3D Structure of the Nucleon & Large-Momentum EFT

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CENTER for NUCLEAR FEMTOGRAPHY

Outline

- Unravelling 3D structure of the nucleon through GPDs and DVCS
- Parton as effective field theory DOF & why it is hard to solve directly.
- Large momentum expansion: A Euclidean effective theory of partons.
- Computing light-cone wave functions without light-cone quantization.

Unravelling 3D structure of the nucleon through GPDs and DVCS

Generalized Parton Distributions

- GPDs were discovered
  - As interesting hadron matrix elements with both DGLAP and ERBL evolutions (Muller et al, 1994)
  - As a way to access to form factors of energy-momentum tensor (mass, spin, stress, etc), particularly the spin structure of the nucleon (Ji, 1996).
- GPDs unite two well-known "observables" of the nucleon structure
  - Electromagnetic form factors, $F_1(t), F_2(t)$
  - Parton distributions functions, $q(x), g(x)$, etc.

Parton distributions in transverse space

- Partons in 3-space

$$\vec{k} = \left(k^z, \vec{k}_\perp \right) \qquad \text{TMD PDF}$$

  - Or $\left(k^z, \vec{b}_\perp \right)$ Hybrid

- Parton distributions in transverse space

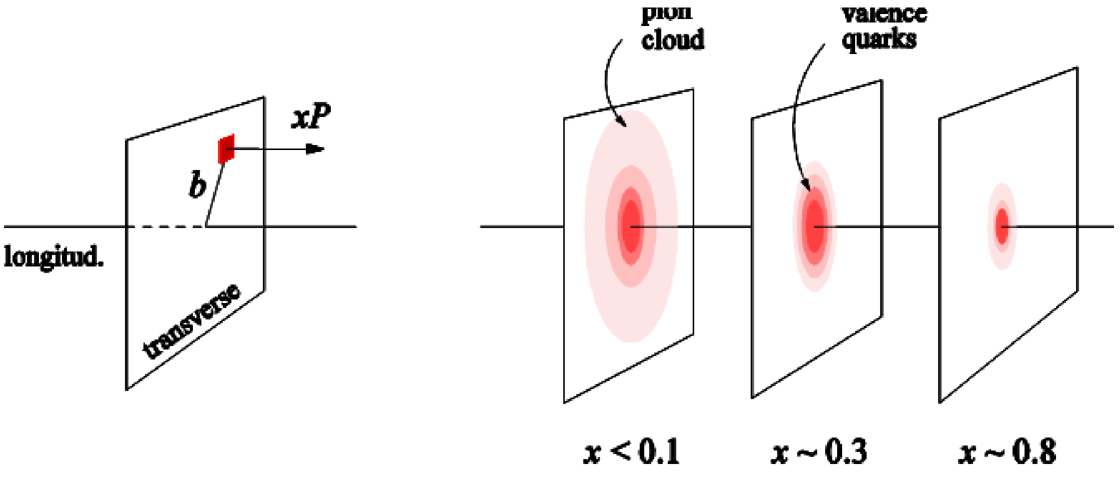
$$f\left(x = k^z/P^z, \vec{b}_\perp\right)$$

  was first introduced by Soper in 1977

- 3D = (2+1)D

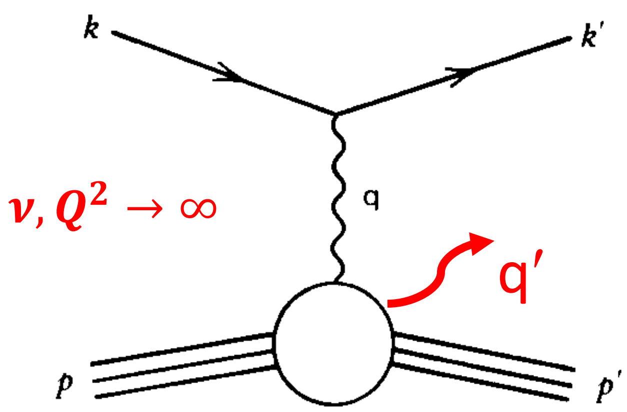
Tomographic picture of the nucleon

- GPDs in $\xi = 0$ limit can be related to coordinate space parton distributions, as shown by M. Burkardt in 2002.
- "Momentum-dissected" 2+1D distributions



