Two- and three-gluon glueballs through QCD sum rules

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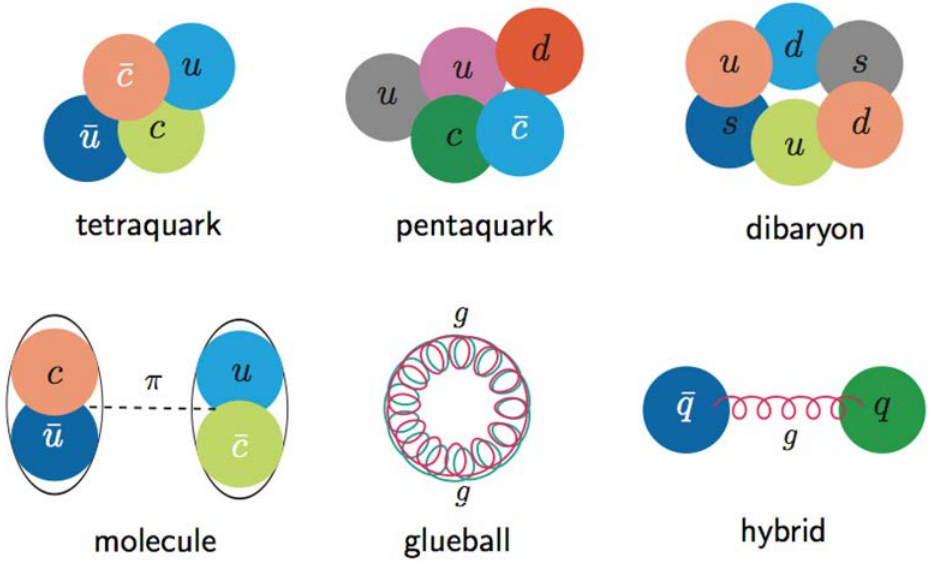
Collaborators: Wei Chen and Shi-Lin Zhu

Hadron spectrum

Meson

Baryon

Conventional Quark Model



Multiquark

Molecular

Glueball

Hybrid

Exotic hadron

Hadron spectrum

glueball

Glueball

Exotic hadron

Previous Studies

MIT bag model: A. Chodos et al., Phys. Rev. D 9, 3471 (1974);
R. L. Jaffe and K. Johnson, Phys. Lett. 60B, 201 (1976).

Flux tube model: N. Isgur and J. E. Paton, Phys. Rev. D 31, 2910 (1985).

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Glueball trajectories: I. Szanyi et al., Nucl. Phys. A 998, 121728 (2020).

Lattice QCD: K. G. Wilson, Phys. Rev. D 10, 2445 (1974);
Y. Chen et al., Phys. Rev. D 73, 014516 (2006);
V. Mathieu, N. Kochelev and V. Vento, IJMPE 18, 1 (2009);
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QCD sum rules: V. A. Novikov, M. A. Shifman, A. I. Vainshtein and V. I. Zakharov, NPB165, 67 (1980);
S. Narison, Z. Phys. C 26, 209 (1984);
S. Narison, Nucl. Phys. B 509, 312 (1998);
J. I. Latorre, S. Narison and S. Paban, Phys. Lett. B 191, 437 (1987);
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G. Hao, C. F. Qiao and A. L. Zhang, Phys. Lett. B 642, 53 (2006);
C. F. Qiao and L. Tang, Phys. Rev. Lett. 113, 221601 (2014);
A. Pimikov, H. J. Lee, N. Kochelev and P. Zhang, Phys. Rev. D 95, 071501(R) (2017).

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Recent D0 and TOTEM experiments

- **There is currently no definite evidence for the glueball's existence.**

- Recently, D0 and TOTEM studied pp and $p\bar{p}$ cross sections, and found them differ with a significance of 3.4σ (which can be increased to be $5.2 - 5.7\sigma$).

  D0 Collaboration, Phys. Rev. D 86, 012009 (2012);

  D0 and TOTEM Collaborations, arXiv:2012.03981 [hep-ex];

  TOTEM Collaboration, Eur. Phys. J. C 79, 785 (2019).

- **The above difference leads to the evidence of a *t*-channel exchanged odderon, i.e., predominantly a three-gluon glueball of C = −.**

  COMPETE Collaboration, Phys. Rev. Lett. 89, 201801 (2002);

  V. A. Khoze, A. D. Martin and M. G. Ryskin, Phys. Rev. D 97, 034019 (2018);

  E. Martynov and B. Nicolescu, Eur. Phys. J. C 79, 461 (2019).

Interests in glueballs are reviving recently!

Recent D0 and TOTEM experiments

- **There is** ence.

- Recently found them dif to be 5.2 – 5.

