EXPERIMENTAL STUDY OF THE HIGGS SECTOR

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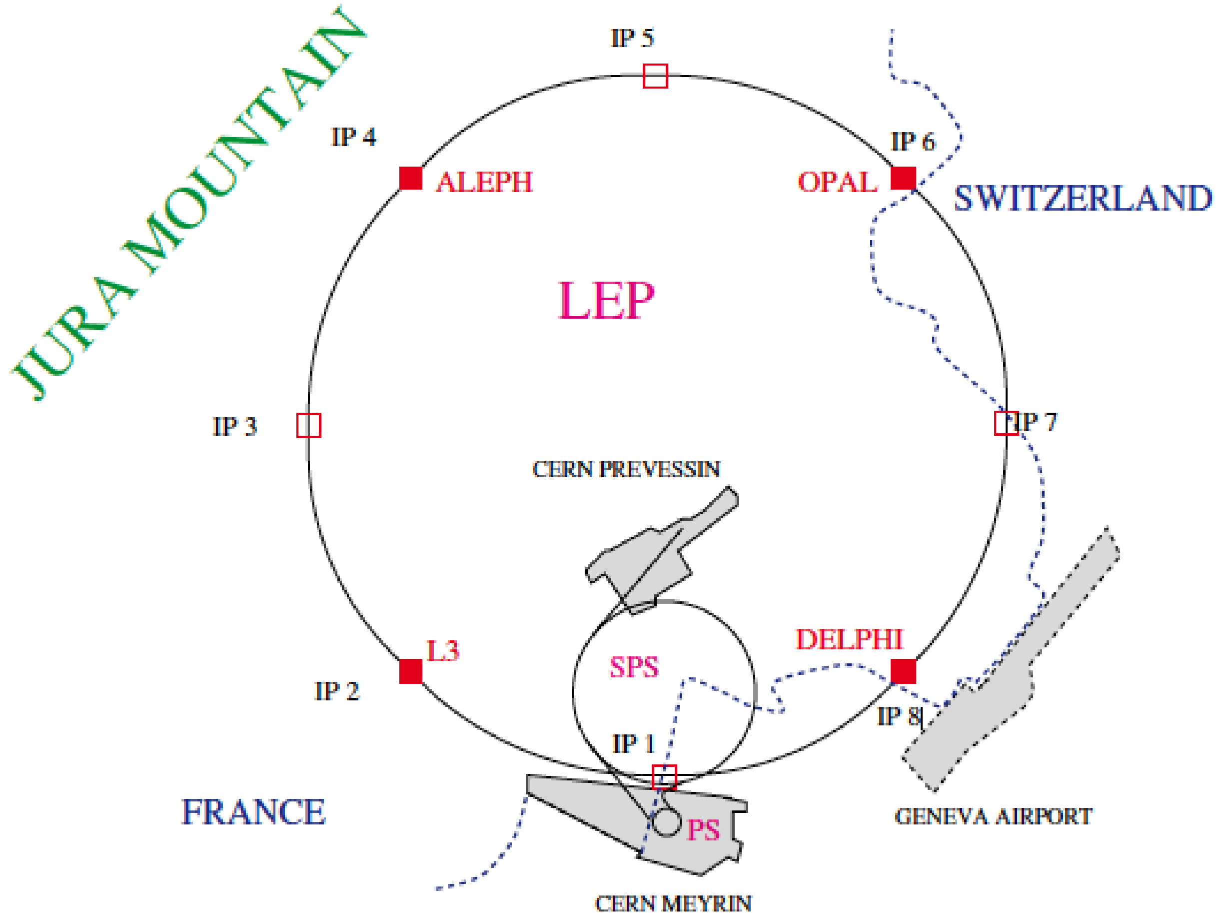
OUTLINE

- Discovery of the H(125) at CMS and ATLAS
  - ATLAS and CMS experiments
  - Reconstruction and particle identifications
  - Statistical methods in HEP
- Latest results related to H(125)

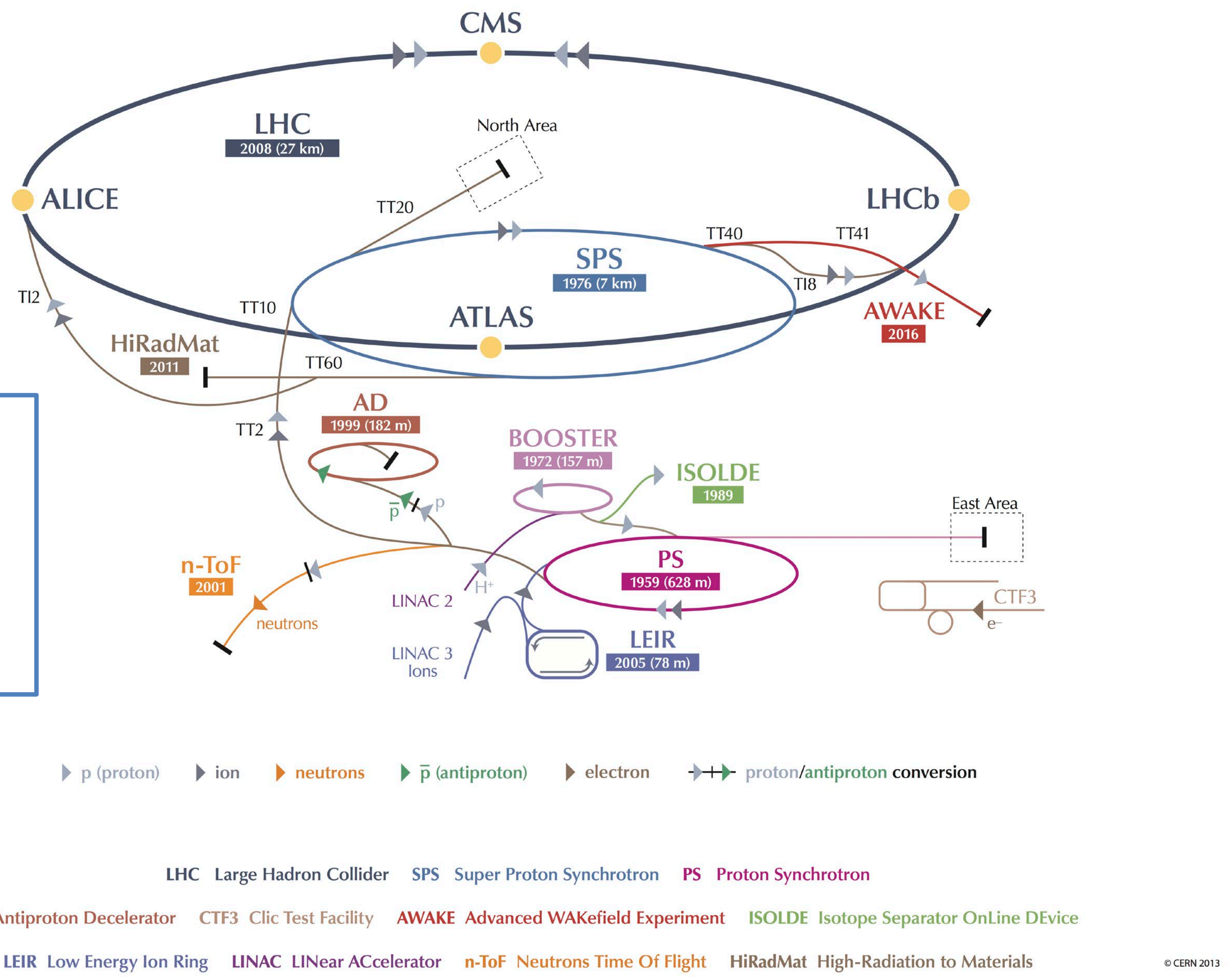
1990

- **1992 LHC Expression of Interest**
- 1995.10 LHC Design report
- 1999.5 LEP reached Ecm = 192 GeV
- **2000 LEP reached Ecm = 200 GeV**
- 2000.11.2 LEP shutdown

