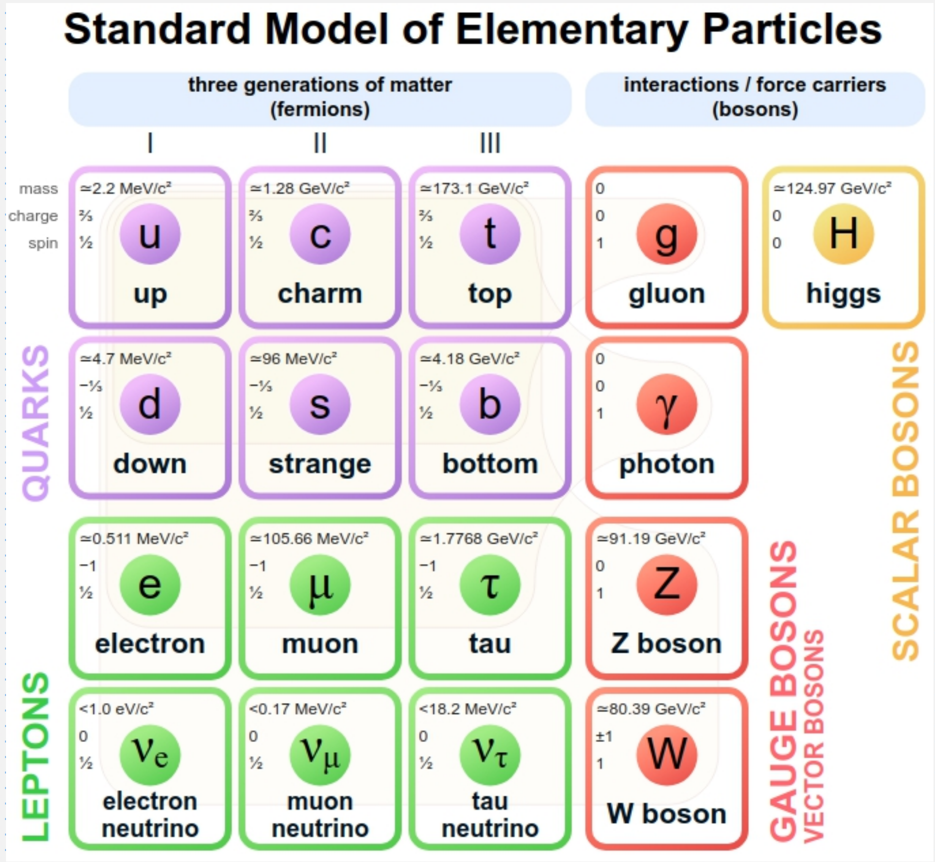
Higgs Potential and Phase Transitions

郭怀珂

University of Utah

August 29, 2021

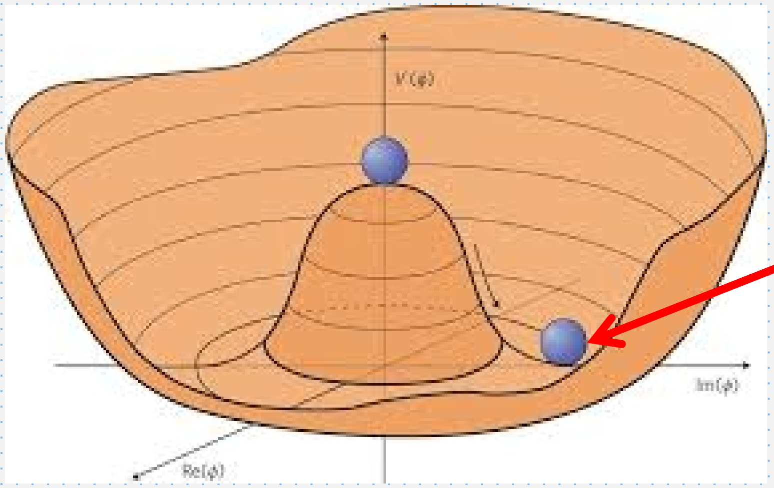
The Standard Model of Particle Physics



The properties of the (complete) SM particle contents are well measured, with some exceptions.

The Higgs Potential

At the core of the SM is the Higgs mechanism, but the Higgs potential is largely (experimentally) unknown.



Collider experiments measure the shape around the vev.

How to measure the full Higgs potential?

How to measure its evolution throughout the cosmic history?

Is there any new physics beyond those in the SM?

Higgs Cubic Coupling

(see Luca Cadamuro, Zihang Jia, Bowen zhang, Junmou Chen's talks)

Non-excluded at 95%CL (ATLAS, see also CMS's result)

single Higgs production $-3.2 < \kappa_\lambda < 11.9$ ($79.8 fb^{-1}$)

double Higgs production $-5.0 < \kappa_\lambda < 12.0$ ($36.1 fb^{-1}$)

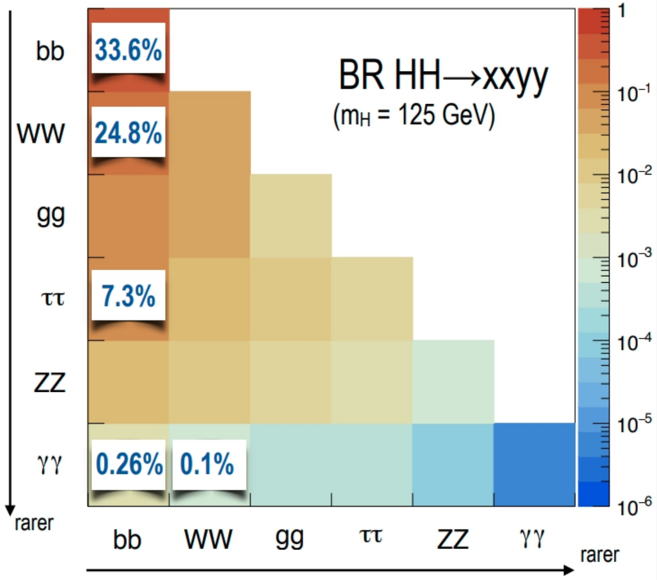
(non-resonant,channels combined)

ATLAS Combined

$$-2.3 < \kappa_\lambda < 10.3$$

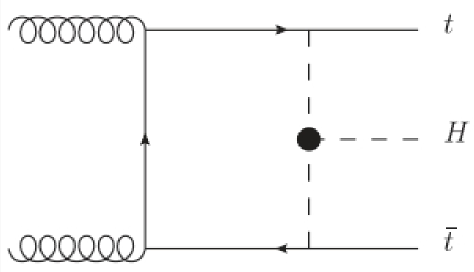
ATL-PHYS-PROC-2020-114

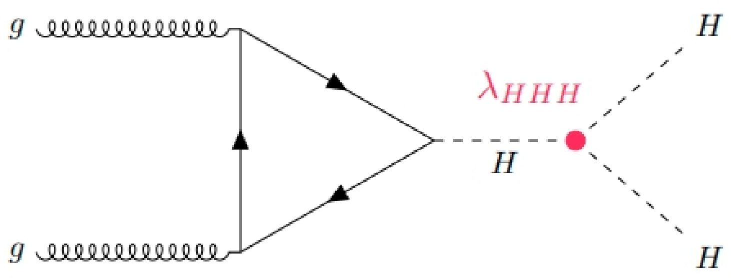
ATL-PHYS-PUB-2019-009, 2019
ATLAS collaboration, Phys.Lett.B 800 (2020) 135103



Morse, arXiv:1708.08249

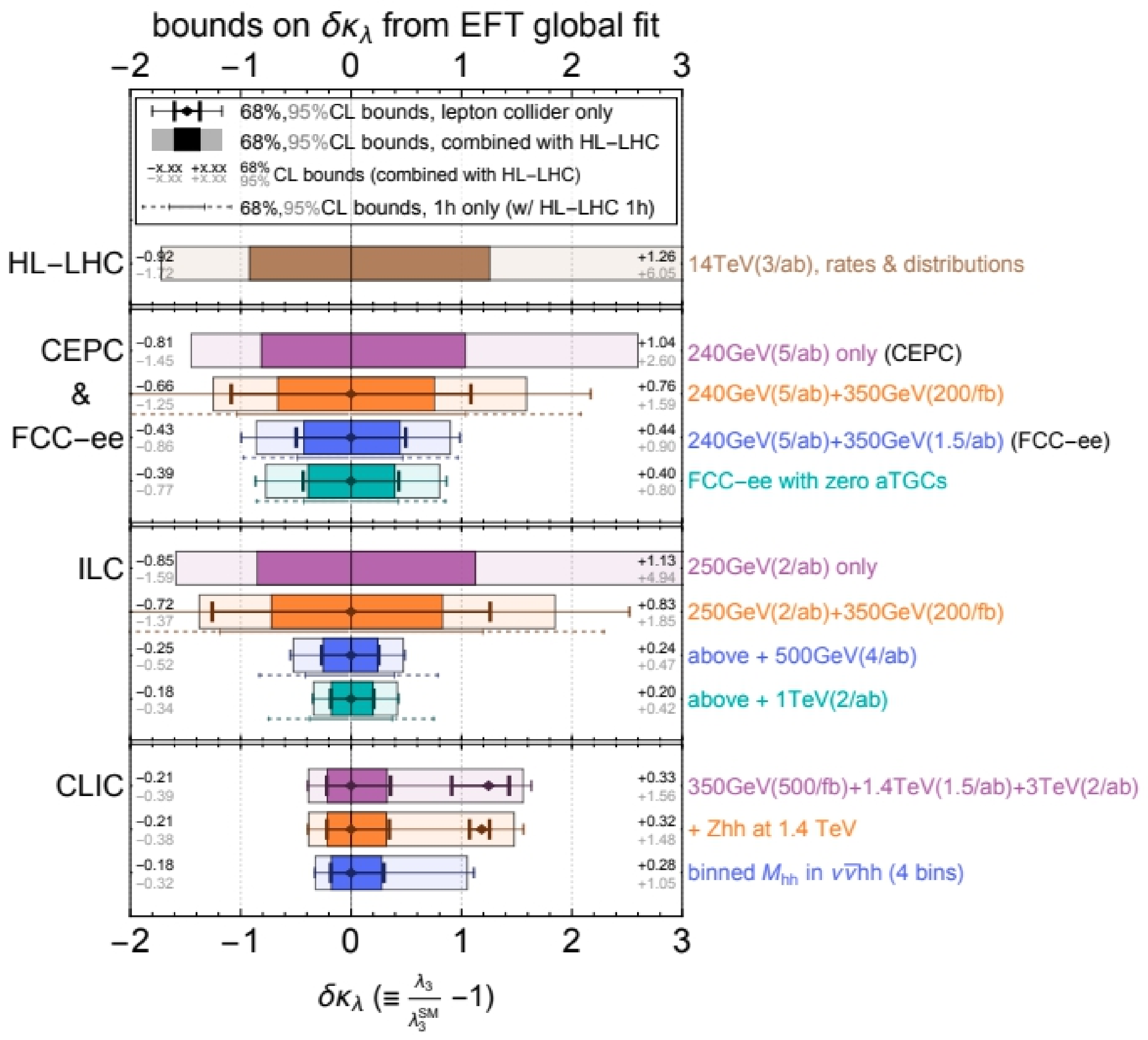
Most recent search in bbττ channel (ATLAS-CONF-2021-030)





(example diagrams for single and double Higgs production)

