On the nature of $X(6900)$ and other structures in the LHCb di-J/ψ spectrum

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Outline

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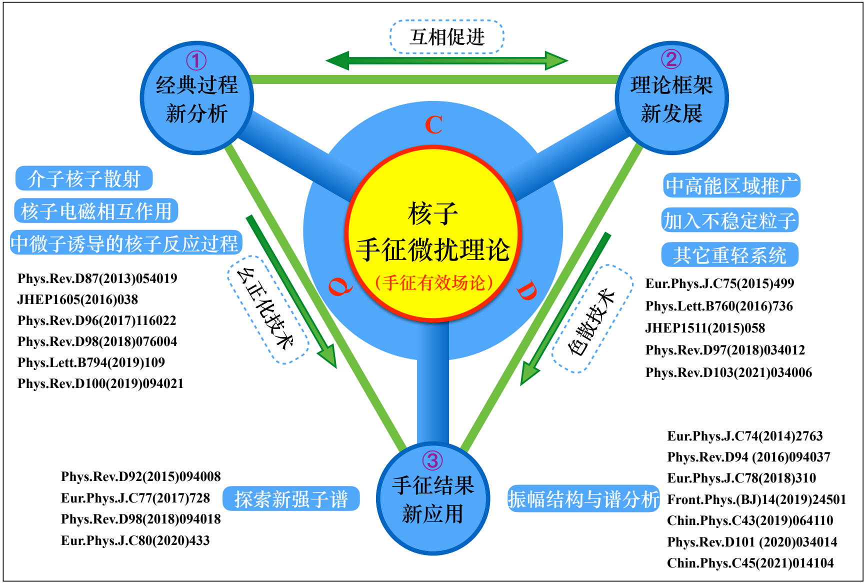
1. Introduction

Background



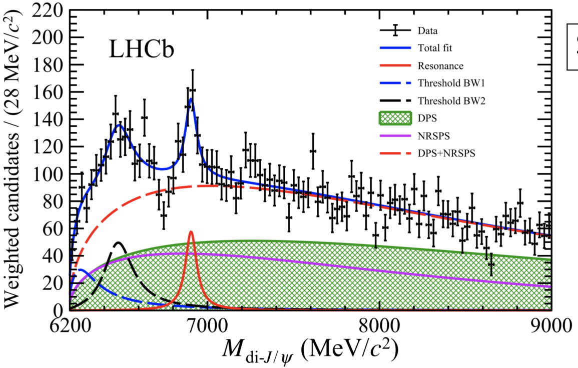
[arXiv: 2009.13495]

"A Review on Partial-wave Dynamics with Chiral Effective Field Theory and Dispersion Relation"

De-Liang Yao, Ling-Yun Dai, Han-Qing Zheng, and Zhi-Yong Zhou

Reports on Progress in Physics, under review.

Background



Sci. Bull. 65 (2020), 1983-1993

- LHCb reported a narrow structure around 6.9 GeV in the di-J/ψ invariant mass spectrum: X(6900).
- A possible board structure at range $[6.2\ \mathrm{GeV}, 6.8\ \mathrm{GeV}]$.
- A hint of another structure around $7.2\ \mathrm{GeV}$.

Observation of structure in the J/ψ -pair mass spectrum #2

LHCb Collaboration • Roel Aaij (NIKHEF, Amsterdam) et al. (Jun 30, 2020)

Published in: *Sci.Bull.* 65 (2020) 23, 1983-1993 • e-Print: 2006.16957 [hep-ex]

pdf links DOI cite 78 citations

Status of theoretical studies

Many works have been accumulated:

- Quark model:
  - R. N. Faustov, V. O. Galkin and E. M. Savchenko, arXiv:2103.01763.
  - X. Jin, X. Liu, Y. Xue, H. Huang and J. Ping, arXiv:2011.12230.
  - Q. F. Lü, D. Y. Chen and Y. B. Dong, Eur. Phys. J. C **80** (2020) no.9, 871.
- QCD sum rules:
  - B. C. Yang, L. Tang and C. F. Qiao, arXiv:2012.04463.
  - B. D. Wan and C. F. Qiao, arXiv:2012.00454.
  - Z. G. Wang, Chin. Phys. C **44** (2020) no.11, 113106.
- NRQCD factorization:
  - F. Feng, Y. Huang, Y. Jia, W. L. Sang and J. Y. Zhang, arXiv:2011.03039.
  - Y. Q. Ma and H. F. Zhang, arXiv:2009.08376.
  - F. Feng, Y. Huang, Y. Jia, W. L. Sang, X. Xiong and J. Y. Zhang, arXiv:2009.08450.

