Hadronic contributions to HVP and HLBL: from amplitude analysis

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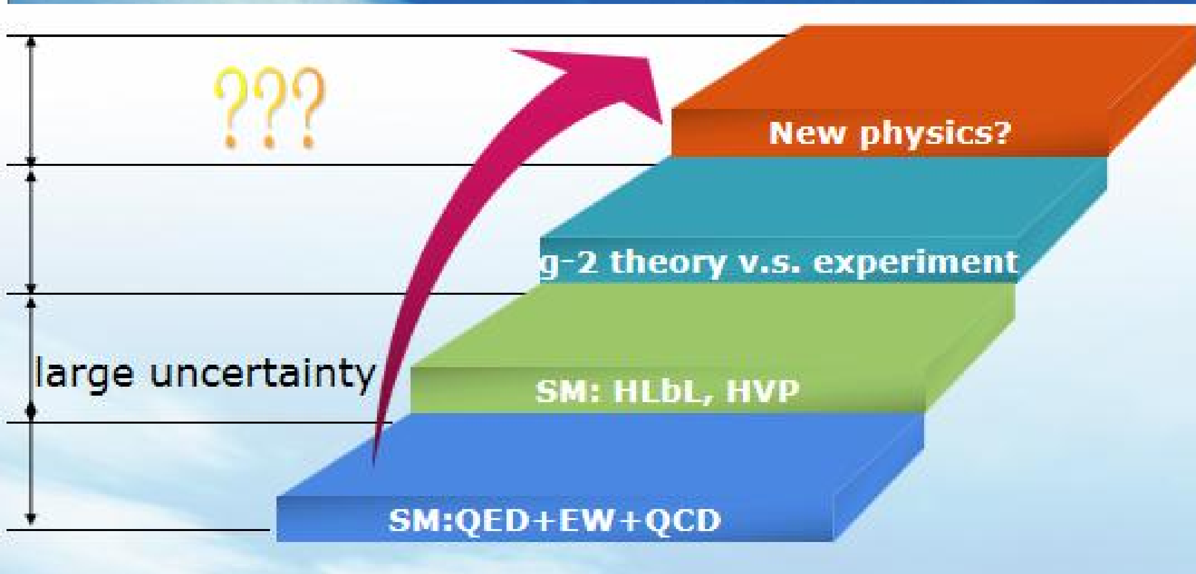
Outlines

1. Introduction
2. HVP
3. HLBL
4. Summary

1.Introduction



- HVP, HLbL?

	values ($\times 10^{-11}$)
QED	116584718.931(104)
EW	153.6(1.0)
HVP	6845(40)
HLBL	92(18)
SM	116591810(43)
exp.(BNL)	116592089(63)
exp.(FNAL)	116592040(54)
exp.(avg.)	116592061(41)
a_μ^{SM}-a_μ^{exp}	251(59)

Phys.Rept.887(2020)1

Phys.Rev.Lett.126, 141801 (2021)
Phys.Rev.D 73, 072003 (2006).

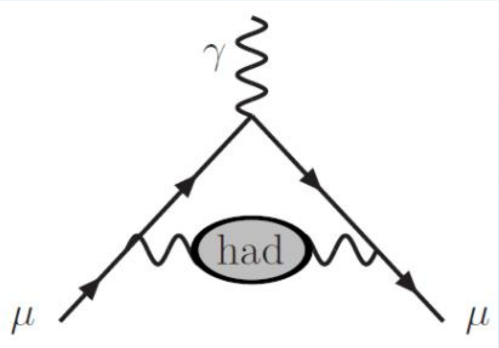
amplitude analysis

- LQCD
- data-driven solutions from experiment
- amplitude analysis? dispersive approach, ChEFT, etc.

    - Only one physical amplitude!
    - It should satisfy the fundamental QFT principles
    - It should be compatible with the exp results

2、HVP

- QCD: high energy region
- Dispersive approach: Roy, KT, PKU, etc., difficult to deal with multi-body rescattering
- ChPT: works in the low energy region
- RChT: extend to a bit higher energy region



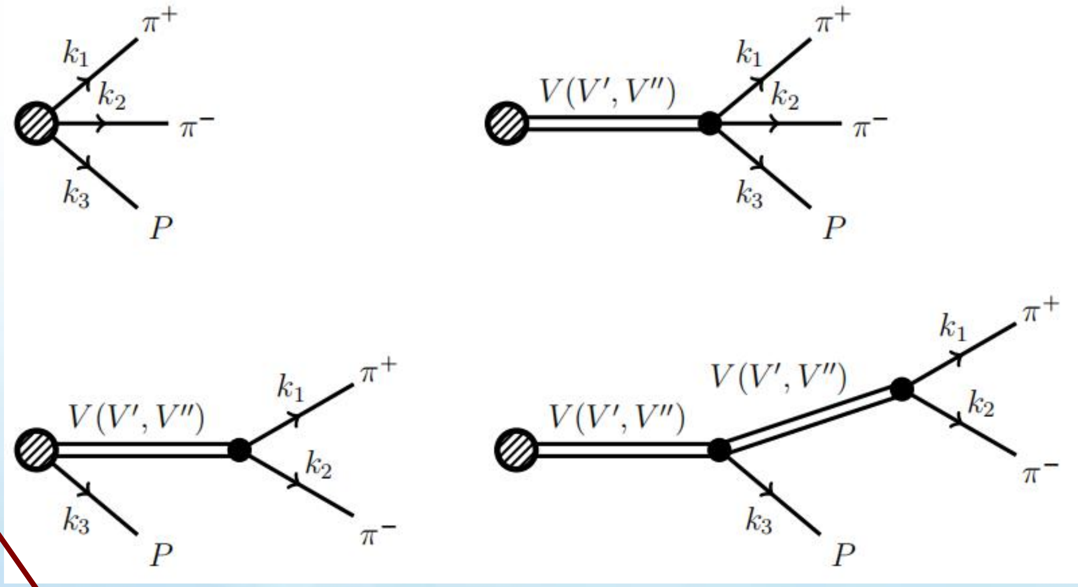
Dai et.al., PRD 99 (2019) 114015;
Guo et.al., JHEP 06 (2007) 030;

- resonances included as new degrees of freedom
- Matching with QCD, DRs to reduce LECs
- 1/Nc expansion

Building amplitudes

- RChT in the resonance region, excited states?

