Recent results from lattice QCD

Ying Chen

Institute of High Energy Physics,
Chinese Academy of Sciences, China

Beijing, July. 09, 2021

Outline

I. Evidence of existence of glueballs at the physical point
II. Glueball component of η_c and its implications
III. Prospects
IV. Summary

I. Existence of glueballs at the physical point

I) Glueball spectrum (previous results)

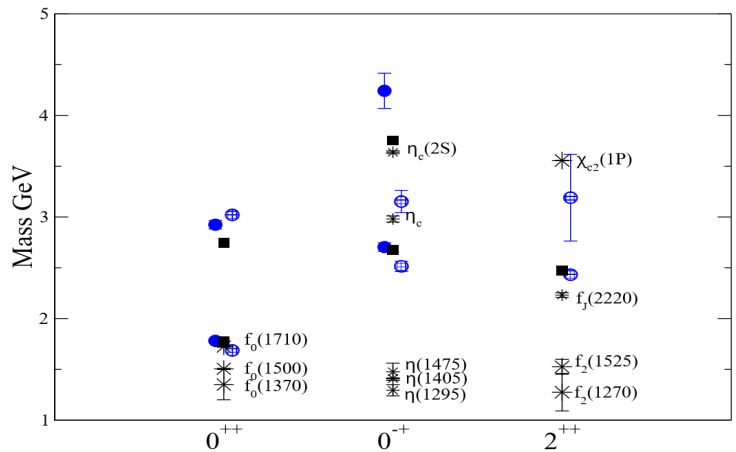
	m_π (MeV)	$m_{0^{++}}$ (MeV)	$m_{2^{++}}$ (MeV)	$m_{0^{-+}}$ (MeV)
$N_f = 2$	938	1417(30)	2363(39)	2573(55)
	650	1498(58)	2384(67)	2585(65)
$N_f = 2+1$ [22]	360	1795(60)	2620(50)	—
quenched [13]	—	1710(50)(80)	2390(30)(120)	2560(35)(120)
quenched [14]	—	1730(50)(80)	2400(25)(120)	2590(40)(130)

Nf=2: W. Sun et al (CLQCD), Chin. Phys. C (in press), arXiv:1702.08174(hep-lat)

[14] C. Morningstar and M. Peardon, Phys. Rev. D 60, 034509, 1999

[13] Y. Chen et al, Phys. Rev. D 73, 014516, 2006

[22] E. Gregory et al., JHEP 10 (2012) 170, arXiv:1208.1858(hep-lat)



Filled Squares: QQCD

Open circles: full QCD, coarse lattice

Closed circles: full QCD, fine lattice

C.M. Richards et al., [UKQCD Collab.], Phys. Rev. D82, 034501 (2010).

II) What is the situation at the physical point

- $N_f = 2 + 1$ **dynamical confiugrations generated by RBC/UKQCD Collaboration.**

- **Accessed through the agreement between χQCD Collaboration (PI: Prof. K.-F. Liu of Univ. Kentucky)**

TABLE I. Parameters of 48I and 64I ensemble.

$L^3 \times T$	a (fm)	m_π (MeV)	La (fm)	$N_{\rm conf}$
$48^3 \times 96$	0.1141(2)	~ 139	~ 5.5	364
$64^3 \times 128$	0.0836(2)	~ 139	~ 5.3	300

- **Physical m_π, m_K, large volume, but small size of ensembles**

III) Efficacy of cluster decomposition

K.-F. Liu, J. Liang, Y.-B. Yang, Phys. Rev. D 97 (2018), 034507

- In Euclidean space, translation invariance, stability of the vacuum, existence of a lowest non-zero mass local commutativity, require that

$$|\langle\Omega|T\mathcal{O}_1(x)\mathcal{O}_2(y)|\Omega\rangle_s| \leq A\, r^{-\frac{3}{2}} e^{-Mr}, \qquad r = |x-y|$$

where $\langle\cdots\rangle_s$ means a the vacuum-subtracted correlator.

$$\int_0^R r^2 dr\; r^{-\frac{3}{2}} e^{-Mr} = \frac{\sqrt{\pi}\,\mathrm{erf}\left(\sqrt{MR}\right)}{M^{3/2}} - \frac{\sqrt{R}e^{-mR}}{M}$$

- In numerical lattice calculations, signals saturate by $r < \mathrm{R} \approx \frac{8}{M}$

