B+L and PMF from First-order phase transition

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Higgs potential and BSM opportunity (Aug 27-31, 2021)

Based on work with Yuefeng Di, Jailong Wang, Ruiyu Zhou, Rong-Gen Cai, and Jing Liu, 2012.15625(Phys.Rev.Lett. 126 (2021) 25, 25); 2107.08978

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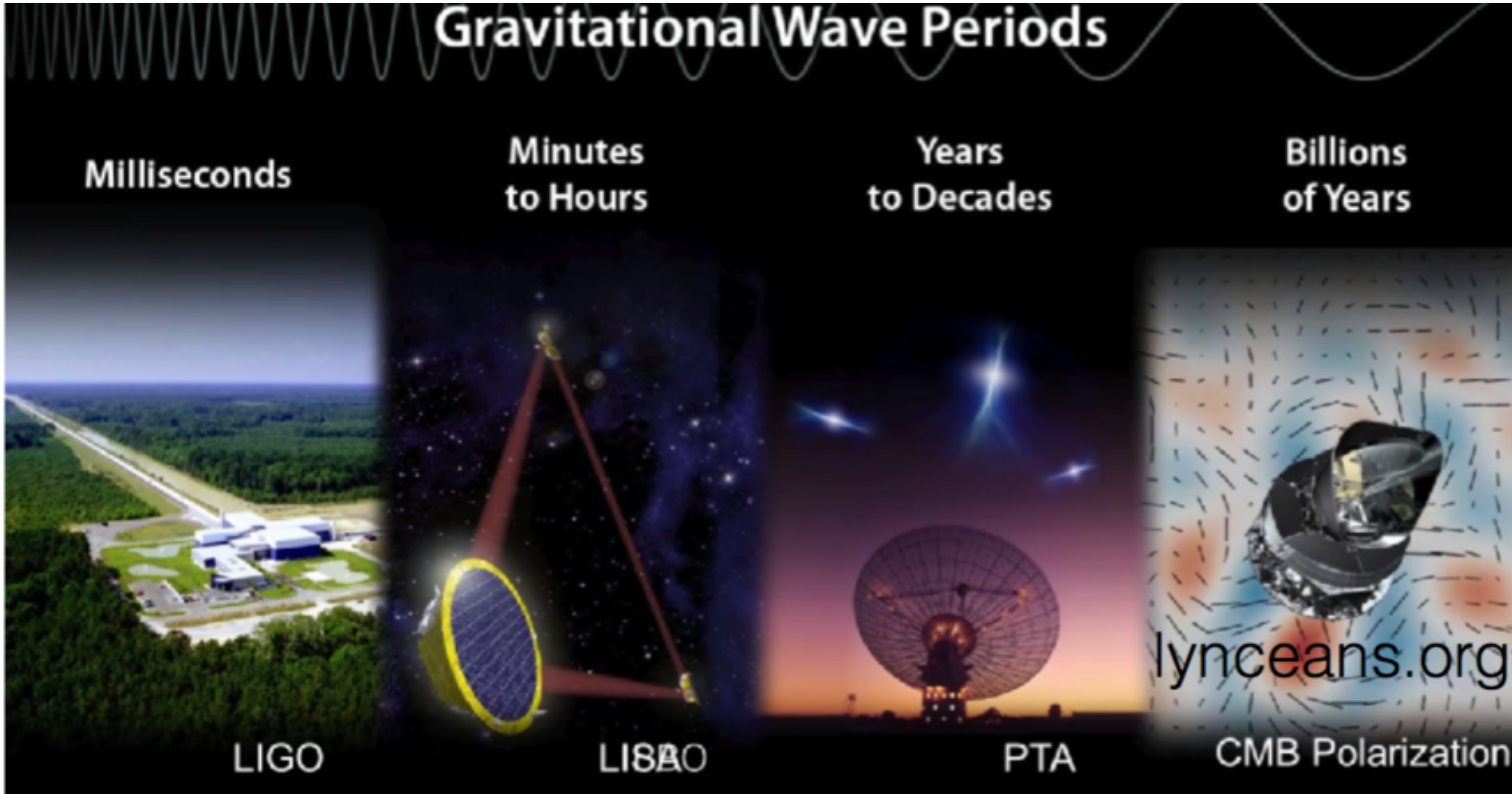
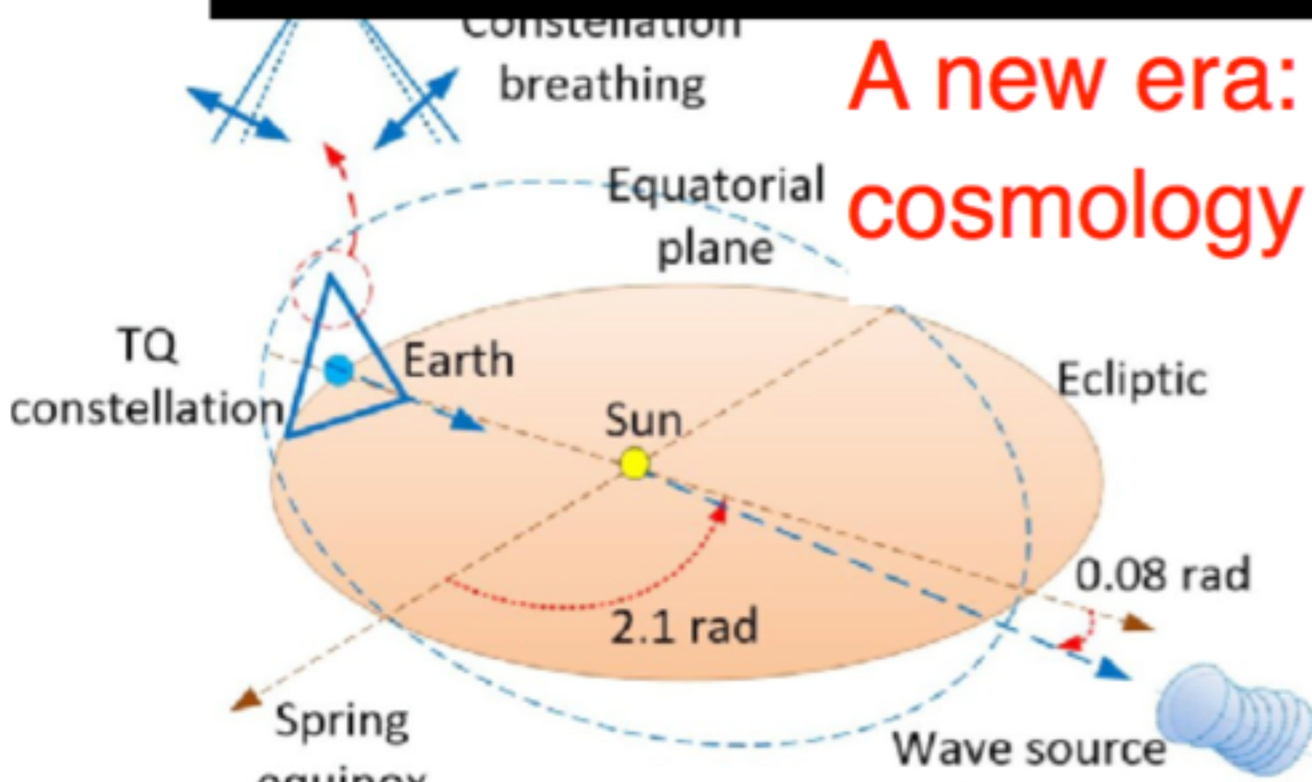


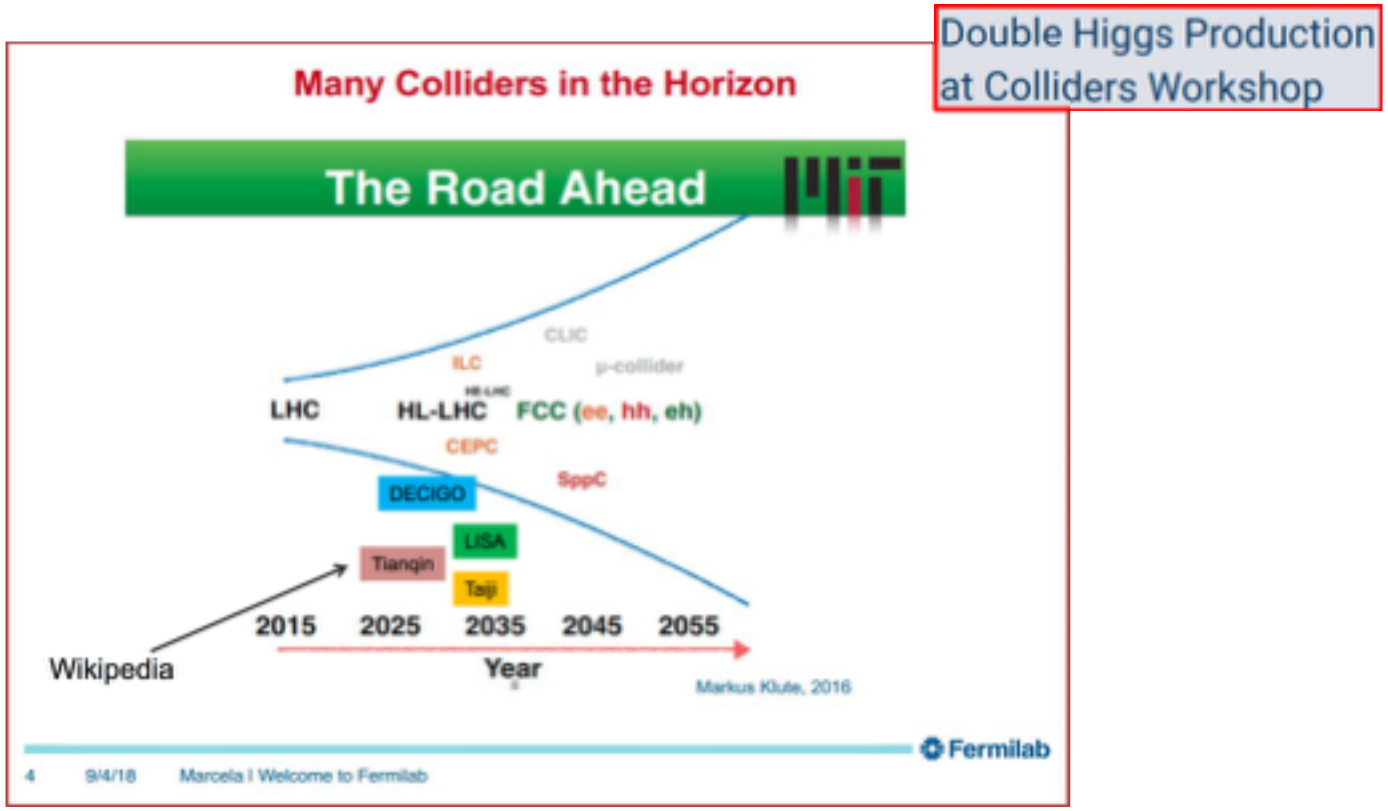


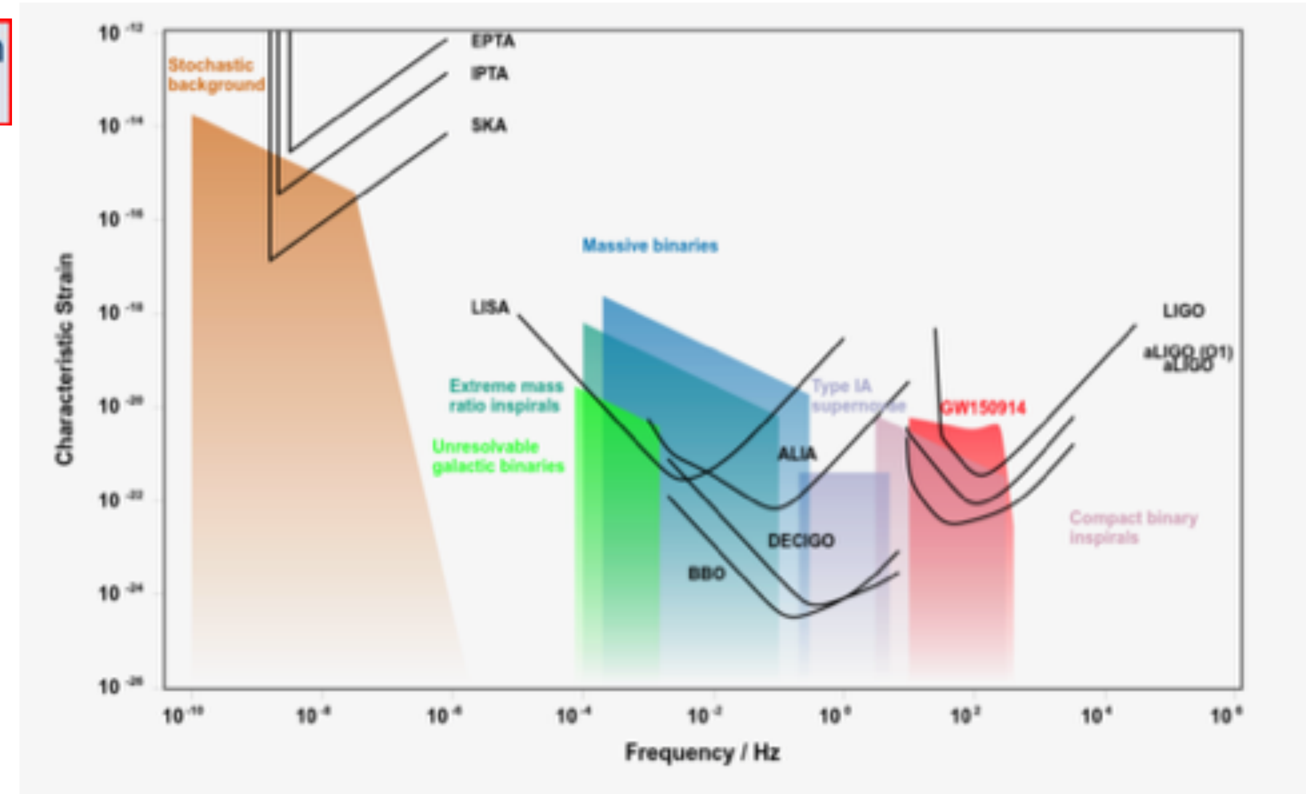
A new era: GWs as a new tool for probing cosmology and high energy physics

Current: LIGO,PTA,…

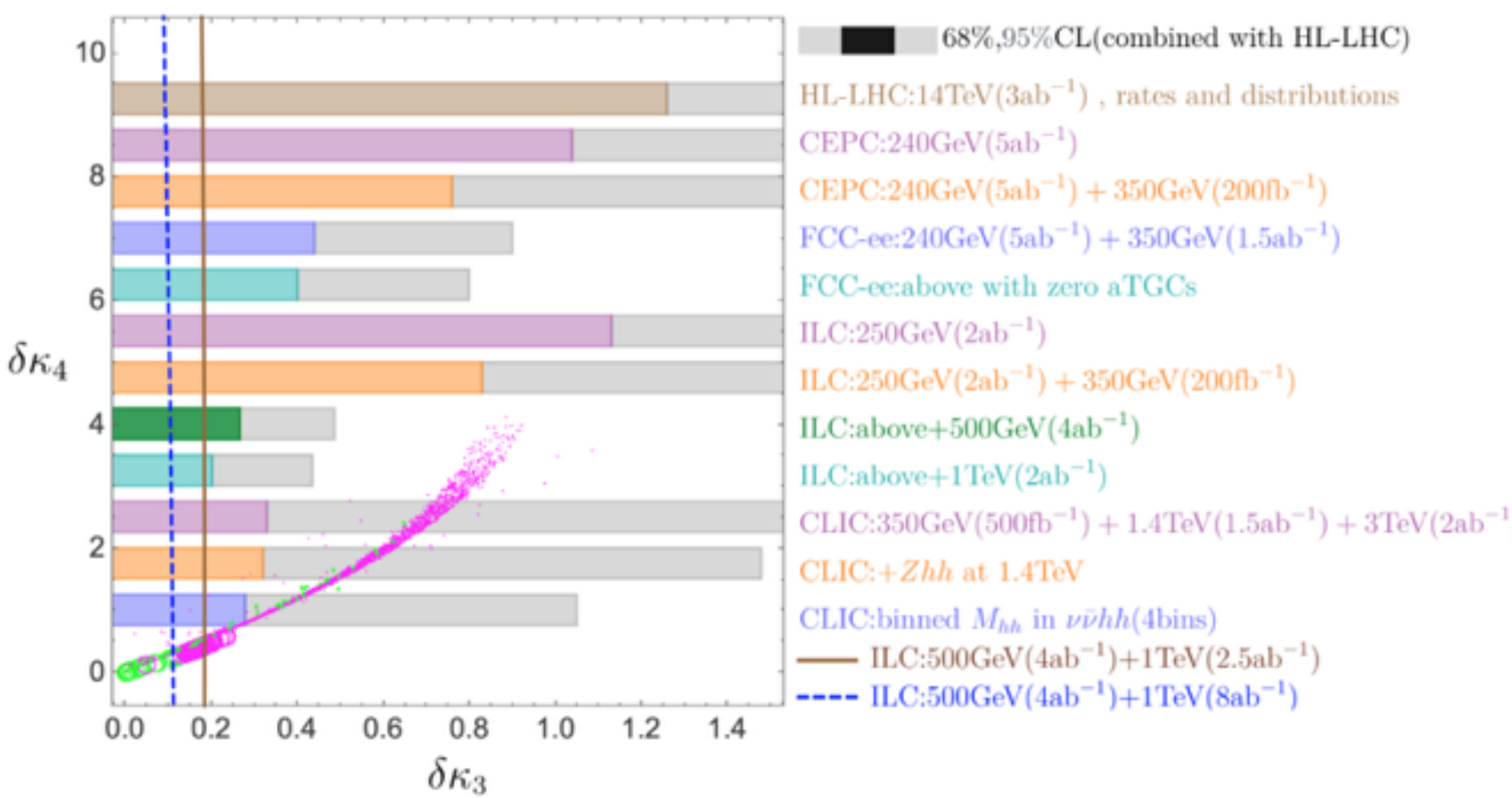
Future GW experiments:
LISA, Taiji, TianQin, BBO, DECIGO, ET, CE,...

GW and collider search





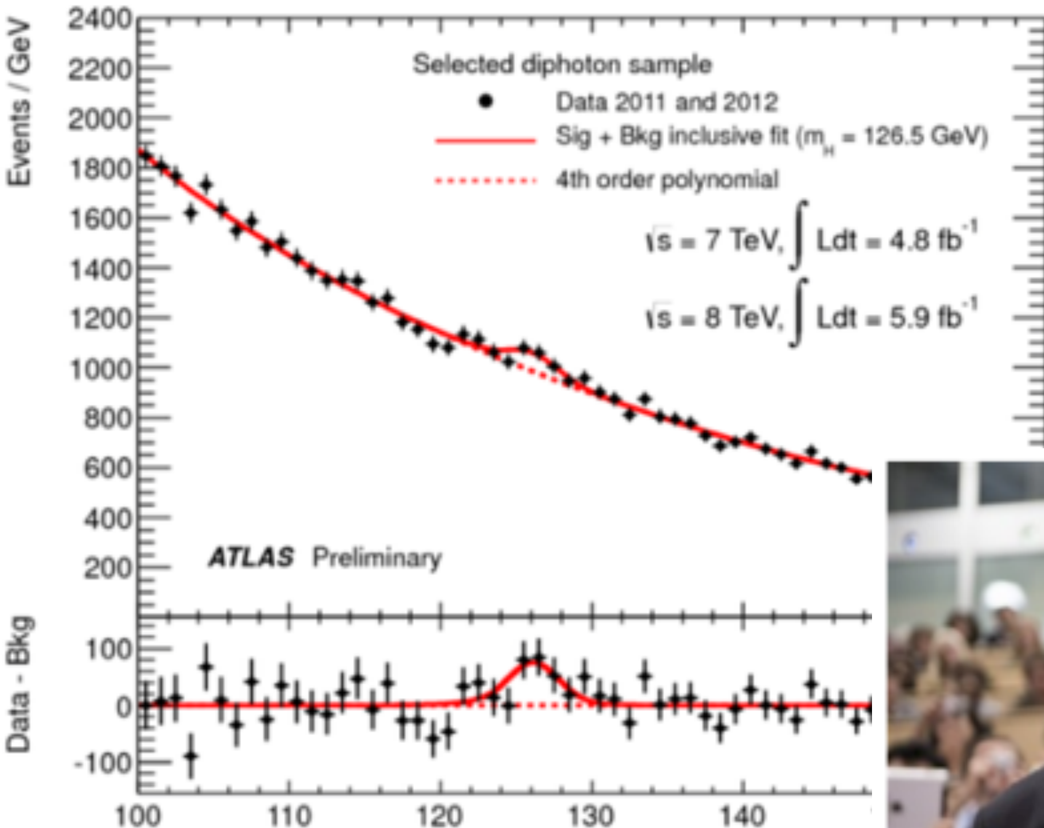
C J Moore et al. Class. Quantum Grav. 32 (2015) 015014.