  - V', V'' has the same topologies as the ground states
  - ππ-KK FSI part by matching with Omens functions



$$\frac{1}{M_V^2 - x} \to \frac{1}{M_V^2 - x} + \frac{\beta'_\pi}{M_{V'}^2 - x} + \frac{\beta''_\pi}{M_{V''}^2 - x}$$

Guerrero, et.al., PLB 412 (1997) 382

Dai, et.al., PRD88 (2013) 056001

Building amplitudes

- We give a combined analysis on four channels:

$$\pi^+\pi^-, K^+K^-, \pi^+\pi^-\pi^0, \pi^+\pi^-\eta$$

  - ππ-KK FSI part by matching with Omnes function
  - ρ-ω mixing, origined from Gasser&Leutwyler's
- Not much freedom for Fit

$$F_V^\pi = \left(1 + \frac{F_V G_V}{F^2} Q^2 \left(BW(M_\rho, \Gamma_{\rho,}, Q^2) + \beta'_{\pi\pi} BW(M_{\rho'}, \Gamma_{\rho',}, Q^2) + \beta''_{\pi\pi} BW(M_{\rho''}, \Gamma_{\rho'',}, Q^2)\right) \left(\frac{1}{\sqrt{3}} \sin\theta_V \sin\delta^\rho + \cos\delta\right)\cos\delta - \frac{F_V G_V}{F^2} Q^2 \left(BW(M_\omega, \Gamma_{\omega,}, Q^2) + \beta'_{\pi\pi} BW(M_{\omega'}, \Gamma_{\omega',}, Q^2) + \beta'''_{\pi\pi} BW(M_{\omega''}, \Gamma_{\omega'',}, Q^2)\right) \left(\frac{1}{\sqrt{3}} \sin\theta_V \cos\delta - \sin\delta^\omega\right)\sin\delta^\omega\right) \exp\left[\frac{-s}{96\pi^2 F^2}\left(\mathrm{Re}\left[A[m_\pi, M_\rho, Q^2] + \frac{1}{2} A[m_K, M_\rho, Q^2]\right]\right)\right]$$

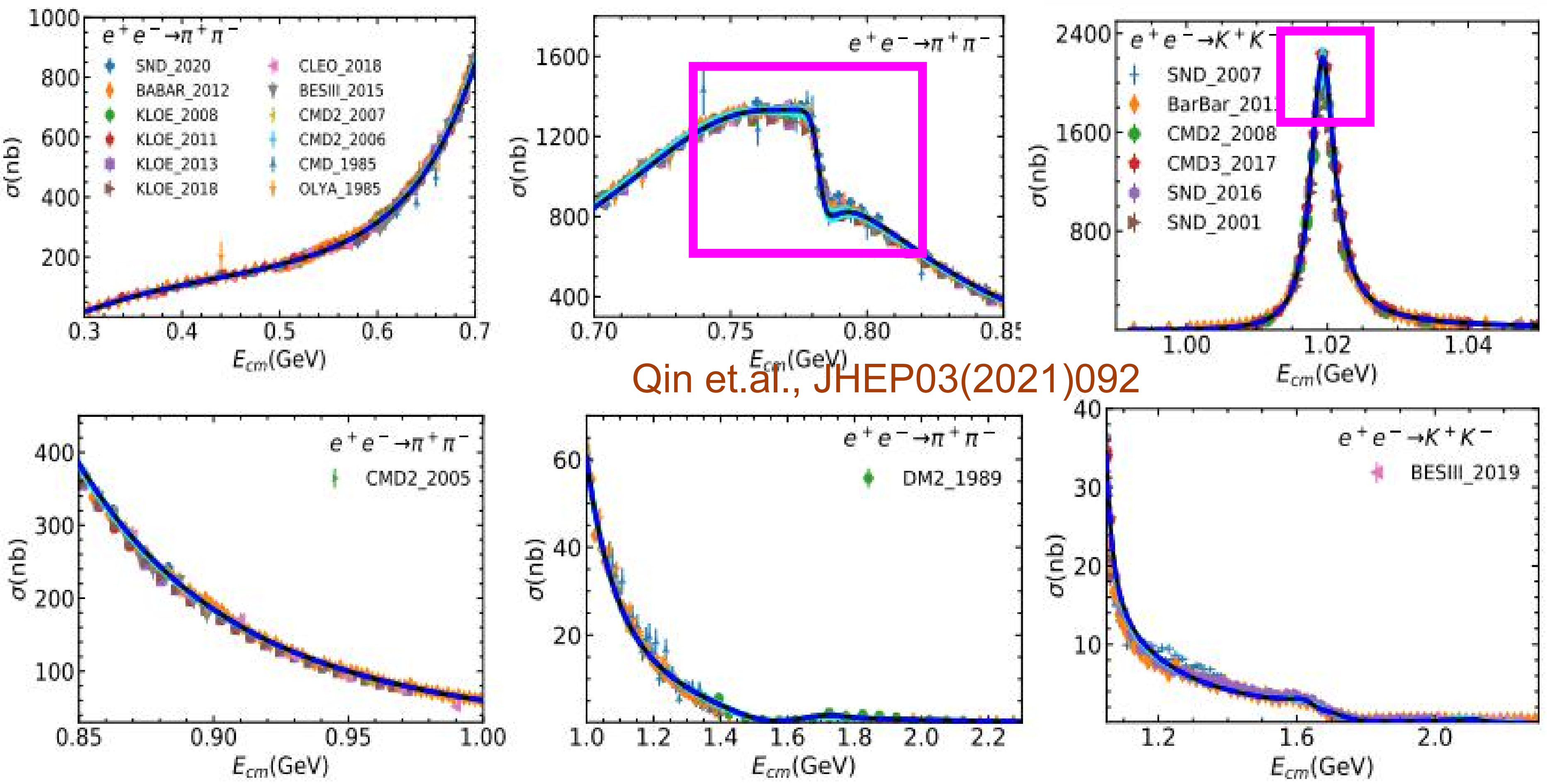
=1, from QCD as well as disersion relation constraints