  D0 Collab

  D0 and T

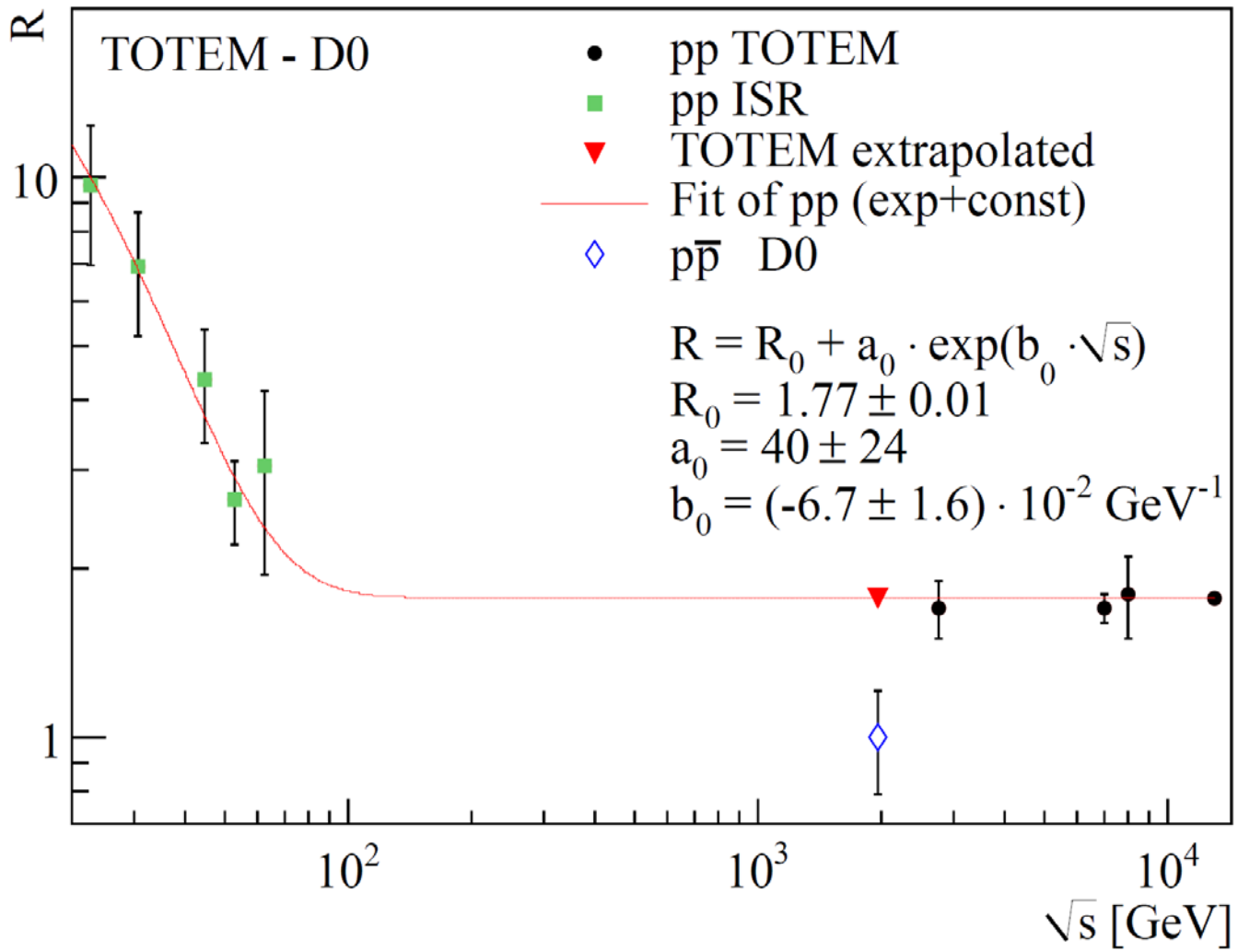
  TOTEM C

- **The abo hanged odderor**

  COMPETI

  V. A. Kho

  E. Martyr



Interests in glueballs are reviving recently!

QCD sum rule approach

- Construct relativistic glueball currents using:

$$G^a_{\mu\nu} \text{ and } \widetilde{G}^a_{\mu\nu}$$

- Perform QCD sum rule calculations

- Compare with Lattice QCD calculations

Non-relativistic operators

R. L. Jaffe, K. Johnson and Z. Ryzak
Annals Phys. 168, 344 (1986)

$$E_i = G^a_{i0} \text{ and } B_i = -\frac{1}{2}\epsilon_{ijk}G^a_{jk}$$

C=-
operators

J^{PC}	Operator
1^{+-}	$d_{abc}(\vec{E}_a \cdot \vec{E}_b)\,\vec{B}_c$
1^{--}	$d_{abc}(\vec{E}_a \cdot \vec{E}_b)\,\vec{E}_c$
2^{+-}	$d_{abc}[E^i_a[E^i_a(\vec{B}_b \times \vec{E}_c)^j + (i \leftrightarrow j)]$
2^{--}	$d_{abc}[B^i_a(\vec{E}_b \times \vec{B}_c)^j + (i \leftrightarrow j)]$
3^{--}	$d_{abc}[E^i_a E^j_b E^k_c - \frac{1}{5}\vec{E}_a \cdot \vec{E}_b(\delta^{ij}E^k_c + \delta^{jk}E^i_c + \delta^{ik}E^j_c)]$
3^{+-}	$d_{abc}[B^i_a B^j_b B^k_c - \frac{1}{5}\vec{B}_a \cdot \vec{B}_b(\delta^{ij}B^k_c + \delta^{jk}B^i_c + \delta^{ik}B^j_c)]$

Relativistic currents

$G^a_{\mu\nu}$ and $\tilde{G}^a_{\mu\nu}$

$$
\begin{aligned}
\tilde{J}_1^{\alpha\beta} &= d^{abc}\tilde{G}_a^{\mu\nu}\tilde{G}_{b,\mu\nu}\tilde{G}_c^{\alpha\beta}\,, \\
J_1^{\alpha\beta} &= d^{abc}G_a^{\mu\nu}G_{b,\mu\nu}G_c^{\alpha\beta}\,, \\
J_2^{\alpha_1\alpha_2,\beta_1\beta_2} &= d^{abc}\mathcal{S}'[G_a^{\alpha_1\beta_1}\tilde{G}_b^{\alpha_2\mu}G_{c,\mu}^{\beta_2} - \{\alpha_2 \leftrightarrow \beta_2\}], \\
\tilde{J}_2^{\alpha_1\alpha_2,\beta_1\beta_2} &= d^{abc}\mathcal{S}'[\tilde{G}_a^{\alpha_1\beta_1}G_b^{\alpha_2\mu}\tilde{G}_{c,\mu}^{\beta_2} - \{\alpha_2 \leftrightarrow \beta_2\}], \\
J_3^{\alpha_1\alpha_2\alpha_3,\beta_1\beta_2\beta_3} &= d^{abc}\mathcal{S}'[G_a^{\alpha_1\beta_1}G_b^{\alpha_2\beta_2}G_c^{\alpha_3\beta_3}]\,, \\
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\end{aligned}
$$

Non-relativistic operators

Relativistic currents

$$E_i = G^a_{i0} \quad \text{and} \quad B_i = -\frac{1}{2}\epsilon_{ijk}G^a_{jk}$$

$$G^a_{\mu\nu} \quad \text{and} \quad \tilde{G}^a_{\mu\nu}$$

J^{PC}	
1^{+-}	$d_{abc}(\vec{E}_a \cdot \vec{E}_b)\,\vec{B}_c$
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2^{+-}	$d_{abc}[E^i_a[E^i_a(\vec{B}_b \times \vec{E}_c)^j -$
2^{--}	$d_{abc}[B^i_a(\vec{E}_b \times \vec{B}_c)^j + (i \cdot$
3^{--}	$d_{abc}[E^i_a E^j_b E^k_c - \frac{1}{5}\vec{E}_a \cdot \vec{E}$
3^{+-}	$d_{abc}[B^i_a B^j_b B^k_c - \frac{1}{5}\vec{B}_a \cdot \vec{B}$

$$\tilde{J}_1^{\alpha\beta} = d^{abc}\tilde{G}^{\mu\nu}_a \tilde{G}_{b,\mu\nu}\tilde{G}^{\alpha\beta}_c\,,$$

$$J_1^{\alpha\beta} = d^{abc}G^{\mu\nu}_a G_{b,\mu\nu}G^{\alpha\beta}_c\,,$$

$$J_2^{\alpha_1\alpha_2,\beta_1\beta_2} = d^{abc}\mathcal{S}'[G^{\alpha_1\beta_1}_a \tilde{G}^{\alpha_2\mu}_b G^{\beta_2}_{c,\mu} - \{\alpha_2 \leftrightarrow \beta_2\}],$$

$$\tilde{J}_2^{\alpha_1\alpha_2,\beta_1\beta_2} = d^{abc}\mathcal{S}'[\tilde{G}^{\alpha_1\beta_1}_a G^{\alpha_2\mu}_b \tilde{G}^{\beta_2}_{c,\mu} - \{\alpha_2 \leftrightarrow \beta_2\}],$$