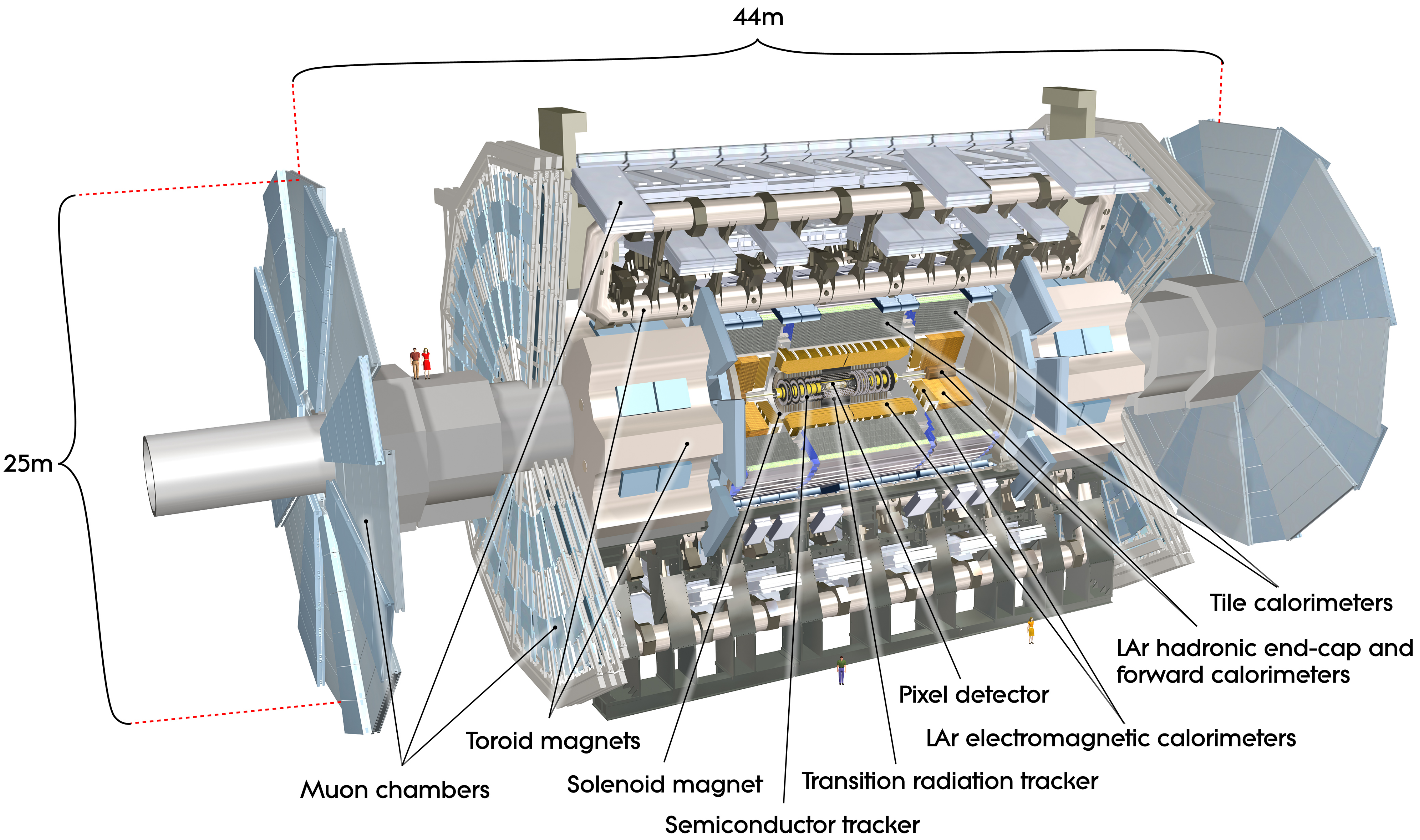


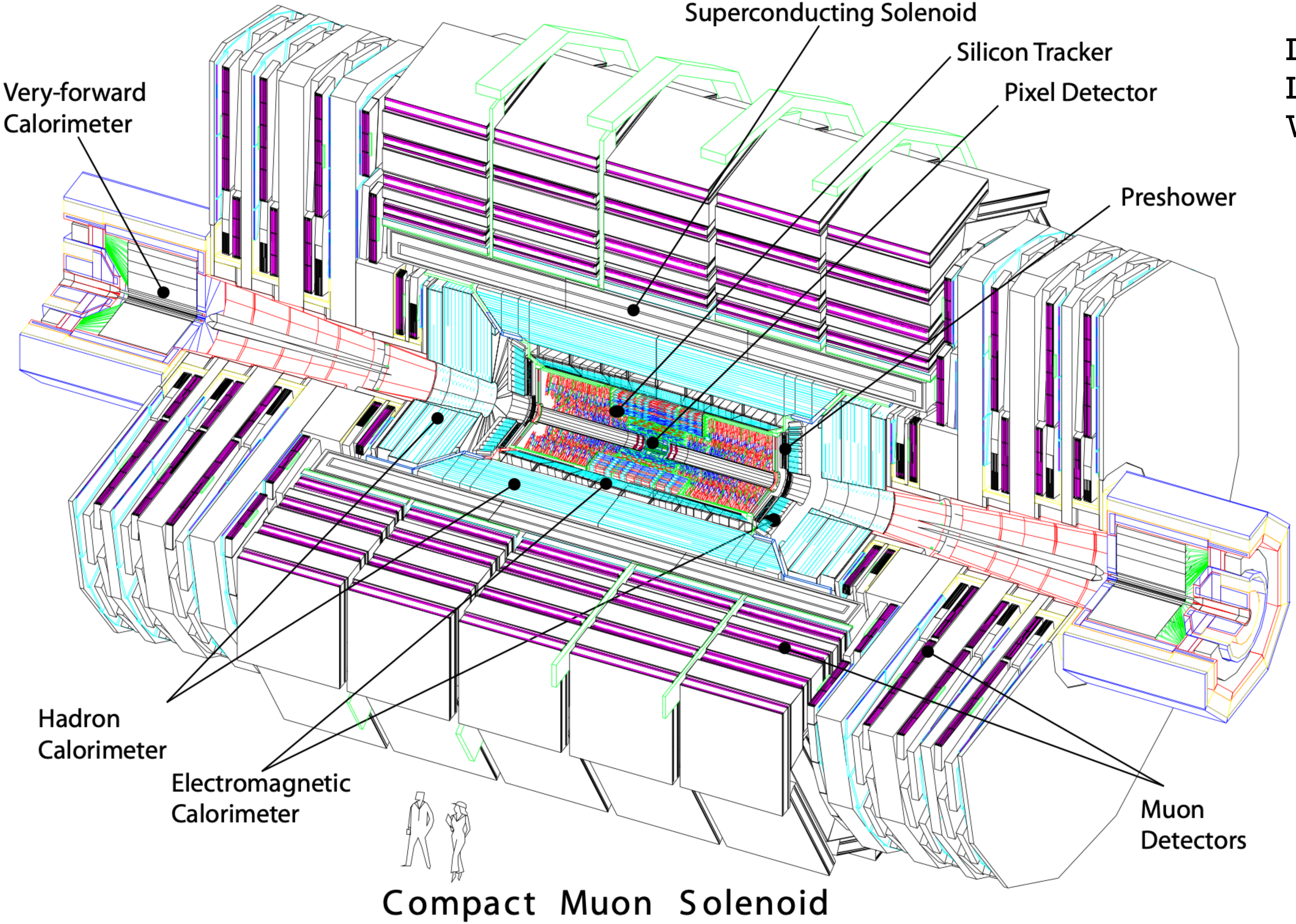
LEP : 1981 – 2000

CERN's Accelerator Complex

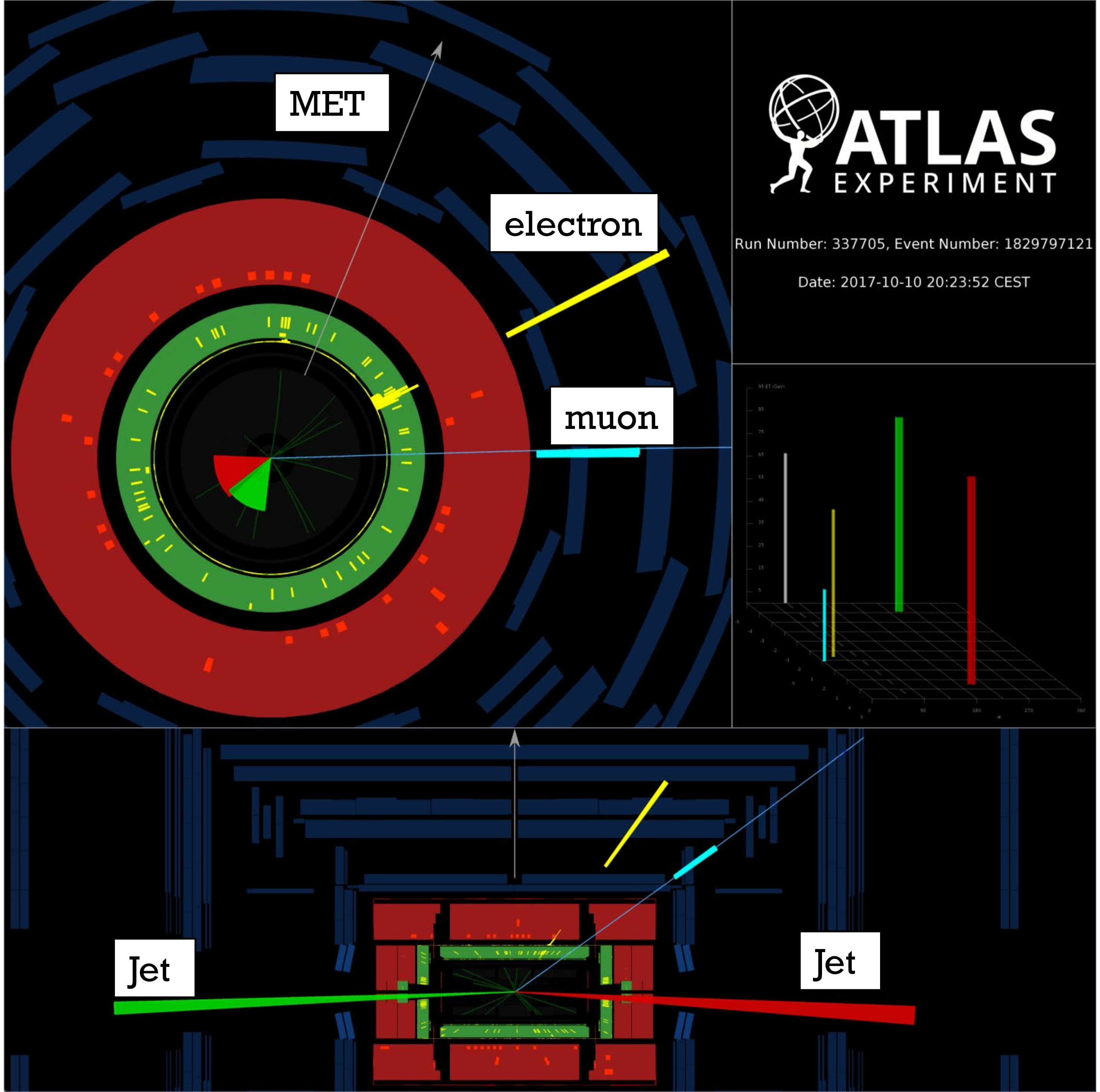


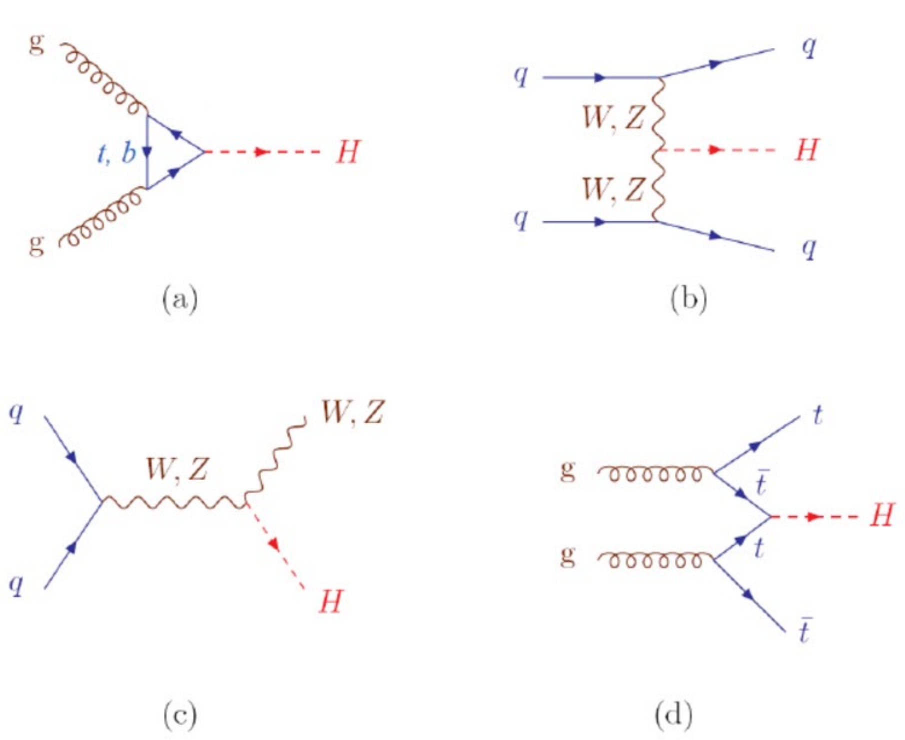
LHC time line :
1992 : expressions of interest
1995.10 : LHC TDR published
2009.11: first pp collisons at CME=900 GeV
2010.2: first pp collisions at CME=7 TeV
2011: CMS and ATLAS recorded ~5/fb of data
2012: CMS and ATLAS ~20/fb at CME=8 TeV
2015 – 2018 data-taking at CME = 13 TeV


44m
25m
Tile calorimeters
LAr hadronic end-cap and forward calorimeters
Pixel detector
LAr electromagnetic calorimeters
Toroid magnets
Muon chambers
Solenoid magnet
Transition radiation tracker
Semiconductor tracker



