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Precise calculation of gravitational wave in electroweak phase transition

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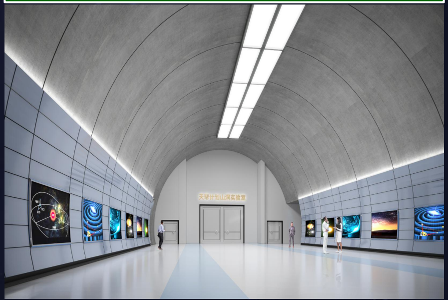

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- Introduction
- Energy Budget
- Gravitational Wave
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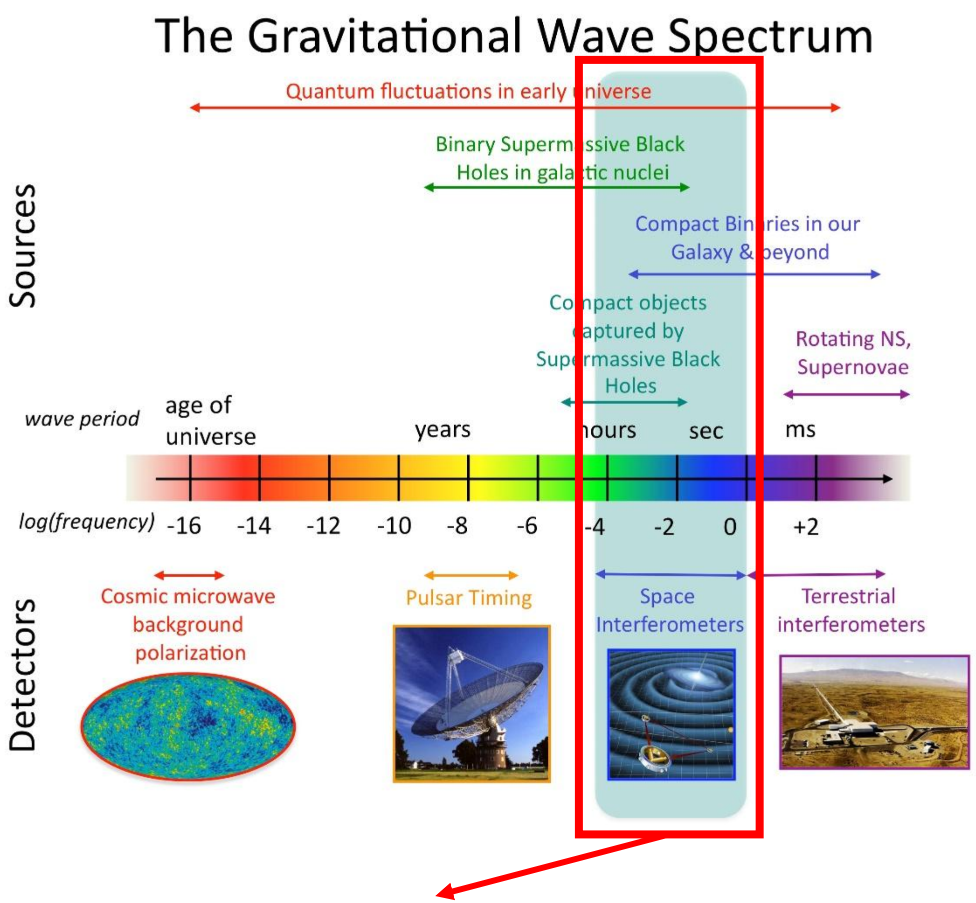
Introduction

After the observation of the gravitational wave by LIGO, the GW detector provides a new technique to study the fundamental physics.

Sources of GWs:

- *Astrophysical origin*: black hole, neutron star, etc.
- *Cosmological origin*: inflation, **FOPT, etc.**



EW FOPT

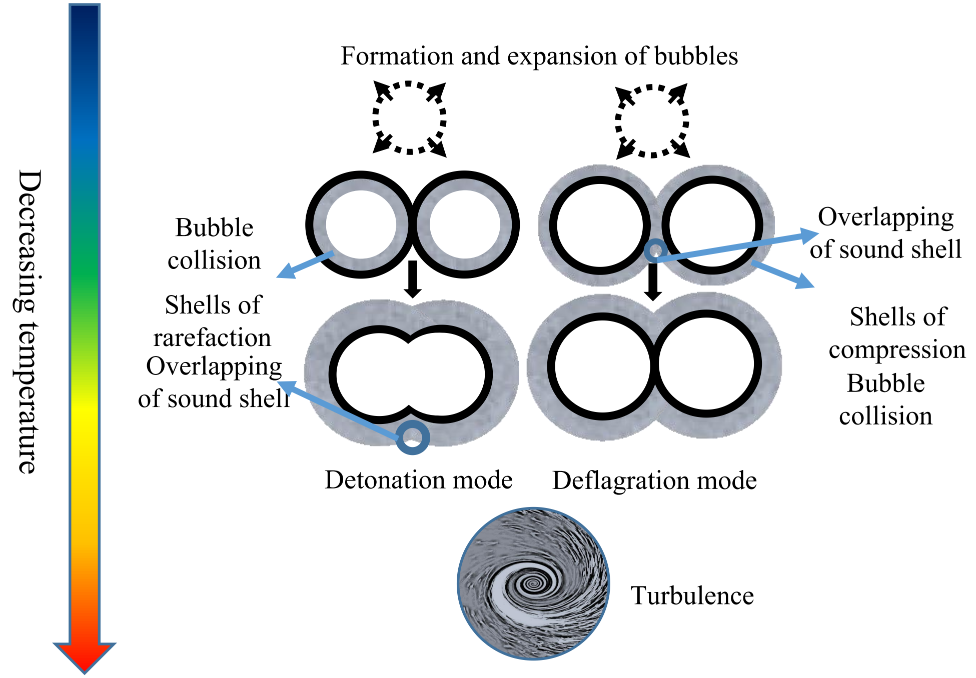
Introduction

GWs sources of FOPT:

- Bubble collisions
- Sound wave
- Turbulence

The amplitude and shape of GW spectrum are strongly related to PT dynamics.