Measuring GPD: DVCS & hard exclusive processes

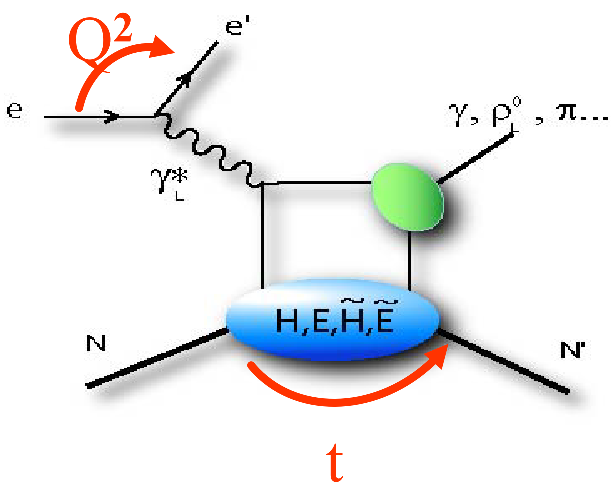
- Deeply-virtual photon diffraction (DVCS) Ji '96

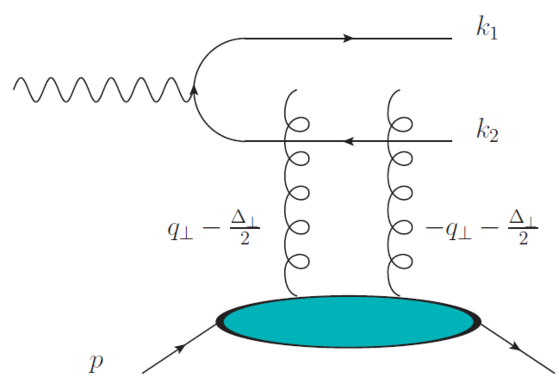


- Two more kinematic variables are now in the play.
- Photon emission can be replaced by other mesons or even jets. (Radyushkin '96, K. Golec-Biernat, Kwiecinski, Martin, '98)

Handbag diagram mechanism

- In the Bjorken limit $Q^2 \rightarrow \infty$, $\nu \rightarrow \infty$, Q^2/ν =finite, the emission mechanism becomes very simple, single-quark & gluon (Compton-ish) scattering





Compton Form Factors

- What experiments measure is the CFF

$$\mathcal{H}(\xi,t) = \sum_f \left[\frac{e_f}{e}\right]^2 \left\{ i\pi \left[H_f(\xi,\xi,t) - H_f(-\xi,\xi,t)\right] \right.$$
$$\left. + \mathcal{P}\int_{-1}^{+1} dx \left[\frac{1}{\xi - x} - \frac{1}{\xi + x}\right] H_f(x,\xi,t) \right\}.$$

  providing 2D information.

- Missing dimension can be provided by more complex processes (DDVCS, two meson production etc.)
- Threshold production of J/ψ quarkonium (gluon EMT)

  (Guo, Ji, Liu, 2021)

- Ab initio QCD theory calculations!

Parton effective field theory & why it is hard to solve directly

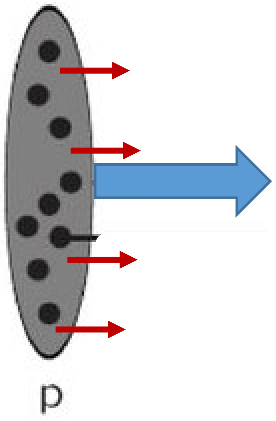
Feynman's parton

- When a high-energy proton travels at $v \to c$, one can assume the proton travels exactly at $v=c$, or the proton momentum is

$$p=E=\infty$$

(Infinite momentum frame, IMF)

- The proton may be considered as a collection of interaction-free particles: partons



PHYSICAL REVIEW VOLUME 185, NUMBER 5 25 SEPTEMBER 1969

Inelastic Electron-Proton and γ-Proton Scattering and the Structure of the Nucleon*

J. D. BJORKEN AND E. A. PASCHOS
Stanford Linear Accelerator Center, Stanford University, Stanford, California 94305
(Received 10 April 1969)

A model for highly inelastic electron-nucleon scattering at high energies is studied and compared with existing data. This model envisages the proton to be composed of pointlike constituents ("partons") from which the electron scatters incoherently. We propose that the model be tested by observing γ rays scattered inelastically in a similar way from the nucleon. The magnitude of this inelastic Compton-scattering cross section can be predicted from existing electron-scattering data, indicating that the experiment is feasible, but difficult, at presently available energies.