Gasser&Leutwyler, Phys.Rept.87 (1982) 77

Guerrero&Pich, PLB 412 (1997) 382

Fit

- $\pi\pi$: Babar has large difference with KLOE and BESIII
- KK: data in the ϕ 'peak' have large discrepancy



Qin et.al., JHEP03(2021)092

Fit

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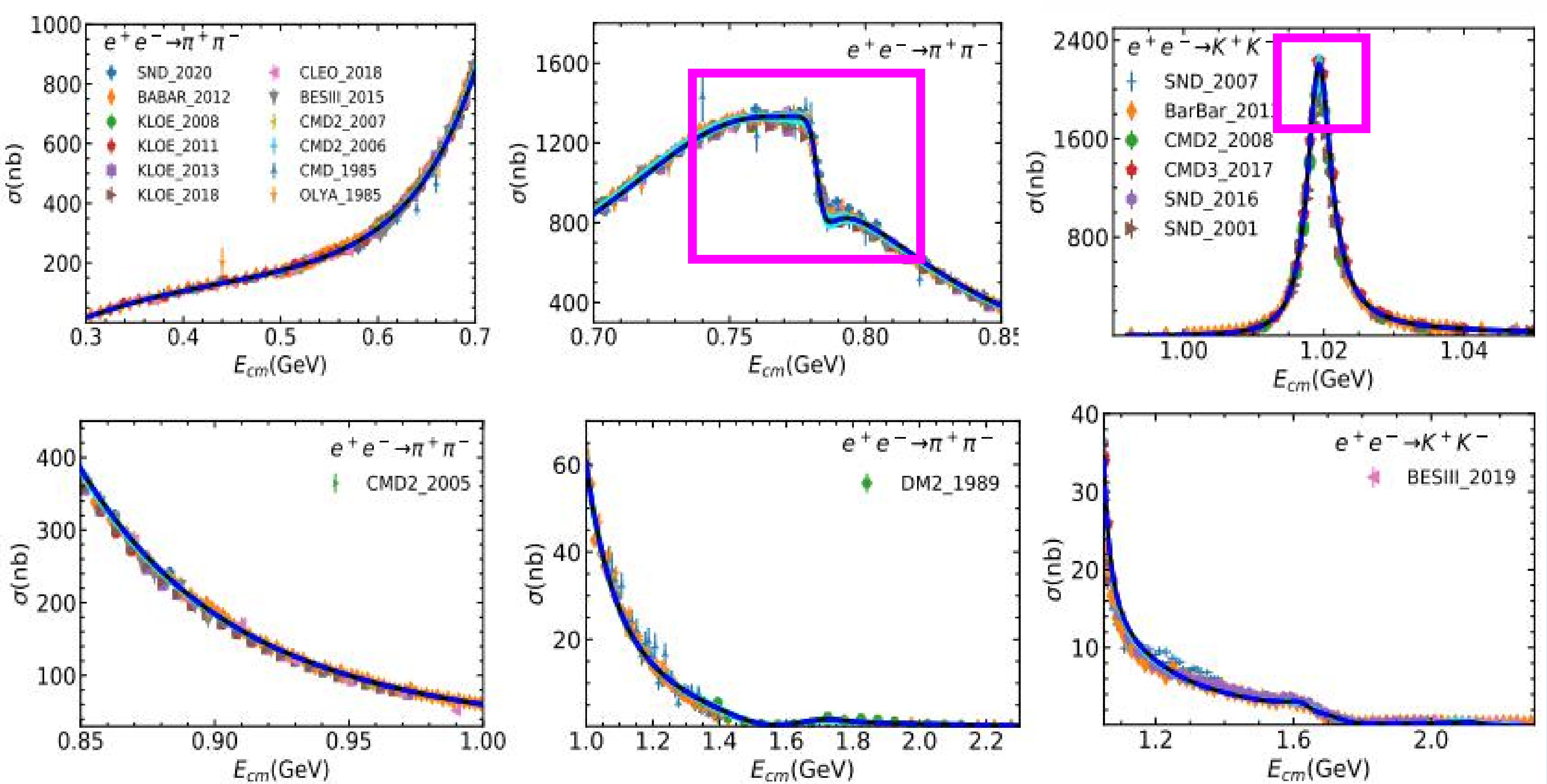
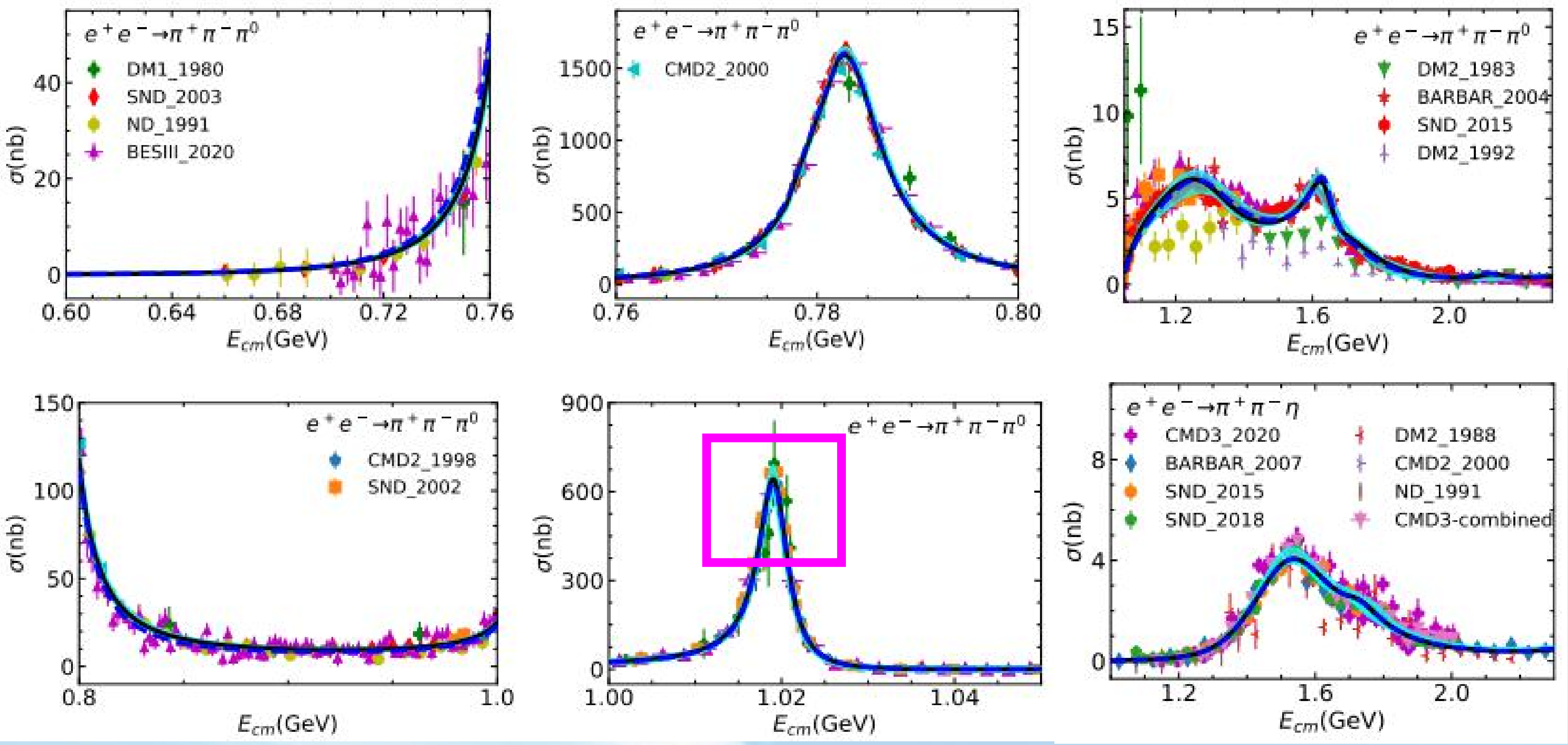


