

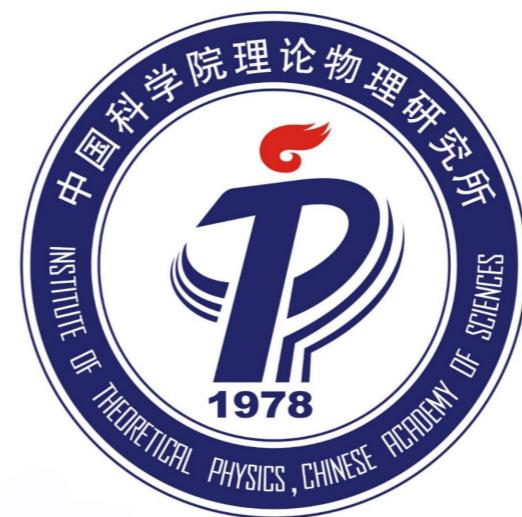
Effective picture of bubble expansion

Shao-Jiang Wang

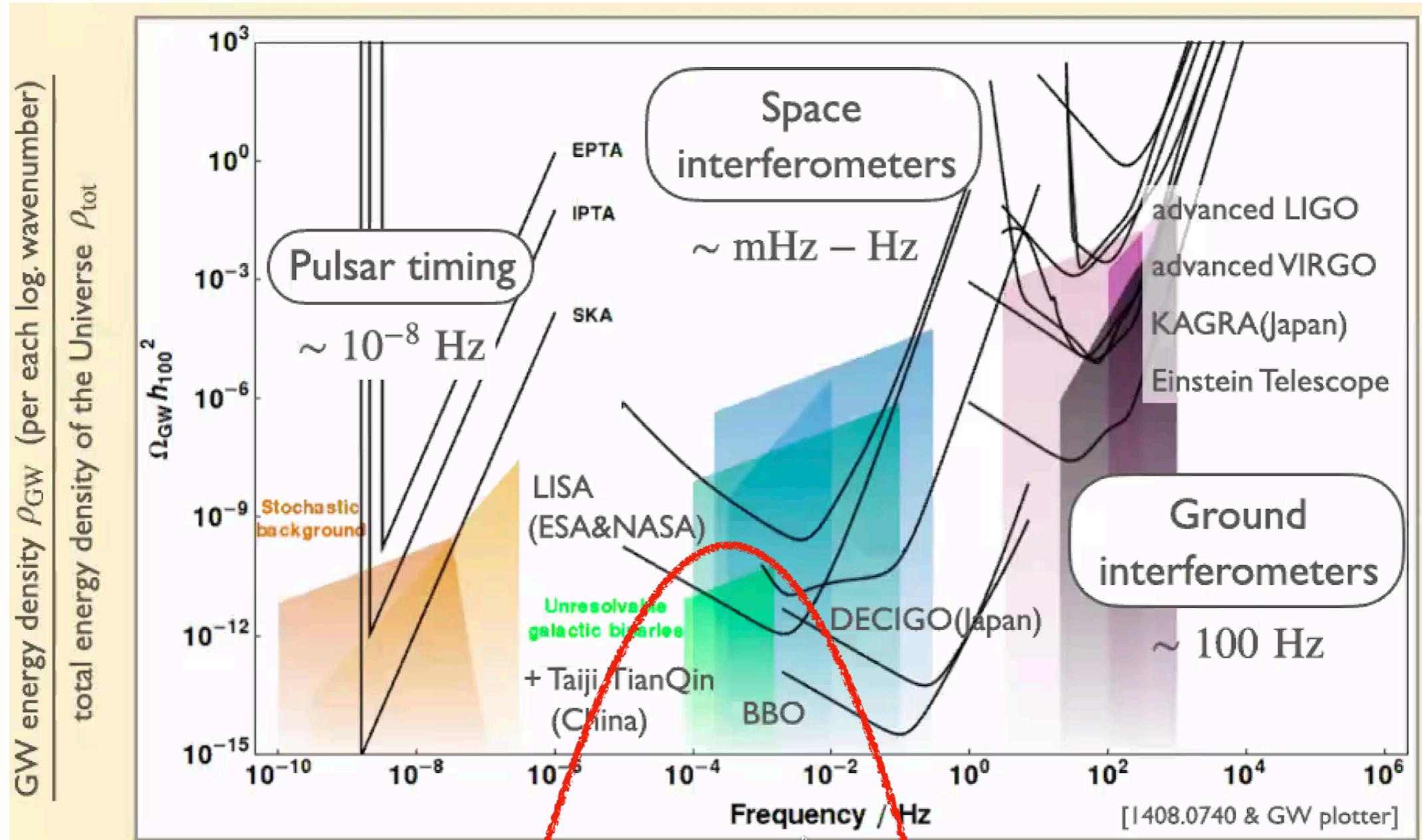
Institute of Theoretical Physics
Chinese Academy of Sciences

Higgs potential and BSM opportunity
@Nanjing @2021/08/29 10:20-10:40

Based on [2011.11451\(JCAP 03 \(2021\) 096\)](#)



GWs from FOPT



T_{PT}

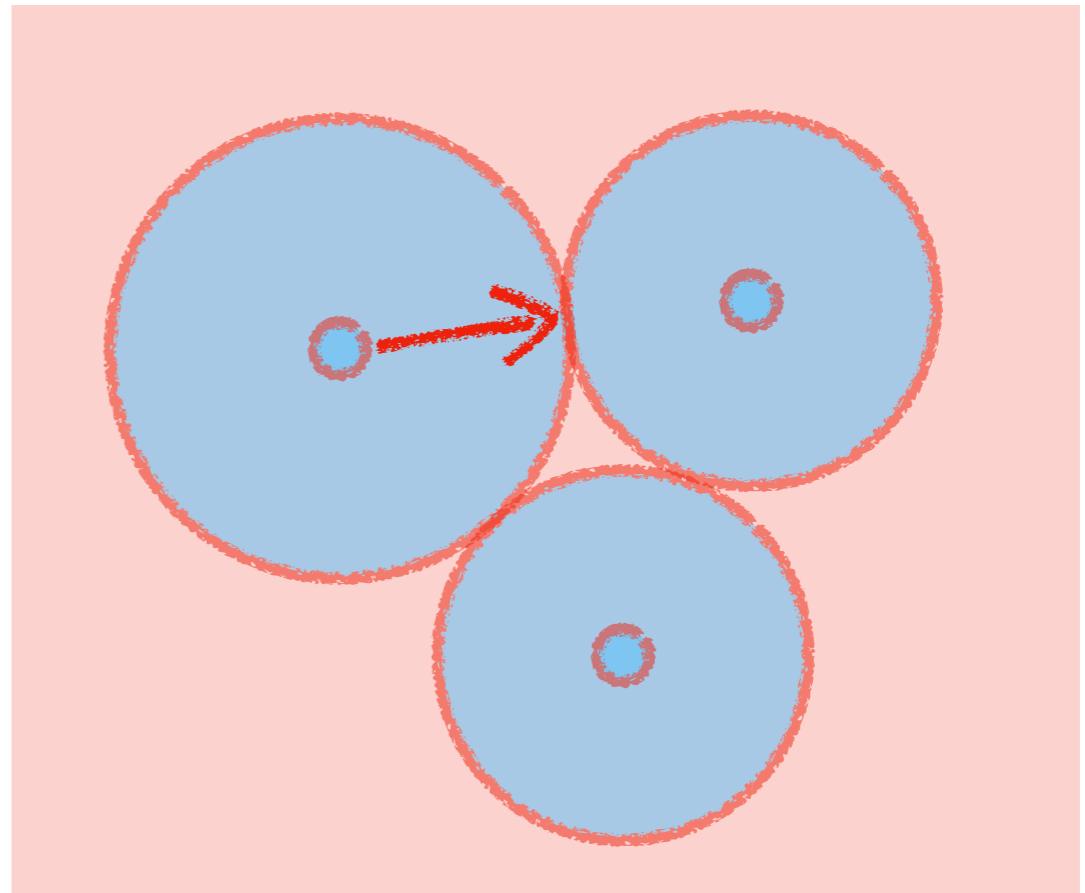
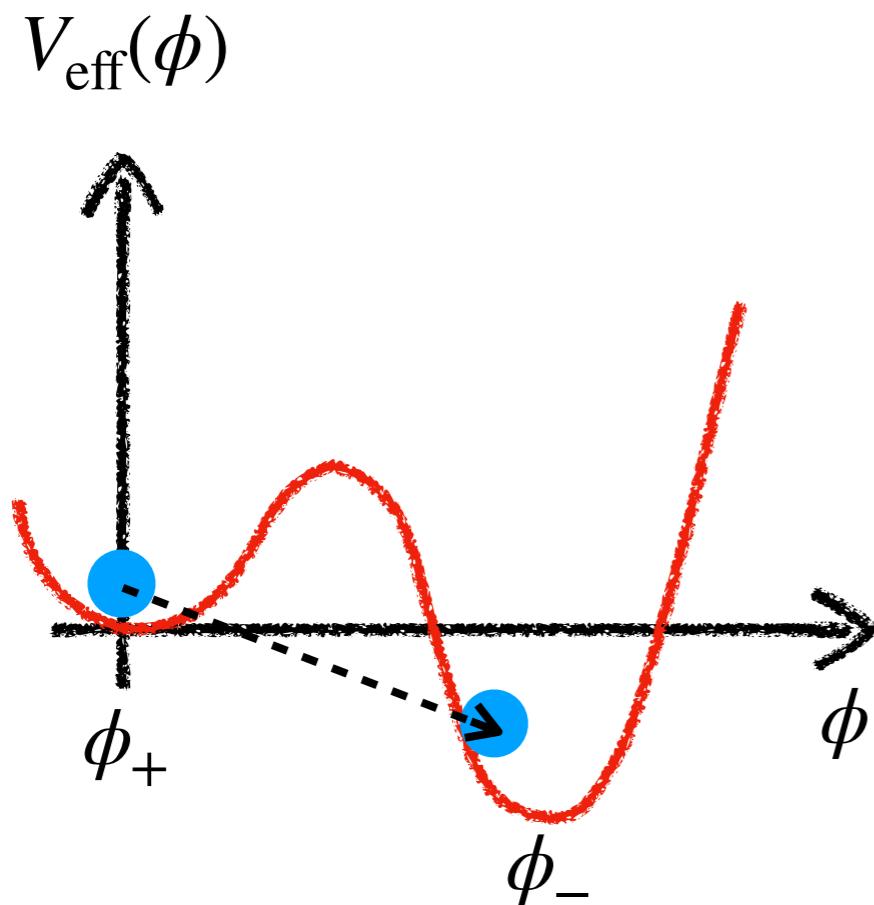
MeV
QCDPT