JCAP 0809, 022 (2008); PRL112, 041301 (2014); PRD92, no. 12,123009 (2015); PRD96, no. 10,103520 (2017); Phys. Rev. D 66, 024030 (2002), Phys. Rev. D 76 (2007) 083002, JCAP 0912, 024 (2009)



$$h^2\Omega_{\rm co}(f) \simeq 1.67\times10^{-5}\left(\frac{H_*R_*}{(8\pi)^{1/3}}\right)^2\left(\frac{\kappa_\phi\alpha}{1+\alpha}\right)^2\left(\frac{100}{g_*}\right)^{1/3}\frac{0.11v_w}{0.42+v_w^2}\frac{3.8(f/f_{\rm co})^{2.8}}{1+2.8(f/f_{\rm co})^{3.8}}$$

Bubble collision

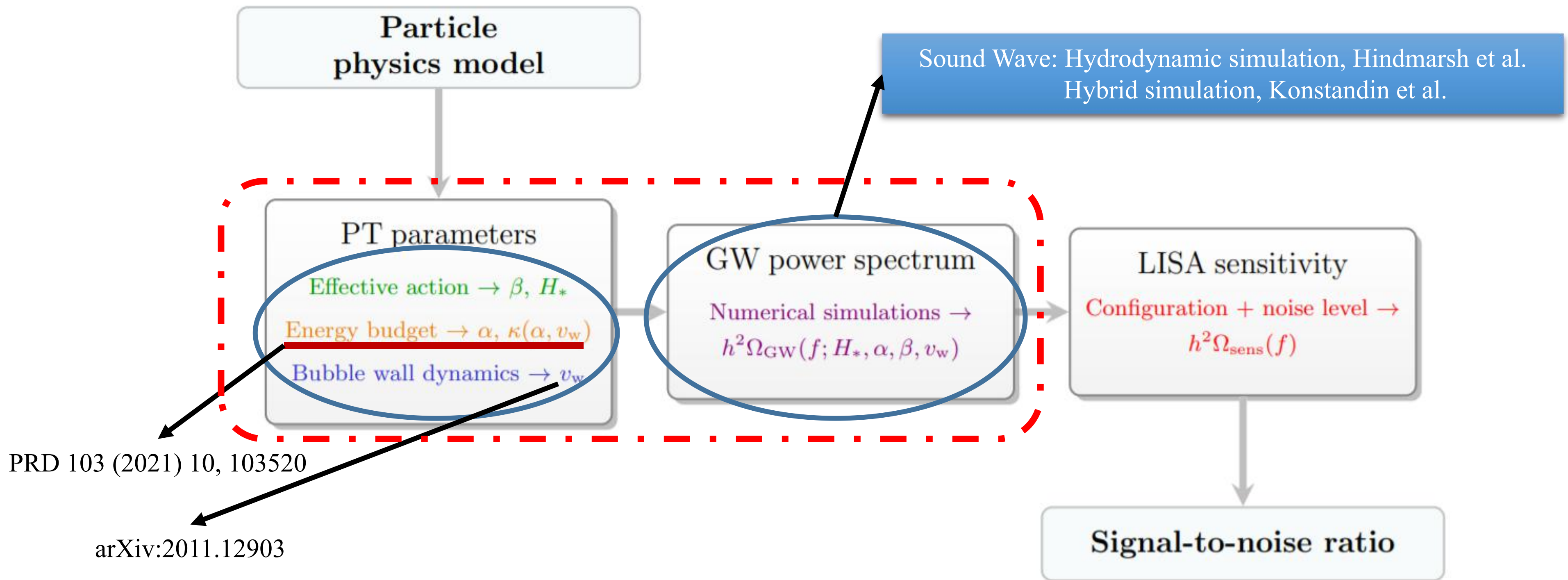
$$h^2\Omega_{\rm sw}(f) \simeq 1.64\times10^{-6}(H_*\tau_{\rm sw})(H_*R_*)\left(\frac{\kappa_v\alpha}{1+\alpha}\right)^2\left(\frac{100}{g_*}\right)^{1/3}(f/f_{\rm sw})^3\left(\frac{7}{4+3(f/f_{\rm sw})^2}\right)^{7/2}$$

Sound wave

$$h^2\Omega_{\rm turb}(f) \simeq 1.14\times10^{-}H_*R_*\left(\frac{\kappa_{\rm turb}\alpha}{1+\alpha}\right)^{3/2}\left(\frac{100}{g_*}\right)^{1/3}\frac{(f/f_{\rm turb})^3}{(1+f/f_{\rm turb})^{11/3}(1+8\pi f/H_*)}$$