Diameter 14.6 m
Length 21.6 m
Weight 12500 t

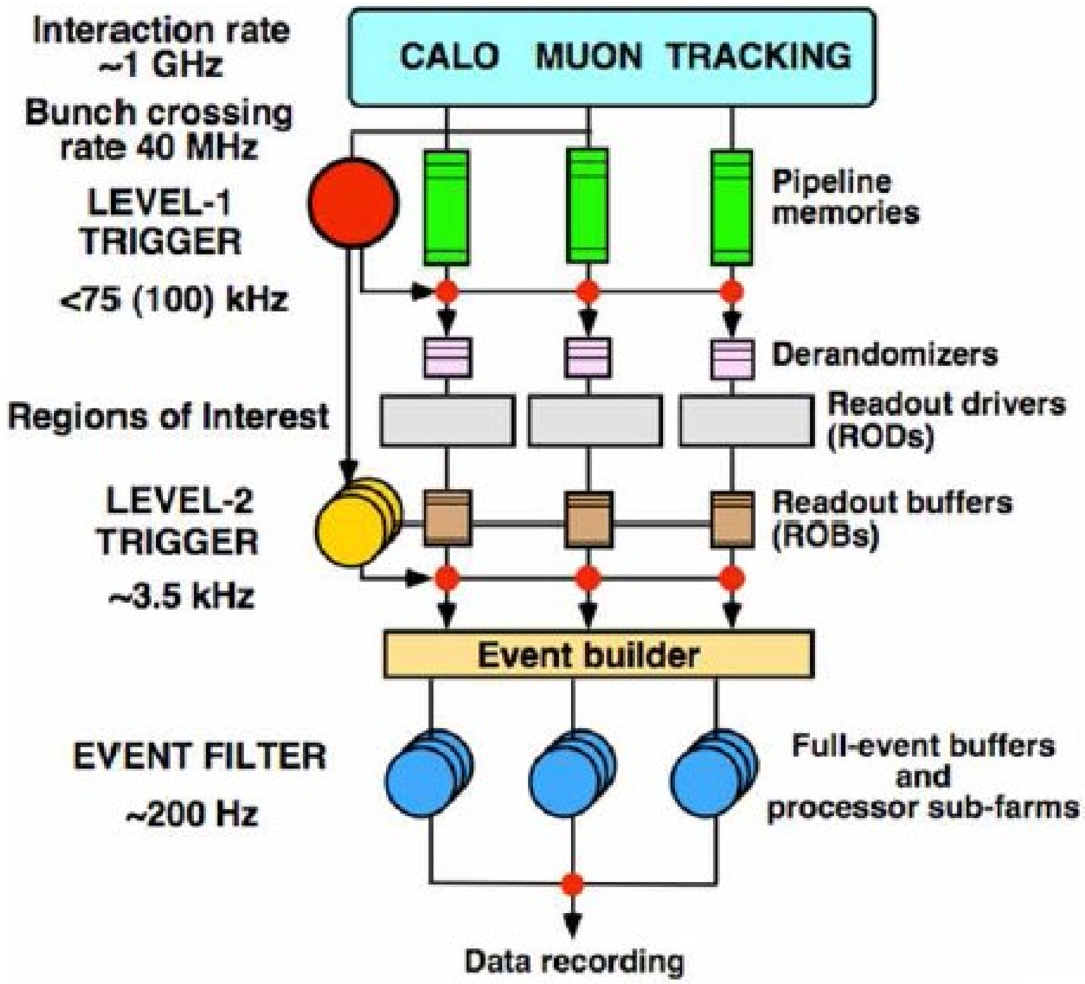




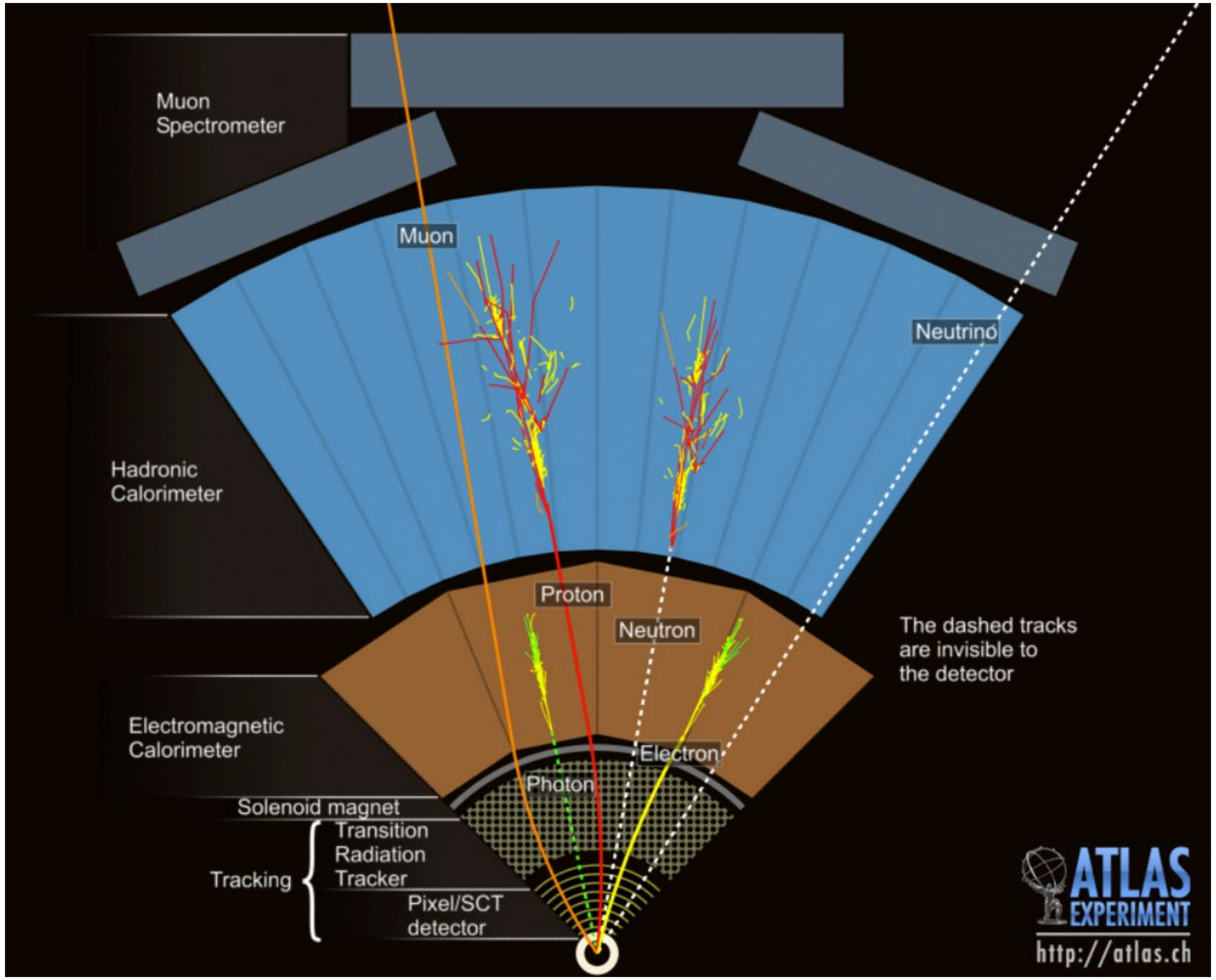
An event display

VBF H, H→W+W-

ATLAS TRIGGER AND DATA ACQUISTION

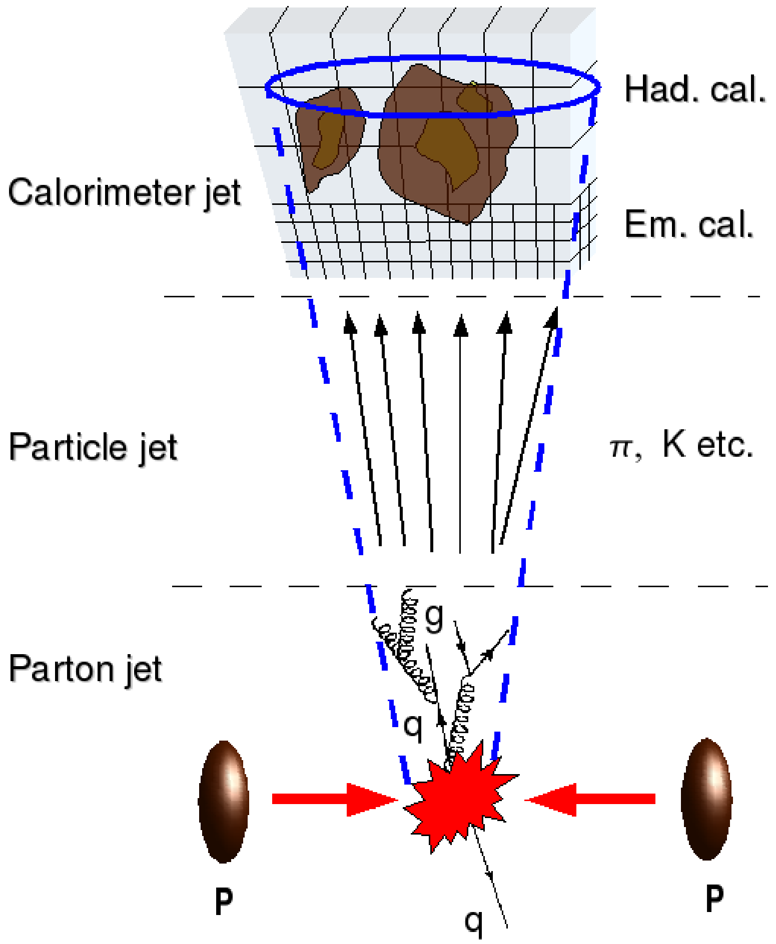


FINAL STATE SIGNATURES



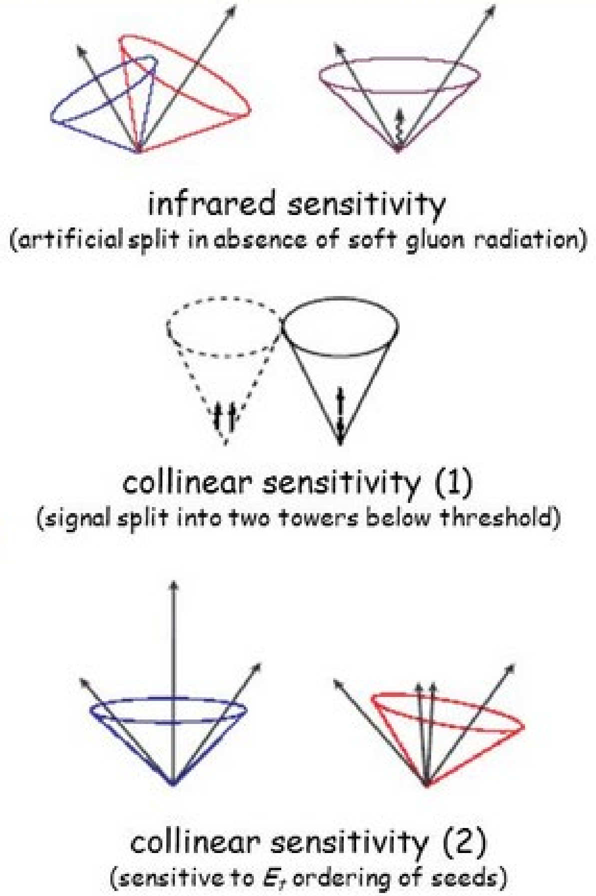
Electron
Photon
Proton
Neutron
Neutrino

JET ALGORITHM



Theoretical requirement to jet algorithm choices

- Infrared safety
  - Adding or removing soft particles should not change the result of jet clustering
- Collinear safety
  - Splitting of large p_T particle into two collinear particles should not affect the jet finding
- Invariance under boost
  - Same jets in lab frame of reference as in collision frame
- Order independence
  - Same jet from partons, particles, detector signals
- Many jet algorithms don't fulfill above requirements!



ParticleID, Nov 30, 2011 37

http://arxiv.org/abs/hep-ph/0512210v2

JET FINDING ALGORITHM (KT)

1. For each pair of particles i, j work out the k_t distance $d_{ij} = \min(k_{ti}^2, k_{tj}^2)R_{ij}^2$ with $R_{ij}^2 = (\eta_i - \eta_j)^2 + (\phi_i - \phi_j)^2$, where k_{ti}, η_i and ϕ_i are the transverse momentum, rapidity and azimuth of particle i; for each parton i also work out the beam distance $d_{iB} = k_{ti}^2$.

2. Find the minimum $d_{\min}$ of all the d_{ij}, d_{iB}. If $d_{\min}$ is a d_{ij} merge particles i and j into a single particle, summing their four-momenta (alternative recombination schemes are possible); if it is a d_{iB} then declare particle i to be a final jet and remove it from the list.

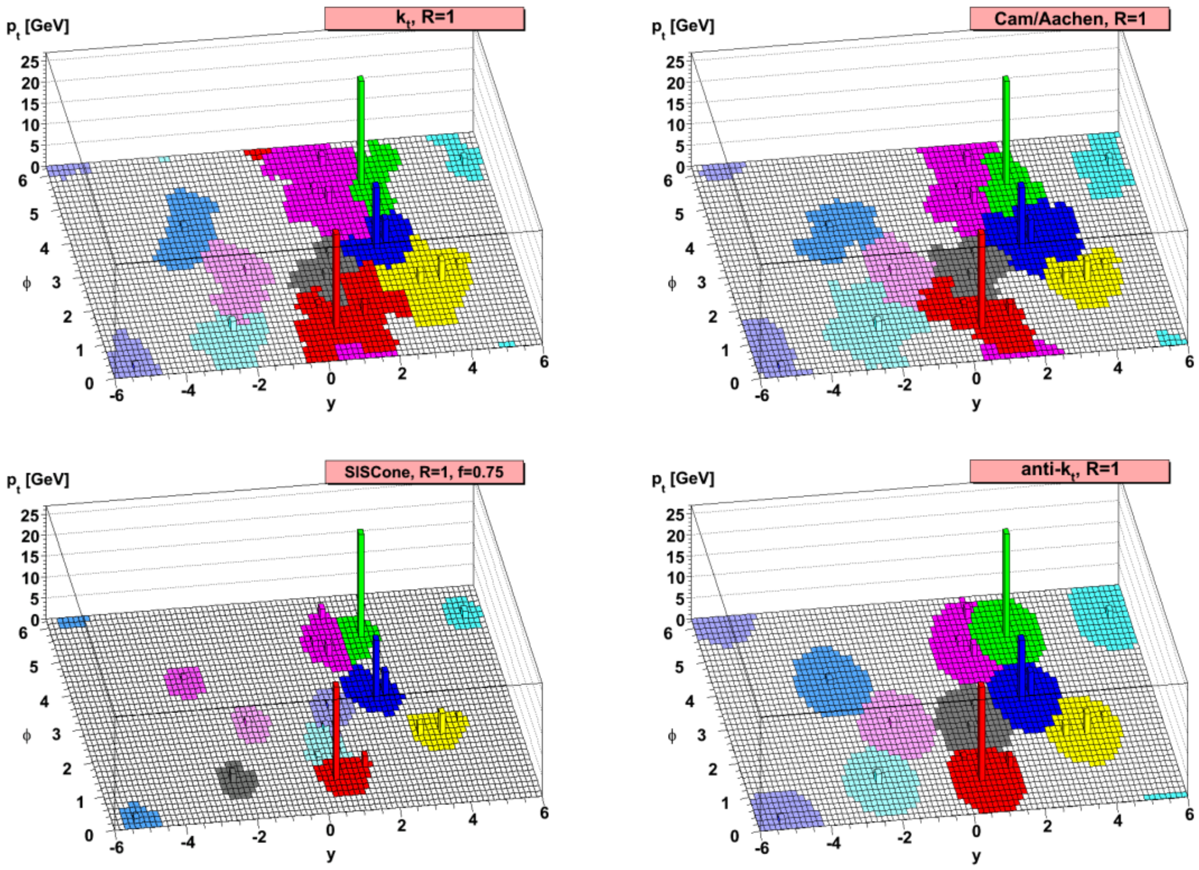
3. Repeat from step 1 until no particles are left.

http://arxiv.org/abs/0802.1189v2

GENERALIZATIONS (ANTI-KT, CAMBRIDGE/AACHEN)

$$d_{ij} = \min(k_{ti}^{2p}, k_{tj}^{2p}) \frac{\Delta_{ij}^2}{R^2},$$

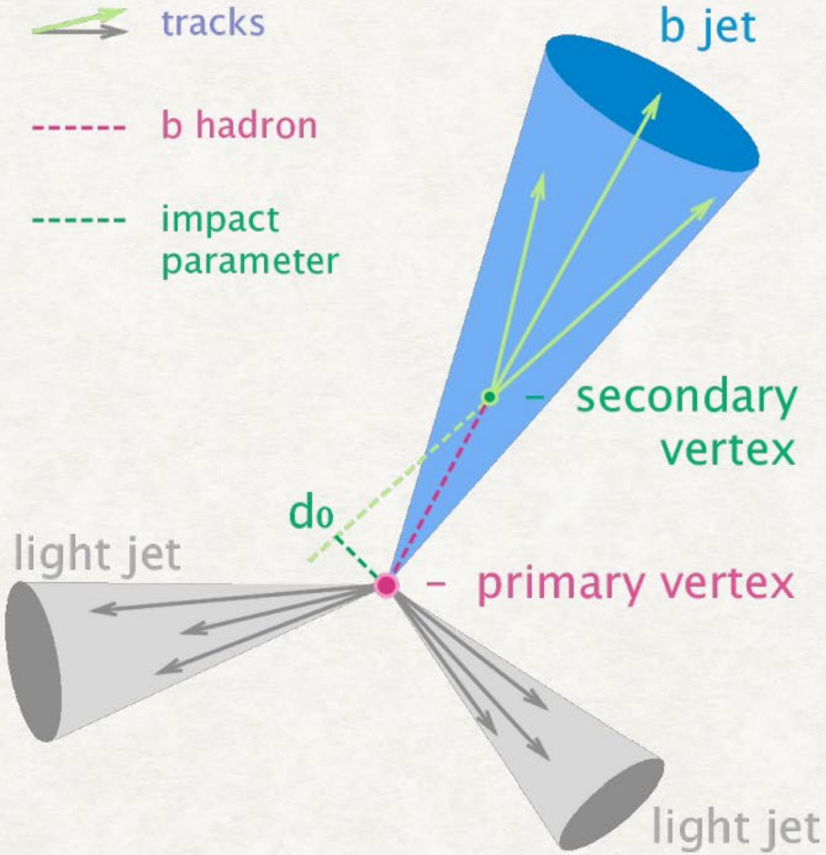
$$d_{iB} = k_{ti}^{2p},$$



p=0 => CA
p=1 => kT
p=-1 => anti-kT

B-TAGGING

- MV2c10 algorithm
  - A **BDT** classifier
  - Training on simulated ttbar events
    -signal: b-jets
    -background: light-jets (90%) and charm-jets (10%)
- **Input variables related to the features of b-jet**
  - Large positive impact parameters (**IP**)
  - Significant secondary vertex (**SV**)
  - B-hadron decay chain inside the jet-cone (**JF**)