TeV
EWPT

PeV
PQPT

BSM

Phase dynamics



$$\Gamma(t) \sim e^{-S(t)}$$

$$\Gamma(T_*) \sim H(T_*)$$

$$\alpha \sim \frac{\Delta V_{\text{eff}}}{\rho_{\text{rad}}}$$

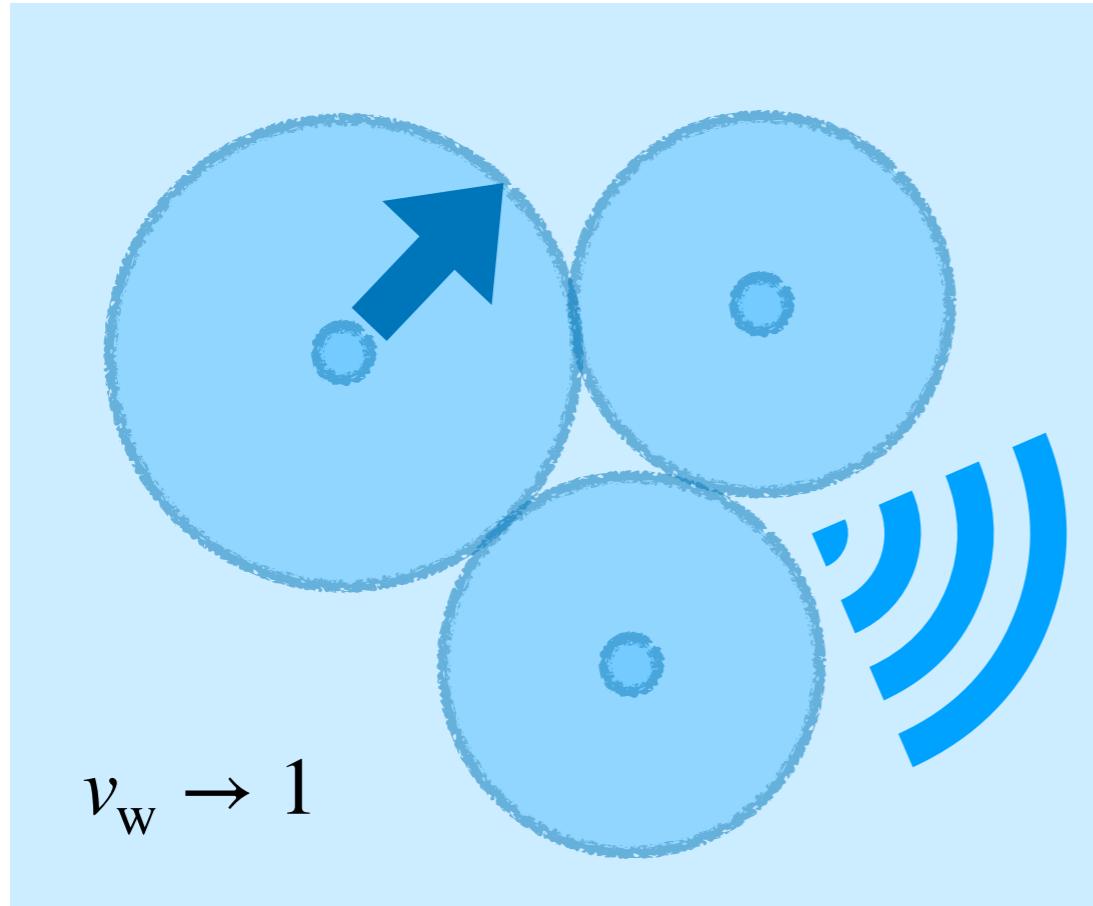
$$\Omega_{\text{peak}} \sim \alpha^2$$

$$R_* \sim \langle R_i(T_*) \rangle$$

$$f_{\text{peak}} \sim \frac{1}{R_*}$$

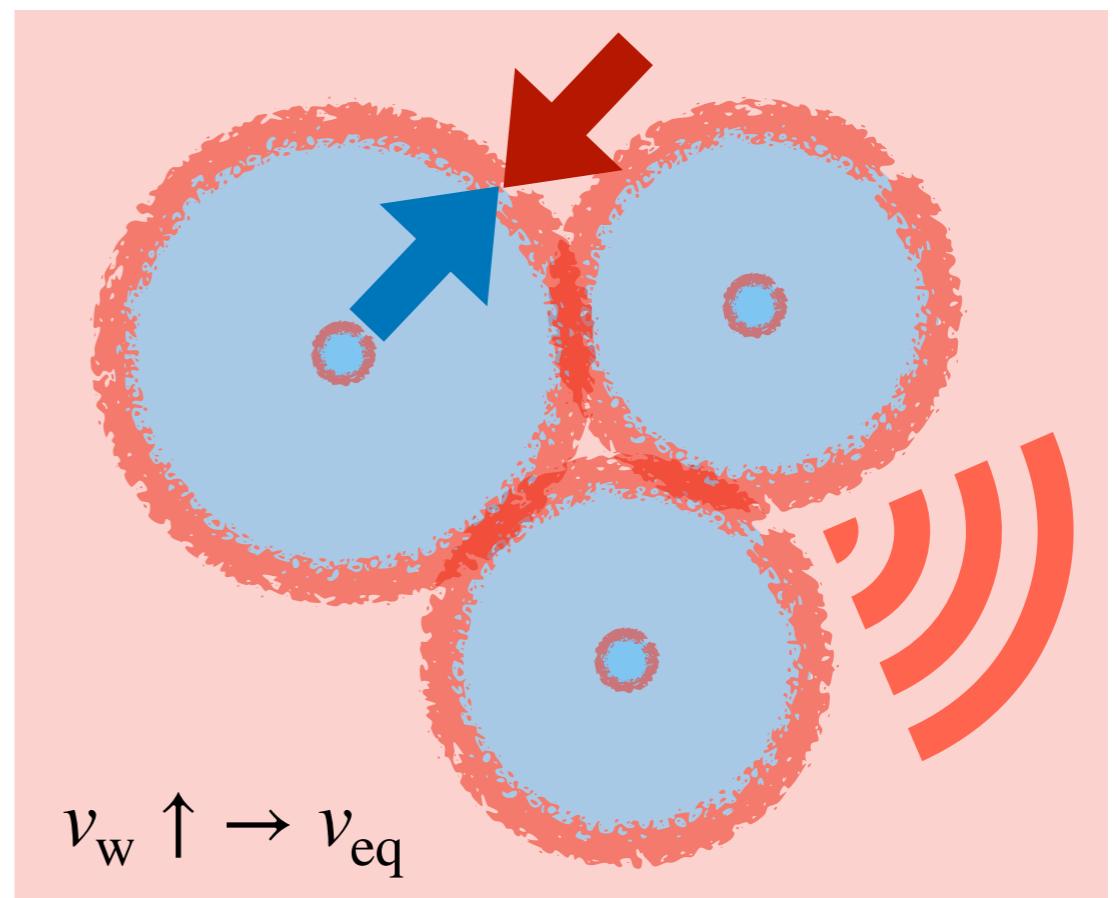
Bubble dynamics

Bubble expansion in vacuum



$$v_w \rightarrow 1$$

vs Bubble expansion in plasma



$$v_w \uparrow \rightarrow v_{eq}$$

Bubble walls collide with each other when they are rapidly accelerating

$$\Omega_{\text{GW}}^{\text{scalar}} \propto \kappa_{\text{collision}}^2 \sim \left(\frac{E_{\text{kinetic}}^{\text{wall}}}{\rho_{\text{vac}}} \right)^2$$

Bubble walls collide with each other when they are steadily expanding

$$\Omega_{\text{GW}}^{\text{plasma}} \propto \kappa_{\text{soundwave}}^2 \sim \left(\frac{E_{\text{kinetic}}^{\text{fluid}}}{\rho_{\text{vac}}} \right)^2$$

GW specturm

Scalar contribution : bubble wall collision