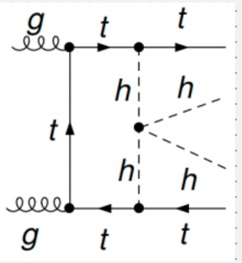
Higgs Cubic Coupling

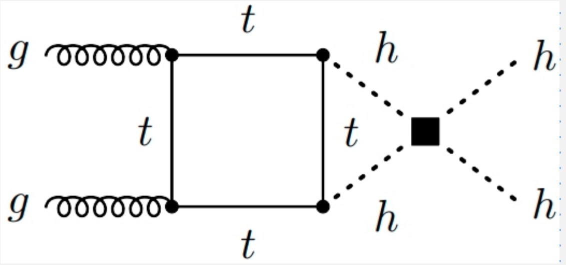


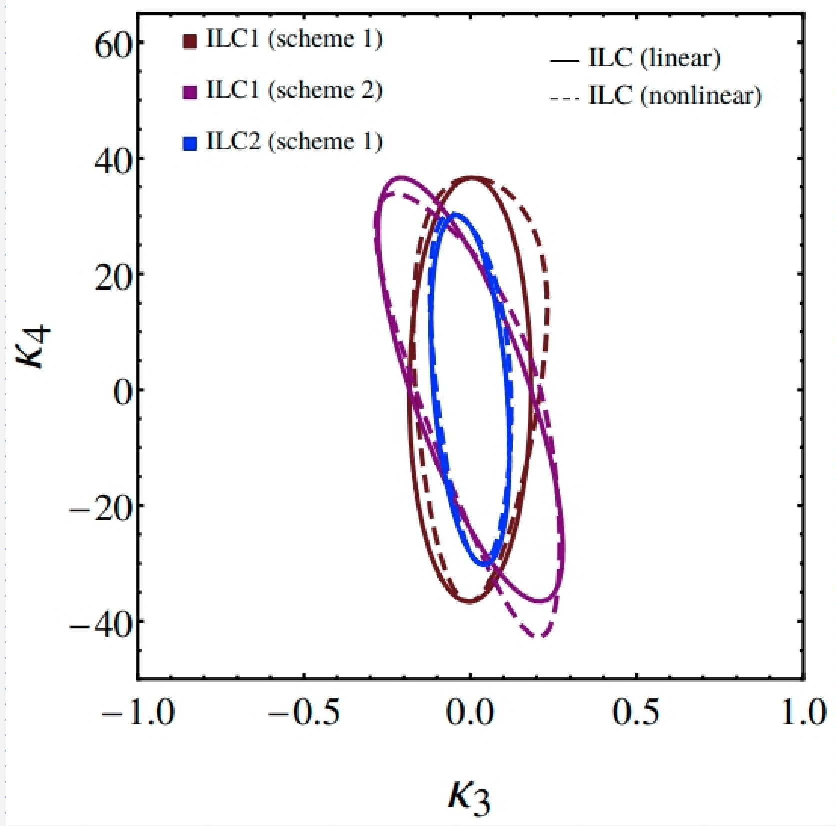
Vita, et al JHEP02(2018)178

Higgs Quartic Coupling

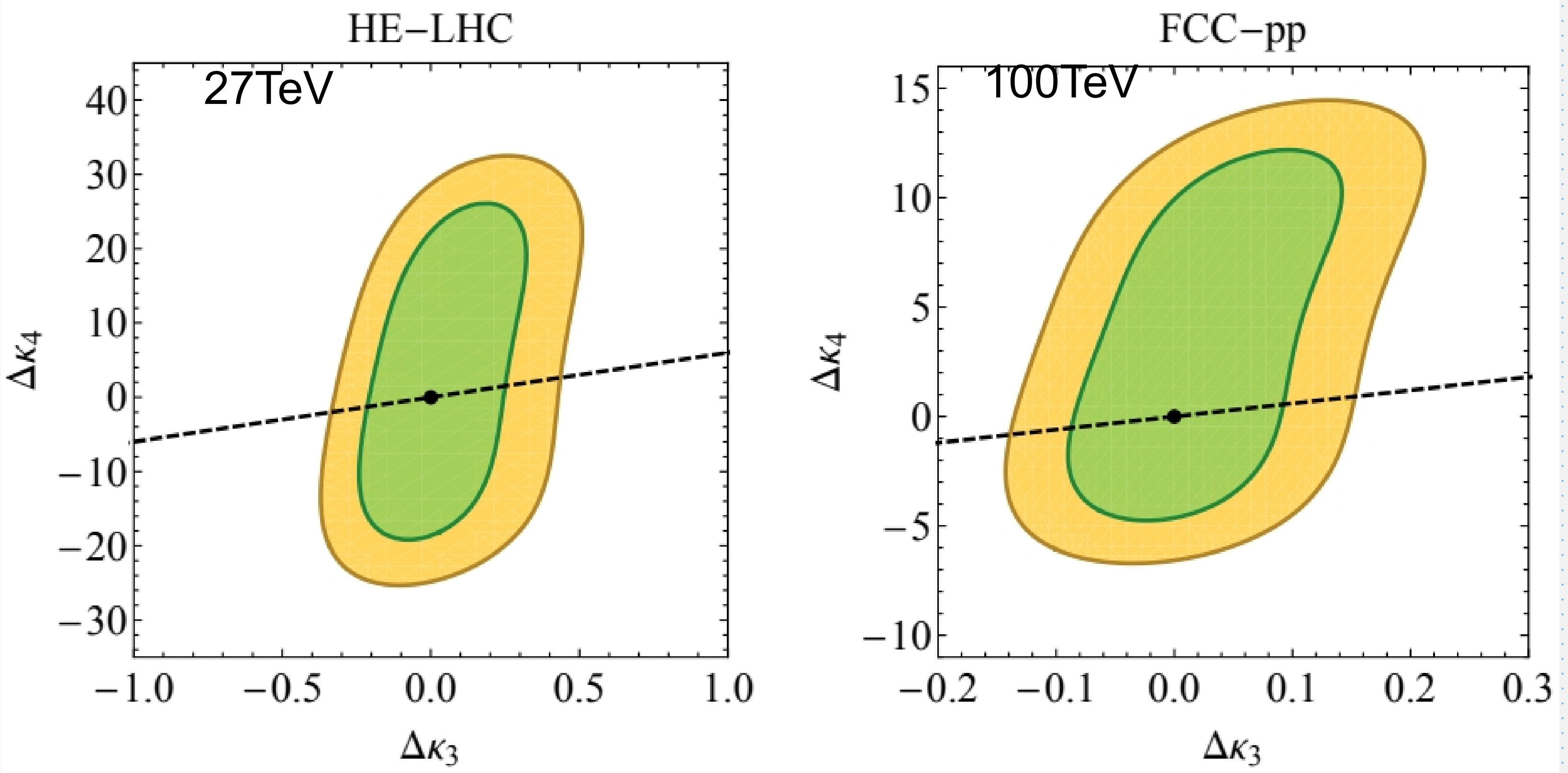
Quartic coupling is (will be) even less constrained







Liu, Lyu, Ren, Zhu, PRD 98 (2018) 9, 093004

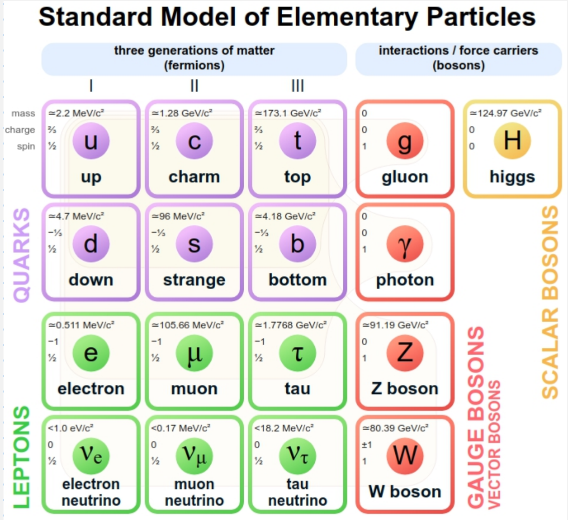


Bizon, Haisch, Rottoli, JHEP 10 (2019) 267

Why do we need new physics?

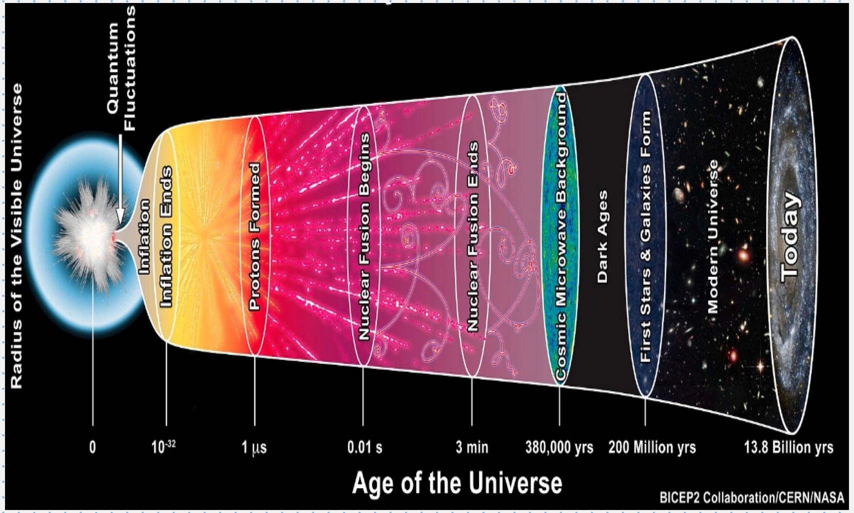
Two Standard Models, hard to reconcile with each other!

Particle Physics



Physics Beyond the SM?

Cosmology

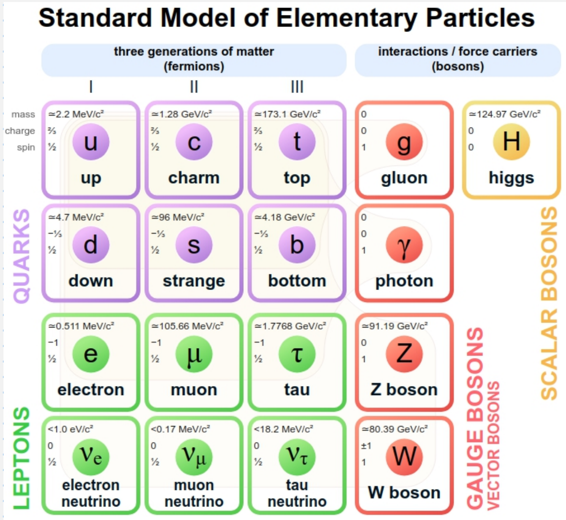


What is dark matter?
Why is there more matter than anti-matter?
What is dark energy?

Why do we need new physics?

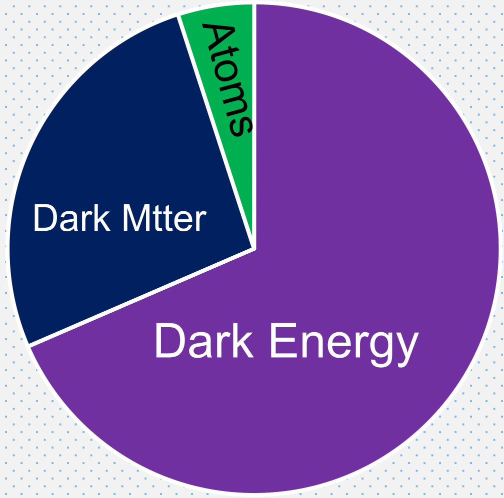
Two Standard Models, hard to reconcile with each other!

Particle Physics



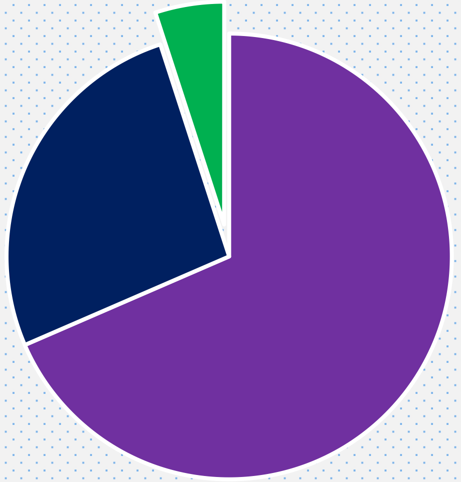
Physics Beyond the SM?

Cosmology



What is dark matter?
Why is there more matter than anti-matter?
What is dark energy?

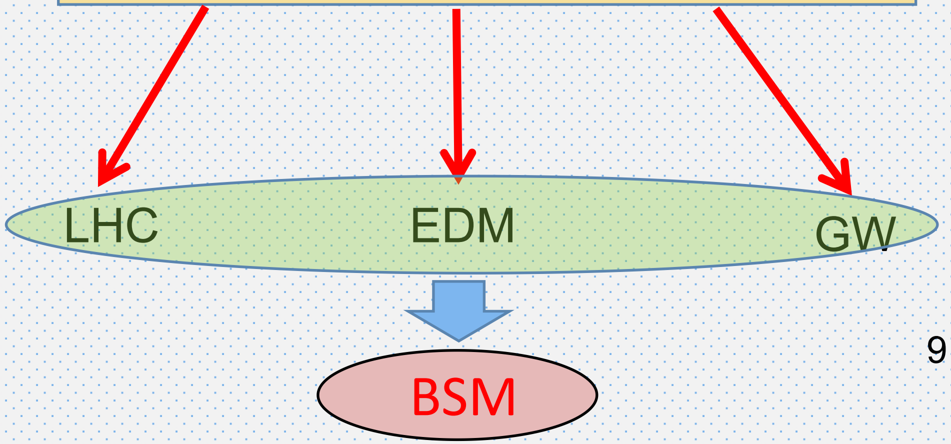
Baryon Asymmetry Generation



Sakharov conditions(1967)

- ✓ B-violation
- ✓ C, CP violation (new physics)
- ✓ Out-of-Equilibrium (new physics)