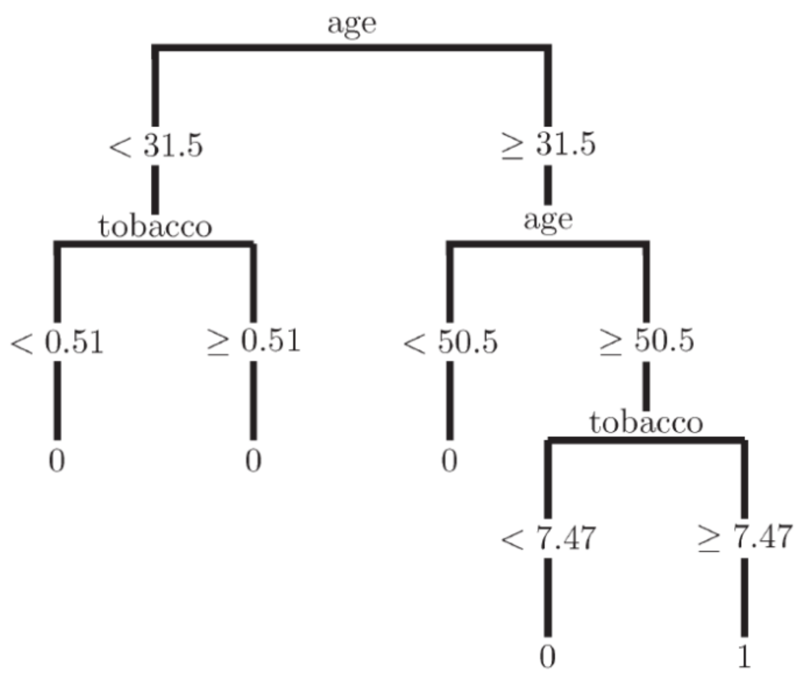


BOOSTED DECISION TREE

树分类

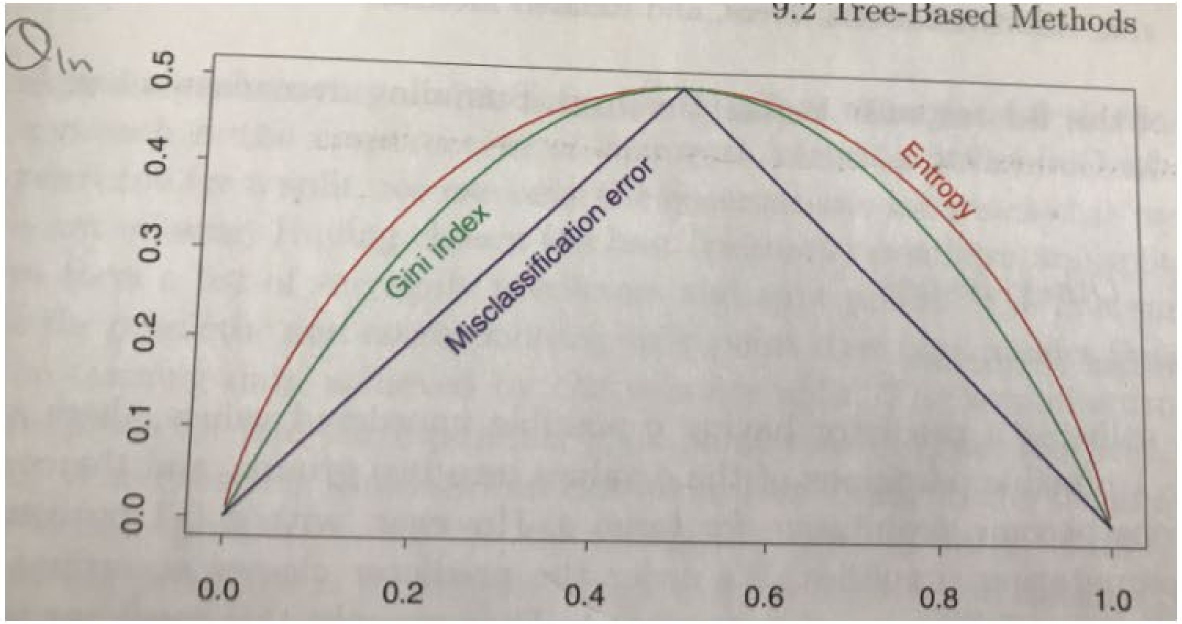
数据形式，(X, Y) 例：X_1 为年龄，X_2为血压，$Y \in \{0, 1\}$表示心脏是否健康

1. 选择一个标准将数据样本二分。
   - 选择协变量，X_i以及分割点t, $A_1 : X_i \geq t$ vs $A_2 : X_i < t$
   - 计算混杂度: A_1中，$Y = 1$的比例为p_1, 混杂度为$\gamma_1 = 2p_1(1 - p_1)$, A_2中，$Y = 1$的比例为p_2, 混杂度为$\gamma_2 = 2p_2(1 - p_2)$, 此二分混杂度为$I(t) = \gamma_1 + \gamma_2$
   - 选择混杂度最低的二分，将样本一分为二。
2. 在A_1,A_2上重复上述步骤。
3. 终止条件
   - 每个"叶子"上的数据数小于n_0
   - 树叶数大于某个事先给定的数。
4. 清点所有树叶，以$Y = 1$为主的树叶，作为1型叶，，以$Y = 1$为主的树叶，作为0型叶



$$I(t) = \sum_{s=1}^{2} \gamma_s$$

$$\gamma_s = \begin{cases} 1 - \max(p, 1-p) & \text{misclassification error} \\ p(1-p) & \text{Gini index} \\ -p \ln p - (1-p)\ln(1-p) & \text{cross entropy} \end{cases}$$



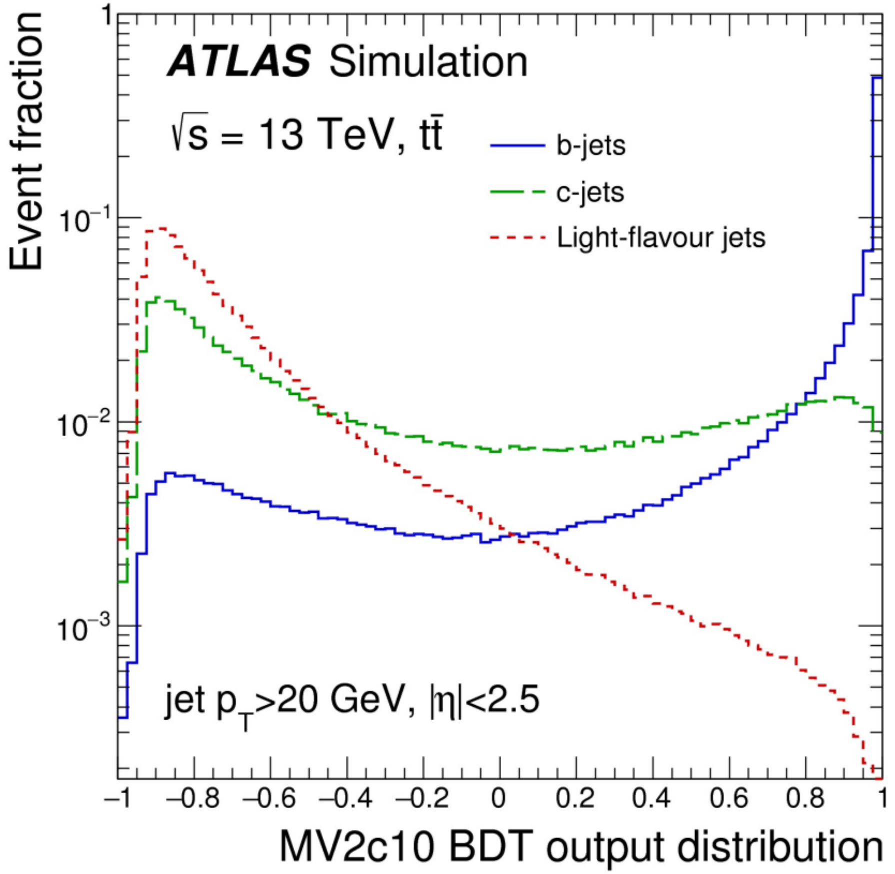
BOOST

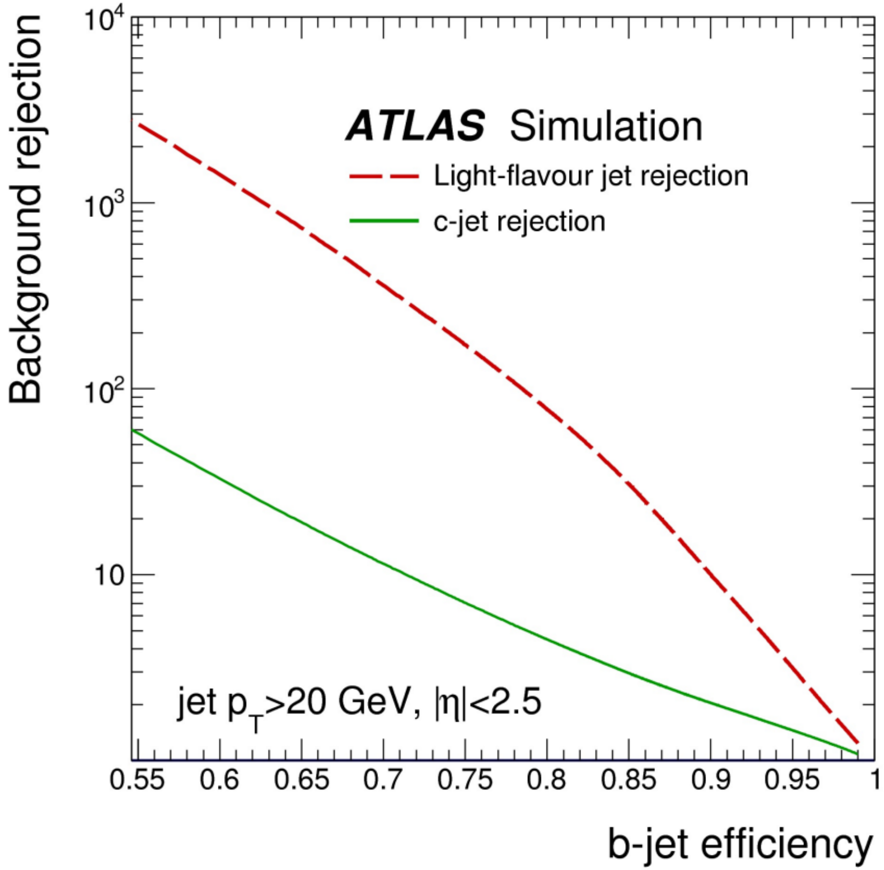
AdaBoost(原始版本)

为了数学上简单，Y的取值范围$\{0,1\} \to \{-1,1\}$,设训练样本大小为n

1. 设定每个数据的权重$w_i = 1/n, i = 1, \cdots, n$
2. 对$j = 1, \cdots, J$执行以下步骤
   - 用权重$w_1, \cdots, w_n$从数据中构造分类器h_j
   - 计算加权误差估计：$\hat{L}_j = \frac{\sum w_i I(y_i \neq h_j(x_i))}{\sum w_i}$
   - 令$\alpha_j = \ln \frac{1-\hat{L}_j}{\hat{L}_j}$
   - 更新数据权重: $w_i \leftarrow w_i e^{\alpha_j I(y_i \neq h_j(x_i))}$
3. 最终分类器为$\hat{h}(x) = \text{sign}(\sum_{j=1}^{J} \alpha_j h_j(x)))$