$$h^2 \Omega_\phi = 1.67 \times 10^{-5} \underbrace{\left(\frac{100}{g_{\text{dof}}} \right)^{\frac{1}{3}}}_{\text{Redshift factor}} \underbrace{(HR)_*^2 \left(\frac{\kappa_\phi \alpha}{1 + \alpha} \right)^2 \frac{0.11 v_w^3}{0.42 + v_w^2}}_{\text{Scaling factor}} \underbrace{\frac{3.8 \left(f/f_\phi \right)^{2.8}}{1 + 2.8 \left(f/f_\phi \right)^{3.8}}}_{\text{Spectrum factor}}$$

Efficiency factor from
bubble wall motion $\kappa_\phi(\alpha, R_*, \text{friction})$

King's College London group & This work **CERN group**

Fluid contribution : sound wave + MHD turbulence (negligible)

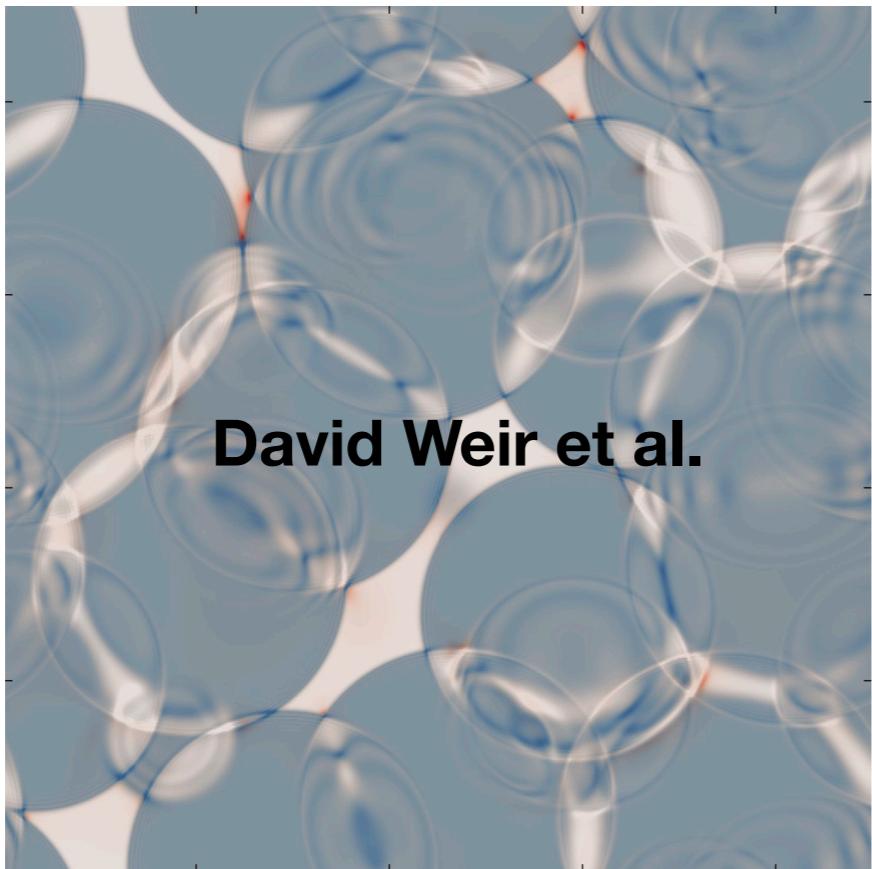
$$h^2 \Omega_{\text{sw}} = 2.65 \times 10^{-6} \underbrace{\left(\frac{100}{g_{\text{dof}}} \right)^{\frac{1}{3}}}_{\text{Redshift factor}} \underbrace{\frac{(HR)_*}{(8\pi)^{1/3}} \left(\frac{\kappa_v \alpha}{1 + \alpha} \right)^2 v_w}_{\text{Scaling factor}} \underbrace{\frac{7^{\frac{7}{2}} \left(f/f_{\text{sw}} \right)^3}{\left(4 + 3 \left(f/f_{\text{sw}} \right)^2 \right)^{\frac{7}{2}}}}_{\text{Spectrum factor}} \underbrace{\left(1 - \frac{1}{\sqrt{1 - 2\tau_{\text{sw}} H}} \right)}_{\text{Expansion suppression}}$$