Status of theoretical studies

- Phenomenological model:
  - Z. Zhao, K. Xu, A. Kaewsnod, X. Liu, A. Limphirat and Y. Yan, arXiv:2012.15554.
  - C. Gong, M. C. Du, B. Zhou, Q. Zhao and X. H. Zhong, arXiv:2011.11374.
  - Q. F. Cao, H. Chen, H. R. Qi and H. Q. Zheng, arXiv:2011.04347.
  - Z. H. Guo and J. A. Oller, PRD**103** (2021)034024.
  - X. K. Dong, V. Baru, F. K. Guo, C. Hanhart and A. Nefediev, PRL**126**(2021)132001
- ...
- **Our work** (Effective Lagrangian & Unitarization)
  Reveal possible states and explore their J^{PC} quantum numbers
  - derive potentials from effective Lagrangians
  - take coupled-channel effects into account
  - employ helicity amplitude formalism and perform partial-wave analysis

2. Theoretical Framework

Effective Lagrangian

$$
\begin{aligned}
\mathcal{L}_{\mathrm{eff.}} &= h_1(J/\psi\cdot J/\psi)^2 \\
&+ h_2(J/\psi\cdot J/\psi)(J/\psi\cdot\psi(2S)) \\
&+ h_3(J/\psi\cdot J/\psi)(J/\psi\cdot\psi(3770)) \\
&+ 2h_4(J/\psi\cdot\psi(2S))^2 \\
&+ 2h_4'(J/\psi\cdot J/\psi)(\psi(2S)\cdot\psi(2S)) \\
&+ h_5(J/\psi\cdot\psi(2S))(J/\psi\cdot\psi(3770)) \\
&+ h_5'(J/\psi\cdot J/\psi)(\psi(2S)\cdot\psi(3770)) \\
&+ h_6(J/\psi\cdot\psi(3770))^2 \\
&+ h_6'(J/\psi\cdot J/\psi)(\psi(3770)\cdot\psi(3770)) \\
&+ h_7(J/\psi\cdot\psi(2S))(\psi(2S)\cdot\psi(2S)) \\
&+ h_8(J/\psi\cdot\psi(3770))(\psi(2S)\cdot\psi(2S)) \\
&+ h_8'(J/\psi\cdot\psi(2S))(\psi(3770)\cdot\psi(2S)) \\
&+ 4h_9(\psi(2S)\cdot\psi(2S))^2
\end{aligned}
$$

- HQSS & meson exchange
- Four channels: $\{J/\psi J/\psi,\ J/\psi\psi(2S),\ J/\psi\psi(3770),\ \psi(2S)\psi(2S)\}$
- The Lagrangian satisfies the basic symmetries, such as Lorentz invariance, P and C parity symmetries and so on.
- The unknown couplings $h_{1,2,..9}$ and $h'_{4,5,6,8}$ need to be determined by experimental data.

Potentials for the scattering processes

- The generic form of the potentials for $V_1(p_1, \epsilon_1) + V_2(p_2, \epsilon_2) \to V_3(p_3, \epsilon_3) + V_4(p_4, \epsilon_4)$

$$V_{ij} = \mathcal{C}_1\, \epsilon_1 \cdot \epsilon_2 \epsilon_3^\dagger \cdot \epsilon_4^\dagger + \mathcal{C}_2\, \epsilon_1 \cdot \epsilon_3^\dagger \epsilon_2 \cdot \epsilon_4^\dagger + \mathcal{C}_3\, \epsilon_1 \cdot \epsilon_4^\dagger \epsilon_2 \cdot \epsilon_3^\dagger$$

- Coefficients for different processes

V_{ij}	Channels	$\mathcal{C}_1$	$\mathcal{C}_2$	$\mathcal{C}_3$
11	$J/\psi J/\psi \to J/\psi J/\psi$	$8h_1$	$8h_1$	$8h_1$
12	$J/\psi J/\psi \to \psi(2S) J/\psi$	$2h_2$	$2h_2$	$2h_2$
13	$J/\psi J/\psi \to \psi(3770) J/\psi$	$2h_3$	$2h_3$	$2h_3$
14	$J/\psi J/\psi \to \psi(2S)\psi(2S)$	$4h_4'$	$2h_4$	$2h_4$
22	$\psi(2S) J/\psi \to \psi(2S) J/\psi$	$2h_4$	$4h_4'$	$2h_4$
23	$\psi(2S) J/\psi \to \psi(3770) J/\psi$	h_5	$2h_5'$	h_5
24	$\psi(2S) J/\psi \to \psi(2S)\psi(2S)$	$2h_7$	$2h_7$	$2h_7$
33	$\psi(3770) J/\psi \to \psi(3770) J/\psi$	$2h_6$	$4h_6'$	$2h_6$
34	$\psi(3770) J/\psi \to \psi(2S)\psi(2S)$	$2h_8$	h_8'	h_8'
44	$\psi(2S)\psi(2S) \to \psi(2S)\psi(2S)$	$8h_9$	$8h_9$	$8h_9$