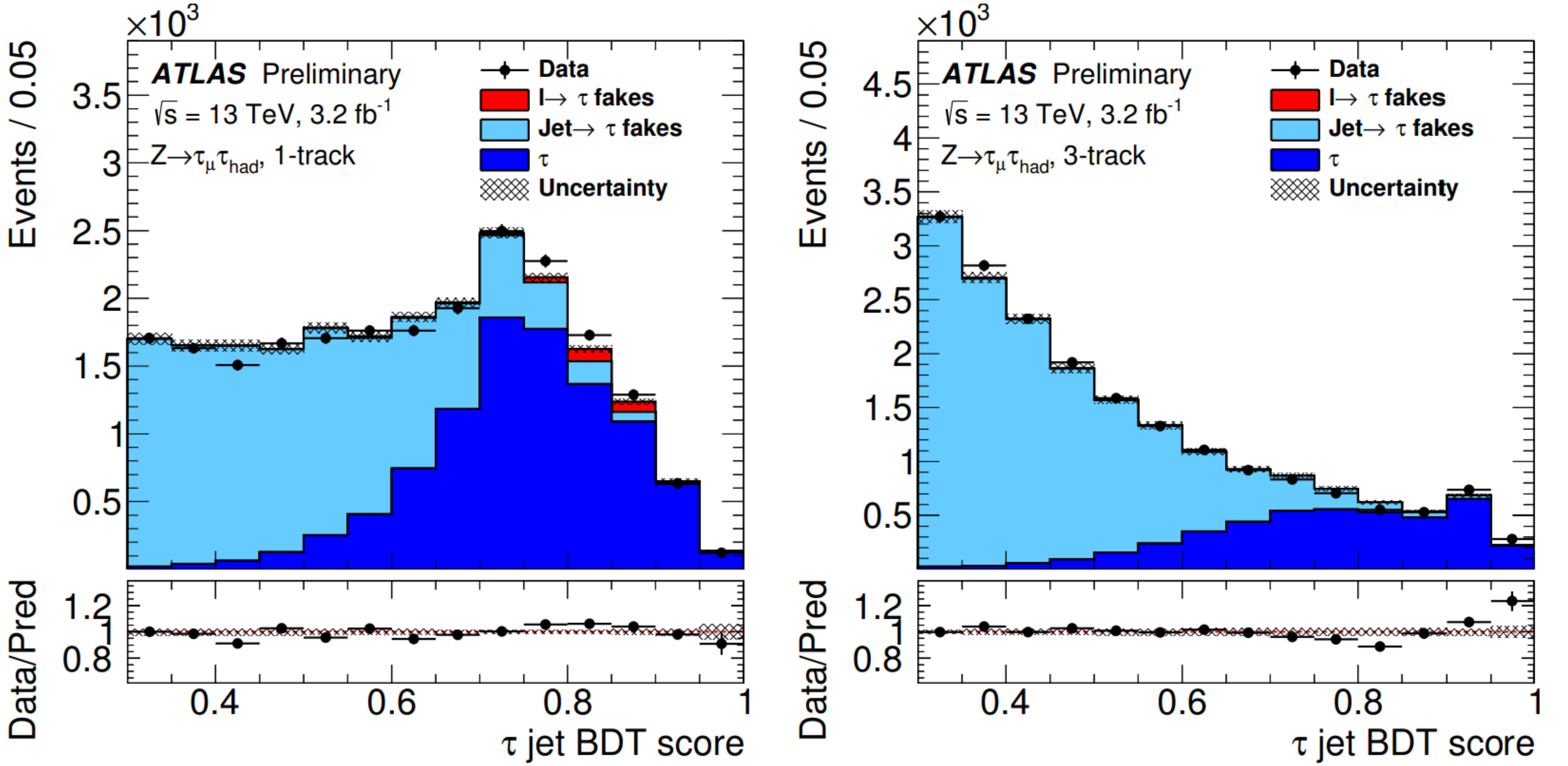
B-TAGGING PERFORMANCE





ATLAS-CONF-2017-029

TAU IDENTIFICATION

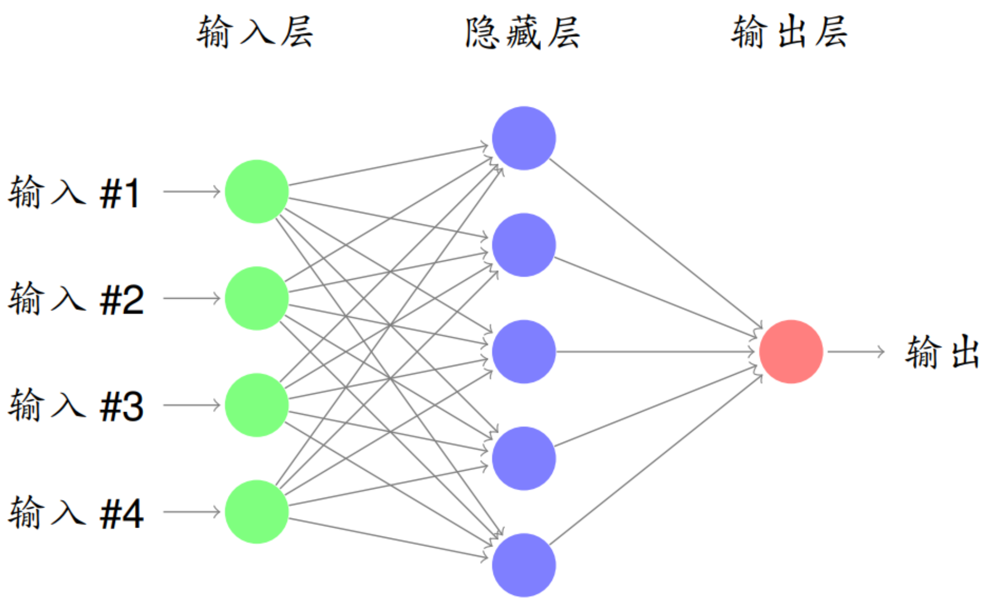


ARTIFICIAL NEURAL NET

统计和人工智能分别发展出来的机器学习方法。将输入参数线性组合,得到"derived feature", 用非线性函数来Model输出与"feature"之间的关系。

Projection Pursuit Regression (PPR)

- 输入X, p维
- 输出Y
- ω_m为线性组合的系数， $m = 1, \cdots, M$,每一个都是p维向量。
- Derived feature : $V_m = \omega_m^T X$
- $Y = f(X) = \sum_{m=1}^{M} g_m(V_m)$
- 拟合模型: 最小化方差 : $\sum_{i=1}^{n}[Y_i - \sum_{m=1}^{M} g_m(\omega_m^T X_i)]^2$ (数值解法)



ANN MATH

- 记输入层神经元为：$X_1\cdots,X_p$, 隐藏层：$Z_1,\cdots,Z_M$ 输出层可以有多个节点,$Y_1,\cdots,Y_K$
- $Z_m=\sigma(\alpha_{0m}+\alpha_m^T X), m=1,\cdots,M$. PPR...
- $T_k=\beta_{0k}+\beta_k^T Z,\ Y_k=g_k(T_k),\ k=1,\cdots,K$. PPR...
- 通常取

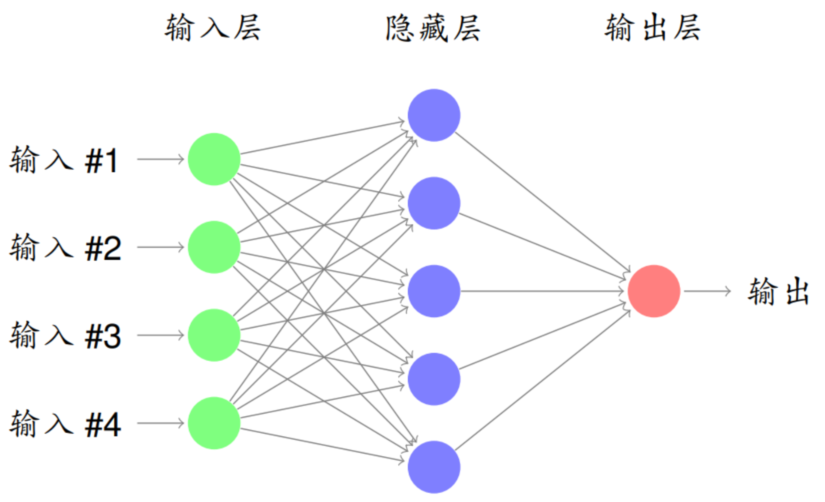
$$\sigma(v)=\frac{1}{1+\exp(-v)}$$

$$g_k(T)=\frac{e^{T_k}}{\sum e^{T_l}}$$

- 系数α,β称作"权重"，神经元网络的训练：确定权重使得方差

$$R(\theta)=\sum_{k=1}^{K}\sum_{i=1}^{N}(y_{ik}-f_k(x_i))^2$$

取最小. 需要数值算法。常用下坡法。



[Deep ANN b-tagger (ATLAS)]

arXiv:1207.7214v2

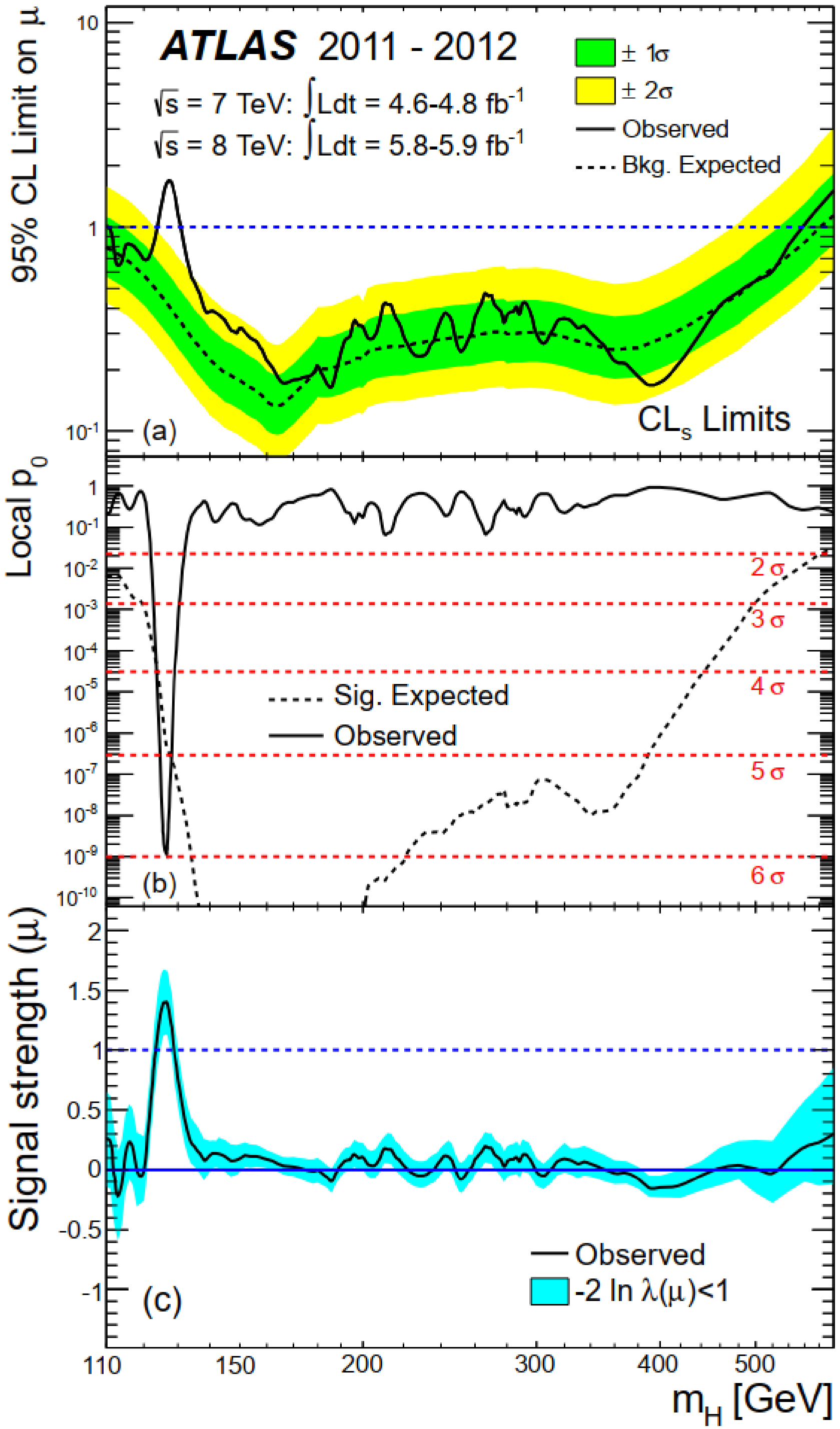
DISCOVERY PAPER(ATLAS)

Dataset	L(fb^{-1})	Higgs decay channels
2011 7 TeV pp	4.8	ZZ*, WW*,bb,$\tau\tau$, 4-lepton(improved), $\gamma\gamma$ (improved)
2012 8 TeV pp	5.8	ZZ*(4-lepton),$\gamma\gamma$, WW* →$e\nu\mu\nu$