Turbulence

Introduction



Flow chart of the calculation of phase transition gravitational wave

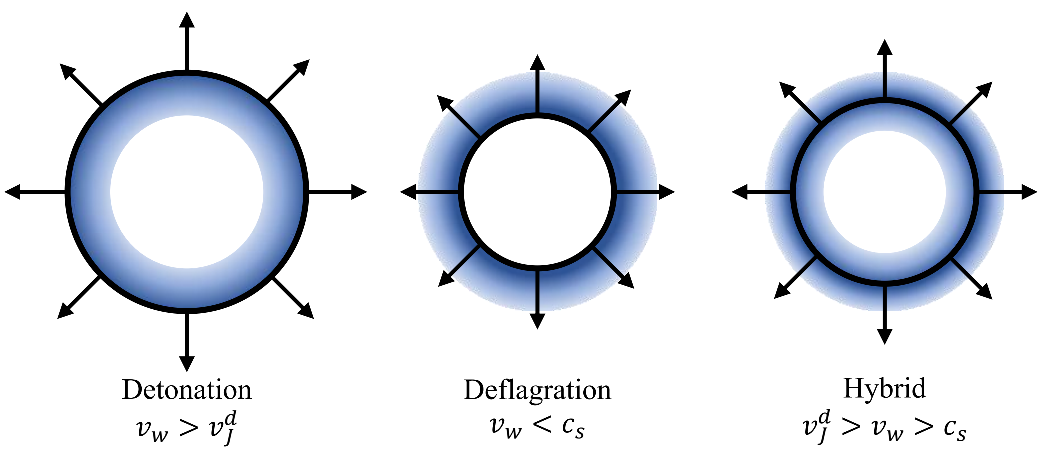
Chiara Caprini *et al,* JCAP03(2020)024

Energy Budget

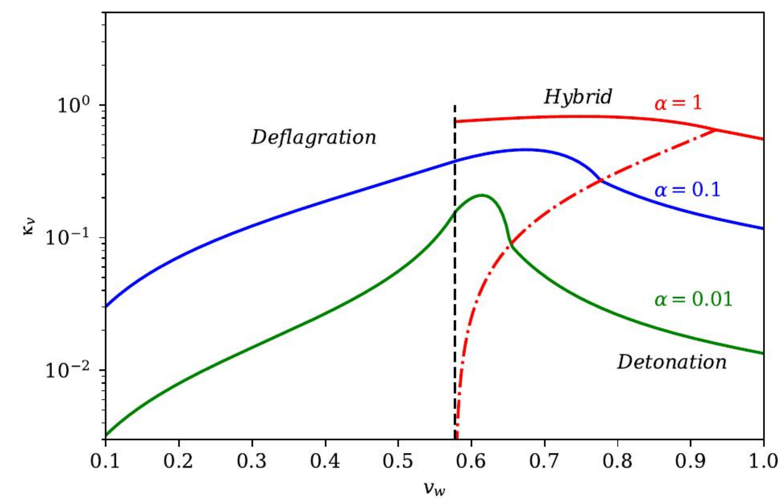
Energy budget (to measure the efficiency of the energy released by PT convert to the kinetic energy of sounding plasma)

Kinetic energy fraction: $K \equiv \frac{\rho_{\rm fl}}{e_+} = \frac{3}{e v_w^3} \int w(\xi)\, v^2 \gamma^2 \xi^2 d\xi \ , \quad \rho_{\rm fl} = \frac{3}{v_w^3} \int \xi^2 v^2 \gamma^2 w d\xi$

Efficiency parameter: $\kappa_v = \frac{3}{\epsilon v_w^3} \int w(\xi) v^2 \gamma^2 \xi^2 d\xi$



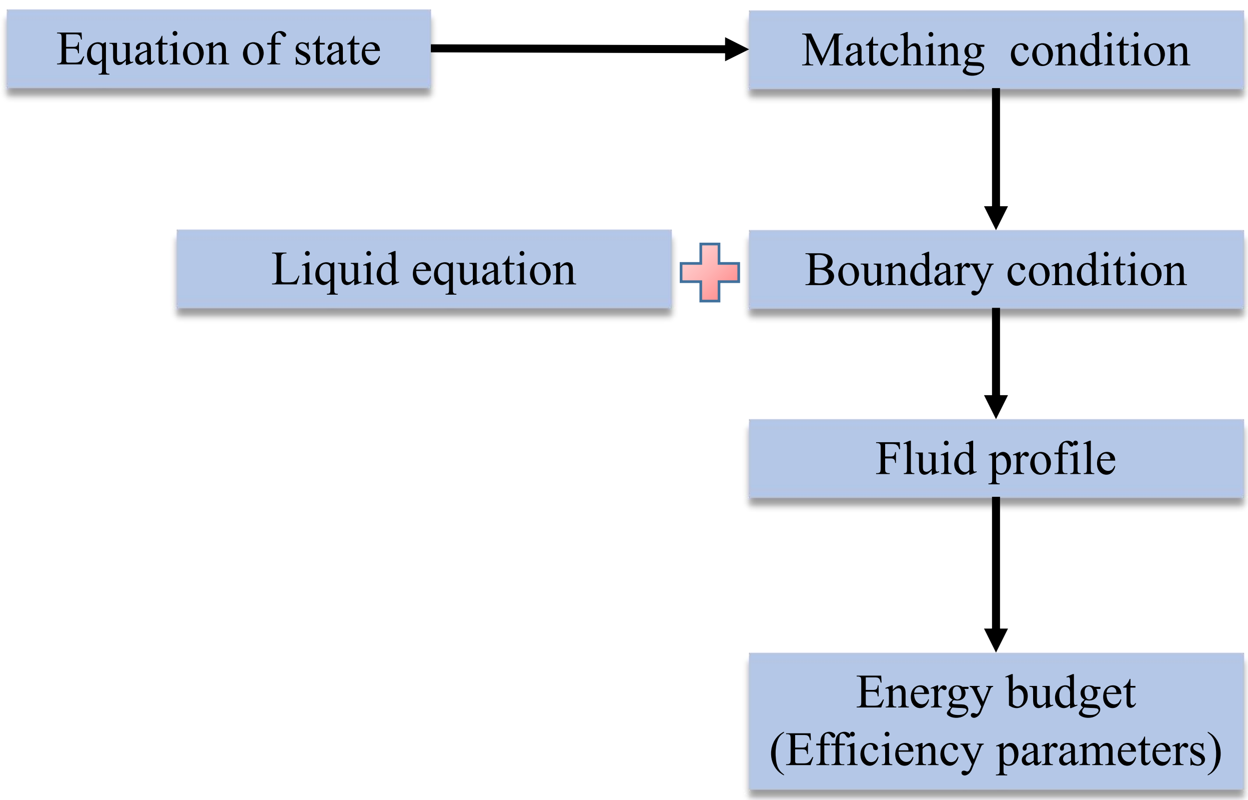
Energy budget is also strongly related the bubble wall velocity