Efficiency factor from
bulk fluid motion $\kappa_v(\alpha, v_w)$

1004.4187 CERN group **Helsinki/Sussex group**

Huai-Ke Guo 2020

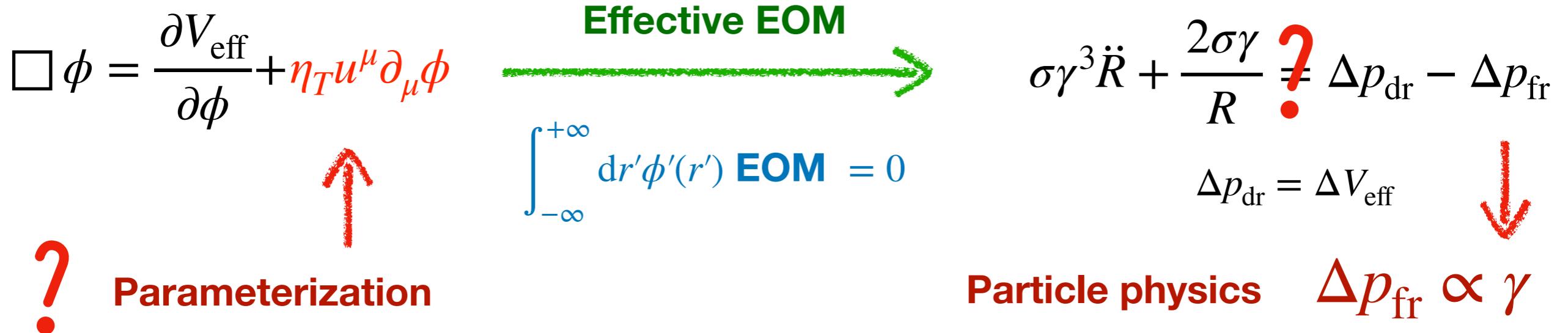
Numerical simulation



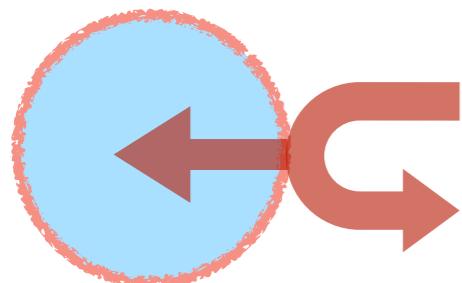
$$\square \phi = \frac{\partial V_{\text{eff}}}{\partial \phi} + \delta f$$

$$\partial_\mu T_p^{\mu\nu} + \partial^\nu \phi \frac{\partial V_T}{\partial \phi} = - \partial^\nu \phi \cdot \delta f$$

$$\delta f = \sum_{i=\text{B,F}} g_i \frac{dm_i^2}{d\phi} \int \frac{d^3 \vec{k}}{(2\pi)^3} \frac{\delta f_i}{2E_i(\vec{k})} ?$$



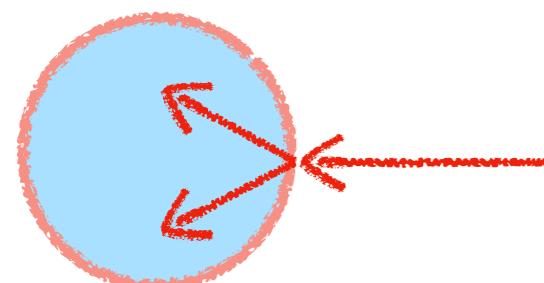
Thermal friction



Bodeker & Moore 09 Particle transmission and reflection

$$P_{1 \rightarrow 1} \approx \frac{\Delta m^2 T^2}{24} \equiv \Delta p_{\text{LO}}$$

Run-away

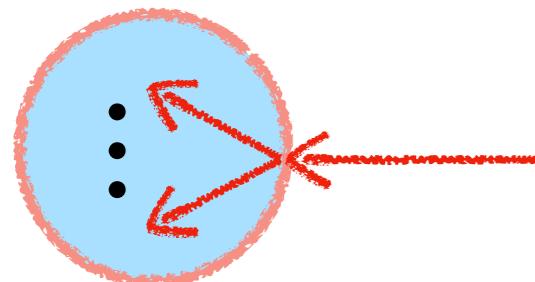


Bodeker & Moore 17

Transition splitting of a fermion emitting a soft vector boson

$$P_{1 \rightarrow 2} \approx \gamma g^2 \Delta m_V T^3 \equiv \gamma \Delta p_{\text{NLO}}$$

Non-run-away

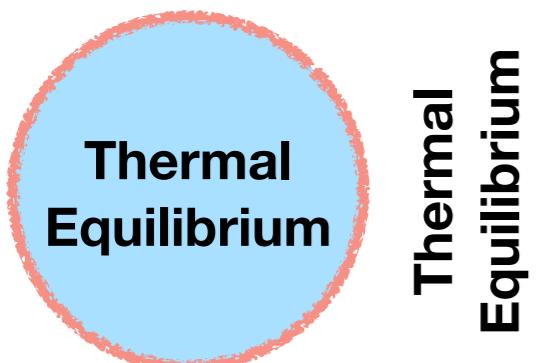


Hoche et al 2007.10343

Re-summing multiple soft gauge bosons scattering to all orders

$$P_{1 \rightarrow N} \approx 0.005 \gamma^2 g^2 T^4 \equiv \gamma^2 \Delta p_{\text{NLO}}$$

Non-run-away



Mancha et al 2005.10875

$$\Delta p = (\gamma^2 - 1) T \Delta s$$

$$\delta f = - \tilde{\eta}_T (u^\mu \partial_\mu \phi)^2$$

$$\Delta p_{\text{fr}} \propto (\gamma^2 - 1)$$

Effective picture

$$\Delta p_{\text{fr}} = \Delta p_{\text{LO}} + h(\gamma) \Delta p_{\text{NLO}}$$

γ -independent friction force



γ -dependent friction force



$$\sigma \gamma^3 \ddot{R} + \frac{2\sigma\gamma}{R} = \Delta p_{\text{dr}} - \Delta p_{\text{fr}}$$



Kinetic energy
of bubble wall

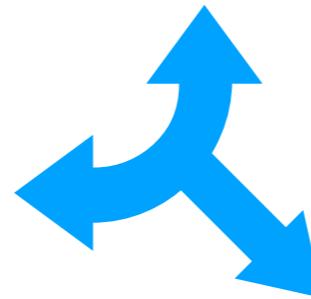
Potential
energy
of bubble

Work done by
friction force on
bubble wall

Conservation Law

$$E = 4\pi\sigma R^2 \gamma - \frac{4}{3}\pi R^3 (\Delta p_{\text{dr}} - \Delta p_{\text{fr}})$$

$$\left(\sigma + \frac{R}{3} \frac{d\Delta p_{\text{fr}}}{d\gamma} \right) \gamma^3 \ddot{R} + \frac{2\sigma\gamma}{R} = \Delta p_{\text{dr}} - \Delta p_{\text{fr}}$$



General solution

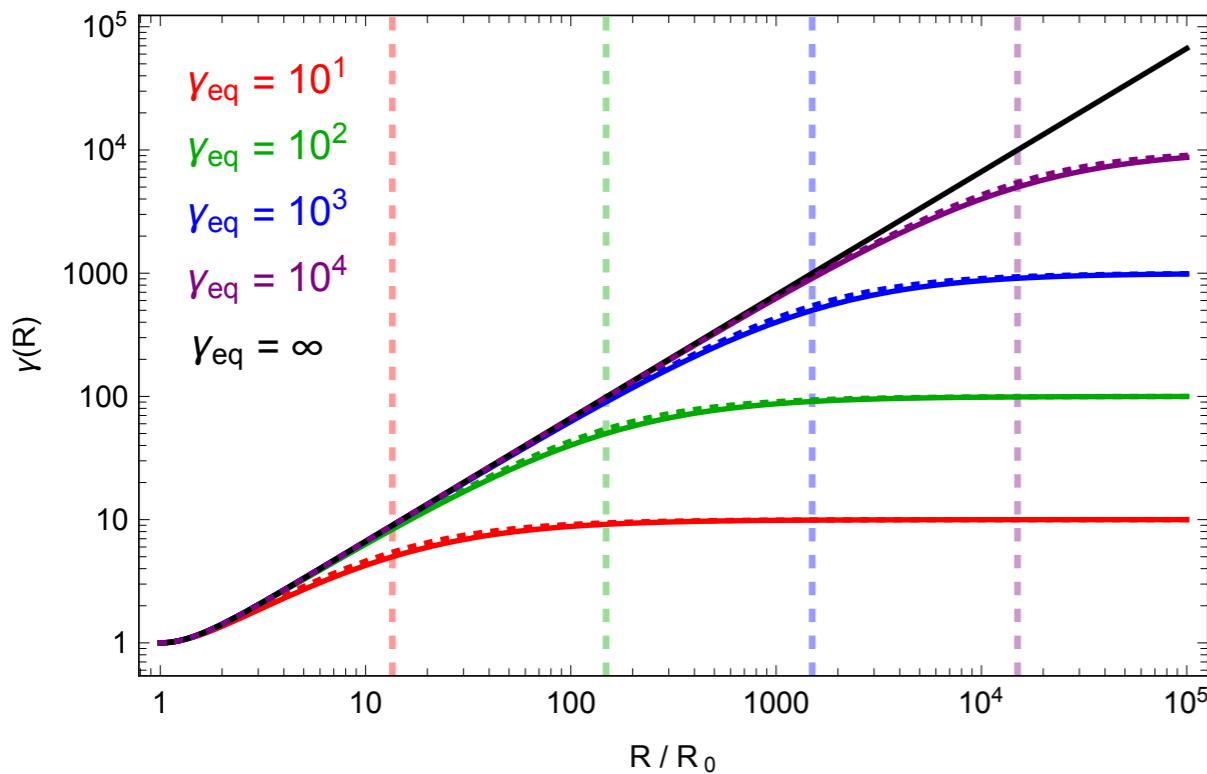
$$R_\sigma = \frac{3}{2}(\gamma_{\text{eq}} - 1)R_0$$