I. INTRODUCTION

ONE of the most interesting results emerging from the study of inelastic lepton-hadron scattering at high energies and large momentum transfers is the possibility of obtaining detailed information about the structure, and about any fundamental constituents, of hadrons. We discuss here an intuitive but powerful model, in which the nucleon is built of fundamental pointlike constituents. The important feature of this model, as developed by Feynman, is its emphasis on the infinite-momentum frame of reference.

Partons in QFT are effective D.O.F.s

- Partons are not just the quarks and gluons in the usual QCD lagrangian.
- Partons are a special type of IR collinear modes with momentum

$$k^\mu = \left(k^0, k^z, \vec{k}_\perp\right)$$

with $k^z \to \infty, k^0 \to \infty, k_\perp \sim \Lambda_{QCD}, k_\mu^2 \sim \Lambda_{QCD}^2$

Collins, Soper and Sterman, QCD factorization, 70'-80's

Bauer, Stewart et al, Soft-Collinear EFT, 00's

Partons are an idealized concept

- Partons do not exist in the real world, as there is no hadron traveling at $P^z = \infty$, but a useful concepts like "ideal gas" or "perfect fluid"
- A similar useful concept or effective d.o.f. in QCD is "infinite mass quark"

$$m_Q = \infty$$

a good approximation for c, b, t quarks.

EFT: heavy-quark effective theory(HQET)

Partonic Effective Theory

- Hamiltonian Formulation

  S. Weinberg (66')

  S. J. Chang and S. K. Ma (69')

  J. Kogut and D. Soper ('70) …

  Dirac's Light-Front Quantization (1949)

  Much work has been devoted to LFQ due to the leadership by S. Brodsky and late Ken Wilson.

- Lagrangian Formulation
  - QCD factorization: light-front correlations (Collins,Soper et al)
  - Using the soft-collinear modes in SCET can construct the bound state of the proton. (Bauer, Stewart et al)

Solving Parton EFT

- Parton EFT has extra divergences compared with full QCD: light-cone singularity or rapidity divergence.
- These divergences mix UV & IR physics, making the effective Hamiltonian difficult to construct (infinite number of renormalization const)
- Even effective H is known, there is no know method to solve it systematically.

Large momentum expansion: A Euclidean effective theory of partons.

EFT for partons: Full Euclidean QCD

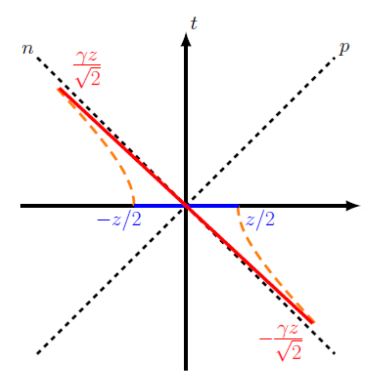
High-energy scattering in Minkowski QCD

Large-momentum Hadrons in Euclidean QCD

QCD factorization Or Soft-Collinear EFT

Partonic Theory (LFQ…)

Large-momentum effective theory



IMF formulation of partons

- Euclidean correlation functions, e. g.

$$\boxed{C(\lambda) = \langle P^z = \infty | \overline{\psi}(z)\Gamma\psi(0) | P^z = \infty \rangle}$$

Correlation distance: $\lambda = \lim_{P^z\to\infty, z\to 0}(zP^z).$

- PDFs: Fourier Transformation of the spatial correlations

$$\boxed{f(x) = \frac{1}{2P^+}\int \frac{d\lambda}{2\pi} e^{ix\lambda} C(\lambda)\ .}$$