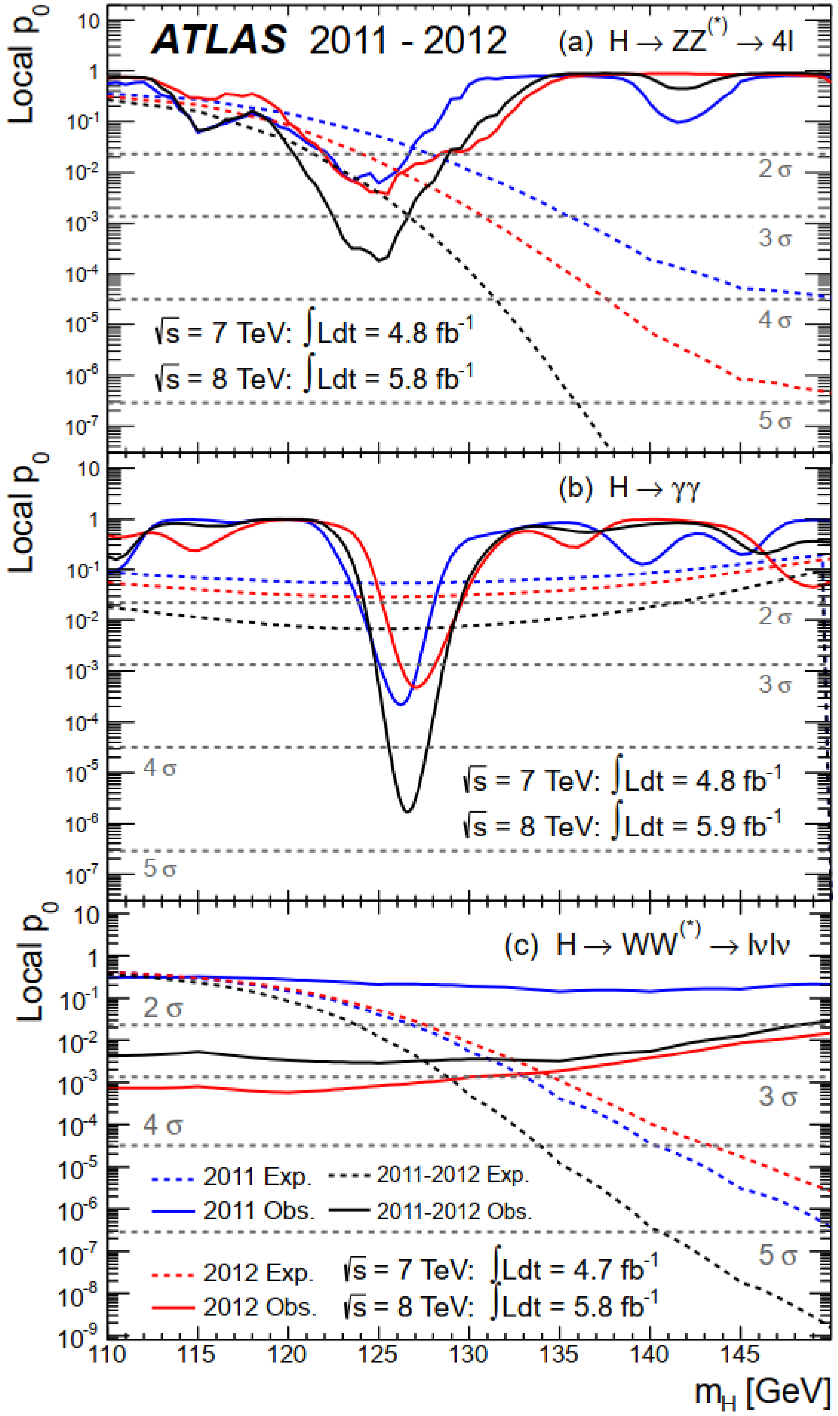
Abstract

A search for the Standard Model Higgs boson in proton-proton collisions with the ATLAS detector at the LHC is presented. The datasets used correspond to integrated luminosities of approximately 4.8 fb^{-1} collected at $\sqrt{s} = 7$ TeV in 2011 and 5.8 fb^{-1} at $\sqrt{s} = 8$ TeV in 2012. Individual searches in the channels $H \to ZZ^{(*)} \to 4\ell$, $H \to \gamma\gamma$ and $H \to WW^{(*)} \to e\nu\mu\nu$ in the 8 TeV data are combined with previously published results of searches for $H \to ZZ^{(*)}$, $WW^{(*)}$, $b\bar{b}$ and $\tau^+\tau^-$ in the 7 TeV data and results from improved analyses of the $H \to ZZ^{(*)} \to 4\ell$ and $H \to \gamma\gamma$ channels in the 7 TeV data. Clear evidence for the production of a neutral boson with a measured mass of 126.0 ± 0.4 (stat) ± 0.4 (sys) GeV is presented. This observation, which has a significance of 5.9 standard deviations, corresponding to a background fluctuation probability of 1.7×10^{-9}, is compatible with the production and decay of the Standard Model Higgs boson.

Higgs Boson Decay	Subsequent Decay	Sub-Channels	m_H Range [GeV]	$\int L\,dt$ [fb^{-1}]
2011 $\sqrt{s}$ =7 TeV				
$H \rightarrow ZZ^{(*)}$	4ℓ	$\{4e, 2e2\mu, 2\mu 2e, 4\mu\}$	110–600	4.8
	$\ell\ell\nu\bar{\nu}$	$\{ee, \mu\mu\} \otimes$ {low, high pile-up}	200–280–600	4.7
	$\ell\ell q\bar{q}$	{b-tagged, untagged}	200–300–600	4.7
$H \rightarrow \gamma\gamma$	–	10 categories $\{p_{\mathrm{Tt}} \otimes \eta_\gamma \otimes \text{conversion}\} \oplus$ {2-jet}	110–150	4.8
$H \rightarrow WW^{(*)}$	$\ell\nu\ell\nu$	$\{ee, e\mu/\mu e, \mu\mu\} \otimes$ {0-jet, 1-jet, 2-jet} $\otimes$ {low, high pile-up}	110–200–300–600	4.7
	$\ell\nu qq'$	$\{e, \mu\} \otimes$ {0-jet, 1-jet, 2-jet}	300–600	4.7
$H \rightarrow \tau\tau$	$\tau_{\mathrm{lep}}\tau_{\mathrm{lep}}$	$\{e\mu\} \otimes$ {0-jet} $\oplus \{\ell\ell\} \otimes$ {1-jet, 2-jet, VH}	110–150	4.7
	$\tau_{\mathrm{lep}}\tau_{\mathrm{had}}$	$\{e, \mu\} \otimes$ {0-jet} $\otimes \{E_{\mathrm{T}}^{\mathrm{miss}} < 20$ GeV, $E_{\mathrm{T}}^{\mathrm{miss}} \geq 20$ GeV} $\oplus \{e, \mu\} \otimes$ {1-jet} $\oplus \{\ell\} \otimes$ {2-jet}	110–150	4.7
	$\tau_{\mathrm{had}}\tau_{\mathrm{had}}$	{1-jet}	110–150	4.7
$VH \rightarrow Vbb$	$Z \rightarrow \nu\nu$	$E_{\mathrm{T}}^{\mathrm{miss}} \in \{120-160, 160-200, \geq 200$ GeV}	110–130	4.6
	$W \rightarrow \ell\nu$	$p_{\mathrm{T}}^{W} \in \{< 50, 50-100, 100-200, \geq 200$ GeV}	110–130	4.7
	$Z \rightarrow \ell\ell$	$p_{\mathrm{T}}^{Z} \in \{< 50, 50-100, 100-200, \geq 200$ GeV}	110–130	4.7
2012 $\sqrt{s}$ =8 TeV				
$H \rightarrow ZZ^{(*)}$	4ℓ	$\{4e, 2e2\mu, 2\mu 2e, 4\mu\}$	110–600	5.8
$H \rightarrow \gamma\gamma$	–	10 categories $\{p_{\mathrm{Tt}} \otimes \eta_\gamma \otimes \text{conversion}\} \oplus$ {2-jet}	110–150	5.9
$H \rightarrow WW^{(*)}$	$e\nu\mu\nu$	$\{e\mu, \mu e\} \otimes$ {0-jet, 1-jet, 2-jet}	110–200	5.8



ATLAS 2011 - 2012
√s = 7 TeV: ∫Ldt = 4.6-4.8 fb-1
√s = 8 TeV: ∫Ldt = 5.8-5.9 fb-1
± 1σ
± 2σ
Observed
Bkg. Expected
95% CL Limit on μ
(a)
CLs Limits
Local p0
Sig. Expected
Observed
2 σ
3 σ
4 σ
5 σ
6 σ
(b)
Signal strength (μ)
Observed
-2 ln λ(μ)<1
(c)
110
150
200
300
400
500
mH [GeV]



ATLAS 2011 - 2012
(a) H → ZZ(*) → 4l
Local p0
√s = 7 TeV: ∫Ldt = 4.8 fb-1
√s = 8 TeV: ∫Ldt = 5.8 fb-1
2 σ
3 σ
4 σ
5 σ
(b) H → γγ
√s = 7 TeV: ∫Ldt = 4.8 fb-1
√s = 8 TeV: ∫Ldt = 5.9 fb-1
(c) H → WW(*) → lνlν
2011 Exp.
2011-2012 Exp.
2011 Obs.
2011-2012 Obs.
2012 Exp.
2012 Obs.
√s = 7 TeV: ∫Ldt = 4.7 fb-1
√s = 8 TeV: ∫Ldt = 5.8 fb-1
110
115
120
125
130
135
140
145
mH [GeV]

置信区间（误差的统计意义）

称参数θ的$1-\alpha$置信区间为$C_n=(a,b)$, 其中 $a=a(X_1,\cdots,X_n)$, $b=b(X_1,\cdots,X_n)$是数据的函数， 如果：

$$P_\theta(\theta\in C_n)\geq 1-\alpha,\theta\in\Theta$$

即(a,b)覆盖参数的概率为$1-\alpha$ 常用$\alpha=0.05$

注意: θ是固定的， C_n是随机的。 并不是说θ 是随机数，以$1-\alpha$的概率落在C_n中。

假设检验

从原假设开始,通过数据是否提供显著性证据来支持拒绝原假设。如果不能拒绝，就保留原假设。

例子：检验硬币是否均匀，令$X_1, \cdots, X_n \sim \text{Bernoulli}(p)$,

$H_0 : p = \frac{1}{2}$	$H_1 : p \neq \frac{1}{2}$
原假设(Null Hypothesis)	备选假设(Alternative Hypothesis)

由数据找出p的95%置信区间，看其是否包含$\frac{1}{2}$

假设检验和新物理的寻找

把参数空间Θ分为两个不相交的子集， Θ_0, Θ_1, 检验：

$H_0 \in \Theta_0$	vs	$H_1 \in \Theta_1$
原假设		备选假设

X是随机变量，令$\mathbb{X}$是X的取值范围， 找出一个称为<u>拒绝域</u>(rejection region)的适当子集$R \subset \mathbb{X}$来检验假设
$X \in R \Rightarrow$拒绝H_0
$X \notin R \Rightarrow$不能拒绝H_0, 原假设被保留。
通常，拒绝域的形式为：

$$R = \{x : T(x) > c\}$$

其中，T是检验统计量，c是临界值。
第一类错误 (Type I error)： H_0真，但被拒绝
第二类错误 (Type II error)： H_0伪， 但被保留

势函数、容度、检验水平(CL)

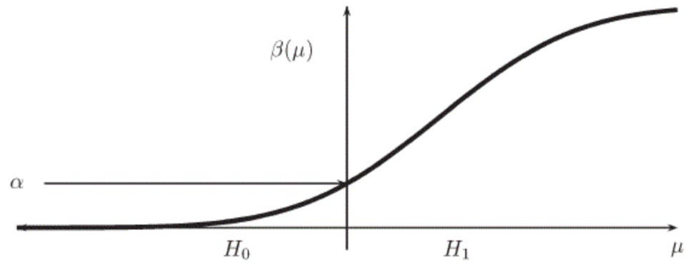
定义拒绝域为R的假设检验的势函数(power function)为：

$$\beta(\theta) = P(X \in R; \theta)$$

“给定θ, 数据落在拒绝域的概率“。定义检验的容度 (sizeof a test),为

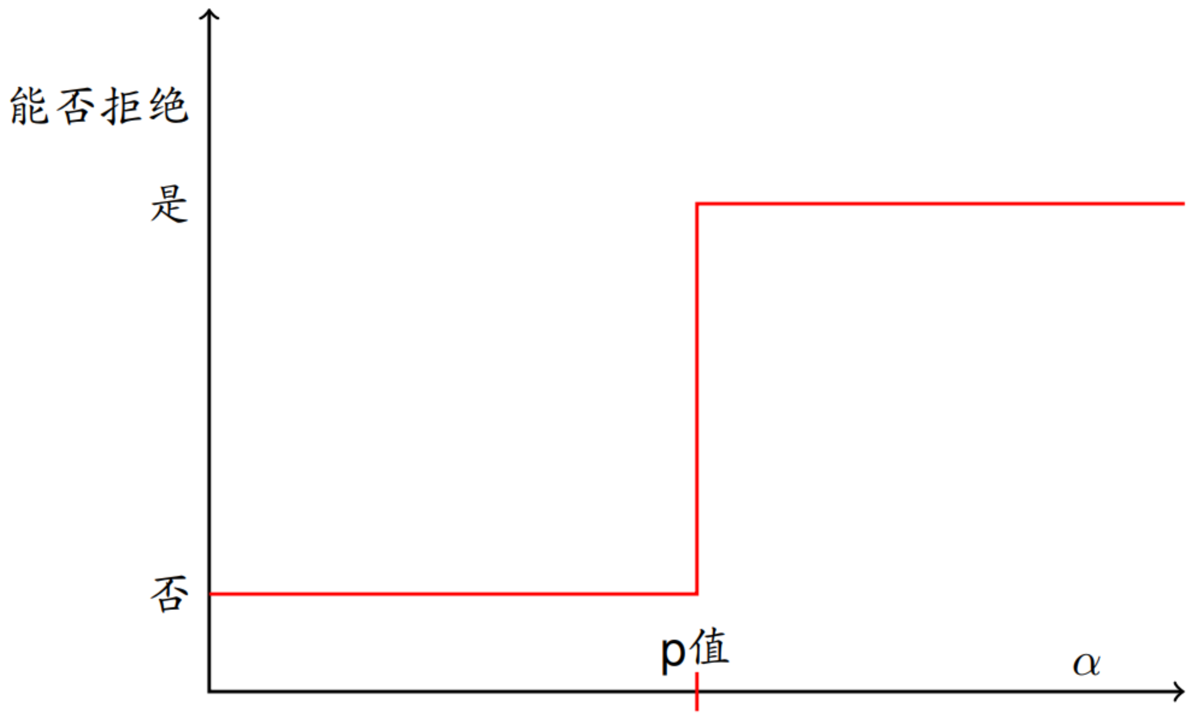
$$\alpha = \sup_{\theta \in \Theta_0} \beta(\theta)$$