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Non-relativistic operators

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3^{+-}	$d_{abc}[B^i_a B^j_b B^k_c - \frac{1}{5}\vec{B}_a \cdot \vec{B}$

$i, j \qquad (\vec{E}\cdot\vec{E} + \vec{B}\cdot\vec{B})\,\vec{B}$

$$\tilde{J}_1^{\alpha\beta} = d^{abc}\tilde{G}^{\mu\nu}_a \tilde{G}_{b,\mu\nu}\tilde{G}^{\alpha\beta}_c\,,$$

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Non-relativistic operators

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$$E_i = G^a_{i0} \quad \text{and} \quad B_i = -\frac{1}{2}\epsilon_{ijk}G^a_{jk}$$

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$$\begin{aligned}
0, i \quad & \left(\vec{E}\cdot\vec{E} + \vec{B}\cdot\vec{B}\right)\vec{E} \\
i, j \quad & \left(\vec{E}\cdot\vec{E} + \vec{B}\cdot\vec{B}\right)\vec{B}
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\end{aligned}$$

$$\boxed{\begin{aligned}
\langle 0|J_1^{\alpha\beta}|X_{1+-}\rangle &= i f_{1+-}\epsilon^{\alpha\beta\mu\nu}\epsilon_\mu p_\nu\,, \\
\langle 0|J_1^{\alpha\beta}|X_{1--}\rangle &= i f_{1--}(p^\alpha\epsilon^\beta - p^\beta\epsilon^\alpha)\,,
\end{aligned}}$$

QCD Sum Rules

- In sum rule analyses, we consider two-point correlation functions:

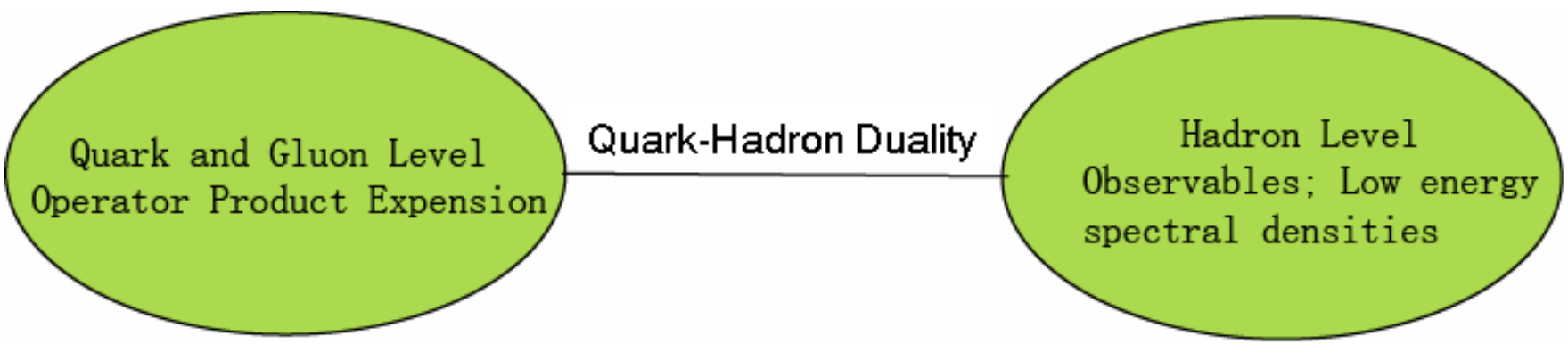
$$\Pi(q^2) \stackrel{\text{def}}{=} i \int d^4x e^{iqx} \langle 0|\mathrm{T}\eta(x)\eta^+(0)|0\rangle$$
$$\approx \sum_n \langle 0|\eta|n\rangle\langle n|\eta^+|0\rangle$$

where η is the current which can couple to hadronic states.

- By using the dispersion relation, we can obtain the spectral density

$$\Pi(q^2) = \int_{s_<}^{\infty} \frac{\rho(s)}{s - q^2 - i\varepsilon} ds$$

- In QCD sum rule, we can calculate these matrix elements from QCD (OPE) and relate them to observables by using dispersion relation.



SVZ sum rule

Quark and Gluon Level

(Convergence of OPE)

$$\Pi_{\mathrm{OPE}}(q^2) \xrightarrow[s = -q^2]{\text{dispersion relation}} \rho_{OPE}(s) = a_n s^n + a_{n-1} s^{n-1}$$

Quark-Hadron Duality

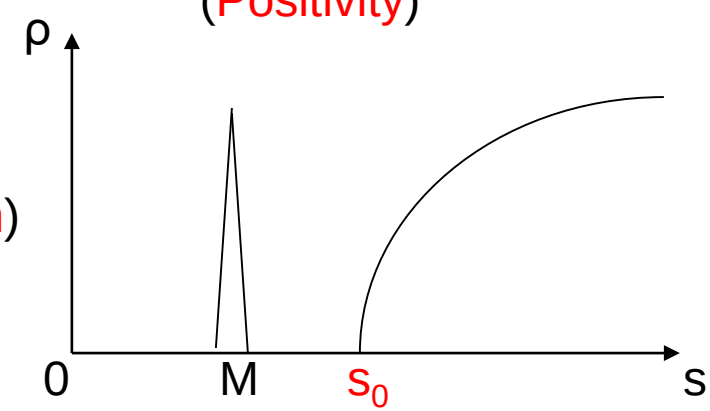
Hadron Level

$$\Pi_{phys}(q^2) = f_G^2 \frac{1}{q^2 - M^2} \longleftrightarrow \rho_{phys}(s) = \lambda_x^2 \delta(s - M_x^2) + \cdots$$

(for boson case)

(Positivity)

(Sufficient amount of Pole contribution)



QCD Sum Rules

- Borel transformation to suppress the higher order terms:

$$\Pi(M_B^2) \equiv f^2\, e^{-M^2/M_B^2} = \int_{s_<}^{s_0} e^{-s/M_B^2} \rho(s) ds$$

- Two free parameters

$$M_B\,, \quad s_0$$

We need to choose certain region of (M_B, s_0).

