

Higgs Portal to Cosmology

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Higgs potential and BSM opportunity

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Higgs plays very important role in physics!

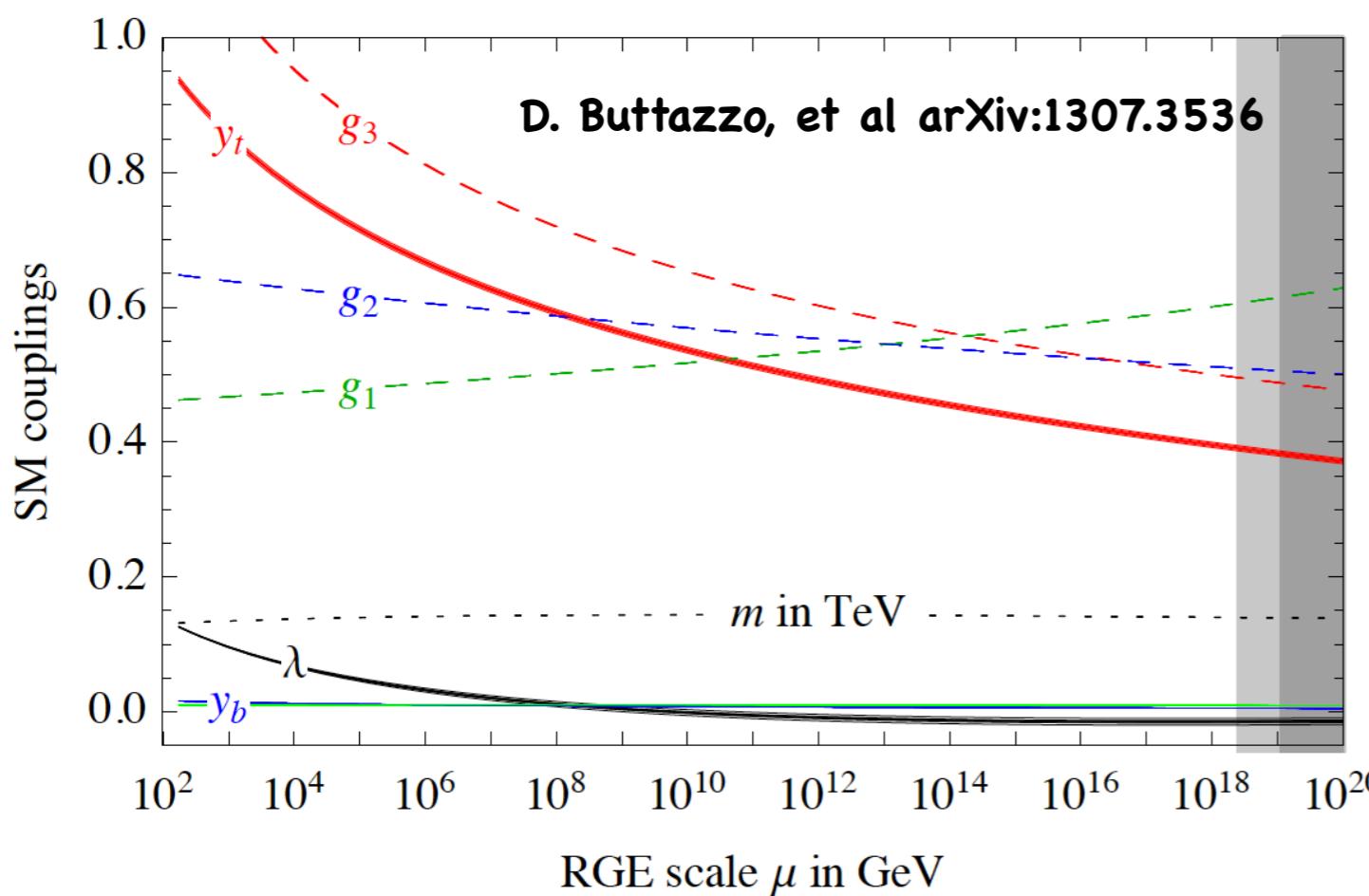
- To understand EWSB
 - The origin of the masses for the fermions
 - ...
- Physics related to cosmology

Looking into Higgs sector more carefully

In SM, the Higgs potential is:

$$V = \lambda(\mathcal{H}^\dagger \mathcal{H} - v^2)^2 \quad \lambda \sim 0.13 \quad v \sim 174 \text{ GeV}$$

$$\frac{d\lambda}{d \ln \mu^2} = \frac{1}{(4\pi)^2} \left[\lambda \left(12\lambda + 6y_t^2 - \frac{9g_2^2}{2} - \frac{9g_1^2}{10} \right) \boxed{-3y_t^4} + \frac{9g_2^4}{16} + \frac{27g_1^4}{400} + \frac{9g_2^2 g_1^2}{40} \right]$$



y_t : top Yukawa coupling

g_2, g_Y : weak gauge coupling

g_3 : strong gauge coupling

Higgs potential and self coupling

To study the vacuum structure of the Higgs, RG-improved effective potential:

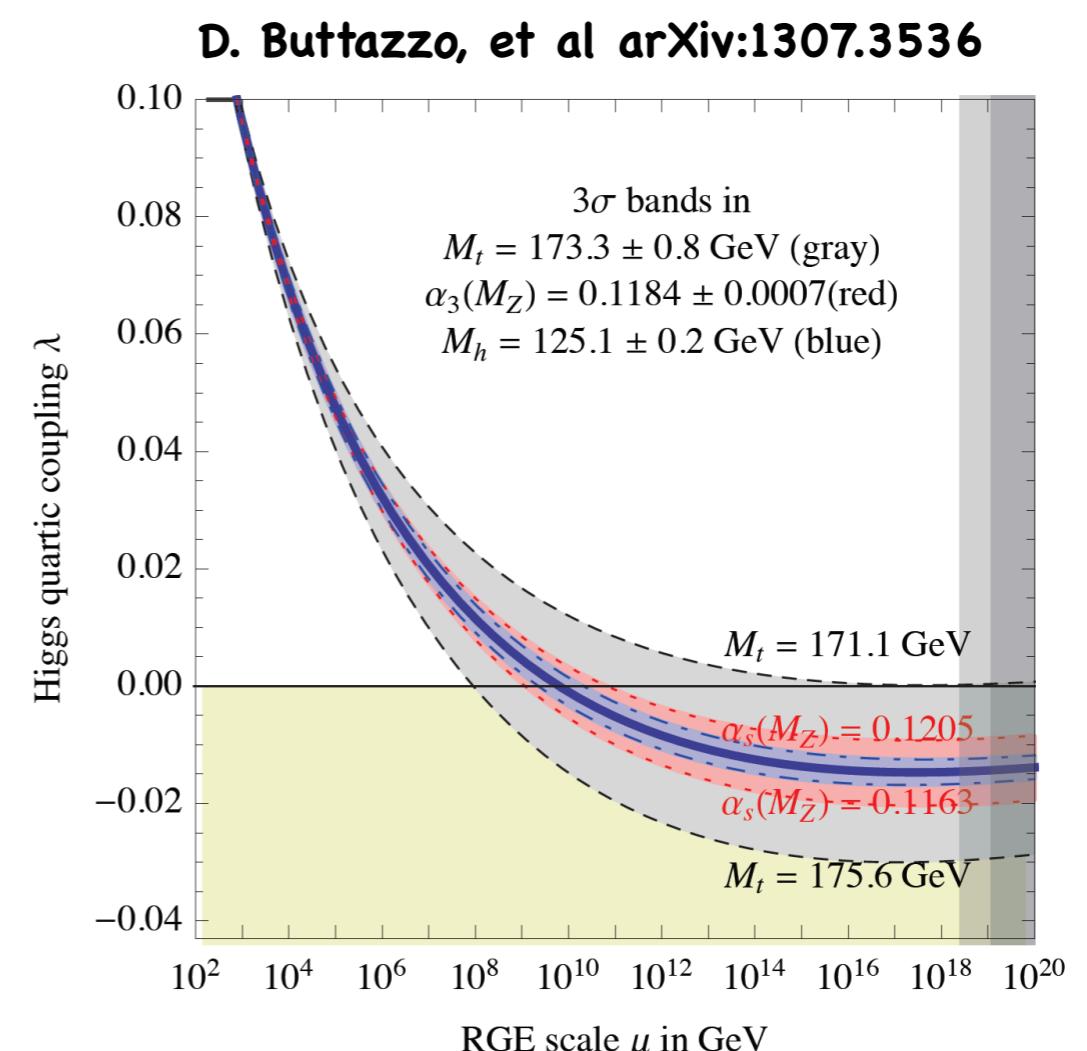
For $h \gg v$

$$V_{\text{eff}} = \lambda_{\text{eff}}(h) \frac{h^4}{4}$$

$$\lambda_{\text{eff}}(h) = e^{4\Gamma(h)} \left[\lambda(\bar{\mu} = h) + \lambda_{\text{eff}}^{(1)}(\bar{\mu} = h) + \lambda_{\text{eff}}^{(2)}(\bar{\mu} = h) \right]$$

$$\Gamma(h) \equiv \int \gamma(\mu) d \ln \mu$$

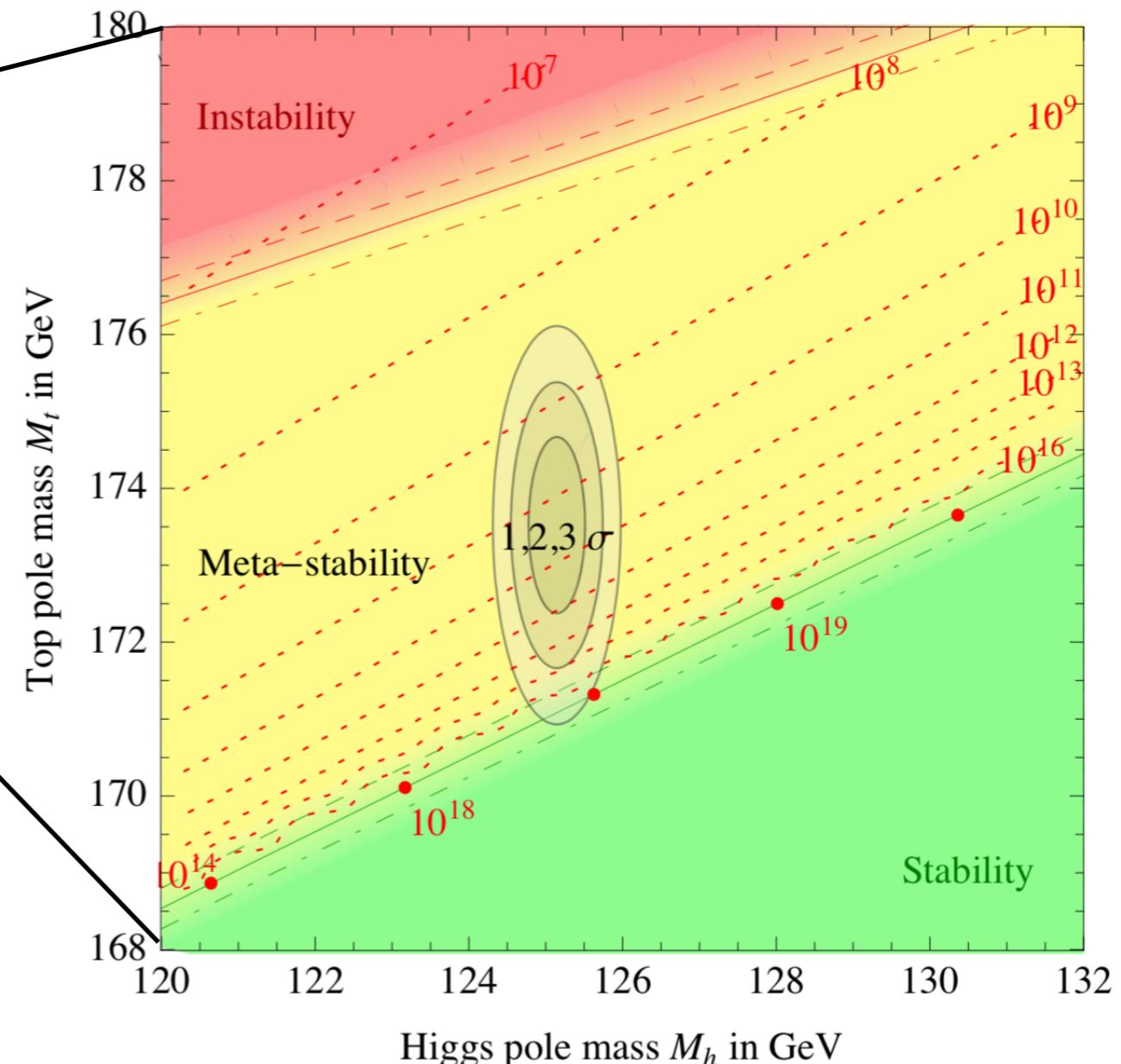
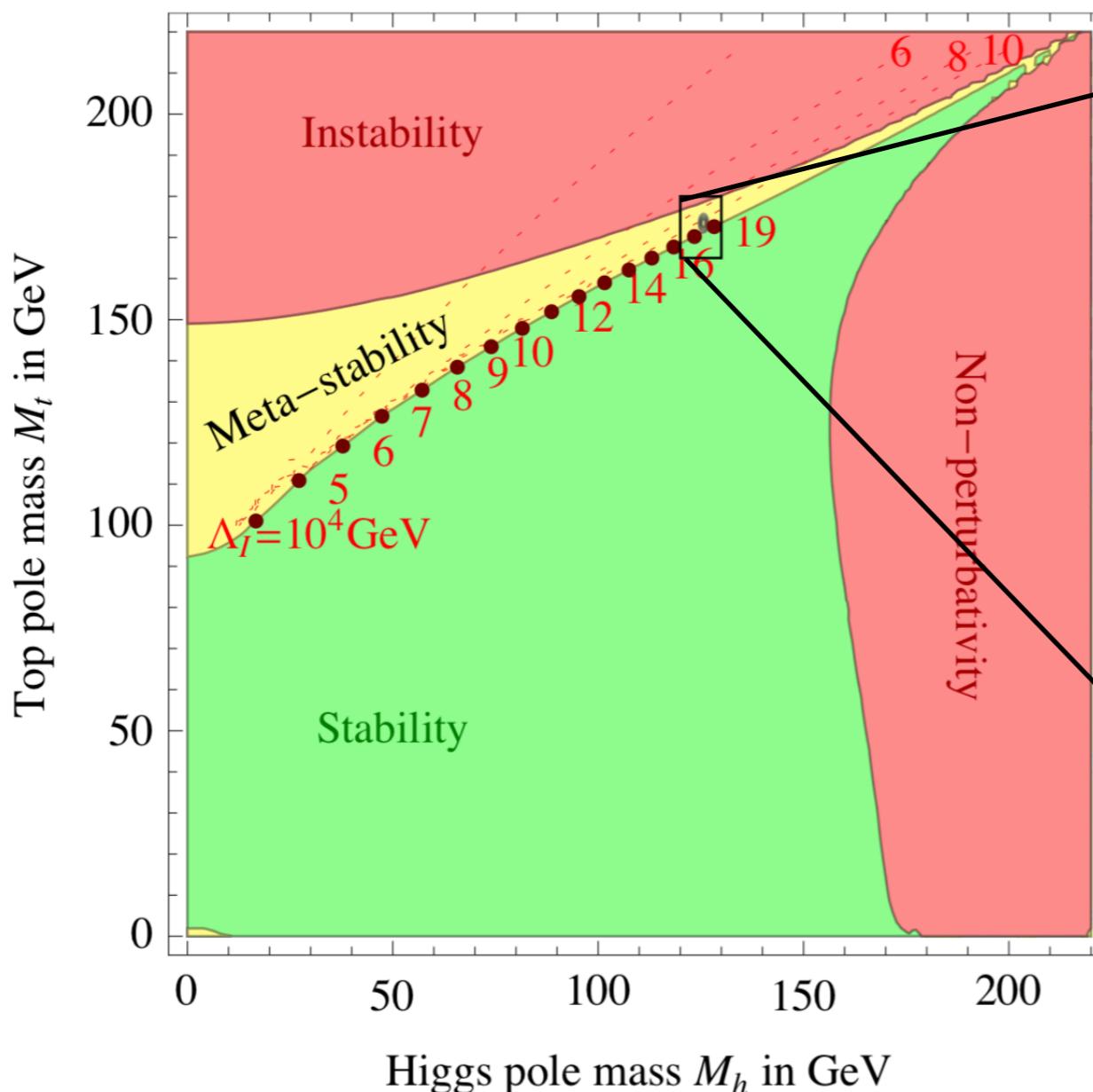
γ is the Higgs field anomalous dimension



New stable vacuum is developed at high energy scale!