$$\frac{h(\gamma) - h(1)}{h(\gamma_{\text{eq}}) - h(1)} + \frac{3\gamma}{2R} = 1 + \frac{1}{2R^3}$$

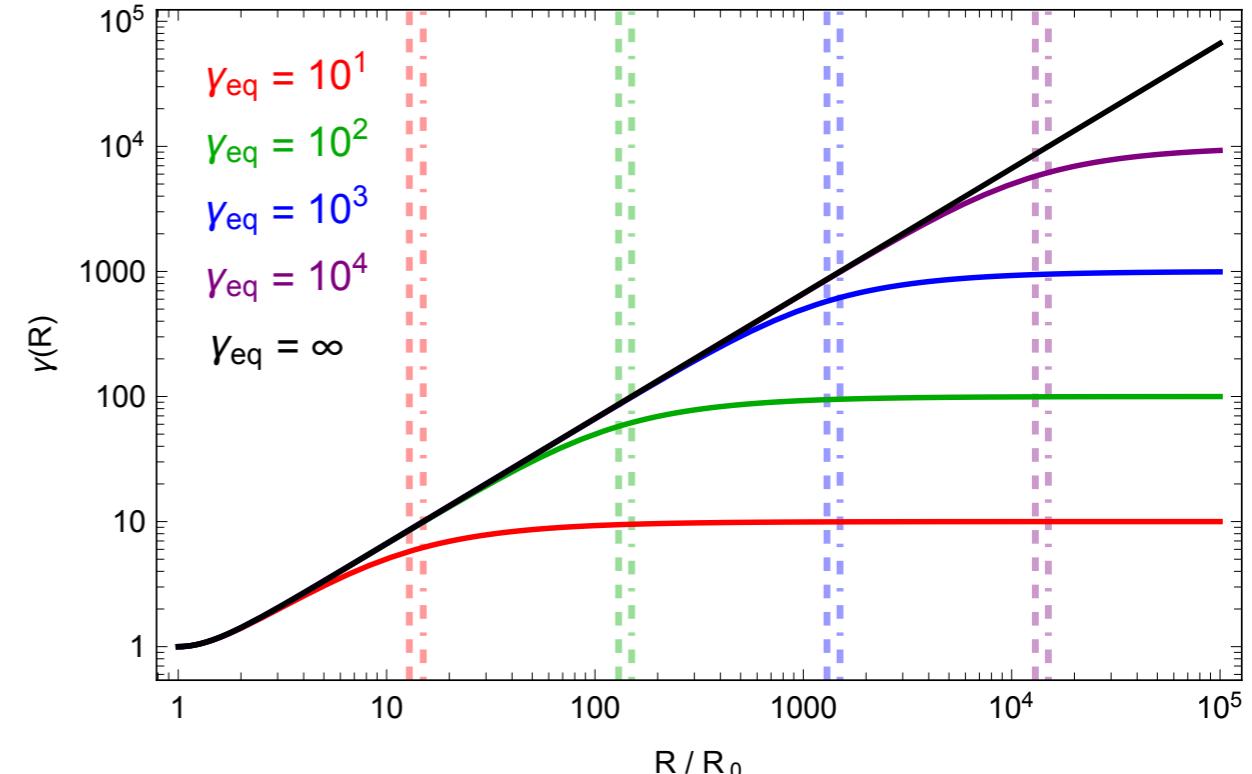
$$h(\gamma_{\text{eq}}) = \frac{\Delta p_{\text{dr}} - \Delta p_{\text{LO}}}{\Delta p_{\text{NLO}}}$$

Wall velocity

$$P_{1 \rightarrow 2} \equiv h(\gamma) \Delta p_{\text{NLO}}, \quad h(\gamma) = \gamma$$



$$P_{1 \rightarrow N} \equiv h(\gamma) \Delta p_{\text{NLO}}, \quad h(\gamma) = \gamma^2$$