Helicity amplitude

- Helicity amplitudes (81 in total), $\lambda_i = \pm 1, 0$

$$V_{\lambda_1\lambda_2\lambda_3\lambda_4} = \epsilon_3^{\rho\dagger}(p_3,\lambda_3)\epsilon_4^{\sigma\dagger}(p_4,\lambda_4)V_{\mu\nu\rho\sigma}\epsilon_1^{\mu}(p_1,\lambda_1)\epsilon_2^{\nu}(p_2,\lambda_2)$$

$$V_{\mu\nu\rho\sigma} = \mathcal{C}_1 g_{\mu\nu}g_{\rho\sigma} + \mathcal{C}_2 g_{\mu\rho}g_{\nu\sigma} + \mathcal{C}_3 g_{\mu\sigma}g_{\nu\rho}$$

According to P and T parity symmetries ($81 \to 25$), e.g.

$$V_{++++} = V_{----} \ ,$$
$$V_{+++0} = -V_{---0} = V_{-0--} = -V_{+0++} \ .$$

- Explicit expressions for helicity amplitudes, e.g.

$$V_{++++} = V_{----} = \mathcal{C}_1 + \frac{1}{4}(\mathcal{C}_3(-1+z_s)^2 + \mathcal{C}_2(1+z_s)^2) \ ,$$

with $z_s = \cos\theta$, θ being scattering angle .

Partial-Wave Projection

- Partial wave amplitudes

$$V^J_{\lambda_1\lambda_2\lambda_3\lambda_4}(s) = \frac{1}{2}\int_{-1}^{+1} dz_s V_{\lambda_1\lambda_2\lambda_3\lambda_4}(s, t(s, z_s)) d^J_{\lambda,\lambda'}(z_s)$$

where $\lambda = \lambda_1 - \lambda_2$, $\lambda' = \lambda_3 - \lambda_4$.
Relations due to P, T symmetries and properties of $d^J_{\lambda\lambda'}$ functions:
e.g.
  - $V^J_{++++} = V^J_{----}$
  - $V^J_{+++0} = V^J_{---0} = V^J_{-0--} = V^J_{+0++}$
- Helicity basis $\to$ JLS basis (definite J^{PC} quantum number)

$$\mathcal{V}^J = \sum_{\substack{\lambda_1\lambda_2 \\ \lambda_3\lambda_4}} U^J_{\lambda_3\lambda_4} \mathcal{V}^J_{\lambda_1\lambda_2\lambda_3\lambda_4} [U^J_{\lambda_1\lambda_2}]^\dagger$$

  - The transformation matrix

$$U^J_{\lambda_1\lambda_2} = \frac{1}{\sqrt{2S+1}} \langle S_1\lambda_1 S_2 - \lambda_2 | S\lambda\rangle \quad => \boxed{\text{C-G coefficient}}$$

S-Wave Amplitude

S-wave: $L=0$, $J=L+S=S \Rightarrow$ For identical particles: $L+S=\text{even}$

- $J^{PC}=0^{++}$

$$\mathcal{V}(0^{++}) = \frac{2}{3}V^{J=0}_{++++} + \frac{2}{3}V^{J=0}_{++--} + \frac{1}{3}V^{J=0}_{0000} - \frac{4}{3}V^{J=0}_{++00}$$