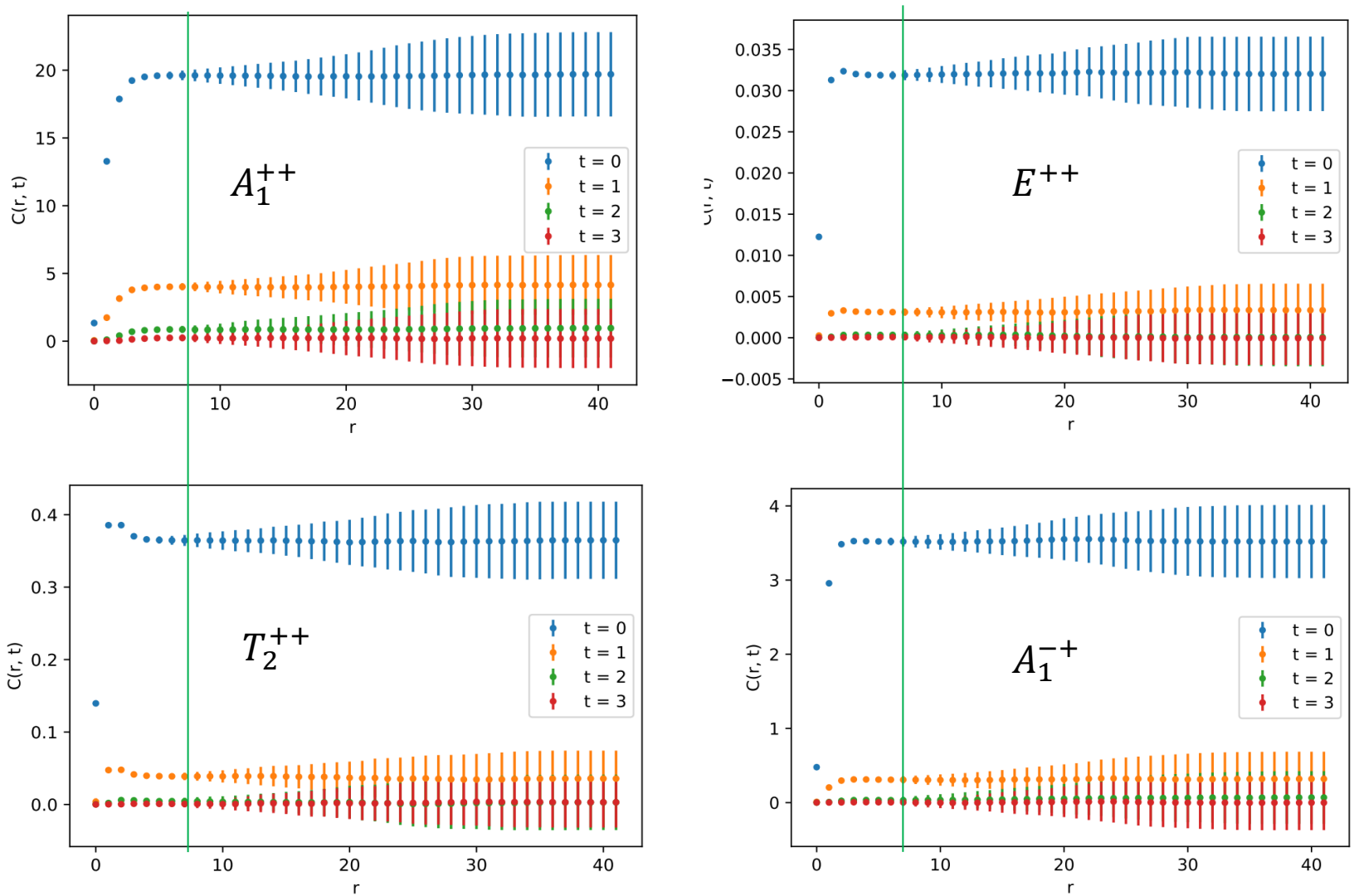
$$C(t) = \sum_{\vec{x},\vec{y}} \langle\Omega|T\mathcal{O}_1(\vec{x},t)\mathcal{O}_2(\vec{y},0)|\Omega\rangle \qquad r = \sqrt{\vec{r}'^2 + t^2}$$

$$\approx \sum_{\vec{x}} \sum_{r<\mathrm{R}} \langle\Omega|T\mathcal{O}_1(\vec{x}+\vec{r}',t)\mathcal{O}_2(\vec{x},0)|\Omega\rangle \equiv C(R,t)$$

$$= \int_{r<R} d^3\vec{r}'\, \langle K(\vec{r}',t)\rangle$$

$$K(\vec{r}',t) \sim \int d^3\vec{p}\; e^{i\vec{p}\cdot\vec{r}'}\mathcal{O}_1(\vec{p},t)\mathcal{O}_2(-\vec{p},t)$$

- $\mathcal{O}_G - \mathcal{O}_G$ correlation function for different cut R (r in the plot)



The saturation R is chosen to be $R = 7a$

IV) AA-operators for glueballs

P. Forcrand, K.-F. Liu, Phys.Rev.Lett. 69 (1992) 245

J. Liang, Y. Chen, W.-F. Chiu, L.-C. Gui, and M. Gong, Phys. Rev. D 91 (2015) 5, 054513

- **Gauge fields:** $A_\mu(x) \sim \ln U_\mu(x)\,,\quad U_\mu(x) = e^{-iagA_\mu(x)}$

$$R^+ U_\mu(x) R \equiv \mathrm{diag}(\lambda_1, \lambda_2, \lambda_3)$$

$$A_\mu(x) \sim R^+ \mathrm{diag}\,(\ln\lambda_1\,, \ln\lambda_2\,, \ln\lambda_3) R$$

- **AA-operators for glueballs**

$$\mathcal{O}_{AA}^{(LM;S)}(\vec{r}) = \frac{1}{N_r}\sum_{|\vec{r}|=r}\sum_{\vec{x}} c_{ij}(S) Y_{LM}(\hat{r}) A_i(\vec{x}+\vec{r}) A_j(\vec{x})$$

S: the total spin of two gauge field;
(LM): the orbital quantum number between two gauge fields;
N_r: the multiplicity of $\vec{r}$ with $|\vec{r}| = r$

- **AA-operators are gauge invariant** → **Coulomb gauge!**

The explicit expressions of $\mathcal{O}_{AA}^{(LM;S)}(\vec{r})$ for $A_1^{++}, A_1^{-+}, E^{++}$ and T_2^{++}

A_1^{++} $\quad L = 0, \quad S = 0$: $\mathcal{O}_{AA}^{A_1^{++}} = [A_1A_1 + A_2A_2 + A_3A_3]$

E^{++} $\quad L = 0, \quad S = 2$: $\mathcal{O}_{AA}^{E^{++},1} = \frac{1}{\sqrt{2}}[A_1A_1 - A_2A_2]$

$L = 0, \quad S = 2$: $\mathcal{O}_{AA}^{E^{++},2} = \frac{1}{\sqrt{6}}[2A_3A_3 - A_1A_1 - A_2A_2]$

T_2^{++} $\quad L = 0, \quad S = 2$: $\mathcal{O}_{AA}^{T_2^{++},1} = \frac{1}{\sqrt{2}}[A_2A_3 + A_3A_2]$

$L = 0, \quad S = 2$: $\mathcal{O}_{AA}^{T_2^{++},2} = \frac{1}{\sqrt{2}}[A_3A_1 + A_1A_3]$

$L = 0, \quad S = 2$: $\mathcal{O}_{AA}^{T_2^{++},3} = \frac{1}{\sqrt{2}}[A_1A_2 + A_2A_1]$

A_1^{-+} $\quad L = 1, \quad S = 1$

$$\mathcal{O}_{AA}^{A_1^{-+}}(r,t) = \frac{1}{N_r}\sum_{\vec{x},|\vec{r}|=r} \epsilon_{ijk}A_i(\vec{x}+\vec{r},t)A_j(\vec{x})\frac{r_k}{|r|}$$