Fit

- $\pi\pi\pi$: needs more precise data in the ω ϕ region
- $\pi\pi\eta$: check our model



g-2:HVP

- Other channels are taken from data-driven or QCD
- Comparing with latest exp's: a 3.3 σ discrepancy

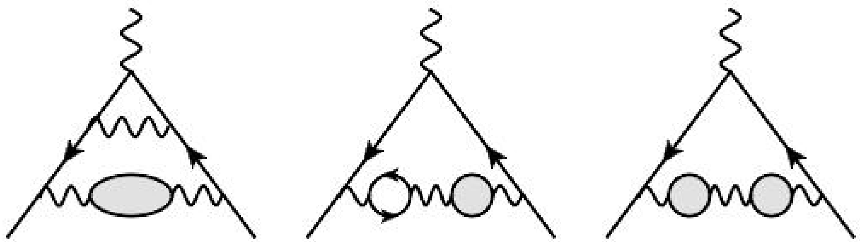
$a_\mu^C \times 10^{-10}$	CHHKS'19	DHMZ'19	Fit I	Fit II
$a_\mu^{\pi\pi}\vert_{\leq 0.63\mathrm{GeV}}$	132.8(0.4)(1.0)	-	132.11 ± 0.63	132.11 ± 0.67
$a_\mu^{\pi\pi}\vert_{\leq 1\mathrm{GeV}}$	495.0(1.5)(2.1)	-	498.48 ± 2.34	498.47 ± 2.33
$a_\mu^{\pi\pi}\vert_{\leq 1.8\mathrm{GeV}}$	-	$507.85\pm0.83\pm3.23\pm0.55$	508.89 ± 2.45	508.89 ± 2.45
$a_\mu^{\pi\pi}\vert_{\leq 2.3\mathrm{GeV}}$	-	-	509.13 ± 2.48	509.13 ± 2.48
$a_\mu^{KK}\vert_{\leq .1.1\mathrm{GeV}}$	-	-	20.73 ± 0.94	20.74 ± 0.88
$a_\mu^{KK}\vert_{\leq .1.8\mathrm{GeV}}$	-	$23.08\pm0.20\pm0.33\pm0.21$	24.35 ± 1.02	24.36 ± 0.97
$a_\mu^{KK}\vert_{\leq 2.3\mathrm{GeV}}$	-	-	24.43 ± 1.03	24.44 ± 1.01
$a_\mu^{\pi\pi\pi}\vert_{\leq 1.8\mathrm{GeV}}$	46.2(8)	$46.21\pm0.40\pm1.10\pm0.86$	48.55 ± 1.42	48.54 ± 1.39
$a_\mu^{\pi\pi\pi}\vert_{\leq 2.3\mathrm{GeV}}$	-	-	48.76 ± 1.45	48.75 ± 1.43
$a_\mu^{\eta\pi\pi}\vert_{\leq 1.8\mathrm{GeV}}$		$1.19\pm0.02\pm0.04\pm0.02$	1.28 ± 0.10	1.29 ± 0.09
$a_\mu^{\eta\pi\pi}\vert_{\leq 2.3\mathrm{GeV}}$	-	-	1.52 ± 0.12	1.53 ± 0.12
$a_\mu^{\mathrm{HVP.LO}}$	-	694.0 ± 4.0	699.46 ± 3.41	699.47 ± 3.39
a_μ^{SM}		11659183.1 ± 4.8	11659187.3 ± 3.8	11659187.3 ± 3.9
Δa_μ		$26.0\pm7.9(3.3\sigma)$	$21.6\pm7.4(2.9\sigma)$	$21.6\pm7.4(2.9\sigma)$