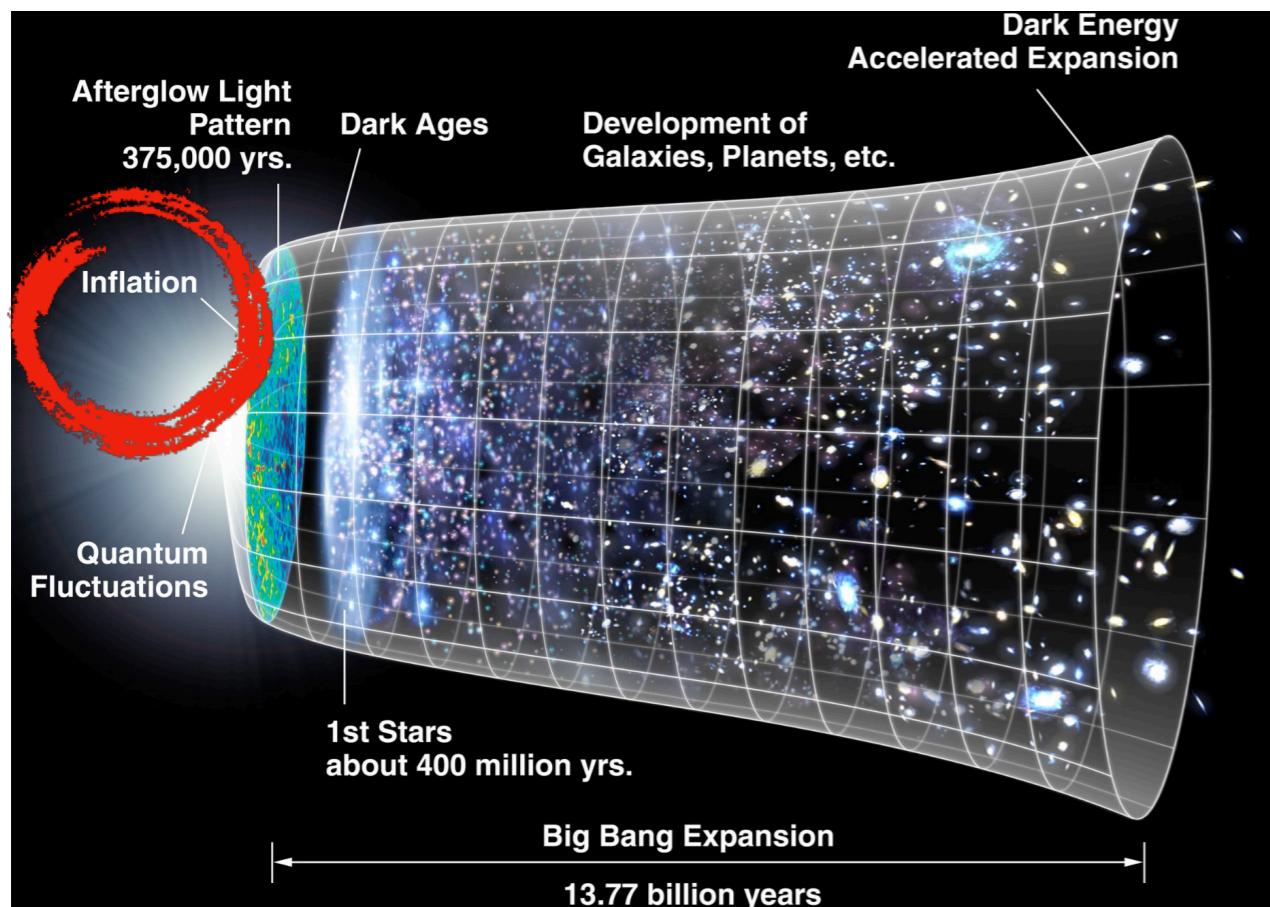
Our universe is at the edge

D. Buttazzo, et al arXiv:1307.3536

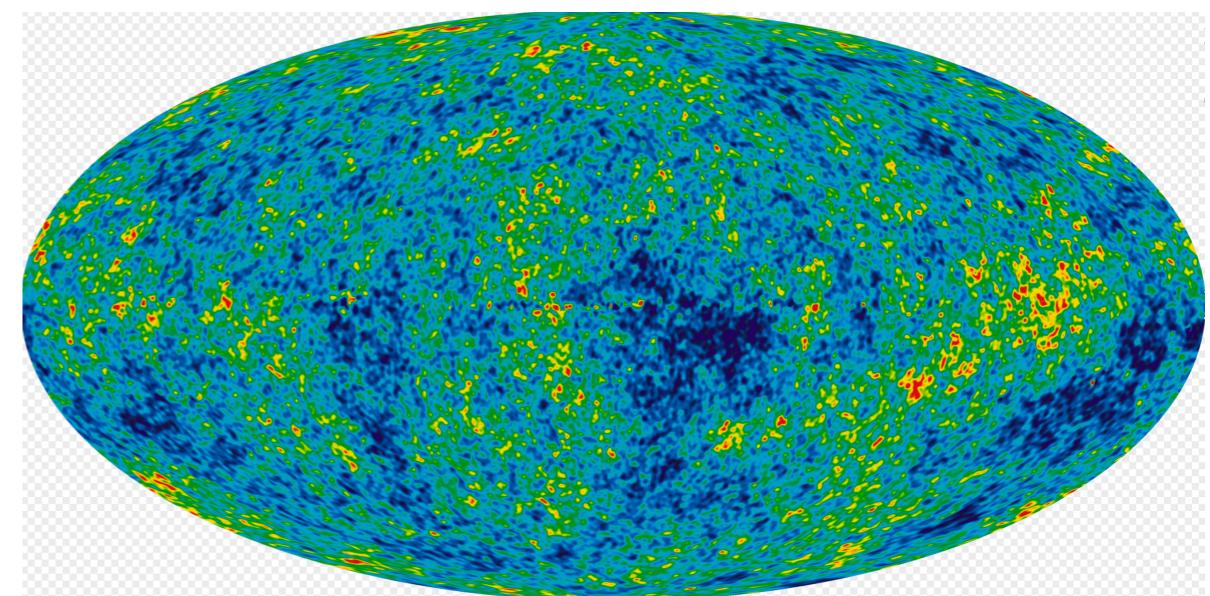


It seems we do not need worry about the
Higgs decay in the late universe

Inflation in the early universe

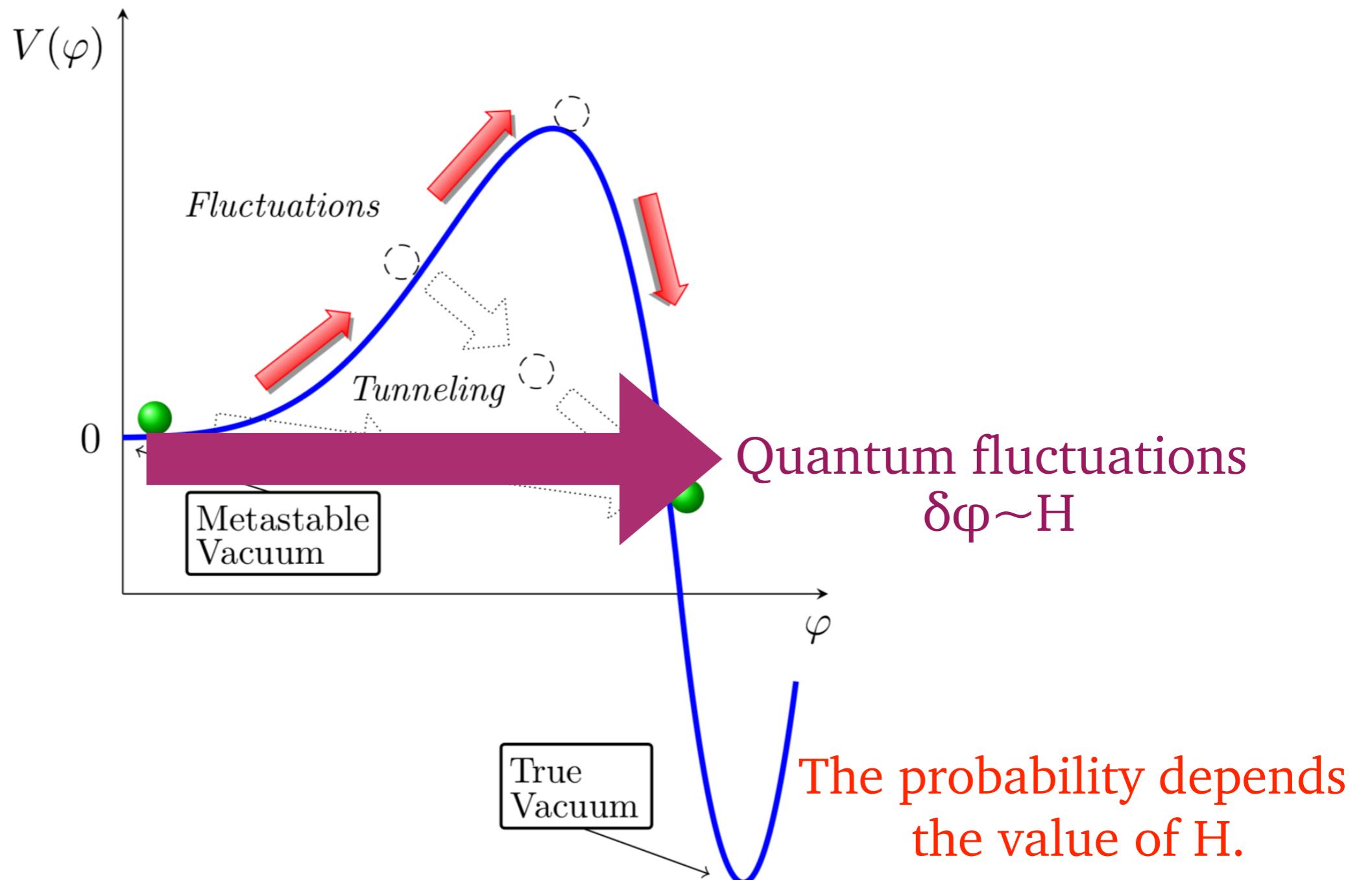


fluctuations in CMB



- ❖ Explains the origin of the large-scale structure of the cosmos.
- ❖ Horizon problem
- ❖ Flatness problem
- ❖ Magnetic-monopole problem ...

Higgs instability during inflation

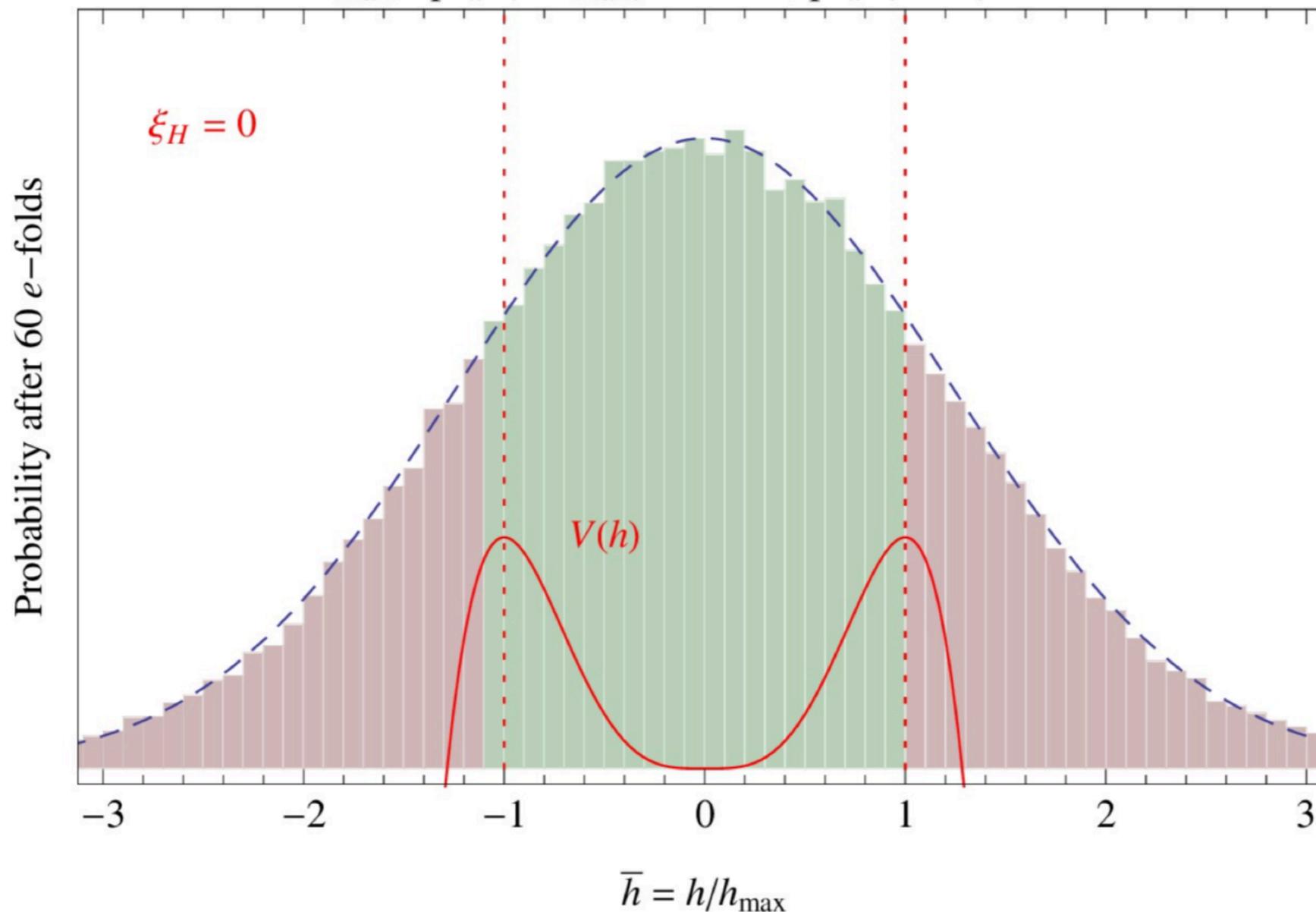


Higgs instability during inflation

- Even if $H \sim h_{\max}$, the probability of ending up in true vacuum is large.

J. R. Espinosa, et al [arXiv:1505.04825](https://arxiv.org/abs/1505.04825)

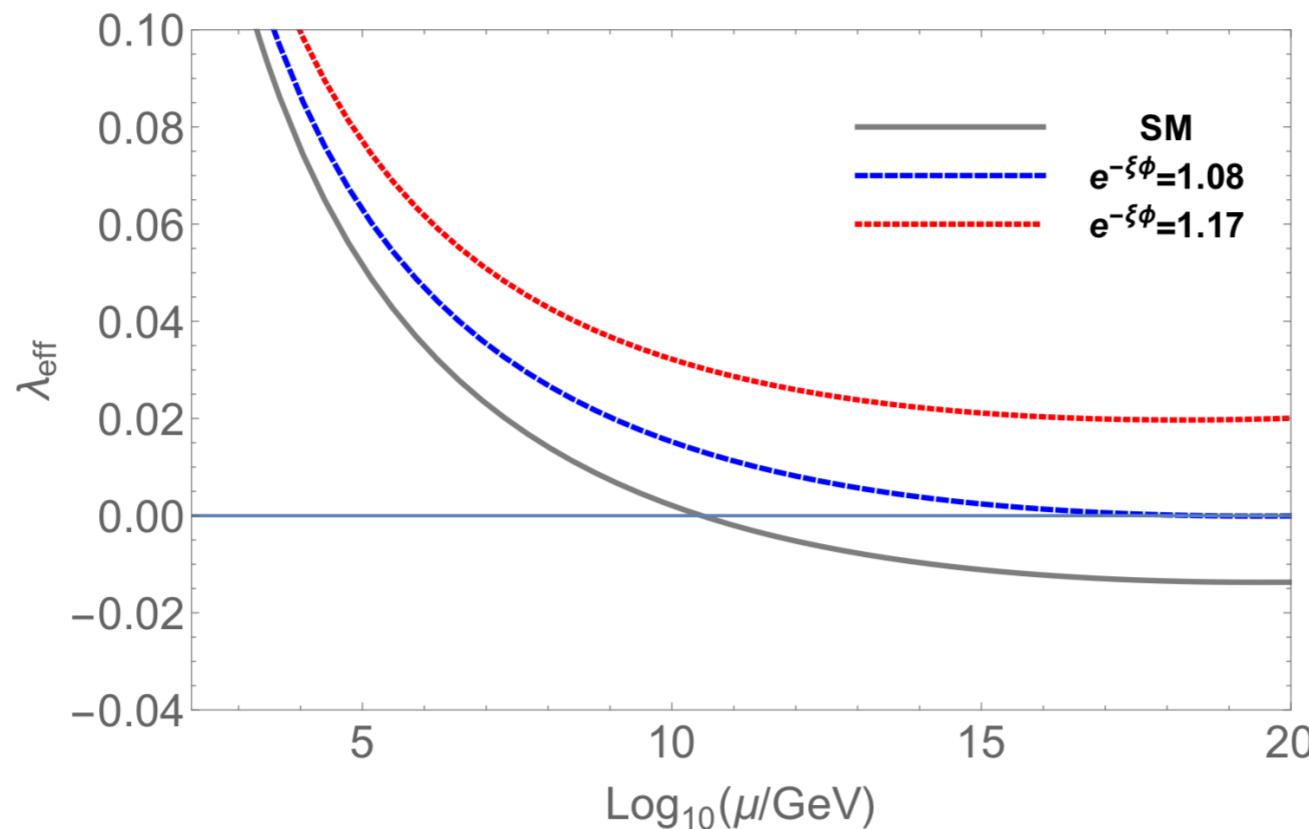
$$H = h_{\max}, p(|h| > h_{\max}) = 0.42, p(|h| \rightarrow \infty) = 0.00016$$



Quintessence-Higgs Model

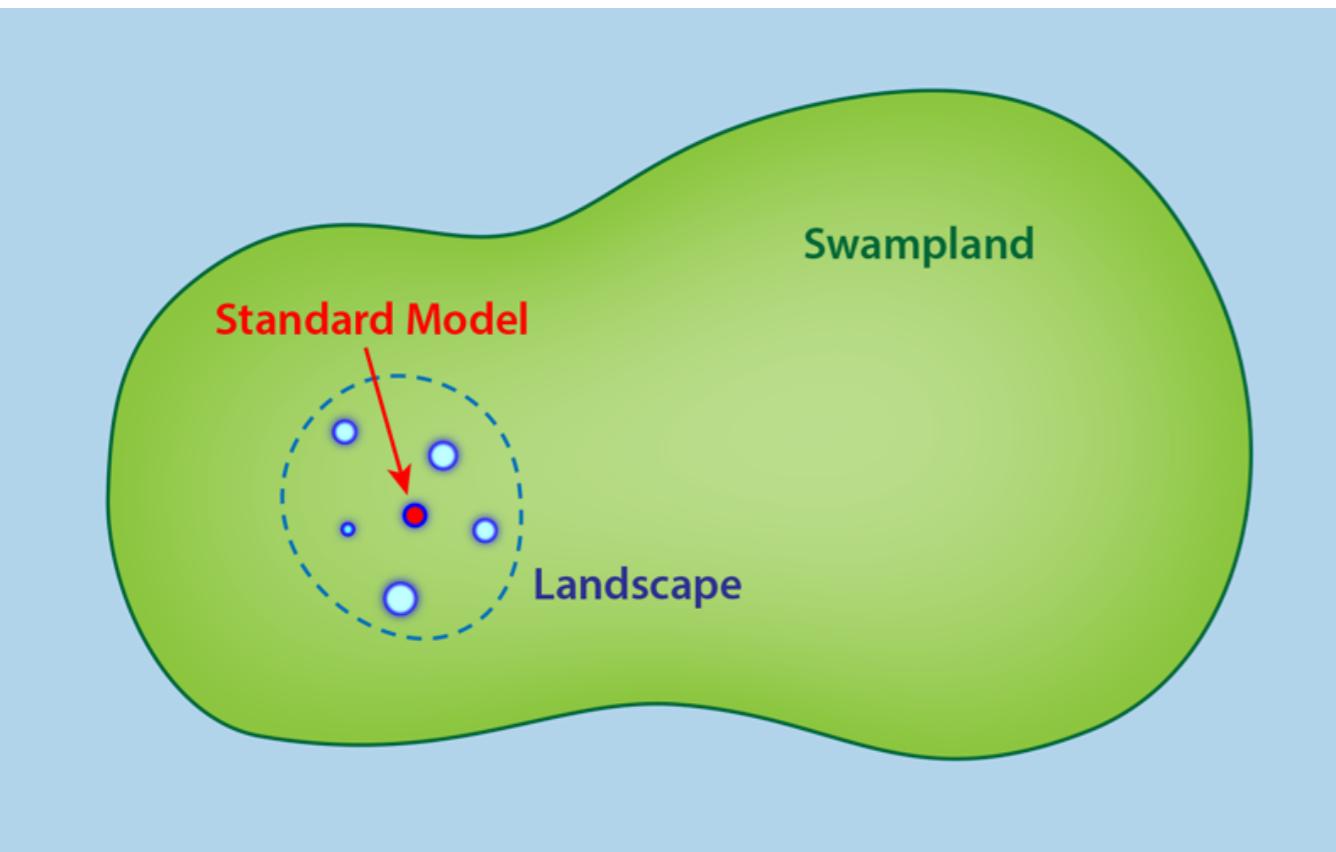
CH, Shi Pi and Misao Sasaki, arXiv:1809.05507

$$V(\phi, \mathcal{H}) = e^{-\xi\phi} \left(\lambda (|\mathcal{H}|^2 - v^2)^2 + \Lambda \right) \quad (\xi > 0)$$



$$e^{-\xi\phi} > 1.08 \pm 0.02 \quad \longrightarrow \quad \xi > 0.35 \pm 0.05$$

Consistent with the Swampland Conjecture



- ❖ What is Swampland?
- ❖ The set of low energy physics models which look consistent but ultimately are not when coupled to gravity, is called the “swampland.” —Vafa