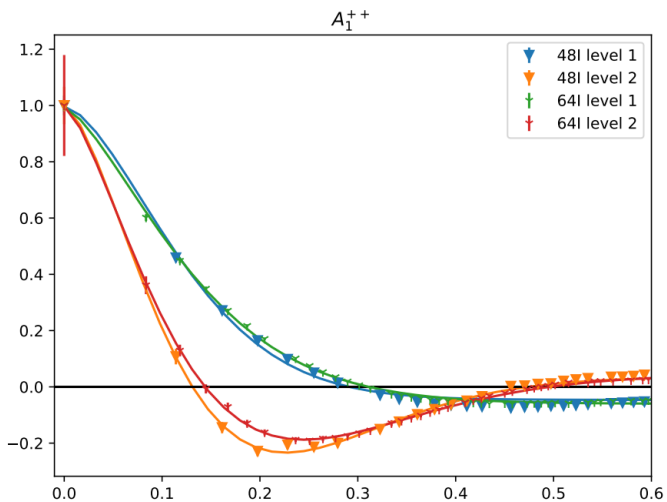
- **Bethe-Salpeter wave functions from the $\mathcal{O}_{AA} - \mathcal{O}_G$ correlation functions**

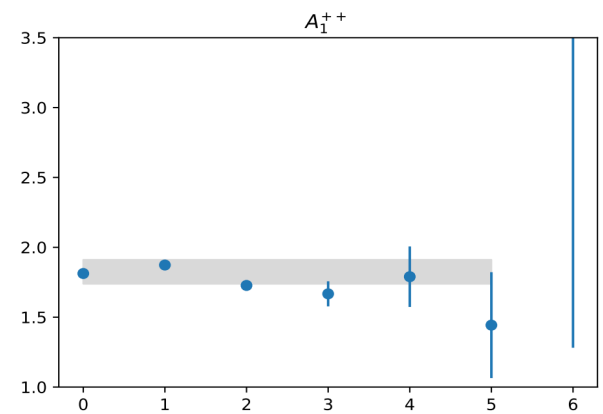
Optimized glueball operators: $\left\langle \mathcal{O}_G^{(n)}(t)\mathcal{O}_G^{(n)}(0)\right\rangle \approx e^{-m_n t} + \cdots$

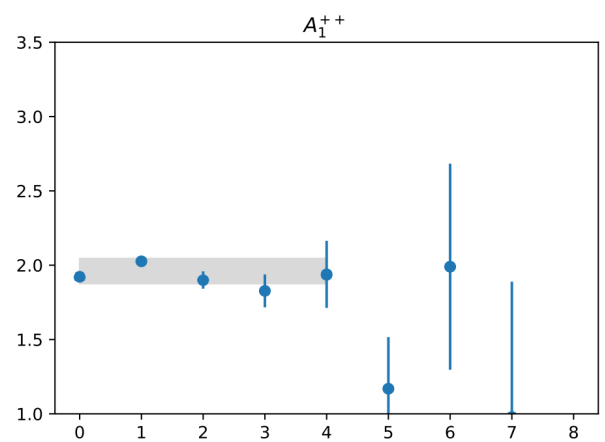
$\mathcal{O}_{AA} - \mathcal{O}_G$ **correlation functions:**

$$\left\langle \mathcal{O}_{AA}(t)\mathcal{O}_G^{(n)}(0)\right\rangle \propto \langle \Omega|\mathcal{O}_{AA}(r)|n\rangle \left\langle n\left|\mathcal{O}_G^{(n)}\right|\Omega\right\rangle e^{-m_n t} \approx \Phi_n(r)e^{-m_n t} + \cdots$$

A) Scalar glueball (A_1^{++})



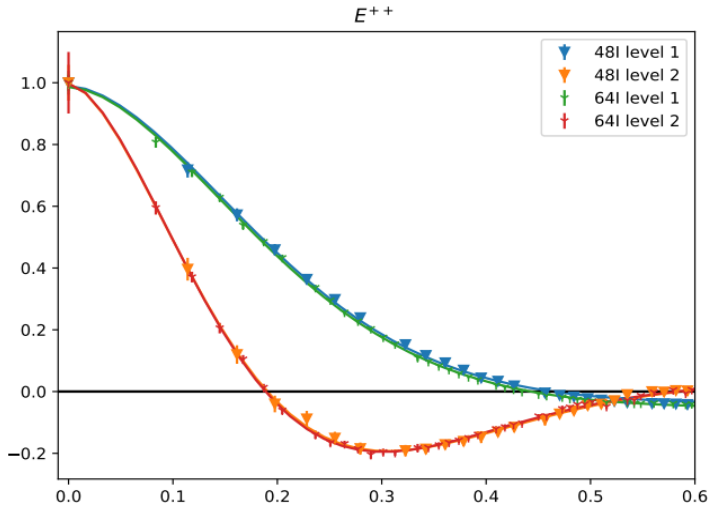


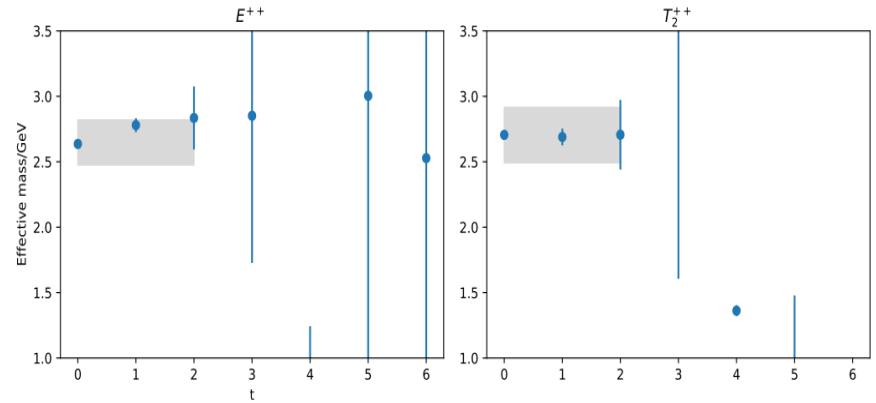


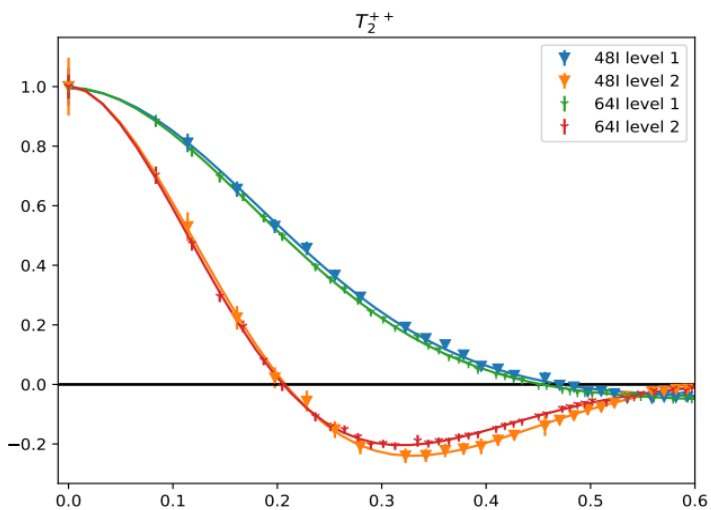
$$\Phi_1(r) = \Phi_1(0)e^{-(r/r_0)^\alpha}$$