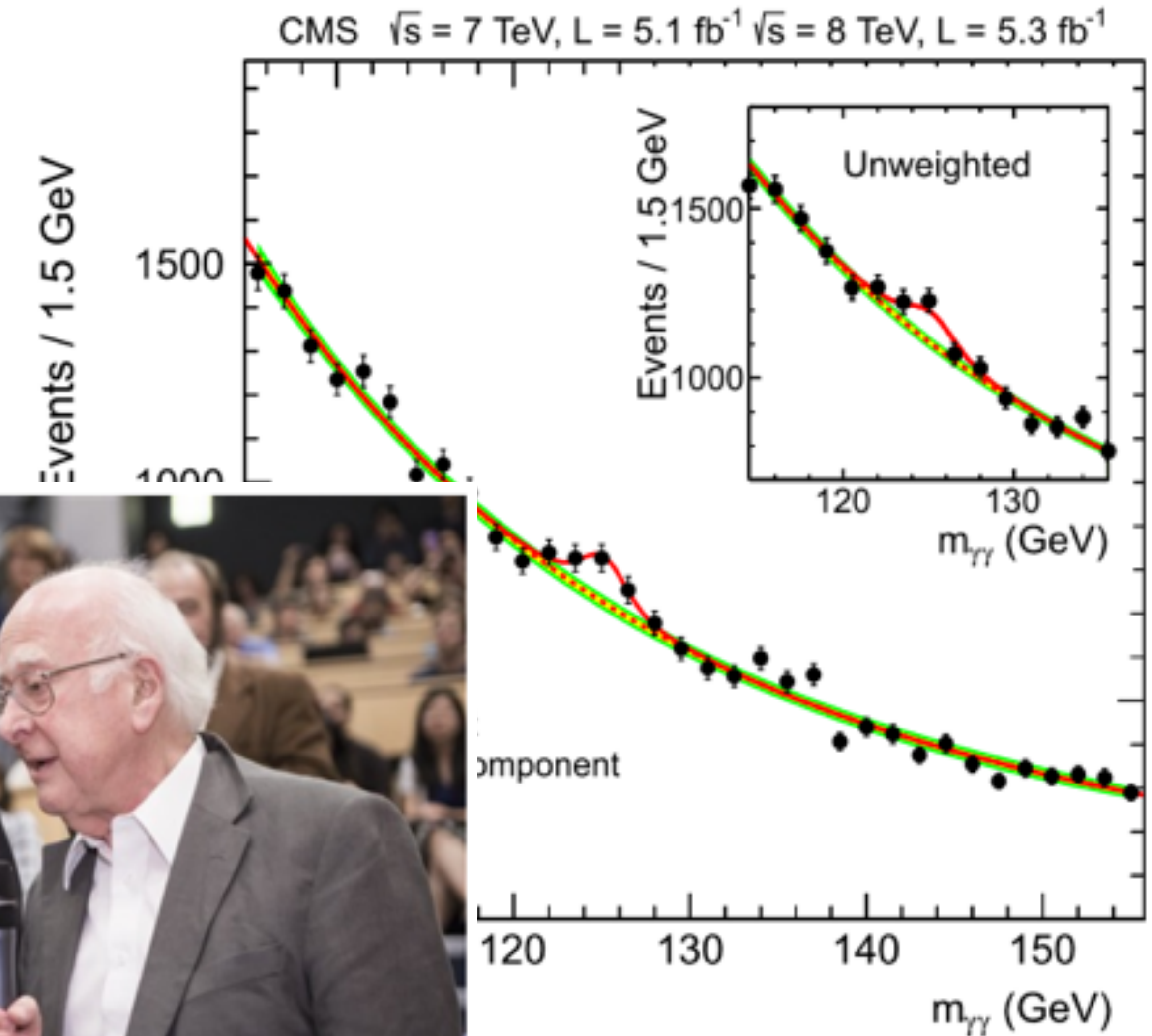


Circles and the dotted points for the GM and xSM scenarios

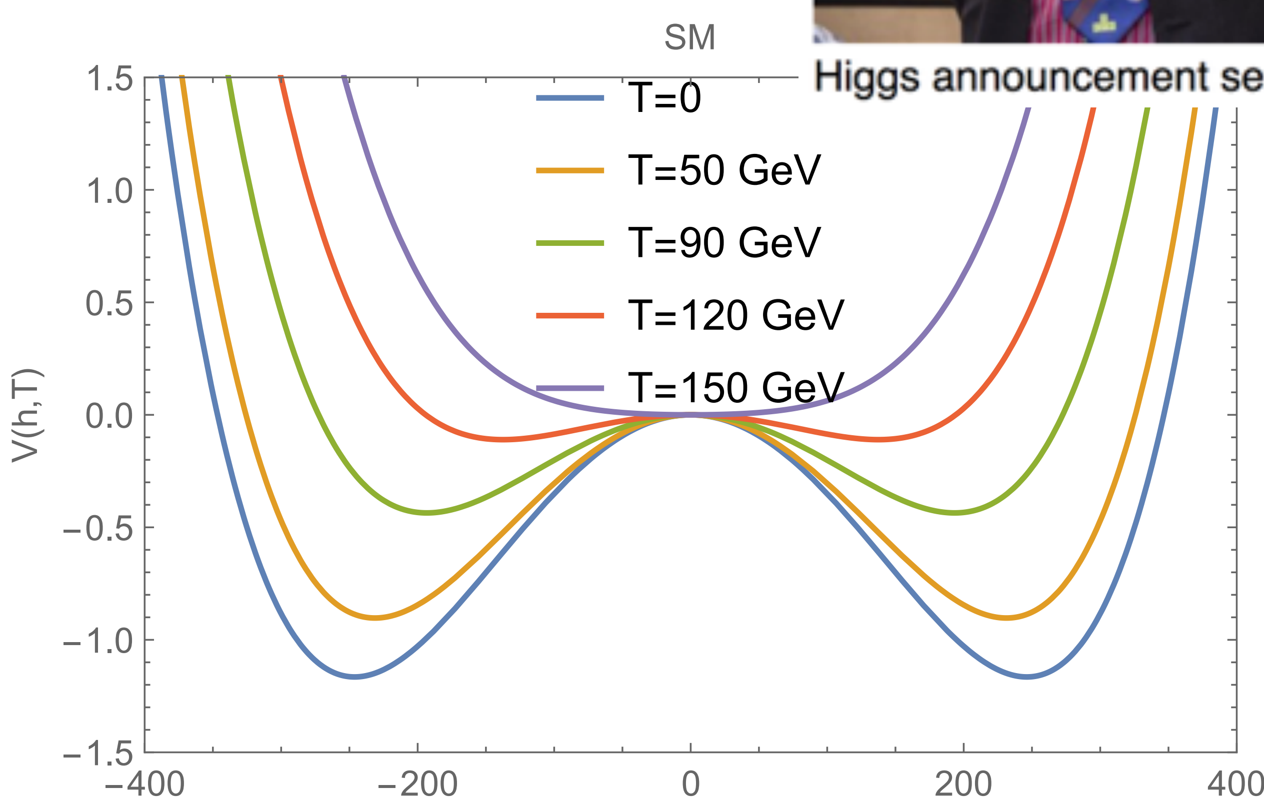
SNR > 10 points for two-step and one-step SFOEWPT

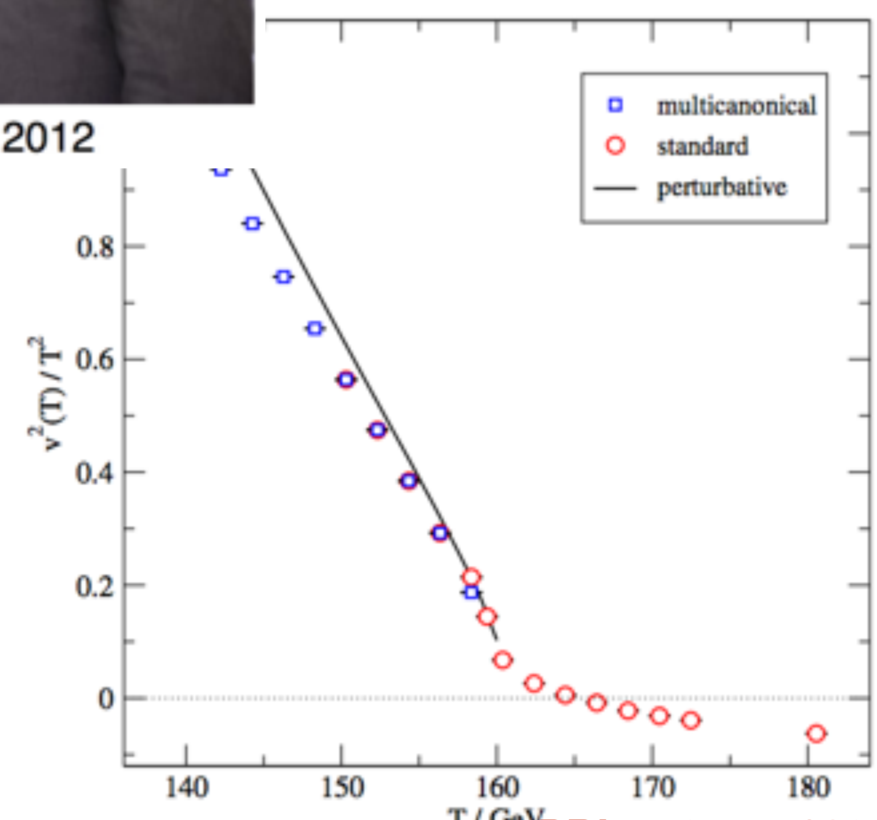
Implication of 125 GeV Higgs





Higgs announcement seminar on 4 July 2012

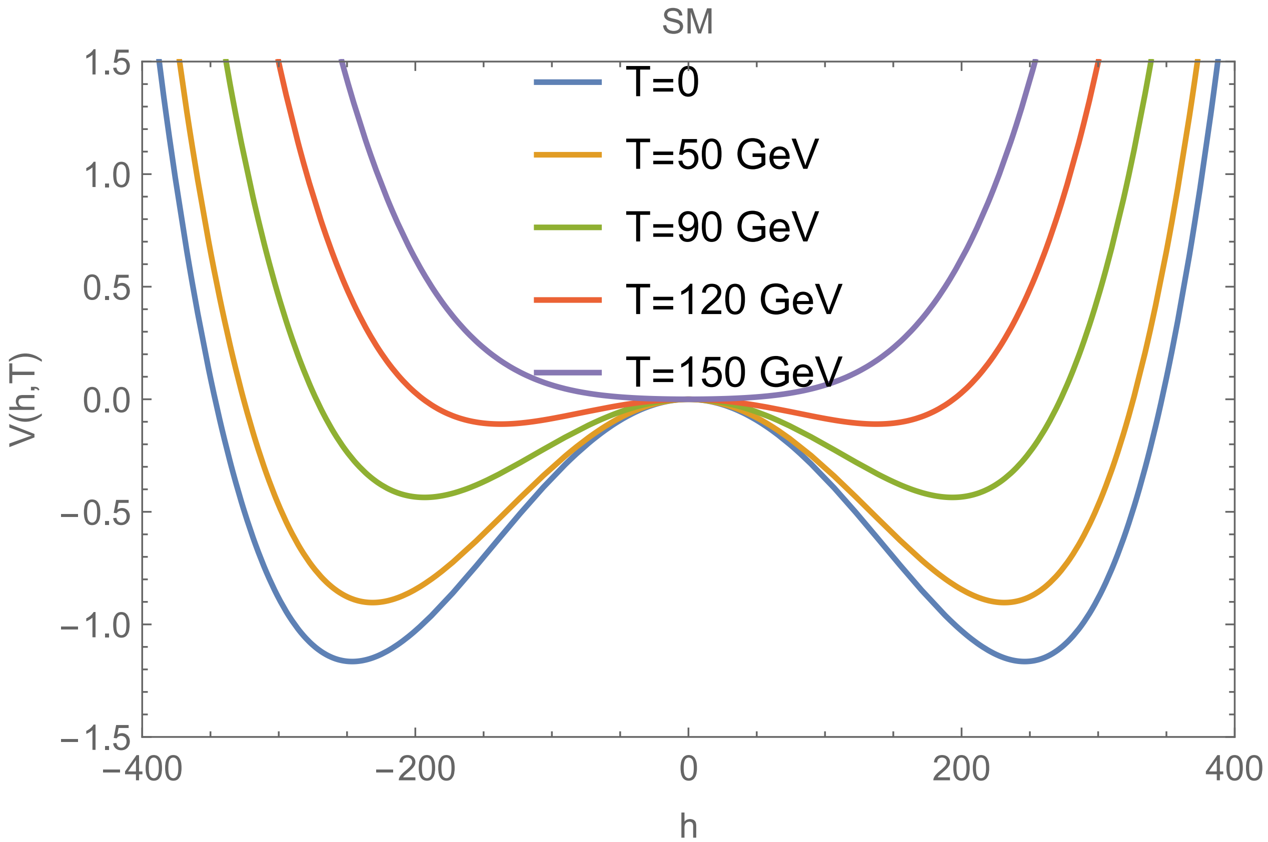


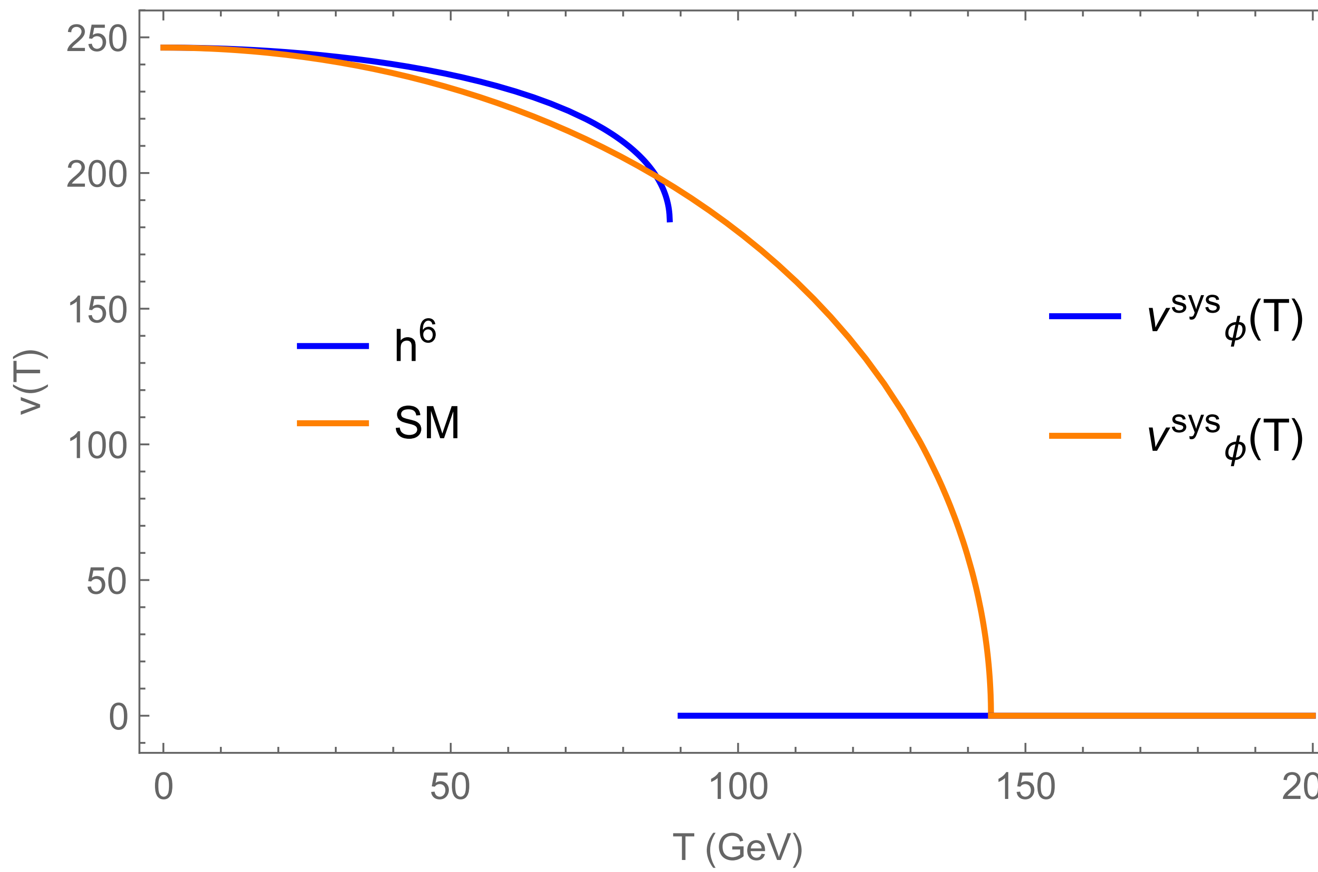


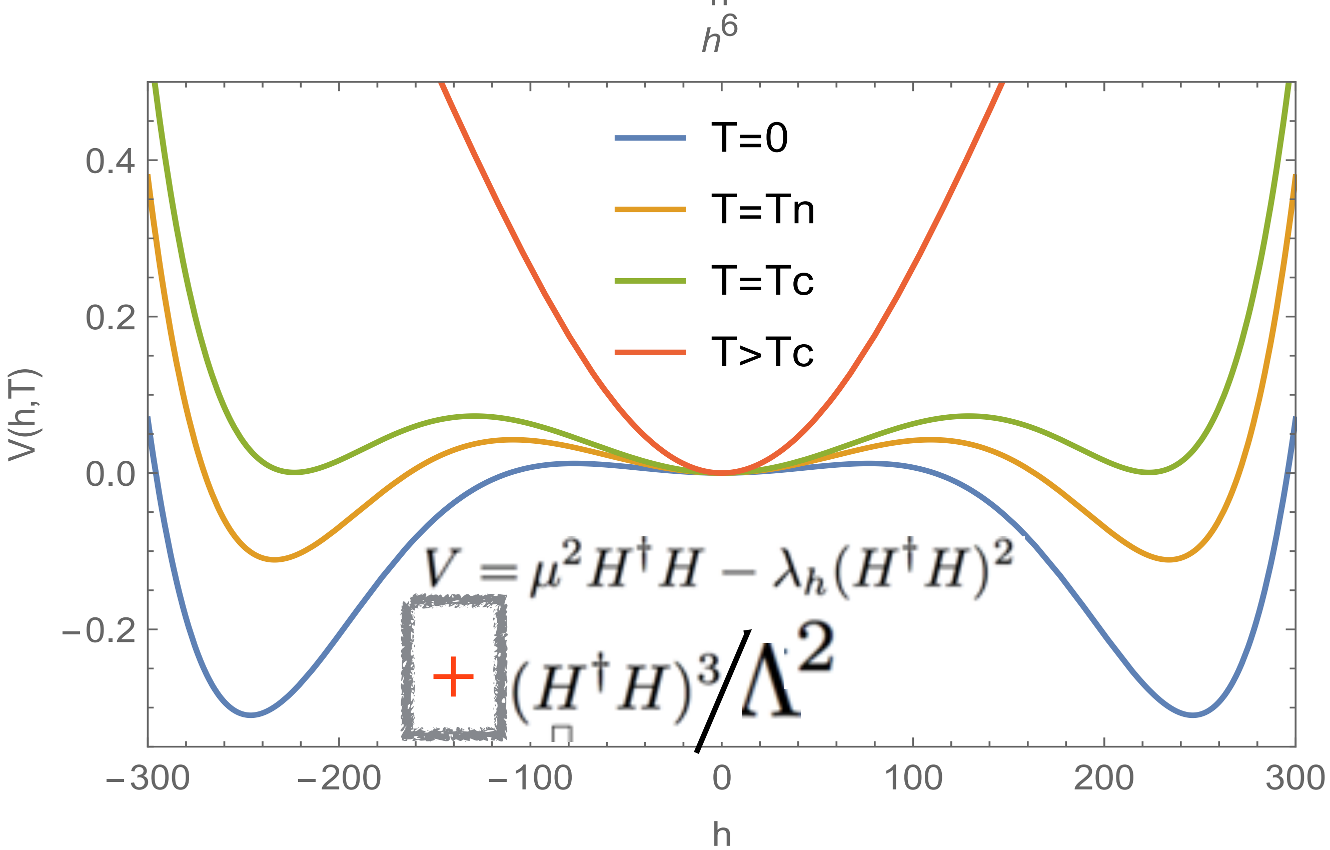
PRL 113, 141602 (2014)

Higgs Potential Shape??? EFT or ???

First or second order



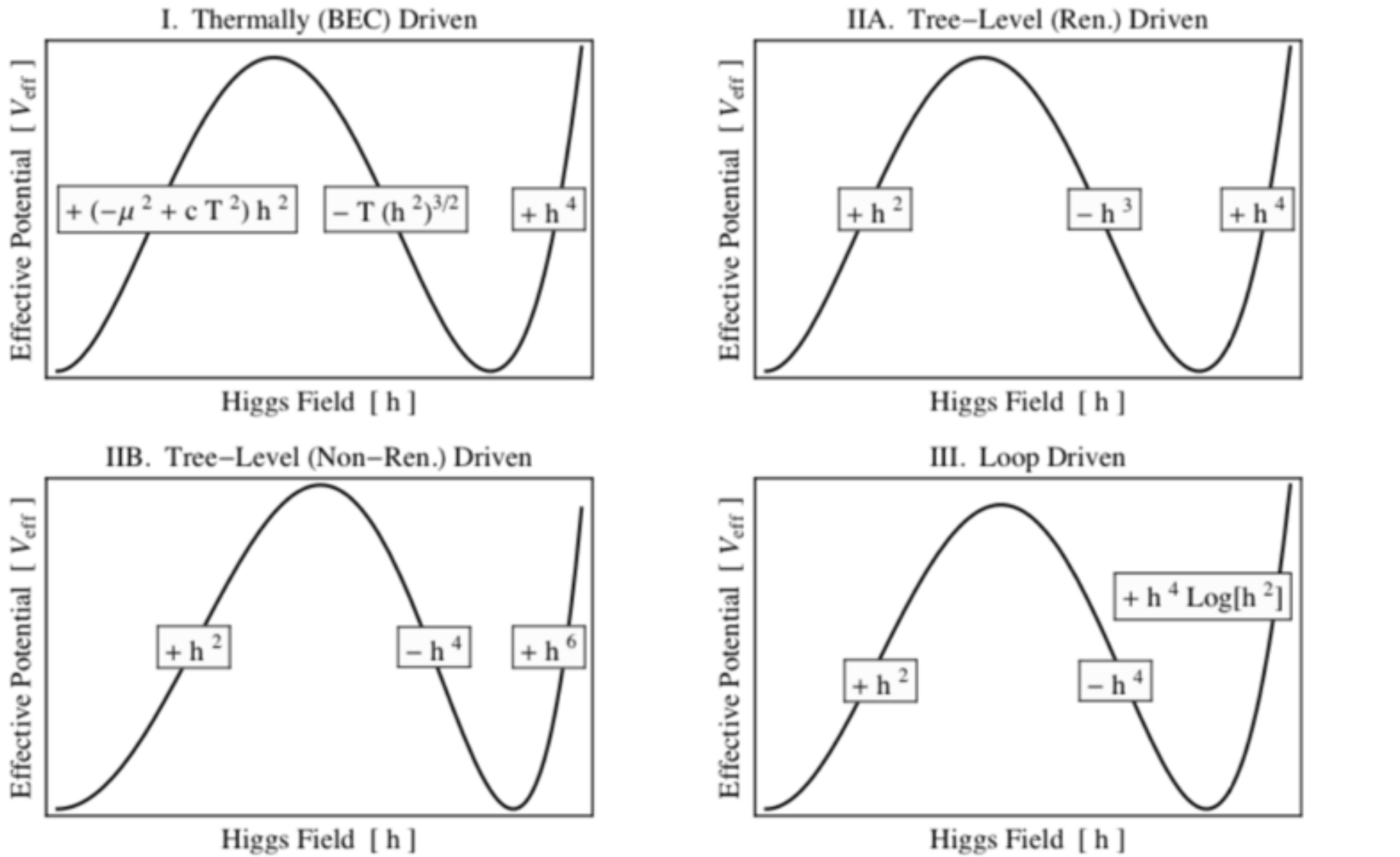




Grojean, Servant, Wells 05, P. Huang, Jokelar, Li, Wagner **(2015)** F.P. Huang, Gu, Yin, Yu, Zhang **(2015)** F.P. Huang, Wan, Wang, Cai, Zhang **(2016)** Cao, F.P. Huang, Xie, & Zhang **(2017)**

LHC say the quantum fluctuation (quadratic oscillation) around h=v with mh=126 GeV, not sensitive to the specifically potential shape