Partons from a large-P expansion of Euclidean observables

- Assuming $P^z \to \infty$ limit exists, parton physics is obtained by expansion (Feynman, 1969)

$$f(k^z, P^z) = f(x) + f_2(x)(M/P^z)^2 + ...$$

- Account for subtlety of non-commuting limits of $\mathrm{P^z} \to \infty$ & $\Lambda_{QCD} \to \infty$ in QFT (Ji, 2013)

$$\tilde{f}(y, P^z) = \int Z(y/x, xP^z/\mu) f(x, \mu) dx$$
$$+ \mathcal{O}\Big(\frac{\Lambda^2_{\rm QCD}}{y^2(P^z)^2}, \frac{\Lambda^2_{\rm QCD}}{(1-y)^2(P^z)^2}\Big),$$

Alternative interpretation

- Approximate $P^z = \infty$ by a finite large P^z.

  We frequently do this in QCD

  Lattice QCD & HQET

- EFT expansion for PDFs in the spirit of Weinberg,

$$\boxed{q(x,\mu) = \int_{-\infty}^{\infty} \frac{dy}{|y|} \tilde{C}\left(\frac{x}{y}, \frac{\mu}{yP^z}\right) \tilde{q}(y, P^z, \mu) + \mathcal{O}\left(\frac{\Lambda_{\rm QCD}^2}{(xP^z)^2}, \frac{\Lambda_{\rm QCD}^2}{((1-x)P^z)^2}\right).}$$

x-dependence of PDF can be calculated from QCD!

Not by fitting as in extracting PDFs from exp. data.

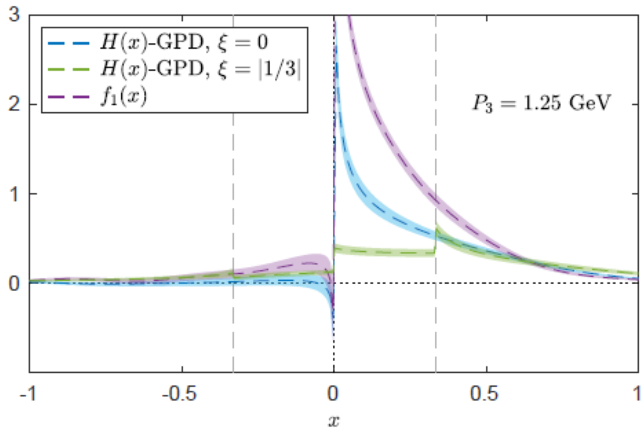
GPDs via LaMET

ETMC

PRL125 (2020)

MSU

2008.12474 (2020)



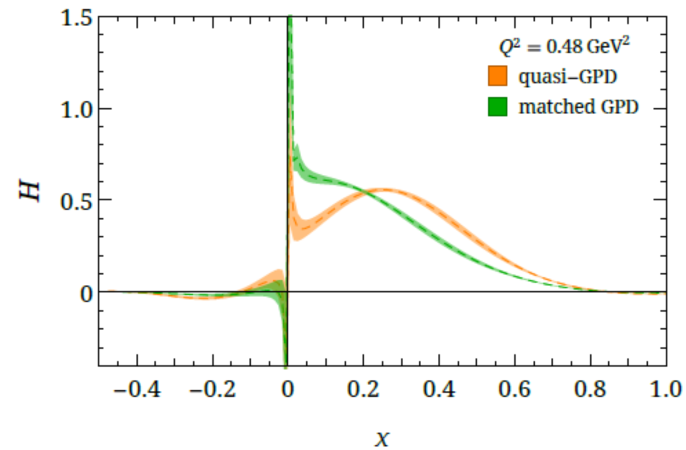
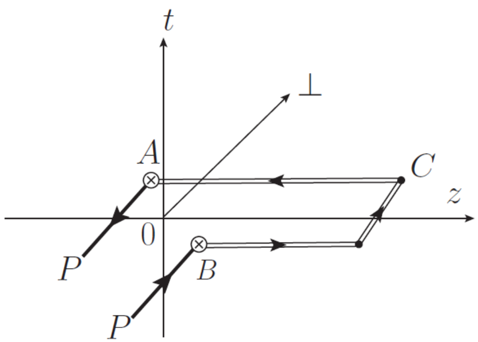
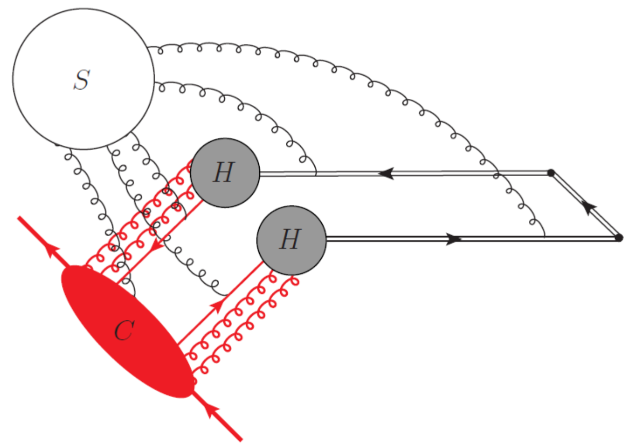