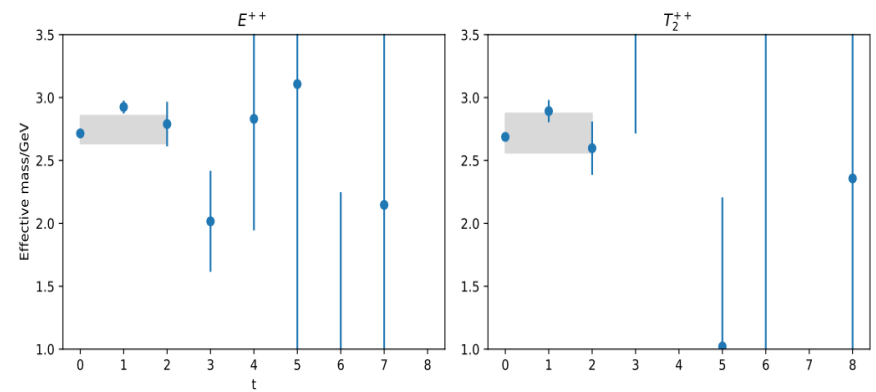
$$\Phi_2(r) = \Phi_2(0)(1 - \beta r^\alpha)e^{-(r/r_0)^\alpha}$$

B) Tensor glueball (E^{++} and T_2^{++})

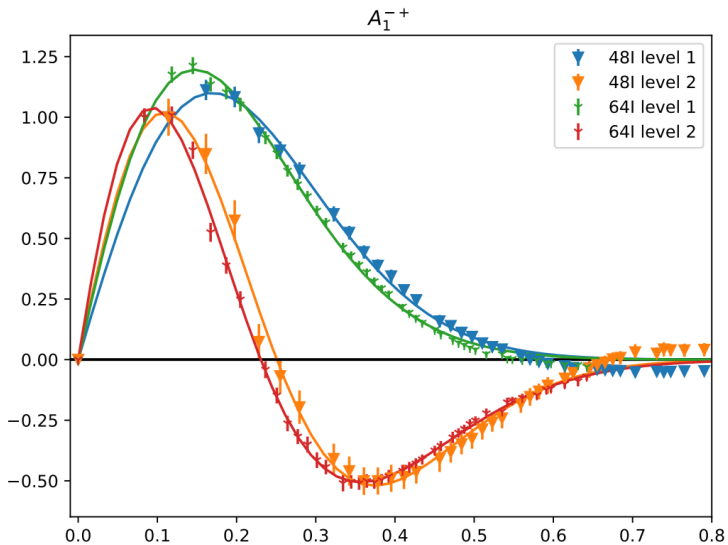


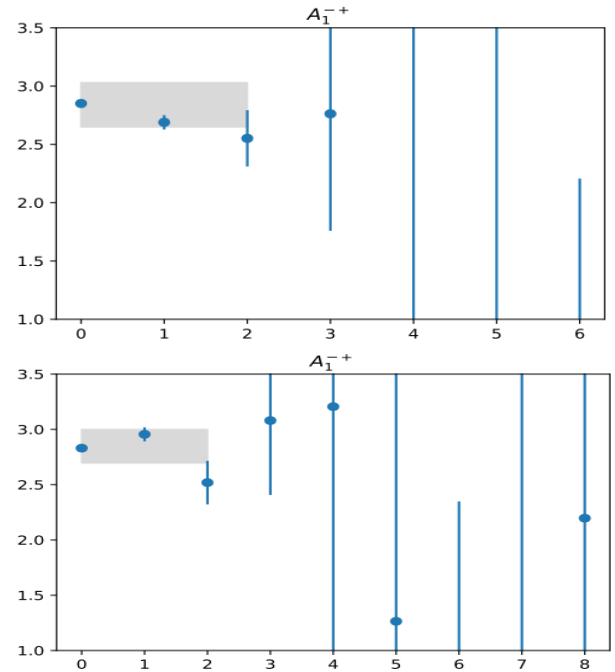






C) Pseudoscalar glueball (A_1^{-+})





D) Preliminary masses of ground state glueballs

0	A_1^{++}	E^{++}	T_2^{++}	A_1^{-+}
48I	1.82 ± 0.09	2.6 ± 0.2	2.7 ± 0.02	2.8 ± 0.2
64I	1.96 ± 0.08	2.7 ± 0.1	2.7 ± 0.2	2.8 ± 0.2

E) Tentative interpretation

- Glueballs are well-defined in the quenched approximation, whose spectrum is also well-established.
- This is not the case for full-QCD.
- Ideally, the energy levels obtained in lattice QCD are the eigenvalues of H_{QCD} of Euclidean lattice,

$$H_{QCD}|\alpha\rangle = E_\alpha|\alpha\rangle, \qquad \sum_\alpha |\alpha\rangle\langle\alpha| = 1$$

- Imagine that there is a complete state set

$$|n\rangle = |G\rangle, |q\bar{q}\rangle, |MM\rangle, \dots, \qquad \sum_n |n\rangle\langle n| = 1$$

here $|n\rangle$ are pure gauge glueballs, $q\bar{q}$ mesons, non-interacting multi-hadron states, etc.