G. Obied, H. Ooguri, L. Spodyneiko, C. Vafa, arXiv:1806.08362

Swampland conjecture: a quantum gravity theory should satisfy:

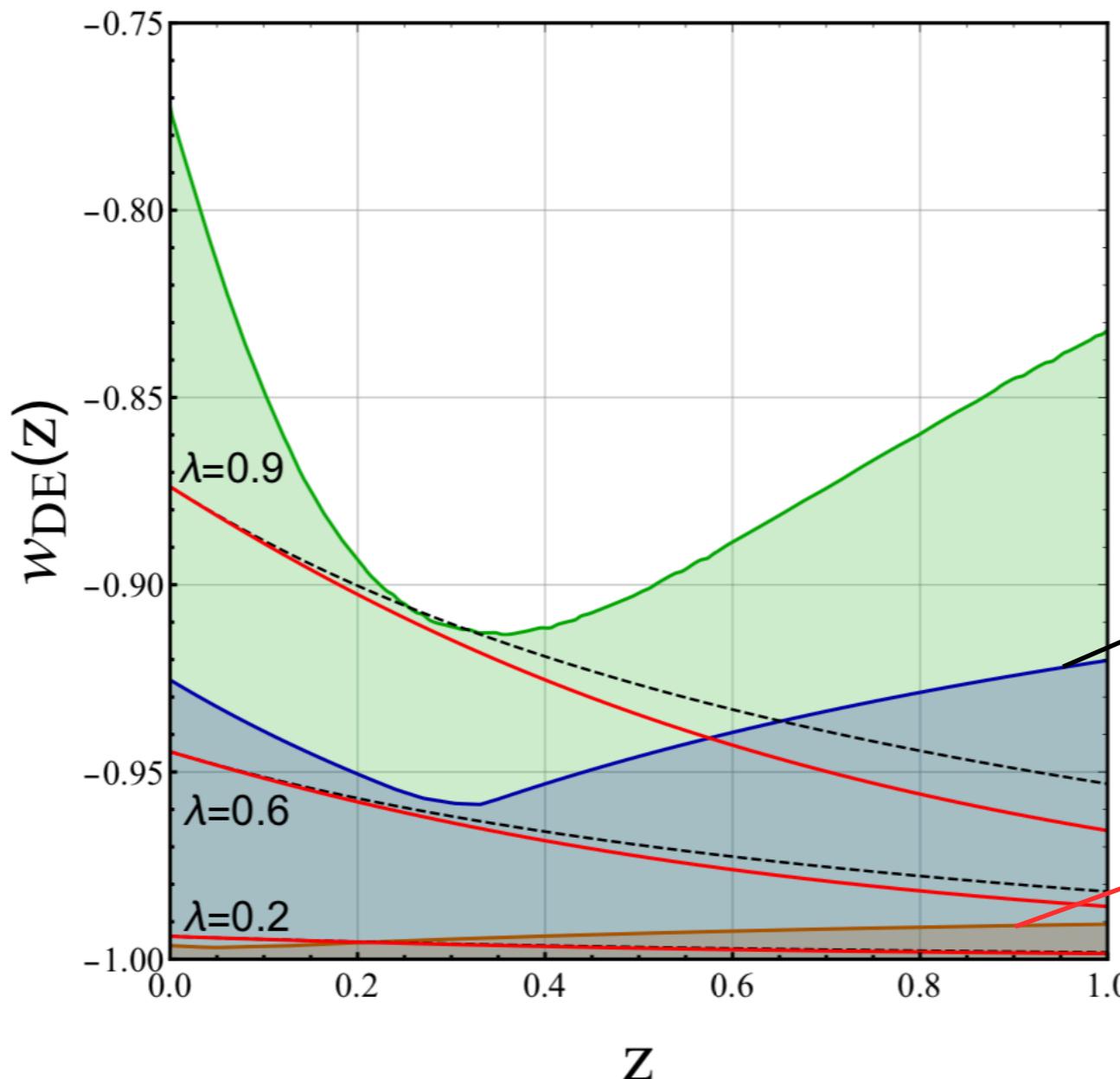
$$|\nabla V|/V > c, c \sim O(1)$$

$$||\nabla V_{\text{total}}|| = \sqrt{\sum_{i,j} g_{\text{conf}}^{ij} (\partial_{\phi_i} V_{\text{total}})(\partial_{\phi_j} V_{\text{total}})} ,$$

Can be tested in near future!

Dark energy experiment already exclude $\xi > 0.6$

Y. Akrami, R. Kallosh, A. Linde, V. Vardanyan, arxiv:1808.09440



Exclusion limit combining
SNeIa, CMB, BAO, H_0

Future reach: Euclid and the SKA
(large-scale structure surveys)
(within 10 years)

Higgs inflation

Higgs is the only scalar field in SM

Bezrukov and Shaposhnikov, Phys.Lett.B 659 (2008) 703-706

$$S_J = \int d^4x \sqrt{-g_J} \left[\frac{M_P^2}{2} \left(1 + \boxed{\frac{\xi\phi^2}{M_P^2}} R_J \right) - \frac{1}{2} |\partial_\mu \phi|^2 - V_J(\phi) \right]$$

$$g_{\mu\nu} = \Omega(\phi)^2 g_{J\mu\nu} \quad \Omega^2 = 1 + \frac{\xi\phi^2}{M_P^2}$$

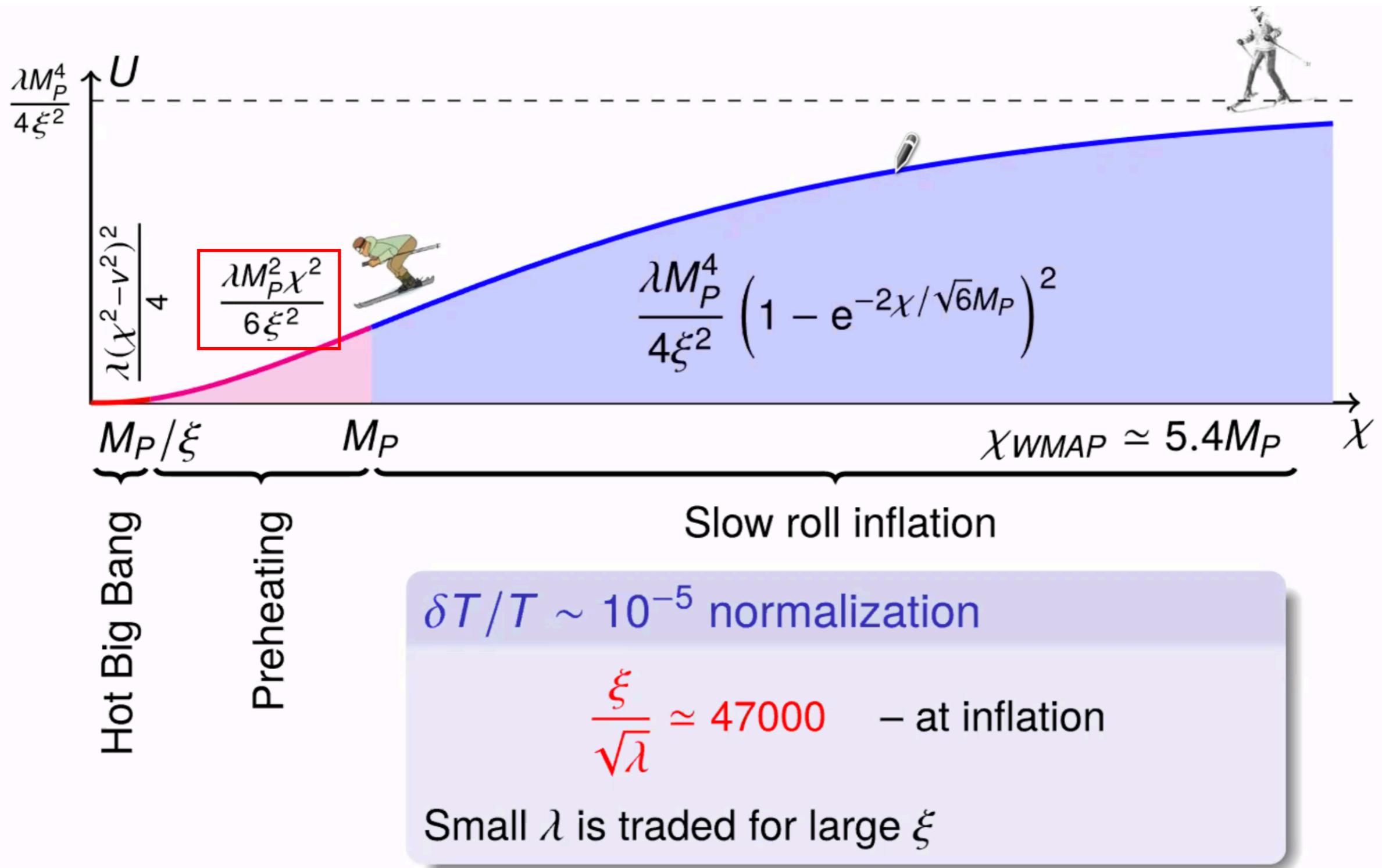
$$\frac{dh}{d\phi} = \left(\frac{1 + \xi(1 + 6\xi)\phi^2/M_P^2}{(1 + \xi\phi^2/M_P^2)^2} \right)^{1/2}$$

$$S = \int d^4x \sqrt{-g} \left[\frac{M_P^2}{2} R - \frac{1}{2} g^{\mu\nu} \partial_\mu h \partial_\nu h - V(h) \right] \quad V(h) \equiv V_J(\phi(h))/\Omega^4(\phi(h))$$

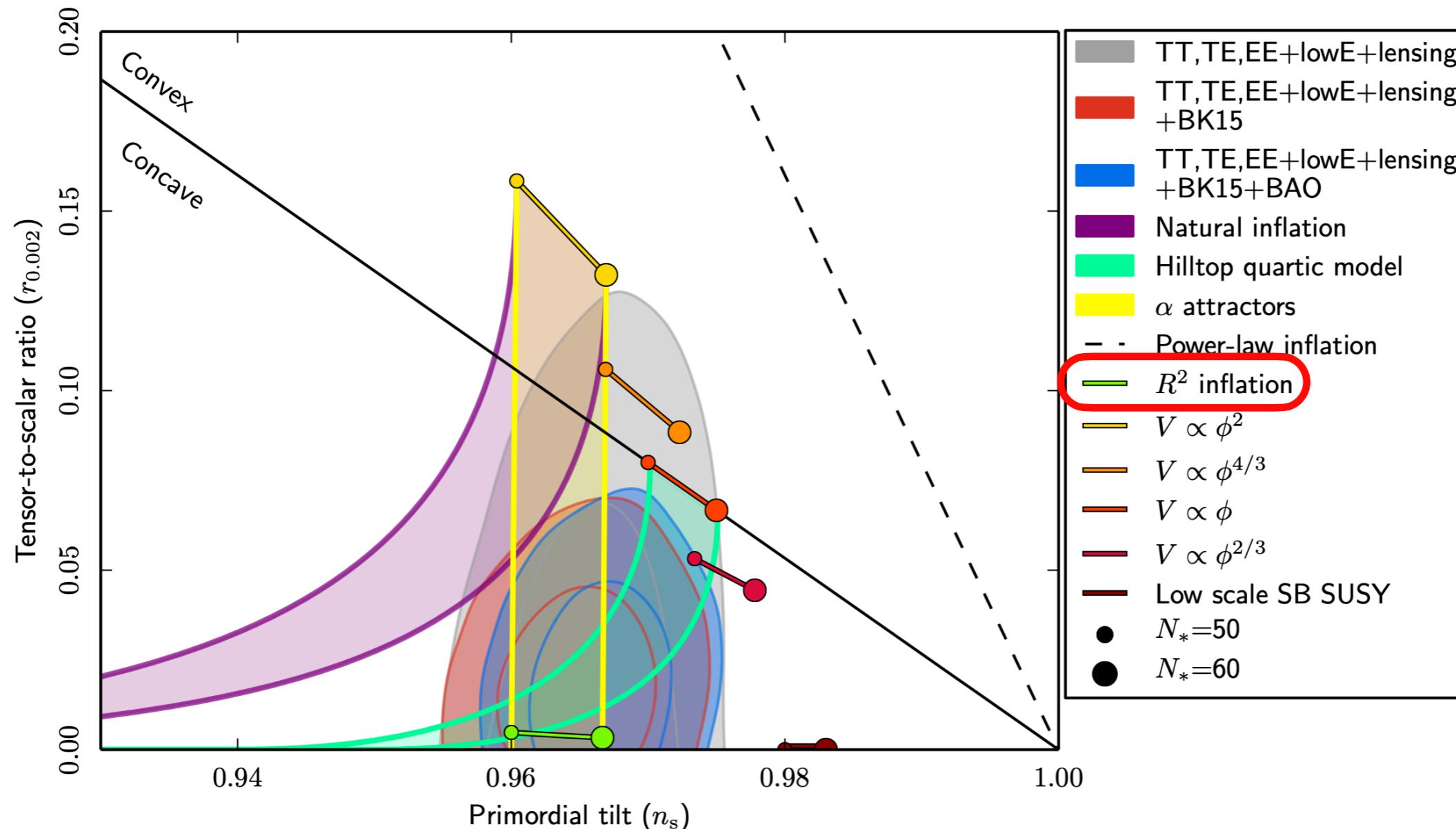
$$V(h) \simeq \begin{cases} \frac{\lambda}{4} h^4 & \text{for } |h| \ll \frac{M_P}{\xi} \\ \frac{\lambda M_P^2}{4\xi^2} \left(1 - e^{-\sqrt{\frac{2}{3}} \frac{h}{M_P}} \right)^2 & \text{for } |h| \gg \frac{M_P}{\xi} \end{cases}$$

Higgs inflation

Plot borrowed from Bezrukov



Current status



Concave potential is preferred by the data

SM + a triplet Higgs

$H(2, 1/2)$, $\Delta(3, 1)$, $L(2, -1/2)$

$$H = \begin{pmatrix} h^+ \\ h \end{pmatrix}, \quad \Delta = \begin{pmatrix} \Delta^+/\sqrt{2} & \Delta^{++} \\ \Delta^0 & -\Delta^+/\sqrt{2} \end{pmatrix}$$

$$\mathcal{L}_{Yukawa} = \mathcal{L}_{Yukawa}^{\text{SM}} - \boxed{\frac{1}{2} y_{ij} \bar{L}_i^c \Delta L_j} + h.c.$$

Giving neutrino mass matrix
Delta get a lepton number -2

SM + a triplet Higgs

Adding non-minimal couplings for inflation

$$\begin{aligned}\frac{\mathcal{L}}{\sqrt{-g}} = & -\frac{1}{2}M_P^2 R - \boxed{f(H, \Delta)R} - g^{\mu\nu}(D_\mu H)^\dagger(D_\nu H) \\ & - g^{\mu\nu}(D_\mu \Delta)^\dagger(D_\nu \Delta) - V(H, \Delta) + \mathcal{L}_{\text{Yukawa}}\end{aligned}$$

$$f(H, \Delta) = \xi_H H^\dagger H + \xi_\Delta \Delta^\dagger \Delta + \dots,$$

SM + a triplet Higgs

During inflation(Oleg Lebedev and Hyun Min Lee, arXiv:1105.2284)

$$\frac{|H|}{|\Delta|} = \tan \alpha \simeq \sqrt{\frac{2\lambda_\Delta \xi_H - \lambda_H \Delta \xi_\Delta}{2\lambda_H \xi_\Delta - \lambda_H \Delta \xi_H}}$$

$$H = \phi \sin \alpha, \quad \Delta = \phi \cos \alpha$$

$$f(H, \Delta) = \frac{\xi_H}{2}|H|^2 + \frac{\xi_\Delta}{2}|\Delta|^2 = \frac{\xi}{2}|\phi|^2$$

$$\xi = \xi_H \sin^2 \alpha + \xi_\Delta \cos^2 \alpha$$

SM + a triplet Higgs

$$H(2, 1/2), \Delta(3, 1), L(2, -1/2)$$

$$\begin{aligned} V(H, \Delta) = & -m_H^2 H^\dagger H + \lambda_H (H^\dagger H)^2 + m_\Delta^2 \text{Tr}(\Delta^\dagger \Delta) \\ & + \lambda_3 \text{Tr}(\Delta^\dagger \Delta)^2 + \lambda_1 (H^\dagger H) \text{Tr}(\Delta^\dagger \Delta) \\ & + \lambda_2 (\text{Tr}(\Delta^\dagger \Delta))^2 + \lambda_4 H^\dagger \Delta \Delta^\dagger H \end{aligned}$$

$$\langle \Delta^0 \rangle \simeq \frac{\mu v_{\text{EW}}^2}{2m_\Delta^2}$$

$$\begin{aligned} & + \left[\mu (H^T i\sigma^2 \Delta^\dagger H) + \frac{\lambda_5}{M_P} (H^T i\sigma^2 \Delta^\dagger H)(H^\dagger H) \right. \\ & \left. + \frac{\lambda'_5}{M_P} (H^T i\sigma^2 \Delta^\dagger H)(\Delta^\dagger \Delta) + h.c. \right] + \dots \end{aligned}$$