$h^2\Omega_{GW} \propto \kappa^2$ or $\kappa^{\frac{3}{2}}$

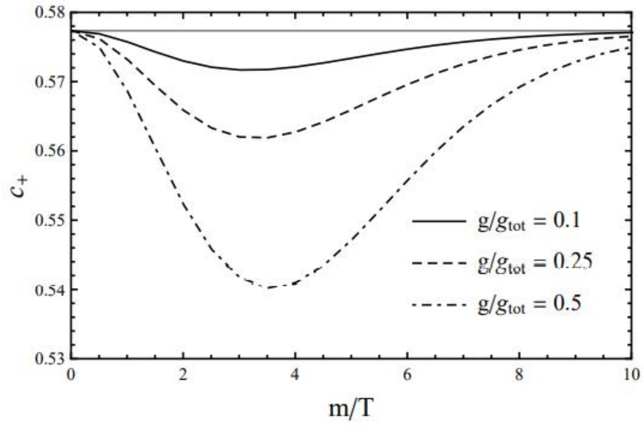
Energy Budget

A particle physics model independent analysis for energy budget:



Energy Budget

- Most of previous studies of the efficiency parameter are based on bag EoS, which assume the sound velocity is $1/\sqrt{3}$ in both phases. For a realistic FOPT, particle can obtain the mass in broken phase. Hence, the sound velocity can deviate from pure radiation phase.



L. Leitao and A. Megevand, Nucl. Phys. B **891**, 159-199 (2015)

Energy Budget

Bag EoS

$$p_+ = \frac{1}{3}a_+T_+^4 - \epsilon_+, \quad e_+ = a_+T_+^4 + \epsilon_+ \ ,$$

$$p_- = \frac{1}{3}a_-T_-^4 - \epsilon_-, \quad e_- = a_-T_-^4 + \epsilon_- \ .$$

$$\alpha_\theta = \frac{4}{3}\frac{\Delta\epsilon}{w_+}, \quad \epsilon_\pm = \frac{1}{4}(e_\pm - 3p_\pm)$$

Strength parameter

$$\partial p/\partial e = c_s^2 = \text{constant}$$

EoS with different sound velocity (DSVM)

$$p_+ = c_+^2 a_+T_+^4 - \epsilon_+, \quad e_+ = a_+T_+^4 + \epsilon_+ \ ,$$

$$p_- = c_-^2 a_-T_-^4 - \epsilon_-, \quad e_- = a_-T_-^4 + \epsilon_- \ ,$$

$$\alpha_{\bar\theta} = \frac{\Delta\bar\theta}{3w_+}, \quad \bar\theta = e - p/c_-^2$$

Strength parameter

Energy Budget

- Energy momentum conservation derives fluid equations:

$$(\xi - v)\frac{\partial_\xi e}{w} = 2\frac{v}{\xi} + \gamma^2(1 - v\xi)\partial_\xi v \ ,$$

$$(1 - v\xi)\frac{\partial_\xi p}{w} = \gamma^2(\xi - v)\partial_\xi v \ .$$

$$2\frac{v}{\xi} = \gamma^2(1 - v\xi)\left[\frac{\mu^2}{c_s^2} - 1\right]\partial_\xi v$$

Velocity profile

$$\frac{\partial_\xi w}{w} = \left(1 + \frac{1}{c_s^2}\right)\mu\gamma^2\partial_\xi v$$

Enthalpy profile

$$\frac{\partial_\xi T}{T} = \gamma^2\mu\partial_\xi v$$

Temperature profile

Different boundary conditions give different hydrodynamical modes.

- Matching condition

$$w_- v_-^2 \gamma_-^2 + p_- = w_+ v_+^2 \gamma_+^2 + p_+,$$
$$w_- v_- \gamma_-^2 = w_+ v_+ \gamma_+^2.$$

$$\longrightarrow \quad v_+ v_- = \frac{p_+ - p_-}{e_+ - e_-}, \frac{v_+}{v_-} = \frac{e_- + p_+}{e_+ + p_-}.$$

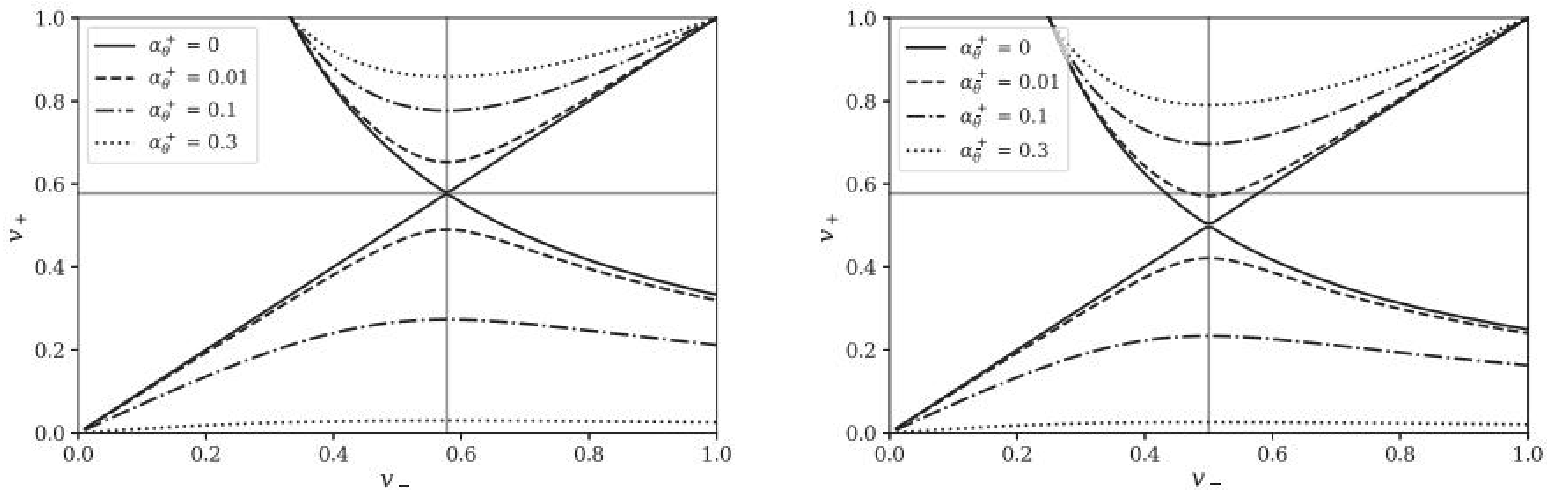
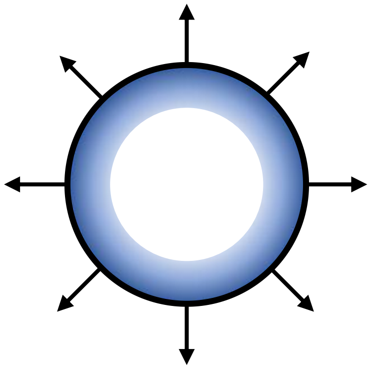