- The eigen states of H_{QCD} can be expanded in terms of $|n\rangle$

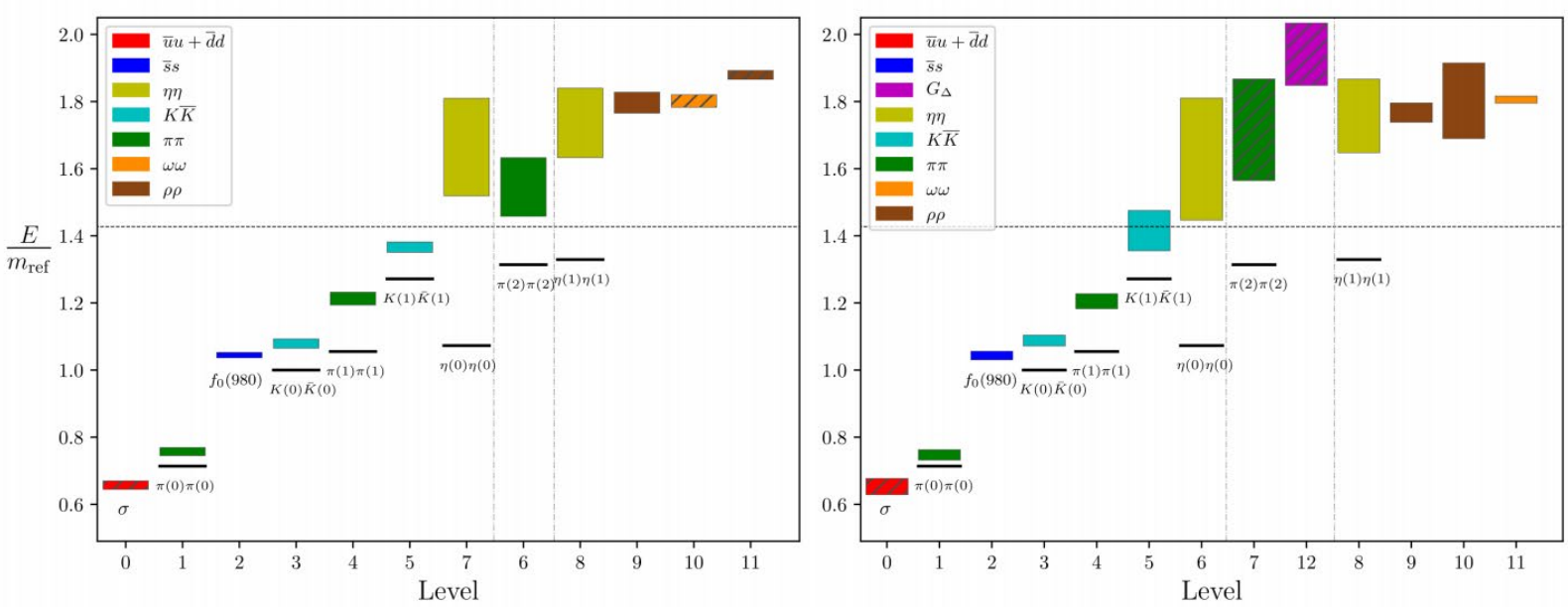
$$|\alpha\rangle = C_n^\alpha|n\rangle$$

- The ground states addressed previously may be (to be checked)

$$|grd\rangle \equiv O_G^{(1)}|\Omega\rangle = (1-\epsilon_1)|G\rangle + \sum_{n,n\neq G} \epsilon_n\,|n\rangle, \qquad \epsilon_n \ll 1$$

Spectroscopy from lattice QCD: The scalar glueball

R. Brett et al. AIP Conf. Proc. 2249 (2020) 030032 (arXiv: 1909.07306(hep-lat)



II. Glueball component of η_c and its implication

- **Large total width of η_c: $\Gamma_{\eta_c} = 32.0(7)$ MeV**

- **This motivates the possibility of sizable gluonic component in η_c**

- **If $\eta(1405)$ and $\eta(1475)$ are the same state, there is no need for a pseudoscalar glueball round 1.3-1.5 GeV**

  J.-J. Wu et al., Phys. Rev. Lett. 108, 081803 (2012), arXiv:1108.3772 [hep-ph].

- **Lattice QCD predict the mass of the pseudoscalar glueball to be around 2.4-2.6 GeV**

- **There may be mixing between $\bar{c}c(\,^1S_0\,)$ and the PS glueball**

  Y.-D. Tsai, H.-n. Li, and Q. Zhao, Phys. Rev. D 85, 034002 (2012), arXiv:1110.6235 [hep-ph]

  W. Qin, Q. Zhao, and X.-H. Zhong, Phys. Rev. D 97, 096002 (2018), arXiv:1712.02550 [hep-ph]

I) Gauge configurations with $N_f = 2$ degenerate charm quarks

ensemble	$L^3 \times T$	β	a_s(fm)	ξ	$m_{J/\psi}$(MeV)	$N_{\rm cfg}$
I	$16^3 \times 128$	2.8	0.1026	5	2743	~ 6000
II	$16^3 \times 128$	2.8	0.1026	5	3068	~ 6000

II) Two point functions of the glueball operator and $c\bar{c}$ operator

$$
\begin{aligned}
C_{XY}(t) &\equiv \frac{1}{Z_T} \mathrm{Tr}\left[e^{-\hat{H}(T-t)} \mathcal{O}_X e^{-\hat{H}t} \mathcal{O}_Y^\dagger\right] \\
&= \frac{1}{Z_T} \sum_{m,n} e^{-E_m(T-t)} \langle m|\mathcal{O}_X|n\rangle \langle n|\mathcal{O}_Y^\dagger|m\rangle e^{-E_n t} \\
&\approx \sum_{n\neq 0} \left[\langle 0|\mathcal{O}_X|n\rangle \langle n|\mathcal{O}_Y^\dagger|0\rangle \left(e^{-E_n t} \pm e^{-E_n(T-t)}\right)\right]
\end{aligned}
$$

III) $c\bar{c}$-glueball mixing model

$$H = \begin{pmatrix} m_{G_1} & x_1 \\ x_1 & m_{(cc)_1} \end{pmatrix} \oplus \begin{pmatrix} m_{G_2} & x_2 \\ x_2 & m_{(cc)_2} \end{pmatrix} \oplus \cdots$$

$$\begin{pmatrix} |g_i\rangle \\ |\eta_i\rangle \end{pmatrix} = \begin{pmatrix} \cos\theta_i & -\sin\theta_i \\ \sin\theta_i & \cos\theta_i \end{pmatrix} \begin{pmatrix} |G_i\rangle \\ |(cc)_i\rangle \end{pmatrix}$$

$$\sin\theta_i \approx \frac{x_i}{m_{cc,i} - m_{G_i}}$$

A) $\mathcal{O}_{\gamma_5} = \bar{c}\gamma_5 c$ operator for pseudoscalar charmonia

Assuming:

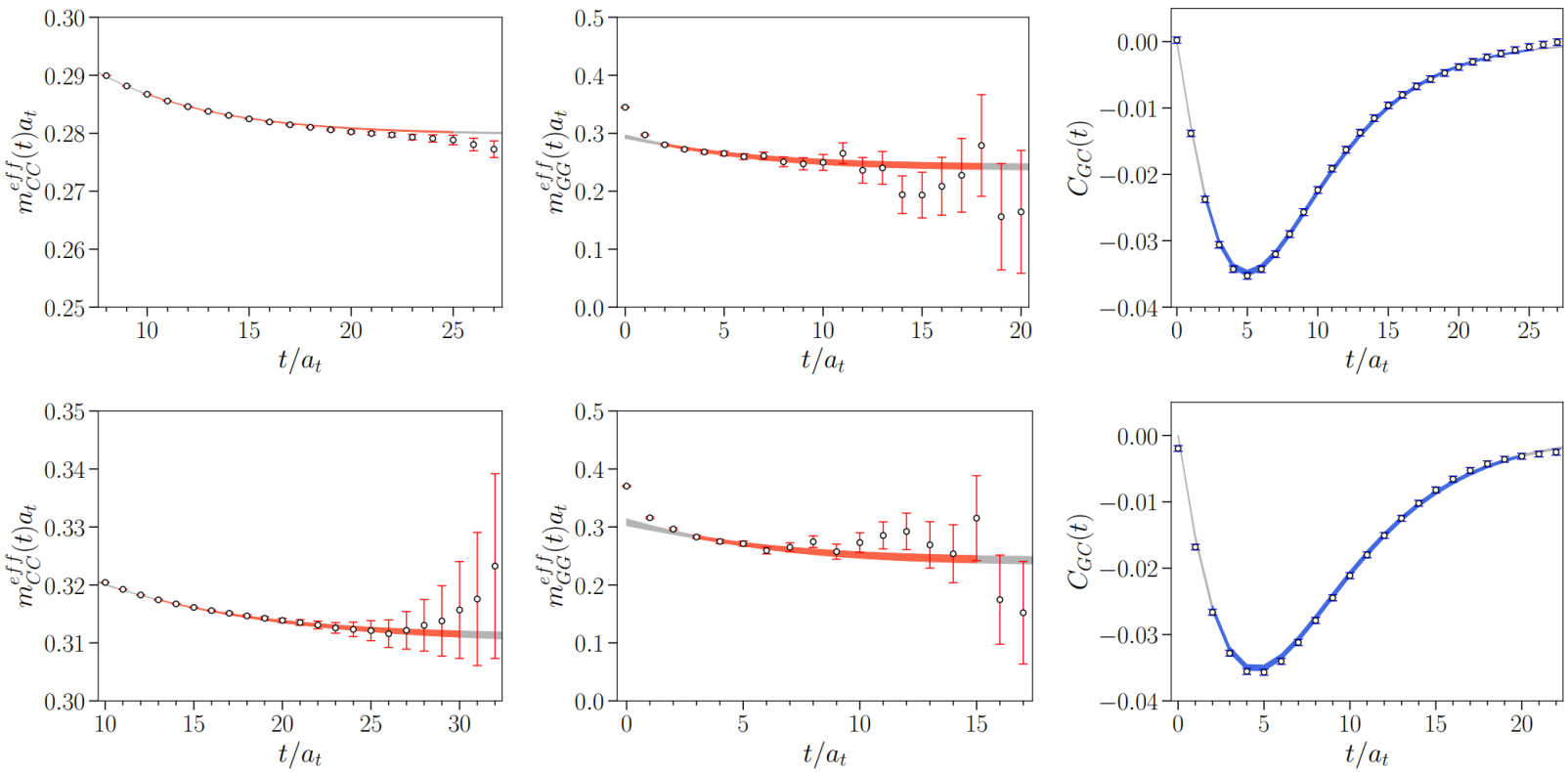
$$\mathcal{O}_G^\dagger|0\rangle = \sum_{n\neq 0} \sqrt{Z_{G_i}}|G_i\rangle$$

$$\mathcal{O}_{\gamma_5}^\dagger|0\rangle = \sum_{n\neq 0} \sqrt{Z_{(\gamma_5)_i}}|(cc)_i\rangle$$

$$C_{G\gamma_5}(t) = -\sum_i \sqrt{Z_{G_i} Z_{(\gamma_5),i}} \cos\theta_i \sin\theta_i \left(e^{-m_{g_i} t} - e^{-m_{g_i}(T-t)} - \left(e^{-m_{\eta_i} t} - e^{-m_{\eta_i}(T-t)}\right)\right)$$

$$C_{GG}(t) = \sum_i Z_{G_i} \left(e^{-m_{g_i} t} + e^{-m_{g_i}(T-t)}\right)$$

$$G_{\gamma_5}(t) = \sum_i Z_{(\gamma_5),i} \left(e^{-m_{\eta_i} t} + e^{-m_{\eta_i}(T-t)}\right)$$



B) $\mathcal{O}_{\gamma_5\gamma_4} = \bar{c}\gamma_5\gamma_4 c$ operator for pseudoscalar charmonia

Anomalous PCAC

$$\partial_\mu J_5^\mu(x) = 2m_c\bar{c}(x)\gamma_5 c(x) + q(x) \qquad q(x) = \frac{g^2}{16\pi^2}\epsilon^{\alpha\beta\rho\sigma}G^a_{\alpha\beta}G^a_{\rho\sigma}$$