U(1)L breaking term



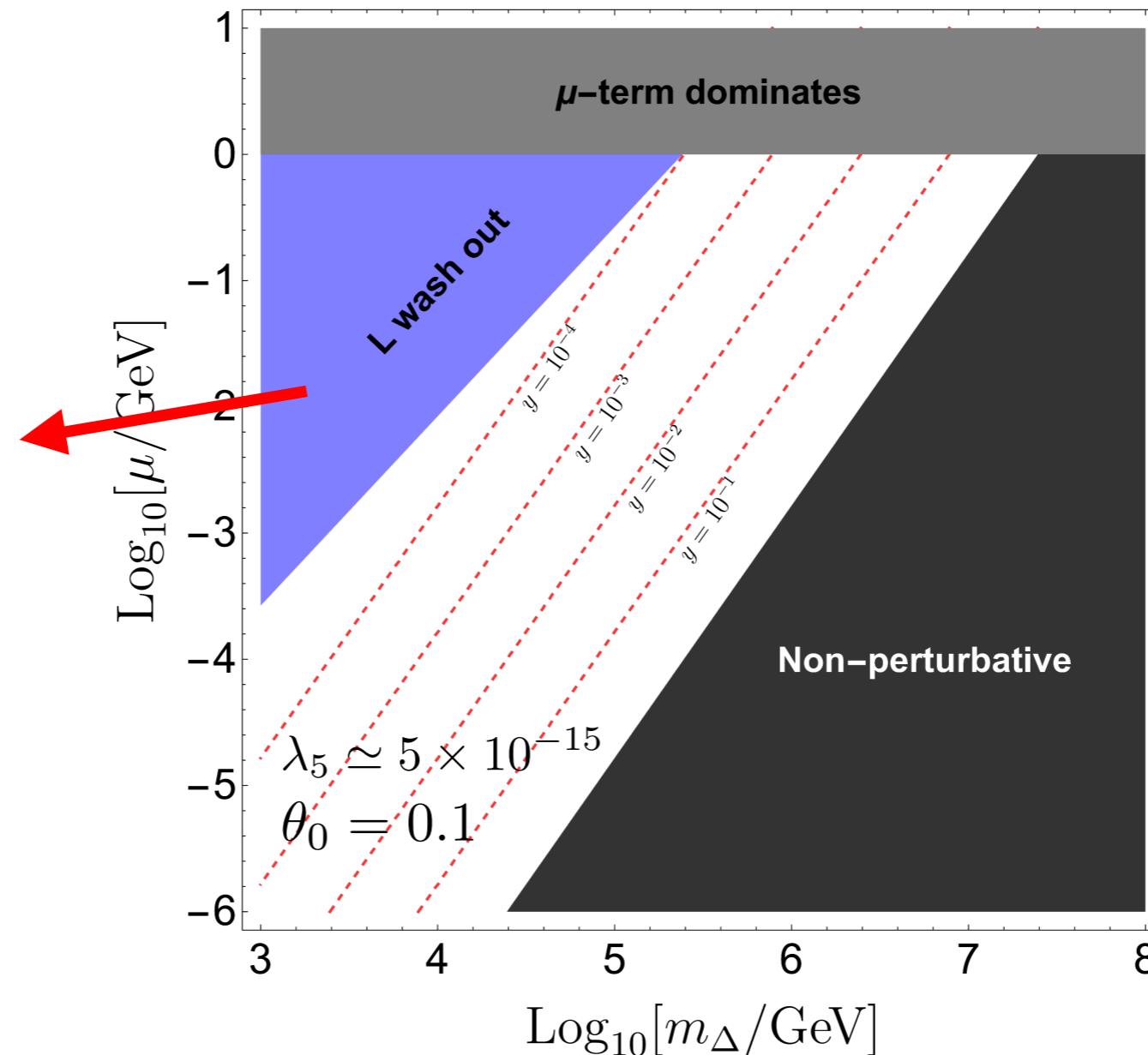
$$V(\phi) = m^2 |\phi|^2 + \lambda |\phi|^4 + (\tilde{\mu} |\phi|^2 \phi + \frac{\tilde{\lambda}_5}{M_P} |\phi|^4 \phi + h.c.)$$

SM + a triplet Higgs

$$\lambda_H \simeq 0.1, \lambda_\Delta \simeq 4.5 \times 10^{-5}, \xi_H \sim \xi_\Delta = 300, \alpha \simeq 0.022$$

Avoid washing out
the lepton asymmetry

$$\Gamma_{ID}(HH \leftrightarrow \Delta)|_{T=m_\Delta} < H|_{T=m_\Delta}$$



$$\langle \Delta^0 \rangle \simeq \frac{\mu v_{\text{EW}}^2}{2m_\Delta^2} \quad (\text{at least one neutrino mass } 0.05 \text{ eV})$$

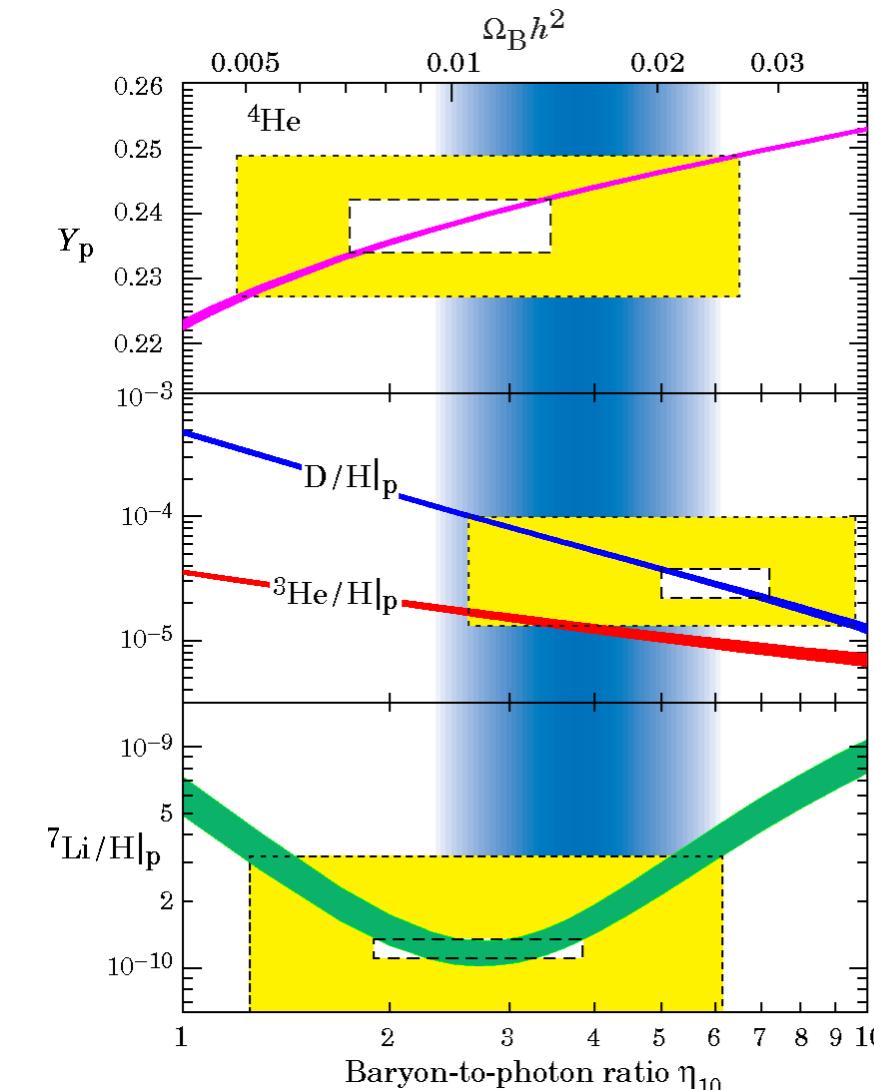
Summary

Higgs not only plays an important role in low energy particle physics, it can be related other important physics(cosmology)

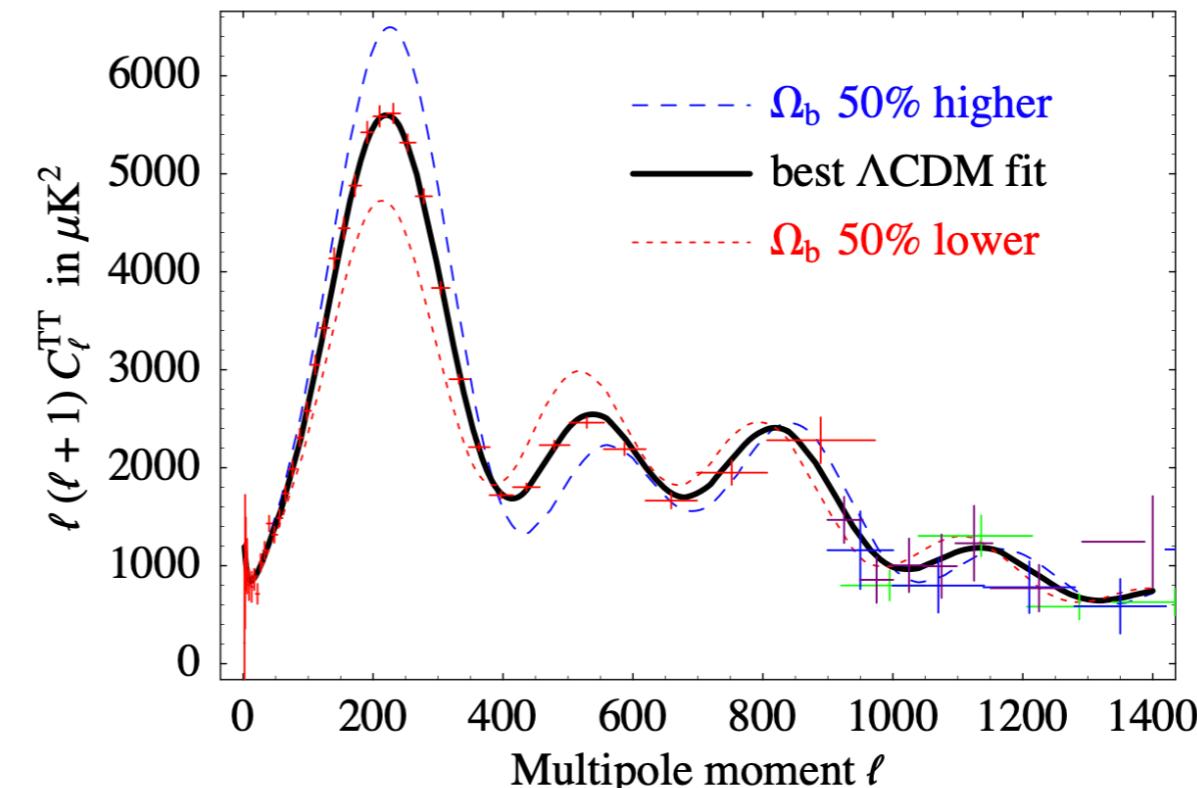
- Inflation
- Baryogenesis
- Dark energy and the fundamental law

Thanks!

Baryon asymmetry of our universe



BBN



Parameter	Plik best fit	Plik [1]	CamSpec [2]	([2] - [1])/ σ_1	Combined
$\Omega_b h^2$	0.022383	0.02237 ± 0.00015	0.02229 ± 0.00015	-0.5	0.02233 ± 0.00015
$\Omega_c h^2$	0.12011	0.1200 ± 0.0012	0.1197 ± 0.0012	-0.3	0.1198 ± 0.0012

$$\frac{n_b - n_{\bar{b}}}{n_\gamma} \sim 10^{-10}$$

How to generate baryon asymmetry?

Assuming no baryon asymmetry in the beginning
(if any, diluted by inflation)

Sakharov conditions

1. B number violation
2. C and CP violation
3. Out of thermal equilibrium

SM has (1) (2) but not enough CP violation, (3) does not

Three popular ways to generate baryon asymmetry

- Electroweak baryogenesis Rubakov and Shaposhnikov, 1996'
D. E. Morrissey and M. J. Ramsey-Musolf, 2012'
First order phase transition (adding scalars) + additional \cancel{CP}
- Baryogenesis via thermal leptogenesis Fukugita and Yanagida, 1986'
Connection to neutrino masses
$$n_B = \frac{28}{79} (\mathcal{B} - \mathcal{L})_i$$
- Baryogenesis from Affleck-Dine mechanism Affleck and Dine, 1985'

A well-known mechanism for high energy physics society

Baryogenesis from Affleck-Dine mechanism

Assuming a complex scalar ϕ taking U(1)B charge

$$\mathcal{L} = |\partial_\mu \phi|^2 - m^2 |\phi|^2$$

$\phi \rightarrow e^{i\alpha} \phi$ symmetry, corresponding current

$$j_B^\mu = i(\phi^* \partial^\mu \phi - \phi \partial^\mu \phi^*)$$

ϕ is spatially constant $n_B = i(\phi^* \dot{\phi} - \phi \dot{\phi}^*)$

We can add a small U(1) breaking term

$$V = m^2 |\phi|^2 + c_n (\phi^n + \phi^{*n}) + \frac{|\phi|^{2m}}{M^{2m-4}}$$

Baryogenesis from Affleck-Dine mechanism

Equation of motion in an expansion of universe

$$\ddot{\phi} + 3H\dot{\phi} + \frac{\partial V}{\partial \phi^*} = 0$$

Only from U(1) breaking term

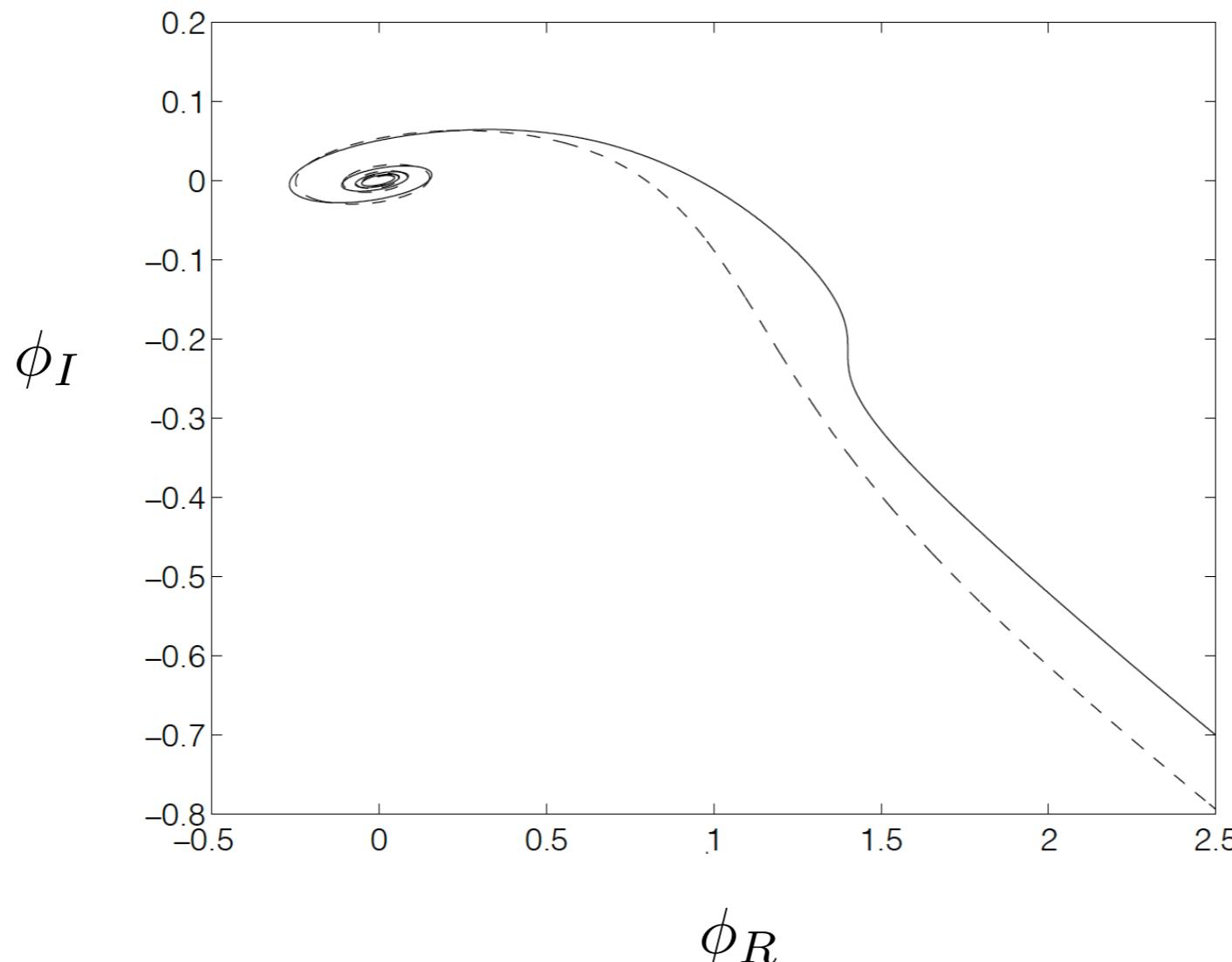
$$\dot{n}_B + 3Hn_B = \boxed{\text{Im} \left(\phi \frac{\partial V}{\partial \phi} \right)}$$

At $t_0 = 1/H \sim 1/m$ $n_B(t_0) \sim \frac{nc_n \phi_0^n}{m}$

$$n_B(t) = n_B(t_0)(a_0/a(t))^3$$

$$\eta = \frac{n_B}{s} \Big|_{t_{rh}} \quad t_{rh} = \frac{1}{H_{rh}} \sim \frac{M_{pl}}{T_{rh}^2} \quad s \sim T_{rh}^3$$

Baryogenesis from Affleck-Dine mechanism



CP violation appears when $\langle \phi_I \rangle \neq 0$

Affleck-Dine mechanism for SUSY

Scalar potential in SUSY

$$V = \sum_i |F_i|^2 + \frac{1}{2} \sum_a g_a^2 D^a D^a$$

$$F_i \equiv \frac{\partial W_{MSSM}}{\partial \phi_i}, \quad D^a = \phi^\dagger T^a \phi.$$

There exist particular vacuum alignment that the potential vanish(flat direction)

For example,

$$H_u = \frac{1}{\sqrt{2}} \begin{pmatrix} 0 \\ \phi \end{pmatrix}, \quad L = \frac{1}{\sqrt{2}} \begin{pmatrix} \phi \\ 0 \end{pmatrix} \quad V(\phi) = 0$$

$$H_u = \phi \sin \alpha \quad L = \phi \cos \alpha \quad \alpha = \frac{\pi}{4}$$

Affleck-Dine mechanism for SUSY

The Flat directions can be lifted by adding high dimension operator
(as required by neutrino mass)

mixing with L and Hu are important

$$W = \frac{1}{M} (LH_u)^2 = \frac{1}{2M} \phi^4 \quad M \sim 10^{15} \text{ GeV}$$

Including the SUSY breaking (supergravity mediation)

$$V(\phi) = m^2 |\phi|^2 + \left(\frac{2A}{M} \phi^4 + h.c \right) + \frac{4}{M^2} |\phi|^6$$

U(1)L breaking term

m, A are SUSY breaking parameters $m, A \sim m_{3/2}$

Coupling with inflaton providing an initial condition

Many problems to answer

- How to generate the “flat directions” without SUSY?
- What is the origin of the U(1) breaking term?
- Why initial (large) value for the scalar field?