Model classes for catalyzing a strongly first order electroweak phase transition



Daniel J H. Chung, Andrew J. Long, and Lian-Tao Wang PRD87, 023509 (2013)

BSM for EWPT

SM+Scalar Singlet

Espinosa, Quiros 93, Benson 93, Choi, Volkas 93, Vergara 96, Branco, Delepine, Emmanuel- Costa, Gonzalez 98, Ham, Jeong, Oh 04, Ahriche 07, Espinosa, Quiros 07, Profumo, Ramsey-Musolf, Shaughnessy 07, Noble, Perelstein 07, Espinosa, Konstandin, No, Quiros 08, Barger, Langacker, McCaskey, Ramsey-Musolf, Shaughnessy 09, Ashoorioon, Konstandin 09, Das, Fox, Kumar, Weiner 09, Espinosa, Konstandin, Riva 11, Chung, Long 11, Barger, Chung, Long, Wang 12, Huang, Shu, Zhang 12, Fairbairn, Hogan 13, Katz, Perelstein 14, Profumo, Ramsey-Musolf, Wainwright, Winslow 14, Jiang, Bian, Huang, Shu 15, Kozaczuk 15, Cline, Kainulainen, Tucker-Smith 17, Kurup, Perelstein 17, Chen, Kozaczuk, Lewis 17,Cheng, Bian 17, Bian, Tang 18,Chen, Li, Wu, Bian,19…

SM+Scalar Doublet

Turok, Zadrozny 92, Davies, Froggatt, Jenkins, Moorhouse 94, Cline, Lemieux 97, Huber 06, Froome, Huber, Seniuch 06, Cline, Kainulainen, Trott 11, Dorsch, Huber, No 13, Dorsch, Huber, Mimasu, No 14, Basler, Krause, Muhlleitner, Wittbrodt, Wlotzka 16, Dorsch, Huber, Mimasu, No 17, Bernon, Bian, Jiang 17, Bian, Liu 18,…

SM + Scalar Triplet

Profumo, Ramsey-Musolf 12, Chiang 14, Zhou, Cheng, Deng, Bian, Wu 18,Zhou, Bian, Guo,Wu 19,…

NMSSM

Pietroni 93, Davies, Froggatt, Moorhouse 95, Huber, Schmidt 01, Ham, Oh, Kim, Yoo, Son 04, Menon, Morrissey, Wagner 04, Funakubo, Tao, Yokoda 05, Huber, Konstandin, Prokopec, Schmidt 07, Chung, Long 10, Kozaczuk, Profumo, Stephenson Haskins, Wainwright 15, Bi, Bian, Huang, Shu, Yin 15, Bian, Guo, Shu 17,…

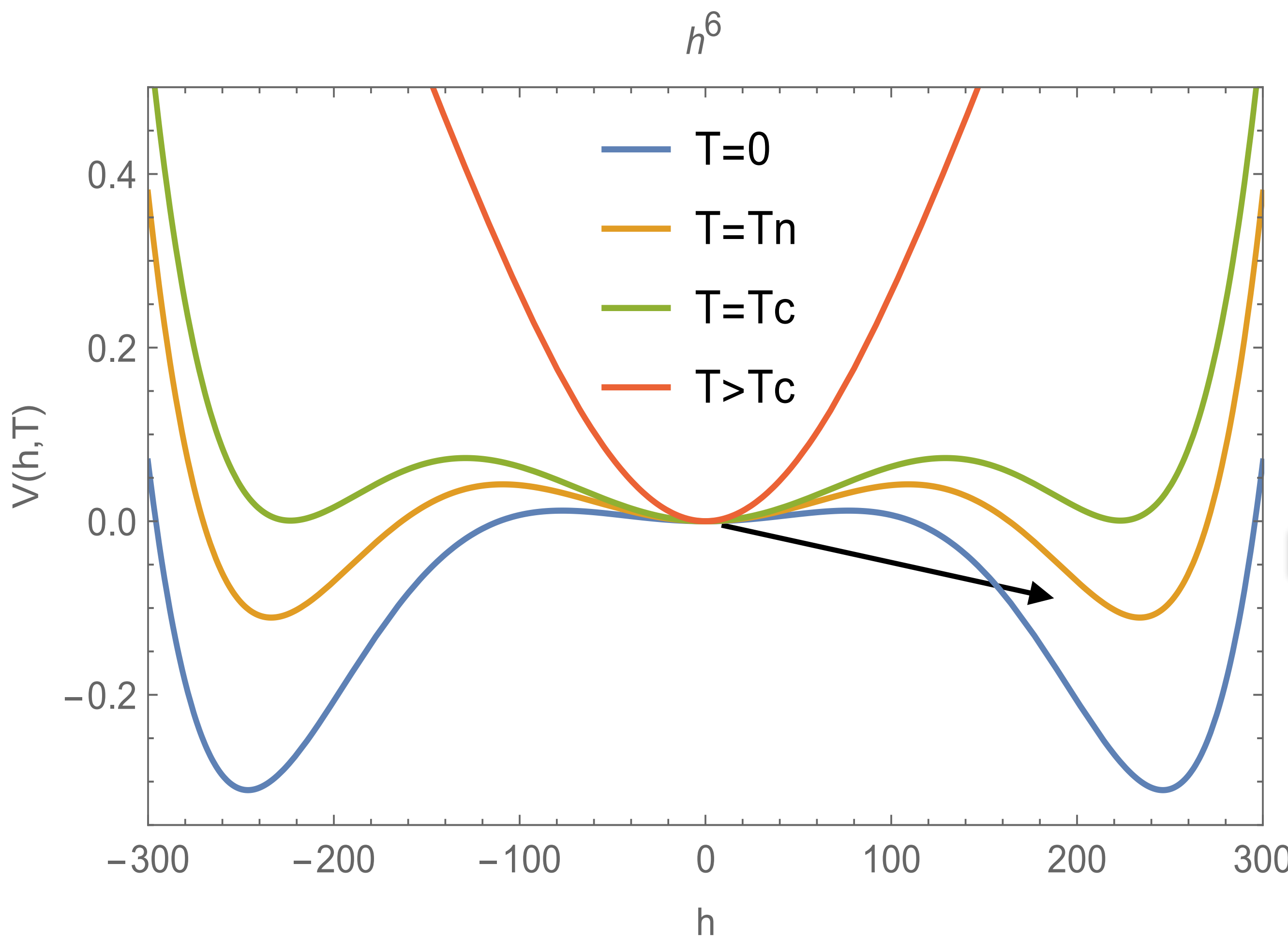
Composite Higgs

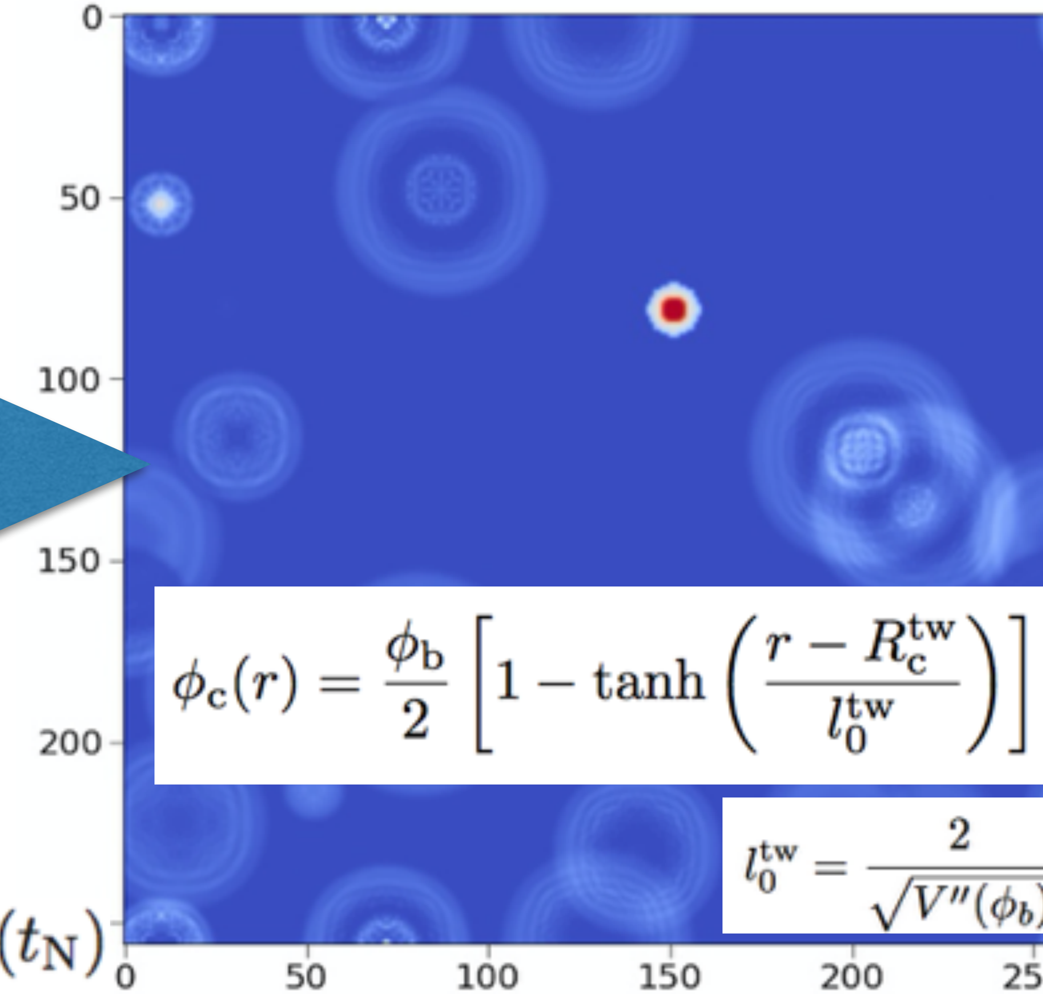
Espinosa, Gripaios, Konstandin, Riva 11, Bruggisser, Von Harling, Matsedonskyi, Servant 18, Bian,Wu,Xie 19, De Curtis, Delle Rose, Panico 19, Bian,Wu,Xie 20,…

EFT

Grojean, Servant, Wells 05, Bodeker, Froome, Huber, Seniuch 05, Huang, Joglekar, Li, Wagner 15,Cai,Sasaki ,Wang17,Zhou, Bian, Guo 19, …

Higgs Potential Shape and the Bubble picture





$$\phi_c(r) = \frac{\phi_b}{2}\left[1 - \tanh\left(\frac{r - R_c^{tw}}{l_0^{tw}}\right)\right]$$

$$l_0^{tw} = \frac{2}{\sqrt{V''(\phi_b)}}$$

$$V = \mu^2 H^\dagger H - \lambda_h (H^\dagger H)^2$$

$$+ \quad (H^\dagger H)^3 / \Lambda^2$$

$$\alpha = \frac{\Delta\rho}{\rho_R} = \frac{1}{\rho_R}\left[-V(\vec{\phi}_b, T) + T\frac{\partial V(\vec{\phi}_b, T)}{\partial T}\right]\Bigg|_{T=T_n}$$

$$p(t_N) = H^4(t_N)$$

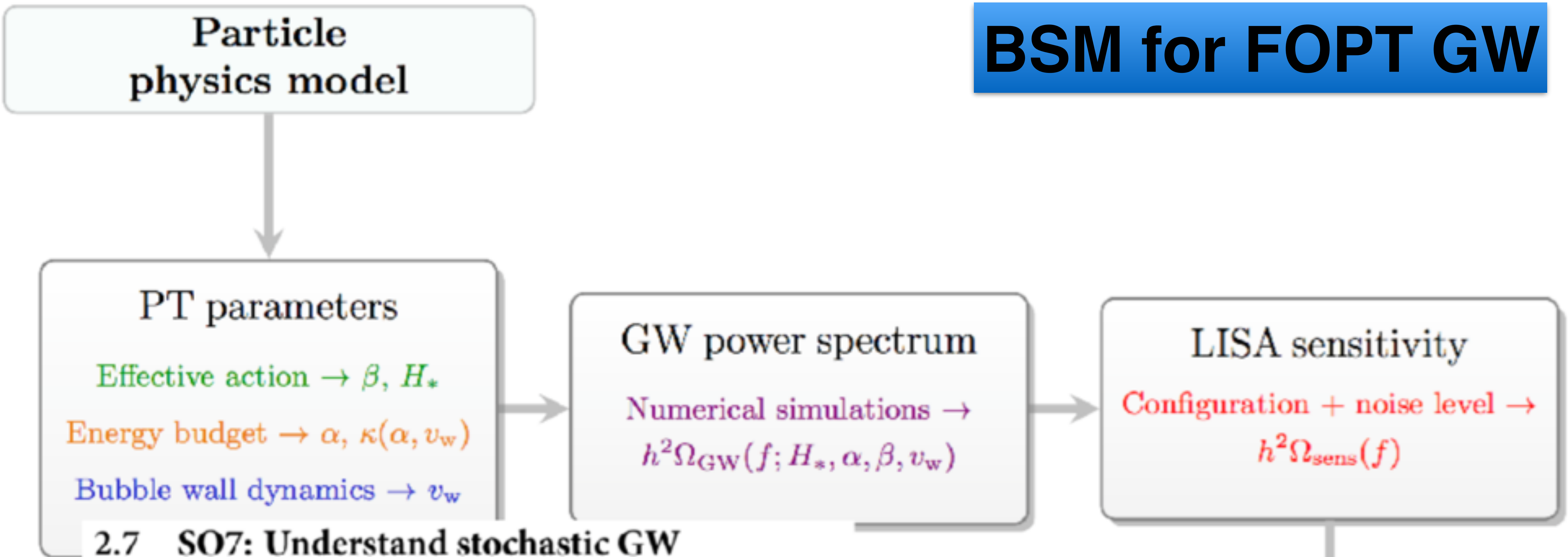
$$p(t) \simeq \Gamma_0 e^{-S(t_N)+\beta(t-t_N)}$$

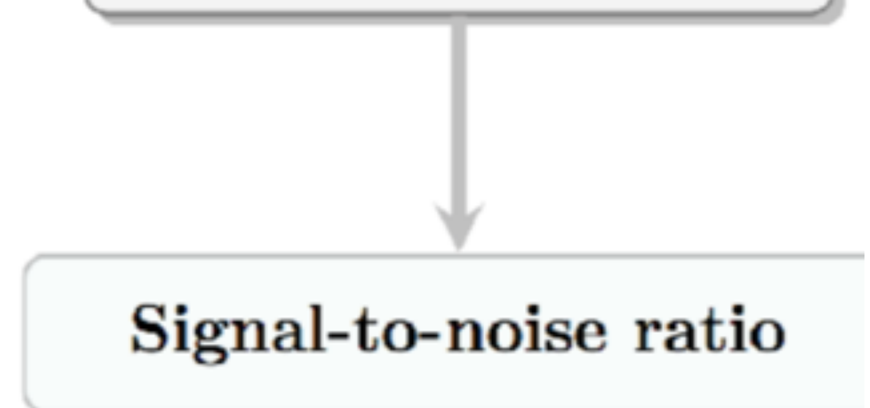
$\Gamma 0 \sim \alpha^5{}_W\, Tc^4$ $\quad S(t_N) \sim 4\ln(m_P/T_N)$

$$\beta = -\, d\ln p(t)/dt\big|_{t_f}$$

$$\frac{\beta}{H_*} \simeq \frac{2S(t_N)}{(1 - T_N/T_c)}$$

BSM for FOPT GW





2.7 SO7: Understand stochastic GW backgrounds and their implications for the early Universe and TeV-scale particle physics

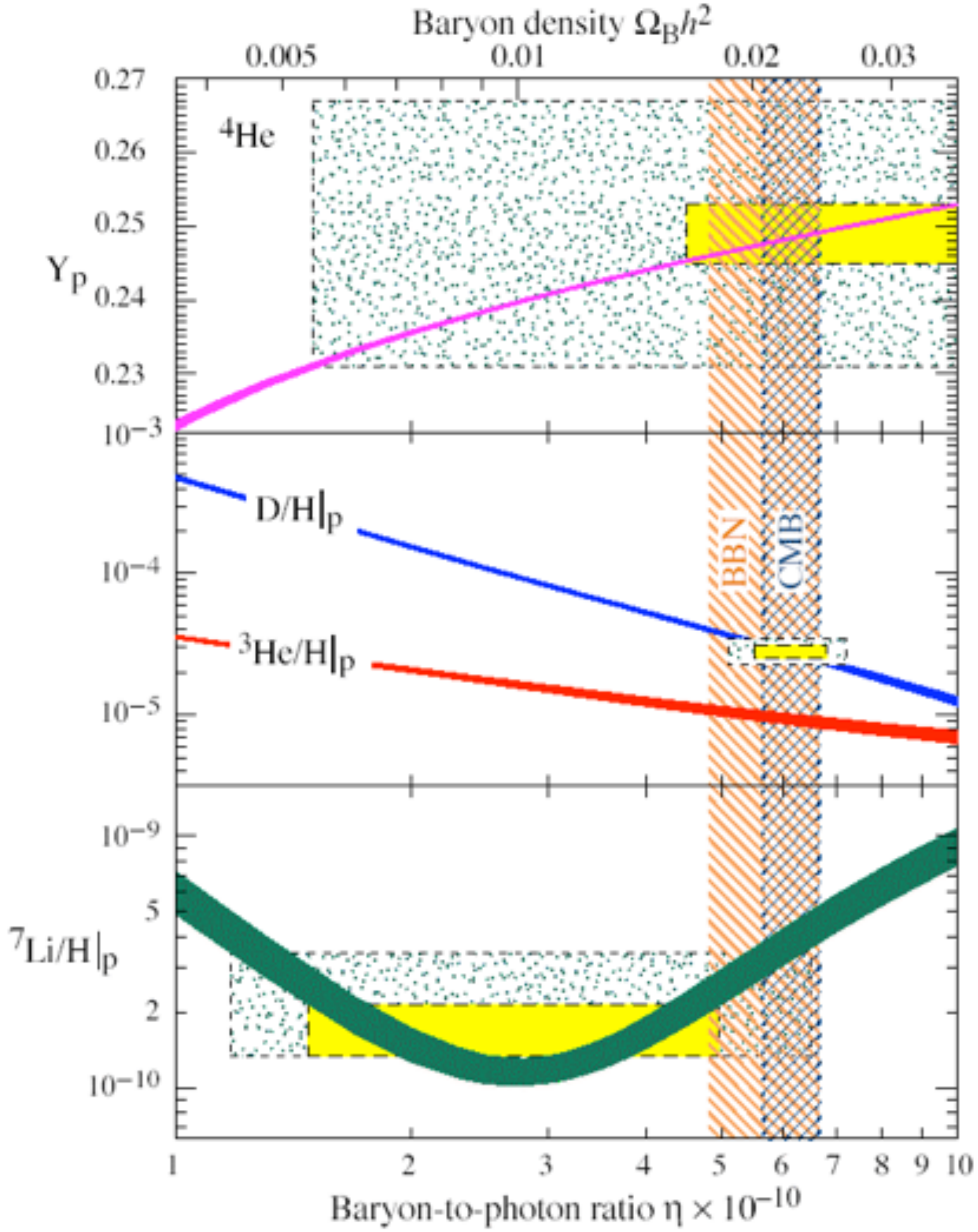
One of the LISA goals is the direct detection of a stochastic GW background of cosmological origin (like for example the one produced by a first-order phase transition around the TeV scale) and stochastic foregrounds. Probing a stochastic GW background of cosmological origin provides information on new physics in the early Universe. The shape of the signal gives an indication of its origin, while an upper limit allows to constrain models of the early Universe and particle physics beyond the standard model.