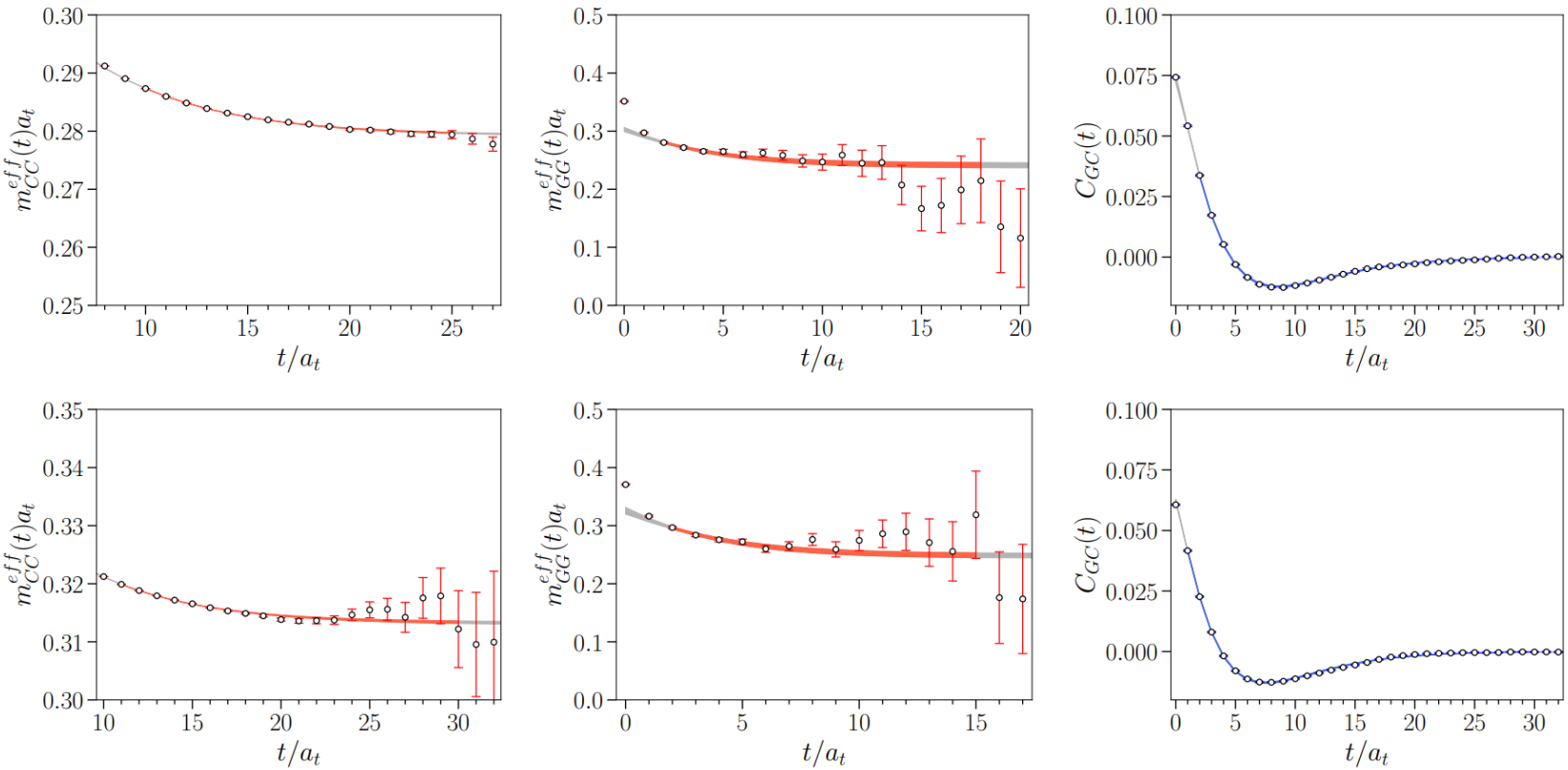
$$\left.\begin{aligned} &\langle 0|\partial_\mu J_5^\mu|G_i\rangle \approx \langle 0|q(x)|G_i\rangle \\ &\langle 0|\partial_\mu j_5^\mu(0)|G_i, \mathbf{p}=0\rangle = m_{G_i}^2 f_{G_i} \end{aligned}\right\} \quad f_{G_i} = \frac{1}{m_{G_i}^2}\langle 0|q(x)|G_i\rangle$$

$\mathcal{O}_{\gamma_5\gamma_4}$ **can couple to glueballs:**

$$\langle 0|\mathcal{O}_{\gamma_5\gamma_4}|G_i, \mathbf{p}=0\rangle \propto \frac{1}{m_{G_i}}\langle 0|q(0)|G_i\rangle$$

$$C_{G\gamma_5\gamma_4}(t) = \sqrt{Z_{G_1}}\langle 0|\mathcal{O}_{\gamma_5\gamma_4}|G_1\rangle \cos^2\theta_1 e^{-m_{g_1}}$$

$$- \sum_{i=1}^{2} \sqrt{Z_{G_i} Z_{(cc)_i}} \cos\theta_i \sin\theta_i \left(e^{-m_{g_i} t} + e^{-m_{g_i}(T-t)} - \left(e^{-m_{\eta_i} t} + e^{-m_{\eta_i}(T-t)}\right)\right)$$



C) Results

ensemble	Γ	m_{η_1}(MeV)	m_{g_1}(MeV)	θ_1	x_1(MeV)
I	γ_5	2691(2)	2317(51)	7.7(1.1)°	48(7)
	$\gamma_5\gamma_4$	2685(1)	2317(43)	6.8(8)°	43(6)
	avg.	2686(1)	2317(46)	7.1(9)°	46(7)
II	γ_5	2987(9)	2308(63)	4.9(6)°	59(8)
	$\gamma_5\gamma_4$	3013(3)	2385(40)	4.2(3)°	46(6)
	avg.	3010(4)	2363(47)	4.3(4)°	49(7)

IV) Physical implications

A) $X(2370)$ observed by BESIIII

M. Ablikim et al. (BESIII), Phys. Rev. Lett. 106, 072002 (2011), arXiv:1012.3510 [hep-ex]
M. Ablikim et al. (BESIII), Eur. Phys. J. C 80, 746 (2020), arXiv:1912.11253 [hep-ex]

$J/\psi \to \gamma X(2370) \to \gamma\pi^+\pi^-\eta'$:

$$M_{X(2370)} = 2341.6 \pm 6.5(\text{stat.}) \pm 5.7(\text{syst.})\ \text{MeV}$$
$$\Gamma_{X(2370)} = 117 \pm 10(\text{stat.}) \pm 8(\text{syst.})\ \text{MeV}$$

$J/\psi \to \gamma X(2370) \to \gamma K\bar{K}\eta'$:

$$M_{X(2370)} = 2376.3 \pm 8.7(\text{stat.})^{+3.2}_{-4.3}(\text{syst.})\ \text{MeV}$$
$$\Gamma_{X(2370)} = 83 \pm 17(\text{stat.})^{+44}_{-6}(\text{syst.})\ \text{MeV},$$

$$Br(\gamma K^+K^-\eta') = (1.79 \pm 0.23(stat.) \pm 0.65(syst.)) \times 10^{-5}$$
$$Br(\gamma K_sK_s\eta') = (1.18 \pm 0.32(stat.) \pm 0.39(syst.)) \times 10^{-5}$$

Lattice QCD predictions

L.-C. Gui et al. Phys. Rev. D 100, 054511 (2019), arXiv:1906.03666 [hep-lat]

$$\Gamma(J/\psi \to \gamma G_{0^-}) = 22(7)\ eV$$
$$\text{Br}(J/\psi \to \gamma G_{0^-}) = 2.3(8) \times 10^{-4}$$

B) $X(2370)$ as (predominantly) a pseudoscalar glueball?