- **Criteria**

  1. Stability
  2. Convergence of OPE
  3. Positivity of spectral density
  4. Sufficient amount of pole contribution

QCD sum rule results

$$G^a_{\mu\nu} = \partial_\mu A^a_\nu - \partial_\nu A^a_\mu + g_s f^{abc} A^b_\mu A^c_\nu$$

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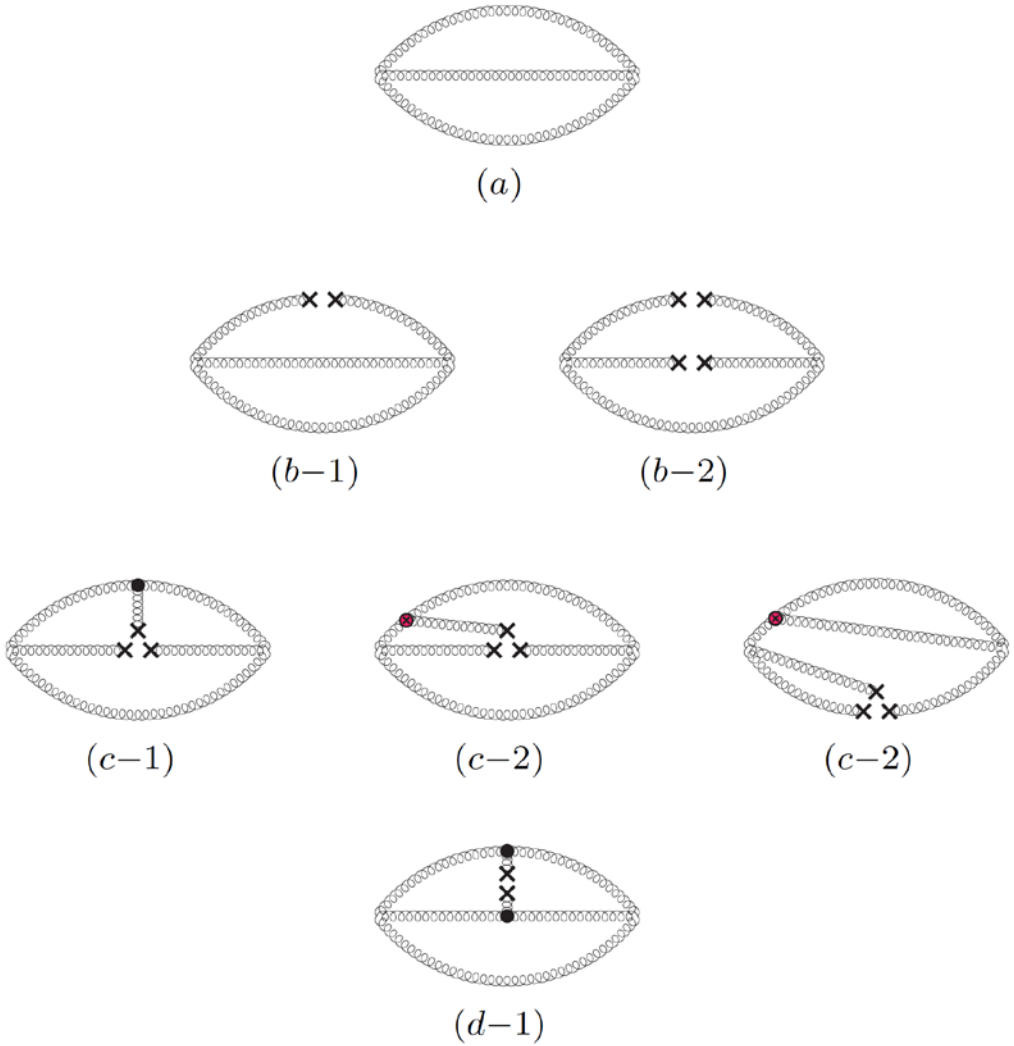

QCD sum rule results

$$J_1^{\alpha\beta} = d^{abc} G_a^{\mu\nu} G_{b,\mu\nu} G_c^{\alpha\beta} ,$$

$$\rho_{1--}(s) = \frac{4\alpha_s^3}{81\pi} s^4 - \frac{10\alpha_s^2 \langle g_s^2 GG \rangle}{9} s^2 + \frac{25\alpha_s^3 \langle g_s^2 GG \rangle}{36\pi} s^2 - \frac{5\alpha_s^2 \langle g_s^3 G^3 \rangle}{27} s$$

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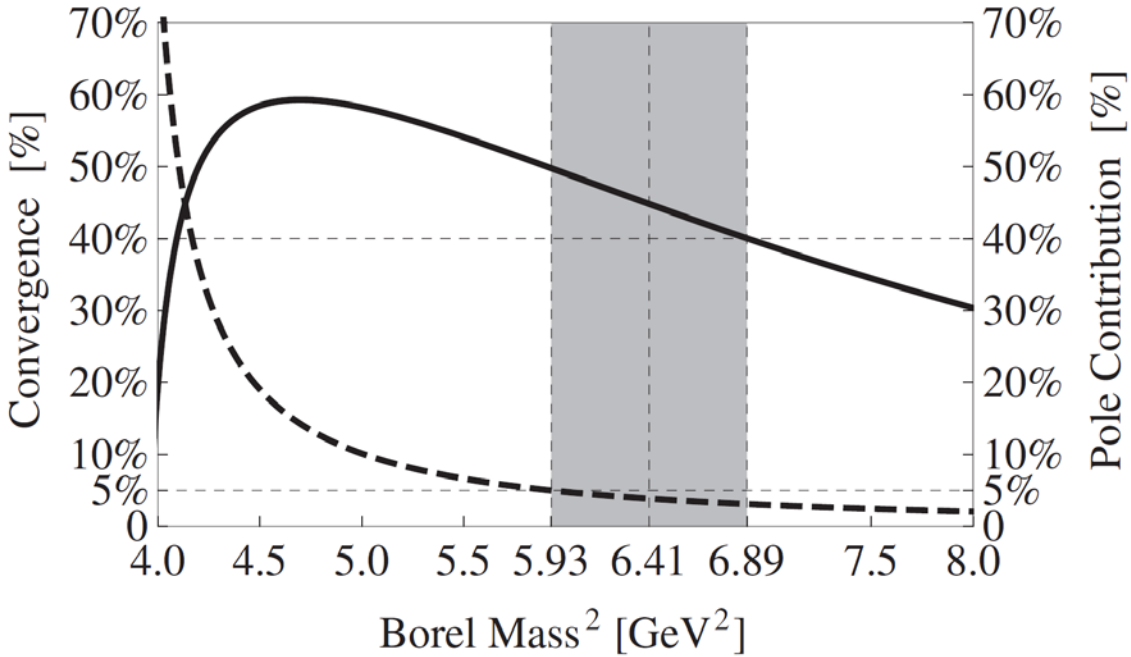
$$\mathrm{CVG} \equiv \left| \frac{\Pi^{\mathrm{D}=8\oplus\alpha_s^{n=3}}(s_0, M_B^2)}{\Pi(s_0, M_B^2)} \right| \leq 5\%$$

$$\text{Pole contribution} \equiv \left| \frac{\Pi(s_0, M_B^2)}{\Pi(\infty, M_B^2)} \right| \geq 40\%$$