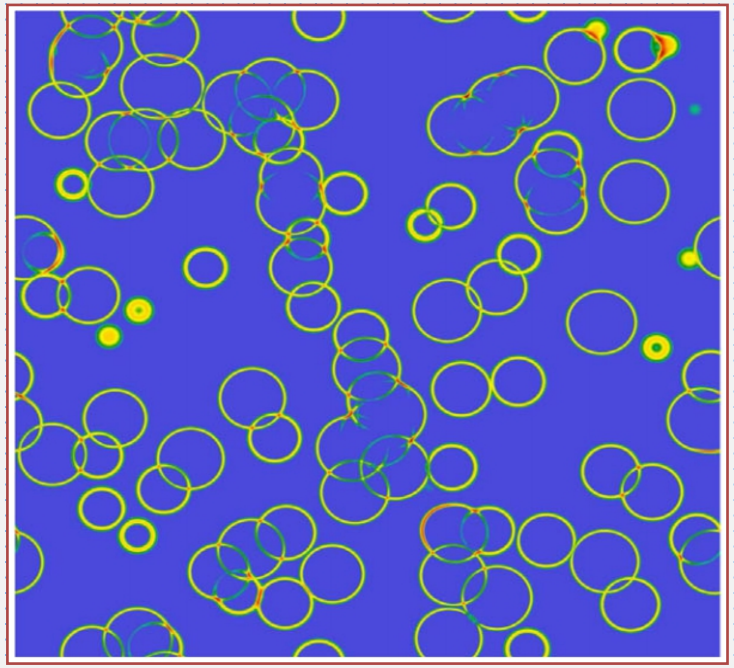
- GUT Baryogenesis
- Affleck-Dine Mechanism
- Leptogenesis
- Spontaneous Baryogenesis
- Electroweak Baryogenesis



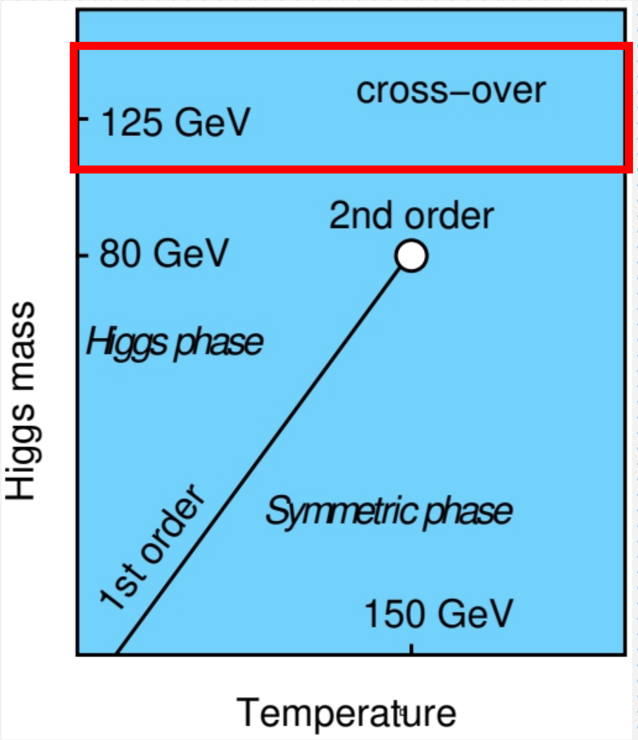
Why a different Higgs potential?

A first order EWPT can provide the non-equilibrium environment.

The SM Higgs potential with a 125GeV Higgs gives a smooth cross-over.



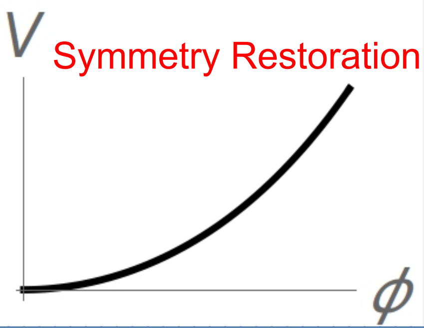
Hindmarsh, et al, 2015

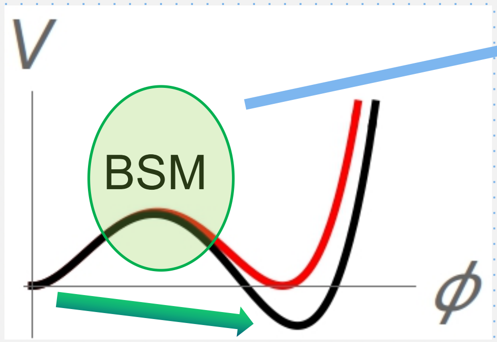


Hindmarsh et al SciPost Phys.Lect.Notes 24 (2021)

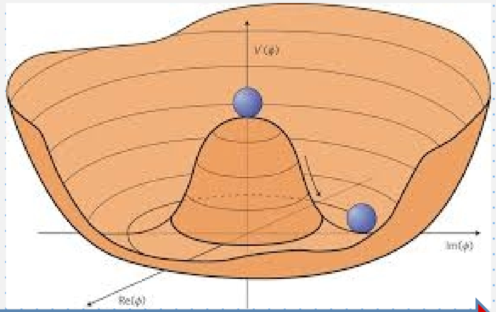
First Order Electroweak Phase Transition

How symmetry-breaking is realized in the early universe?

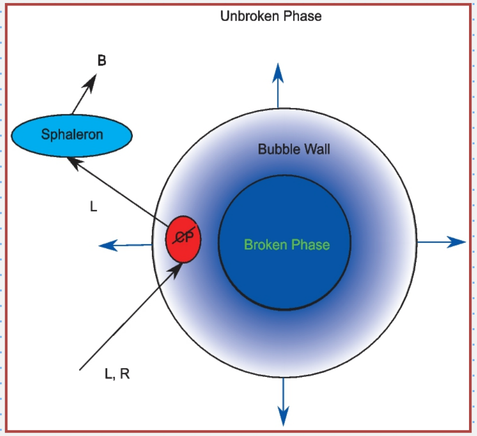


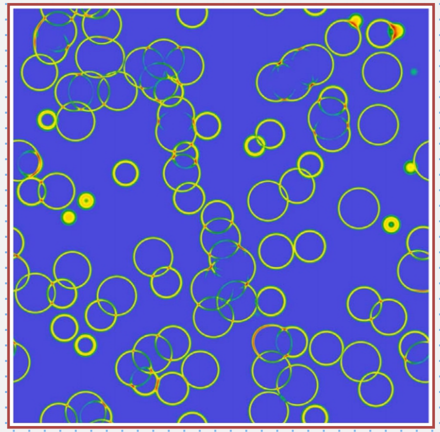


modification of Higgs properties



Temperature decreases





Hindmarsh, et al, 2015

(For primordial black hole formation, see Kepan Xie's talk)

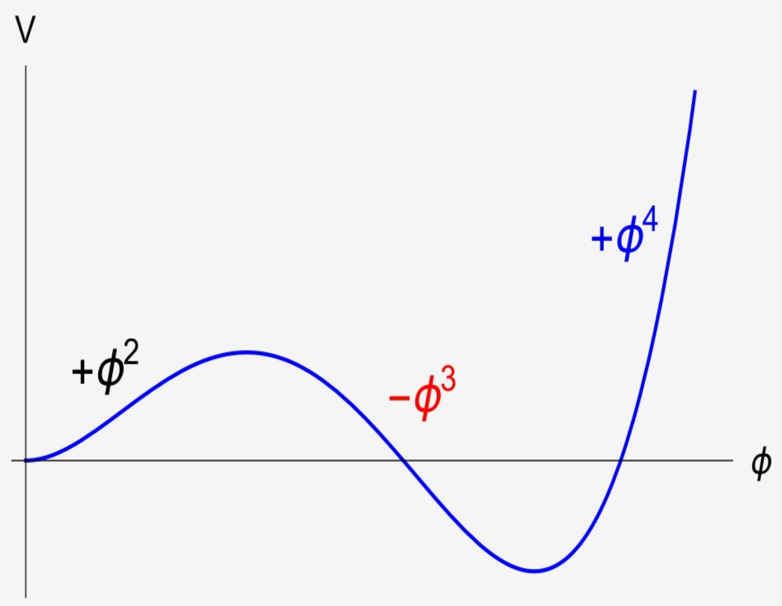
Models	Strong 1^{st} order phase transition	GW signal	Cold DM	Dark Radiation and small scale structure
SM charged				
Triplet [20–22]	✓	✓	✓	✗
complex and real Triplet [23] (Georgi-Machacek model)	✓	✓	✓	✗
Multiplet [24]	✓	✓	✓	
2HDM [25–30]	✓	✓		✗
MLRSM [31]	✓	✓	✗	✗
NMSSM [32–36]	✓	✓	✓	✗
SM uncharged				
S_r (xSM) [37–49]	✓	✓	✗	✗
2 S_r's [50]	✓	✓	✓	✗
S_c (cxSM) [49, 51–54]	✓	✓	✓	✗
$\mathrm{U(1)_D}$ (no interaction with SM) [55]	✓	✓	✓	✗
$\mathrm{U(1)_D}$ (Higgs Portal) [56]	✓	✓	✓	
$\mathrm{U(1)_D}$ (Kinetic Mixing) [57]	✓	✓	✓	
Composite SU(7)/SU(6) [58]	✓	✓	✓	
$\mathrm{U(1)_L}$ [59]	✓	✓	✓	✗
$\mathrm{SU(2)_D} \to$ global $SO(3)$ by a doublet [60–62]			✓	✗
$\mathrm{SU(2)_D} \to \mathrm{U(1)_D}$ by a triplet [63–65]			✓	✓
$\mathrm{SU(2)_D} \to Z_2$ by two triplets [66]			✓	✗
$\mathrm{SU(2)_D} \to Z_3$ by a quadruplet [67, 68]			✓	✗
$\mathrm{SU(2)_D} \times \mathrm{U(1)_{B\text{-}L}} \to Z_2 \times Z_2$ by a quintuplet and a S_c [69]			✓	✗
$\mathrm{SU(2)_D}$ with two dark Higgs doublets [70]	✓	✓	✗	✗
$\mathrm{SU(3)_D} \to Z_2 \times Z_2$ by two triplets [62, 71]			✓	✗
$\mathrm{SU(3)_D}$ (dark QCD) (Higgs Portal) [72, 73]	✓	✓	✓	
$G_{\mathrm{SM}} \times G_{\mathrm{D,SM}} \times Z_2$ [74]	✓	✓	✓	
$G_{\mathrm{SM}} \times G_{\mathrm{D,SM}} \times G_{\mathrm{D,SM}} \cdots$ [75]	✓	✓	✓	
Current work				
$\mathrm{SU(2)_D} \to \mathrm{U(1)_D}$ (see the text)	✓	✓	✓	✓

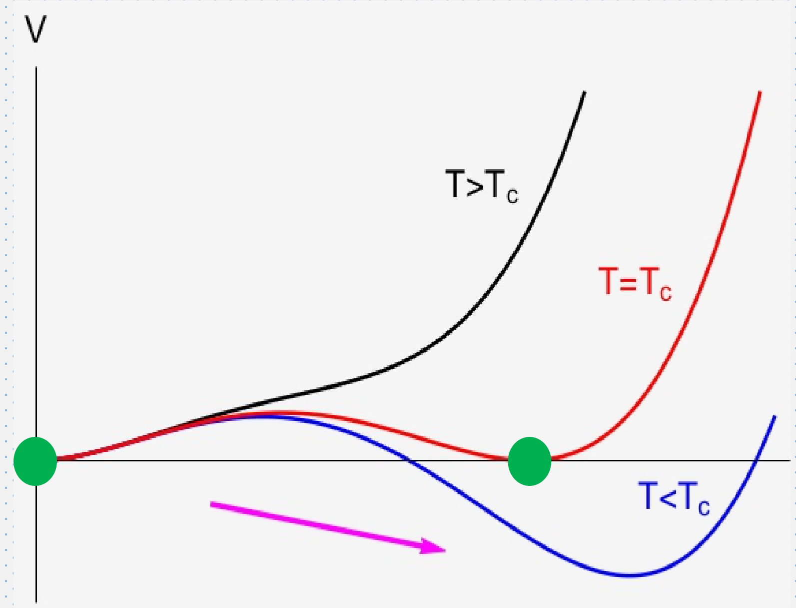
Ghosh, Guo, Han, Liu,
JHEP 07 (2021) 045

Finite Temperature Effective Potential

Illustration with a simple potential ansatz:

$$V(\phi, T) = D(T^2 - T_0^2)\phi^2 - ET\phi^3 + \frac{\lambda}{4}\phi^4$$





The point is to generate a barrier between the stable and meta-stable vacua

Finite Temperature Effective Potential

At finite T, there are thermal corrections due to interactions between the scalar and the other particles.

Perturbative calculations (analytical control under high-T approximation)

$$V_T = V_{\rm tree} + V_{\rm CW} + \frac{T^2}{24}\sum_i c_i M_i^2(\phi) - \frac{T}{12\pi}\sum_j d_j [M_j^2(\phi)]^{3/2}$$

symmetry restoration

cubic term, barrier

Problems with the perturbative approach:

gauge-dependence (see Patel, Ramsey-Musolf, JHEP07(2011)029),
infrared-problem (Linde, Phys. Lett. B 96 (1980) 289)

non-perturbative method
(but computationally expensive)

xSM (Gould et al, PRD 100, 115024)
2HDM (Andersen et al, PRL121.191802)

How to get a barrier?