Figure 2: Examples of the first GPD data from lattice QCD. The left plot shows the matched GPD H at $\xi = 0$ and $|\xi| = 1/3$ with momentum transfer $Q^2 = 0.69$ GeV^2 [18]. The right one shows the quasi-GPD and matched GPD H at momentum transfer $Q^2 = 0.48$ GeV^2 and $\xi = 0$ [19].

TMDPDF: LaMET Factorization

Ji, Liu, Liu, Phys.Lett.B 811 (2020)

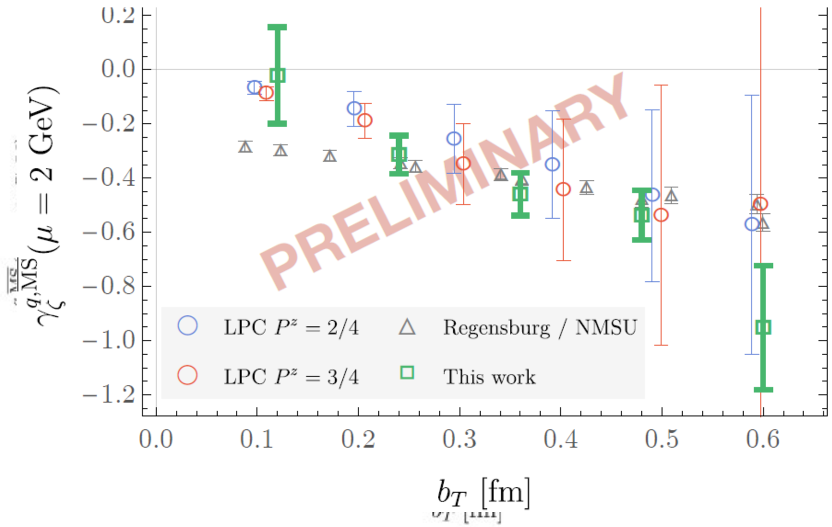


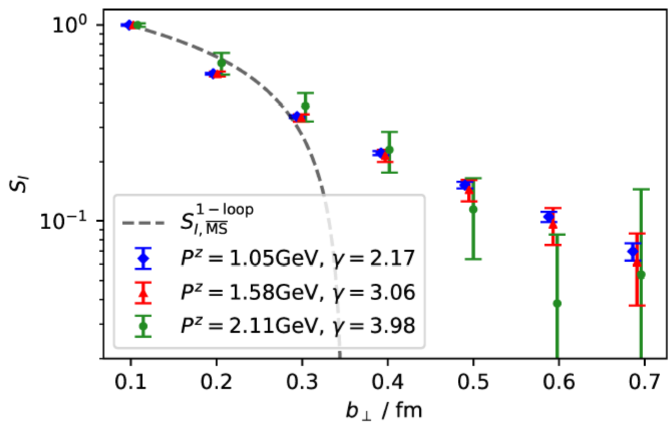


$$\boxed{\begin{aligned}&\tilde{f}(x, b_\perp, \mu, \zeta_z)\sqrt{S_r(b_\perp, \mu)} \\ &= H\left(\frac{\zeta_z}{\mu^2}\right) e^{K(b_\perp,\mu)\ln(\frac{\zeta_z}{\zeta})} f^{\mathrm{TMD}}(x, b_\perp, \mu, \zeta) + \ldots\end{aligned}}$$

$$\mu \frac{d}{d\mu} \ln H\left(\frac{\zeta_z}{\mu^2}\right) = \Gamma_{\mathrm{cusp}} \ln \frac{\zeta_z}{\mu^2} + \gamma_C$$