Borrowed the idea of Higgs inflation

Higgs is the only scalar field in SM

Bezrukov and Shaposhnikov, Phys.Lett.B 659 (2008) 703-706

$$S_J = \int d^4x \sqrt{-g_J} \left[\frac{M_P^2}{2} \left(1 + \boxed{\frac{\xi\phi^2}{M_P^2}} R_J \right) - \frac{1}{2} |\partial_\mu \phi|^2 - V_J(\phi) \right]$$

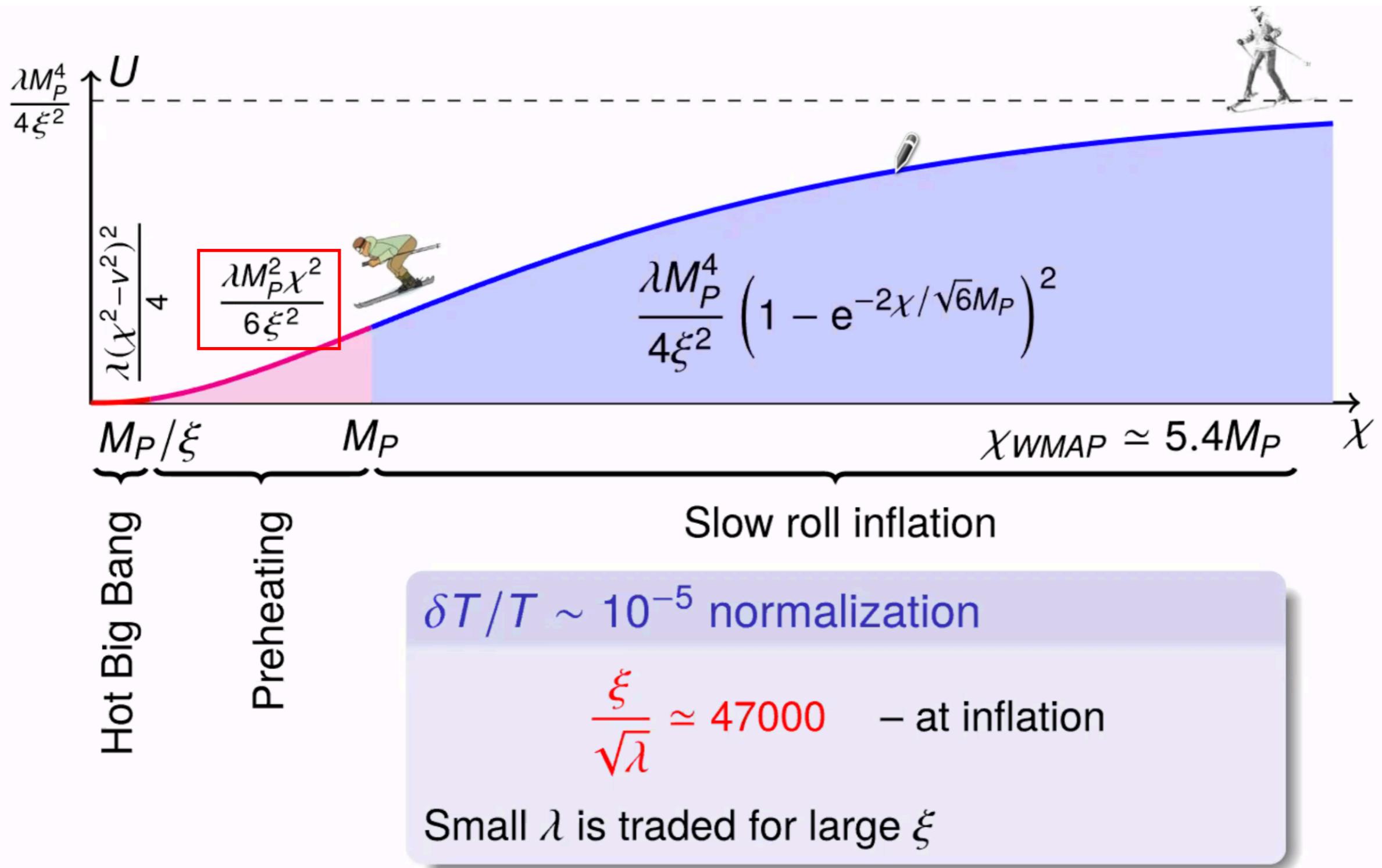
$$g_{\mu\nu} = \Omega(\phi)^2 g_{J\mu\nu} \quad \Omega^2 = 1 + \frac{\xi\phi^2}{M_P^2}$$

$$\frac{d\chi}{d\phi} = \left(\frac{1 + \xi(1 + 6\xi)\phi^2/M_P^2}{(1 + \xi\phi^2/M_P^2)^2} \right)^{1/2}$$

$$S = \int d^4x \sqrt{-g} \left[\frac{M_P^2}{2} R - \frac{1}{2} g^{\mu\nu} \partial_\mu \chi \partial_\nu \chi - V(\chi) \right] \quad V(\chi) \equiv V_J(\phi(\chi))/\Omega^4(\phi(\chi))$$

Higgs inflation

Plot borrowed from Bezrukov



SM + a triplet Higgs

Adding non-minimal couplings for inflation

$$\begin{aligned}\frac{\mathcal{L}}{\sqrt{-g}} = & -\frac{1}{2}M_P^2 R - \boxed{f(H, \Delta)R} - g^{\mu\nu}(D_\mu H)^\dagger(D_\nu H) \\ & - g^{\mu\nu}(D_\mu \Delta)^\dagger(D_\nu \Delta) - V(H, \Delta) + \mathcal{L}_{\text{Yukawa}}\end{aligned}$$

$$f(H, \Delta) = \xi_H H^\dagger H + \xi_\Delta \Delta^\dagger \Delta + \dots,$$

SM + a triplet Higgs

During inflation(Oleg Lebedev and Hyun Min Lee, arXiv:1105.2284)

$$\frac{|H|}{|\Delta|} = \tan \alpha \simeq \sqrt{\frac{2\lambda_\Delta \xi_H - \lambda_H \Delta \xi_\Delta}{2\lambda_H \xi_\Delta - \lambda_H \Delta \xi_H}}$$

$$H = \phi \sin \alpha, \quad \Delta = \phi \cos \alpha$$

$$f(H, \Delta) = \frac{\xi_H}{2}|H|^2 + \frac{\xi_\Delta}{2}|\Delta|^2 = \frac{\xi}{2}|\phi|^2$$

$$\xi = \xi_H \sin^2 \alpha + \xi_\Delta \cos^2 \alpha$$

Similar to SUSY case, mixing with an angle alpha

SM + a triplet Higgs

$H(2, 1/2)$, $\Delta(3, 1)$, $L(2, -1/2)$

$$H = \begin{pmatrix} h^+ \\ h \end{pmatrix}, \quad \Delta = \begin{pmatrix} \Delta^+/\sqrt{2} & \Delta^{++} \\ \Delta^0 & -\Delta^+/\sqrt{2} \end{pmatrix}$$

$$\mathcal{L}_{Yukawa} = \mathcal{L}_{Yukawa}^{\text{SM}} - \boxed{\frac{1}{2} y_{ij} \bar{L}_i^c \Delta L_j} + h.c.$$

Giving neutrino mass matrix
Delta get a lepton number -2

SM + a triplet Higgs

$$H(2, 1/2), \Delta(3, 1), L(2, -1/2)$$

$$\begin{aligned} V(H, \Delta) = & -m_H^2 H^\dagger H + \lambda_H (H^\dagger H)^2 + m_\Delta^2 \text{Tr}(\Delta^\dagger \Delta) \\ & + \lambda_3 \text{Tr}(\Delta^\dagger \Delta)^2 + \lambda_1 (H^\dagger H) \text{Tr}(\Delta^\dagger \Delta) \\ & + \lambda_2 (\text{Tr}(\Delta^\dagger \Delta))^2 + \lambda_4 H^\dagger \Delta \Delta^\dagger H \end{aligned}$$

$$\langle \Delta^0 \rangle \simeq \frac{\mu v_{\text{EW}}^2}{2m_\Delta^2}$$

$$\begin{aligned} & + \left[\mu (H^T i\sigma^2 \Delta^\dagger H) + \frac{\lambda_5}{M_P} (H^T i\sigma^2 \Delta^\dagger H)(H^\dagger H) \right. \\ & \left. + \frac{\lambda'_5}{M_P} (H^T i\sigma^2 \Delta^\dagger H)(\Delta^\dagger \Delta) + h.c. \right] + \dots \end{aligned}$$

U(1)L breaking term



$$V(\phi) = m^2 |\phi|^2 + \lambda |\phi|^4 + (\tilde{\mu} |\phi|^2 \phi + \frac{\tilde{\lambda}_5}{M_P} |\phi|^4 \phi + h.c.)$$

SM + a triplet Higgs

In practice

$$\phi = \frac{1}{\sqrt{2}} \varphi \exp(i\theta)$$

$$\begin{aligned} \frac{\mathcal{L}}{\sqrt{-g}} = & -\frac{1}{2} M_P^2 R - \frac{1}{2} \xi \varphi^2 R - \frac{1}{2} g^{\mu\nu} \partial_\mu \varphi \partial_\nu \varphi \\ & - \frac{1}{2} \varphi^2 g^{\mu\nu} \partial_\mu \theta \partial_\nu \theta - V(\varphi, \theta) , \end{aligned}$$

$$V(\varphi, \theta) = \frac{1}{2} m^2 \varphi^2 + \frac{\lambda}{4} \varphi^4 + 2\varphi^3 \left(\tilde{\mu} + \frac{\tilde{\lambda}_5}{M_P} \varphi^2 \right) \cos \theta$$

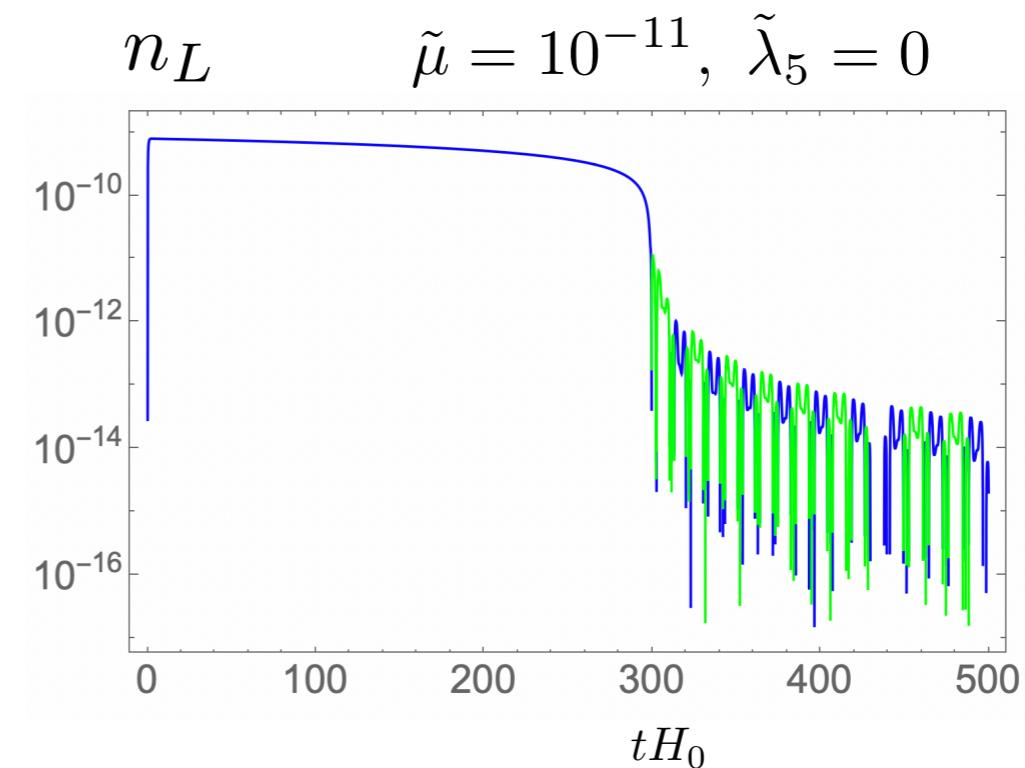
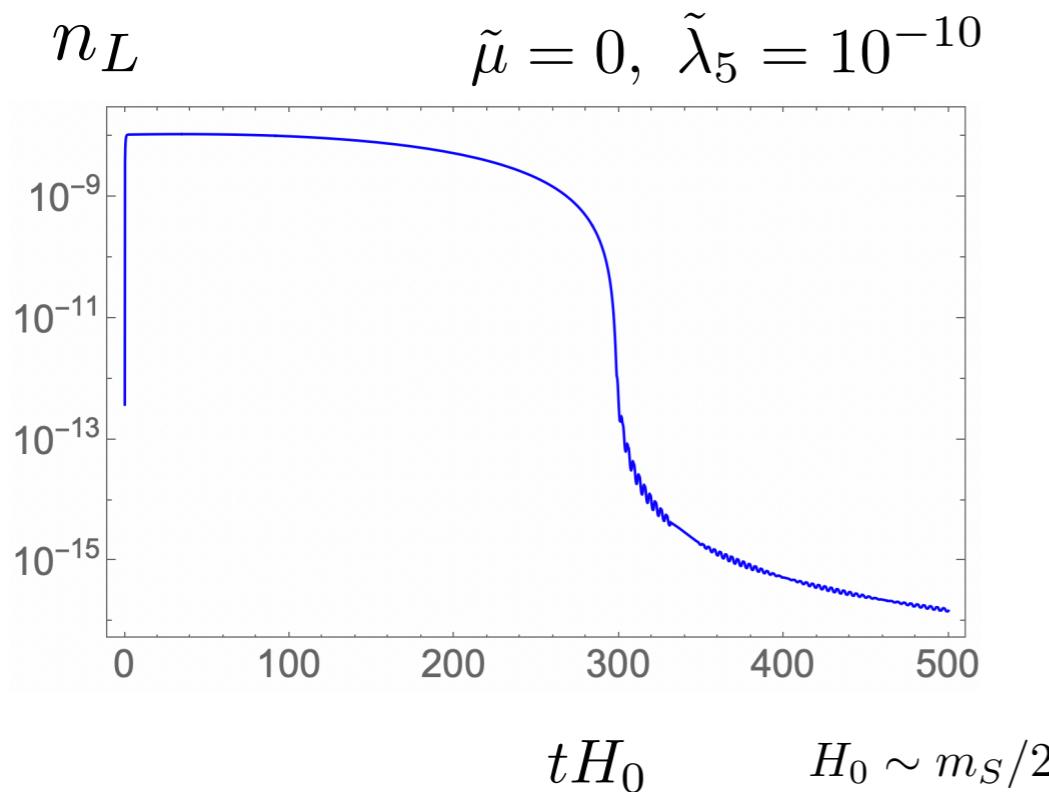
SM + a triplet Higgs

$$\frac{\chi}{M_p} \approx \begin{cases} \frac{\varphi}{M_p} & \text{for } \frac{\varphi}{M_p} \ll \frac{1}{\xi} \quad (\text{after reheating}) \\ \sqrt{\frac{3}{2}} \xi \left(\frac{\varphi}{M_p} \right)^2 & \text{for } \frac{1}{\xi} \ll \frac{\varphi}{M_p} \ll \frac{1}{\sqrt{\xi}} \quad (\text{reheating}) \\ \sqrt{\frac{3}{2}} \ln \Omega^2 = \sqrt{\frac{3}{2}} \ln \left[1 + \xi \left(\frac{\varphi}{M_p} \right)^2 \right] & \text{for } \frac{1}{\sqrt{\xi}} \ll \frac{\varphi}{M_p} \quad (\text{inflation}) \end{cases}$$

$$U(\chi) \approx \begin{cases} \frac{1}{4} \lambda \chi^4 & \text{for } \frac{\chi}{M_p} \ll \frac{1}{\xi} \quad (\text{after reheating}) \\ \frac{1}{2} m_S^2 \chi^2 & \text{for } \frac{1}{\xi} \ll \frac{\chi}{M_p} \ll 1 \quad (\text{reheating}) \\ \frac{3}{4} m_S^2 M_p^2 \left(1 - e^{-\sqrt{\frac{2}{3}}(\chi/M_p)} \right)^2 & \text{for } 1 \ll \frac{\chi}{M_p} \quad (\text{inflation}) \end{cases}$$

SM + a triplet Higgs: baryon number

$$n_L = Q_L \phi^2(\chi) \dot{\theta} \cos^2 \alpha$$



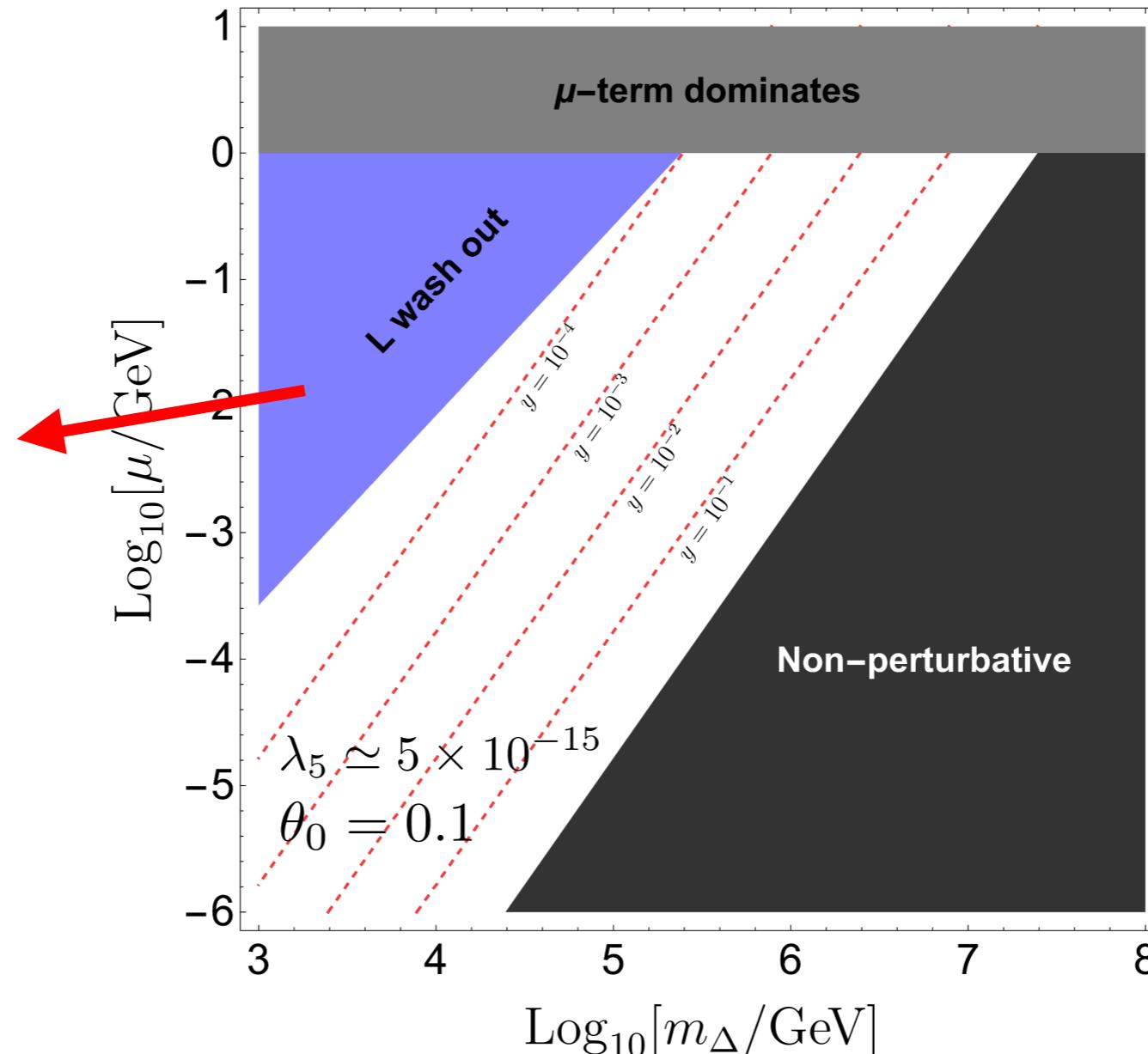
Cubic term should not be too large, otherwise baryon number oscillates
A small cubic term also benefit to avoid washing out