- $J^{PC}=2^{++}$

$$\begin{aligned}
\mathcal{V}(2^{++}) &= \frac{2}{15}V^{J=2}_{0000} + \frac{4}{15}V^{J=2}_{00++} + \frac{4\sqrt{6}}{15}V^{J=2}_{00+-} + \frac{2\sqrt{6}}{15}(V^{J=2}_{+-++} + V^{J=2}_{-+++}) \\
&+ \frac{4\sqrt{3}}{15}(V^{J=2}_{00+0} + V^{J=2}_{000+}) + \frac{2\sqrt{3}}{15}(V^{J=2}_{+0++} + V^{J=2}_{0+++} + V^{J=2}_{0-++} + V^{J=2}_{-0++}) \\
&+ \frac{1}{5}(V^{J=2}_{+0+0} + V^{J=2}_{-0+0} + V^{J=2}_{0+0+} + V^{J=2}_{0-0+}) \\
&+ \frac{2\sqrt{2}}{5}(V^{J=2}_{+-+0} + V^{J=2}_{0++-} + V^{J=2}_{-++0} + V^{J=2}_{0-+-}) \\
&+ \frac{2}{5}(V^{J=2}_{0++0} + V^{J=2}_{0-+0} + V^{J=2}_{+-+-} + V^{J=2}_{-++-}) + \frac{1}{15}(V^{J=2}_{++++} + V^{J=2}_{--++})
\end{aligned}$$

Unitarization

- Requirement of unitarity
  $\longrightarrow$ Bethe-Salpeter equation method is employed to restore unitarity.
- The unitarized amplitude under on-shell approximation

$$\mathcal{T}_J(s) = \mathcal{V}^J(s) \cdot \left[1 - \mathcal{G}(s) \cdot \mathcal{V}^J(s)\right]^{-1}$$

Graphic representation

- For coupled-channel:

$$\mathcal{V}^J(s) = \begin{pmatrix} V^J_{11}(s) & V^J_{12}(s) & V^J_{13}(s) & V^J_{14}(s) \\ V^J_{12}(s) & V^J_{22}(s) & V^J_{23}(s) & V^J_{24}(s) \\ V^J_{13}(s) & V^J_{23}(s) & V^J_{33}(s) & V^J_{34}(s) \\ V^J_{14}(s) & V^J_{24}(s) & V^J_{34}(s) & V^J_{44}(s) \end{pmatrix}$$

- two-point functions

$$\mathcal{G}(s) = \begin{pmatrix} g_1(s) & & & \\ & g_2(s) & & \\ & & g_3(s) & \\ & & & g_4(s) \end{pmatrix}$$

Unitarization

- The explicit form of two-point loop function

$$
\begin{aligned}
g_i(s) =& \frac{1}{16\pi^2}\Big\{ a(\mu) + \ln\frac{M_{V_1}^2}{\mu^2} + \frac{s - M_{V_1}^2 + M_{V_2}^2}{2s}\ln\frac{M_{V_2}^2}{M_{V_1}^2} \\
&+ \frac{\sigma(s)}{2s}\Big[\ln(s - M_{V_2}^2 + M_{V_1}^2 + \sigma(s)) - \ln(-s + M_{V_2}^2 - M_{V_1}^2 + \sigma(s)) \\
&+ \ln(s + M_{V_2}^2 - M_{V_1}^2 + \sigma(s)) - \ln(-s - M_{V_2}^2 + M_{V_1}^2 + \sigma(s))\Big]\Big\}
\end{aligned}
$$

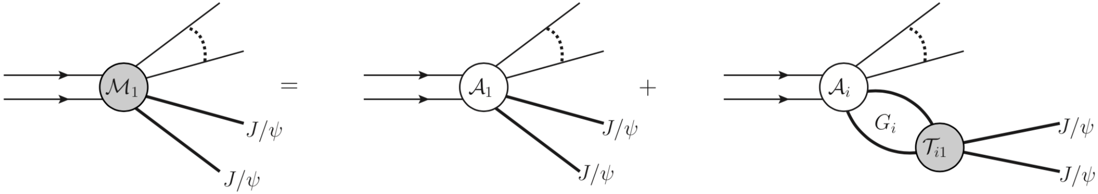
where

- $\sigma(s) = \{[s - (M_{V_2} + M_{V_1})^2][s - (M_{V_2} - M_{V_1})^2]\}^{1/2}$.
- The subtraction scale $a(\mu) = -3.0$ at renormalization scale $\mu = 1\ \mathrm{GeV}$.
- Analytic continuation (Riemann sheet)

$$g^{\mathrm{II}}(s) = g^{\mathrm{I}}(s) + 2i\rho(s)$$

Production Amplitudes

- Production of di-J/ψ via pp collision



- The production amplitude

$$\mathcal{M}_1(s) = \mathcal{A}_1 + \sum \mathcal{A}_i \mathcal{G}_i(s) \mathcal{T}_{i1}(s) = \mathcal{A}_1 \left[1 + \sum r_i \mathcal{G}_i(s) \mathcal{T}_{i1}(s)\right]$$

  - $\gamma_i = \mathcal{A}_i / \mathcal{A}_1$
  - $\mathcal{A}_1$ represents direct production of J/ψ pairs.