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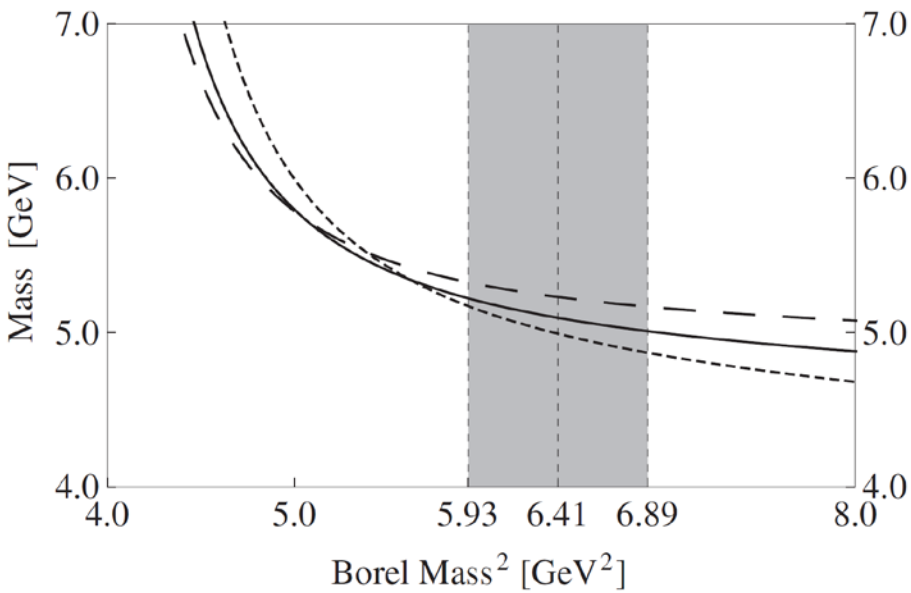
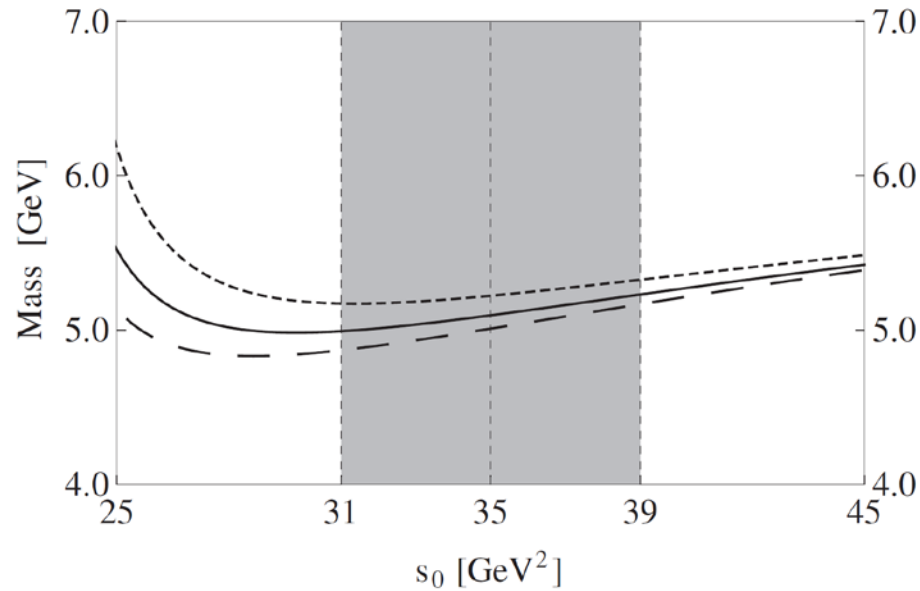
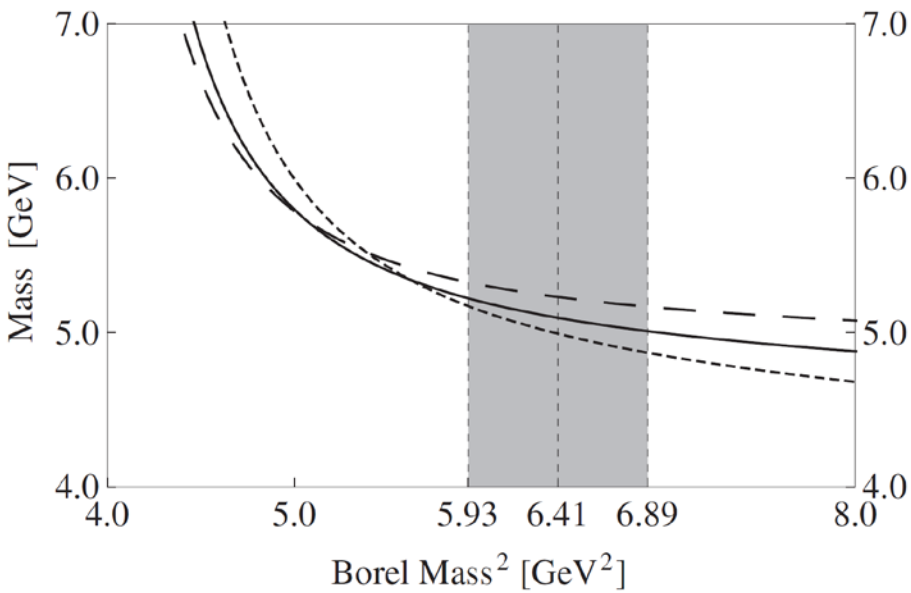
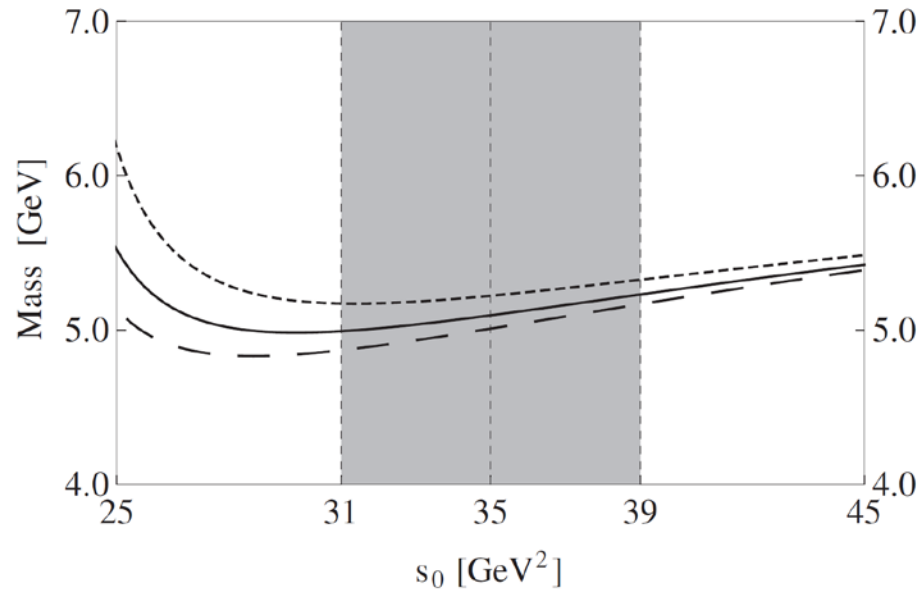