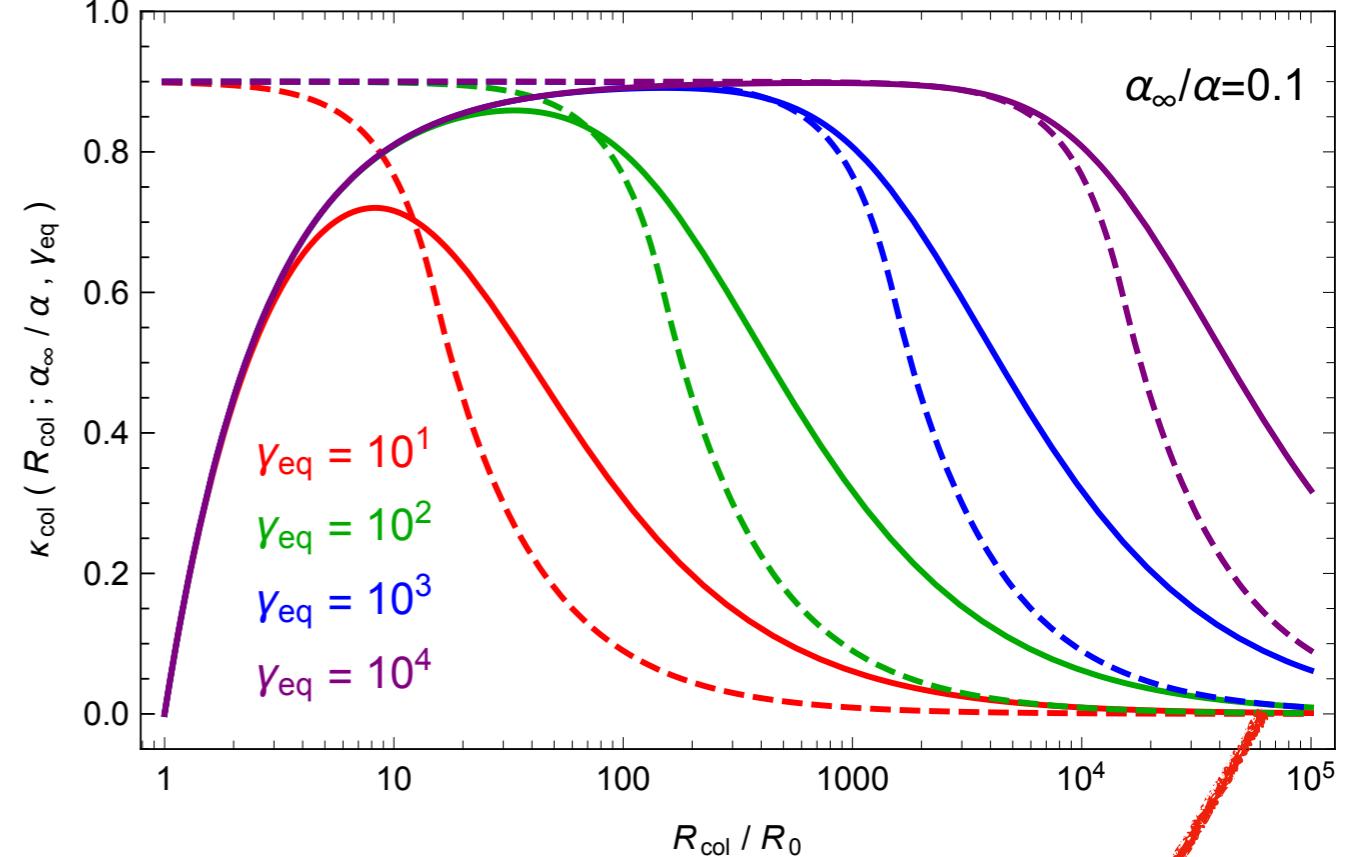
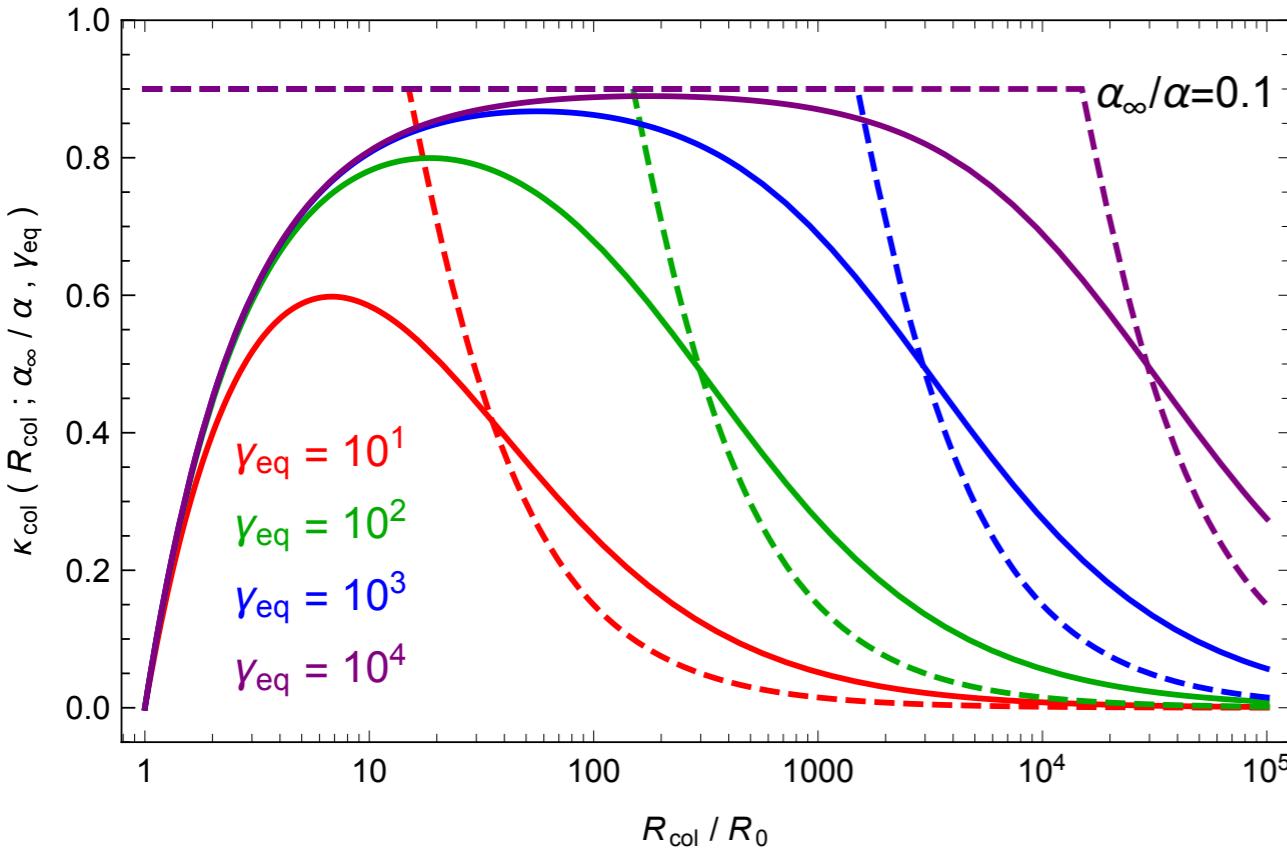
$$\gamma(R) = \frac{2\gamma_{\text{eq}}R^3 + \gamma_{\text{eq}} - 1}{2R^3 + 3(\gamma_{\text{eq}} - 1)R^2}$$

$$\lim_{\gamma_{\text{eq}} \rightarrow \infty} \gamma(R) = \frac{2}{3}R + \frac{1}{3R^2}$$

$$\gamma(R) = \sqrt{\gamma_{\text{eq}}^2 + \frac{9(\gamma_{\text{eq}}^2 - 1)^2}{16R^2} + \frac{\gamma_{\text{eq}}^2 - 1}{2R^3}} - \frac{3(\gamma_{\text{eq}}^2 - 1)}{4R}$$

$$\lim_{R \rightarrow \infty} \gamma(R) = \gamma_{\text{eq}}$$

Efficiency factor



For dash lines, see KCL group' works

1. Previous estimations are reproduced when bubble walls collide with each other long after they have been approaching the terminal velocity $R_{\text{col}} \gtrsim \mathcal{O}(10^3)R_{\sigma}$
2. Important for strong FOPT : bubble walls collide with each other just around the time when they are starting to approach the terminal velocity $R_{\text{col}} \lesssim \mathcal{O}(10^2)R_{\sigma}$

An example

$$\gamma_{\text{eq}} = 10 \quad R_{\sigma} \approx 15R_0$$

$$v_{\text{eq}} \approx 0.995 \quad R_{\text{col}} = 100R_0$$

$$h(\gamma) = \gamma^2$$

10/10

$$\begin{aligned} \kappa_{\text{col}}^{\text{old}} &\approx 0.1 \\ \kappa_{\text{col}}^{\text{new}} &\approx 0.3 \\ \frac{\Omega_{\text{col}}^{\text{new}}}{\Omega_{\text{col}}^{\text{old}}} &\sim \left(\frac{\kappa_{\text{col}}^{\text{new}}}{\kappa_{\text{col}}^{\text{old}}} \right)^2 \approx 9 \end{aligned}$$

Take-home message

Effective EOM for an expanding bubble wall in thermal plasma out-of equilibrium

$$\left(\sigma + \frac{R}{3} \frac{d\Delta p_{fr}}{d\gamma} \right) \frac{d\gamma}{dR} + \frac{2\sigma\gamma}{R} = \Delta p_{dr} - \Delta p_{fr}$$

General expansion solution with arbitrary γ -scaling friction $\Delta p_{fr} \equiv \Delta p_{LO} + h(\gamma)\Delta p_{NLO}$

$$\frac{h(\gamma) - h(1)}{h(\gamma_{eq}) - h(1)} + \frac{3\gamma}{2R} = 1 + \frac{1}{2R^3}$$

General expression for the efficiency factor from bubble wall collisions

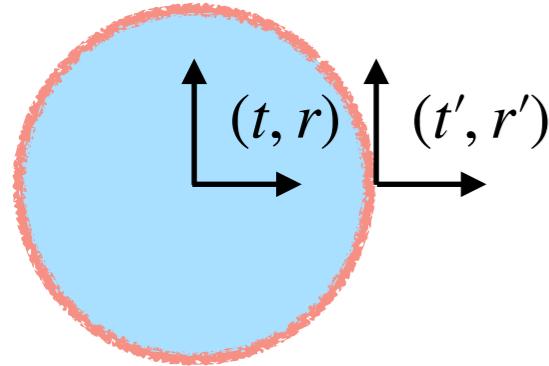
$$\kappa_{col} = \left(1 - \frac{\alpha_\infty}{\alpha} \right) \int_1^{R_{col}} \frac{dR}{R_{col}} \left[1 - \frac{h(\gamma(R))}{h(\gamma_{eq})} \right]$$