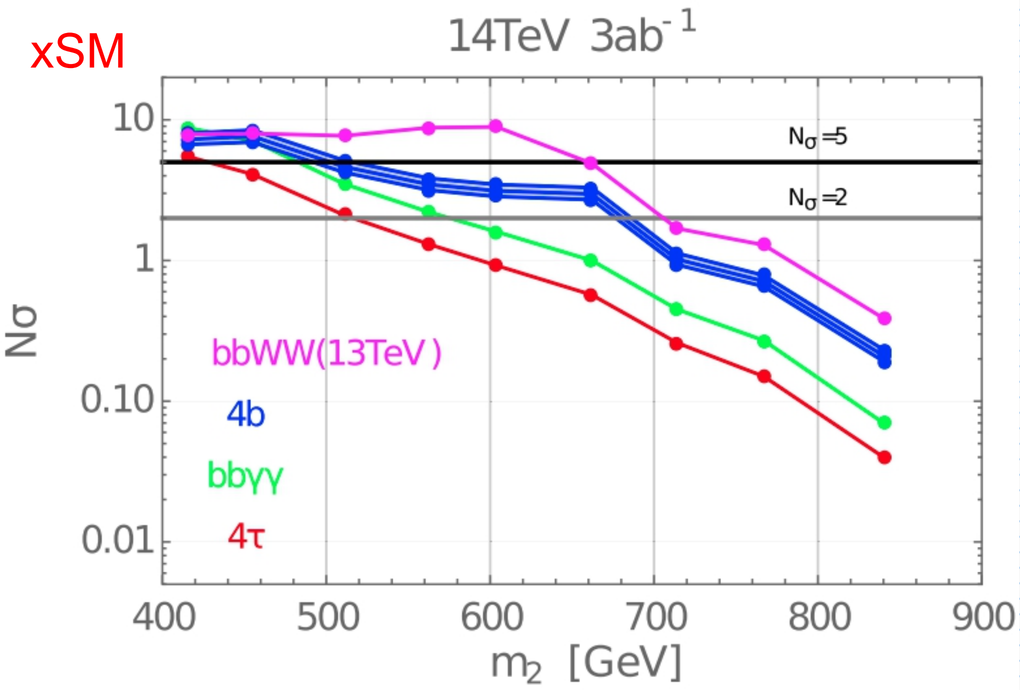
(see also Chung, Long, Wang, PRD 87 (2013) 2, 023509)

- Generating a large cubic term from thermal correrctions (loop level)
- Add new scalars (tree level) (see Wei Liu, Mengchao Zhang's talks)
- Including non-renormalizable operators (tree level)

Resonant Di-Higgs Production

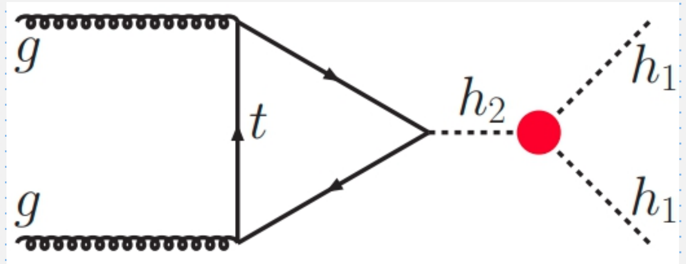
- Much easier to have vacua coexistence with multiple scalars
- Generically leading to a resonant di-Higgs production
- Correlation between EWPT and di-Higgs measurements

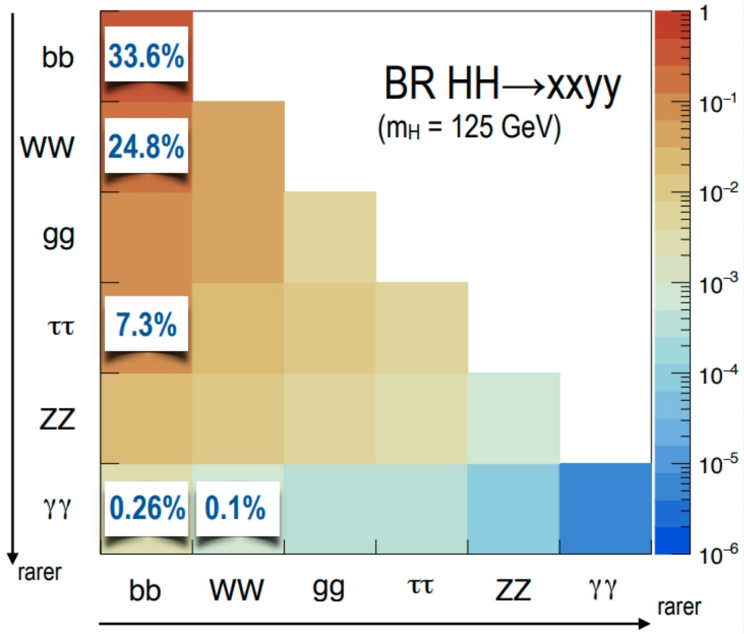
  Many such studies have been done.



Li, Ramsey-Musolf, Willocq, PRD 100, 075035 (2019)

(see also Wei Liu, Zihang Jia, Bowen zhang, Bruce Mellado's talks)



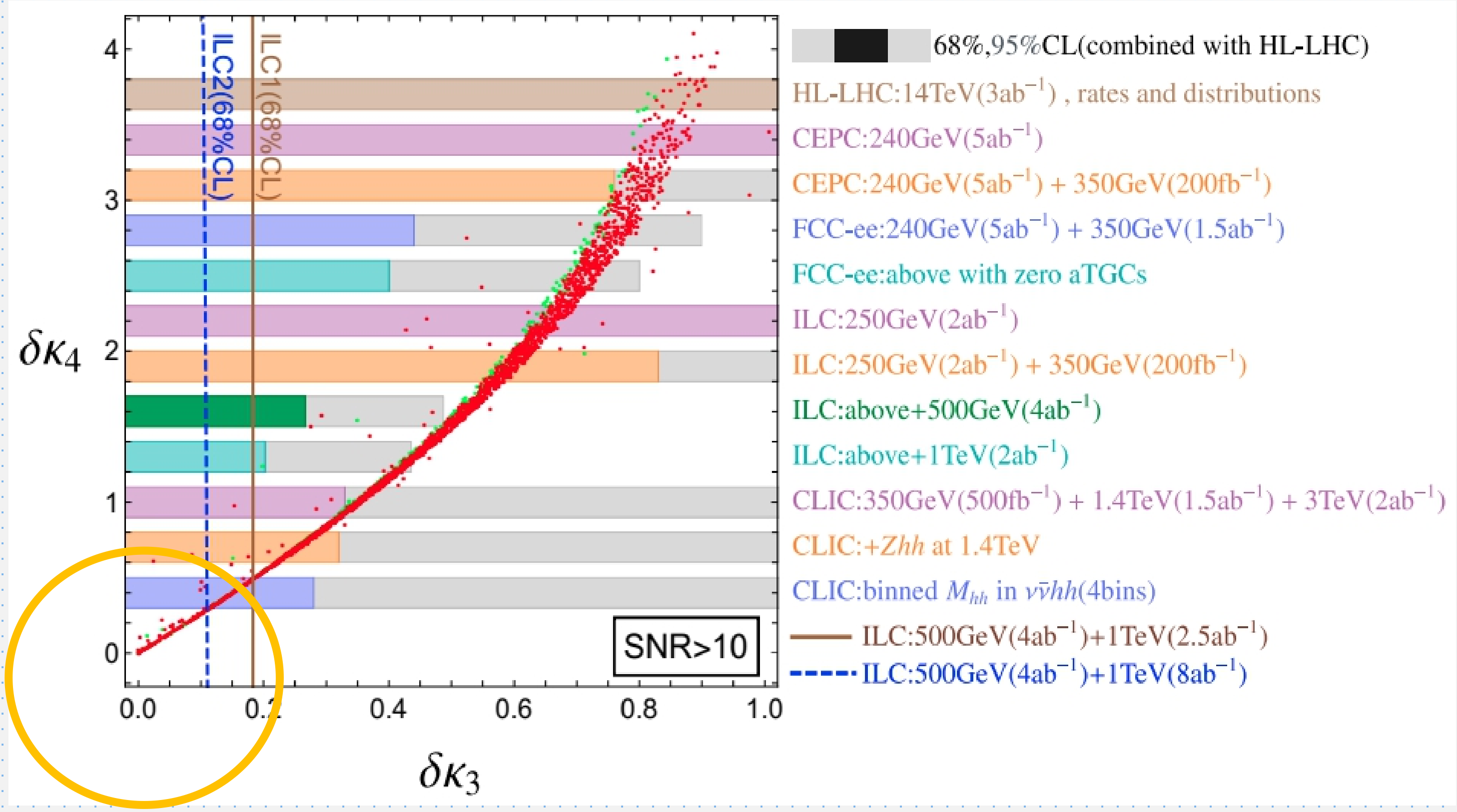


Morse, arXiv:1708.08249

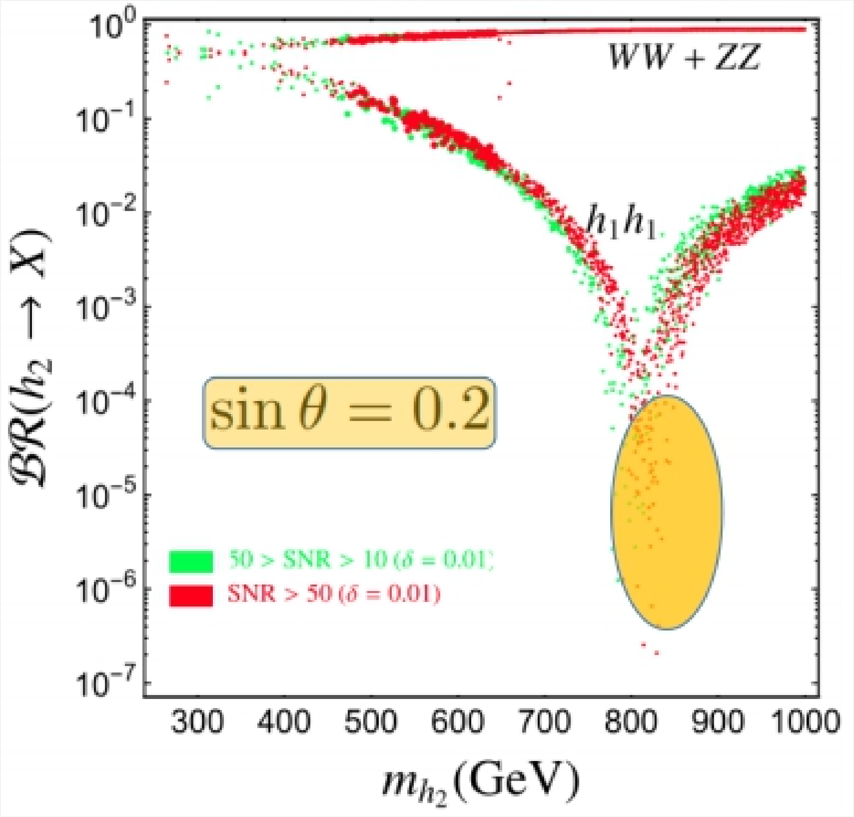
Di-Higgs Blindspot?

$$\Delta\mathcal{L} = -\frac{1}{2}\frac{m_{h_1}^2}{v}(1+\delta\kappa_3)h_1^3 - \frac{1}{8}\frac{m_{h_1}^2}{v^2}(1+\delta\kappa_4)h_1^4$$

check for other channels (WW, ZZ)



Alves, Ghosh, Guo, Sinha, Vagie, JHEP04(2019)052



Alves, Gonçalves, Ghosh, Guo, Sinha, Vagie, JHEP04(2019)052

Bubble Nucleations

Essentially the vacuum decay at finite temperature

- **Nucleation rate**

fluctuations

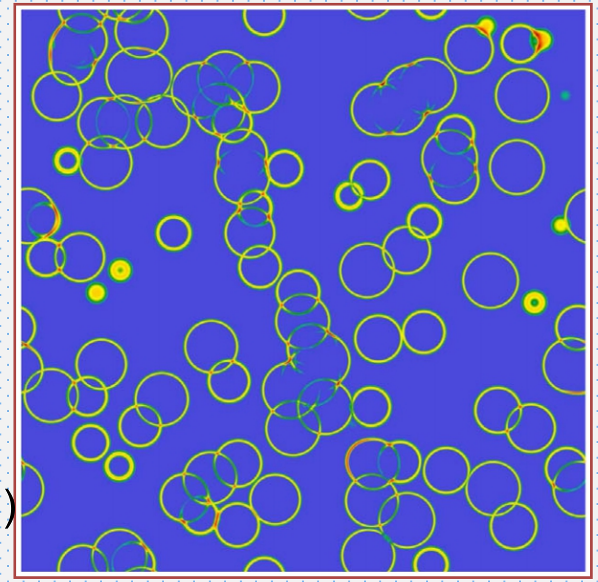
$$p = p_0 \exp\left[-\frac{S_{3,b}(T)}{T}\right]$$

critical bubble: bounce solution (CosmoTransitions, AnyBubble, BubbleProfiler...)