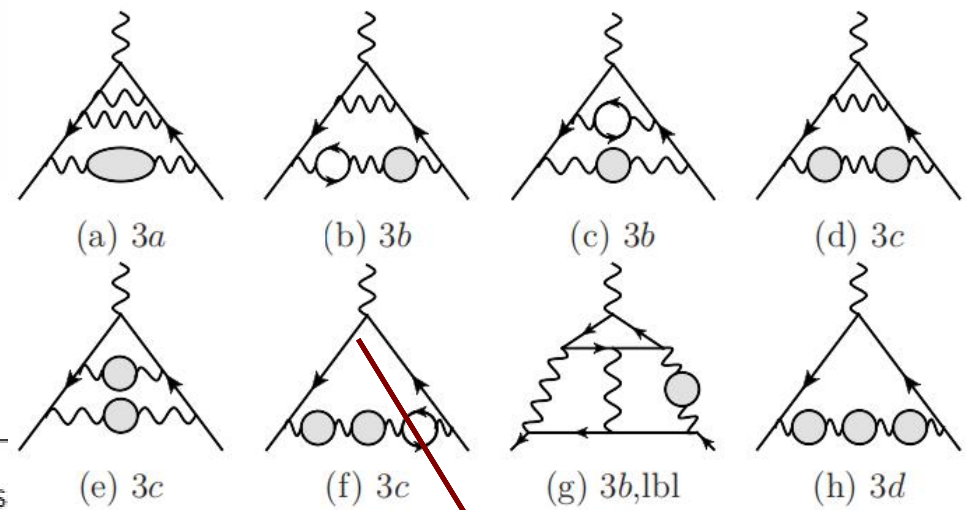
$708.7(5.3)\times10^{-10}$

Nature(2021)

HVP: NLO, NNLO?

- More channels (also high energy ones) to give a complete estimation?





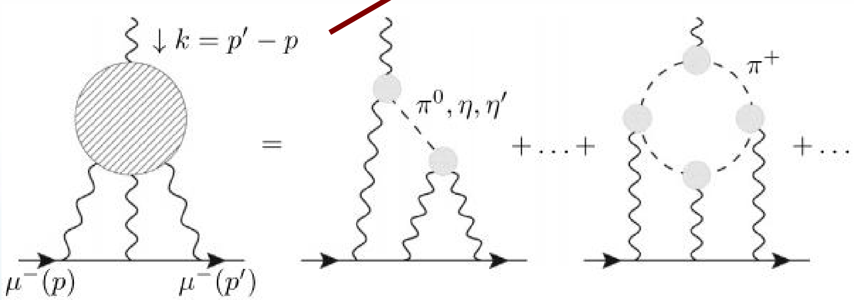
$\times 10^{-12}$	$\pi\pi$	KK	$\pi\pi\pi$	$\pi\pi\eta$	S	(PLB 734,144)
2a	-1369±8	-79.8±2.8	-145±3	-5.93±0.46	-1600±9	-2090
2b	776±5	37.6±1.3	74.7±1.8	2.37±0.18	891±5	1068
2c			22.4±0.2		22.4±0.2	35
$a_\mu^{\rm NLO}$					-687±10	-987±9
3a	45.4±0.3	3.11±0.11	5.20±0.12	0.267±0.021	54.0±0.3	80
3b	-24.8±0.2	-1.62±0.06	-2.78±0.06	-0.131±0.010	-29.3±0.2	-41
3bLBL	58.0±0.3	3.47±0.12	6.19±0.14	0.268±0.021	67.9±0.4	91
3c			-2.34±0.02		-2.34±0.02	-6
3d			0.0249±0.0004		0.0249±0.0004	0.05
$a_\mu^{\rm NNLO}$					90.3±0.5	124±1