Use our general formula if bubble wall collisions dominate over sound waves

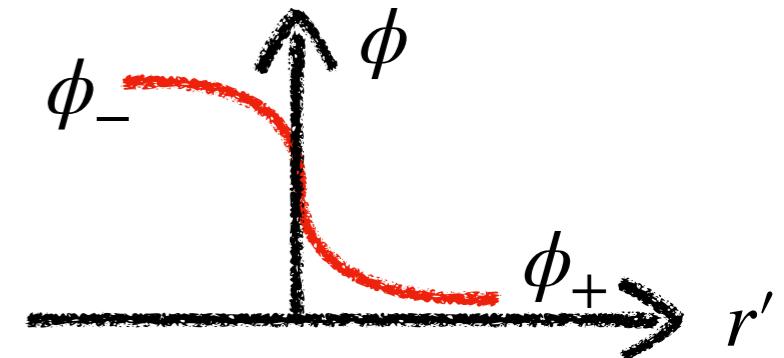
Use KCL's asymptotic expression if sound waves dominate over bubble wall collisions

Thank you
11/11

Backup slide



$$\begin{aligned}\phi(t, r) &= \phi(\gamma_w(t)[r - r_w(t)]) \equiv \phi(r') \\ t' &= \gamma_w(t)[t - v_w(t)r] \quad \phi(r' = -\infty) = \phi_- \\ r' &= \gamma_w(t)[r - r_w(t)] \quad \phi(r' = +\infty) = \phi_+\end{aligned}$$



Convert field derivative in bubble wall frame

$$\frac{\partial \phi}{\partial r} = \frac{d\phi}{dr'} \frac{\partial r'}{\partial r} = \phi'(r') \gamma_w$$

$$\frac{\partial^2 \phi}{\partial r^2} = \frac{d^2 \phi}{dr'^2} \left(\frac{\partial r'}{\partial r} \right)^2 + \frac{d\phi}{dr'} \frac{\partial^2 r'}{\partial r^2} = \phi''(r') \gamma_w^2$$

Surface tension $\sigma = \int_{-\infty}^{+\infty} dr' \phi'(r')^2$

Averaged value $\langle F \rangle = \frac{1}{\sigma} \int_{-\infty}^{+\infty} dr' \phi'(r')^2 F(r')$

$\langle F \rangle = 0$ if $F(r')$ is odd function

$$\begin{aligned}\frac{\partial \phi}{\partial t} &= \frac{d\phi}{dr'} \frac{\partial r'}{\partial t} \\ &= \phi'(r') [\dot{\gamma}_w(r - r_w) - \gamma_w v_w] \\ &= \phi'(r') [(\dot{\gamma}_w / \gamma_w) r' - \gamma_w v_w] \\ &= \phi'(r') (\gamma_w^2 v_w \ddot{r}_w r' - \gamma_w v_w) \\ &= \phi'(r') \gamma_w v_w (\gamma_w \ddot{r}_w r' - 1)\end{aligned}$$

$$\begin{aligned}\frac{\partial^2 \phi}{\partial t^2} &= \frac{d^2 \phi}{dr'^2} \left(\frac{\partial r'}{\partial t} \right)^2 + \frac{d\phi}{dr'} \frac{\partial^2 r'}{\partial t^2} \\ &= \phi''(r') [\dot{\gamma}_w(r - r_w) - \gamma_w v_w]^2 \\ &\quad + \phi'(r') [\dot{\gamma}_w(r - r_w) - 2\dot{\gamma}_w v_w - \gamma_w \ddot{r}_w] \\ &= \phi''(r') \gamma_w^2 v_w^2 (\gamma_w \ddot{r}_w r' - 1)^2 \\ &\quad + \phi'(r') [(\dot{\gamma}_w / \gamma_w) r' - \gamma_w (2\gamma_w^2 - 1) \ddot{r}_w]\end{aligned}$$

Backup slide

Thermal bubble in thermal equilibrium

$$\nabla_\mu \nabla^\mu \phi \equiv \frac{\partial^2 \phi}{\partial r^2} + \frac{2}{r} \frac{\partial \phi}{\partial r} - \frac{\partial^2 \phi}{\partial t^2} = \frac{\partial V_{\text{eff}}}{\partial \phi}$$

$$\int_{-\infty}^{+\infty} dr' \phi'(r') \text{ EOM } = 0$$

$$\int_{-\infty}^{+\infty} dr' \phi'(r') \frac{\partial V_{\text{eff}}}{\partial \phi} = \int_{\phi_-}^{\phi_+} d\phi \left(\frac{\partial V_0}{\partial \phi} + \frac{\partial V_T}{\partial \phi} \right)$$

Effective EOM
of bubble wall

$$\begin{aligned} \int_{-\infty}^{+\infty} dr' \phi'(r') \frac{\partial^2 \phi}{\partial r^2} &= \int d\left(\frac{\phi'^2}{2}\right) \gamma_w^2 = \frac{\gamma_w^2}{2} \phi'(r')^2 \Big|_{-\infty}^{+\infty} = 0 \\ \int_{-\infty}^{+\infty} dr' \phi'(r') \frac{2}{r} \frac{\partial \phi}{\partial r} &= \int_{-\infty}^{+\infty} dr' \phi'(r') \frac{2}{r} \phi'(r') \gamma_w \simeq \frac{2\sigma\gamma_w}{r_w} \end{aligned} \quad \begin{aligned} &= \int_{\phi_-}^{\phi_+} d\phi \frac{dV_0}{d\phi} + \sum_{i=B,F} g_i T^4 \int_{\phi_-}^{\phi_+} d\phi J'_i \left(\frac{m_i^2}{T^2} \right) \frac{1}{T^2} \frac{dm_i^2}{d\phi} \\ &= V_0(\phi) \Big|_{\phi_-}^{\phi_+} + \sum_{i=B,F} g_i T^4 J_i \left(\frac{m_i^2(\phi)}{T^2} \right) \Big|_{\phi_-}^{\phi_+} \\ &\equiv -\Delta V_0 - \Delta V_T = -\Delta V_{\text{eff}} \equiv V_{\text{eff}}(\phi_+) - V_{\text{eff}}(\phi_-) \end{aligned}$$

\downarrow

$$\begin{aligned} \sigma \gamma_w^3 \ddot{r}_w + \frac{2\sigma\gamma_w}{r_w} &= \Delta p_{\text{dr}} \\ \frac{d\gamma_w}{dr_w} + \frac{2\gamma_w}{r_w} &= \frac{\Delta p_{\text{dr}}}{\sigma} \end{aligned} \quad \text{or} \quad \begin{aligned} \int_{-\infty}^{+\infty} dr' \phi'(r') \frac{\partial^2 \phi}{\partial t^2} &= \int_{-\infty}^{+\infty} d\left(\frac{\phi'^2}{2}\right) \gamma_w^2 v_w^2 (\gamma_w \ddot{r}_w r' - 1)^2 + \int_{-\infty}^{+\infty} dr' \phi'(r')^2 [(\dot{\gamma}_w/\gamma_w)r' - \gamma_w(2\gamma_w^2 - 1)\ddot{r}_w] \\ &= \frac{1}{2} \phi'(r')^2 \gamma_w^2 v_w^2 (\gamma_w \ddot{r}_w r' - 1)^2 \Big|_{-\infty}^{+\infty} - \int_{-\infty}^{+\infty} \phi'(r')^2 \gamma_w^2 v_w^2 (\gamma_w \ddot{r}_w r' - 1) \gamma_w \dot{r}_w dr' \\ &\quad + \int_{-\infty}^{+\infty} dr' \phi'(r')^2 [(\dot{\gamma}_w/\gamma_w)r' - \gamma_w(2\gamma_w^2 - 1)\ddot{r}_w] \\ &= 0 + \sigma \gamma_w^3 v_w^2 \ddot{r}_w - \sigma \gamma_w (2\gamma_w^2 - 1) \ddot{r}_w = -\sigma \gamma_w^3 \ddot{r}_w, \end{aligned}$$

