

# Selected results in the 3HDMs

Igor Ivanov

School of Physics and Astronomy, SYSU, Zhuhai

Higgs potential and BSM opportunity Workshop

August 30th, 2021



**中山大學 物理与天文学院**  
SUN YAT-SEN UNIVERSITY SCHOOL OF PHYSICS AND ASTRONOMY

# Several Higgs generations

Higgses can come in **generations** → *N*-Higgs-doublet models (NHDMs)

- **T.D. Lee, 1973**: 2HDM as a new source of *CP*-violation (CPV);
- **Weinberg, 1976**: 3HDM with natural flavour conservation (NFC) and CPV;
- Intense activity in **70–80's**: trying to reconstruct hierarchical quark and lepton **masses and mixing** patterns from **symmetries** and their breaking;
- **1990–2000's**: **MSSM** requires two Higgs doublets;
- Around **2000's**: cosmological consequences of extra scalar fields: scalar **dark matter candidates** (protected by symmetries), strong first-order phase transitions leading to **baryogenesis**.
- In total,  $\mathcal{O}(10^4)$  papers over 40 years; see e.g. [[Ivanov, 1702.03776](#)].

# 3HDM vs 2HDM

2HDMs explored in thousands of papers. What new can 3HDMs bring?

- More options for model-building (scalar and fermion)  $\Rightarrow$  richer pheno!  
Many symmetry options [many classic papers], exact or approximate;

That's the single most powerful novelty of the 3HDMs.

- Just one example of consequences: automatic scalar alignment from large symmetry groups [Pramanick, Raychaudhuri, 2017; Ferreira et al, 2017].

Scalar alignment: the directions of  $v_i$  and  $h_{125}$  coincide  $\rightarrow h_{125}VV$  coupling exactly as in the SM, while  $H_iVV = 0$  for other Higgses.

A simpler observation:

$$V = -m^2 \sum_i \phi_i^\dagger \phi_i + V_4$$

displays scalar alignment for arbitrary  $V_4$ .

Any group with Higgs triplets does the jobs in 3HDM.

# 3HDM vs 2HDM

more options for model-building (scalar and fermion)  $\Rightarrow$  richer pheno;

- new options for  $CP$  violation [Branco, Gerard, Grimus, 1984];
- exotic  $CP$  symmetry of order 4 [Ivanov, Silva, 2015];
- combining features of 2HDM: NFC + CPV [Weinberg, 1976; Branco, 1979], scalar DM + CPV [Grzadkowski et al, 2009].

astroparticle consequences:

- more options for dark sectors [Cordero et al, 2017];
- new options for baryon asymmetry [Davoudiasl, Lewis, Sullivan, 2019];
- many minima  $\rightarrow$  multi-step phase transitions  $\rightarrow$  GW signals.

3HDMs bring numerous **model-building opportunities!**

- These opportunities are **barely explored**; phenomenology still limited to isolated cases.
- 3HDMs involve **technical challenges** which require **new tools** beyond straightforward numerical scans.
- I will give examples of directions to explore.

# (Approximate) symmetries

# NHDM in a nutshell

$N$  Higgs doublets  $\phi_a$ ,  $a = 1, \dots, N$ , with equal quantum numbers.

- The general NHDM potential

$$V = Y_{ab}(\phi_a^\dagger \phi_b) + Z_{ab,cd}(\phi_a^\dagger \phi_b)(\phi_c^\dagger \phi_d),$$

with  $N^2(N^2 + 3)/2$  free parameters (14 for the 2HDM, 54 for the 3HDM).

- Quark Yukawa sector

$$-\mathcal{L}_Y = \bar{Q}_{Li} \Gamma_{ij}^{(a)} \phi_a d_{Rj} + \bar{Q}_{Li} \Delta_{ij}^{(a)} \tilde{\phi}_a u_{Rj} + h.c.$$

Substituting vevs  $\langle \phi_a^0 \rangle = v_a / \sqrt{2}$ , we get

$$M_d = \frac{1}{\sqrt{2}} \sum_a \Gamma^{(a)} v_a, \quad M_u = \frac{1}{\sqrt{2}} \sum_a \Delta^{(a)} v_a^*,$$

and eventually to  $m_q$ ,  $V_{CKM}$ , and FCNCs.

# Symmetries in 3HDM

## The curse of dimensionality

**No chance** to cover the full 3HDM parameter space with a random scan!

New global **symmetries** help make sense of the full parameter space.

The 3HDM potential with  $\phi_a$ ,  $a = 1, 2, 3$ ,

$$V = Y_{ab}(\phi_a^\dagger \phi_b) + Z_{abcd}(\phi_a^\dagger \phi_b)(\phi_c^\dagger \phi_d)$$

can be invariant under global symmetries  $\phi_a \rightarrow U_{ab}\phi_b$  forming a group  $G$ .

- General 3HDM  $\rightarrow$  **54 free parameters** in the scalar sector alone.
- Impose **group  $G$**   $\rightarrow$  reduce free parameters in scalar and Yukawa sectors.
- Pick up a minimum  $\rightarrow$  deduce scalar and fermion properties, explore pheno consequences.