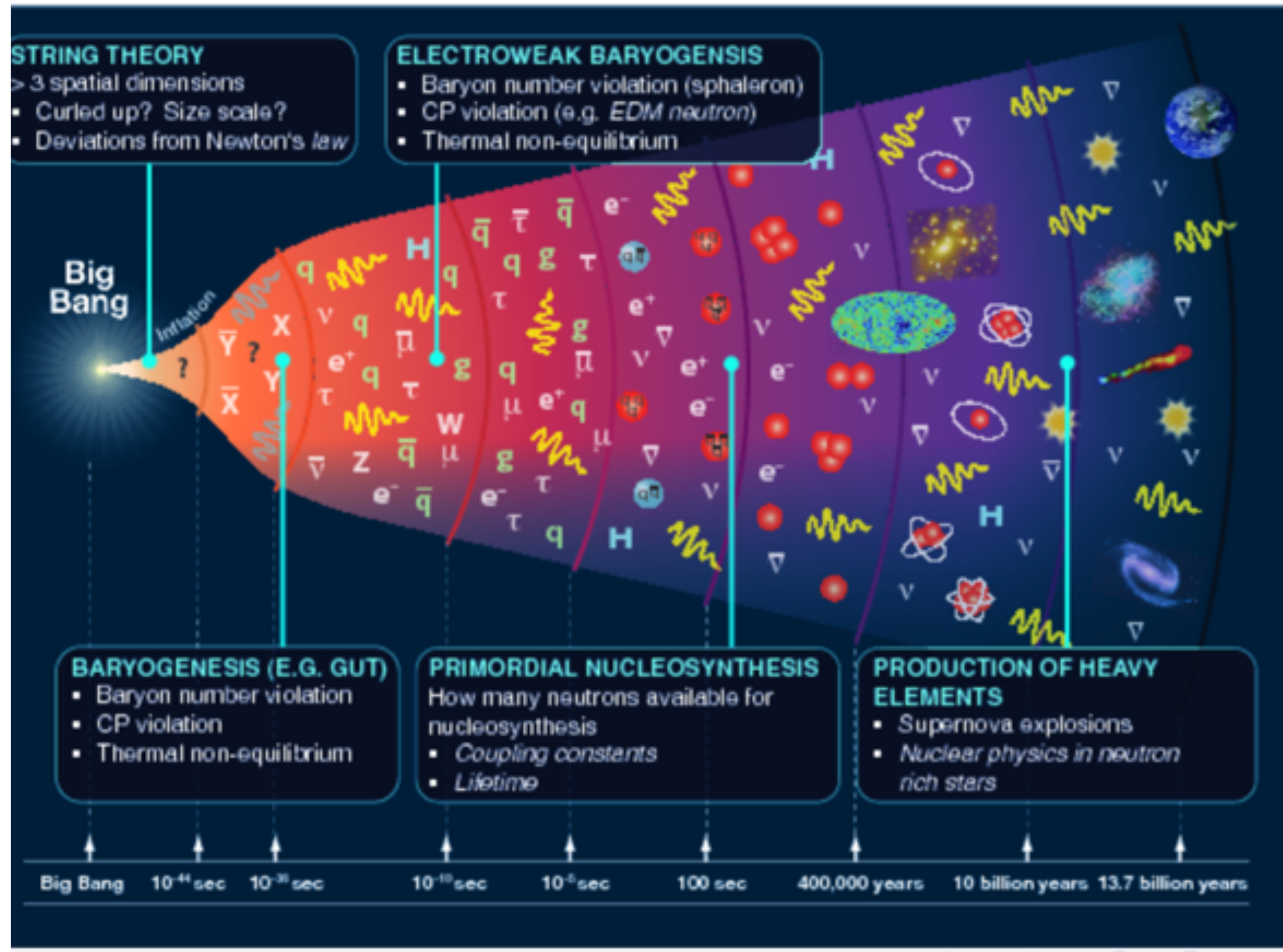
1702.00786

$$\mathrm{SNR} = \sqrt{\mathcal{T} \int_{f_{\min}}^{f_{\max}} \mathrm{d}f \left[\frac{h^2\Omega_{\mathrm{GW}}(f)}{h^2\Omega_{\mathrm{Sens}}(f)} \right]^2}$$

JCAP03(2020)024

BAU



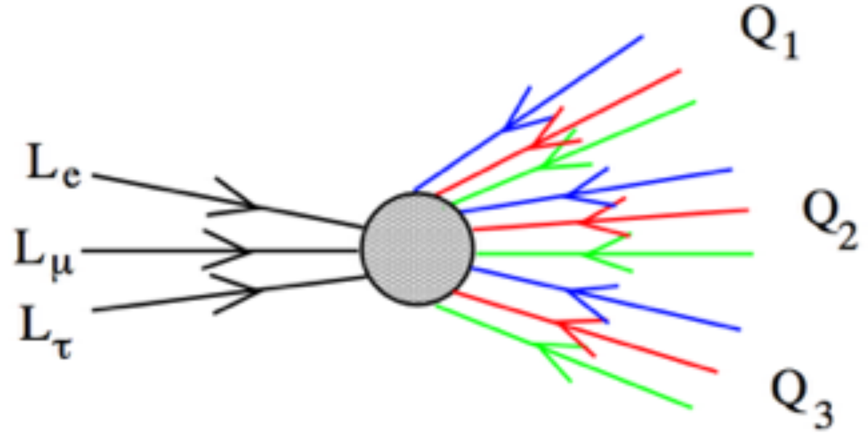


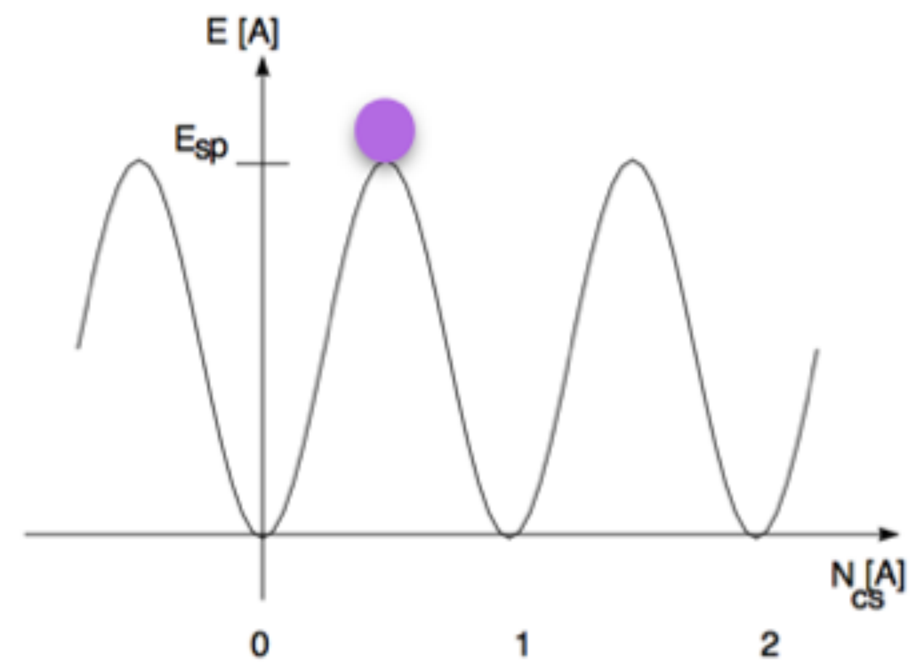
more baryons than anti-baryons (BBN & CMB, etc)

$$\frac{n_b}{s} \approx (0.7 - 0.9) \times 10^{-10} \neq 0$$

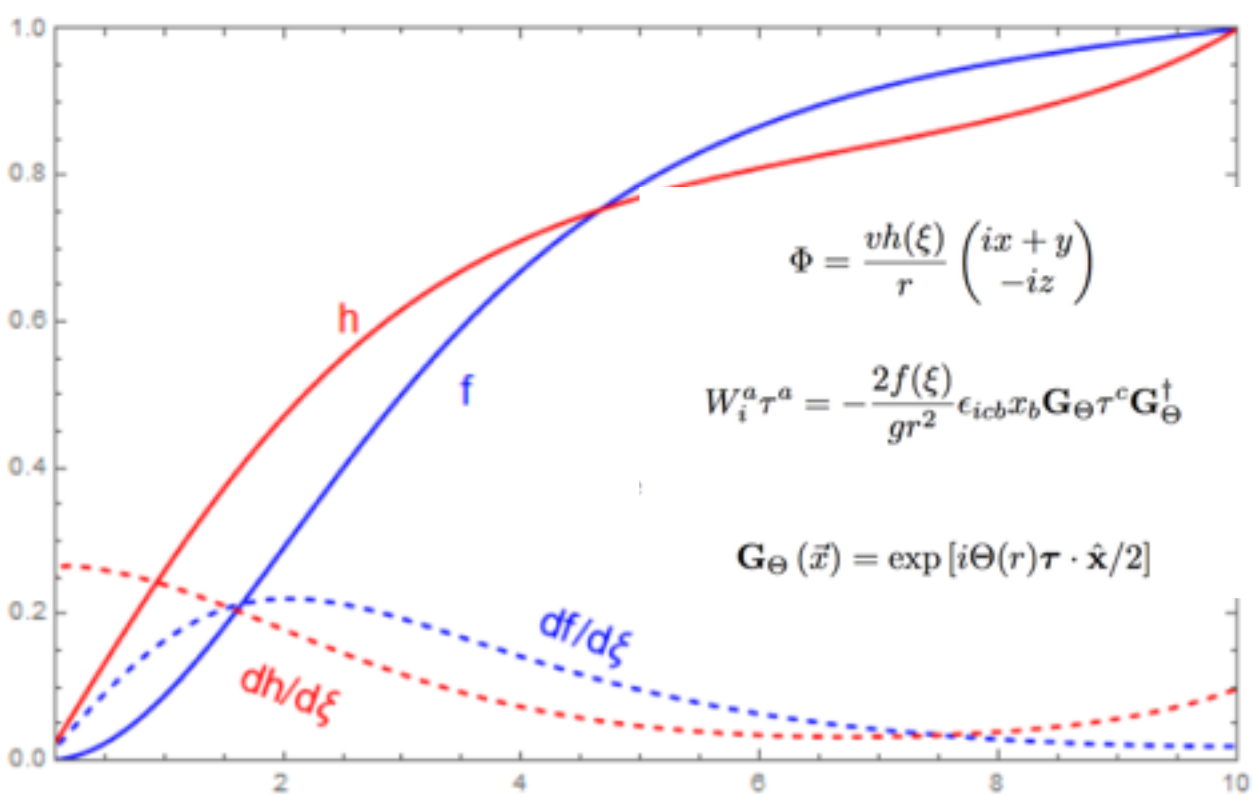
B violation and sphaleron

The Standard Model already contains a process that violates B-number. It
known as the electroweak sphaleron (“sphaleros” is Greek for “ready to fal





Klinkhammer & Manton (1984); Kuzmin, Rubakov, & Shaposhnikov (1985); Harvey & Tu...
but also identified earlier by Dashen, Hasslacher, & Neveu (1974) and Boguta (1983)



$$\Phi = \frac{vh(\xi)}{r}\begin{pmatrix} ix+y \\ -iz \end{pmatrix}$$

$$W_i^a\tau^a = -\frac{2f(\xi)}{gr^2}\epsilon_{icb}x_b\mathbf{G}_\Theta\tau^c\mathbf{G}_\Theta^\dagger$$

$$\mathbf{G}_\Theta(\vec{x}) = \exp\left[i\Theta(r)\boldsymbol{\tau}\cdot\hat{\mathbf{x}}/2\right]$$

ξ =gvr

$$\partial_\mu J_B^\mu = i\frac{N_F}{32\pi^2}\left(-g_2^2 F^{a\mu\nu}\widetilde{F}_{\mu\nu}^a + g_1^2 f^{\mu\nu}\widetilde{f}_{\mu\nu}\right),$$

$$\Delta B = N_F(\Delta N_{CS} - \Delta n_{CS}),$$

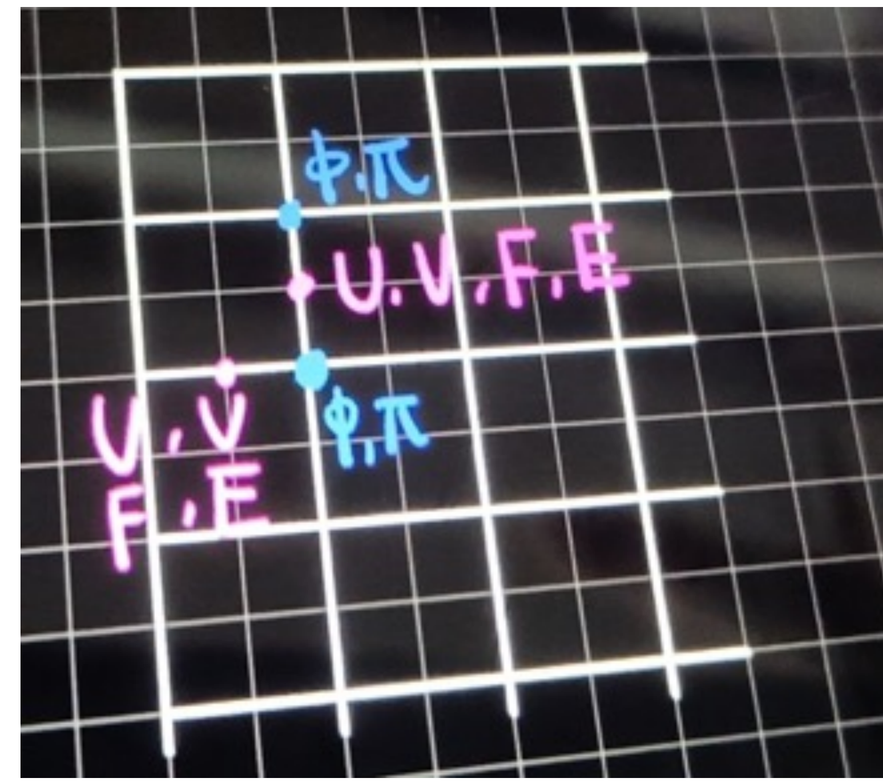
$$N_{CS} = -\frac{g_2^2}{16\pi^2}\int d^3x\, 2\epsilon^{ijk}\,\mathrm{Tr}\left[\partial_i A_j A_k + i\frac{2}{3}g_2 A_i A_j A_k\right]$$

$$n_{CS} = -\frac{g_1^2}{16\pi^2}\int d^3x\, \epsilon^{ijk}\,\partial_i B_j B_k,$$

Lattice EW field foundation

$\Phi(t, x)$: Higgs field doublet defined on sites; $U_i(t, x)$ and $V_i(t, x)$: SU(2) and U(1) link fields, defined on the link between the neighboring sites x and x + i , $\Phi(t, x)$, $U_i(t, x)$ and $V_i(t, x)$ are defined at time steps t + Δt, t + 2Δt, . . .; conjugate momentum fields, $\Pi(t+\Delta t/2, x)$, $F(t+\Delta t/2, x)$ and $E(t+\Delta t/2, x)$, are defined at time steps t + Δt/2, t + 3Δt/2.

leapfrog



$$U_i(t,x) = \exp\left(-\frac{i}{2} g \Delta x \sigma^a W_i^a\right) \qquad D_i\Phi = \frac{1}{\Delta x}\left[U_i(t,x)V_i(t,x)\Phi(t,x+i) - \Phi(t,x)\right]$$

$$U_0(t,x) = \exp\left(-\frac{i}{2} g \Delta t \sigma^a W_0^a\right) \qquad D_0\Phi = \frac{1}{\Delta t}\left[U_0(t,x)V_0(t,x)\Phi(t+\Delta t,x) - \Phi(t,x)\right].$$

$$V_i(t,x) = \exp\left(-\frac{i}{2} g \Delta x B_i\right)$$

$$V_0(t,x) = \exp\left(-\frac{i}{2} g \Delta t B_0\right).$$

$$\Phi(t+\Delta t, x) = \Phi(t,x) + \Delta t \Pi(t+\Delta t/2, x)$$

$$V_i(t+\Delta t, x) = \frac{1}{2} g' \Delta x \Delta t E_i(t+\Delta t/2, x) V_i(t,x)$$

$$U_i(t+\Delta t, x) = g \Delta x \Delta t F_i(t+\Delta t/2, x) U_i(t,x),$$

$U_0(t, x) = I_2$ and $V_0(t, x) = 1$

Temporal gauge

$$\Pi(t+\Delta t/2, x) = \Pi(t-\Delta t/2, x) + \Delta t\Big\{\frac{1}{\Delta x^2}\sum_i \big[U_i(t,x)V_i(t,x)\Phi(t,x+i) - 2\Phi(t,x) + U_i^\dagger(t,x-i)V_i^\dagger(t,x-i)\Phi(t,x-i)\big] - \frac{\partial U}{\partial \Phi^\dagger}\Big\}$$

$$\mathrm{Im}[E_k(t+\Delta t/2, x)] = \mathrm{Im}[E_k(t-\Delta t/2, x)] + \Delta t\Big\{\frac{g'}{\Delta x}\mathrm{Im}[\Phi^\dagger(t,x+k)U_k^\dagger(t,x)V_k^\dagger(t,x)\Phi(t,x)] - \frac{2}{g'\Delta x^3}\sum_i \mathrm{Im}[V_k(t,x)V_i(t,x+k)V_k^\dagger(t,x+i)V_i^\dagger(t,x) + V_i(t,x-i)V_k(t,x)V_i^\dagger(t,x+k-i)V_k^\dagger(t,x-i)]\Big\}$$

$$\mathrm{Tr}[i\sigma^m F_k(t+\Delta t/2, x)] = \mathrm{Tr}[i\sigma^m F_k(t-\Delta t/2, x)] + \Delta t\Big\{\frac{g}{\Delta x}\mathrm{Re}[\Phi^\dagger(t,x+k)U_k^\dagger(t,x)V_k^\dagger(t,x)i\sigma^m\Phi(t,x)] - \frac{1}{g\Delta x^3}\sum_i \mathrm{Tr}[i\sigma^m U_k(t,x)U_i(t,x+k)U_k^\dagger(t,x+i)U_i^\dagger(t,x) + i\sigma^m U_k(t,x)U_i^\dagger(t,x+k-i)U_k^\dagger(t,x-i)U_i(t,x-i)]\Big\},$$

Bubble with sphaleron

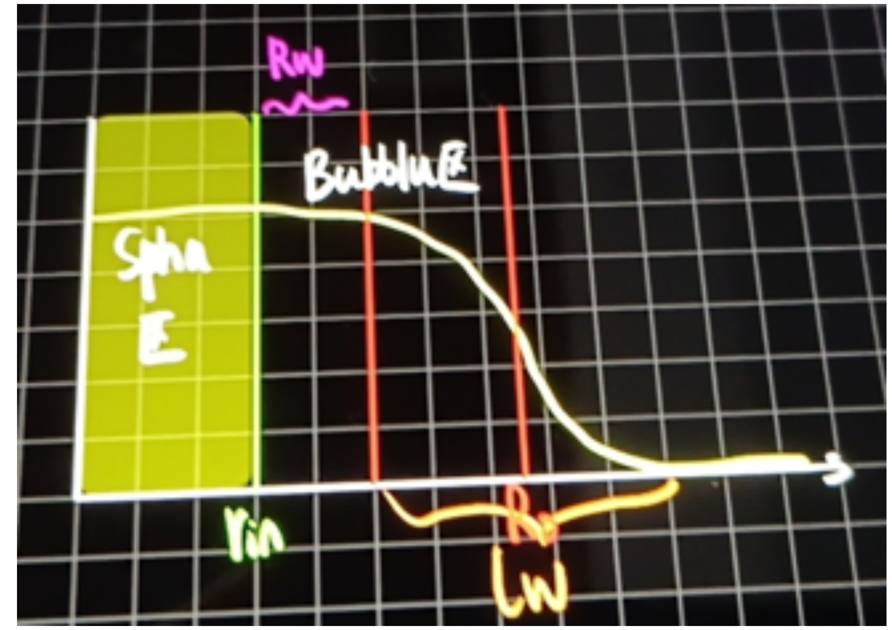
$$\partial_0^2 \Phi = D_i D_i \Phi - 2\lambda(|\Phi|^2 - \eta^2)\Phi - 3(\Phi^\dagger \Phi)^2 \Phi/\Lambda^2,$$

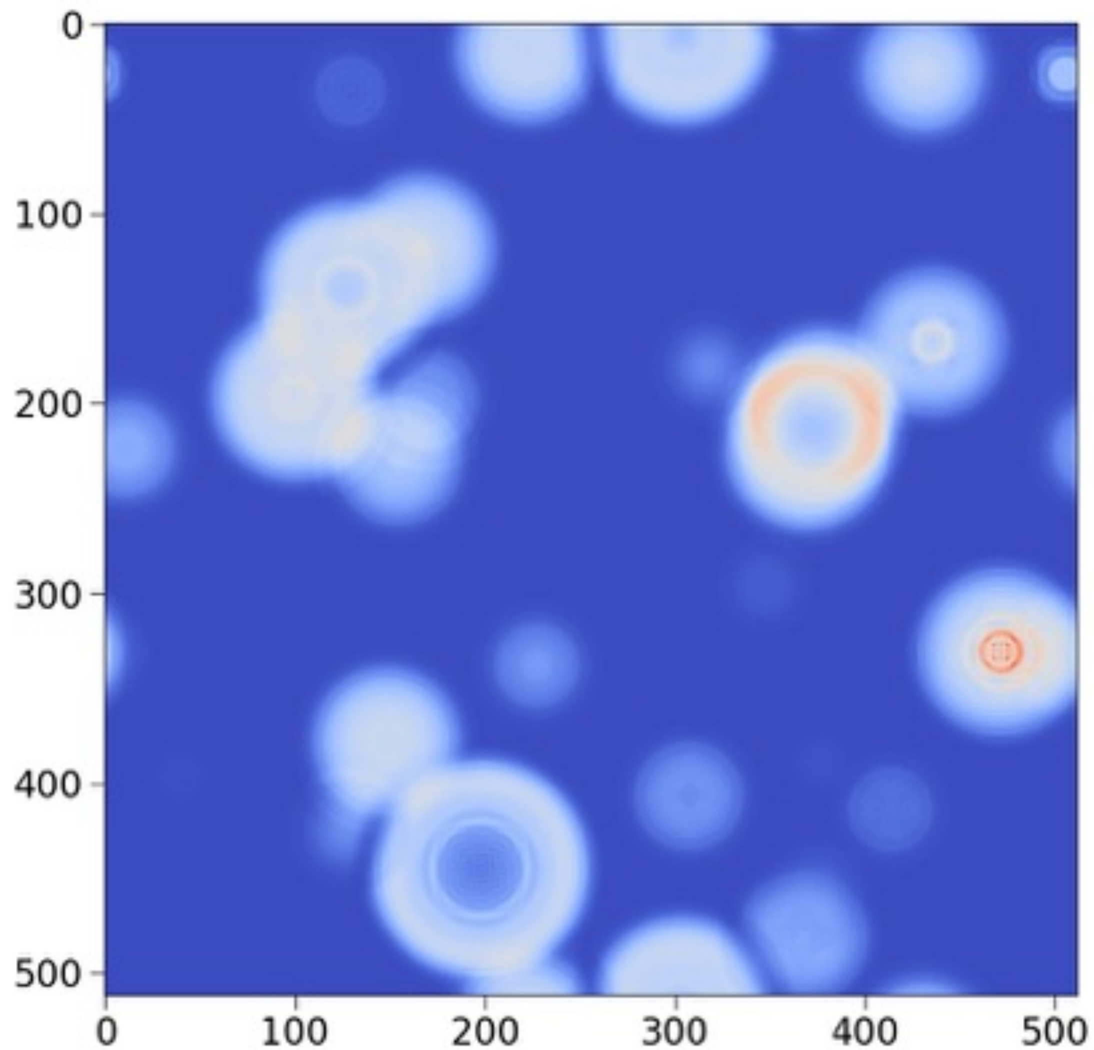
$$\partial_0^2 B_i = -\partial_j B_{ij} + g' \, \mathrm{Im}[\Phi^\dagger D_i \Phi],$$