FIG. 1. The fluid velocities v_+ and v_- in the reference frame of bubble wall for different definitions and values of phase transition strength parameter. The horizontal and vertical gray lines indicate the sound velocities of symmetric and broken phase. Left panel: the bag model. Right panel: the DSVM with $c_+^2 = 1/3$ and $c_-^2 = 0.25$.

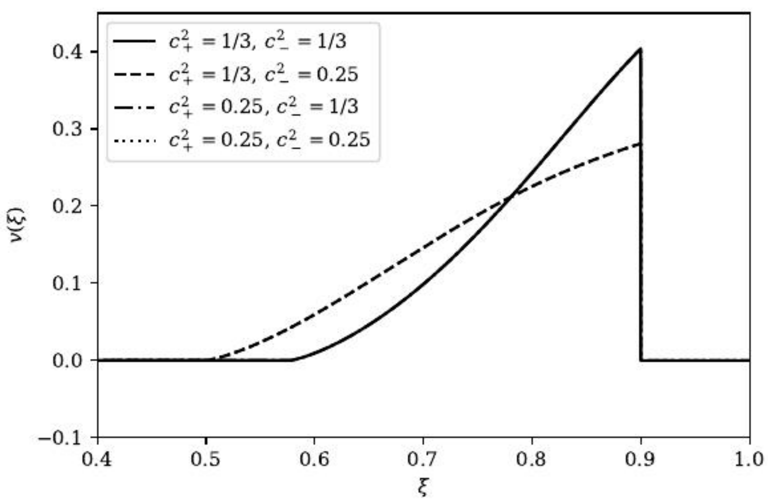
X. Wang, F. P. Huang and X. Zhang, PRD 103 (2021) 10, 103520

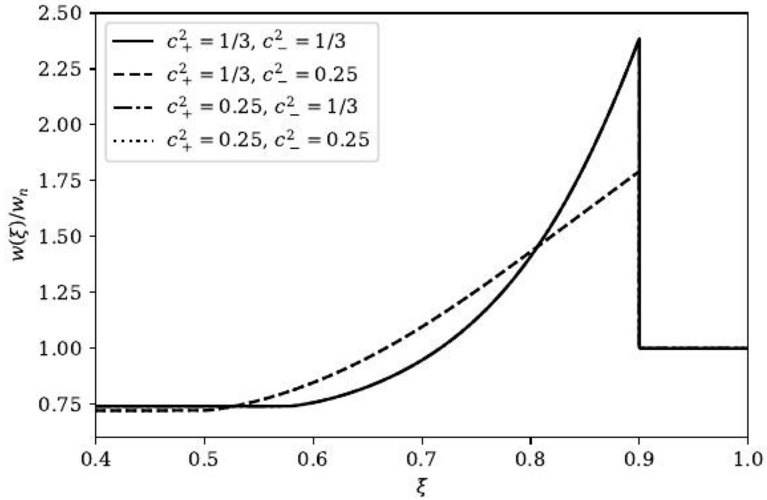
Energy Budget

✓ Boundary conditions of Detonation and the relevant velocity and enthalpy profiles:



$$\tilde{v}_+ = 0, \quad v_+ = v_w, \quad v_- = v_-(\alpha_{\bar{\theta}+}, v_+), \quad v(v_w) = \tilde{v}_- = \mu(v_w, v_-)$$





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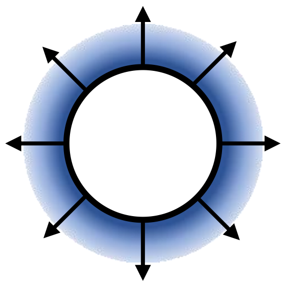
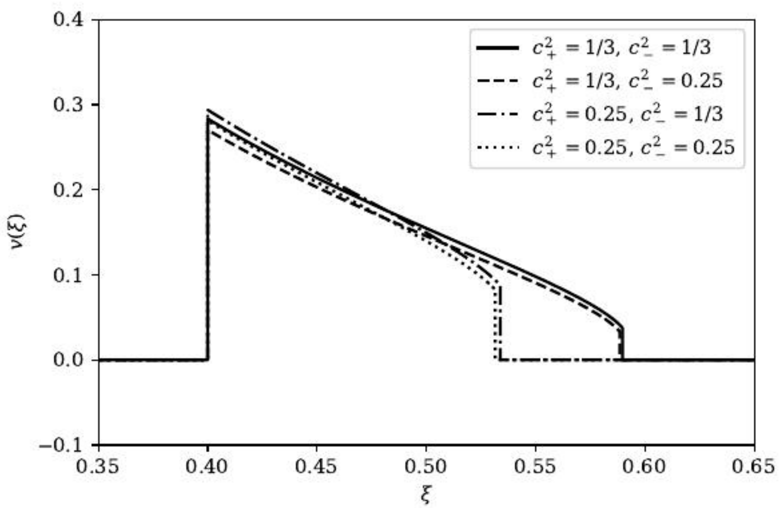
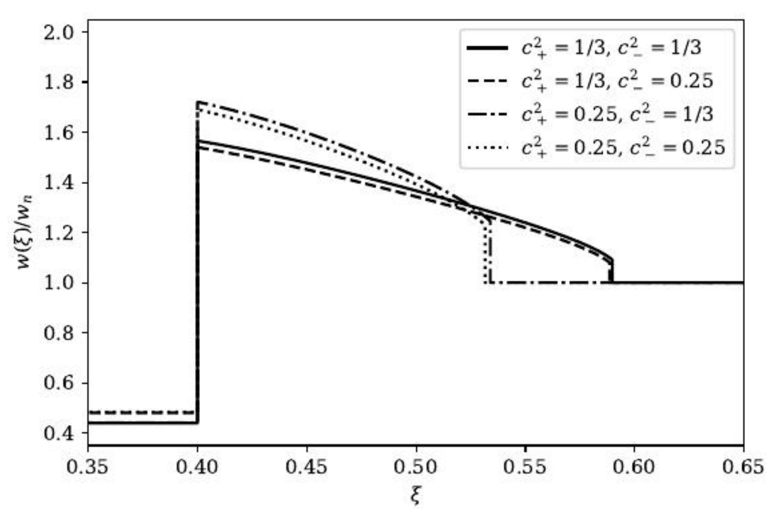
Energy Budget