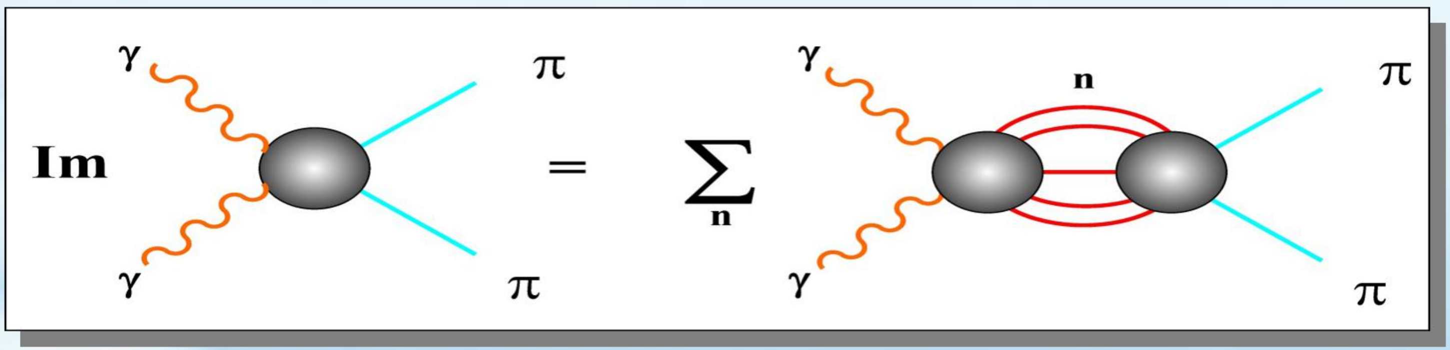
Kurz, et.al.
PLB 734 (2014) 144

HLBL

- Final State Interaction Theorem
- Dispersion relations
- ChPT constraints

Phys.Rept.887(2020)1



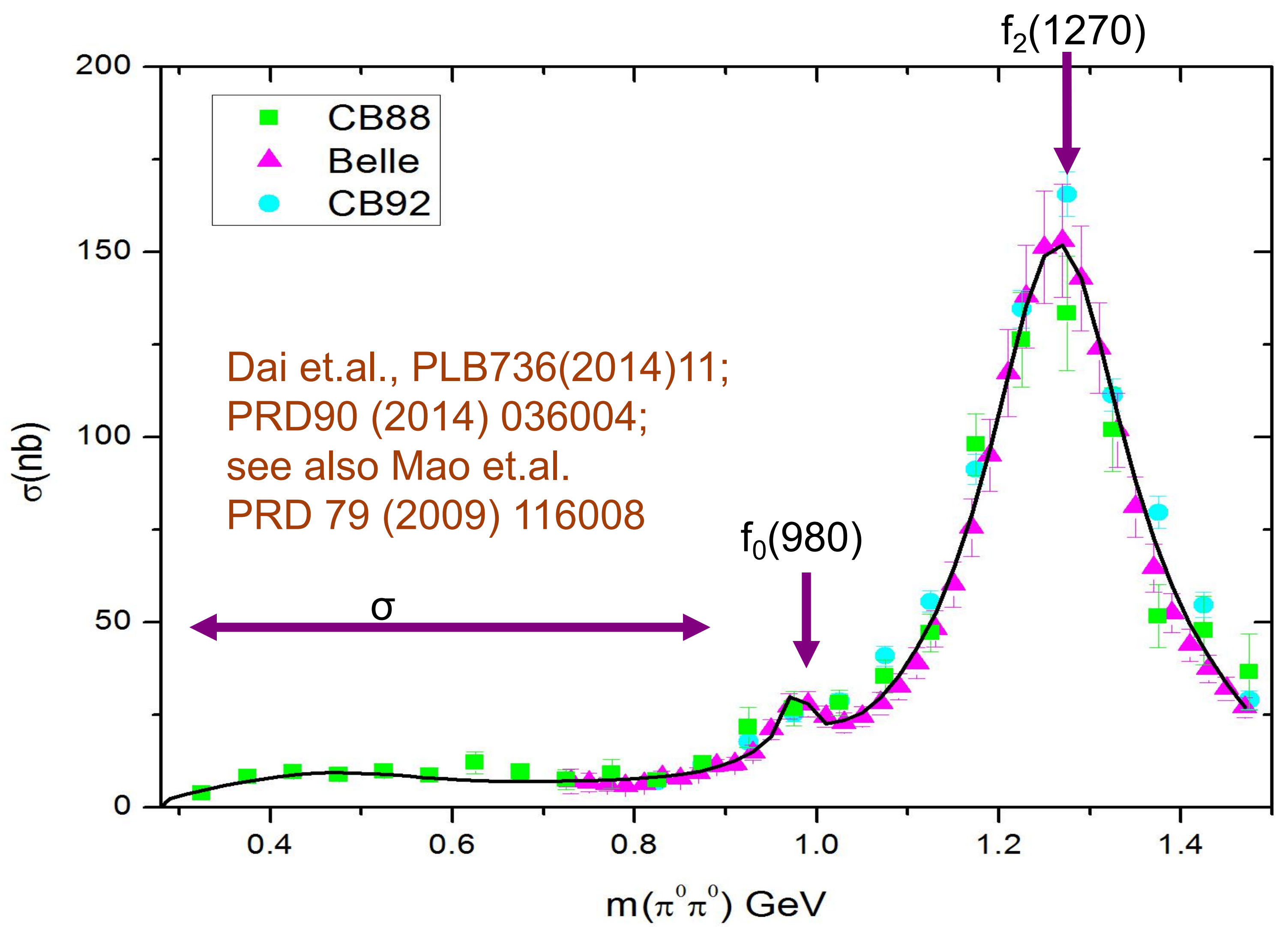


$$\mathcal{F}^I_{00}(s) = \mathcal{B}^I_{00}(s) + b^I s\,\Omega^I_{00}(s) + \frac{s^2\,\Omega^I_{00}(s)}{\pi}\int_L ds' \frac{\mathrm{Im}\left[\mathcal{L}^I_{00}(s')\right]\Omega^I_{00}(s')^{-1}}{s'^2(s'-s)}$$

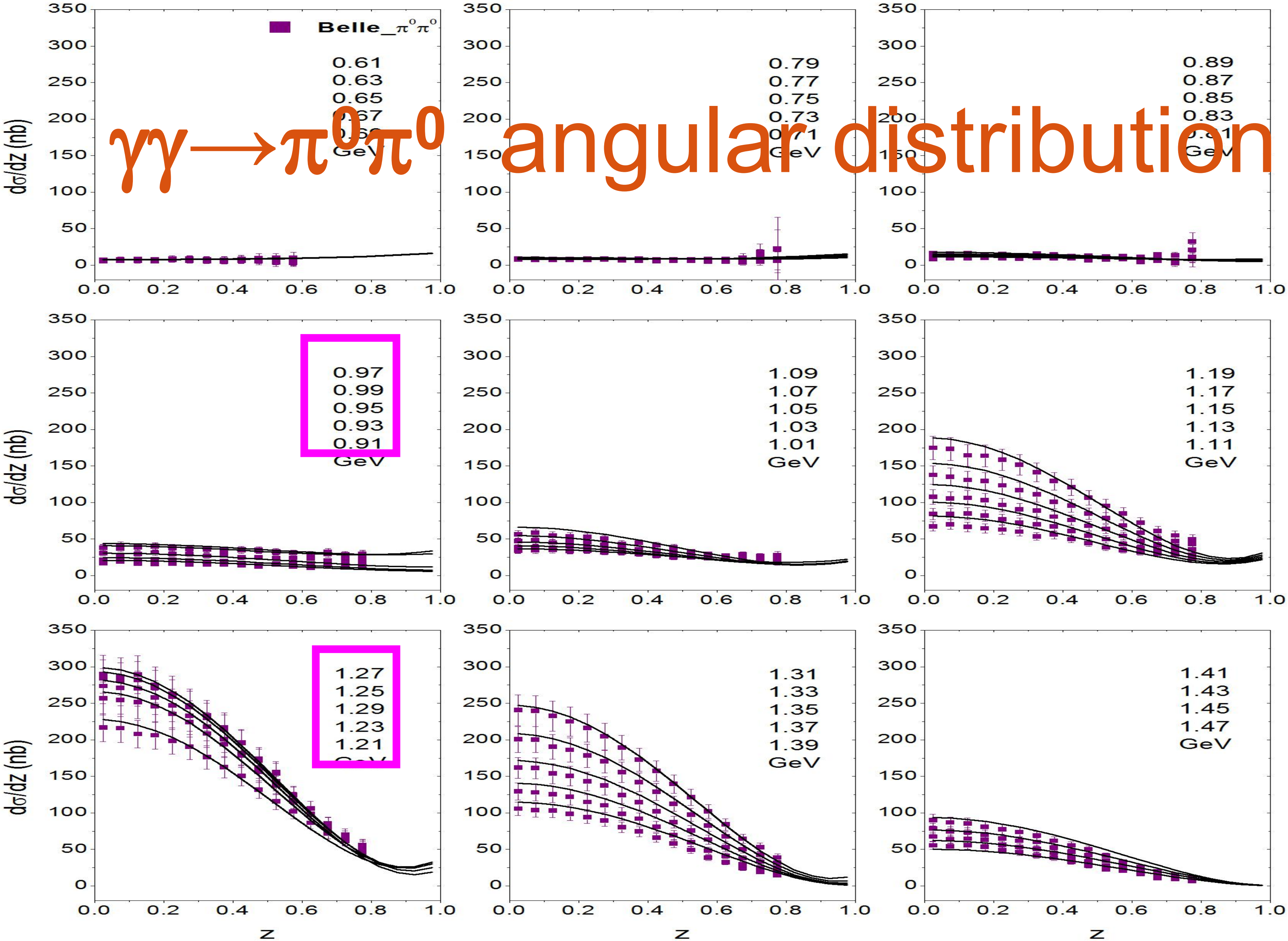
$$- \frac{s^2\,\Omega^I_{00}(s)}{\pi}\int_R ds' \frac{\mathcal{B}^I_{00}(s')\,\mathrm{Im}\left[\Omega^I_{00}(s')^{-1}\right]}{s'^2(s'-s)}$$

