$$

no flavons

Example: modular A4 (2)

After spontaneous gauge symmetry breaking, one is left with flavor textures:

$$M_l = \begin{pmatrix} Y_1^* & Y_2^* & Y_3^* \\ Y_3^* & Y_1^* & Y_2^* \\ Y_2^* & Y_3^* & Y_1^* \end{pmatrix} \begin{pmatrix} \alpha_l & 0 & 0 \\ 0 & \beta_l & 0 \\ 0 & 0 & \gamma_l \end{pmatrix}$$

lepton flavor mixing

$$M_{\mathrm{R}} = M_0 \begin{pmatrix} 2Y_1^* & -Y_3^* & -Y_2^* \\ -Y_3^* & 2Y_2^* & -Y_1^* \\ -Y_2^* & -Y_1^* & 2Y_3^* \end{pmatrix} ,$$

$$M_{\mathrm{D}} = v_2 \begin{pmatrix} 2g_1^* Y_1^* & -\left(g_1^* + g_2^*\right) Y_3^* & \left(g_2^* - g_1^*\right) Y_2^* \\ \left(g_2^* - g_1^*\right) Y_3^* & 2g_1^* Y_2^* & -\left(g_1^* + g_2^*\right) Y_1^* \\ -\left(g_1^* + g_2^*\right) Y_2^* & \left(g_2^* - g_1^*\right) Y_1^* & 2g_1^* Y_3^* \end{pmatrix}$$

$$\boxed{M_\nu \simeq -M_{\mathrm{D}} M_{\mathrm{R}}^{-1} M_{\mathrm{D}}^T}$$

neutrino seesaw

Comment A: **physical meaning of the complex modular parameter τ is unclear.**

Comment B: **the flavor textures are not transparent at all, and the number of free parameters is still unsatisfactory. A careful numerical fitting is needed.**

Comment C: **no good reason for the strong mass hierarchy of charged leptons**

Recent development (an incomplete list)

A hot direction: a lot of papers have been published in the past four years.

Topic 1: lepton and quark flavor issues

Modular S3: Kobayashi et al, 2018, 2019;

Modular A4: Criado, Feruglio, 2018; Kobayashi et al, 2018; Okada, Tanimoto, 2019; Ding et al, 2019; **Zhang**, 2020; **Wang**, 2020; King, King, 2020; …

Modular S4: Penedo, Petcov, 2019; Kobayashi et al, 2020; **Wang**, **Zhou**, 2020; **Zhang**, **Zhou**, 2021; …

Modular A5: Novichkov et al, 2019; Ding et al, 2019; Criado et al, 2020; …

Topic 2: multiple modular symmetries

Varzielas et al, 2020; King, Zhou, 2020, King,Zhou, 2021; …

Topic 3: double covering of ΓN

Modular A4′: Liu, Ding, 2019; Lu et al, 2020

Modular S4′: Novichkov et al, 2021; Liu et al, 2021; …

Modular A5′: **Wang**, **Yu**, **Zhou**, 2021; Yao et al, 2021; …

Topic 4: fixed points + residual symmetry

Novichkov et al, 2019; Ding et al, 2019; Varzielas et al, 2020; Feruglio et al, 2021; Okada, Tanimoto, 2021; **Wang**, **Zhou**, 2021

Topic 5: modular symmetry and gCP

Novichkov et al, 2019; Baul et al, 2019; …

Other topics: GUT, top-down approach, …

Summary

★ **In the coming 20 years, precision tests of fundamental Yukawa interactions of charged fermions remain a big challenge at the energy frontier.**

★ **The trivial neutrino Yukawa interactions (i.e., *Dirac* neutrinos) would make an experimental test impossible. This possibility is theoretically unnatural.**

★ **The Majorana nature of neutrinos is so appealing, and the seesaw picture is consistent with the spirit of Weinberg's EFT and thus in the landscape.**

★ **The modular symmetry, different from the conventional flavor symmetry in several aspects (modulus parameter versus CG coefficients, …), offers a new string + SUSY possibility to look at the neutrino Yukawa interactions.**

THANK YOU FOR YOUR ATTENTION