- **Nucleation types** (e.g., Hindmarsh, Hijazi JCAP12(2019)062)

exponential $$p(t) = p_0 \exp\left[-S_* + \beta(t - t_*)\right]$$

simultaneous $$p(t) = p_0 \exp\left[-S_* - \frac{1}{2}\beta_2^2(t - t_*)^2\right]$$

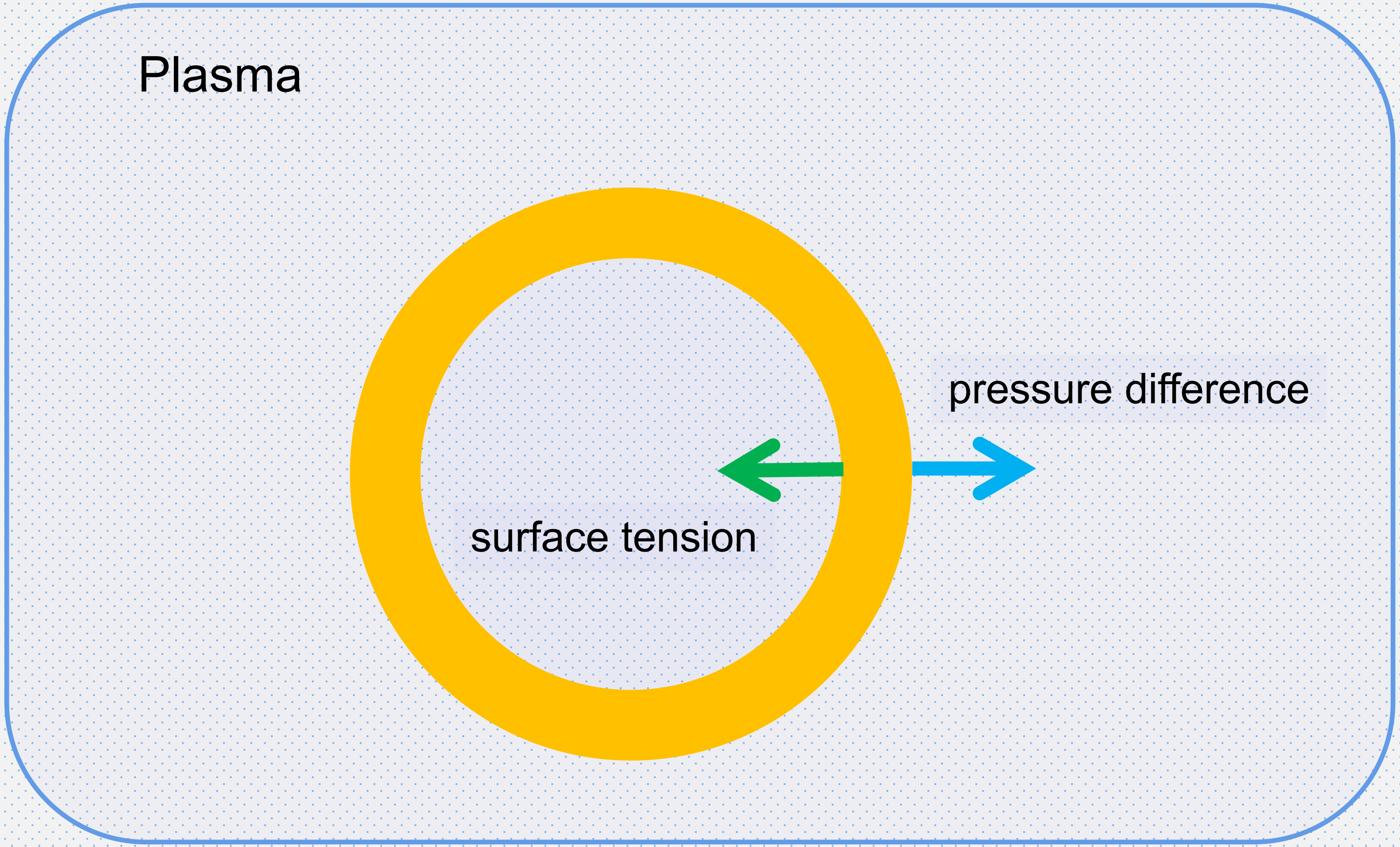


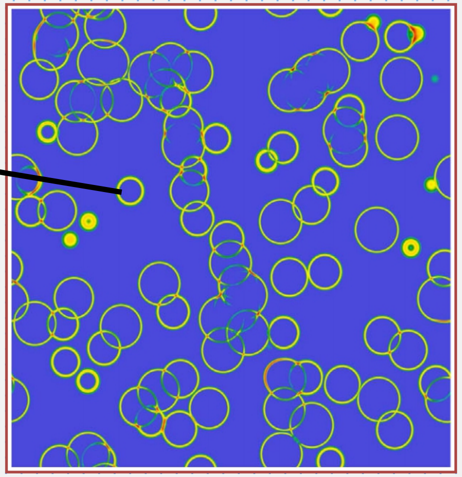
Hindmarsh, et al, 2015

Similar to the picture of boiling water

Kinetics of the Phase Transition: A Single Bubble

(see also Shaojiang Wang's talk)





Hindmarsh, et al, 2015

Magnetic field generation (see Ligong Bian's talk).

Relativistic Hydrodynamics

The fluid-scalar field model (Ignatius et al, PRD49, 3854(1994), Kurki-Suonio, Laine, PRD 54, 7163)

- Plasma: relativistic species $$T_{\mu\nu}^{\text{plasma}} = w\, u_\mu u_\nu - g_{\mu\nu}\, p,$$

- Scalar field: ϕ $$T_{\mu\nu}^{\phi} = \partial_\mu \phi \partial_\nu \phi - g_{\mu\nu} \left[\frac{1}{2} \partial_\rho \phi \partial^\rho \phi - V_0(\phi) \right]$$

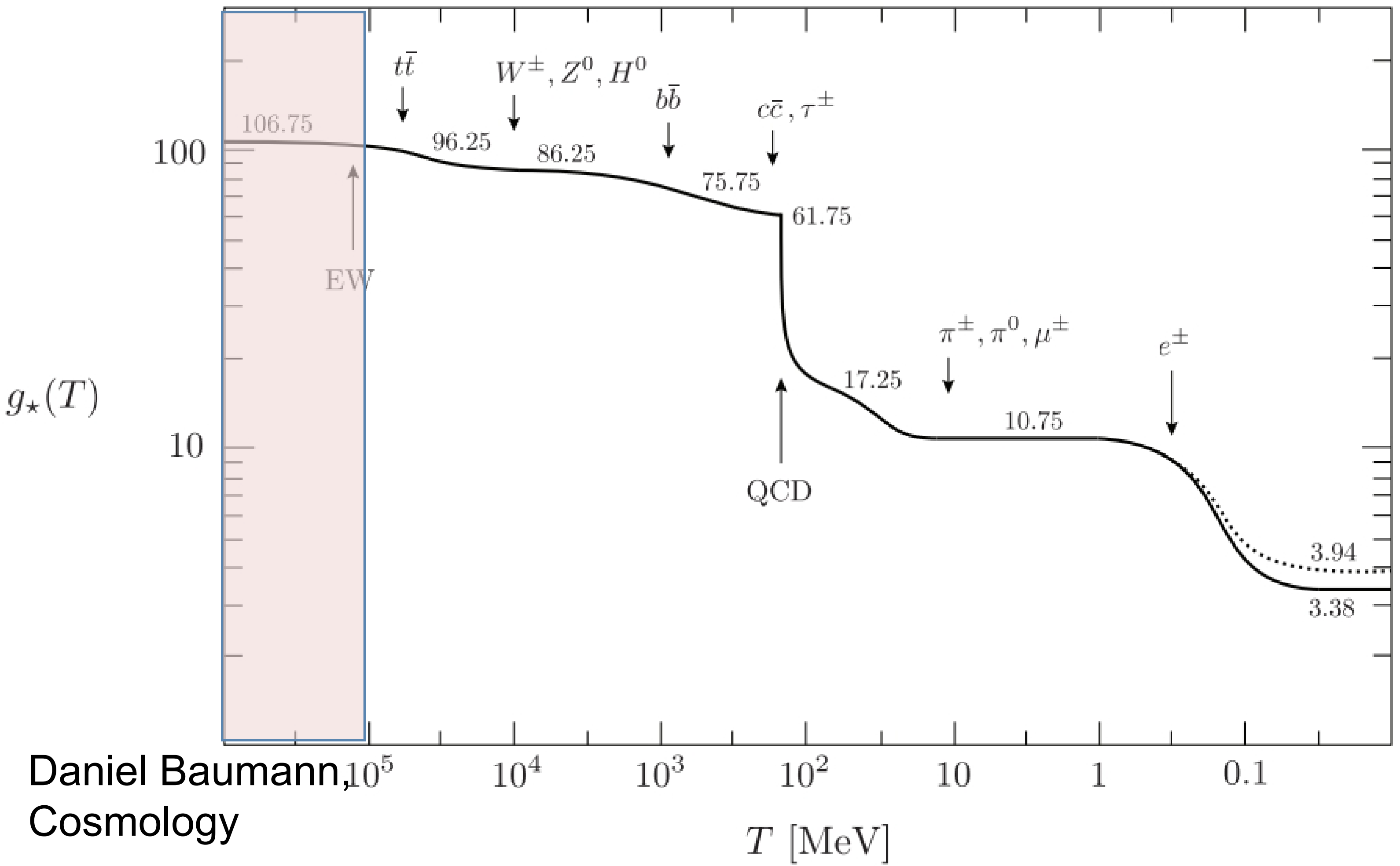
Equations of motion (numerical simulations, theoretical modellings, gravitational waves, etc)