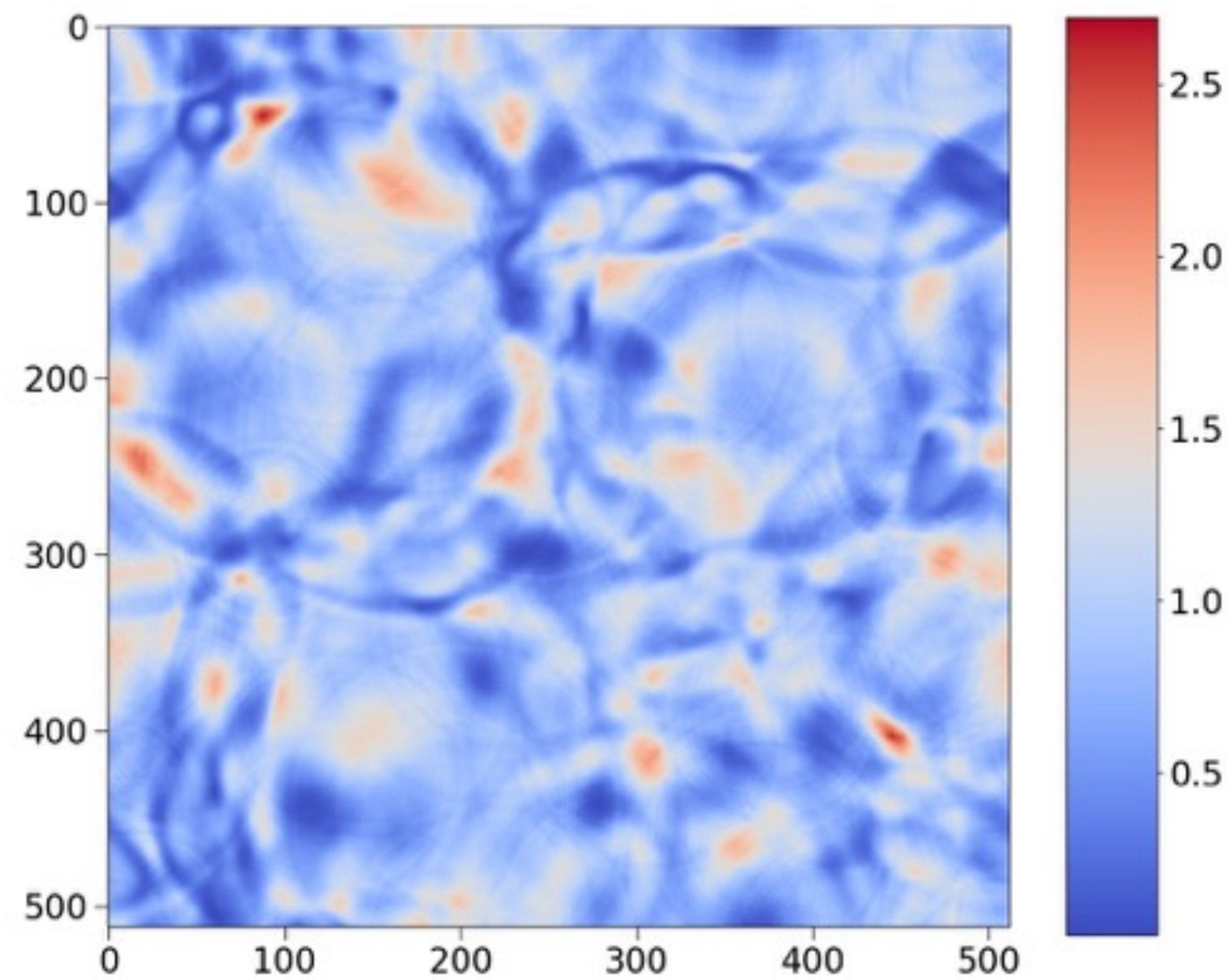
$$\partial_0^2 W_i^a = -\partial_k W_{ik}^a - g\,\epsilon^{abc} W_k^b W_{ik}^c + g\,\mathrm{Im}[\Phi^\dagger \sigma^a D_i \Phi].$$

$$\partial_0 \partial_j B_j - g' \, \mathrm{Im}[\Phi^\dagger \partial_0 \Phi] = 0,$$

$$\partial_0 \partial_j W_j^a + g\,\epsilon^{abc} W_j^b \partial_0 W_j^c - g\,\mathrm{Im}[\Phi^\dagger \sigma^a \partial_0 \Phi] = 0.$$







2012.15625

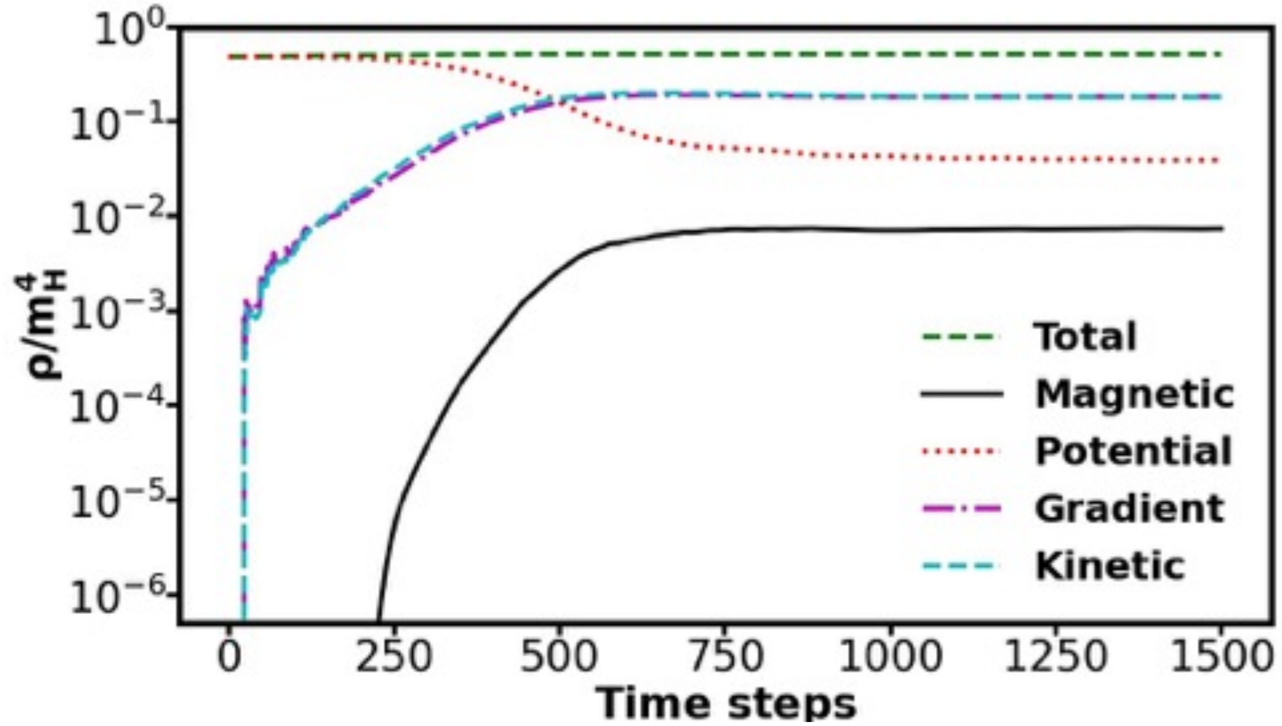
GW from Bubble collisions

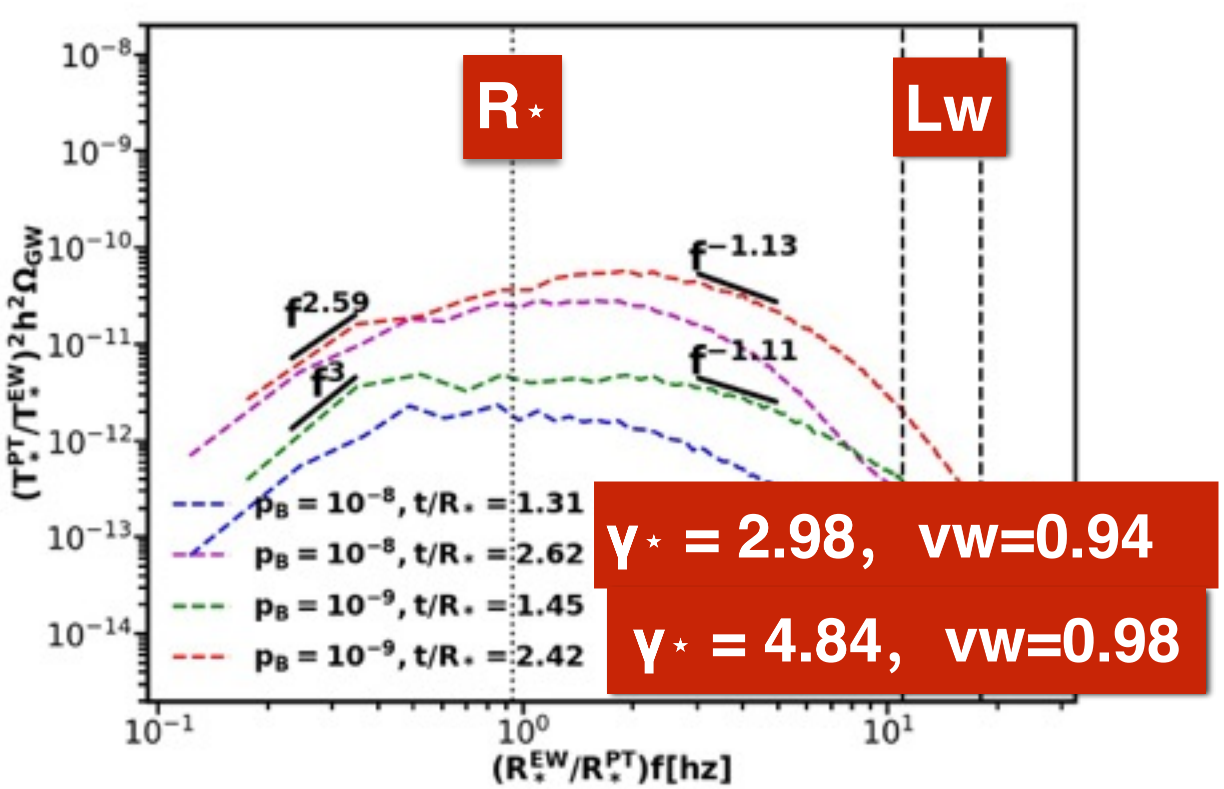
$$T_{\mu\nu} = \partial_\mu \Phi^\dagger \partial_\nu \Phi - g_{\mu\nu}\frac{1}{2}\mathrm{Re}[(\partial_i \Phi^\dagger \partial^i \Phi)^2]$$

$$\ddot{h}_{ij} - \nabla^2 h_{ij} = 16\pi G T_{ij}^{\mathrm{TT}}$$

$$\langle \dot{h}_{ij}^{TT}(\mathbf{k},t)\dot{h}_{ij}^{TT}(\mathbf{k}',t)\rangle = P_{\dot{h}}(\mathbf{k},t)(2\pi)^3\delta(\mathbf{k}+\mathbf{k}')$$

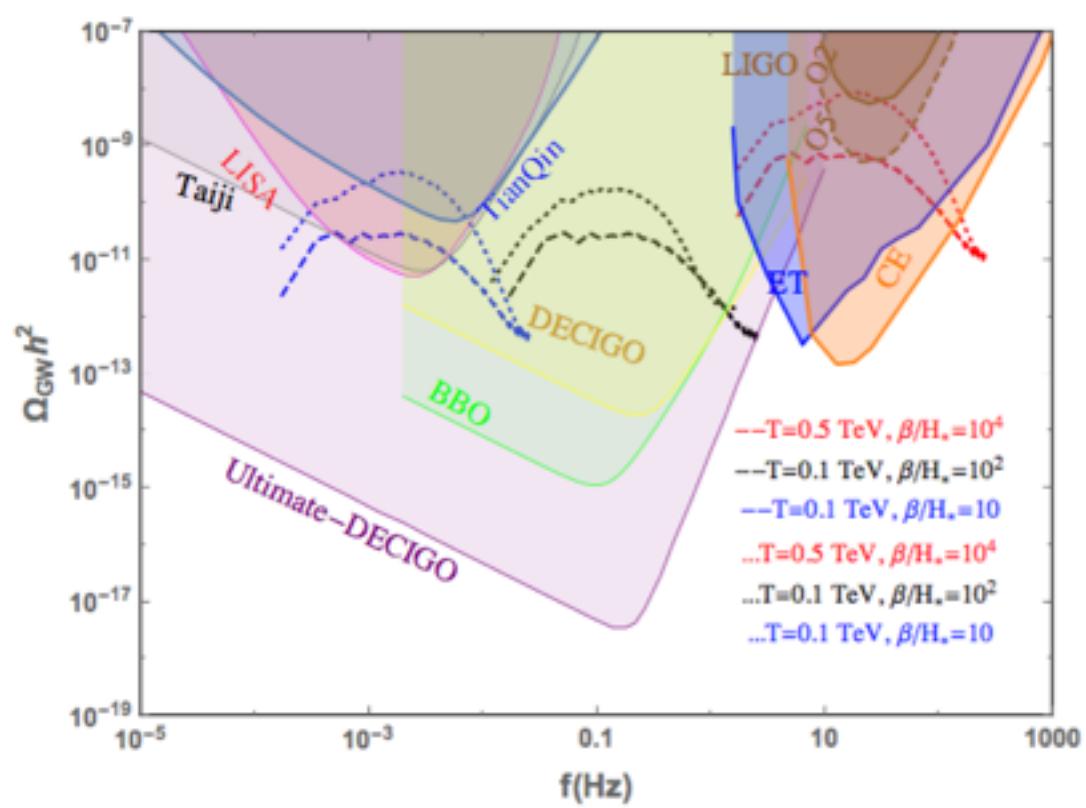
$$\frac{d\Omega_{\mathrm{gw}}}{d\ln(k)} = \frac{1}{32\pi G\rho_c}\frac{k^3}{2\pi^2}P_{\dot{h}}(\mathbf{k},t)$$





γ⋆ = 2.98, vw=0.94

γ⋆ = 4.84, vw=0.98

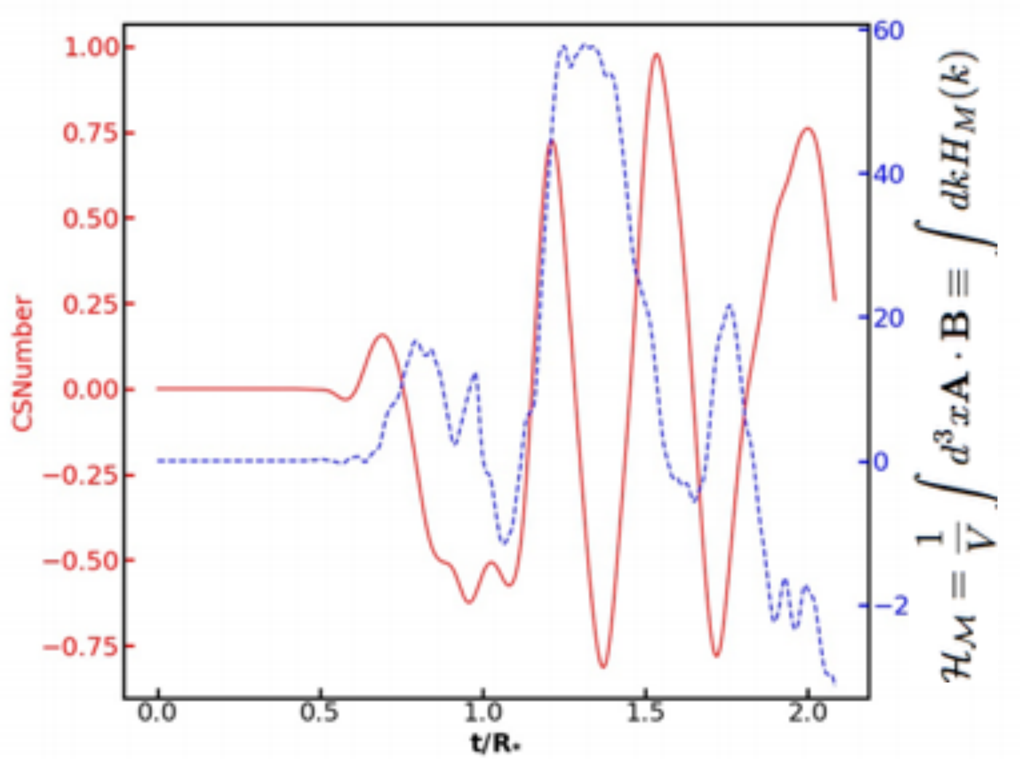


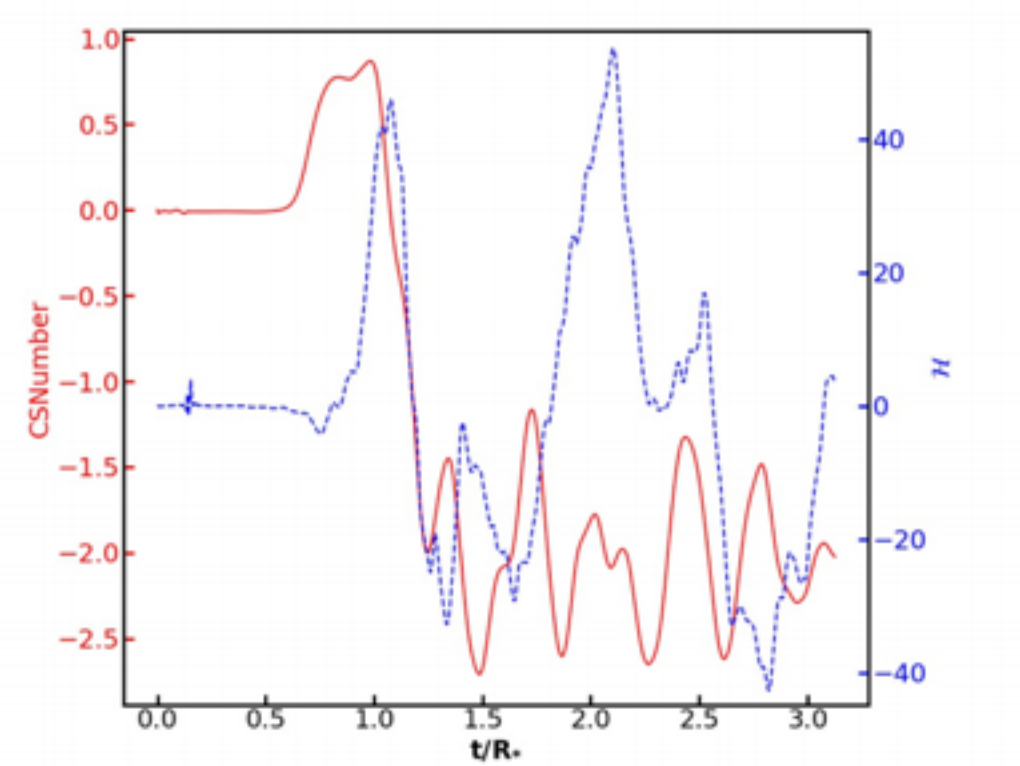
γ⋆ = R⋆ /(2Rc)

$$R_* = \left(\frac{\mathcal{V}}{N_b}\right)^{1/3}$$

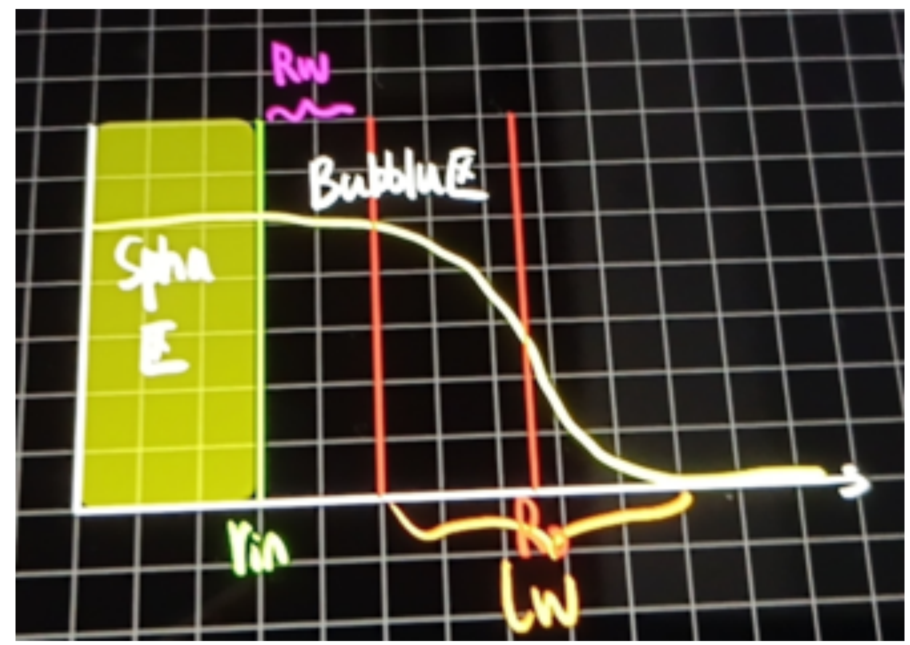
2012.15625

CS number and the magnetic helicity





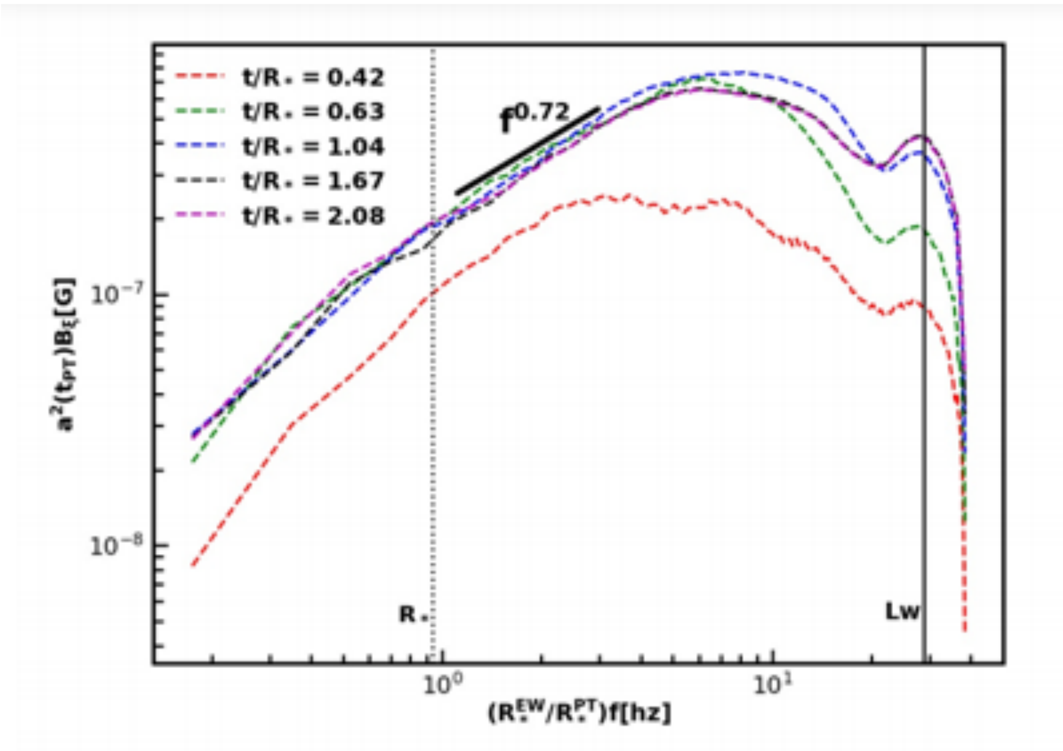
Without sphaleron, the CS number oscillates around zero throughout the PT process