Model with triplet Higgs

$$\lambda_H \simeq 0.1, \lambda_\Delta \simeq 4.5 \times 10^{-5}, \xi_H \sim \xi_\Delta = 300, \alpha \simeq 0.022$$

Avoid washing out
the lepton asymmetry

$$\Gamma_{ID}(HH \leftrightarrow \Delta)|_{T=m_\Delta} < H|_{T=m_\Delta}$$



$$\langle \Delta^0 \rangle \simeq \frac{\mu v_{\text{EW}}^2}{2m_\Delta^2}$$

(at least one neutrino mass 0.05 eV)

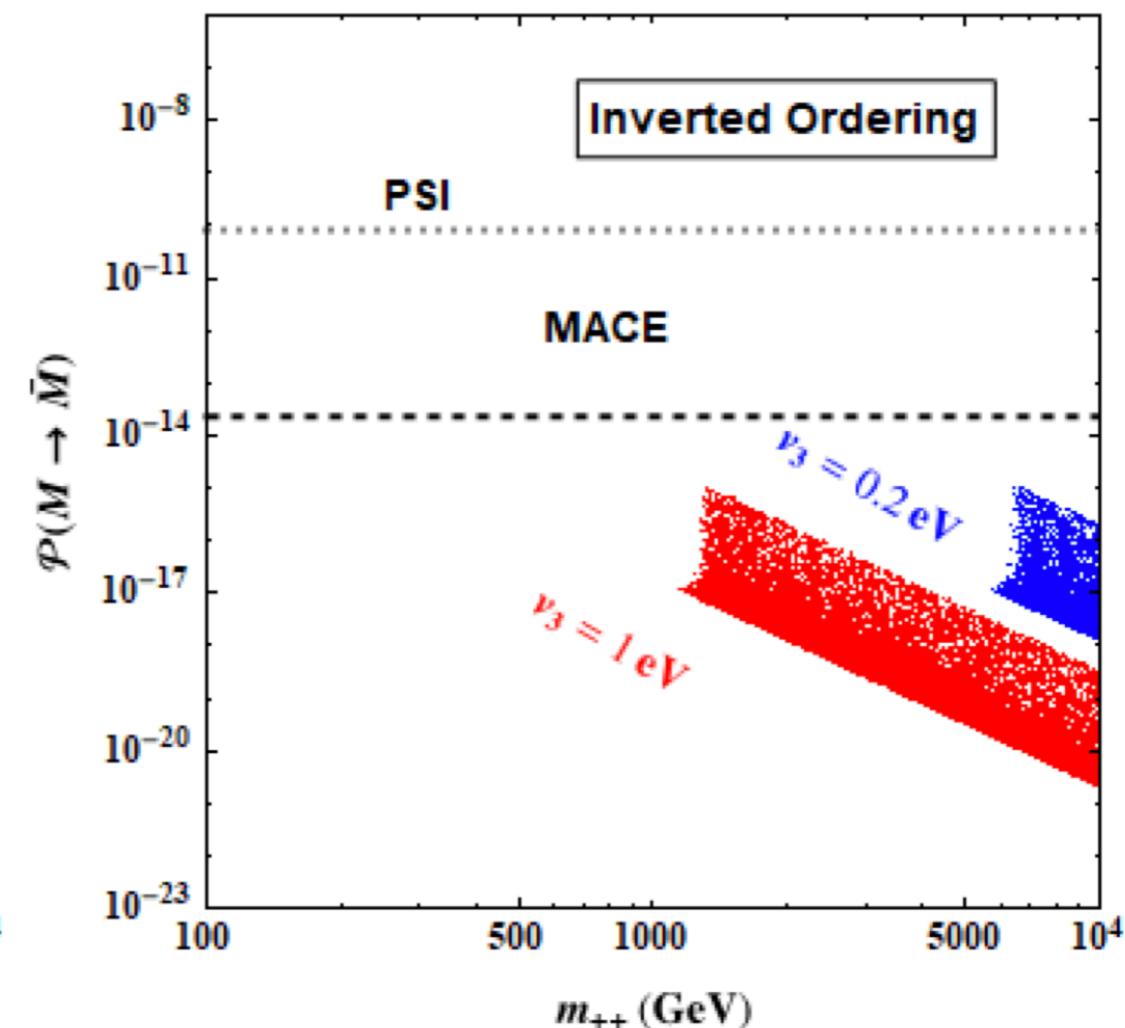
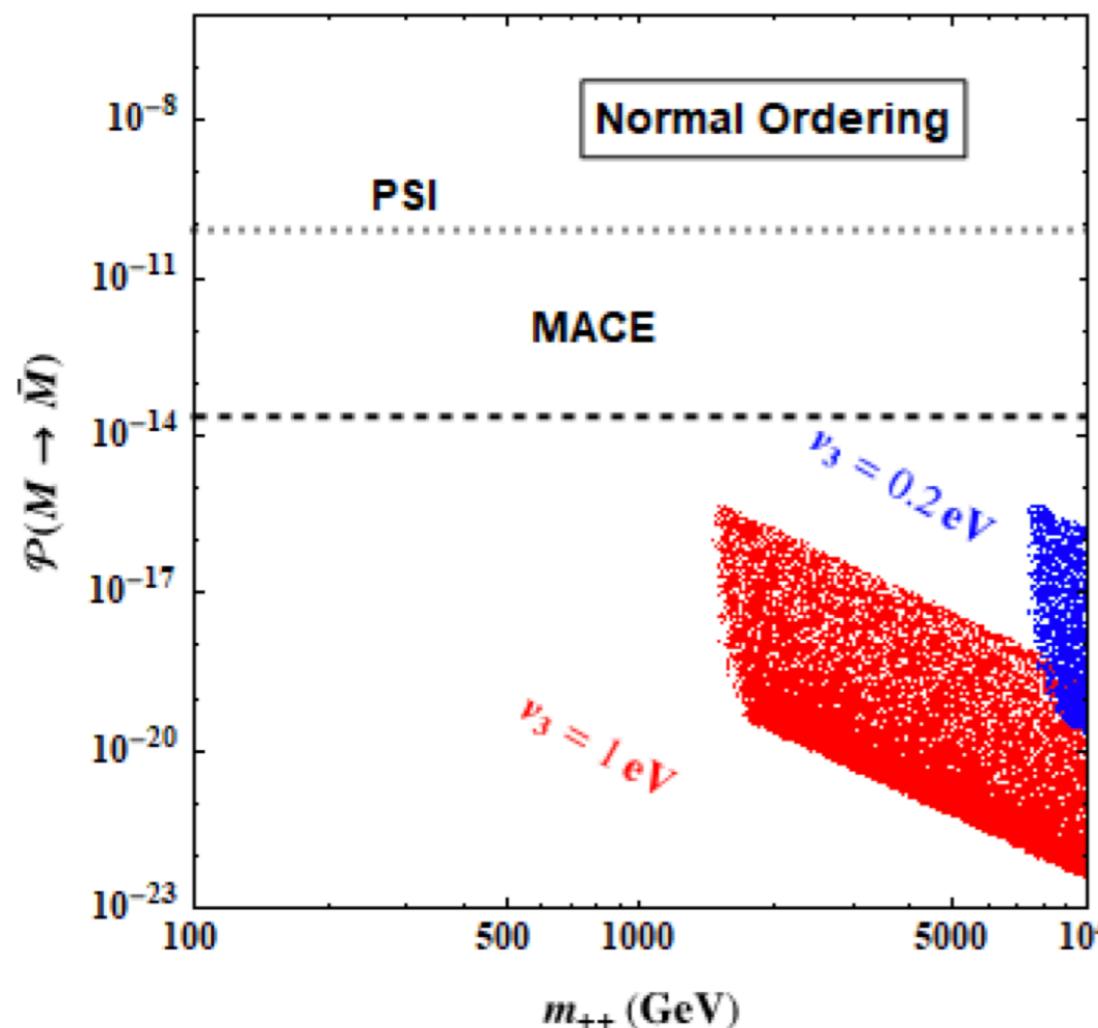
Indication for low energy physics

$$\mathcal{B}(\mu^+ \rightarrow e^+ e^- e^+) = \frac{|(y_N)_{\mu e}(y_N^\dagger)_{ee}|^2}{16G_F^2 m_{++}^4}$$

$$\mathcal{B}(\mu^+ \rightarrow e^+ e^- e^+) \leq 1.0 \times 10^{-12}$$

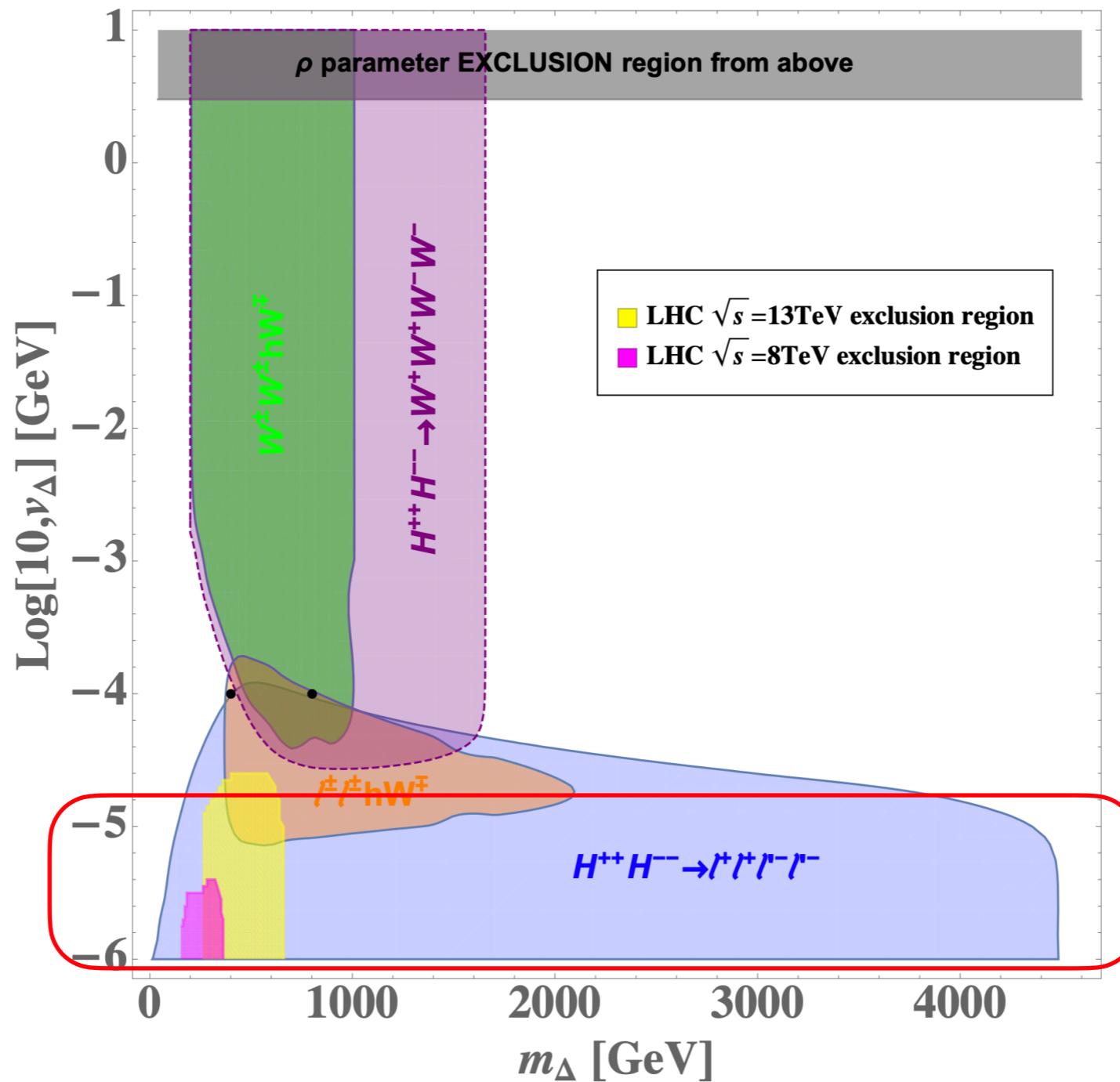
$$\mathcal{B}(\mu \rightarrow e\gamma) \simeq \frac{\alpha}{768\pi} \frac{\left|(y_N^\dagger y_N)_{e\mu}\right|^2}{G_F^2} \left(\frac{1}{m_+^2} + \frac{8}{m_{++}^2}\right)^2 \quad \mathcal{B}(\mu \rightarrow e\gamma) < 4.2 \times 10^{-13}$$

CH, D. Huang, J. Tang, Y. Zhang, Phys. Rev. D 103, 055023 (2021)



Indication for low energy physics

Y. Du, A. Dunbrack, M. J. Ramsey-Musolf, J. Yu, JHEP01(2019)101



5 sigma discover region @100 TeV collider

Summary

- We present a simple extension of SM to resolve three important problems: inflation, baryon asymmetry and neutrino masses
- Neutrino masses are majorana-type: $0\nu\beta\beta$
- A sizable tensor to scalar ratio: $r \sim 0.005$, which can be reached by next generation CMB measurement
- Leaving a light triplet Higgs at low energy scale: which might be probed by collider physics and LFV measurement

SM + a triplet Higgs

$$\tilde{g}_{\mu\nu} = \Omega^2 g_{\mu\nu}, \quad \Omega^2 = 1 + \xi \varphi^2 / M_P^2 .$$

$$\chi(\varphi) = 1/\sqrt{\xi}(\sqrt{1+6\xi} \sinh^{-1}(\sqrt{\xi+6\xi^2}\varphi) - \sqrt{6\xi} \sinh^{-1}(\sqrt{6\xi^2}\varphi/\sqrt{1+\xi\varphi^2})$$

$$\frac{\mathcal{L}}{\sqrt{-g}} = -\frac{M_P^2}{2}R - \frac{1}{2}g^{\mu\nu}\partial_\mu\chi\partial_\nu\chi - \frac{1}{2}f(\chi)g^{\mu\nu}\partial_\mu\theta\partial_\nu\theta - U(\chi, \theta)$$

$$f(\chi) \equiv \frac{\varphi(\chi)^2}{\Omega^2(\chi)} \qquad U(\chi, \theta) \equiv \frac{V(\varphi(\chi), \theta)}{\Omega^4(\chi)}$$

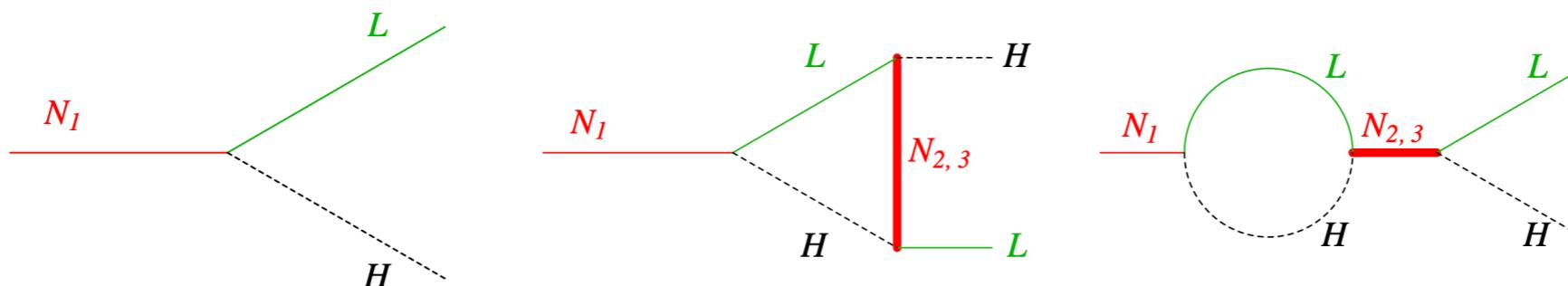
Baryogenesis via thermal leptogenesis

SM + 3 right-handed neutrinos

Fukugita and Yanagida, 1986'

$$\begin{aligned}\mathcal{L} = & \mathcal{L}_{\text{SM}} + \bar{N}_1 i\partial N_1 + \lambda_1 N_1 H L + \frac{M_1}{2} N_1^2 + \\ & + \bar{N}_{2,3} i\partial N_{2,3} + \lambda_{2,3} N_{2,3} H L + \frac{M_{2,3}}{2} N_{2,3}^2 + \text{h.c.}\end{aligned}$$

$$Y_{\Delta B} \simeq \frac{135\zeta(3)}{4\pi^4 g_*} \epsilon \times \eta \times C$$



$$\Gamma(N_1 \rightarrow LH) \propto |\lambda_1 + A\lambda_1^* \lambda_{2,3}^2|^2, \quad \Gamma(N_1 \rightarrow \bar{L}\bar{H}) \propto |\lambda_1^* + A\lambda_1 \lambda_{2,3}^{2*}|^2$$