The invariant mass formula

- The invariant mass

$$\frac{\mathrm{d}\mathcal{N}}{\mathrm{d}\sqrt{s}} \propto \rho(s)|\mathcal{M}_1(s)|^2 = \rho(s)|\mathcal{A}_1(s)|^2 \left|\gamma + \sum_{i=1}^{3} \mathcal{G}_i(s)\mathcal{T}_{i1}(s)\right|^2$$

  with γ coherent background.
- Phase factor

$$\rho(s) = \frac{p_1(s)}{8\pi\sqrt{s}} = \frac{\lambda^{1/2}(s, m_{J/\psi}^2, m_{J/\psi}^2)}{16\pi s}$$

- The direct production amplitude is parameterized as

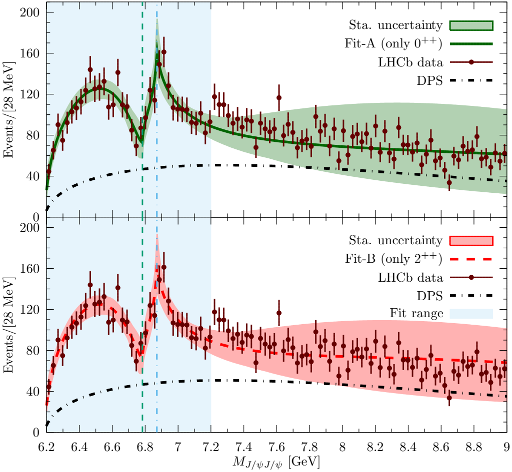
$$|\mathcal{A}_1(s)|^2 = \alpha^2 e^{-2\beta s}$$

  - Overall factor α
  - The slope parameter β is fixed ($\beta = 0.0123$)
    [X. K. Dong, V. Baru, F. K. Guo, C. Hanhart and A. Nefediev, PRL126(2021).]

3. Numerical Results

Fit with three coupled-channels

- Description of LHCb data:



- consider three coupled-channels: $J/\psi J/\psi$, $J/\psi\psi(2S)$, $J/\psi\psi(3770)$;
- perform two different kinds of fits 0^{++} and 2^{++} within range [6.2 GeV, 7.2 GeV] ;
- Both fits well describe the di-J/ψ spectrum within 1-σ uncertainty.

Fit with three coupled-channels

- Pole positions and residues:
  - 0^{++}

RS	Position $\sqrt{s_{\rm pole}}$ [MeV]	$\lvert\text{Residue}\rvert^{1/2}$ [GeV] $J/\psi J/\psi$	$J/\psi\psi(2S)$	$J/\psi\psi(3770)$
I	$6172.4^{+21.3}_{-30.4}$	$17.2^{+4.5}_{-11.7}$	$22.4^{+4.1}_{-14.0}$	$5.0^{+4.2}_{-4.3}$
II	$6191.2^{+2.5}_{-3.5}$	$4.3^{+5.5}_{-1.0}$	$5.6^{+12.4}_{-1.3}$	$1.2^{+3.9}_{-0.9}$
II	$6959.1^{+37.1}_{-58.7} - i161.2^{+125.9}_{-93.3}$	$27.8^{+7.1}_{-7.7}$	$37.2^{+4.0}_{-8.2}$	$20.3^{+16.1}_{-5.9}$

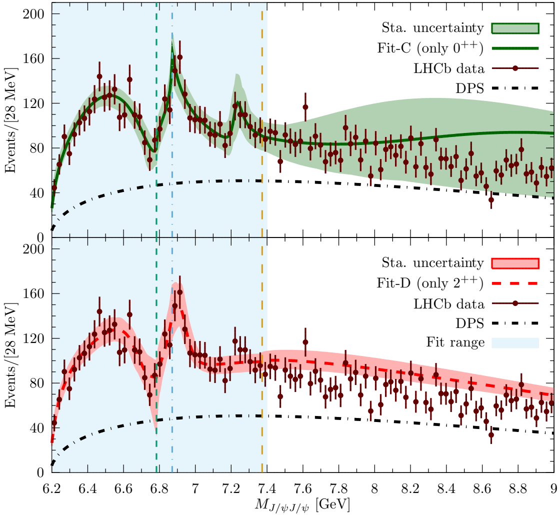
  - 2^{++}

RS	Position $\sqrt{s_{\rm pole}}$ [MeV]	$\lvert\text{Residues}\rvert^{1/2}$ [GeV] $J/\psi J/\psi$	$J/\psi\psi(2S)$	$J/\psi\psi(3770)$
I	$6170.1^{+19.5}_{-32.1}$	$17.5^{+4.4}_{-6.0}$	$22.6^{+2.8}_{-5.9}$	$5.3^{+3.4}_{-2.9}$
II	$6190.9^{+2.4}_{-3.8}$	$4.4^{+1.0}_{-1.1}$	$5.7^{+0.8}_{-1.0}$	$1.3^{+0.9}_{-0.7}$
II	$6994.1^{+53.8}_{-92.7} - i156.9^{+150.6}_{-127.8}$	$28.4^{+8.7}_{-8.2}$	$36.9^{+4.6}_{-8.7}$	$19.7^{+13.8}_{-7.6}$