Solved by ChPT

$\gamma\gamma\to\pi^0\pi^0$ integrated cross section



Angular distribution is helpful to seperate each partial wave.



Constraints to light-by-light sumrule

- The contribution to PV sumrule is certainly not zero.
- 4π channel's contribution is significant for HLBL
- I=0:150–200 nb, I=2: 50nb

Phys.Rept.887(2020)1

HLBL	92(18)

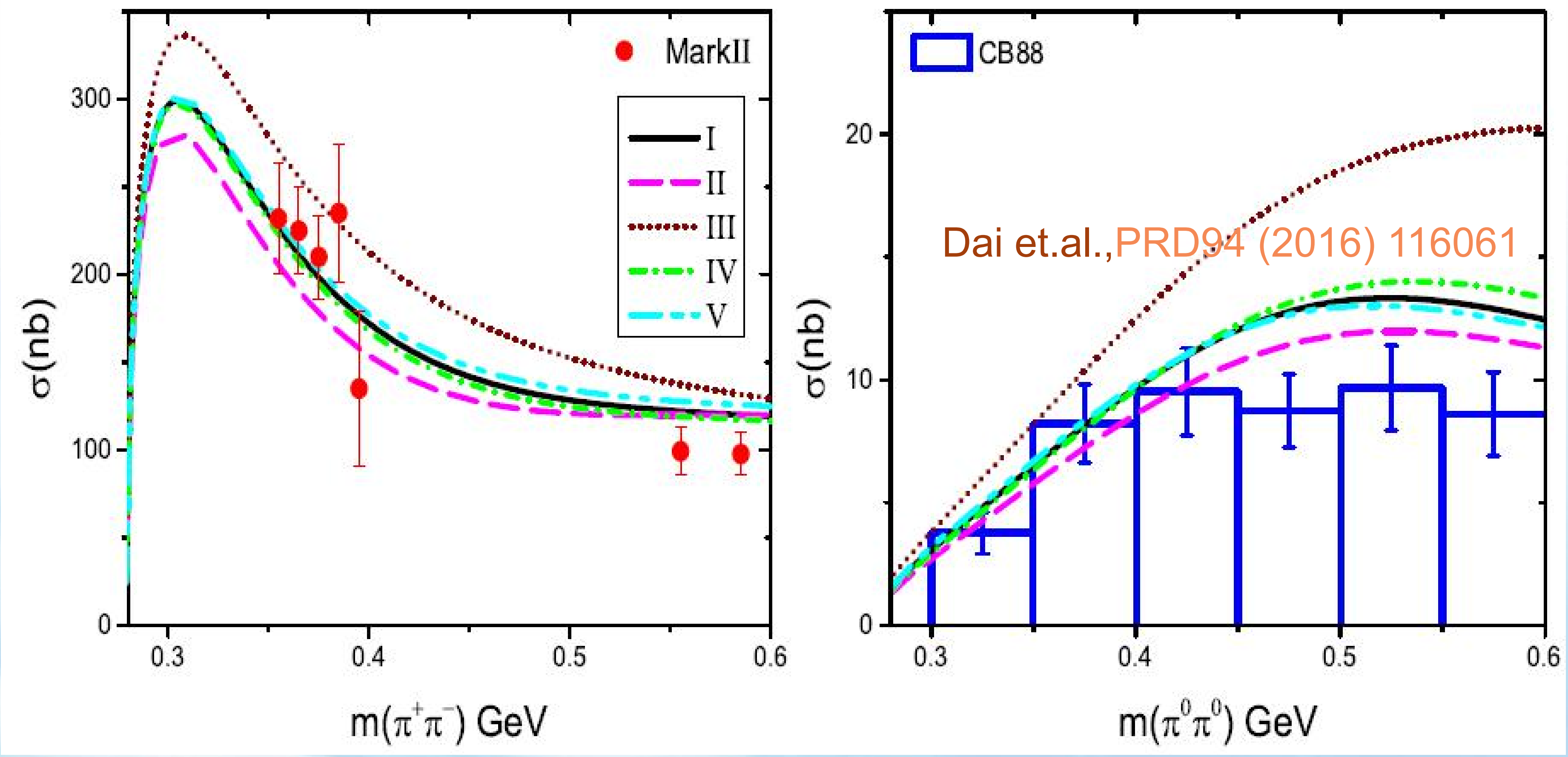
evaluation of $\Delta^I(4m_\pi^2, \infty, Z=1)$	$I=0$	$I=1$	$I=2$
$\gamma\gamma \to \pi^0$ [6] (nb)	-	-190.9±4.0	-
$\gamma\gamma \to \eta, \eta'$ [6] (nb)	-497.7±19.3	-	-
$\gamma\gamma \to a_2(1320)$ [6] (nb)	-	*135.0±12±25 †*	-
$\gamma\gamma \to \pi\pi$ (nb)	308.0±41.5	-	-44.2±6.1
$\gamma\gamma \to \overline{K}K$ (nb)	23.7±7.5	18.1±4.9	-
SUM (nb)	-166.0±46.4	-37.8±28.4	-44.2±6.1

BESIII? BelleII?