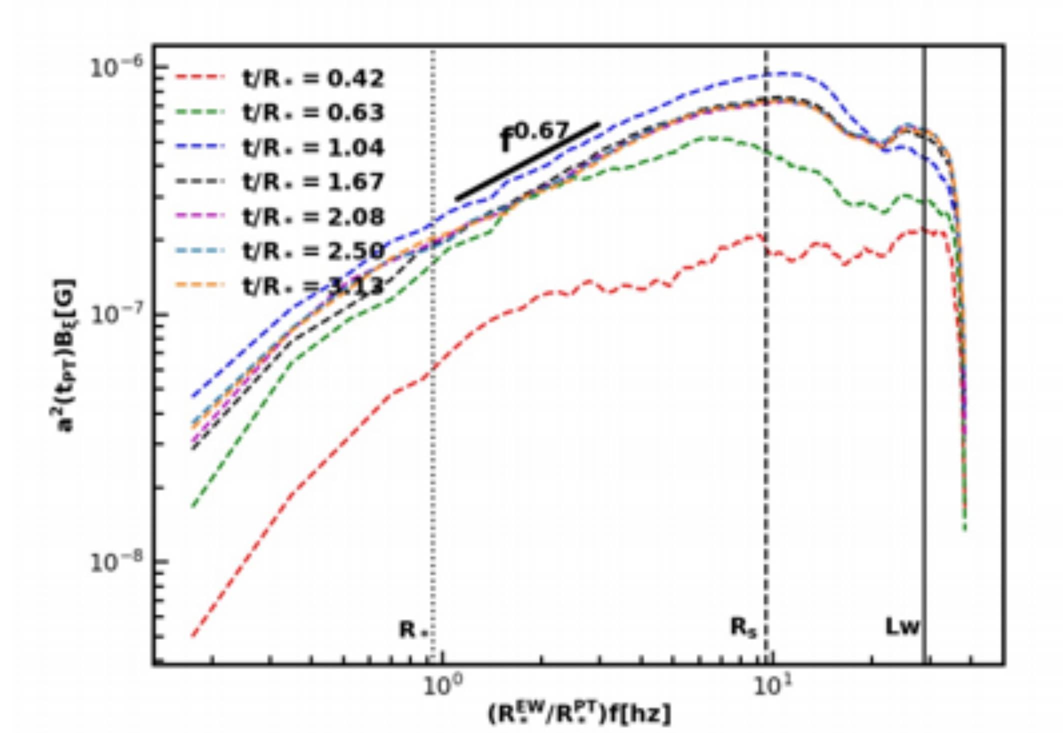


B + L anomaly: ΔNB = 3 ΔNCS~3

Magnetic helicity and NCS: |H|~18ΔNCS(~ 6ΔNB)

MF versus Sphaleron





Non-vanishing gradients of the Higgs fields can generate magnetic fields when the bubbles collide

$$A_{\mu\nu} = \sin\theta_w n^a W^a_{\mu\nu} + \cos\theta_w B_{\mu\nu} - i\frac{2}{gv^2}\sin\theta_w \left[(D_\mu\Phi)^\dagger(D_\nu\Phi) - (D_\nu\Phi)^\dagger(D_\mu\Phi)\right].$$

$$\langle B_i^*(\boldsymbol{k},t)B_j(\boldsymbol{k}',t)\rangle = (2\pi)^3\delta^{(3)}(\boldsymbol{k}-\boldsymbol{k}')F_{ij}(\boldsymbol{k},t)$$

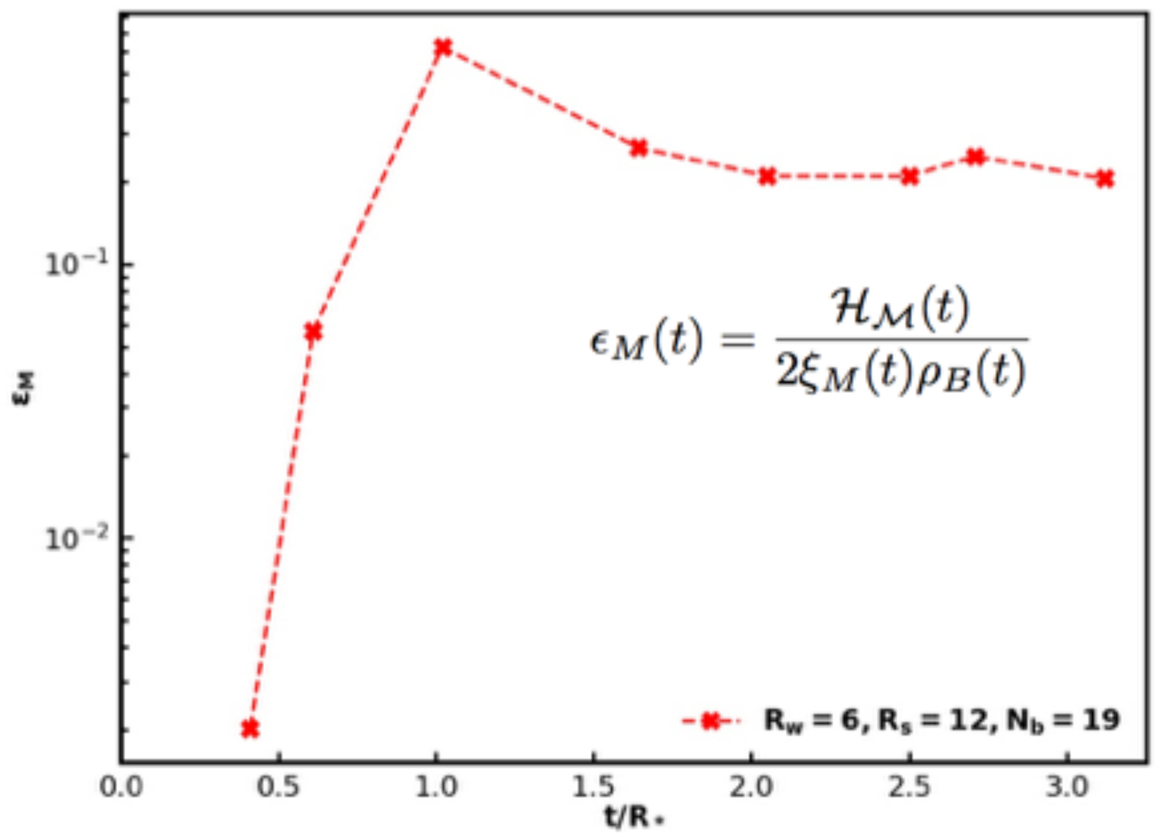
$$F_{ij}(\boldsymbol{k},t) = (\delta_{ij} - \hat{k}_i\hat{k}_j)E_M(k,t)/(4\pi k^2) + i\epsilon_{ijl}k_l H_M/(8\pi k^2)$$

$$\rho_B(t) = \int_0^\infty E_M(k,t)\,dk$$

$$B_\xi = \sqrt{2d\rho_B/d\log(k)} \qquad \xi_M(t) = \int dk k^{-1} E_M(k,t)/\rho_B(t)$$

2107.08978

MF versus Sphaleron

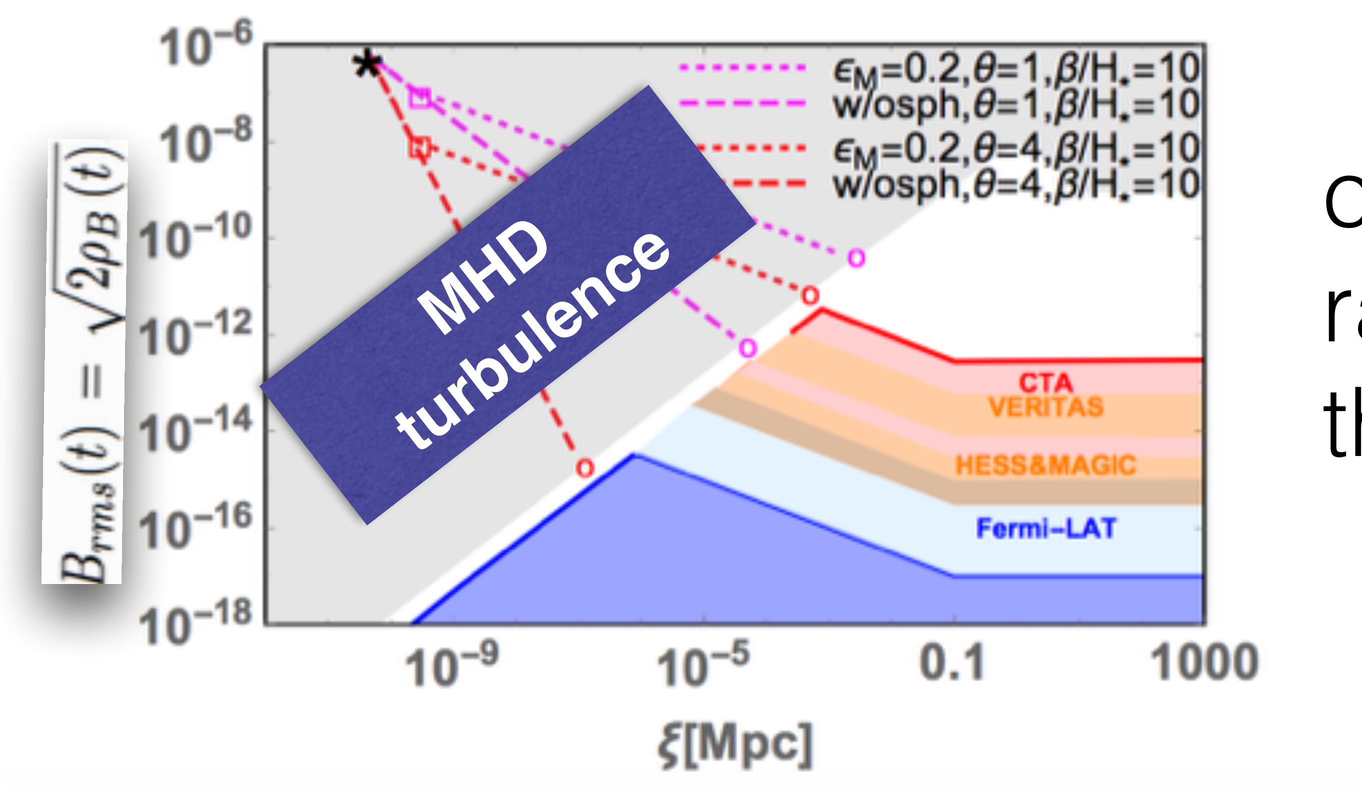


MF helicity evolution with PT proceeding

$$\Phi = \frac{vh(\xi)}{r}\begin{pmatrix} ix + y \\ -iz \end{pmatrix}$$

$$W_i^a \tau^a = -\frac{2f(\xi)}{gr^2}\epsilon_{icb}x_b \mathbf{G}_\Theta \tau^c \mathbf{G}_\Theta^\dagger$$

$$\mathbf{G}_\Theta(\vec{x}) = \exp\left[i\Theta(r)\boldsymbol{\tau}\cdot\hat{\mathbf{x}}/2\right]$$



cosmic-ray and gamma ray observations can tell the helicity of the MF

Summary and future

Observation of the cosmic Magnetic field seeded by phase transition, with GW production may hint the B+L violation

Interaction between the bubble wall and Plasma, and interaction among different bubbles

1) Magnetic field feedback to the phase transition
2) Baryogenesis and/or at fast-wall request by the GW

Higgs Potential shape

1) The future collider prospect, with dihiggs, Zh and/or Zhh production

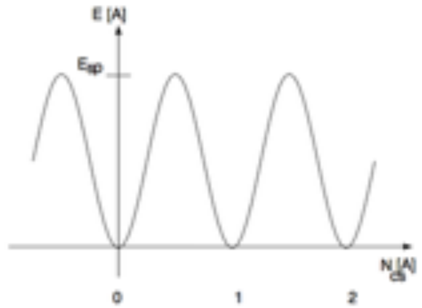
2) Thin wall or thick wall tell by gravitational wave, wall profile and GW spectrum

Thanks

谢谢！

Backup

BNPC, v/T and EW sphaleron



$$\partial_\mu J^\mu_B = i\frac{N_F}{32\pi^2}\left(-g_2^2 F^{a\mu\nu}\widetilde{F}^a_{\mu\nu} + g_1^2 f^{\mu\nu}\widetilde{f}_{\mu\nu}\right),$$

$$\Delta B = N_F(\Delta N_{CS} - \Delta n_{CS}),$$

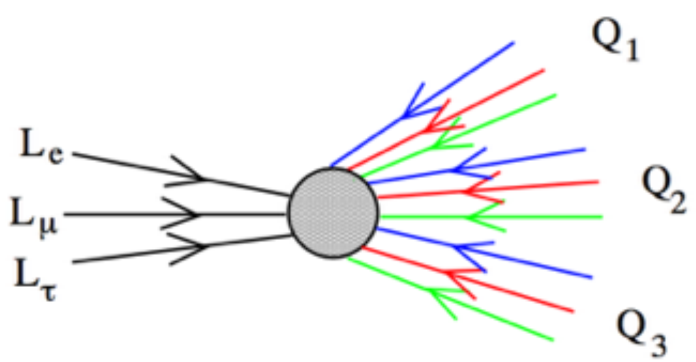
$$N_{CS} = -\frac{g_2^2}{16\pi^2}\int d^3x\, 2\epsilon^{ijk}\,\mathrm{Tr}\left[\partial_i A_j A_k + i\frac{2}{3}g_2 A_i A_j A_k\right],$$

$$n_{CS} = -\frac{g_1^2}{16\pi^2}\int d^3x\, \epsilon^{ijk}\,\partial_i B_j B_k,$$

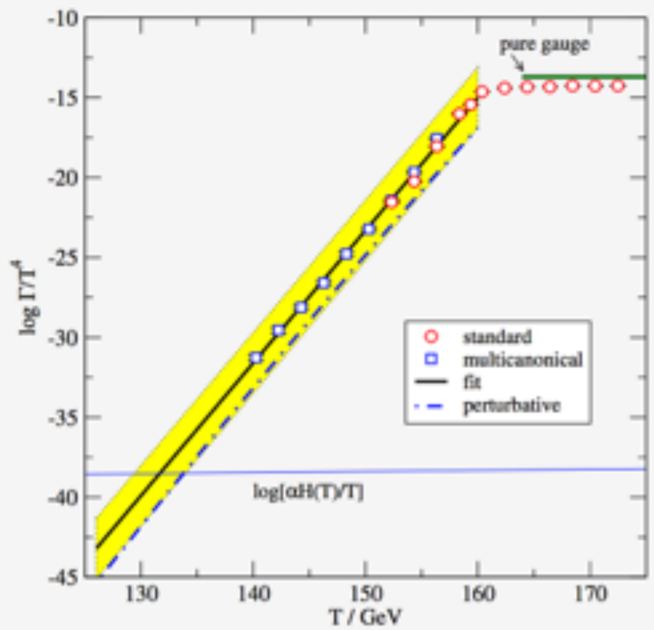
$$A_i \to U A_i U^{-1} + \frac{i}{g_2}(\partial_i U)U^{-1},$$

$$\delta N_{CS} = \frac{1}{24\pi^2}\int d^3x\,\mathrm{Tr}\left[(\partial_i U)U^{-1}(\partial_j U)U^{-1}(\partial_k U)U^{-1}\right]\epsilon^{ijk}.$$

The Standard Model already contains a process that violates B-number. It is known as the electroweak sphaleron ("sphaleros" is Greek for "ready to fall").



Klinkhammer & Manton (1984); Kuzmin, Rubakov, & Shaposhnikov (1985); Harvey & Turner (1990) but also identified earlier by Dashen, Hasslacher, & Neveu (1974) and Boguta (1983)



Lattice result, $T_C = (159.5 \pm 1.5)\mathrm{GeV}$, Phys.Rev.Lett,113, 141602 (2014

$$\Gamma^{\rm sym} \approx 6\times(18\pm3)\alpha_W^5 T^4, \qquad \Gamma^{\rm brok} \sim T^4 \exp\left(-\frac{E_{\rm sph}}{T}\right)$$

Washout avoidance,BNPC

$$\Gamma_{\rm sph} = A_{\rm sph}(T)\exp[-E_{\rm sph}(T)/T] < \mathrm{H(T)}$$

$$PT_{sph} \equiv \frac{E_{\rm sph}(T)}{T} - 7\ln\frac{v(T)}{T} + \ln\frac{T}{100\ \mathrm{GeV}} \qquad PT_{sph} > (35.9 - 42.8)$$

$$E_{\rm sph}(T) \approx E_{\rm sph,0}\frac{v(T)}{v} \qquad \frac{v(T)}{T} > (0.973 - 1.16)\left(\frac{E_{\rm sph,0}}{1.916\times 4\pi v/g}\right)^{-1}$$

Sphaleron details

SM, one higgs

$$E_{\mathrm{sph},T} = \frac{4\pi v[T]}{g}\int_0^\infty d\xi \left[4\left(\frac{df}{d\xi}\right)^2 s_\mu^2 + \frac{8}{\xi^2} f^2(1-f)^2 s_\mu^4 + \frac{\xi^2}{2}\left(\frac{dh}{d\xi}\right)^2 s_\mu^2 + s_\mu^2((1-f)^2 h^2 - 2fh(1-f)(1-h)c_\mu^2 + f^2(1-h)^2 c_\mu^2) + \frac{\xi^2}{g^2 v[T]^4}(V_{eff}[\mu,h,T])\right] \quad (\!$$

$$\frac{d^2 f}{d\xi^2} = \frac{2}{\xi^2} f(1-f)(1-2f)s_\mu^2 - \frac{1}{8}(2h^2(1-f) - 2h(1-h)(1-2f)c_\mu^2 + 2f(1-h)^2 c_\mu^2),$$
$$\frac{d}{d\xi}\left(\xi^2 \frac{dh}{d\xi}\right) = 2h(1-f)^2 - 2f(1-f)(1-2h)c_\mu^2 - 2f^2(1-h)c_\mu^2 + \frac{\xi^2}{g_2^2}\frac{1}{v[T]^4}\frac{\partial V_{\rm eff}}{\partial h},$$

SM+S

$$E_{\rm sph}[f,h,k] = \frac{4\pi v}{g_2}\int_0^\infty d\xi \left[4\left(\frac{df}{d\xi}\right)^2 + \frac{8}{\xi^2}(f-f^2)^2 + \frac{\xi^2}{2}\left(\frac{dh}{d\xi}\right)^2 + h^2(1-f)^2 + \frac{\xi^2}{2}\frac{v_S^2}{v^2}\left(\frac{dk}{d\xi}\right)^2 + \frac{\xi^2}{g_2^2 v^4} V_{\rm eff}(h,k,T)\right]$$

For the xSM model, we consider the following sphaleron field ansatz [1]:

$$A_i(\mu,r,\theta,\phi) = -\frac{i}{g} f(r)\partial_i U(\mu,\theta,\phi)U^{-1}(\mu,\theta,\phi), \quad (1)$$

$$H(\mu,r,\theta,\phi) = \frac{v(T)}{\sqrt{2}}\left[(1-h(r))\begin{pmatrix} 0 \\ e^{-i\mu}\cos\mu \end{pmatrix} + h(r)U(\mu,\theta,\phi)\begin{pmatrix} 0 \\ 1 \end{pmatrix}\right], \quad (2)$$

$$S(\mu,r,\theta,\phi) = v_S(T)k(r), \quad (3)$$

where A_i are SU(2) gauge fields, and the matrix U is defined as

$$U(\mu,\theta,\phi) = \begin{pmatrix} e^{i\mu}(c_\mu - i s_\mu c_\theta) & e^{i\phi}s_\mu s_\theta \\ -e^{-i\phi}s_\mu s_\theta & e^{-i\mu}(c_\mu + i s_\mu c_\theta) \end{pmatrix}, \quad (4)$$

where the $s_{\mu(\theta)} = \sin\mu(\theta)$ and $c_{\mu(\theta)} = \cos\mu(\theta)$. The sphaleron energy is obtained for $\mu = \pi/2$ [2]. From the sphaleron energy in the main body of this paper, the equations of motion can be found:

$$\frac{d^2 f}{d\xi^2} = \frac{2}{\xi^2} f(1-f)(1-2f) - \frac{v[T]^2 h^2}{4\Omega[T]^2}(1-f), \quad (5)$$

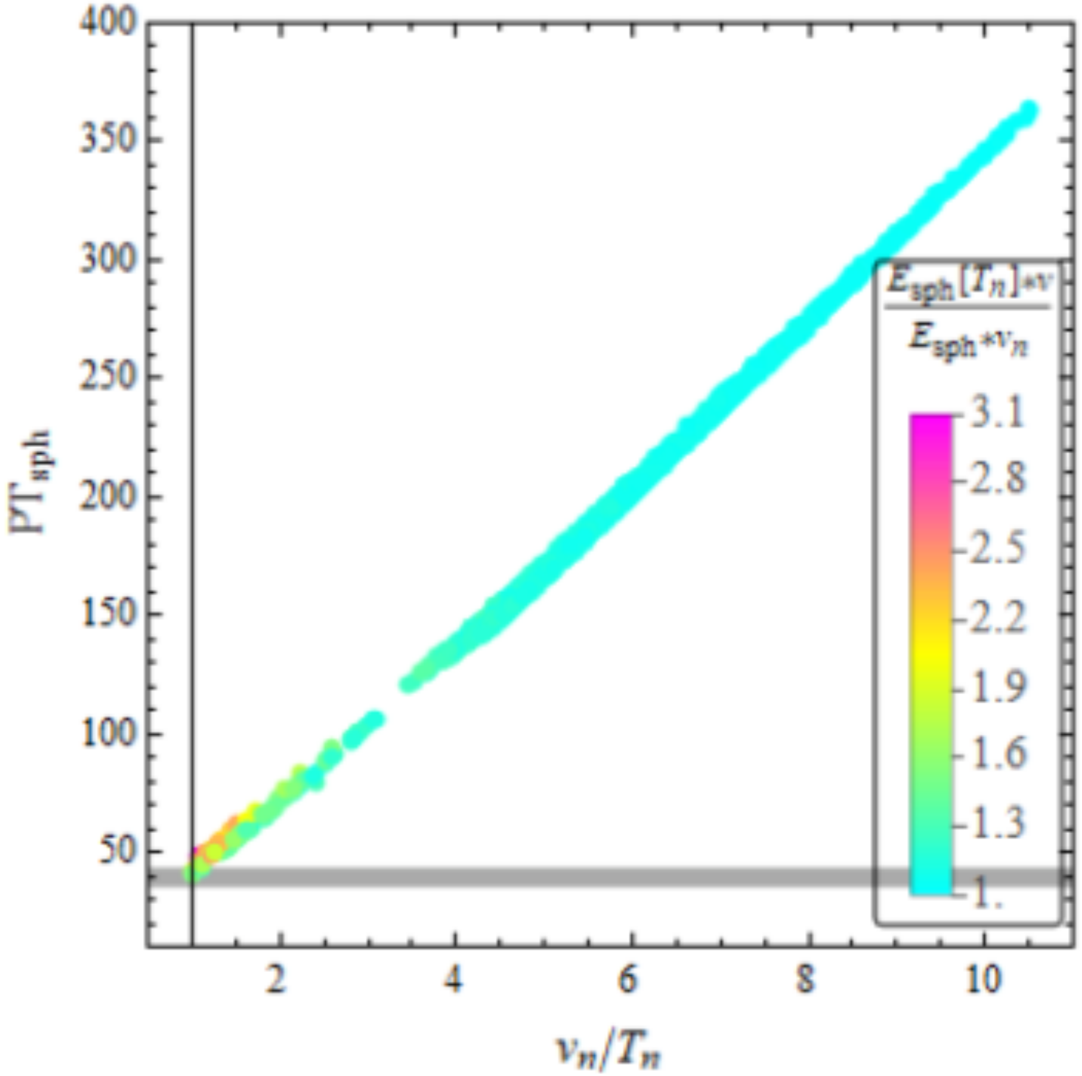
$$\frac{d}{d\xi}\left(\xi^2\frac{dh}{d\xi}\right) = 2h(1-f)^2 + \frac{\xi^2}{g^2}\frac{1}{v[T]^2\Omega[T]^2}\frac{\partial V_{\rm eff}(h,k,T)}{\partial h}, \quad (6)$$

$$\frac{d}{d\xi}\left(\xi^2\frac{dk}{d\xi}\right) = \frac{\xi^2}{g^2}\frac{1}{v_S[T]^2\Omega[T]^2}\frac{\partial V_{\rm eff}(h,k,T)}{\partial k}. \quad (7)$$