1. The total energy and momentum are conserved, not separately (friction).

2. Metric evolution governed by Einstein equation, coupled with matter equations.

Relativistic Hydrodynamics

- Plasma: relativistic species



Daniel Baumann, Cosmology

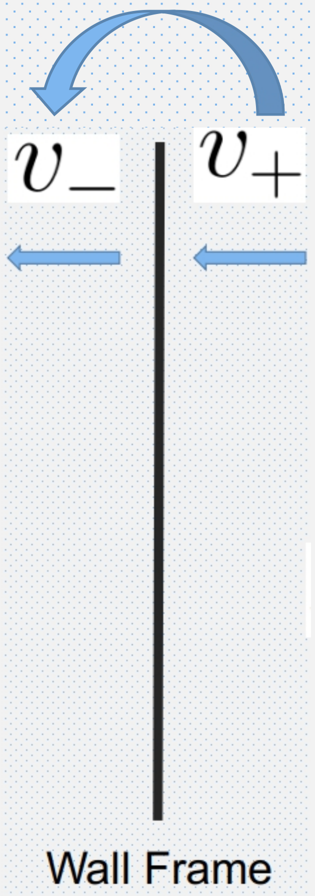
- Scalar field: ϕ

The Bag Model

- Plasma: relativistic species

- Scalar field: ϕ

$$p_- = \frac{1}{3} a_- T_-^4 \,, \quad e_- = a_- T_-^4$$



$$p_+ = \frac{1}{3} a_+ T_+^4 - \epsilon \quad e_+ = a_+ T_+^4 + \epsilon$$

bag constant

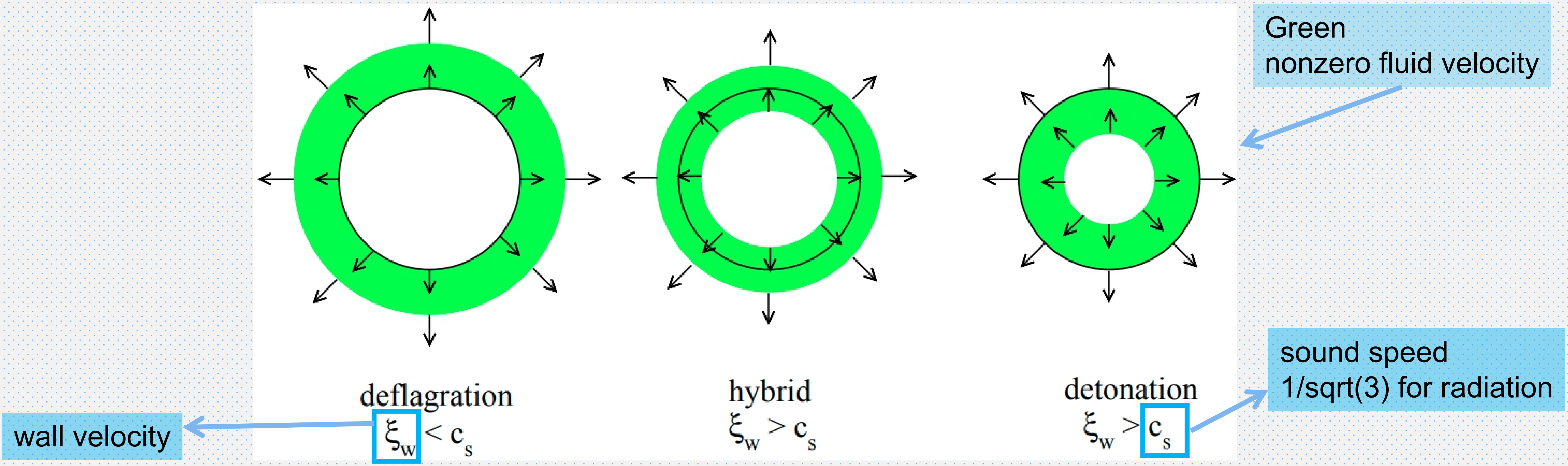
Beyond the bag model

Wang, Huang, Zhang, PRD103, 103520.
Giese, Konstandin, Vis, JCAP07(2020)057

see Xiao Wang's talk

Relativistic Combustion

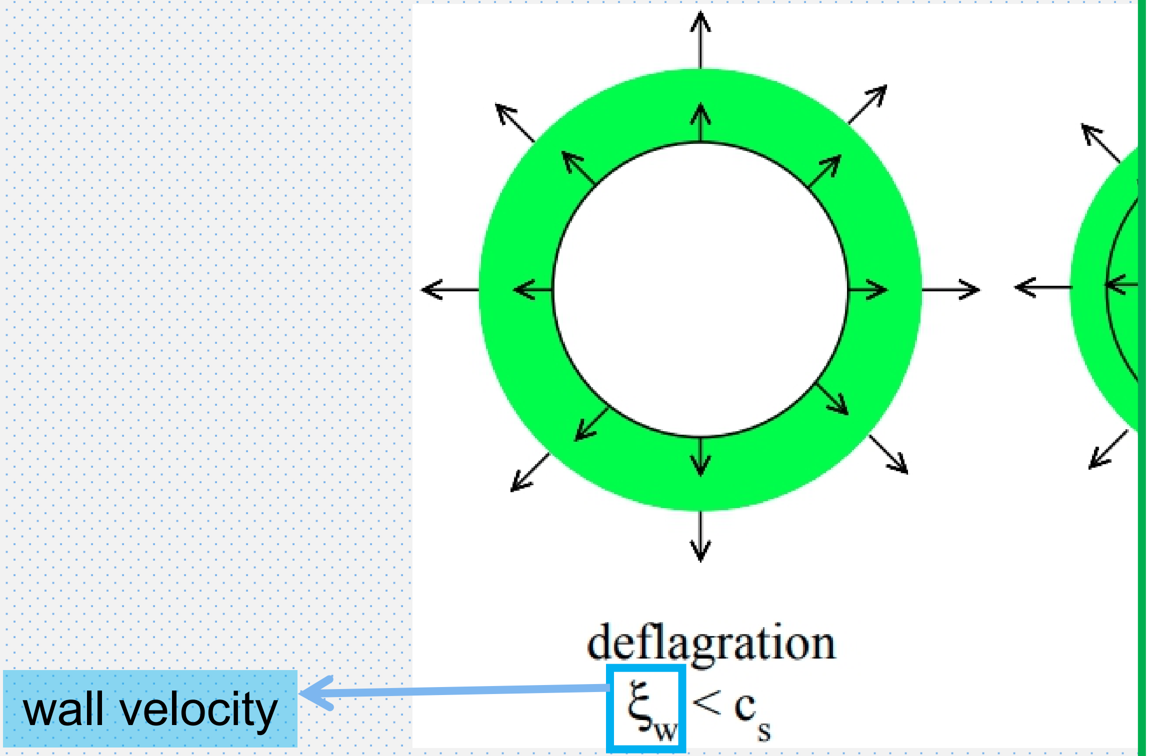
The fluid motion admits 3 modes:



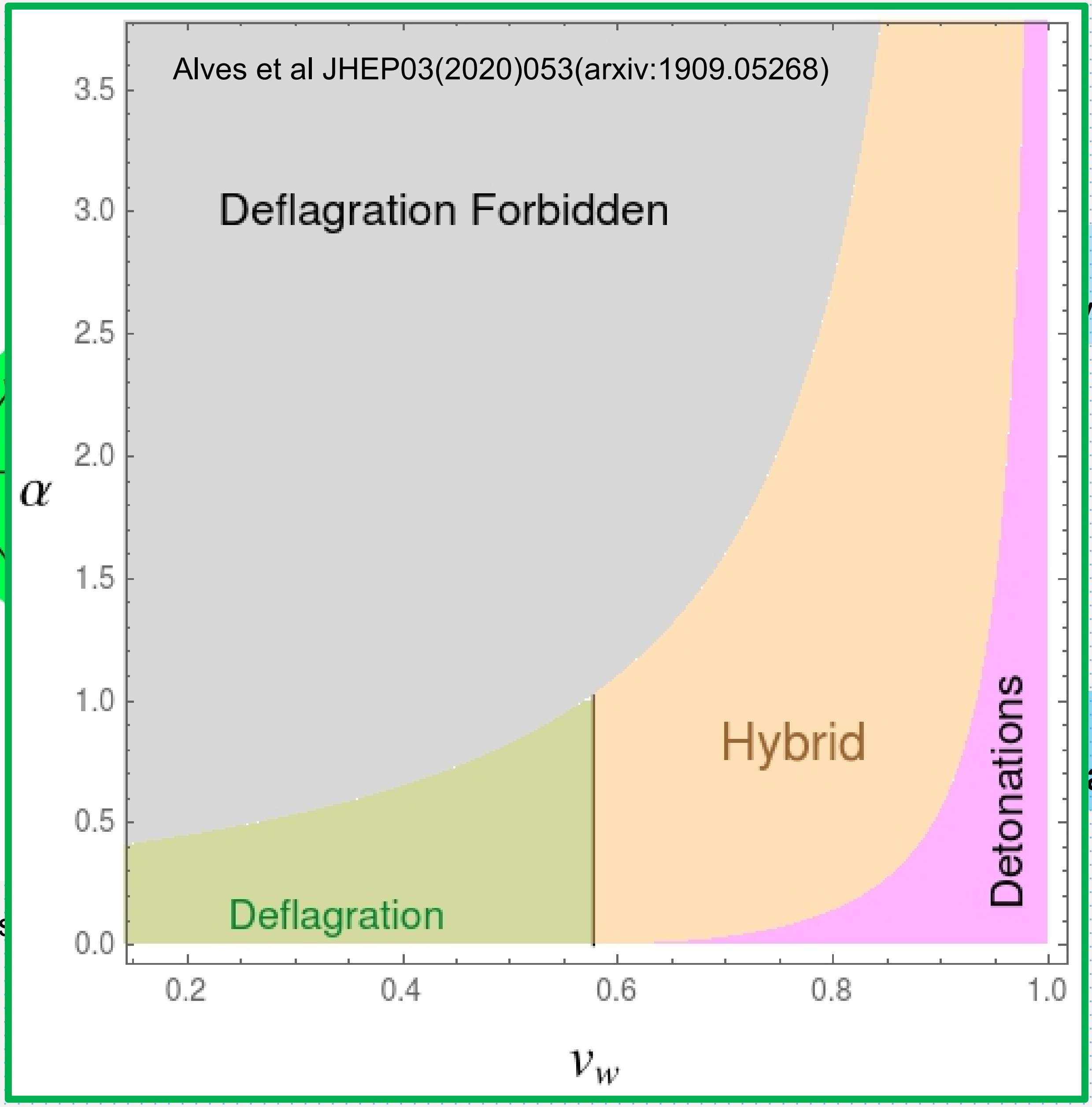
Recent summary of all modes: JCAP06(2010)028 (arxiv: 1004.4187)

Relativistic Combustion

The fluid motion admits 3 modes:

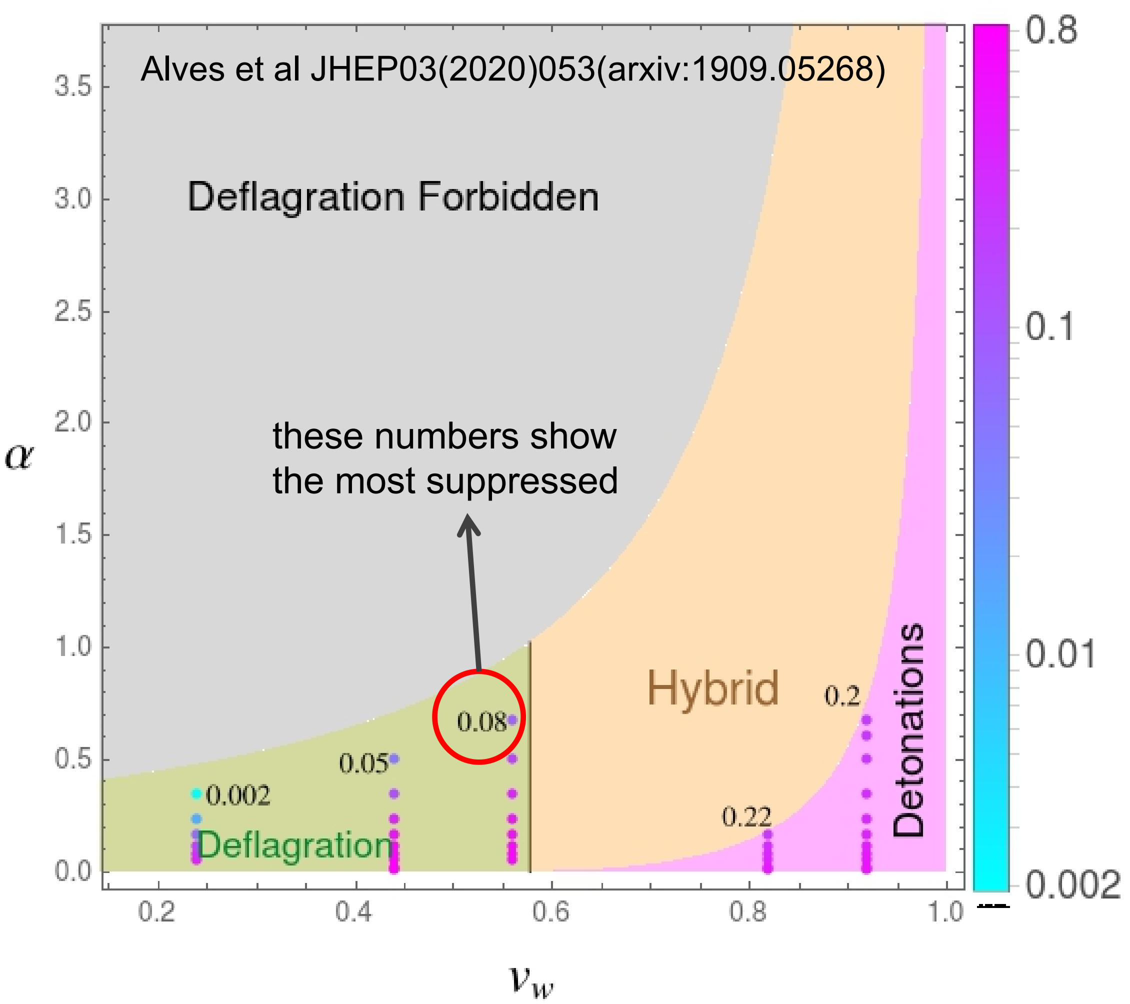


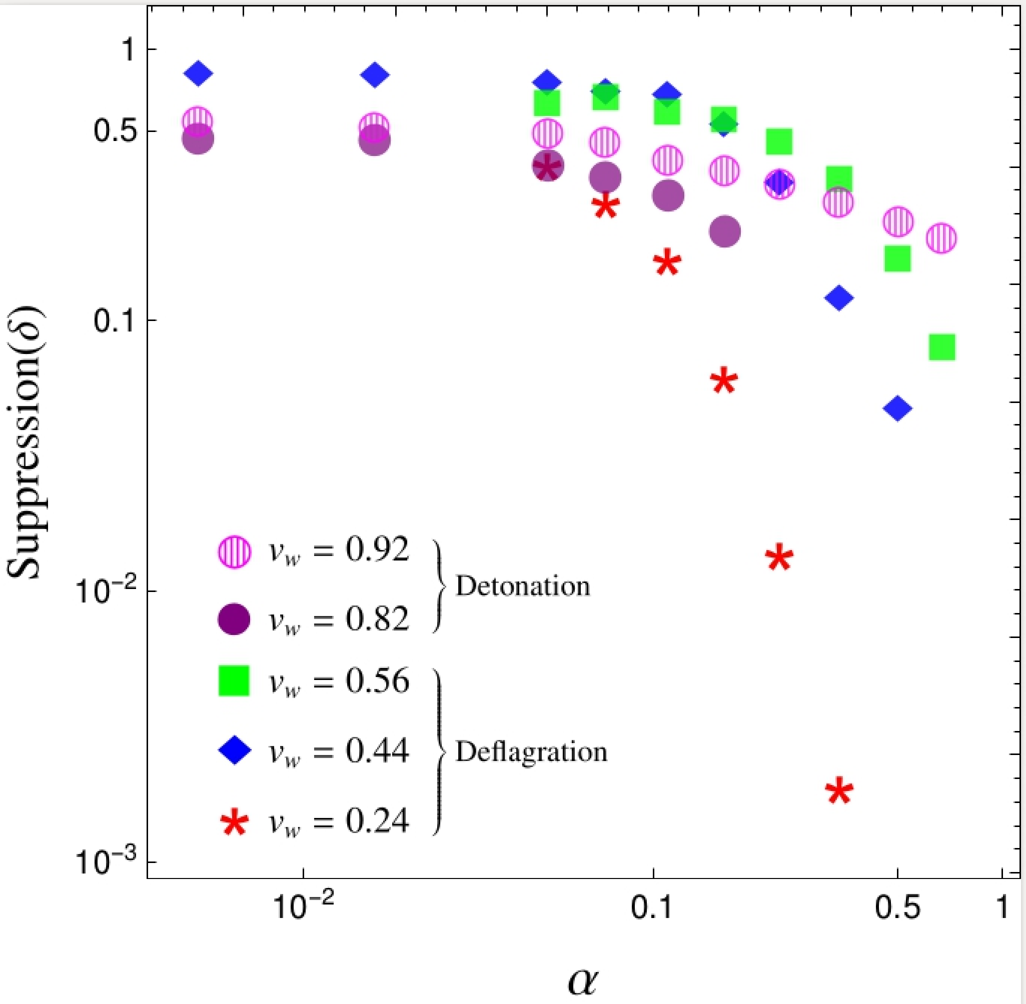
Recent summary of all modes



New Phenomena in Simulations

Reduction of GW from sound waves Cuttings, et al, Phys.Rev.Lett. 125 (2020) 2, 021302
mostly for deflagrations, hybrid not available, detonation less affected



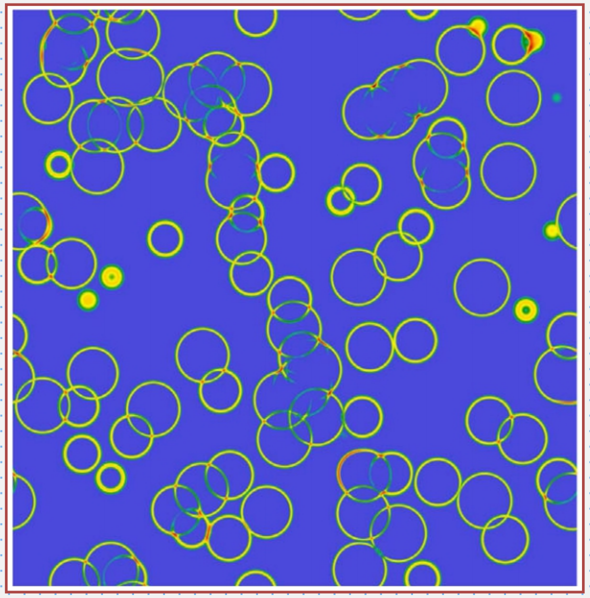


Quantifying the Properties of the Bubbles

A quantitative understanding of the phase transition process.

(Hindmarsh, Hijazi, JCAP12(2019)062, Guo, Sinha, Vagie, White, JCAP01(2021)001)

- Bubble Nucleation Rate
- False Vacuum Fraction
- Unbroken Wall Area
- Bubble Lifetime Distribution
- Bubble Number Density and Mean Bubble Separation(R*)



Hindmarsh, et al, 2015

These are needed for calculating observables of the transition.

Gravitational Waves

$$ds^2 = -dt^2 + a^2(\delta_{ij} + h_{ij}(\mathbf{x}))d\mathbf{x}^2$$

$h_{ij}(\mathbf{x})$ → Tensor Mode

$$\langle \dot{h}_{ij}(t,\mathbf{q})\dot{h}_{ij}(t,\mathbf{k})\rangle = (2\pi)^{-3}\delta^3(\mathbf{k}+\mathbf{q})P_{\dot{h}}(k,t)$$

$$\frac{d\rho_{\mathrm{GW}}(t)}{d\ln k} = \frac{1}{64\pi^3 G}k^3 P_{\dot{h}}(t,k)$$

→ GW Spectrum

Einstein equation

$$h_q'' + 2\frac{a'}{a}h_q' + q^2 h_q = 16\pi G a^2 \pi_q^T$$

Source evolutions

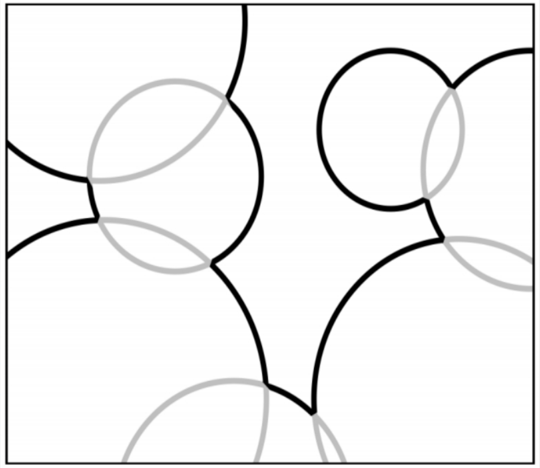
Plasma(relativistic species), Matter(non-relativistic), Scalar field, EM

Energy-momentum conservation (hydrodynamic limit)

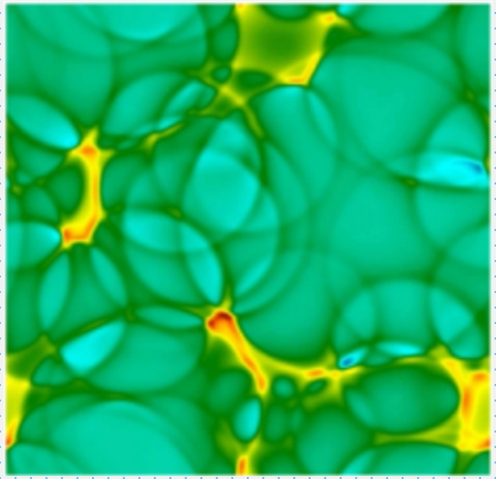
Sources for Gravitational Wave Production

The current understanding:

Bubble Collisions

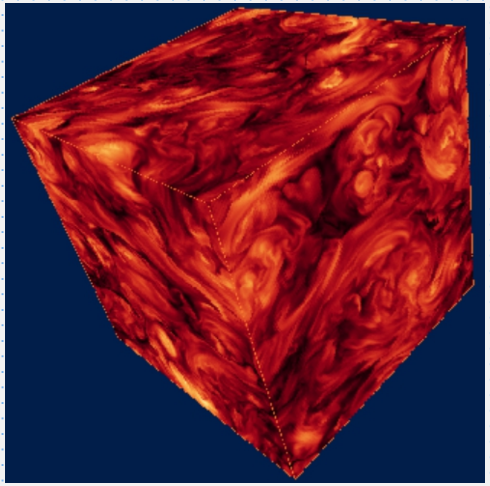


Sound Waves



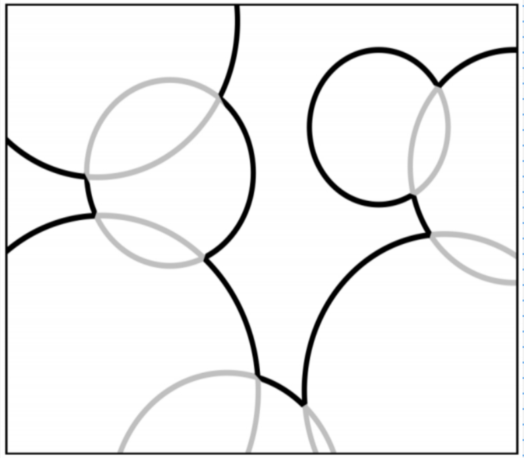
Hindmarsh, et al,PRL112,041301(2013)

MagnetoHydrodynamic Turbulence



https://home.mpcdf.mpg.de/~wcm/projects/homog-mhd/mhd.html

Bubble Collisions



Envelope Approximation

Simulations:
Kosowsky, Turner, Watkins, Kamionkowski, PRL69,2026(1992), PRD45,4514(1992), PRD47,4372(1993), PRD49,2837(1994), Huber, Konstandin, JCAP09(2008)022
Analytical Modelling:
Jinno, Takimoto, PRD95,024009(2017)

Beyond the Envelope Approximation

Bulk flow model: Konstandin, JCAP03(2018)047, Jinno, Takimoto, JCAP01(2019)060
Direct large scalar lattice simulations: Cutting, Escartin, Hindmarsh, Weir, PRD97,123513(2018), arXiv:2005.13537

Expanding Universe: Zhong, Gong, Qiu, arxiv:2107.01845

New Phenomena

Di, Wang, Zhou, Bian, Cai, Liu, Phys.Rev.Lett. 126 (2021) 25, 251102
Lewicki, Vaskonen, EPJC 80,1003(2020)

negligible (dominant) when other sources are present (absent)

$$\Omega_{\rm coll}(f)h^2 = 1.67\times 10^{-5}\Delta\left(\frac{H_{\rm pt}}{\beta}\right)^2\left(\frac{\kappa_\phi\alpha}{1+\alpha}\right)^2 \times\left(\frac{100}{g_*}\right)^{1/3} S_{\rm env}(f),$$

$$f_{\rm env} = 16.5\left(\frac{f_{\rm bc}}{\beta}\right)\left(\frac{\beta}{H_{\rm pt}}\right)\left(\frac{T_{\rm pt}}{100\ {\rm GeV}}\right)\left(\frac{g_*}{100}\right)^{1/6}\ \mu{\rm Hz},$$

Sound Waves

Numerical Simulations:
Hindmarsh, Huber, Rummukainen, Weir,
PRL112, 041301 (2014), PRD92, 123009 (2015), PRD96, 103520 (2017)

Analytical Modelling(sound shell model)
Minkowski: Hindmarsh, 120, 071301 (2018)
Hindmarsh, Hijazi, JCAP12(2019)062
FLRW: HG,Sinha,Vagie,White,JCAP 01 (2021) 001

- Solve the fluid velocity profile
  modes: detonation, deflagration, hybrid
  Espinosa, Konstandin, No, Servant (JCAP06,028)
- Reduction found for alpha~1 and small vw
  Cutting, Hindmarsh, Weir, PRL125, 021302 (2020)

$$\Omega_{\rm sw}(f)h^2 = 2.65\times10^{-6}\left(\frac{H_{\rm pt}}{\beta}\right)\left(\frac{\kappa_{\rm sw}\alpha}{1+\alpha}\right)^2\left(\frac{100}{g_*}\right)^{1/3} \times v_w\left(\frac{f}{f_{\rm sw}}\right)^3\left(\frac{7}{4+3(f/f_{\rm sw})^2}\right)^{7/2}\Upsilon(\tau_{\rm sw}),$$

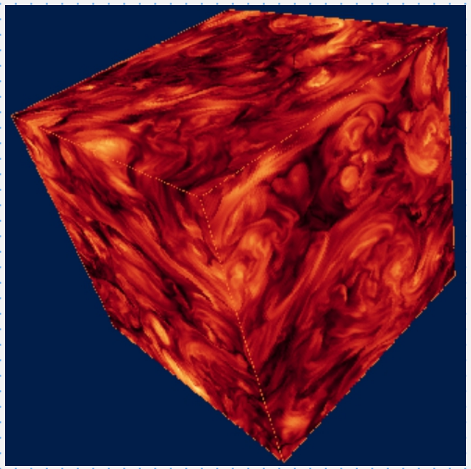
The dominant source for a FOPT in a thermal plasma.

$$\Upsilon = 1-(1+2\tau_{\rm sw}H_{\rm pt})^{-1/2} \quad \text{(RD)}$$

HG,Sinha,Vagie,White,JCAP 01 (2021) 001

Previous formula mistakenly enforces an infinite lifetime.