$$\mathrm{Mass}_{|\mathrm{GGG};1^{--}\rangle} = 5.10^{+0.21}_{-0.16}\ \mathrm{GeV}$$

QCD sum rule results





$$\text{Mass}_{|GGG;1^{--}\rangle} = 5.10^{+0.21}_{-0.16}\ \text{GeV}$$

$$\begin{aligned} \langle \alpha_s GG \rangle &= (6.35 \pm 0.35) \times 10^{-2}\ \text{GeV}^4\,, \\ \langle g_s^3 G^3 \rangle &= \langle \alpha_s GG \rangle \times (8.2 \pm 1.0)\ \text{GeV}^2\,. \end{aligned}$$

S. Narison, Int. J. Mod. Phys. A 33, 1850045 (2018)

QCD sum rule results

$$\begin{aligned}\langle \alpha_s GG \rangle &= (0.005 \pm 0.004) \times \pi \ \mathrm{GeV}^4 \,, \\ \langle g_s^3 G^3 \rangle &= \langle \alpha_s GG \rangle \times (8.2 \pm 1.0) \ \mathrm{GeV}^2 \,.\end{aligned}$$

B. L. Ioffe, Prog. Part. Nucl. Phys. 56, 232 (2006)

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QCD sum rule results

Glueball	Current	s_0^{min} [GeV^2]	Working Regions		Pole [%]	Mass [GeV]
			s_0 [GeV^2]	M_B^2 [GeV^2]		
$\|GG;0^{++}\rangle$	J_0	7.8	9.0 ± 1.0	3.71–4.18	40–48	$1.79^{+0.14}_{-0.16}$
$\|GG;2^{++}\rangle$	$J_2^{\alpha_1\alpha_2,\beta_1\beta_2}$	8.5	10.0 ± 1.0	3.99–4.60	40–50	$1.86^{+0.14}_{-0.17}$
$\|GG;0^{-+}\rangle$	$\tilde{J}_0$	7.9	9.0 ± 1.0	3.16–3.72	40–50	$2.15^{+0.11}_{-0.11}$
$\|GG;2^{-+}\rangle$	$\tilde{J}_2^{\alpha_1\alpha_2,\beta_1\beta_2}$	7.6	9.0 ± 1.0	3.06–3.76	40–52	$2.12^{+0.11}_{-0.12}$
$\|GGG;0^{++}\rangle$	η_0	17.1	19.0 ± 2.0	4.22–4.67	40–49	$3.14^{+0.17}_{-0.19}$
$\|GGG;2^{++}\rangle$	$\eta_2^{\alpha_1\alpha_2,\beta_1\beta_2}$	26.9	29.0 ± 3.0	6.35–6.82	40–46	$3.95^{+0.21}_{-0.24}$
$\|GGG;0^{-+}\rangle$	$\tilde{\eta}_0$	27.2	30.0 ± 3.0	6.25–6.99	40–49	$4.21^{+0.18}_{-0.20}$
$\|GGG;2^{-+}\rangle$	$\tilde{\eta}_2^{\alpha_1\alpha_2,\beta_1\beta_2}$	17.6	30.0 ± 3.0	4.98–7.81	40–78	$3.90^{+0.18}_{-0.26}$
$\|GGG;1^{+-}\rangle$	$\xi_1^{\alpha\beta}$	17.7	20.0 ± 2.0	5.04–5.62	40–49	$3.19^{+0.15}_{-0.17}$
$\|GGG;2^{+-}\rangle$	$\xi_2^{\alpha_1\alpha_2,\beta_1\beta_2}$	23.2	26.0 ± 3.0	6.24–6.90	40–48	$3.67^{+0.20}_{-0.23}$
$\|GGG;3^{+-}\rangle$	$\xi_3^{\alpha_1\alpha_2\alpha_3,\beta_1\beta_2\beta_3}$	23.8	26.0 ± 3.0	6.68–7.18	40–46	$3.63^{+0.21}_{-0.23}$
$\|GGG;1^{--}\rangle$	$\tilde{\xi}_1^{\alpha\beta}$	32.5	35.0 ± 4.0	5.93–6.89	40–50	$5.10^{+0.21}_{-0.16}$
$\|GGG;2^{--}\rangle$	$\tilde{\xi}_2^{\alpha_1\alpha_2,\beta_1\beta_2}$	35.7	38.0 ± 4.0	7.83–8.45	40–46	$4.81^{+0.21}_{-0.24}$
$\|GGG;3^{--}\rangle$	$\tilde{\xi}_3^{\alpha_1\alpha_2\alpha_3,\beta_1\beta_2\beta_3}$	34.9	37.0 ± 4.0	6.07–7.02	40–48	$5.47^{+0.28}_{-0.19}$