# Symmetries in 3HDM

Which **symmetry groups**  $G$  are possible within 3HDM?

- **abelian** groups: [Ferreira, Silva, 1012.2874; Ivanov, Keus, Vdovin, 1112.1660]

$$\mathbb{Z}_2, \quad \mathbb{Z}_3, \quad \mathbb{Z}_4, \quad \mathbb{Z}_2 \times \mathbb{Z}_2, \quad U(1), \quad U(1) \times \mathbb{Z}_2, \quad U(1) \times U(1).$$

- discrete **non-abelian** groups: [Ivanov, Vdovin, 1210.6553]:

$$S_3, \quad D_4, \quad A_4, \quad S_4, \quad \Delta(54), \quad \Sigma(36).$$

- The classification is **exhaustive**: imposing any other discrete group in the 3HDM scalar sector will produce an accidental continuous symmetry.
- symmetry breaking patterns  $G \rightarrow G_V$ : [Ivanov, Nishi, 1410.6139]
- interplay between  $G$  and  $CP$  [many classical works].
- accidental symmetries of the potential: [Darvishi, Pilaftsis, 1912.00887].

# The symmetry dilemma of the 3HDM

Some history:

- The original idea from 1970's: pick up a **large  $G$** , extend it to the fermion sector, observe  $G \rightarrow G_v$  at the minimum  $\rightarrow$  **derive masses/mixing/CPV**.
- Many combinations of  **$G + \text{irreps} + \text{vevs}$**  were tested  $\rightarrow$  severe problems in the quark sector;  $A_4, S_4$  illustrations in [Gonzales Felipe et al, 2013].
- The fundamental obstacle [Leurer, Nir, Seiberg, 1993]:  
If the (active) Higgs sector is equipped with  $G$ , **vevs must break  $G$  completely** in order to produce physical  $m_q$ 's and CKM.
- For large  $G$ , this is **algebraically impossible** [Gonzales Felipe et al, 2014]
- For small  $G$ , **too many free parameters**  $\rightarrow$  poor predictive power.

3HDMs with **approximate symmetries** seem to be perfectly viable candidates.

# The symmetry dilemma of the 3HDM

Some history:

- The original idea from 1970's: pick up a **large  $G$** , extend it to the fermion sector, observe  $G \rightarrow G_v$  at the minimum  $\rightarrow$  **derive masses/mixing/CPV**.
- Many combinations of  **$G + \text{irreps} + \text{vevs}$**  were tested  $\rightarrow$  severe problems in the quark sector;  $A_4, S_4$  illustrations in [Gonzales Felipe et al, 2013].
- The fundamental obstacle [Leurer, Nir, Seiberg, 1993]:  
If the (active) Higgs sector is equipped with  $G$ , **vevs must break  $G$  completely** in order to produce physical  $m_q$ 's and CKM.
- For large  $G$ , this is **algebraically impossible** [Gonzales Felipe et al, 2014]
- For small  $G$ , **too many free parameters**  $\rightarrow$  poor predictive power.

3HDMs with **approximate symmetries** seem to be perfectly viable candidates.

# Working in the vicinity of large symmetry groups

If symmetry is not exact  $\rightarrow$  many more parameters appear.

We need to learn how to **work efficiently in the large parameter space**  $\rightarrow$  make sense which parameters are important for which observable.

A recent illustration:

3HDM with softly broken  $\Sigma(36)$  [Varzielas, Ivanov, Levy, 2107.08227]

- $\Sigma(36)$  group leads to a **very rigid model** with 4 free parameters ( $\rightarrow$  Higgs mass relations, exact alignment, no CP violation, DM candidates).
- **8 soft breaking terms**, with clear consequences.
- $\rightarrow$  **benchmark models** with desired phenomenological features.

# CP4 3HDM

# The freedom of defining CP

General CP transformation:

$$J: \phi_a \xrightarrow{CP} X_{ab} \phi_b^*, \quad X \in U(N),$$

[Grimus, Rebelo, 1997; Branco, Lavoura, Silva, 1999].

Applying  $J$  twice leads to family transformation  $J^2 = XX^*$  which may be non-trivial. It may happen that only  $J^k = \mathbb{I}$  ( $k = \text{power of } 2$ ).

*CP*-symmetry does not have to be of order 2

The usual CP = CP<sub>2</sub>, the first non-trivial is CP<sub>4</sub>, then CP<sub>8</sub>, CP<sub>16</sub>, etc.

CP4 3HDM [Ivanov, Silva, 2015], which is physically distinct from the usual CP [Haber, OGREID, Osland, Rebelo, 2018].