✓ Boundary conditions of Deflagration and the relevant velocity and enthalpy profiles:



$$\tilde{v}_- = 0, \quad v_- = v_w, \quad v_+ = v_+(\alpha_{\bar{\theta}+}, v_-), \quad \tilde{v}_+ = \mu(v_w, v_+)$$





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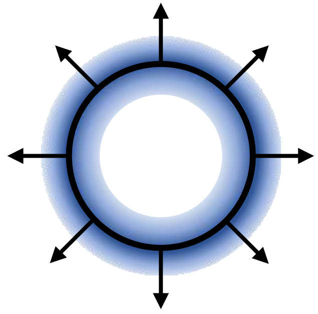
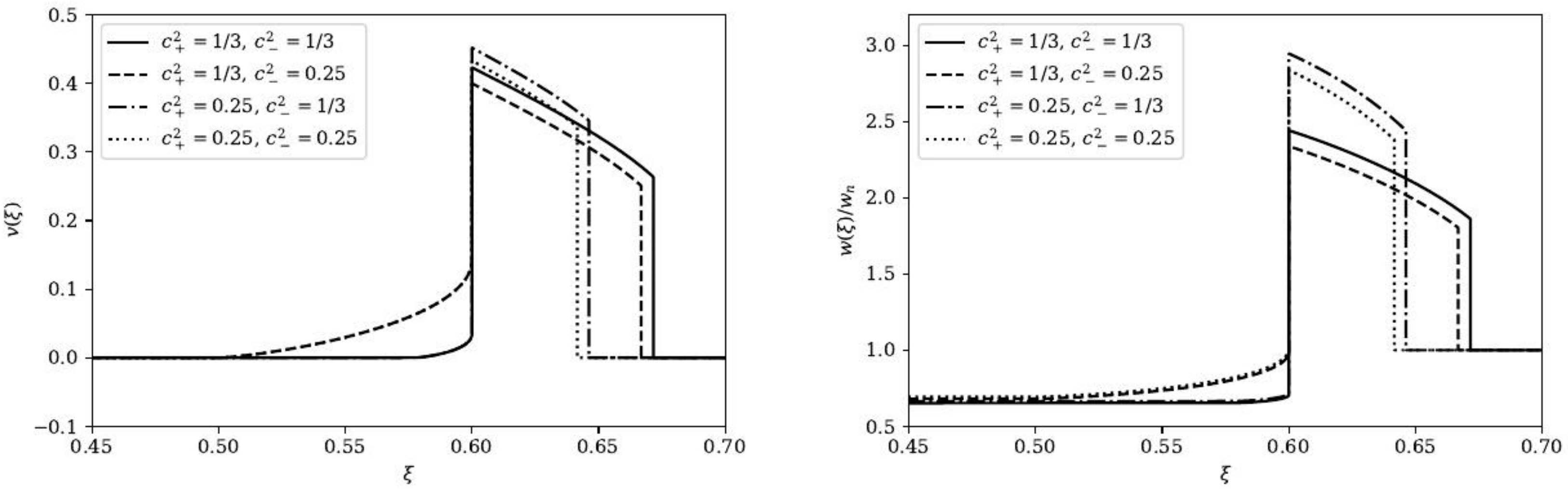
Energy Budget

✓ Boundary conditions of Hybrid and the relevant velocity and enthalpy profiles:

$$v_- = c_-, \quad \tilde{v}_- = \mu(v_w, v_-), \quad v_+ = v_J^{\text{def}}(\alpha_{\bar{\theta}+}), \quad \tilde{v}_+ = \mu(v_w, v_+)$$





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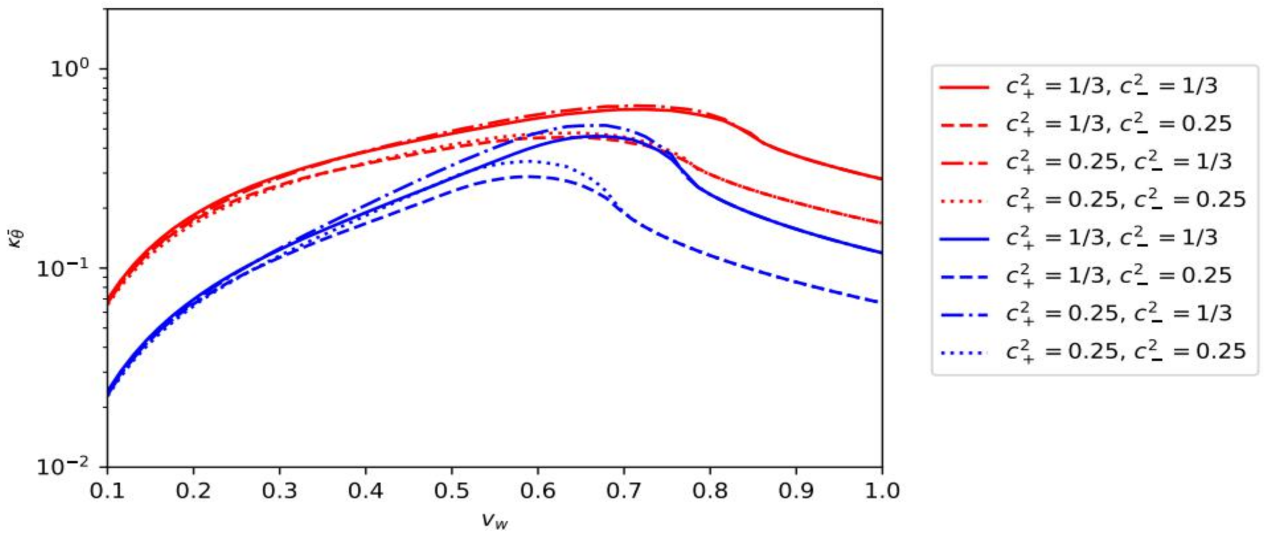
Energy Budget