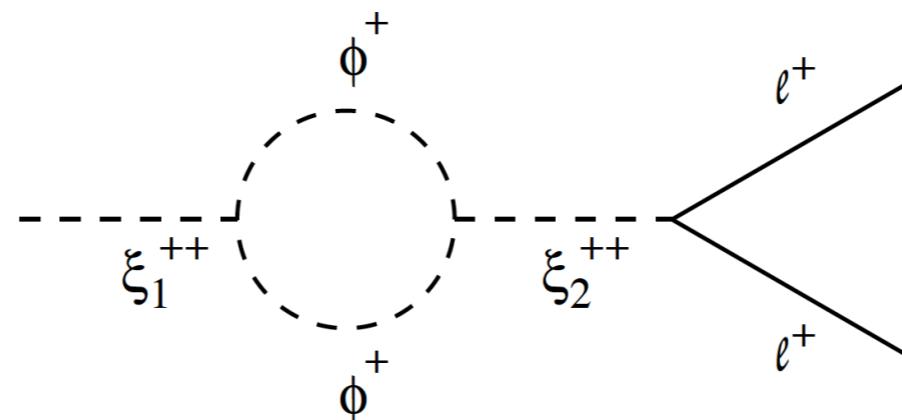
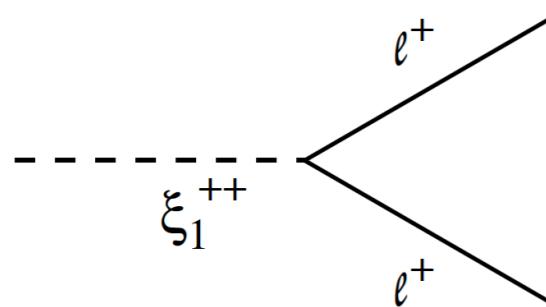
$$\epsilon \equiv \frac{\Gamma(N_1 \rightarrow LH) - \Gamma(N_1 \rightarrow \bar{L}\bar{H})}{\Gamma(N_1 \rightarrow LH) + \Gamma(N_1 \rightarrow \bar{L}\bar{H})} \sim \frac{1}{4\pi} \frac{M_1}{M_{2,3}} \text{Im} \lambda_{2,3}^2$$

Thermal leptogenesis from triplet Higgs

Type II seesaw

$M \sim 10^{13}$ GeV

Neutrino Masses and Leptogenesis with Heavy Higgs Triplets,
E. Ma, U. Sarkar, Phys.Rev.Lett. 80 (1998) 5716-5719

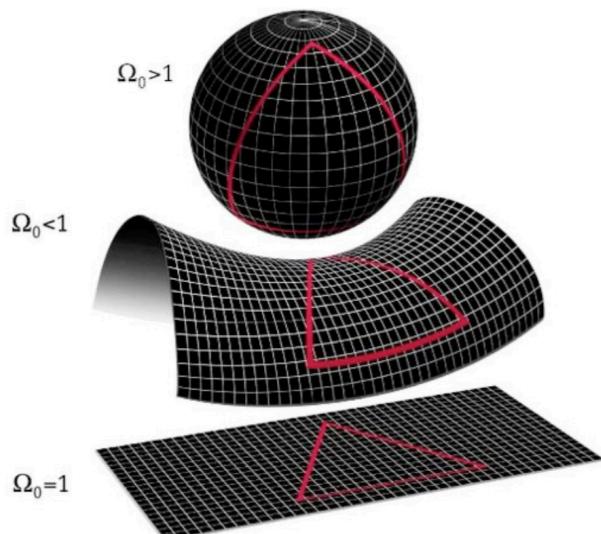
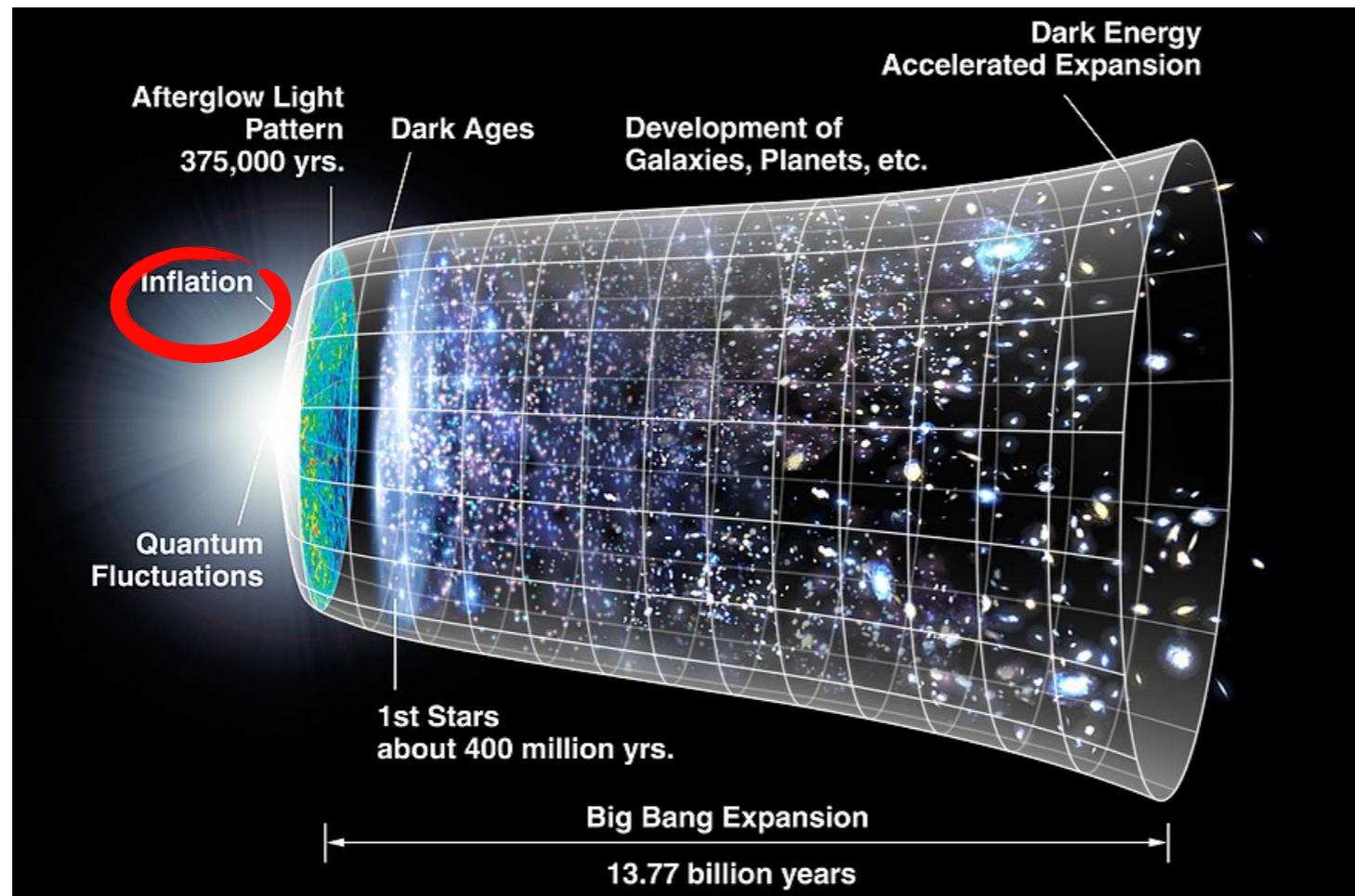


- Unfortunately, one triplet Higgs can not generate the lepton asymmetry
- Two triplet Higgs are needed to generate the baryon asymmetry
- Or one triplet Higgs + right-handed neutrino or ...

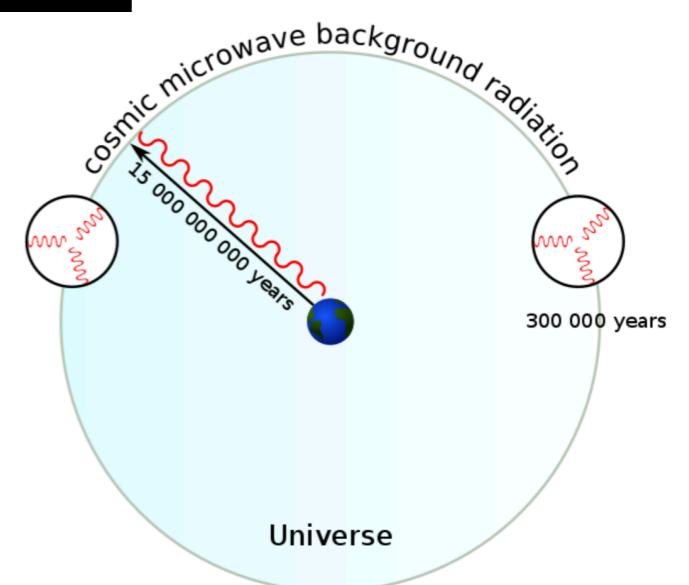
Pei-Hong Gu, He Zhang, Shun Zhou, PhysRevD.74.076002(2006)

Inflation

Expansion of the universe in the early time



- Flatness problem
- Horizon problem
- Monopole problem?

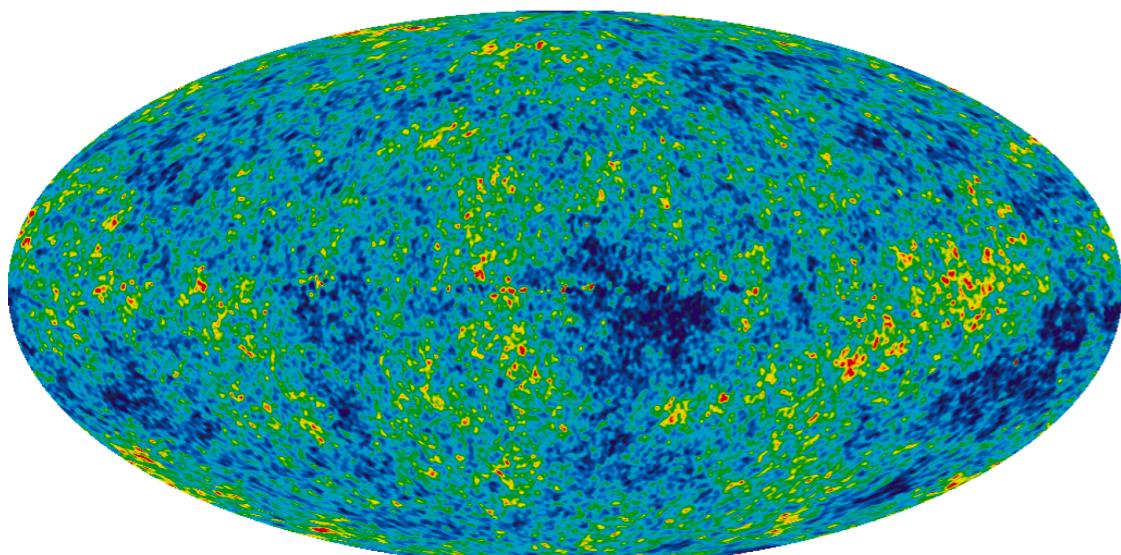


Expansion of the universe in the early time

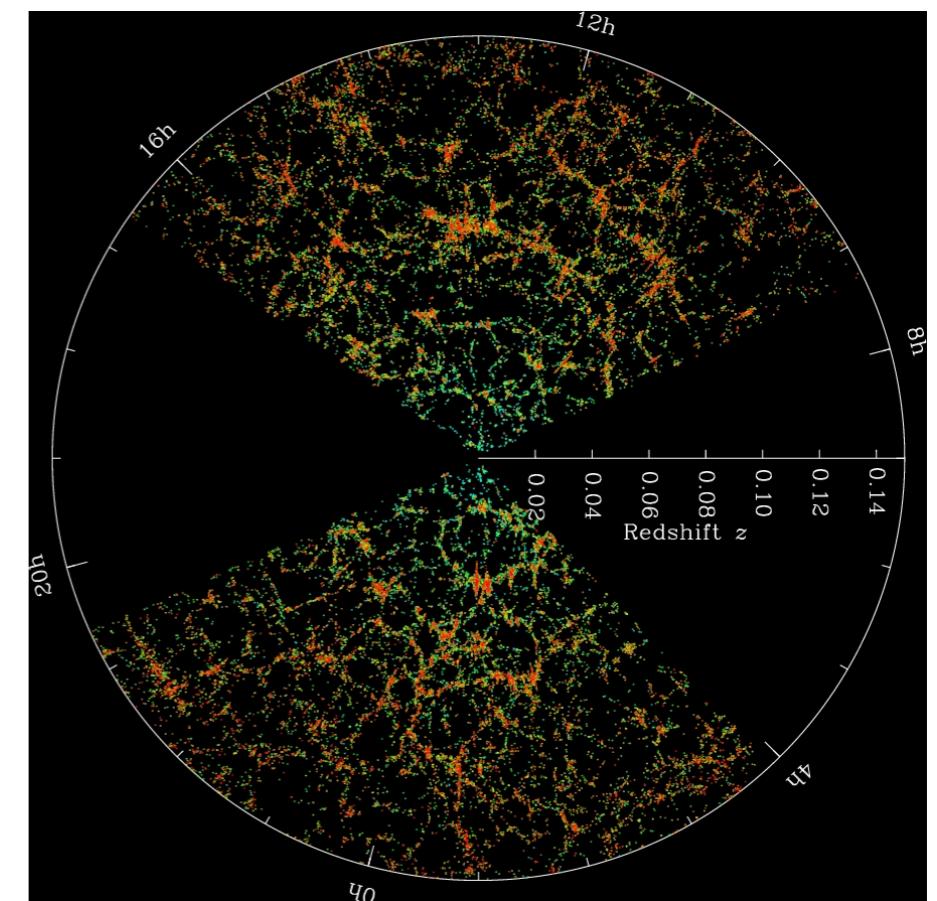
- Flatness problem
- Horizon problem
- Monopole problem?
- Seeding the primordial anisotropies in CMB

Inflation

Generating quantum fluctuations(anisotropies in CMB)



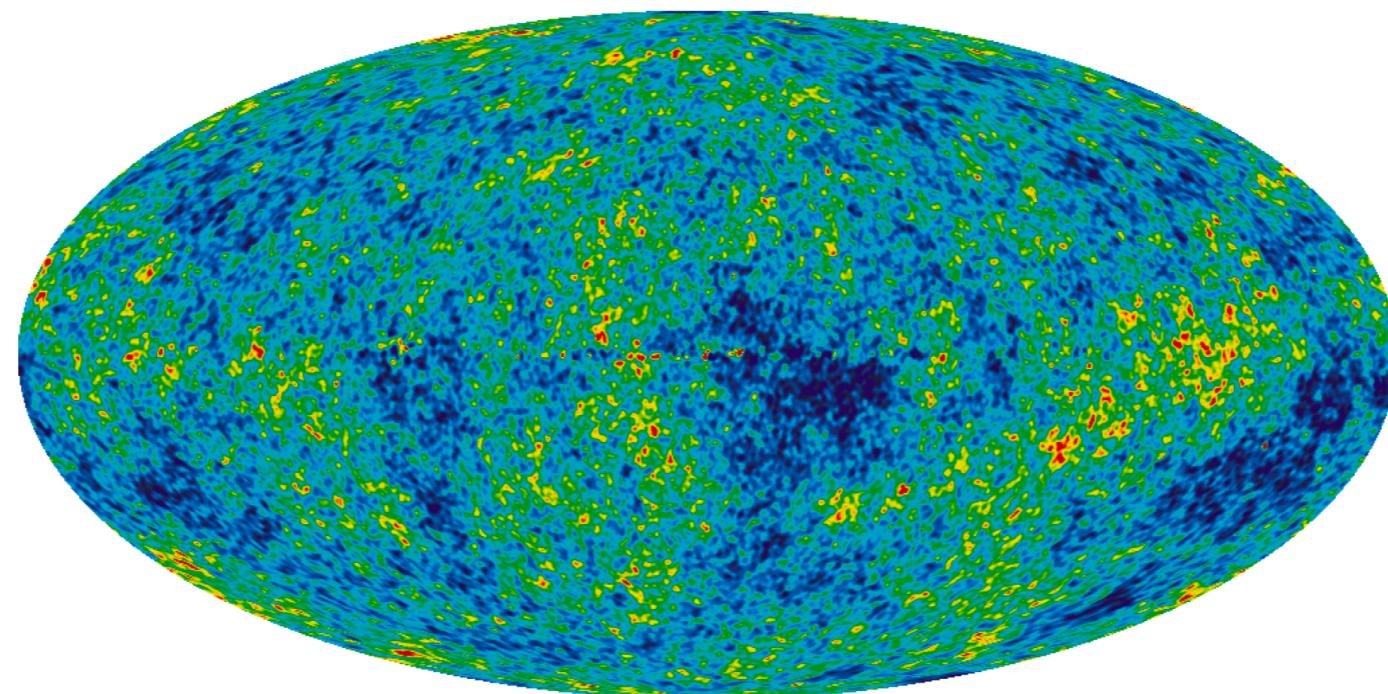
$$\frac{\delta T}{T} \sim 10^{-5}$$



Such small fluctuations finally develops the large structure of our universe

Inflation

Generating quantum fluctuations(anisotropies in CMB)



$$\frac{\delta T}{T} \sim 10^{-5}$$

Such small fluctuations finally develops the large structure of our universe

Slow roll inflation

Assume a scalar field, with equation of motion

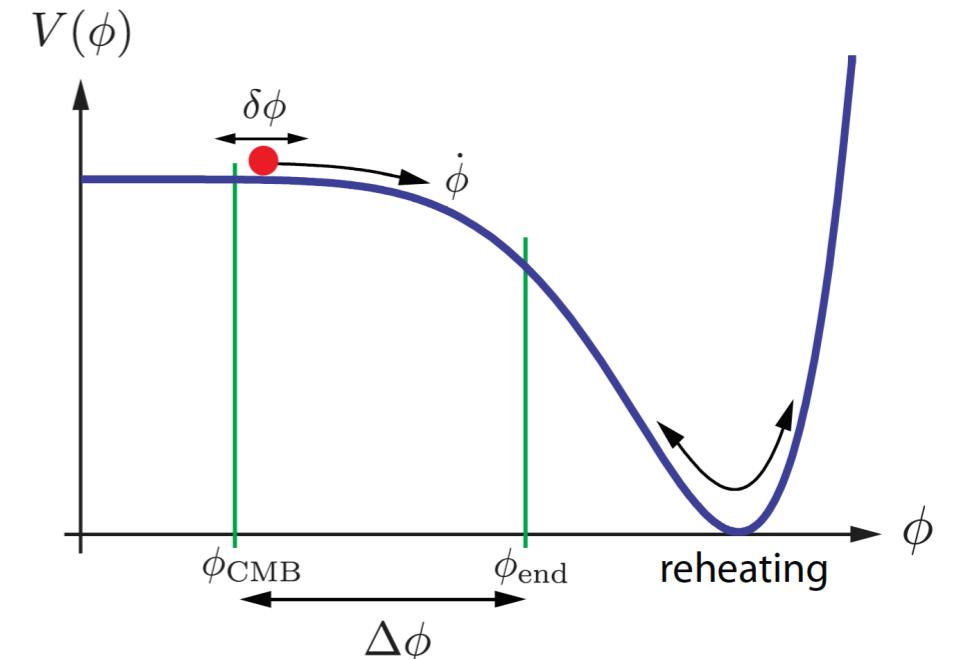
$$\ddot{\phi} + 3H\dot{\phi} + \frac{\partial V}{\partial \phi} = 0$$
$$H^2 = \frac{1}{3} \left(\frac{1}{2}\dot{\phi}^2 + V(\phi) \right)$$