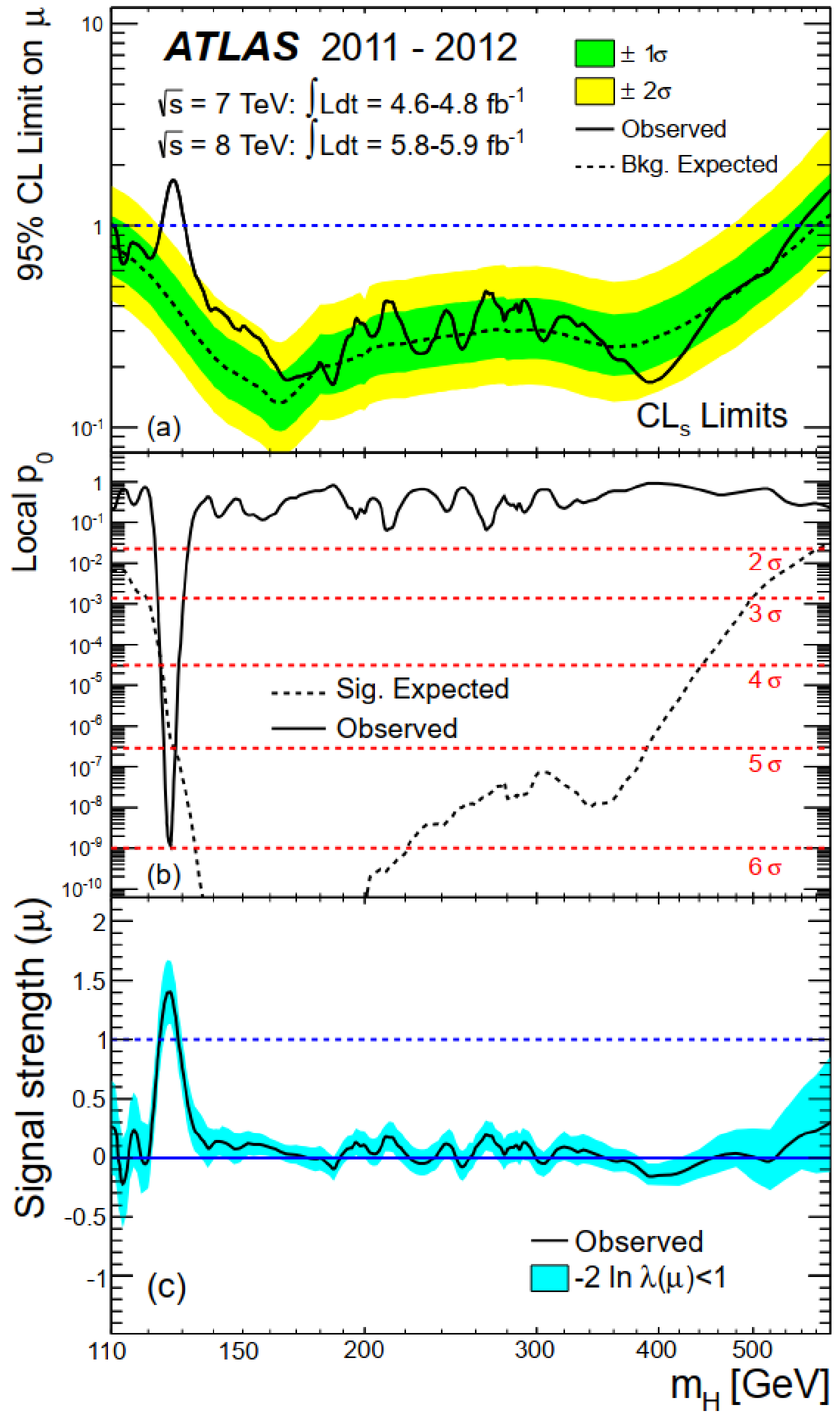
“H_0为真，被拒绝的最大可能性“，即犯第一类错误的最大可能性。若一个检验的容度$\leq \alpha$,则称检验的水平为α.



P-VALUE

拒绝或保留H_0并不能给出很多信息。有时需要讨论，对于任意的显著水平，是否拒绝假设。拒绝原假设的最小显著水平，称为p值。





多项分布

$X \sim \text{Multinomial}(n, p)$, ($p$为向量),坛子里放着$k$种颜色的球，随机抽取一个，抽到颜色$i$的概率是$p_i$, $i = 1, \cdots, k$. 抽取n个，其中颜色为i的球的个数记作X_i 则: $X = (X_1, \cdots, X_k)$为k维随机变量，

$$f(x) = \frac{n!}{x_1! \cdots x_n!} p_1^{x_1} \cdots p_k^{x_k}$$

注：若n不确定，而是服从泊松分布的随机数。则$X_1, \cdots, X_k$是相互独立的泊松分布的随机数。

PEARSON卡方检验

$Z_1,\cdots,Z_k \sim N(0,1)$,

$$V=\sum_{i=1}^{k} Z_i^2 \sim \chi_k^2$$

自由度为k的卡方分布。PDF: $f(v)=\frac{v^{k/2-1}e^{-v/2}}{2^{k/2}\Gamma(k/2)}, v>0$

$$\mathbb{E}(V)=k; V[V]=2k$$

如果$X=(X_1,X_2,\cdots,X_k)\sim$multinomial$(n,p)$, p的极大似然估计为：

$$\hat{p}=(\hat{p}_1,\cdots,\hat{p}_k)=(\frac{X_1}{n},\cdots,\frac{X_k}{n})$$

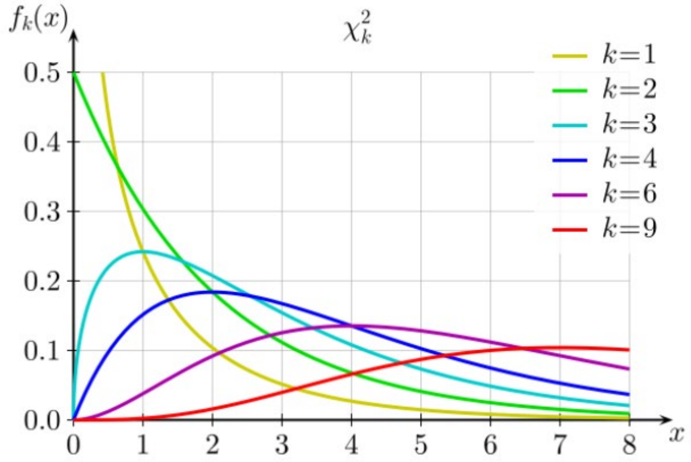
令$p_0=(p_{01},\cdots,p_{0k})$, 希望检验:

$$H_0: p=p_0 \text{ vs. } H_1: p\neq p_0$$

检验统计量：

$$T=\sum_{j=1}^{k}\frac{(X_j-np_{0j})^2}{np_{0j}}=\sum_{j=1}^{k}\frac{(X_j-E_j)^2}{E_j}$$

其中，$E_j=\mathbb{E}(X_j)=np_{0j}$,是$X_j$在$H_0$成立时的期望值。
H_0成立时，$T\sim\chi_{k-1}^2$
若$T>\chi_{k-1,\alpha}^2$，显著水平为α的检验拒绝H_0,对于特定的t, p值为$P(\chi_{k-1}^2>t)$



例：MENDEL豌豆实验（卡方）

饱满的黄颜色豌豆与皮皱的绿颜色豌豆杂交，后代有四种可能：饱满黄、皮皱黄、饱满绿、皮皱绿. Mendel的遗传理论预言，每一类型的概率为$p_0 = \{9/16, 3/16, 3/16, 1/16\}$.在556次试验中，观察到$X = (315, 101, 108, 32)$

$$H_0 : p = p_0 \text{ vs. } H_1 : p \neq p_0$$

$$T = \frac{(315-312.75)^2}{312.75} + \frac{(101-104.25)^2}{104.25} + \frac{(108-104.25)^2}{104.25} + \frac{(32-34.75)^2}{34.75} = 0.47$$

显著水平为$\alpha = 0.05$的χ_3^2的临界值为7.815,所以不能拒绝原假设。 P值$= P(\chi_3^2 > 0.47) = 0.93$

似然函数（极大似然估计）

最常用的参数估计方法。令$X_1, \cdots, X_n$独立同分布，PDF为$f(x;\theta)$, 定义似然函数为

$$\mathcal{L}_n(\theta) = \prod_{i=1}^{n} f(x_i, \theta)$$

对数似然函数为:

$$l_n(\theta) = \ln \mathcal{L}_n$$

注意：似然函数不是数据的联合分布密度函数，只能看作参数θ的函数。

定义: 极大似然估计MLE, 记为$\hat{\theta}_n$是使得函数$\mathcal{L}_n(\theta)$取最大的θ值。MLE只能用于参数模型。思想来自于高斯误差理论。1912年费歇尔提出(发表论文),之后被大量研究。MLE一般为最优良的估计(MVU:Minimum Variance Unbiased).

例：高斯分布的参数估计

"拟合"

设$X_1, \cdots, X_n \sim N(\mu, \sigma^2)$ 似然函数为(忽略常数项)

$$\mathcal{L}_n(\mu, \sigma^2) = \prod_{i=1}^{n} \frac{1}{\sigma} \exp\{-\frac{(x_i - \mu)^2}{2\sigma^2}\} = \sigma^{-n} \exp\{-\frac{1}{2\sigma^2} \sum_{i=1}^{n} (x_i - \mu)^2\}$$

令

$$\bar{x} = \frac{1}{n} \sum_{i=1}^{n} x_i; s^2 = \frac{1}{n} \sum_{i=1}^{n} (x_i - \bar{x})^2$$

则，

$$\mathcal{L}_n(\mu, \sigma^2) = \sigma^{-n} \exp\{-\frac{ns^2}{2\sigma^2}\} \exp\{-\frac{n(\bar{x} - \mu)^2}{2\sigma^2}\}$$

$$l_n(\mu, \sigma) = -n \ln \sigma - \frac{s^2}{2\sigma^2} - \frac{n(\bar{x} - \mu)^2}{2\sigma^2}$$

$$\begin{cases} \frac{\partial l_n(\mu,\sigma)}{\partial \mu} = 0 \\ \frac{\partial l_n(\mu,\sigma)}{\partial \sigma} = 0 \end{cases} \Rightarrow \begin{cases} \hat{\mu} = \bar{x} \\ \hat{\sigma} = s \end{cases}$$

一点细节：s不是σ的无偏估计，$\hat{\sigma} = \sqrt{\frac{1}{n-1} \sum_{i=1}^{n} (x_i - \bar{x})^2}$才是。

似然比检验

$$H_0:\theta\in\Theta_0 \text{vs.} H_1:\theta\notin\Theta_0$$

似然比检验统计量为

$$\lambda=2\ln\frac{\sup\limits_{\theta\in\Theta}\mathcal{L}(\theta)}{\sup\limits_{\theta\in\Theta_0}\mathcal{L}(\theta)}=2\ln\frac{\mathcal{L}(\hat{\theta})}{\mathcal{L}(\hat{\theta}_0)}$$

其中，$\hat{\theta}$表示极大似然估计。$\hat{\theta}_0$表示θ限制在Θ_0上的极大似然估计。假设$\theta=(\theta_1,\cdots,\theta_q,\theta_{q+1},\cdots,\theta_r)$，
令$\Theta_0=\{\theta:(\theta_{q+1},\cdots,\theta_r)=(\theta_{0,q+1},\cdots,\theta_{0,r})\}$,即后面的$r-q$个参数取确定的值。在$H_0$成立的前提下，$\lambda$渐进趋向于卡方分布。

$$\lambda\sim\chi^2_{r-q,\alpha}$$

检验的P值为$P(\chi^2_{r-q}>\lambda)$

例：MENDEL豌豆实验（似然比）

$$f(x) = \frac{n!}{x_1! \cdots x_n!} p_1^{x_1} \cdots p_k^{x_k}$$

$$\lambda = 2\ln\frac{\mathcal{L}(\hat{p})}{\mathcal{L}(\hat{p}_0)} = 2\ln\sum_{i=1}^{4} X_i \ln\frac{\hat{p}_i}{p_{0i}}$$

$$= 2(315\ln\frac{315/556}{9/16} + 101\ln\frac{101/556}{3/16}$$

$$+108\ln\frac{108/556}{3/16} + 32\ln\frac{32/556}{1/16}) = 0.48$$

H_0没有自由参数，H_1：4个参数但和必须为1，三个自由参数。

$\lambda \sim \chi_3^2$, P值=$P(\chi_3^2 > 0.48)$ = 0.92, 结果与卡方检验一致。

COUNTING EXPERIMENT(COWAN FORMULA)