The efficiency parameters of DSVM：

$$\rho_{\text{fl}} = \frac{3}{v_w^3} \int \xi^2 v^2 \gamma^2 w d\xi \quad \kappa_{\bar{\theta}} = \frac{4\rho_{\text{fl}}}{\Delta\bar{\theta}}$$



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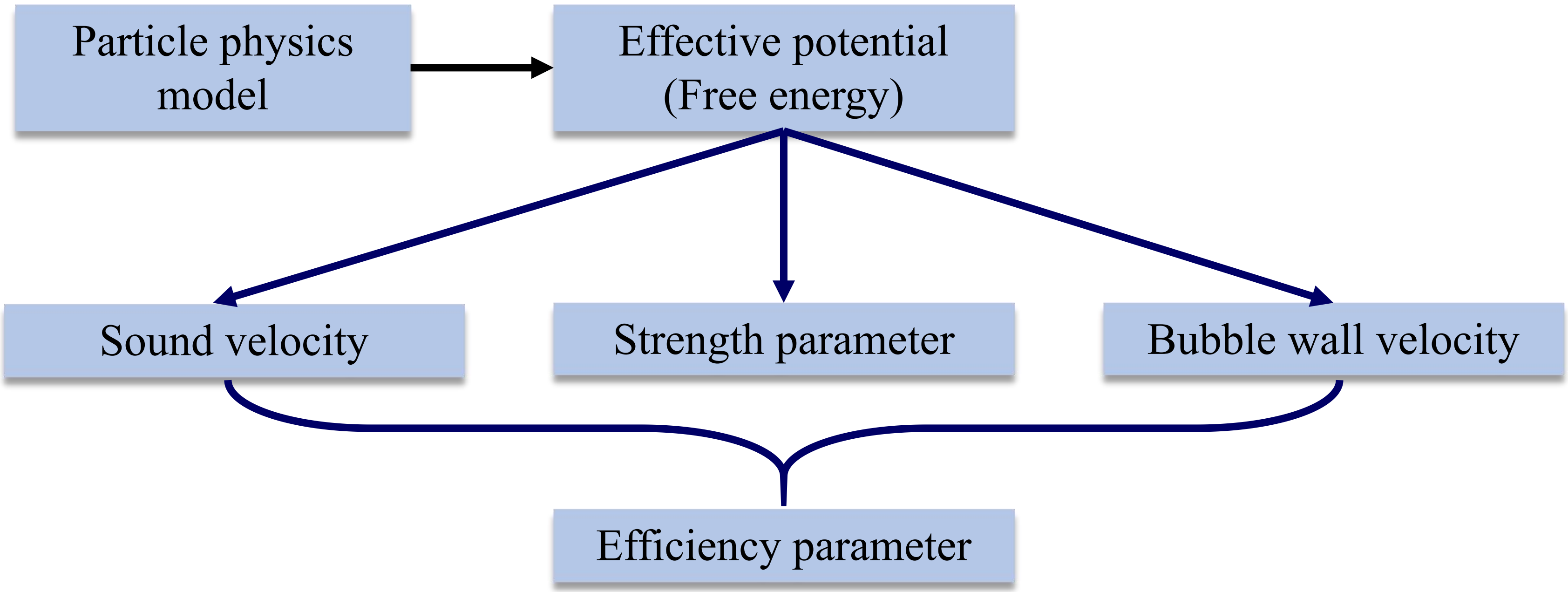
Gravitational Wave

- Mapping a particle physics model on the DSVM to get efficiency parameter:



Gravitational Wave

- The sound velocity of broken phase and symmetric phase in Dim-6 effective model:

$$\mathcal{F}(\phi,T) \approx -\frac{a_\pm}{3}T^4 + \frac{\mu^2+cT^2}{2}\phi^2 + \frac{\lambda}{4}\phi^4 + \frac{\kappa}{8\Lambda^2}\phi^6$$

Sound velocity：

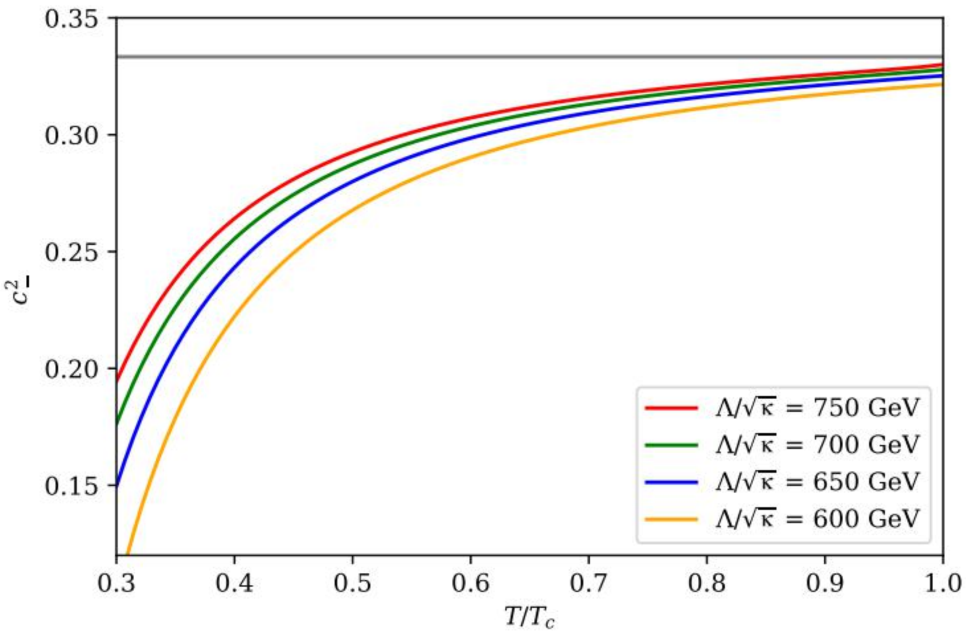
$$c_s^2 = \frac{4a_\pm T^3 - 3cT\phi^2}{12a_\pm T^3 - 3cT\phi^2}$$