    - A near-threshold bound state is found in these two cases, referred as to $X(6200)$.
    - A peak located in 6.9 GeV appears due to the $J/\psi\psi(3770)$ threshold effects.

Fit with four coupled-channels

- Description of LHCb data:



- consider four coupled-channels $J/\psi J/\psi$, $J/\psi\psi(2S)$, $J/\psi\psi(3770)$, $\psi(2S)\psi(2S)$.
- perform two different kinds of fits 0^{++} and 2^{++} and extend the energy range to [6.2 GeV,7.4 GeV] ;
- Fit-C (0^{++}) and Fit-D (2^{++}) behave differently. A peak around 7.2 GeV is observed in 0^{++} case.

Fits with four coupled-channels

- Pole positions and residues:

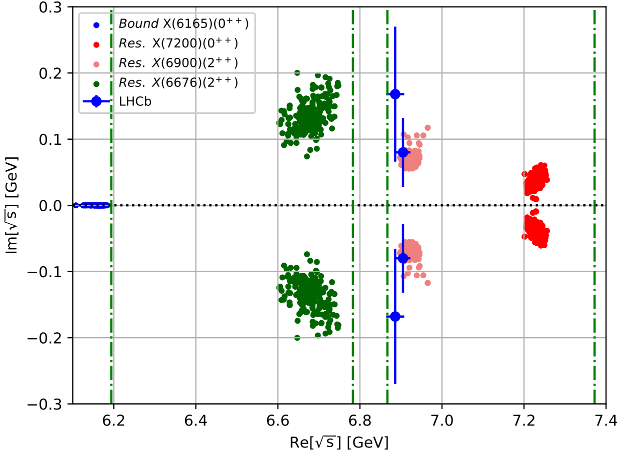
RS	Position $\sqrt{s_{\rm pole}}$ [MeV]	$\lvert\text{Residue}\rvert^{1/2}$ [GeV] $J/\psi J/\psi$	$J/\psi\psi(2S)$	$J/\psi\psi(3770)$	$\psi(2S)\psi(2S)$
I (0^{++})	$6165.6^{+18.8}_{-57.7}$	$19.8^{+5.5}_{-5.0}$	$15.8^{+6.1}_{-3.5}$	$0.7^{+0.5}_{-0.7}$	$1.9^{+1.7}_{-1.9}$
IV (0^{++})	$7225.4^{+30.9}_{-23.7} - i30.4^{+30.4}_{-21.4}$	$4.4^{+2.3}_{-2.5}$	$5.7^{+1.4}_{-2.0}$	$0.9^{+0.6}_{-0.5}$	$37.8^{+2.1}_{-2.7}$
II (2^{++})	$6676.7^{+70.8}_{-73.4} - i136.9^{+63.3}_{-63.0}$	$15.0^{+2.4}_{-2.9}$	$24.4^{+2.6}_{-11.6}$	$13.5^{+18.8}_{-12.0}$	$37.2^{+5.2}_{-5.5}$
IV (2^{++})	$6920.7^{+44.7}_{-22.9} - i66.1^{+51.2}_{-10.7}$	$6.2^{+5.3}_{-1.7}$	$9.6^{+3.2}_{-6.4}$	$5.2^{+4.1}_{-4.5}$	$50.1^{+3.8}_{-1.4}$

- Fit-C:
  (1) A resonance X(7200) ;
  (2) A bound state $X(6200)$;
  (3) Their J^{PC} numbers are 0^{++}.

- Fit-D:
  (1) A narrow resonance X(6900);
  (2) A broad resonance X(6670);
  (3) Their J^{PC} numbers are 2^{++}.