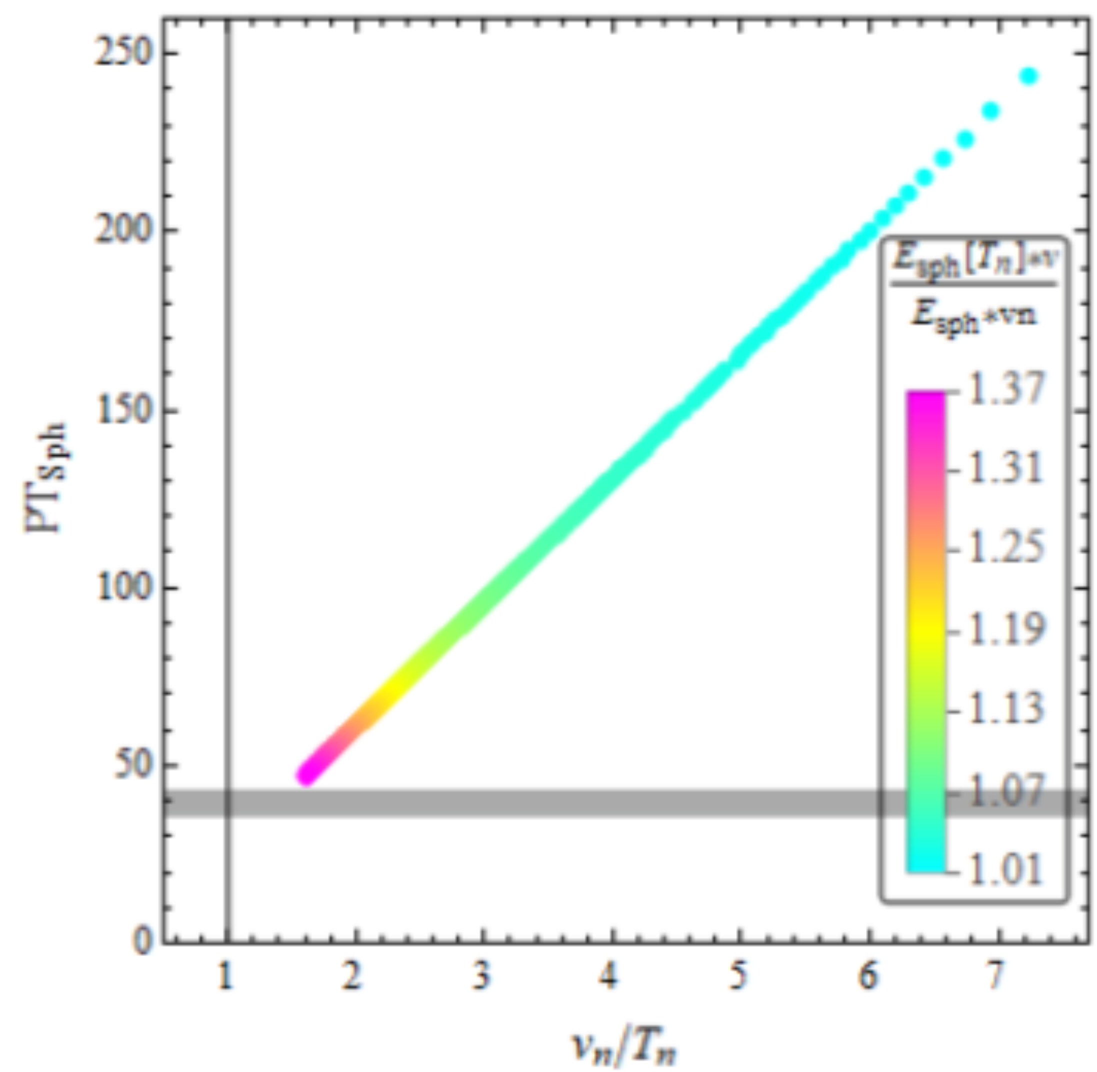
The sphaleron solutions can be obtained with the following boundary conditions,

$$\lim_{\xi\to 0} f(\xi) = 0, \lim_{\xi\to 0} h(\xi) = 0, \quad \lim_{\xi\to 0} k'(\xi) = 0,$$
$$\lim_{\xi\to\infty} f(\xi) = 1, \lim_{\xi\to\infty} h(\xi) = 1, \lim_{\xi\to\infty} k(\xi) = 1. \quad (8)$$

SFOPT condition



SM+singlet



SMEFT

For the "xSM" model, the gauge invariant finite temperature effective potential is found to be:

$$V(h,s,T) = -\frac{1}{2}[\mu^2 - \Pi_h(T)]h^2 - \frac{1}{2}[-b_2 - \Pi_s(T)]s^2 + \frac{1}{4}\lambda h^4 + \frac{1}{4}a_1 h^2 s + \frac{1}{4}a_2 h^2 s^2 + \frac{b_3}{3}s^3 + \frac{b_4}{4}s^4, \quad \text{(C1)}$$

with the thermal masses given by

$$\Pi_h(T) = \left(\frac{2m_W^2 + m_Z^2 + 2m_t^2}{4v^2} + \frac{\lambda}{2} + \frac{a_2}{24}\right)T^2,$$

$$\Pi_s(T) = \left(\frac{a_2}{6} + \frac{b_4}{4}\right)T^2, \quad \text{(C2)}$$

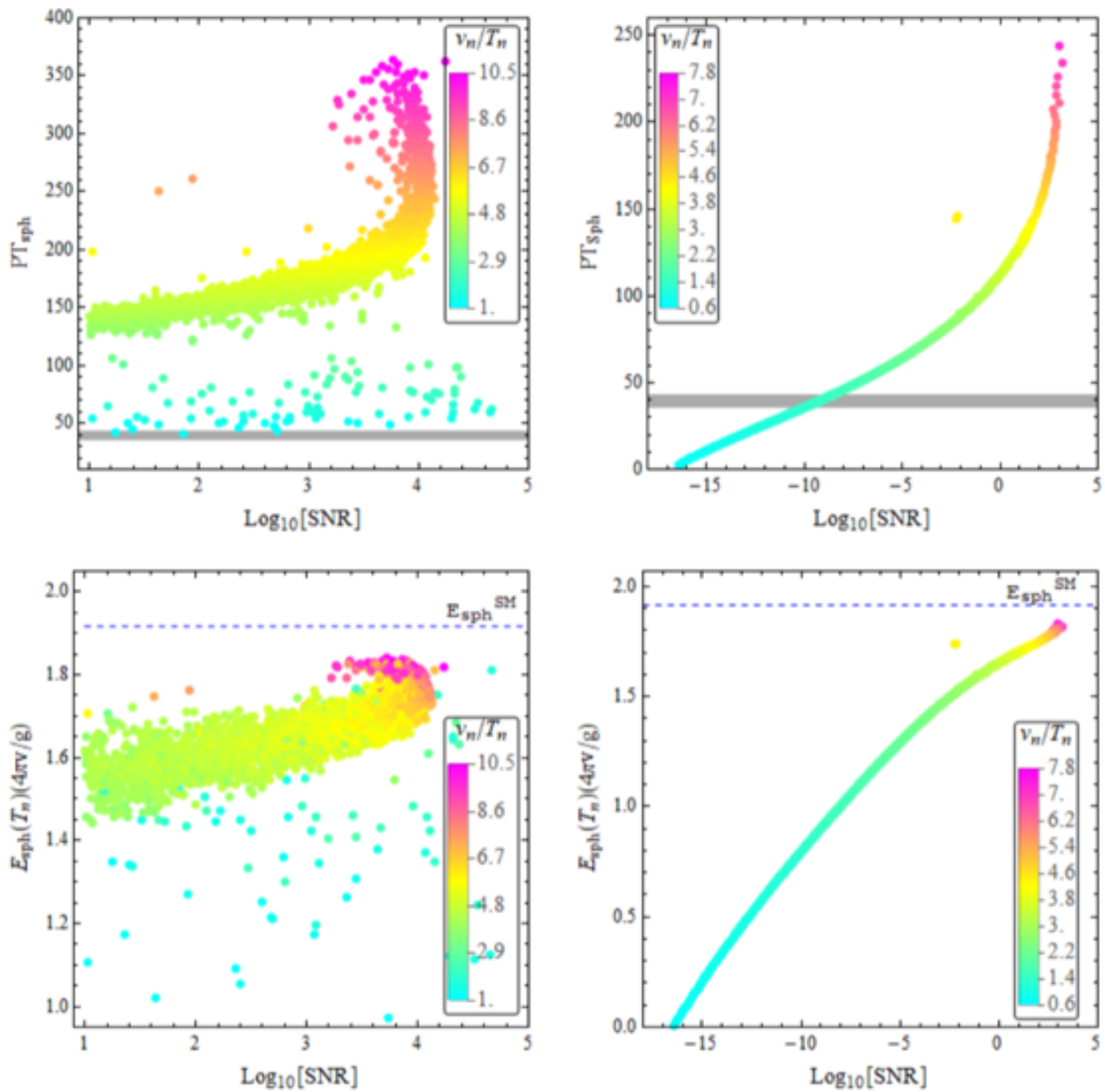
$$V_T(h,T) = V(h) + \frac{1}{2}c_{hT}h^2$$

$$V(H) = -m^2(H^\dagger H) + \lambda(H^\dagger H)^2 + \frac{(H^\dagger H)^3}{\Lambda^2}$$

$$c_{hT} = (4y_t^2 + 3g^2 + g'^2 + 8\lambda)T^2/16$$

Ruiyu Zhou, Ligong Bian, Huai-Ke Guo 1910.00234

Search for sphaleron with GW



Gravitational waves can be searched for by cross-correlating outputs from two or more detectors, with the resulting signal-to- noise ratio(SNR)

$$\mathrm{SNR} = \sqrt{\mathcal{T} \int df \left[\frac{h^2\Omega_{\mathrm{GW}}(f)}{h^2\Omega_{\mathrm{exp}}(f)}\right]^2}$$

where T is the duration of the data in years and Ω_{exp} the power spectral density of the detector.

GW sources

$$\Omega_{GW}(f) = \begin{cases} \Omega_{GW*}\left(\frac{f}{f_*}\right)^{n_{GW1}} & \text{for } f < f_*, \\ \Omega_{GW*}\left(\frac{f}{f_*}\right)^{n_{GW2}} & \text{for } f > f_*, \end{cases}$$

1807.00786

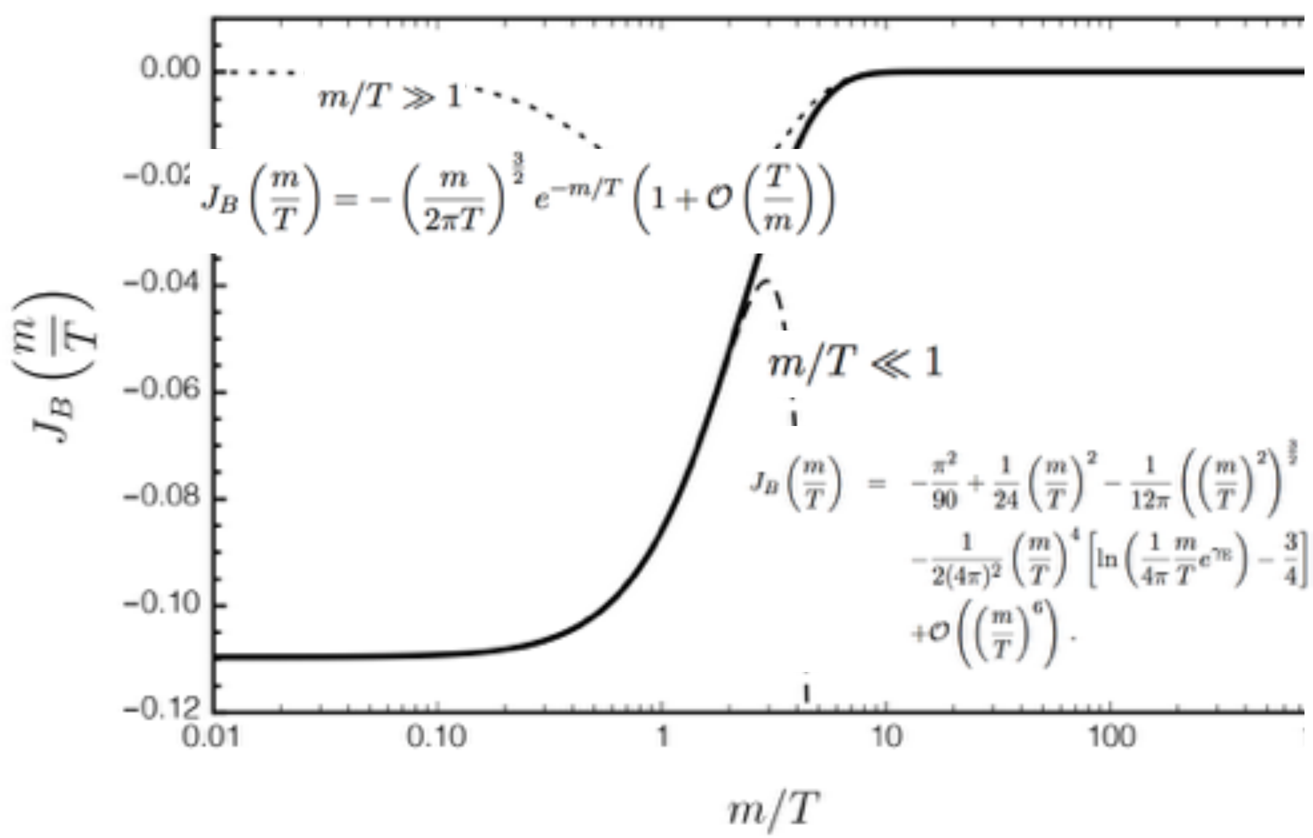
Table 1. Cosmological GW sources

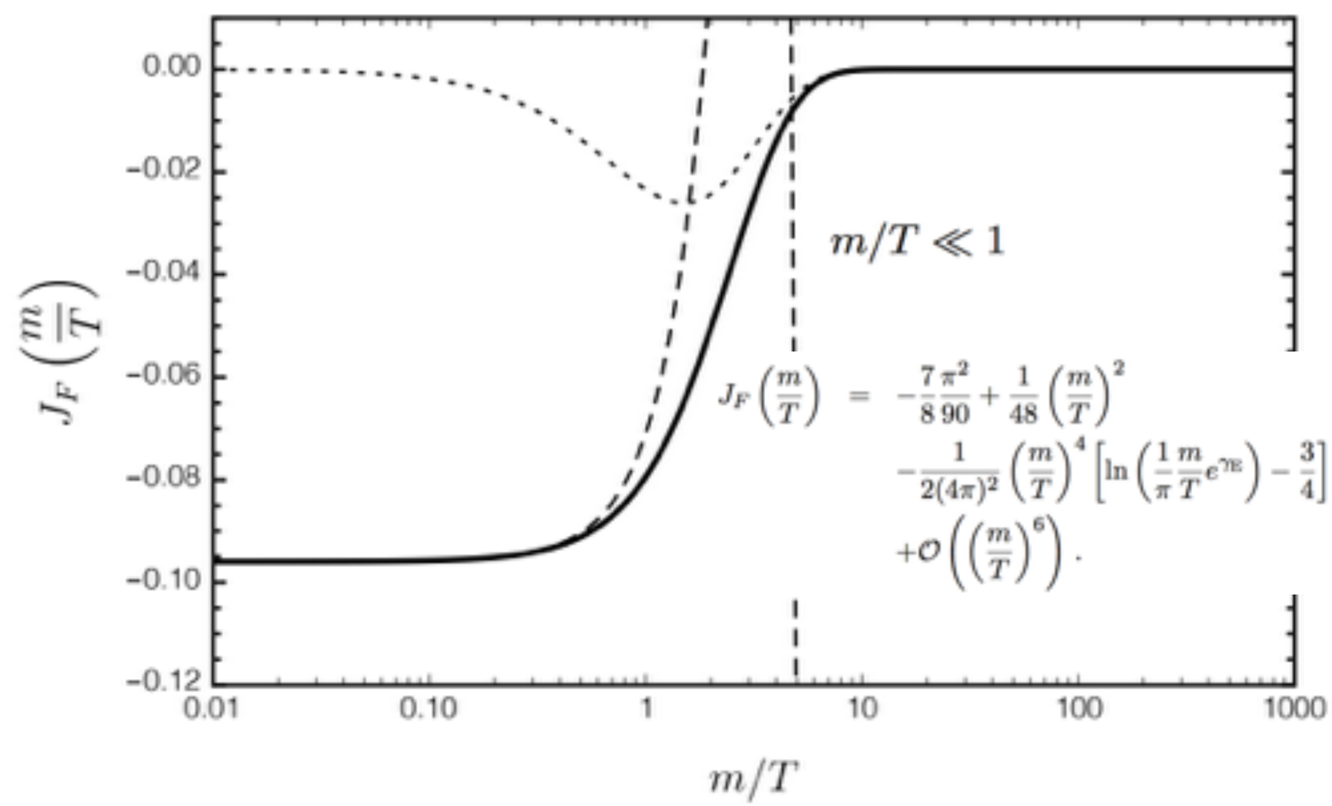
source	n_{GW1}	n_{GW2}	f_* [Hz]	Ω_{GW}
Phase transition (bubble collision)	2.8	-2	$\sim 10^{-5}\left(\frac{f_{PT}}{\beta}\right)\left(\frac{\beta}{H_{PT}}\right)\left(\frac{T_{PT}}{100\text{ GeV}}\right)$	$\sim 10^{-5}\left(\frac{H_{PT}}{\beta}\right)^2\left(\frac{\kappa_\phi\alpha}{1+\alpha}\right)^2\left(\frac{0.11v_w^3}{0.42+v_w^2}\right)$
Phase transition (turbulence)	3	$-5/3$	$\sim 3\times10^{-5}\left(\frac{1}{v_w}\right)\left(\frac{\beta}{H_{PT}}\right)\left(\frac{T_{PT}}{100\text{ GeV}}\right)$	$\sim 3\times10^{-4}\left(\frac{H_{PT}}{\beta}\right)\left(\frac{\kappa_{turb}\alpha}{1+\alpha}\right)^{3/2} v_w$
Phase transition (sound waves)	3	-4	$\sim 2\times10^{-5}\left(\frac{1}{v_w}\right)\left(\frac{\beta}{H_{PT}}\right)\left(\frac{T_{PT}}{100\text{ GeV}}\right)$	$\sim 3\times10^{-6}\left(\frac{H_{PT}}{\beta}\right)\left(\frac{\kappa_v\alpha}{1+\alpha}\right)^2 v_w$
Preheating ($\lambda\phi^4$)	3	cutoff	$\sim 10^7$	$\sim 10^{-11}\left(\frac{g^2/\lambda}{100}\right)^{-0.5}$
Preheating (hybrid)	2	cutoff	$\sim \frac{g}{\sqrt{\lambda}}\lambda^{1/4}10^{10.25}$	$\sim 10^{-5}\left(\frac{\lambda}{g^2}\right)^{1.16}\left(\frac{v}{M_{pl}}\right)^2$
Cosmic strings (loops 1)	$[1,2]$	$[-1,-0.1]$	$\sim 3\times10^{-8}\left(\frac{G\mu}{10^{-11}}\right)^{-1}$	$\sim 10^{-9}\left(\frac{G\mu}{10^{-12}}\right)\left(\frac{\alpha_{loop}}{10^{-1}}\right)^{-1/2}$ (for $\alpha_{loop} \gg \Gamma G\mu$)
Cosmic strings (loops 2)	$[-1,-0.1]$	0	$\sim 3\times10^{-8}\left(\frac{G\mu}{10^{-11}}\right)^{-1}$	$\sim 10^{-9.5}\left(\frac{G\mu}{10^{-12}}\right)\left(\frac{\alpha_{loop}}{10^{-1}}\right)^{-1/2}$ (for $\alpha_{loop} \gg \Gamma G\mu$)
Cosmic strings (infinite strings)	$[0,0.2]$	$[0,0.2]$	—	$\sim 10^{-[11,13]}\left(\frac{G\mu}{10^{-8}}\right)$
Domain walls	3	-1	$\sim 10^{-9}\left(\frac{T_{ann}}{10^{-2}\text{GeV}}\right)$	$\sim 10^{-17}\left(\frac{\sigma}{1\text{TeV}^3}\right)^2\left(\frac{T_{ann}}{10^{-2}\text{GeV}}\right)^{-4}$
Self-ordering scalar fields	0	0	—	$\sim \frac{511}{N}\Omega_{rad}\left(\frac{v}{M_{pl}}\right)^4$
Self-ordering scalar + reheating	0	-2	$\sim 0.4\left(\frac{T_R}{10^7\text{ GeV}}\right)$	$\sim \frac{511}{N}\Omega_{rad}\left(\frac{v}{M_{pl}}\right)^4$
Magnetic fields	3	α_B+1	$\sim 10^{-6}\left(\frac{T_*}{10^2\text{GeV}}\right)$	$\sim 10^{-16}\left(\frac{B}{10^{-10}\text{G}}\right)$
Inflation+reheating	~ 0	-2	$\sim 0.3\left(\frac{T_R}{10^7\text{ GeV}}\right)$	$\sim 2\times10^{-17}\left(\frac{r}{0.01}\right)$
Inflation+kination	~ 0	1	$\sim 0.3\left(\frac{T_R}{10^7\text{ GeV}}\right)$	$\sim 2\times10^{-17}\left(\frac{r}{0.01}\right)$
Particle prod. during inf.	-2ϵ	$-4\epsilon(4\pi\xi-6)(\epsilon-\eta)$	—	$\sim 2\times10^{-17}\left(\frac{r}{0.01}\right)$
2nd-order (inflation)	1	drop-off	$\sim 7\times10^5\left(\frac{T_{reh}}{10^9\text{ GeV}}\right)^{1/3}\left(\frac{M_{inf}}{10^{16}\text{ GeV}}\right)^{2/3}$	$\sim 10^{-12}\left(\frac{T_{reh}}{10^9\text{ GeV}}\right)^{-4/3}\left(\frac{M_{inf}}{10^{16}\text{ GeV}}\right)^{4/3}$
2nd-order (PBHs)	2	drop-off	$\sim 4\times10^{-2}\left(\frac{M_{PBH}}{10^{20}\text{ g}}\right)^{-1/2}$	$\sim 7\times10^{-9}\left(\frac{\mathcal{A}^2}{10^{-3}}\right)^2$
Pre-Big-Bang	3	$3-2\mu$	—	$\sim 1.4\times10^{-6}\left(\frac{H_s}{0.15M_{pl}}\right)^4$