Backup slide

Thermal bubble out-of thermal equilibrium

$$\nabla_\mu \nabla^\mu \phi \equiv \frac{\partial^2 \phi}{\partial r^2} + \frac{2}{r} \frac{\partial \phi}{\partial r} - \frac{\partial^2 \phi}{\partial t^2} = \frac{\partial V_{\text{eff}}}{\partial \phi} + \sum_{i=B,F} g_i \frac{dm_i^2}{d\phi} \int \frac{d^3 \vec{k}}{(2\pi)^3} \frac{\delta f_i}{2E_i(\vec{k})} \begin{cases} \eta_T u^\mu \partial_\mu \phi \\ -\eta_T (u^\mu \partial_\mu \phi)^2 \end{cases}$$

$$\int_{-\infty}^{+\infty} dr' \phi'(r') \text{ EOM } = 0$$

~~Effective EOM
of bubble wall~~

$$\sigma \gamma_w^3 \ddot{r}_w + \frac{2\sigma \gamma_w}{r_w} = \Delta p_{\text{dr}} + \Delta p_{\text{fr}}$$



$$\begin{aligned} \Delta p_{\text{fr}} &= \int_{-\infty}^{+\infty} dr' \phi'(r') \eta_T u^\mu \partial_\mu \phi \\ &= \int_{-\infty}^{+\infty} dr' \phi'(r') \eta_T \left(\gamma_p \frac{\partial \phi}{\partial t} + \gamma_p v_p \frac{\partial \phi}{\partial r} \right) \\ &= \int_{-\infty}^{+\infty} dr' \phi'(r')^2 \eta_T [\gamma_p \gamma_w v_w (\gamma_w \ddot{r}_w r' - 1) + \gamma_p v_p \gamma_w] \\ &= - \int_{-\infty}^{+\infty} dr' \phi'(r')^2 \eta_T \gamma_p \gamma_w (v_w - v_p) \\ \downarrow \text{detonation} \quad v'_p &= -v_w, \quad v_p = 0, \quad \gamma_p = 1 \\ &= -\gamma_w v_w \sigma \langle \eta_T \rangle \propto -\gamma_w v_w \end{aligned}$$

$$\begin{aligned} \Delta p_{\text{fr}} &= - \int_{-\infty}^{+\infty} dr' \phi'(r')^2 \eta_T \gamma_w^2 v_w^2 (\gamma_w \ddot{r}_w r' - 1)^2 \\ &= - \int_{-\infty}^{+\infty} dr' \phi'(r')^2 \eta_T \gamma_w^2 v_w^2 (1 + \gamma_w^2 \dot{r}_w^2 r'^2) \\ &= -\gamma_w^2 v_w^2 \sigma \langle \eta_T \rangle \propto -(\gamma_w^2 - 1) \end{aligned}$$

late-time $\dot{r}_w \approx 0$

$$P_{1 \rightarrow N} = \gamma_w^2 \Delta p_{\text{NLO}}$$

$$P_{1 \rightarrow 2} = \gamma_w \Delta p_{\text{NLO}}$$



Backup slide

Thermal bubble in thermal equilibrium

Effective Lagrangian $L = -4\pi\sigma R^2 \sqrt{1 - \dot{R}^2} + \frac{4}{3}\pi R^3 \Delta p_{\text{dr}}$

Euler-Lagrangian Eq. $\ddot{R} + 2\frac{1 - \dot{R}^2}{R} = \frac{\Delta p_{\text{dr}}}{\sigma} (1 - \dot{R}^2)^{\frac{3}{2}}$ $\frac{d\gamma}{dR} = \dot{R}\gamma^3 = \frac{\Delta p_{\text{dr}}}{\sigma} - \frac{2\gamma}{R}$ $\gamma = \frac{1}{\sqrt{1 - \dot{R}^2}}$

Solution to effective EOM $\gamma(R) = \frac{\Delta p_{\text{dr}}}{3\sigma}R + \frac{C}{R^2}$ C is fixed by $\gamma(R_0) = 1$

Bubble energy $E = (\partial L / \partial \dot{R})\dot{R} - L = 4\pi\sigma R^2 \gamma - \frac{4}{3}\pi R^3 \Delta p_{\text{dr}}$ $\left. \frac{dE}{dR} \right|_{\gamma(t=0)=1} = 0$ $R_0 = \frac{2\sigma}{\Delta p_{\text{dr}}}$

Solution to effective EOM $\gamma(R) = \frac{R_0^2}{R^2} + \frac{\Delta p_{\text{dr}}}{\sigma} \frac{R^3 - R_0^3}{3R^2} = \frac{2}{3}R + \frac{1}{3R^2}$ $R_0 \equiv 1$

Bubble wall velocity $\dot{R}(t) = \sqrt{1 - \frac{9R^4}{(1 + 2R^3)^2}} \rightarrow 1, \quad R \rightarrow \infty$

Backup slide

Thermal bubble out-of thermal equilibrium

$$L = -4\pi\sigma R^2 \sqrt{1 - \dot{R}^2} + \frac{4}{3}\pi R^3 (\Delta p_{\text{dr}} - \Delta p_{\text{LO}} + f(R)g(\dot{R}))$$

$$\left(1 + \frac{f}{3\sigma} \frac{g''}{\gamma^3} R\right) \frac{d\gamma}{dR} + \frac{2\gamma}{R} = \frac{\Delta p_{\text{dr}} - \Delta p_{\text{LO}}}{\sigma} - \frac{3f + Rf'}{3\sigma} (\dot{R}g' - g)$$

$$\downarrow \quad \eta \equiv \frac{\Delta p_{\text{NLO}}}{\sigma}, \quad h(\gamma_{\text{eq}}) \equiv \frac{\Delta p_{\text{dr}} - \Delta p_{\text{LO}}}{\Delta p_{\text{NLO}}}$$

$$\left(1 + \frac{\eta R}{3} h'(\gamma)\right) \frac{d\gamma}{dR} + \frac{2\gamma}{R} = \eta(h(\gamma_{\text{eq}}) - h(\gamma))$$

Same solution

$$E = \frac{\partial L}{\partial \dot{R}} \dot{R} - L = 4\pi\sigma R^2 \gamma + \frac{4}{3}\pi R^3 \sigma \eta (h(\gamma) - h(\gamma_{\text{eq}})) + \frac{4}{3}\pi R^3 \sigma \eta (g''(\dot{R}) - \gamma^3 h'(\gamma)) \dot{R}^2$$

$$= 4\pi\sigma R^2 \gamma + \frac{4}{3}\pi R^3 \sigma \eta (h(\gamma) - h(\gamma_{\text{eq}}))$$

$$\xrightarrow{\quad} \frac{dE}{dR} \Big|_{\gamma(R_0)=1} = 0 \Rightarrow R_0 = \frac{2}{\eta(h(\gamma_{\text{eq}}) - h(1))}$$

Our proposal of effective description

$$\curvearrowright f(R) = \Delta p_{\text{NLO}}$$

$$3f(R) + Rf'(R) = 3\Delta p_{\text{NLO}} \xrightarrow{\quad} \frac{h(\gamma) \Delta p_{\text{NLO}}}{\sigma}$$

$$\dot{R}g'(\dot{R}) - g(\dot{R}) = h(\gamma(\dot{R}))$$

$$\curvearrowright g''(\dot{R}) = \gamma^3 h'(\gamma)$$

$$L = -4\pi\sigma R^2 \sqrt{1 - \dot{R}^2} + \frac{4}{3}\pi R^3 \sigma \eta (h(\gamma_{\text{eq}}) + \dot{R}g'(\dot{R}) - h(\gamma(\dot{R})))$$