Fits with four coupled-channels

- The locations of poles:



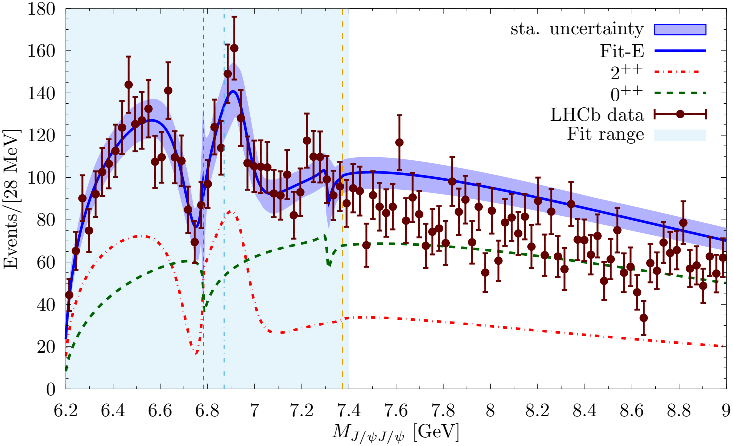
- Green $\rightarrow$ the 2^{++} X(6670) resonance
- Pink $\rightarrow$ the 2^{++} X(6900) resonance
- Red $\rightarrow$ the 0^{++} X(7200) resonance
- Blue $\rightarrow$ the 0^{++} X(6200) bound state

LHCb results (the blue error bar):

- $m = 6905 \pm 11 \pm 7\ \mathrm{MeV}$
  $\Gamma = 80 \pm 19 \pm 33\ \mathrm{MeV}$ (w/o)
- $m = 6886 \pm 11 \pm 11\ \mathrm{MeV}$
  $\Gamma = 168 \pm 33 \pm 69\ \mathrm{MeV}$ (w)

Combined fit

- Description of LHCb data:



- three coupled-channels $J/\psi J/\psi$, $J/\psi\psi(2S)$, $\psi(2S)\psi(2S)$
- To judge which partial-wave is dominant $\rightarrow$ both are sizeable

Combined fit

- Pole positions and residue

RS (J^{PC})	Position $\sqrt{s_{\rm pole}}$ [MeV]	$\lvert\mathrm{Residue}\rvert^{1/2}$ [GeV] $J/\psi J/\psi$	$J/\psi\psi(2S)$	$\psi(2S)\psi(2S)$
I (0^{++})	$6059.6^{+98.0}_{-21.0}$	$29.5^{+2.0}_{-11.2}$	$16.2^{+10.0}_{-2.6}$	$4.5^{+1.4}_{-0.4}$
II (2^{++})	$6759.2^{+84.8}_{-63.5} - i120.1^{+91.8}_{-54.6}$	$17.0^{+6.5}_{-5.8}$	$28.7^{+9.9}_{-7.0}$	$42.9^{+21.4}_{-16.6}$
III (2^{++})	$6950.0^{+32.6}_{-22.1} - i88.7^{+15.8}_{-10.8}$	$9.0^{+1.3}_{-1.6}$	$9.0^{+1.3}_{-2.2}$	$50.7^{+0.9}_{-1.5}$

- The $X(6200)$, $X(6670)$ and $X(6900)$ states still exist.
- Their pole locations are shifted, due to the absence of the $J/\psi\psi(3770)$ channel.
- The $X(7200)$ disappears, however, there still exists an enhancement around 7.2 GeV.
  $\rightarrow$ The $J/\psi\psi(3770)$ channel is important for the existence of the $X(7200)$ state.

4. Summary and Outlook

Summary and Outlook

- Exploring all possible states in $[6.2\ \mathrm{GeV}, 7.4\ \mathrm{GeV}]$ by means of partial-wave analysis.
- Four states are found

  - X(6200): a bound state with $J^{PC} = 0^{++}$, $\sqrt{s_{\mathrm{pole}}} = 6165.6^{+18.8}_{-57.7}\ \mathrm{MeV}$
  - X(6670): a broad resonant state with $J^{PC} = 2^{++}$, $\sqrt{s_{\mathrm{pole}}} = 6676.7^{+70.8}_{-73.4} - i136.9^{+63.3}_{-63.0}\ \mathrm{MeV}$
  - X(6900): a narrow resonant state with $J^{PC} = 2^{++}$, $\sqrt{s_{\mathrm{pole}}} = 6920.7^{+44.7}_{-22.9} - i66.1^{+51.2}_{-10.7}\ \mathrm{MeV}$
  - X(7200): a narrow resonant state with $J^{PC} = 0^{++}$, $\sqrt{s_{\mathrm{pole}}} = 7225.4^{+30.9}_{-23.7} - i30.4^{+30.4}_{-21.4}\ \mathrm{MeV}$

  - Need to be confirmed by more experimental data (in precision and in amount);
  - Study the structures of these states in future.