Thermal effective scalar potential for PT study

$$V_T(\phi, T) = \quad V_0(\phi) + T^4\left[\sum_B J_B\left(\frac{M_B}{T}\right) + \sum_F J_F\left(\frac{M_F}{T}\right)\right]$$

all fermions F and bosons B that are relativistic at temperature T





High-T expansion

$m/T \ll 1$

$$V_T(\phi) = V_0(\phi) + \frac{T^2}{24}\left(\sum_S M_S^2(\phi) + 3\sum_V M_V^2(\phi) + 2\sum_F M_F^2(\phi)\right) - \frac{T}{12\pi}\left(\sum_S \left(M_S^2(\phi)\right)^{\frac{3}{2}} + \sum_V \left(M_V^2(\phi)\right)^{\frac{3}{2}}\right) + \text{higher order terms}.$$

MS, MV , MF are the masses of the scalar fields S, vector fields V and fermonic fields F

Bubble, Sphaleron and BAU

Instanton

$$\frac{\Gamma}{V} = A(T)e^{-S_3/T}.$$

$$\frac{S_3(T_N)}{T_N} - \frac{3}{2}\ln\left(\frac{S_3(T_N)}{T_N}\right) = 152.59 - 2\ln g_*(T_N) - 4\ln\left(\frac{T_N}{100\ \mathrm{GeV}}\right)$$

Bubble nucleation

S3 (TN)/TN ~140-150

Washout avoid

$$\Gamma_{\mathrm{sph}} = A_{\mathrm{sph}}(T)\exp[-E_{\mathrm{sph}}(T)/T] < H(T)$$

$$E_{\mathrm{sph}}(T) \approx E_{\mathrm{sph},0}\frac{v(T)}{v}$$

$$\frac{v(T)}{T} > (0.973 - 1.16)\left(\frac{E_{\mathrm{sph},0}}{1.916\times 4\pi v/g}\right)^{-1}$$

1708.03061

GW parameters and FOPT

Bounce solution

$$S_3(T) = \int 4\pi r^2 dr \left[\frac{1}{2}(\frac{d\phi_b}{dr})^2 + V(\phi_b, T)\right]$$

$$\lim_{r\to\infty} \phi_b = 0\ , \qquad \frac{d\phi_b}{dr}|_{r=0} = 0$$

Bubble nucleation:

$$\Gamma \approx A(T)e^{-S_3/T} \sim 1$$

Latent heat:

$$\alpha = \frac{1}{\rho_R}\left[-(V_{\rm EW} - V_f) + T\left(\frac{dV_{\rm EW}}{dT} - \frac{dV_f}{dT}\right)\right]\Big|_{T=T_*}$$

phase transition inverse duration:

$$\frac{\beta}{H_n} = T\frac{d(S_3(T)/T)}{dT}|_{T=T_n}$$

GW from FOPT

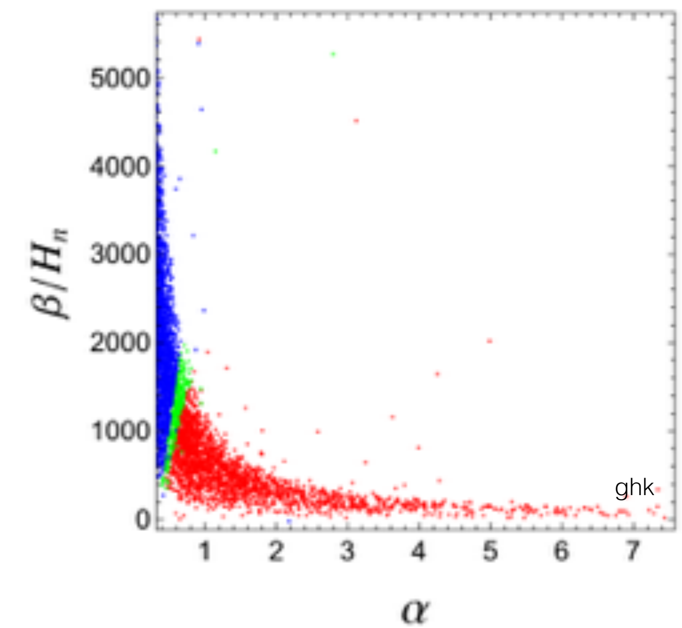
$$\Omega_{\rm GW}(f)h^2 \approx \Omega_{\rm sw}(f)h^2 + \Omega_{\rm turb}(f)h^2$$

Sound Wave: $$\Omega h^2_{\rm sw}(f) = 2.65\times10^{-6}(H_*\tau_{sw})\left(\frac{\beta}{H}\right)^{-1} v_b \left(\frac{\kappa_\nu\alpha}{1+\alpha}\right)^2\left(\frac{g_*}{100}\right)^{-\frac{1}{3}}\left(\frac{f}{f_{\rm sw}}\right)^3\left(\frac{7}{4+3\,(f/f_{\rm sw})^2}\right)^{7/2}$$

phase transition duration: $$\tau_{sw} = min\left[\frac{1}{H_*}, \frac{R_*}{\bar{U}_f}\right],\ H_*R_* = v_b(8\pi)^{1/3}(\beta/H)^{-1}$$

Root-mean-square four-velocity of the plasma $$\bar{U}_f^2 \approx \frac{3}{4}\frac{\kappa_\nu\alpha}{1+\alpha}$$

$$f_{\rm sw} = 1.9\times10^{-5}\frac{\beta}{H}\frac{1}{v_b}\frac{T_*}{100}\left(\frac{g_*}{100}\right)^{\frac{1}{6}}\ {\rm Hz}$$



MHD turbulence: $$\Omega h^2_{\rm turb}(f) = 3.35\times10^{-4}\left(\frac{\beta}{H}\right)^{-1}\left(\frac{\epsilon\kappa_\nu\alpha}{1+\alpha}\right)^{\frac{3}{2}}\left(\frac{g_*}{100}\right)^{-\frac{1}{3}} v_b \frac{(f/f_{\rm turb})^3\,(1+f/f_{\rm turb})^{-\frac{11}{3}}}{[1+8\pi f a_0/(a_*H_*)]}$$

$$f_{\rm turb} = 2.7\times10^{-5}\frac{\beta}{H}\frac{1}{v_b}\frac{T_*}{100}\left(\frac{g_*}{100}\right)^{\frac{1}{6}}\ {\rm Hz}$$

3.1. *Models of time-correlated processes*

The principal results of this paper are referred to a fiducial power-law spectrum of characteristic GW strain

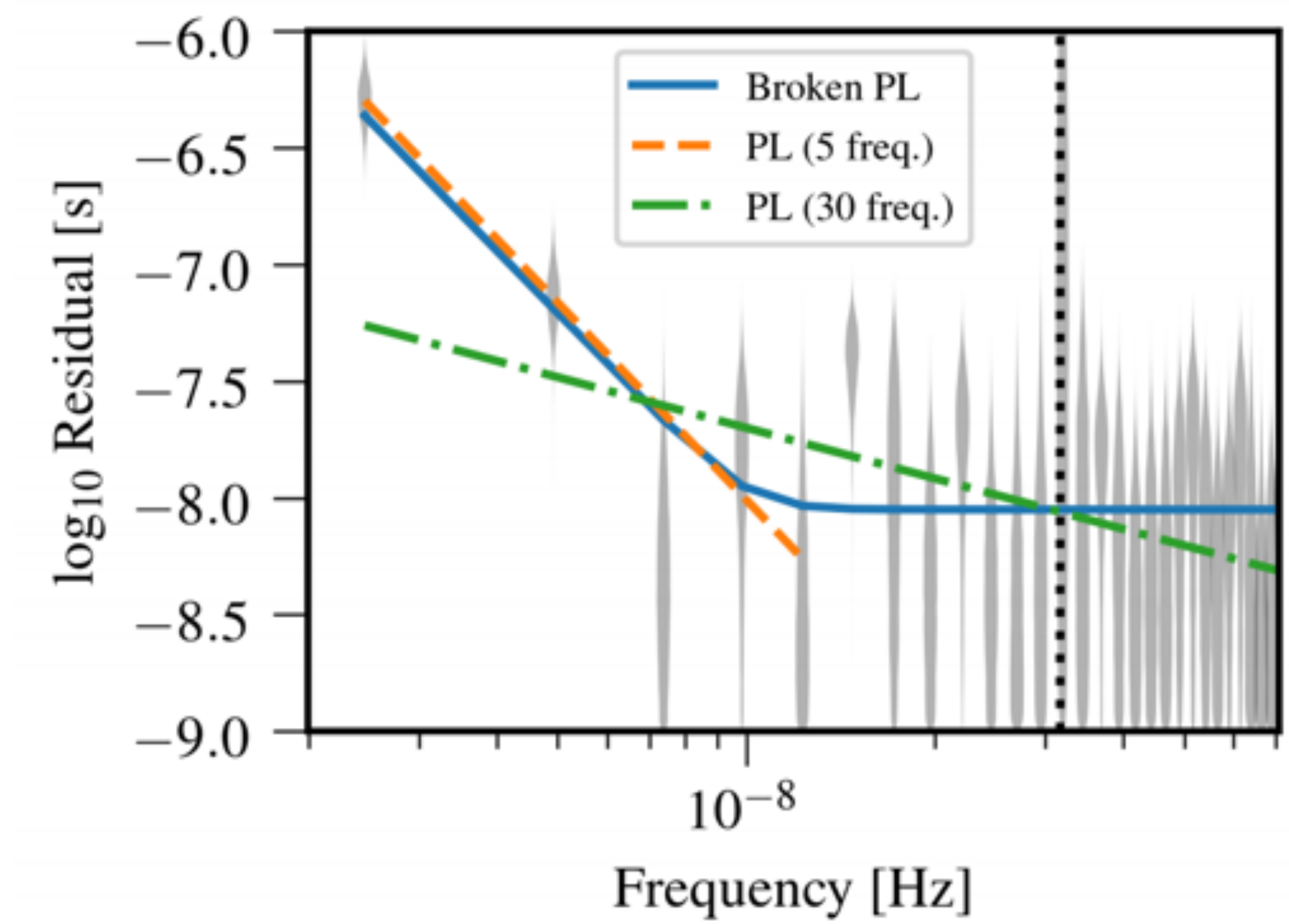
$$h_c(f) = A_{\rm GWB}\left(\frac{f}{f_{\rm yr}}\right)^{\alpha}, \qquad (1)$$

with $\alpha = -2/3$ for a population of inspiraling SMBHBs in circular orbits whose evolution is dominated by GW emission (Phinney 2001). We performed our analysis in terms of the timing-residual cross-power spectral density

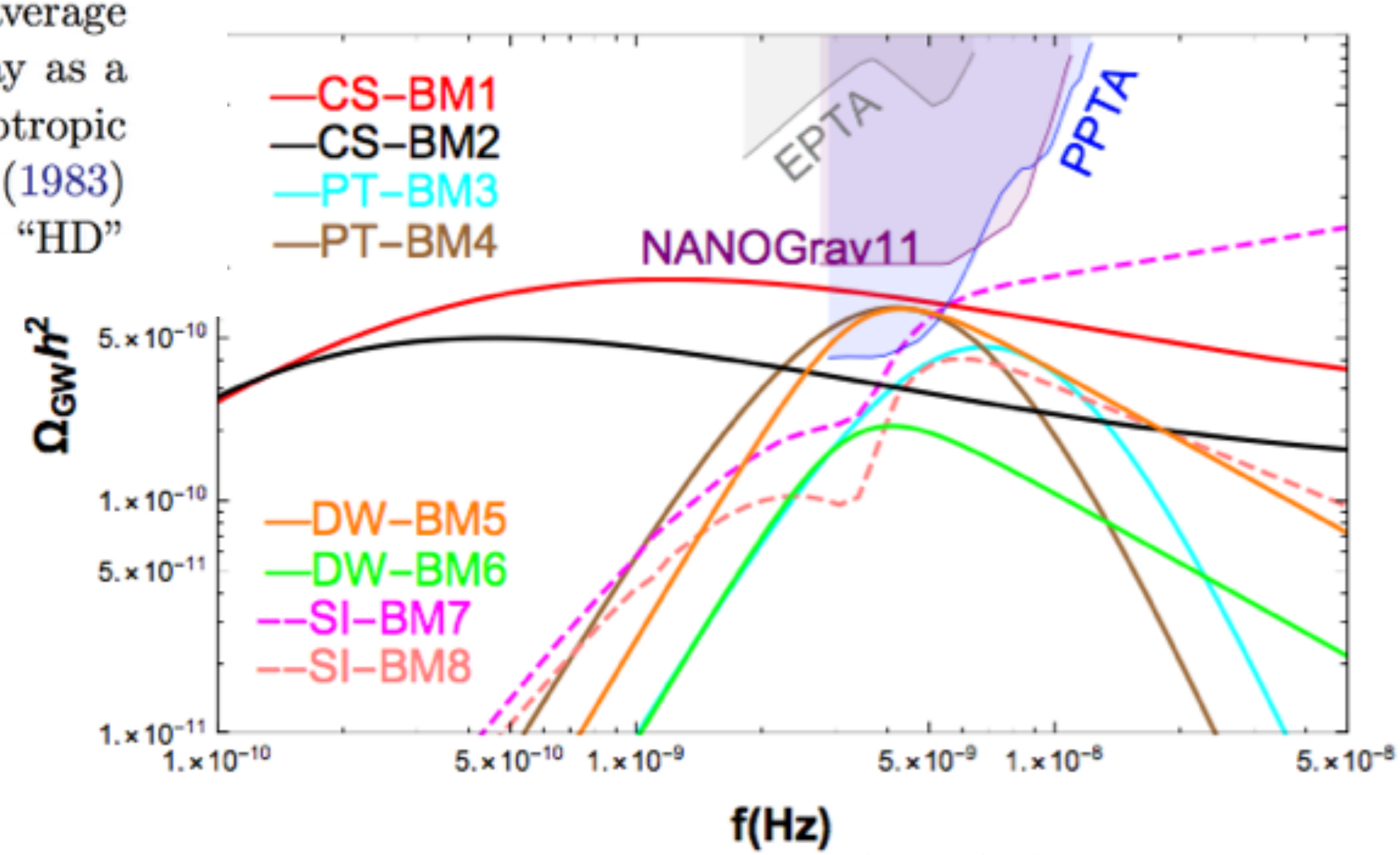
$$S_{ab}(f) = \Gamma_{ab}\,\frac{{A_{\rm GWB}}^2}{12\pi^2}\left(\frac{f}{f_{\rm yr}}\right)^{-\gamma} f_{\rm yr}^{-3}. \qquad (2)$$

where $\gamma = 3 - 2\alpha$ (so the fiducial SMBHB $\alpha = -2/3$ corresponds to $\gamma = 13/3$), and where Γ_{ab} is the overlap reduction function (ORF), which describes average correlations between pulsars a and b in the array as a function of the angle between them. For an isotropic GWB, the ORF is given by Hellings & Downs (1983) and we refer to it casually as "quadrupolar" or "HD" correlations.

2009.04496



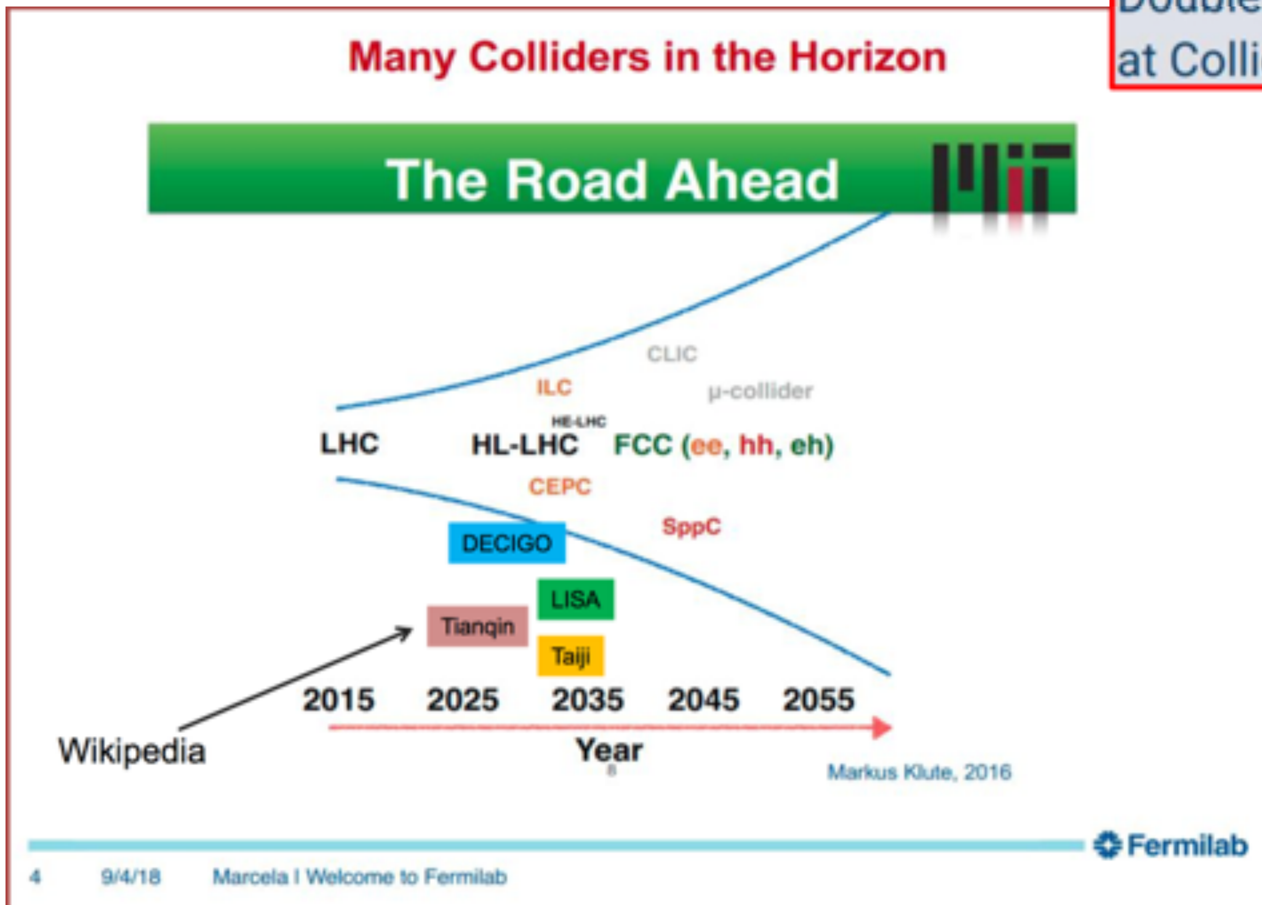
GW for NanoGrav ???

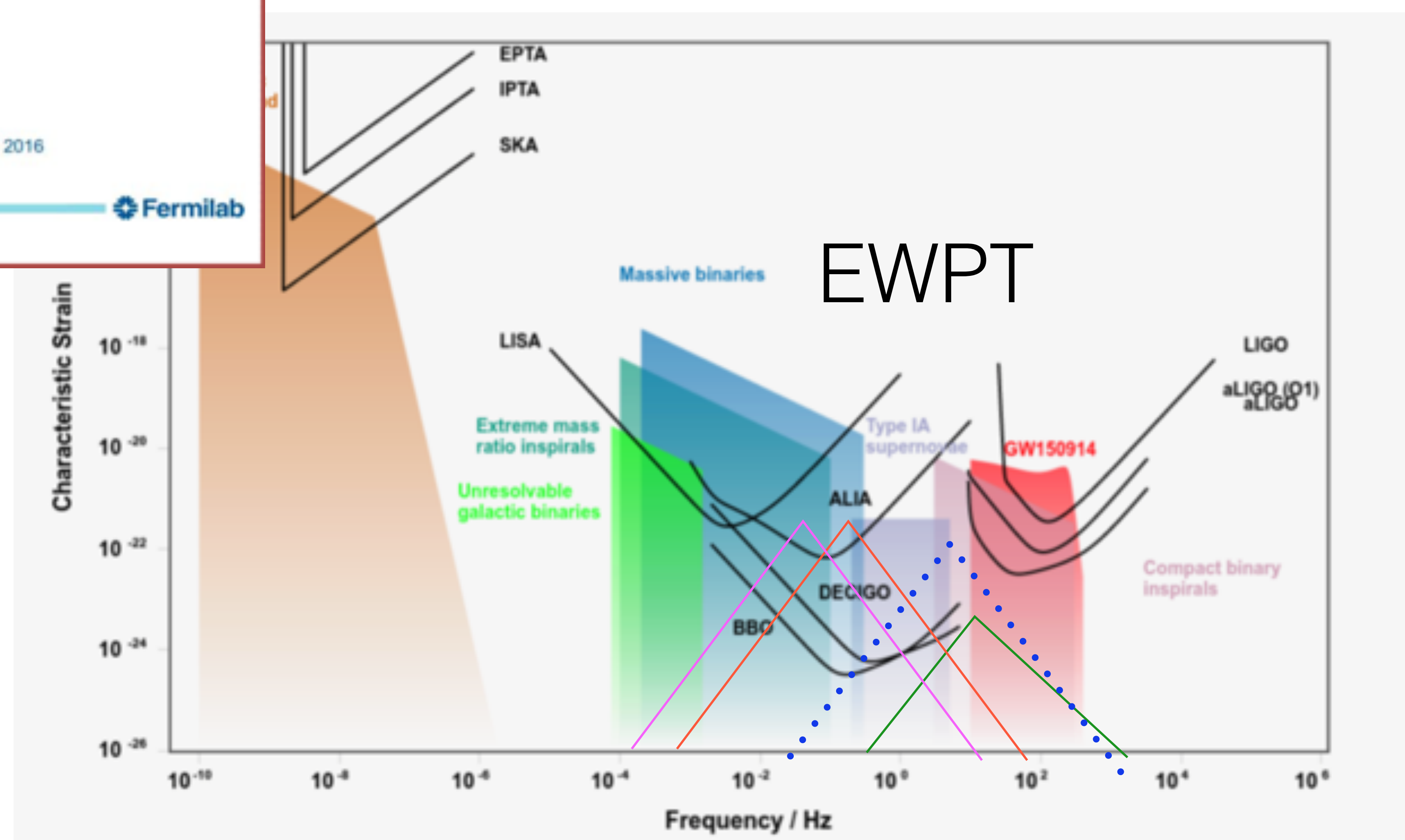


LGB, R.G. Cai, JL, RYZ,2009.13893

Test of SFOEWPT

Double Higgs Production at Colliders Workshop





C J Moore et al. Class. Quantum Grav. 32 (2015) 015014.

Why SFOEWPT

$$\frac{n_b}{s} \approx (0.7 - 0.9) \times 10^{-10} \neq 0$$