QCD sum rule results

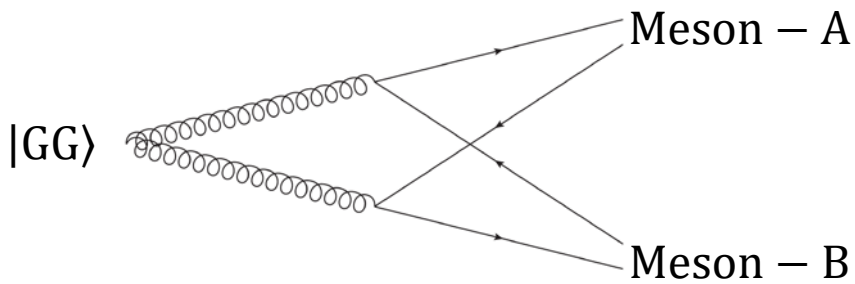
Lattice QCD results

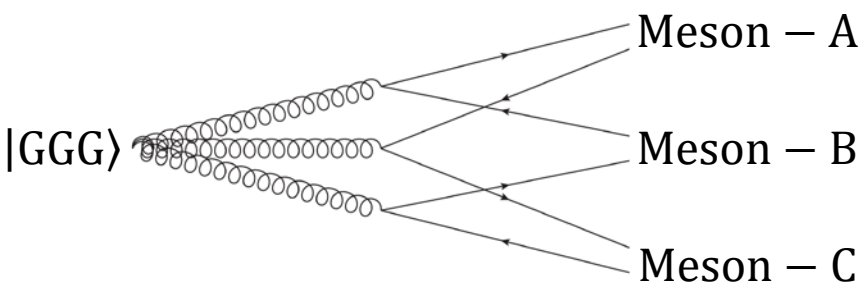
Glueball	QCD sum rules	Ref. [10]	Ref. [11]	Ref. [12]	Ref. [13]
$\|GG; 0^{++}\rangle$	$1.79^{+0.14}_{-0.16}$	$1.71 \pm 0.05 \pm 0.08$	$1.73 \pm 0.05 \pm 0.08$	$1.48 \pm 0.03 \pm 0.07$	1.80 ± 0.06
$\|GG; 2^{++}\rangle$	$1.86^{+0.14}_{-0.17}$	$2.39 \pm 0.03 \pm 0.12$	$2.40 \pm 0.03 \pm 0.12$	$2.15 \pm 0.03 \pm 0.10$	2.62 ± 0.05
$\|GG; 0^{-+}\rangle$	$2.15^{+0.11}_{-0.11}$	$2.56 \pm 0.04 \pm 0.12$	$2.59 \pm 0.04 \pm 0.13$	$2.25 \pm 0.06 \pm 0.10$	–
$\|GG; 2^{-+}\rangle$	$2.12^{+0.11}_{-0.12}$	$3.04 \pm 0.04 \pm 0.15$	$3.10 \pm 0.03 \pm 0.15$	$2.78 \pm 0.05 \pm 0.13$	3.46 ± 0.32
$\|GGG; 0^{++}\rangle$	$3.14^{+0.17}_{-0.19}$	–	$2.67 \pm 0.18 \pm 0.13$	$2.76 \pm 0.03 \pm 0.12$	3.76 ± 0.24
$\|GGG; 2^{++}\rangle$	$3.95^{+0.21}_{-0.24}$	–	–	$2.88 \pm 0.10 \pm 0.13$	–
$\|GGG; 0^{-+}\rangle$	$4.21^{+0.18}_{-0.20}$	–	$3.64 \pm 0.06 \pm 0.18$	$3.37 \pm 0.15 \pm 0.15$	4.49 ± 0.59
$\|GGG; 2^{-+}\rangle$	$3.90^{+0.18}_{-0.26}$	–	–	$3.48 \pm 0.14 \pm 0.16$	–
$\|GGG; 1^{+-}\rangle$	$3.19^{+0.15}_{-0.17}$	$2.98 \pm 0.03 \pm 0.14$	$2.94 \pm 0.03 \pm 0.14$	$2.67 \pm 0.07 \pm 0.12$	3.27 ± 0.34
$\|GGG; 2^{+-}\rangle$	$3.67^{+0.20}_{-0.23}$	$4.23 \pm 0.05 \pm 0.20$	$4.14 \pm 0.05 \pm 0.20$	–	–
$\|GGG; 3^{+-}\rangle$	$3.63^{+0.21}_{-0.23}$	$3.60 \pm 0.04 \pm 0.17$	$3.55 \pm 0.04 \pm 0.17$	$3.27 \pm 0.09 \pm 0.15$	3.85 ± 0.35
$\|GGG; 1^{--}\rangle$	$5.10^{+0.21}_{-0.16}$	$3.83 \pm 0.04 \pm 0.19$	$3.85 \pm 0.05 \pm 0.19$	$3.24 \pm 0.33 \pm 0.15$	–
$\|GGG; 2^{--}\rangle$	$4.81^{+0.21}_{-0.24}$	$4.01 \pm 0.05 \pm 0.20$	$3.93 \pm 0.04 \pm 0.19$	$3.66 \pm 0.13 \pm 0.17$	4.59 ± 0.74
$\|GGG; 3^{--}\rangle$	$5.47^{+0.28}_{-0.19}$	$4.20 \pm 0.05 \pm 0.20$	$4.13 \pm 0.09 \pm 0.20$	$4.33 \pm 0.26 \pm 0.20$	–