$$c_+^2 = \frac{4a_+T^3}{12a_+^2T^3} = \frac{1}{3}$$

$$c_-^2 = \frac{4a_-T^3 - 3cT\phi_{\text{true}}^2}{12a_-T^3 - 3cT\phi_{\text{true}}^2}$$

Strength parameter:

$$\alpha_{\bar{\theta}n} = \frac{(1+1/c_-^2)\Delta V_{\text{eff}} - T\frac{\partial \Delta V_{\text{eff}}}{\partial T}}{3(1+c_+^2)\rho_{\text{R}}}, \quad \rho_{\text{R}} = a_+T_+^4$$



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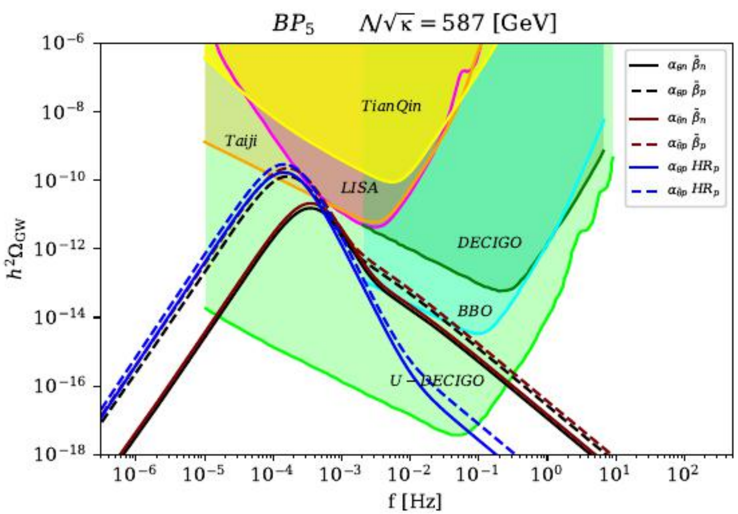
Gravitational Wave

GW spectrum and SNR for different EoS with different parameter combination:



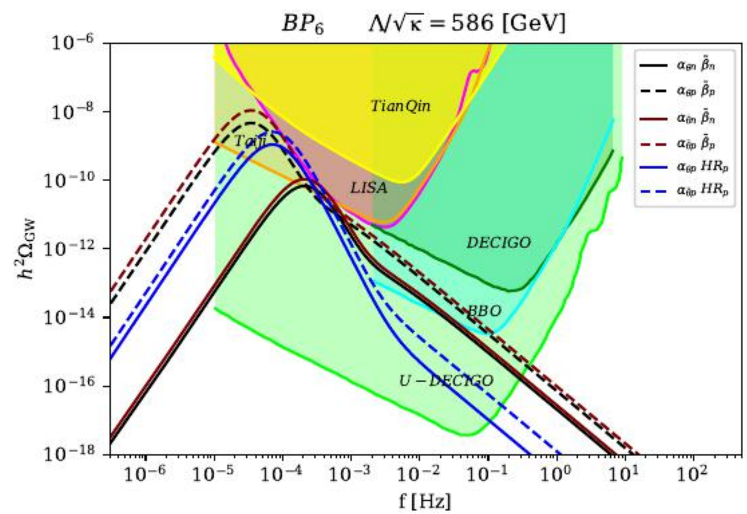


TABLE II. The SNR of BP_5 for different experiment configurations with different combinations of phase transition parameters and models of EOS.

	$\alpha_{\theta n}\,\tilde{\beta}_n$	$\alpha_{\theta p}\,\tilde{\beta}_p$	$\alpha_{\bar{\theta} n}\,\tilde{\beta}_n$	$\alpha_{\bar{\theta} p}\,\tilde{\beta}_p$	$\alpha_{\theta p}\,HR_p$	$\alpha_{\bar{\theta} p}\,HR_p$
$SNR_{(LISA)}$	7.949	16.930	10.913	28.836	16.009	27.468
$SNR_{(Taiji)}$	14.760	58.607	20.271	100.343	66.216	113.609
$SNR_{(TianQin)}$	0.452	1.506	0.620	2.576	1.629	2.794

TABLE III. The SNR of BP_6 for different experiment configurations with different combinations of phase transition parameters and models of EOS.

	$\alpha_{\theta n}\,\tilde{\beta}_n$	$\alpha_{\theta p}\,\tilde{\beta}_p$	$\alpha_{\bar{\theta} n}\,\tilde{\beta}_n$	$\alpha_{\bar{\theta} p}\,\tilde{\beta}_p$	$\alpha_{\theta p}\,HR_p$	$\alpha_{\bar{\theta} p}\,HR_p$
$SNR_{(LISA)}$	14.230	15.368	22.470	26.382	17.367	40.816
$SNR_{(Taiji)}$	38.666	427.813	61.208	1000.501	213.123	500.668
$SNR_{(TianQin)}$	1.060	5.569	1.678	12.934	3.973	9.333

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Conclusions:

- The different sound velocity in different phase can give a non negligible correction to energy budget of phase transition; hence it can affect the strength of GW signal;
- To get a precise prediction of phase transition GWs, a more valid calculation of energy budget is crucial.
- The effect of reheating phenomena are not well incorporated in calculations;
- Comparisons with a full particle physics model dependent calculation deserves a further study.

Thank you!!!