Assuming $\Gamma_X \approx 100$ **MeV**

$$\sin\theta = \frac{x}{m_{\eta_c} - m_X} \approx 0.075(8)$$

$$\delta m_{c\bar{c}} = \frac{x^2}{m_{\eta_c} - m_X} \approx 3.6(5) \text{ MeV}$$

Theoretically

$$\frac{|\mathcal{M}(X \to \mathrm{LH})|}{|\mathcal{M}(c\bar{c} \to \mathrm{LH})|} \approx \left(\frac{m_X \Gamma_X}{m_{\eta_c}\Gamma_{c\bar{c}}}\right)^{1/2}$$

$$\frac{\Gamma_{\eta_c}}{\Gamma_{c\bar{c}}} \approx \left|\cos\theta + \sin\theta \frac{|\mathcal{M}(X \to \mathrm{LH})|}{|\mathcal{M}(c\bar{c} \to \mathrm{LH})|}\right|^2$$

$$\approx 1 + 2\sin\theta \left(\frac{m_X \Gamma_X}{m_{\eta_c}\Gamma_{\eta_c}}\right)^{1/2} \left(\frac{\Gamma_{\eta_c}}{\Gamma_{c\bar{c}}}\right)^{1/2}$$

$$\Gamma_{\eta_c}/\Gamma_{c\bar{c}} \approx 1.27(4), \qquad \Gamma_{c\bar{c}} \approx 25(2) \text{ MeV}$$

To the leading order of QCD,

$$\alpha_s(m_c) \approx 0.30 - 0.35$$

$$\alpha(m_c) \approx \frac{1}{134}$$

$$\frac{\Gamma(c\bar{c} \to \gamma\gamma)}{\Gamma(c\bar{c} \to gg)} \approx \frac{8}{9}\left(\frac{\alpha}{\alpha_s}\right)^2 \frac{1 - 3.4\alpha_s/\pi}{1 + 4.8\alpha_s/\pi}$$

$$\approx (1.7 - 2.6) \times 10^{-4}$$

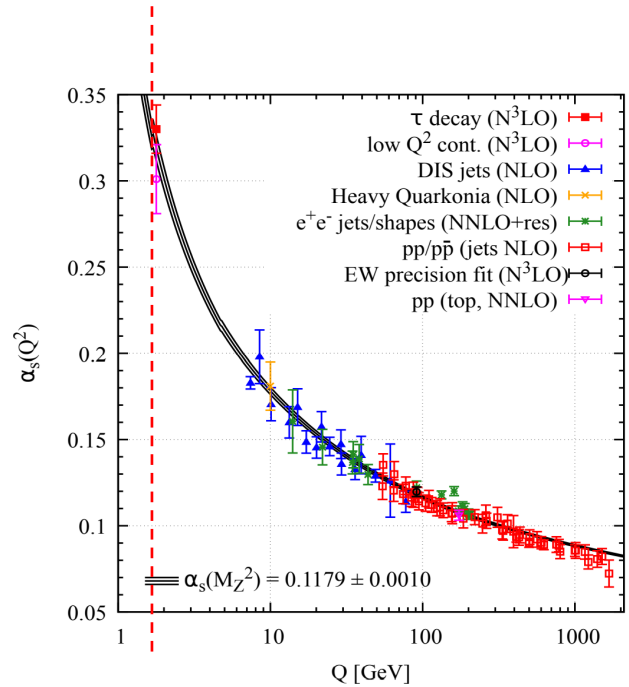


Figure 9.3: Summary of measurements of α_s as a function of the energy scale Q. The respective degree of QCD perturbation theory used in the extraction of α_s is indicated in brackets (NLO: next-to-leading order; NNLO: next-to-next-to-leading order; NNLO+res.: NNLO matched to a resummed calculation; N^{3}LO: next-to-NNLO).

Experimentally (PDG 2020) ,

$$\frac{\Gamma(\eta_c \to \gamma\gamma)}{\Gamma(\eta_c \to all)} \approx (1.61 \pm 0.12) \times 10^{-4}$$

If $\Gamma(\eta_c \to \gamma\gamma) \approx \Gamma((c\bar{c}) \to \gamma\gamma)$, $\Gamma(\eta_c \to all) \approx \Gamma(\eta_c \to gg)$ **,**

$$\Gamma_{\eta_c}/\Gamma_{c\bar{c}} \approx 1.27(4) \quad \Rightarrow \quad \frac{\Gamma(c\bar{c} \to \gamma\gamma)}{\Gamma(c\bar{c} \to gg)} \approx (2.04 \pm 0.14) \times 10^{-4}$$

C) The experiment-theory tension on $J/\psi \to \gamma\eta_c$

Potential model and LQCD predict: $\Gamma(J/\psi \to \gamma\eta_c) \approx 2.4 - 2.9$ **keV**

Experiments (PDG 2020) : $\Gamma(J/\psi \to \gamma\eta_c) \approx 1.6 \pm 0.4$ **keV**

The $(c\bar{c})$-glueball can not alleviate this tension yet.

III. Prospects

- Partial width $J/\psi \to \gamma\eta,\ \gamma\eta_1(1^{-+})$
  (hopefully by the end of 2021)

- Scalar glueball-$s\bar{s}$ meson mixing (2022)

- Partial width $J/\psi \to \gamma s\bar{s}(0^{++}), \gamma s\bar{s}(2^{++})$ (2022)

VI. Summary

A) Glueballs at the physical point

- For $A_1^{++}, E^{++}, T_2^{++}$ and A_1^{-+} channels, the radial behaviors of the BS wave functions are similar to the Schroedinger wave functions of two-body systems in a central potential.

- The masses of the ground states in these channels are close to the corresponding pure gauge glueballs.

- Do the wave functions reflect the glueball-like nature of these states?

- Many open questions, no solid conclusion.

B) Gluball component of η_c

- Possible explanation of the large total width of η_c
- Compatible with the experimental result of $\eta_c \to \gamma\gamma$
- Can not explain the discrepancy of theory and experiments on $\Gamma(J/\psi \to \gamma\eta_c)$

Thanks!

Present status of lattice QCD study on hadron spectroscopy