Slow roll condition

$$\dot{\phi}^2 \ll V(\phi) \quad |\ddot{\phi}| \ll |3H\dot{\phi}|, |V_{,\phi}|$$

$$\epsilon_v(\phi) \equiv \frac{M_{\text{pl}}^2}{2} \left(\frac{V_{,\phi}}{V} \right)^2 \quad \eta_v(\phi) \equiv M_{\text{pl}}^2 \frac{V_{,\phi\phi}}{V}$$

$$\epsilon_v, |\eta_v| \ll 1$$



$$H^2 \approx \frac{1}{3}V(\phi) \approx \text{const.}$$

$$\dot{\phi} \approx -\frac{V_{,\phi}}{3H},$$



$$a(t) \sim e^{Ht}$$

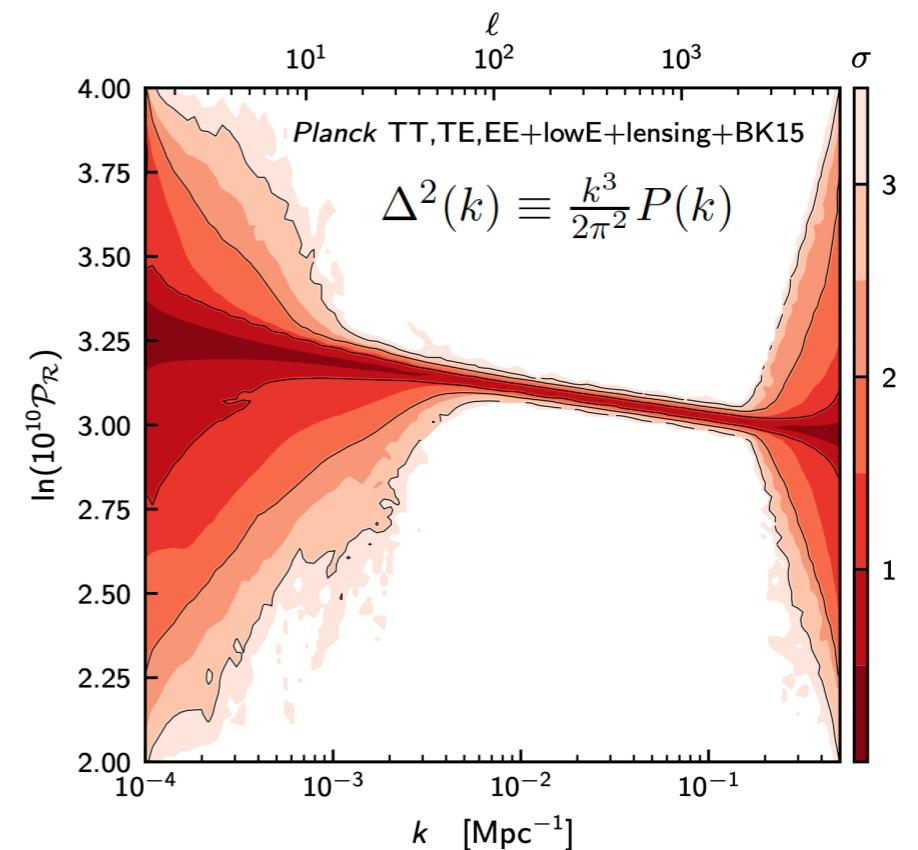
Daniel Baumann, TASI Lectures on Inflation

Slow roll inflation

Power spectrum $\Delta_s^2(k) \equiv \frac{k^3}{2\pi^2} \langle \delta\phi(k) \delta\phi(k') \rangle$

$$\Delta_s^2(k) \approx \frac{1}{24\pi^2} \frac{V}{M_{\text{pl}}^4} \frac{1}{\epsilon_V} \Big|_{k=aH}$$

$$\Delta_t^2(k) \approx \frac{2}{3\pi^2} \frac{V}{M_{\text{pl}}^4} \Big|_{k=aH}$$



$$n_s - 1 \equiv \frac{d \ln \Delta_s^2}{d \ln k} = 2\eta_V - 6\epsilon_V$$

$$r \equiv \frac{\Delta_t^2}{\Delta_s^2} = 16\epsilon_V$$

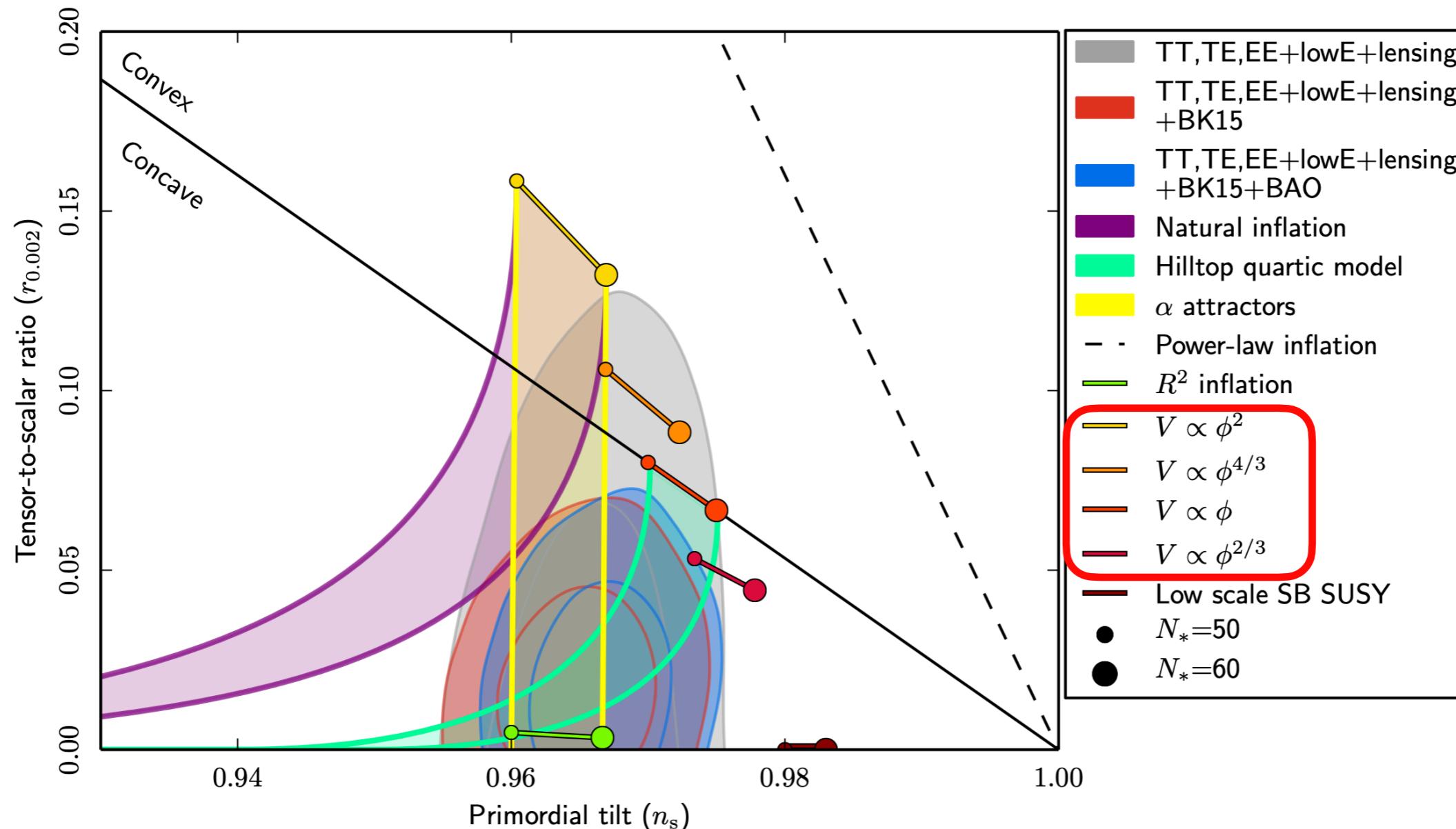
$$n_s \simeq 0.965$$

$$r \lesssim 0.056$$

$n=1$ to be scale invariant

tensor-scalar ratio

Current status



Concave potential is preferred by the data

Higgs inflation

Higgs is the only scalar field in SM

Bezrukov and Shaposhnikov, Phys.Lett.B 659 (2008) 703-706

$$S_J = \int d^4x \sqrt{-g_J} \left[\frac{M_P^2}{2} \left(1 + \boxed{\frac{\xi\phi^2}{M_P^2}} R_J \right) - \frac{1}{2} |\partial_\mu \phi|^2 - V_J(\phi) \right]$$

$$g_{\mu\nu} = \Omega(\phi)^2 g_{J\mu\nu} \quad \Omega^2 = 1 + \frac{\xi\phi^2}{M_P^2}$$

$$\frac{d\chi}{d\phi} = \left(\frac{1 + \xi(1 + 6\xi)\phi^2/M_P^2}{(1 + \xi\phi^2/M_P^2)^2} \right)^{1/2}$$

$$S = \int d^4x \sqrt{-g} \left[\frac{M_P^2}{2} R - \frac{1}{2} g^{\mu\nu} \partial_\mu \chi \partial_\nu \chi - V(\chi) \right] \quad V(\chi) \equiv V_J(\phi(\chi))/\Omega^4(\phi(\chi))$$

One word after inflation

When phi becomes small

$$V(\phi) = \frac{1}{2}m^2\phi^2$$

$$\ddot{\phi} + 3H\dot{\phi} + \frac{\partial V}{\partial \phi} = 0$$

$$H^2 = \frac{1}{3} \left(\frac{1}{2}\dot{\phi}^2 + V(\phi) \right)$$

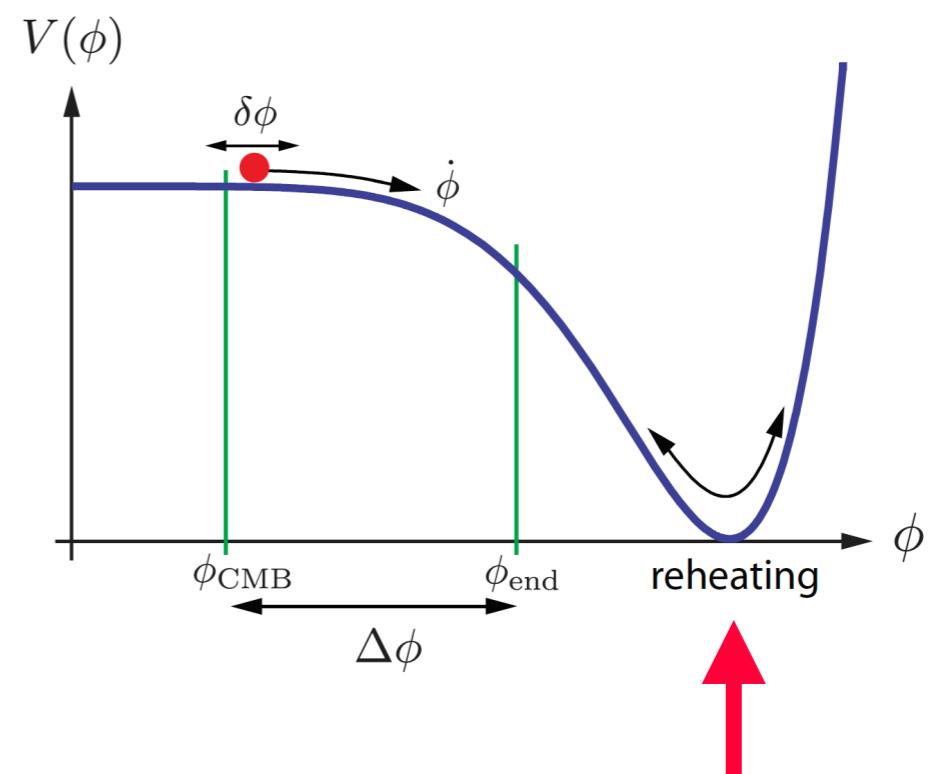
$$\phi(t) \sim \Phi(t) \cos(mt) \quad \Phi(t) = \frac{1}{mt}$$

$$H(t) \equiv \frac{\dot{a}}{a} \sim \frac{2}{3t} \quad \text{similar to matter dominate universe} \quad a(t) = t^{2/3}$$

Reheating at $\frac{1}{\Gamma_\phi} = \frac{1}{H} \sim \frac{M_P}{T_{rh}^2}$

$$T_{rh} \sim \sqrt{M_P \Gamma_\phi}$$

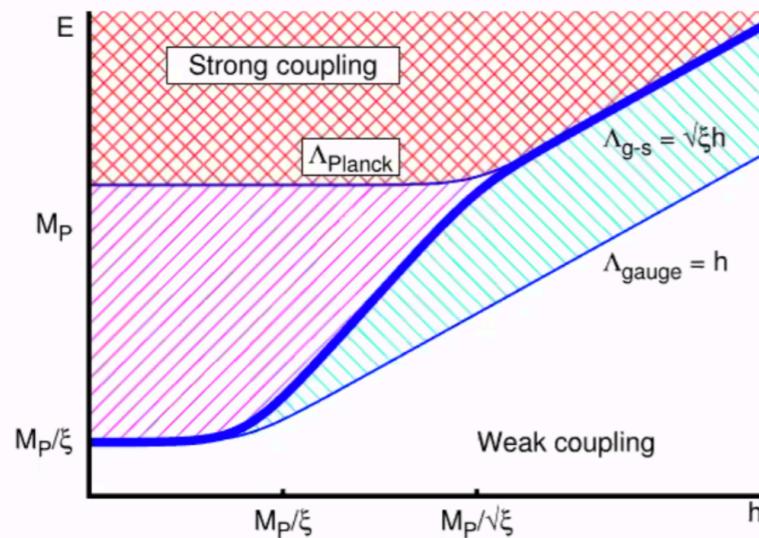
More complicated case: parametric resonance or tachyonic reheating



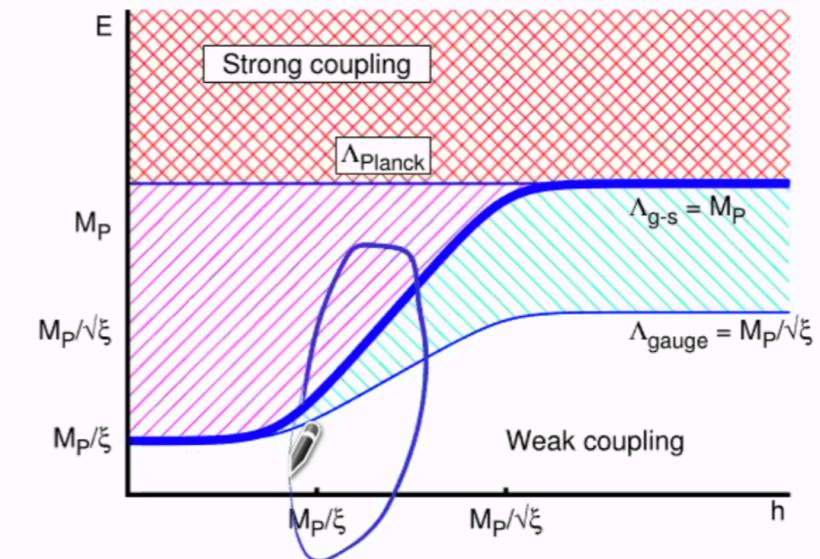
Unitary problem for Higgs inflation

Cut-off grows with the field background

Jordan frame



Einstein frame



Relation between cut-offs in different frames:

$$\Lambda_{\text{Jordan}} = \Lambda_{\text{Einstein}} \Omega$$

Relevant scales at inflation

$$\text{Hubble scale } H \sim \lambda^{1/2} \frac{M_P}{\xi}$$

Energy density at inflation

$$V^{1/4} \sim \lambda^{1/4} \frac{M_P}{\sqrt{\xi}}$$

Reheating temperature $M_P/\xi < T_{\text{reheating}} < M_P/\sqrt{\xi}$

Problems during reheating

SM + a triplet Higgs

In practice

$$\phi = \frac{1}{\sqrt{2}} \varphi \exp(i\theta)$$

$$\begin{aligned} \frac{\mathcal{L}}{\sqrt{-g}} = & -\frac{1}{2} M_P^2 R - \frac{1}{2} \xi \varphi^2 R - \frac{1}{2} g^{\mu\nu} \partial_\mu \varphi \partial_\nu \varphi \\ & - \frac{1}{2} \varphi^2 g^{\mu\nu} \partial_\mu \theta \partial_\nu \theta - V(\varphi, \theta) , \end{aligned}$$

$$V(\varphi, \theta) = \frac{1}{2} m^2 \varphi^2 + \frac{\lambda}{4} \varphi^4 + 2\varphi^3 \left(\tilde{\mu} + \frac{\tilde{\lambda}_5}{M_P} \varphi^2 \right) \cos \theta$$

SM + a triplet Higgs

$$\frac{\chi}{M_p} \approx \begin{cases} \frac{\varphi}{M_p} & \text{for } \frac{\varphi}{M_p} \ll \frac{1}{\xi} \quad (\text{after reheating}) \\ \sqrt{\frac{3}{2}} \xi \left(\frac{\varphi}{M_p} \right)^2 & \text{for } \frac{1}{\xi} \ll \frac{\varphi}{M_p} \ll \frac{1}{\sqrt{\xi}} \quad (\text{reheating}) \\ \sqrt{\frac{3}{2}} \ln \Omega^2 = \sqrt{\frac{3}{2}} \ln \left[1 + \xi \left(\frac{\varphi}{M_p} \right)^2 \right] & \text{for } \frac{1}{\sqrt{\xi}} \ll \frac{\varphi}{M_p} \quad (\text{inflation}) \end{cases}$$

$$U(\chi) \approx \begin{cases} \frac{1}{4} \lambda \chi^4 & \text{for } \frac{\chi}{M_p} \ll \frac{1}{\xi} \quad (\text{after reheating}) \\ \frac{1}{2} m_S^2 \chi^2 & \text{for } \frac{1}{\xi} \ll \frac{\chi}{M_p} \ll 1 \quad (\text{reheating}) \\ \frac{3}{4} m_S^2 M_p^2 \left(1 - e^{-\sqrt{\frac{2}{3}}(\chi/M_p)} \right)^2 & \text{for } 1 \ll \frac{\chi}{M_p} \quad (\text{inflation}) \end{cases}$$