Dai et.al., PRD95 (2017) 056007;

Polarizabilities

Polarizabilities plays important role on HLbL DRs



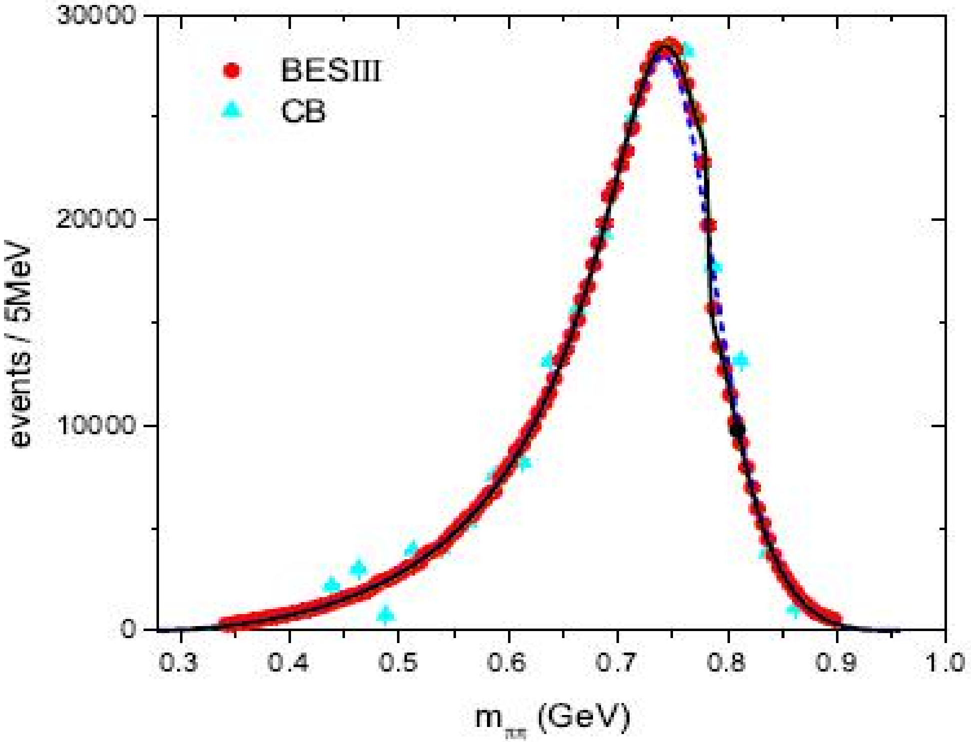
$(\alpha_1-\beta_1)_{\pi+}$ = 11.6, has been exclude by CB's data

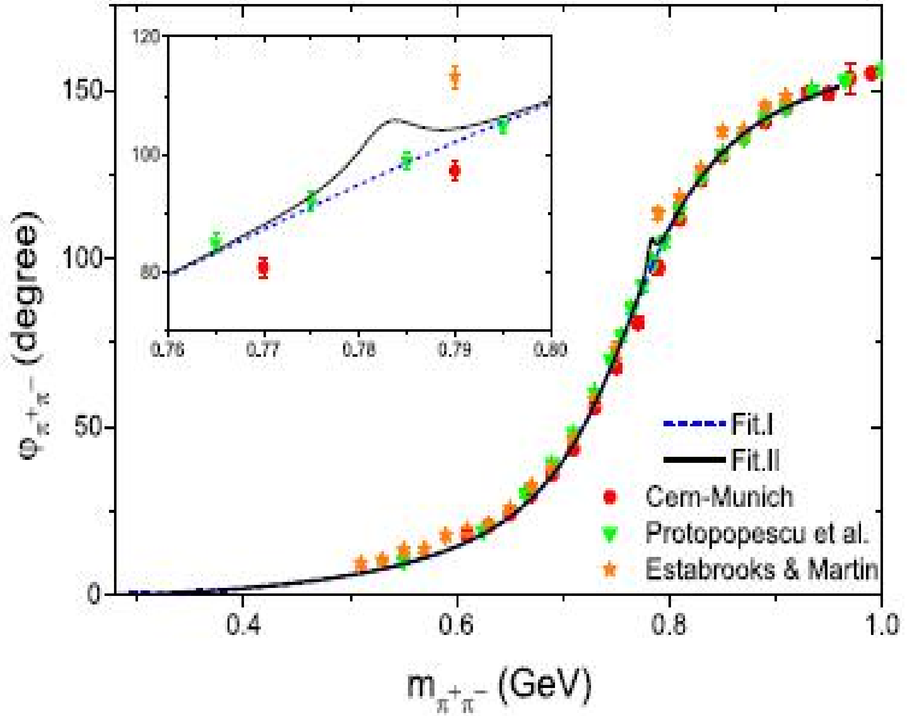
Amplitude analysis

- $\pi^+\pi^-$ P-wave phase-shift (extracted by Exp) should be taken into isospin violation

$\gamma^* \to \pi^+\pi^-$

$\gamma^*\gamma \to \pi^+\pi^-$





Dai et.al.,PRD97 (2018) 036012

6. Summary

Amplitudes	Amplitude analsysis connects QFT principles and Exp.
HVP	Ours has a 3.3 σ discrepancy with the latest Exp
HLBL	Ours is a constraint to HLBL amplitudes. 4π's can not be ignored.
Next?	Light hadron physics are related with each other. Further study is neccessary to give a more reliable answer to muon g-2.

Thank You For your patience !