Many thanks for your attention!

Backup

Details of the 3CC fit

- Couplings

$$h_i = h_i' \ , \qquad h_i = \bar{h}_i \cdot \sum_{j=1}^{4} \sqrt{2m_j} \ ,$$

- Results of fitting parameter

	Fit-A (0^{++})	Fit-B (2^{++})
$\bar{h}_1$	$0.16^{+0.09}_{-0.02}$	$0.48^{+0.24}_{-0.07}$
$\bar{h}_2$	$1.4^{+0.6}_{-0.2}$	$3.6^{+1.4}_{-0.6}$
$\bar{h}_3$	$0.01^{+0.15}_{-0.20}$	$-0.04^{+0.39}_{-0.47}$
$\bar{h}_4$	$1.8^{+0.6}_{-0.4}$	$2.9^{+1.0}_{-0.6}$
$\bar{h}_5$	$-0.57^{+0.40}_{-0.46}$	$-1.0^{+0.7}_{-0.7}$
$\bar{h}_6$	$-0.25^{+0.17}_{-0.07}$	$-0.46^{+0.28}_{-0.13}$
α	208^{+79}_{-21}	228^{+74}_{-28}
β	0.0123^*	0.0123^*
γ	$-0.28^{+0.35}_{-0.11}$	$-0.19^{+0.29}_{-0.12}$
$\chi^2/\mathrm{d.o.f}$	$\frac{26.6}{36-8} \simeq 0.96$	$\frac{26.9}{36-8} \simeq 0.96$

Details of the 4CC fit

	Fit-C (0^{++})	Fit-D (2^{++})
$\bar{h}_1$	$-4.2^{+1.3}_{-0.7}$	$-5.1^{+1.1}_{-2.4}$
$\bar{h}_2$	$15.3^{+4.7}_{-2.7}$	$-22.8^{+6.2}_{-11.5}$
$\bar{h}_3$	$1.0^{+0.5}_{-0.7}$	$-22.7^{+6.1}_{-22.4}$
$\bar{h}_4$	$-11.4^{+3.6}_{-2.0}$	$-18.2^{+3.5}_{-10.1}$
$\bar{h}_5$	$1.4^{+0.7}_{-1.1}$	$-20.6^{+30.6}_{-18.7}$
$\bar{h}_6$	$-0.3^{+0.1}_{-0.1}$	$-43.8^{+16.9}_{-160.5}$
$\bar{h}_7$	$-16.1^{+4.6}_{-2.7}$	$-21.0^{+5.3}_{-10.9}$
$\bar{h}_8$	$-1.2^{+0.5}_{-0.9}$	$-52.4^{+15.9}_{-62.8}$
$\bar{h}_9$	$-4.0^{+1.4}_{-0.7}$	$-3.8^{+0.7}_{-2.8}$
α	1118^{+381}_{-375}	391^{+144}_{-77}
β	0.0123^{*}	0.0123^{*}
γ	$1.22^{+0.10}_{-0.06}$	$0.51^{+0.13}_{-0.13}$
$\chi^2/\mathrm{d.o.f}$	$\frac{24.5}{43-11} \simeq 0.77$	$\frac{26.8}{43-11} \simeq 0.84$

Details of the combined fit

- The invariant mass spectrum is recast as

$$\frac{\mathrm{d}\mathcal{N}}{\mathrm{d}\sqrt{s}} = \sum_J \rho(s)|\mathcal{A}_1^J(s)|^2 \left| \gamma_J + \sum_i \mathcal{G}_i(s)\mathcal{T}_{i1}^J(s) \right|^2 ,$$

with

$$|\mathcal{A}_1^J(s)|^2 = \alpha^2 e^{-2\beta s} .$$

- Results of fitting parameters

	Fit-E (0^{++}&2^{++})
$\bar{h}_1$	$-3.1^{+0.3}_{-0.5}$
$\bar{h}_2$	$-13.0^{+1.4}_{-2.2}$
$\bar{h}_4$	$-10.7^{+1.2}_{-1.5}$
$\bar{h}_7$	$-12.8^{+1.5}_{-1.9}$
$\bar{h}_9$	$-1.9^{+0.3}_{-0.3}$
α	643^{+161}_{-98}
β	0.0123^*
γ_0	$1.2^{+0.1}_{-0.1}$
γ_2	$0.8^{+0.1}_{-0.1}$
$\chi^2/\mathrm{d.o.f}$	$\frac{28.3}{43-8} \simeq 0.81$