Magnetohydrodynamic Turbulence



https://home.mpcdf.mpg.de/~wcm/projects/homog-mhd/mhd.html

Analytical Modelling

Kolmogorov spectrum:
Kosowsky, Mack,Kahniashvili, PRD66,024030(2002)
Gogoberidze, Kahniashvili, Kosowsky, PRD76,083002(2007)
Caprini, Durrer, Servant, JCAP12(2009)024

5%~10% but uncertain

$$h^2\Omega_{\rm turb}(f) = 3.35\times 10^{-4}\left(\frac{H_*}{\beta}\right)\left(\frac{\kappa_{\rm turb}\,\alpha}{1+\alpha}\right)^{\frac{3}{2}}\left(\frac{100}{g_*}\right)^{1/3} v_w\, S_{\rm turb}(f)$$

Caprini, Durrer, Servant, JCAP12(2009)024 (adopted by the LISA Cosmology Working group, JCAP04(2016)001)

$$S_{\rm turb}(f) = \frac{(f/f_{\rm turb})^3}{[1+(f/f_{\rm turb})]^{\frac{11}{3}}\,(1+8\pi f/h_*)}$$

$$h_* = 16.5\times 10^{-3}\,{\rm mHz}\left(\frac{T_*}{100\,{\rm GeV}}\right)\left(\frac{g_*}{100}\right)^{\frac{1}{6}}$$

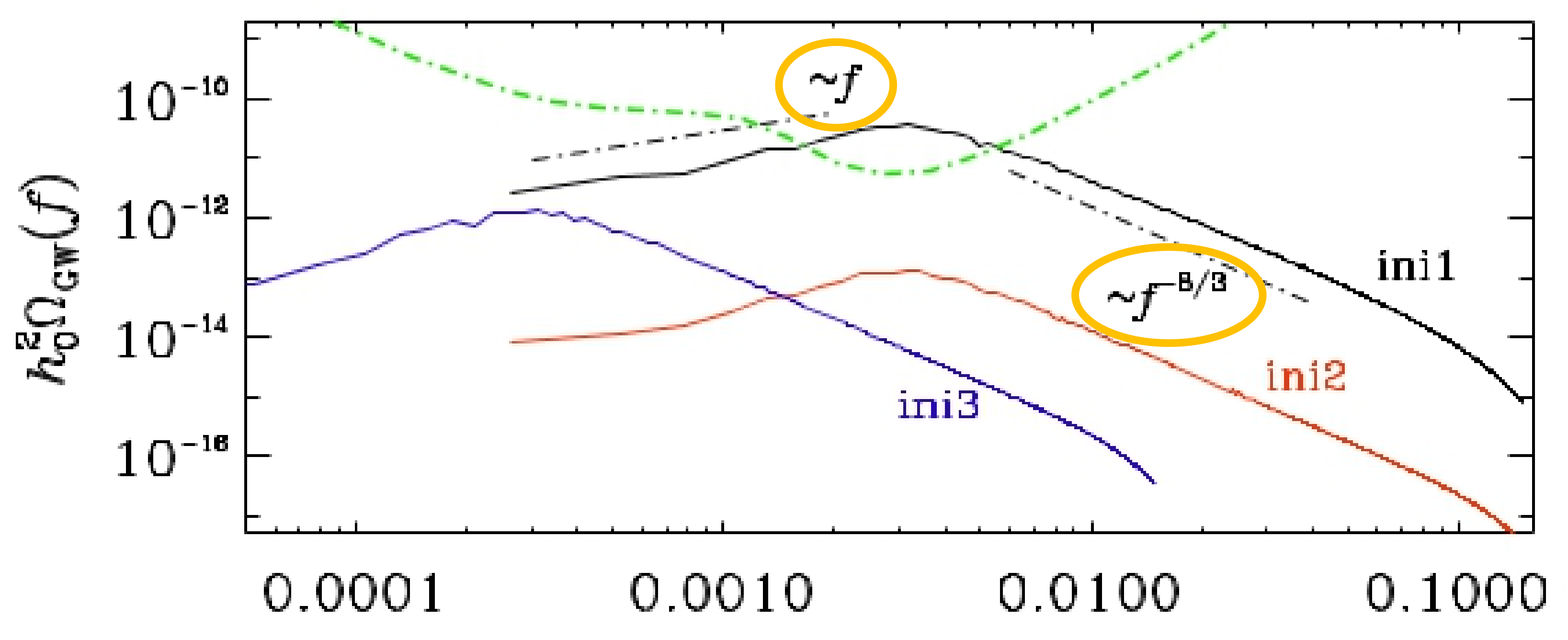
$$f_{\rm turb} = 2.7\times 10^{-2}\,{\rm mHz}\,\frac{1}{v_w}\left(\frac{\beta}{H_*}\right)\left(\frac{T_*}{100\,{\rm GeV}}\right)\left(\frac{g_*}{100}\right)^{\frac{1}{6}}$$

Magnetohydrodynamic Turbulence

Numerical Simulations

Pol, Mandal, Brandenburg, Kahniashvili, Kosowsky, PRD 102, 083512 (2020) (arxiv:1903.08585)

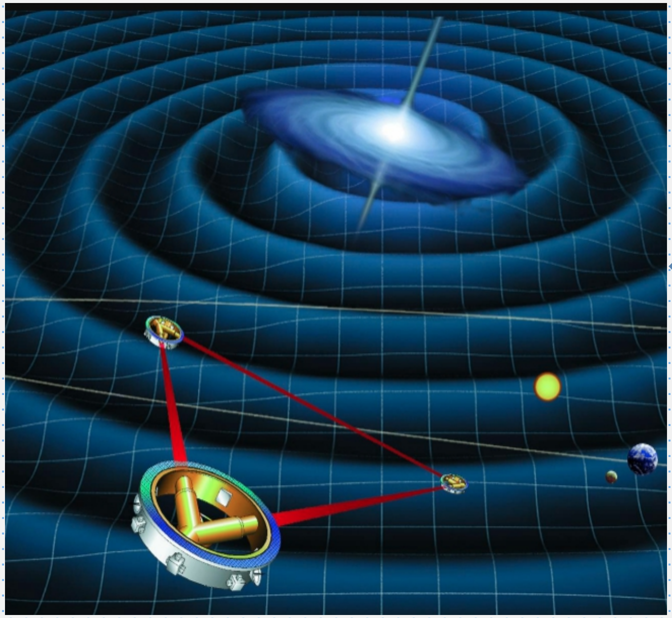
- Result significantly different from spectrum in previous slide
- Result significantly dependent on initial conditions



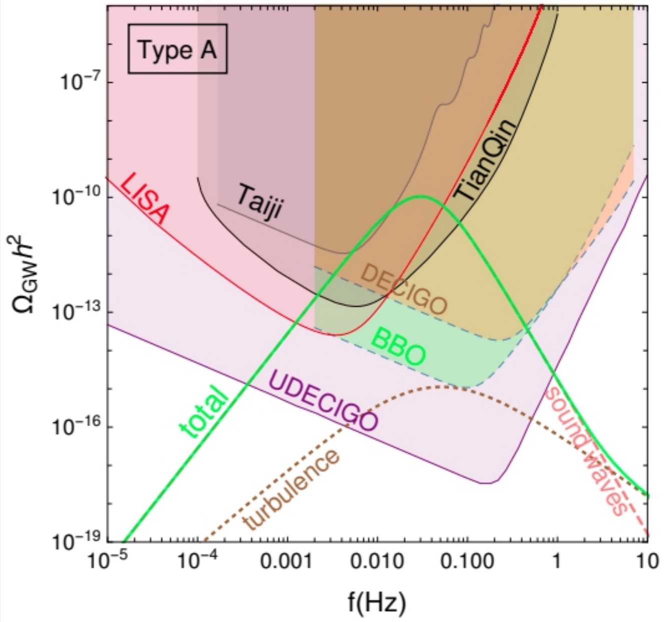
Pol et al, PRD 102, 083512 (2020)

Gravitational Waves

theoretical calculation of gravitational wave spectrum and detector simulation



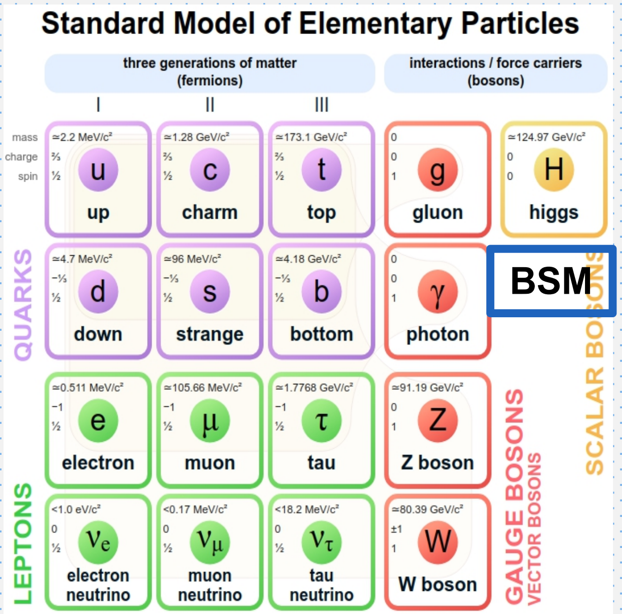
LIGO, LISA, Taiji, Tianqin...



Gravitational Wave Spectrum

α
β
v_w
T_*
g_s
...

Phase Transition Parameters



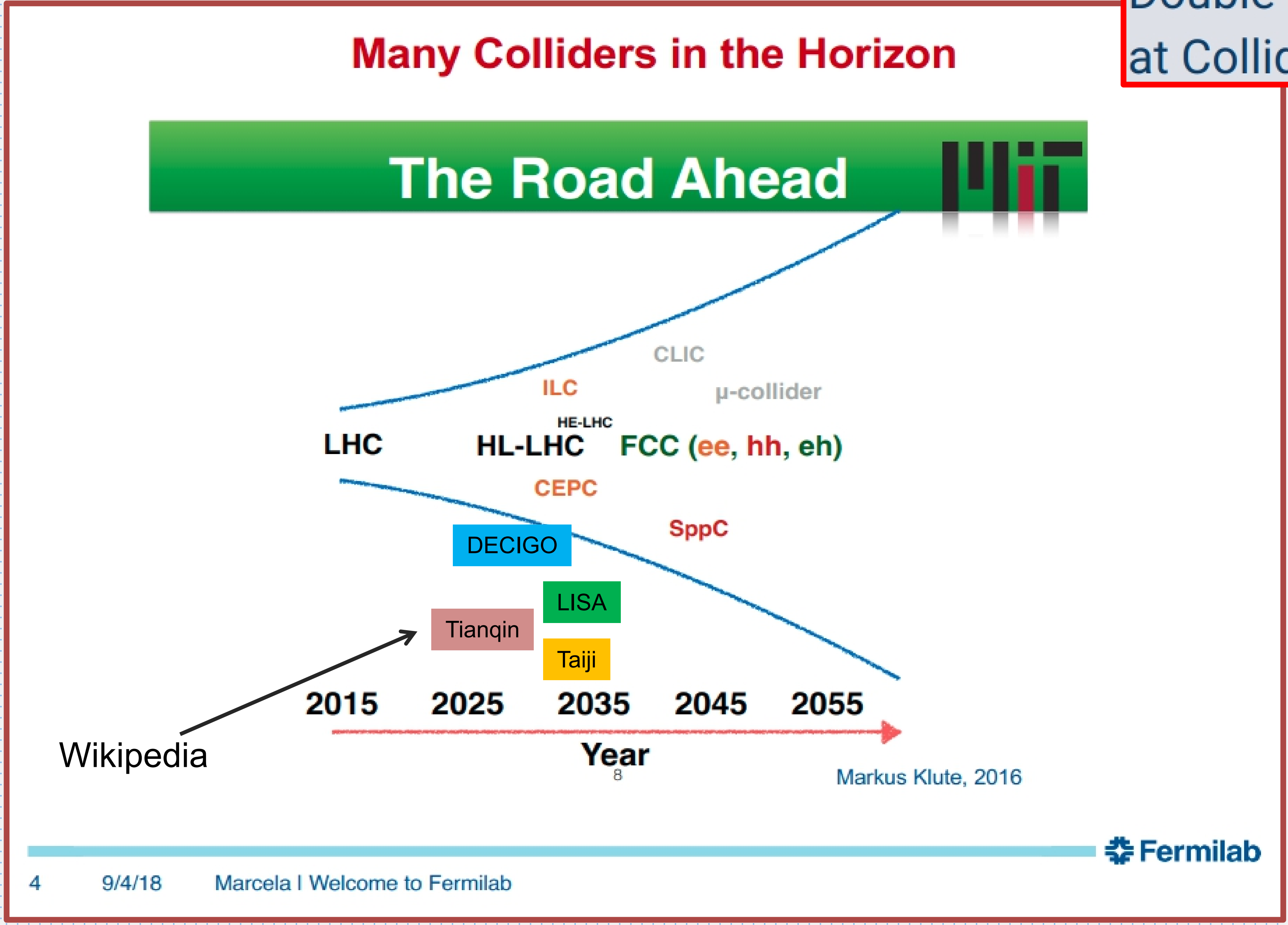
Particle Physics Model

data analysis, constraints or discovery(parameter estimation)

Phys. Rev. Lett. 126, 151301,
A. Romero, K. Martinovic, T. Callister, H.G, M. Martínez, M. Sakellariadou, F.W. Yang, Y. Zhao

Double Higgs Production at Colliders Workshop

2018@Fermilab



Summary

- The Higgs potential remains largely unconstrained in the near future.
- A first order EWPT is well motivated and realized in many new physics models.
- Gravitational waves from a first order EWPT can help reconstruct the Higgs potential

Thanks!