通过事例数寻找新物理，泊松统计

$$N \sim \text{Poisson}(s+b)$$

b已知，实验结果n.
原假设：$s=0$
似然函数：

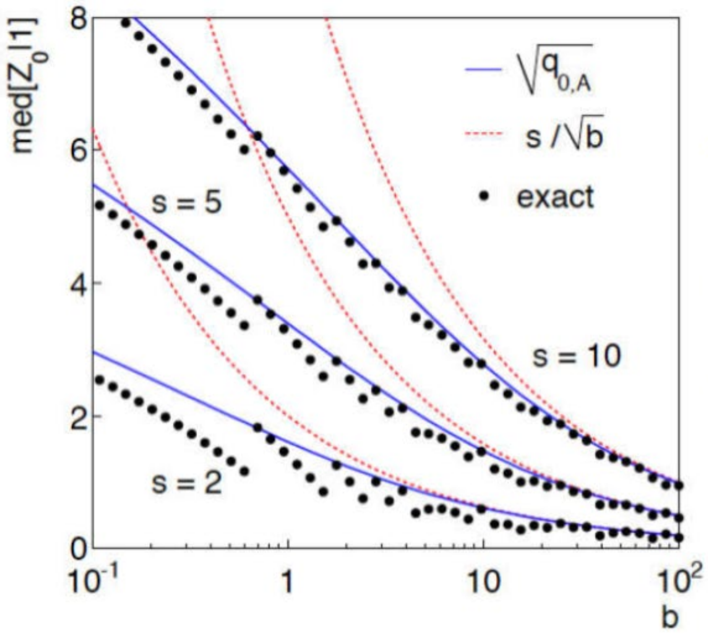
$$\mathcal{L}(s) = \frac{\exp(-s-b)(s+b)^n}{n!}$$

s的极大似然估计值：$\hat{s} = n - b$

$$\lambda = 2\ln\frac{\mathcal{L}(s=\hat{s})}{\mathcal{L}(s=0)} = 2(n\ln\frac{n}{b} - s)$$

预期的统计显著性,对应于$n=s+b$:
$\sqrt{\lambda} = \sqrt{2(s+b)\ln(1+s/b) - s}$

CCGV, EPJC 71 (2011) 1554, arXiv:1007.1727



"Exact" values from MC, jumps due to discrete data.

Asimov $\sqrt{q_{0,A}}$ good approx. for broad range of s, b.

$s/\sqrt{b}$ only good for $s \ll b$.

7. Statistical procedure

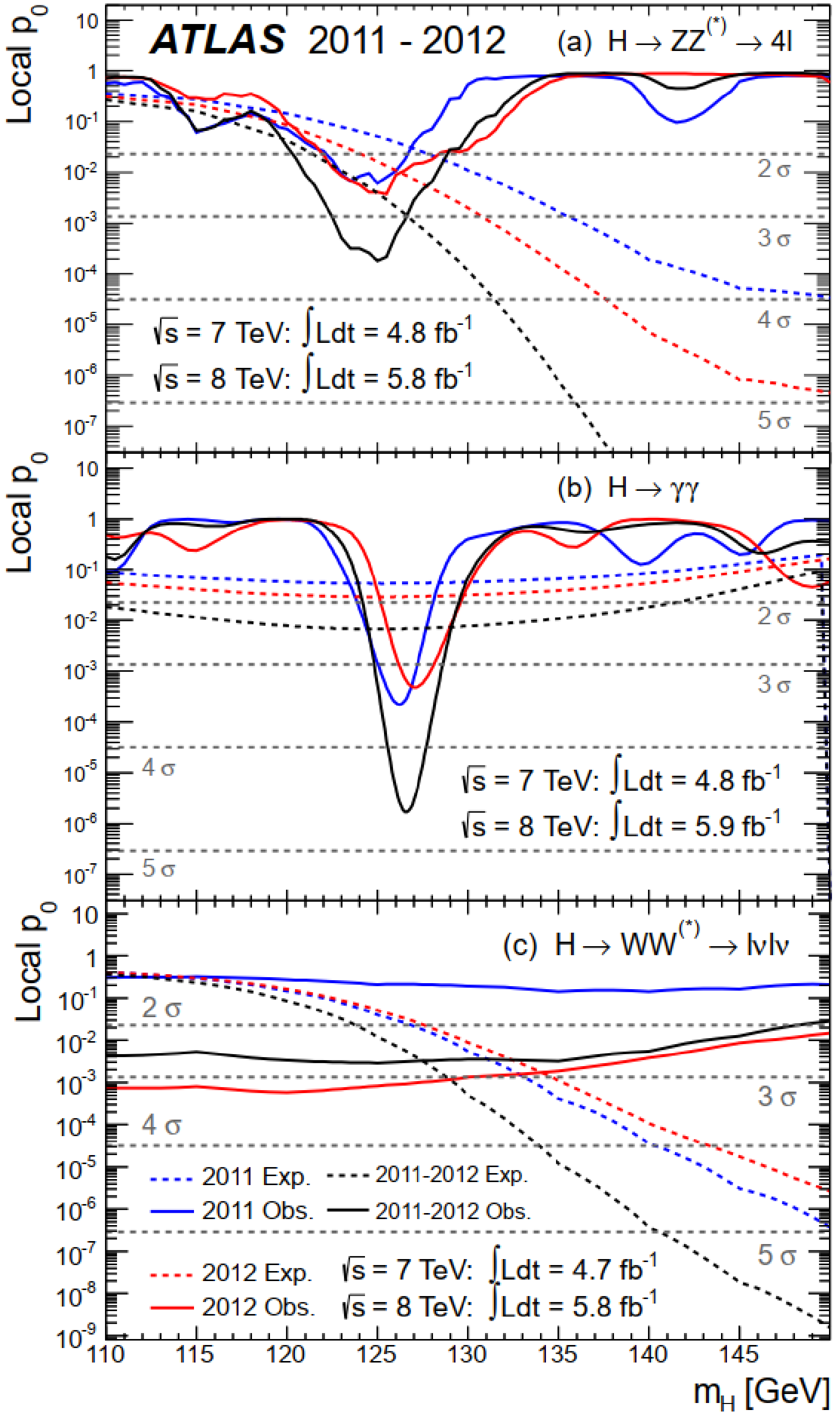
The statistical procedure used to interpret the data is described in Refs. [17, 118–121]. The parameter of interest is the global signal strength factor μ, which acts as a scale factor on the total number of events predicted by the Standard Model for the Higgs boson signal. This factor is defined such that $\mu = 0$ corresponds to the background-only hypothesis and $\mu = 1$ corresponds to the SM Higgs boson signal in addition to the background. Hypothesised values of μ are tested with a statistic $\lambda(\mu)$ based on the profile likelihood ratio [122]. This test statistic extracts the information on the signal strength from a full likelihood fit to the data. The likelihood function includes all the parameters that describe the systematic uncertainties and their correlations.

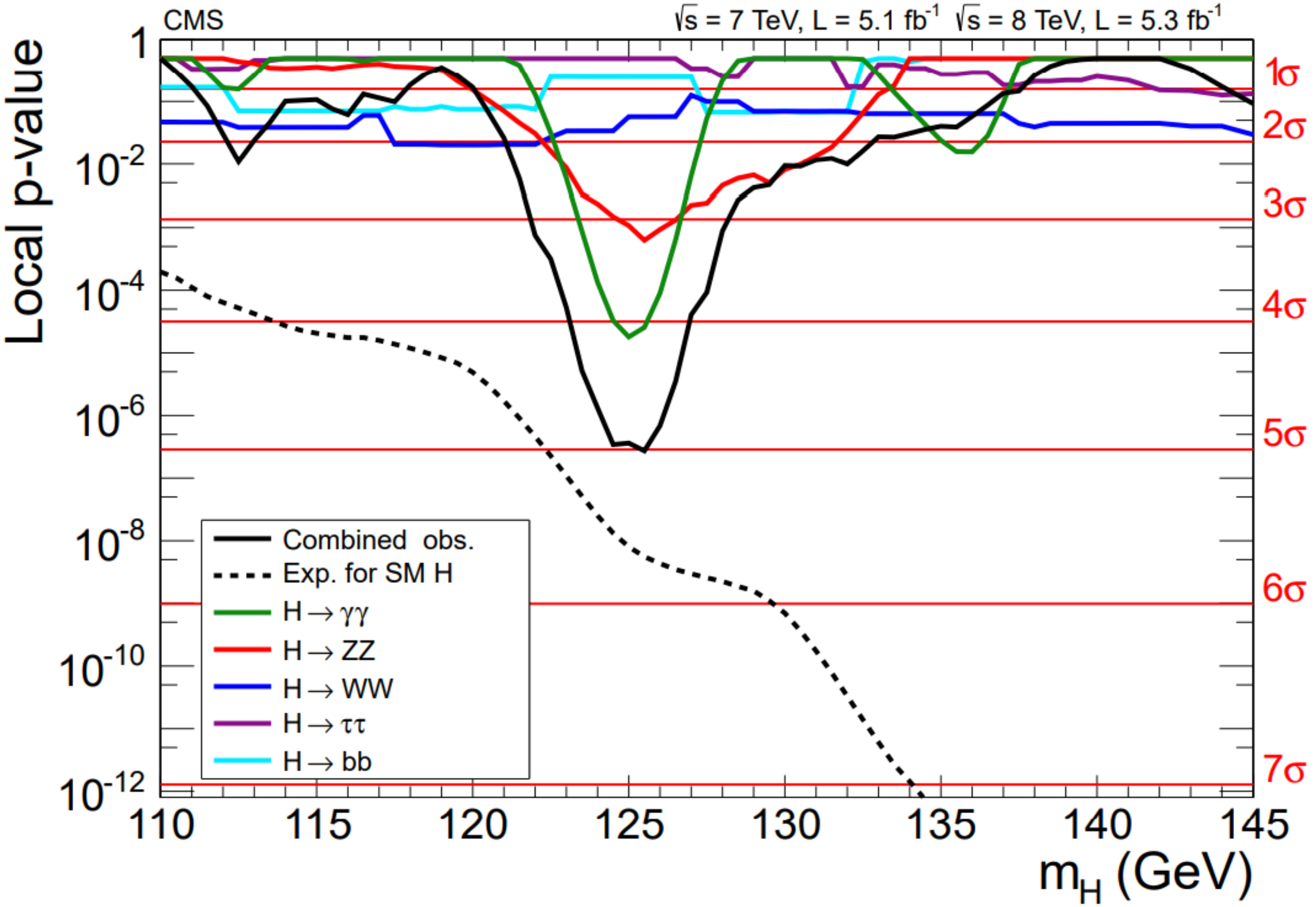
LIKELIHOOD FUNCTION AND LLR

$$\mathcal{L} = \underbrace{\prod_{i,b}^{\text{Table XXIa}} f\Big(N_{ib} \,\Big|\, \mu \cdot S_{ib} \cdot \prod_{r}^{\text{Syst. in Sec. V}} \nu_{br}(\theta_r) + \sum_{k}^{\text{Table I}} \beta_k \cdot B_{kib} \cdot \prod_{s}^{\text{Syst. in Sec. VII C}} \nu_{bs}(\theta_s)\Big)}_{\text{Poisson for SR with signal strength } \mu\text{; predictions } S,\, B}$$

$$\cdot \underbrace{\prod_{l}^{\text{Table XXIb}\,\bullet} f\Big(N_l \,\Big|\, \sum_{k}^{\text{Table I}} \beta_k \cdot B_{kl}\Big)}_{\text{Poisson for profiled CRs}} \cdot \underbrace{\prod_{t}^{\text{Syst. in }\{r,\, s\}} g(\vartheta_t | \theta_t)}_{\text{Gauss. for syst.}} \cdot \underbrace{\prod_{k}^{\text{Table I}} f(\xi_k \,|\, \zeta_k \cdot \theta_k)}_{\text{Poiss. for MC stats}}$$

$$q(\mu) = -2 \ln \frac{\mathcal{L}(\mu, \boldsymbol{\theta})}{\mathcal{L}_{\max}} \Bigg|_{\boldsymbol{\theta} = \hat{\boldsymbol{\theta}}_\mu}$$


ATLAS 2011 - 2012
(a) H → ZZ(*) → 4l
√s = 7 TeV: ∫Ldt = 4.8 fb-1
√s = 8 TeV: ∫Ldt = 5.8 fb-1
(b) H → γγ
√s = 7 TeV: ∫Ldt = 4.8 fb-1
√s = 8 TeV: ∫Ldt = 5.9 fb-1
(c) H → WW(*) → lνlν
2011 Exp.
2011-2012 Exp.
2011 Obs.
2011-2012 Obs.
2012 Exp.
2012 Obs.
√s = 7 TeV: ∫Ldt = 4.7 fb-1
√s = 8 TeV: ∫Ldt = 5.8 fb-1
Local p0
mH [GeV]


CMS
√s = 7 TeV, L = 5.1 fb-1 √s = 8 TeV, L = 5.3 fb-1
Local p-value
1σ
2σ
3σ
4σ
5σ
6σ
7σ
Combined obs.
Exp. for SM H
H → γγ
H → ZZ
H → WW
H → ττ
H → bb
mH (GeV)

END OF PART 1