Consider 3HDM with the following potential  $V = V_0 + V_1$  (notation:  $a \equiv \phi_a$ ):

$$V_0 = -m_{11}^2(1^\dagger 1) - m_{22}^2(2^\dagger 2 + 3^\dagger 3) + \lambda_1(1^\dagger 1)^2 + \lambda_2 \left[ (2^\dagger 2)^2 + (3^\dagger 3)^2 \right] \\ + \lambda_3(1^\dagger 1)(2^\dagger 2 + 3^\dagger 3) + \lambda'_3(2^\dagger 2)(3^\dagger 3) + \lambda_4 \left[ (1^\dagger 2)(2^\dagger 1) + (1^\dagger 3)(3^\dagger 1) \right] + \lambda'_4(2^\dagger 3)(3^\dagger 2),$$

with all parameters real, and

$$V_1 = \frac{\lambda_6}{2} \left[ (2^\dagger 1)^2 - (3^\dagger 1)^2 \right] + \lambda_8(2^\dagger 3)^2 + \lambda_9(2^\dagger 3) \left[ (2^\dagger 2) - (3^\dagger 3) \right] + h.c.$$

with real  $\lambda_6$  and **complex**  $\lambda_{8,9}$ . It is invariant under CP4  $\phi_a \xrightarrow{CP} X_{ab}\phi_b^*$  with

$$X = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & i \\ 0 & -i & 0 \end{pmatrix}, \quad CP4^2 = \text{diag}(1, -1, -1), \quad CP4^4 = \mathbb{I}.$$

# Phenomenology of CP4 3HDM

If vevs conserve CP4  $\rightarrow$  scalar DM candidates stabilized by exotic CP [Koepeke, 2018; Ivanov, Laletin, 2018].

flavored CP4 3HDM:

- CP4 can be extended to the Yukawa sector in four ways [Aranda, Ivanov, Jimenez, 2017; Ferreira et al, 2017];
- CP4 must be spontaneously broken  $\rightarrow$  peculiar patterns in the flavor sector;
- Parameter space scan of [Ferreira et al, 2017] identified many points compatible with theory constraints, EWPT, fermion masses and mixing, meson oscillation parameters.
- However, the scan of [Ferreira et al, 2017] produced many points with  $H_i^\pm$  lighter than top, leading to

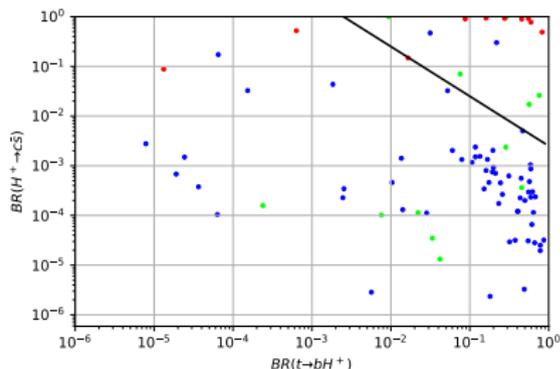
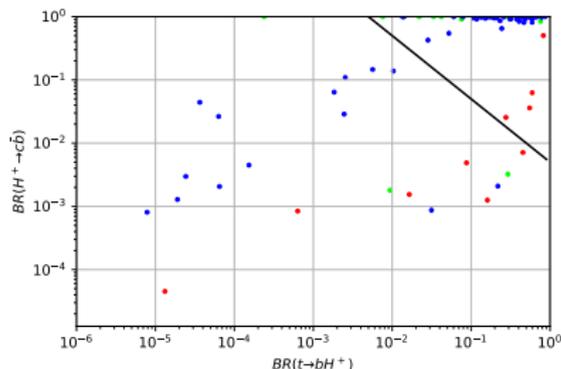
$$t \rightarrow H^+ d_i, \quad H^+ \rightarrow \bar{d}_i u_j,$$

with a variety of  $H^+ d_i u_j$  coupling patterns.

# Phenomenology of CP4 3HDM

In [Ivanov, Obodenko, 2021] we took all these points and checked for

- the total  $\Gamma_t = 1.42_{-0.15}^{+0.19}$  GeV [PDG];
- $Br(t \rightarrow H^+ b) \times Br(H^+ \rightarrow c\bar{b}) < 0.5\%$  based on [CMS, 2018];
- $Br(t \rightarrow H^+ b) \times Br(H^+ \rightarrow c\bar{s}) < 0.25\%$  based on [CMS, 2020].



Almost all points were excluded. Exotic cases survived:  $H^+ \rightarrow u\bar{b}$  as the dominant decay mode or  $t \rightarrow H^+ s$  as the main production mode.

# Phenomenology of CP4 3HDM

That's just first attempt at discovering all CP4 3HDM features.

## CP4 3HDM

Single assumption → numerous consequences (colliders + astroparticle) → requires further study.

# Conclusions

- **3HDMs** can offer much more than 2HDMs. So far, only isolated 3HDM examples were explored.
- Navigating the huge parameter space of the general 3HDM is **challenging** and requires new methods.
- All **3HDM symmetries** are now known. Efficient **basis-invariant methods** were recently developed. → **Systematic exploration** of phenomenologically distinct 3HDMs are now possible, leading to **collider and astroparticle predictions**.
- A team at **SYSU, Zhuhai**, is being set up to investigate these issues → we welcome all sort of collaborations.