QCD sum rule results | **Lattice QCD results**

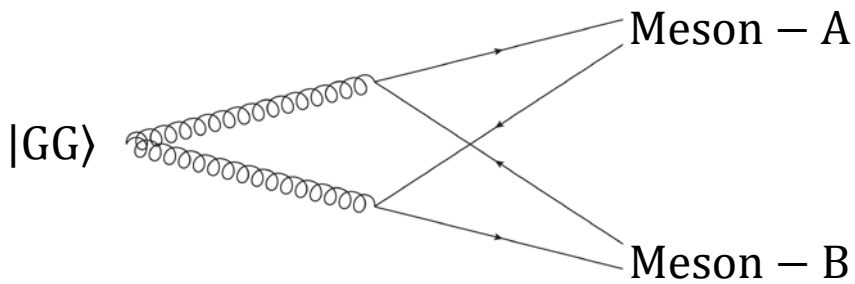
		quenched			unquenched
Glueball	QCD sum rules	Ref. [10]	Ref. [11]	Ref. [12]	Ref. [13]
$\|GG;0^{++}\rangle$	$1.79^{+0.14}_{-0.16}$	$1.71 \pm 0.05 \pm 0.08$	$1.73 \pm 0.05 \pm 0.08$	$1.48 \pm 0.03 \pm 0.07$	1.80 ± 0.06
$\|GG;2^{++}\rangle$	$1.86^{+0.14}_{-0.17}$	$2.39 \pm 0.03 \pm 0.12$	$2.40 \pm 0.03 \pm 0.12$	$2.15 \pm 0.03 \pm 0.10$	2.62 ± 0.05
$\|GG;0^{-+}\rangle$	$2.15^{+0.11}_{-0.11}$	$2.56 \pm 0.04 \pm 0.12$	$2.59 \pm 0.04 \pm 0.13$	$2.25 \pm 0.06 \pm 0.10$	–
$\|GG;2^{-+}\rangle$	$2.12^{+0.11}_{-0.12}$	$3.04 \pm 0.04 \pm 0.15$	$3.10 \pm 0.03 \pm 0.15$	$2.78 \pm 0.05 \pm 0.13$	3.46 ± 0.32
$\|GGG;0^{++}\rangle$	$3.14^{+0.17}_{-0.19}$	–	$2.67 \pm 0.18 \pm 0.13$	$2.76 \pm 0.03 \pm 0.12$	3.76 ± 0.24
$\|GGG;2^{++}\rangle$	$3.95^{+0.21}_{-0.24}$			$3.88 \pm 0.10 \pm 0.1$	–
$\|GGG;0^{-+}\rangle$	$4.21^{+0.18}_{-0.20}$	–	$3.64 \pm 0.06 \pm 0.18$	$3.37 \pm 0.15 \pm 0.15$	4.49 ± 0.59
$\|GGG;2^{-+}\rangle$	$3.90^{+0.18}_{-0.26}$	–	–	$3.48 \pm 0.14 \pm 0.16$	–
$\|GGG;1^{+-}\rangle$	$3.19^{+0.15}_{-0.17}$	$2.98 \pm 0.03 \pm 0.14$	$2.94 \pm 0.03 \pm 0.14$	$2.67 \pm 0.07 \pm 0.12$	3.27 ± 0.34
$\|GGG;2^{+-}\rangle$	$3.67^{+0.20}_{-0.23}$	$4.23 \pm 0.05 \pm 0.20$	$4.14 \pm 0.05 \pm 0.20$	–	–
$\|GGG;3^{+-}\rangle$	$3.63^{+0.21}_{-0.23}$	$3.60 \pm 0.04 \pm 0.17$	$3.55 \pm 0.04 \pm 0.17$	$3.27 \pm 0.09 \pm 0.15$	3.85 ± 0.35
$\|GGG;1^{--}\rangle$	$5.10^{+0.21}_{-0.16}$	$3.83 \pm 0.04 \pm 0.19$	$3.85 \pm 0.05 \pm 0.19$	$3.24 \pm 0.33 \pm 0.15$	–
$\|GGG;2^{--}\rangle$	$4.81^{+0.21}_{-0.24}$	$4.01 \pm 0.05 \pm 0.20$	$3.93 \pm 0.04 \pm 0.19$	$3.66 \pm 0.13 \pm 0.17$	4.59 ± 0.74
$\|GGG;3^{--}\rangle$	$5.47^{+0.28}_{-0.19}$	$4.20 \pm 0.05 \pm 0.20$	$4.13 \pm 0.09 \pm 0.20$	$4.33 \pm 0.26 \pm 0.20$	–

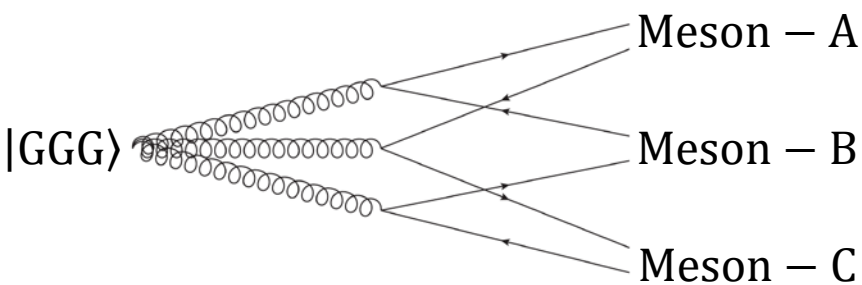
Decay analyses





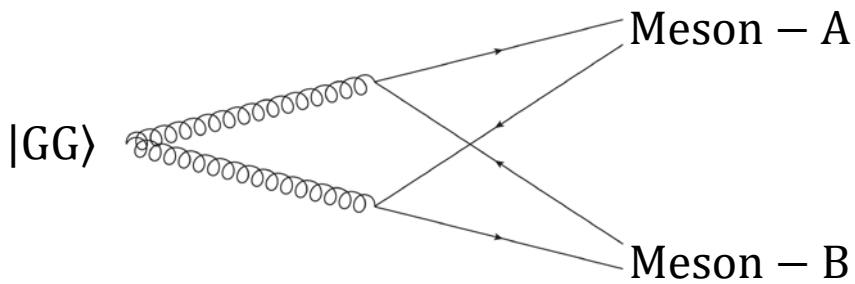
Decay analyses

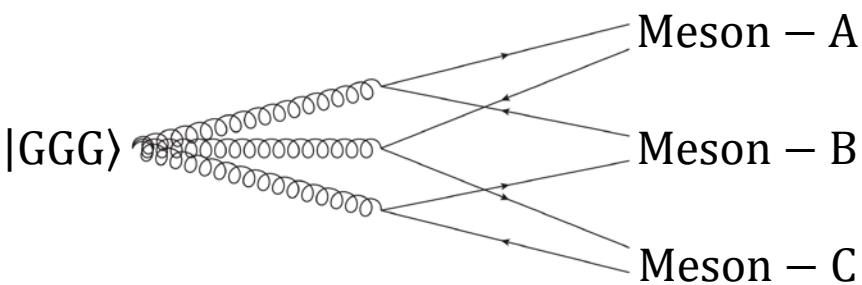




0^{-+}	$\rightarrow$	VVP, VVV	(S-wave)
0^{++}	$\rightarrow$	VPP, VVP, VVV	(P-wave)
1^{--}	$\rightarrow$	VPP, VVP, VVV	(S-wave)
1^{+-}	$\rightarrow$	PPP, VPP, VVP, VVV	(P-wave)
$2^{-\pm}$	$\rightarrow$	VVP, VVV	(S-wave)
$2^{+\pm}$	$\rightarrow$	VPP, VVP, VVV	(P-wave)
3^{--}	$\rightarrow$	VVV	(S-wave)
3^{+-}	$\rightarrow$	VVP, VVV	(P-wave)

Decay analyses





0^{-+}	$\rightarrow$	VVP, VVV	(S-wave)
0^{++}	$\rightarrow$	VPP, VVP, VVV	(P-wave)
1^{--}	$\rightarrow$	VPP, VVP, VVV	(S-wave)
1^{+-}	$\rightarrow$	PPP, VPP, VVP, VVV	(P-wave)
$2^{-\pm}$	$\rightarrow$	VVP, VVV	(S-wave)
$2^{+\pm}$	$\rightarrow$	VPP, VVP, VVV	(P-wave)
3^{--}	$\rightarrow$	VVV	(S-wave)
3^{+-}	$\rightarrow$	VVP, VVV	(P-wave)

Summary

- We study mass spectra of two- and three-gluon glueballs through QCD sum rules.
- Our QCD sum rule results are generally consistent with Lattice QCD calculations.
- Honestly speaking, we still know little about glueballs.

Thank you very much!