Collins-Soper Kernel and Soft Factor





P. Shanahan et al, PRD 102 (2020) & unpublished
Q.A.Zhang et al, PRL125 (2020)
M.Schlemmer, 2103.16991 (2021)

Q. A. Zhang et al, PRL125 (2020) (LPC)

Light-Front Wave-Functions without Light-Front Quantization

Light-Front Wave Function

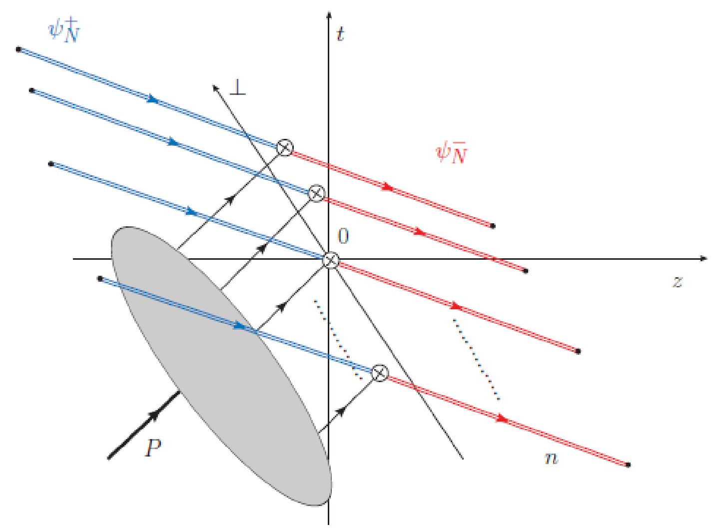
$$\boxed{\hat{P}^{-}|\Psi_n\rangle = \frac{M_n^2}{2P^+}|\Psi_n\rangle}$$

- LF quantization focuses on the WFs, from which everything can be calculated: a very ambitious goal! Brodsky et al. Phys. Rept. 301 (1998)
- Hamiltonian hard to construct
- Truncation is difficult

Light-Front Wave-Functions without Light-Front Quantization

- LaMET provides the practical way to calculate non-perturbative WF

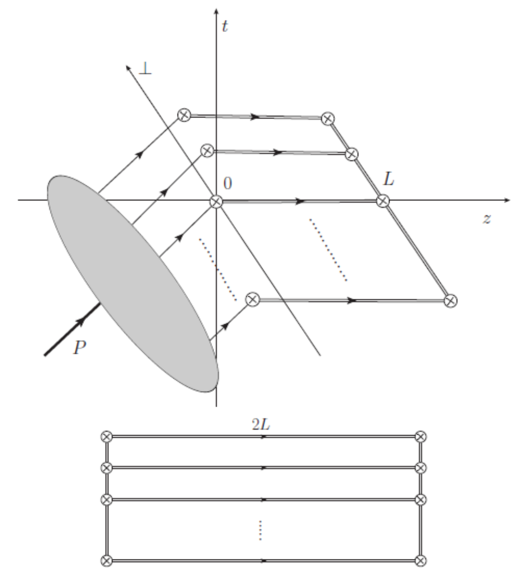
$$\psi_N^{\pm 0}(x_i, \vec{b}_{i\perp}, \mu) = \int \prod_{i=1}^{N} d\lambda_i e^{i\lambda_i x_i} \times e^{+i\lambda_0 x_0}$$
$$\times \langle 0|\mathcal{P}_N \prod_{i=1}^{N} \Phi_i^{\pm}(\lambda_i n + \vec{b}_{i\perp}) \Phi_0^{\pm}(\lambda_0 n + \vec{b}_{0\perp})|P\rangle \ .$$



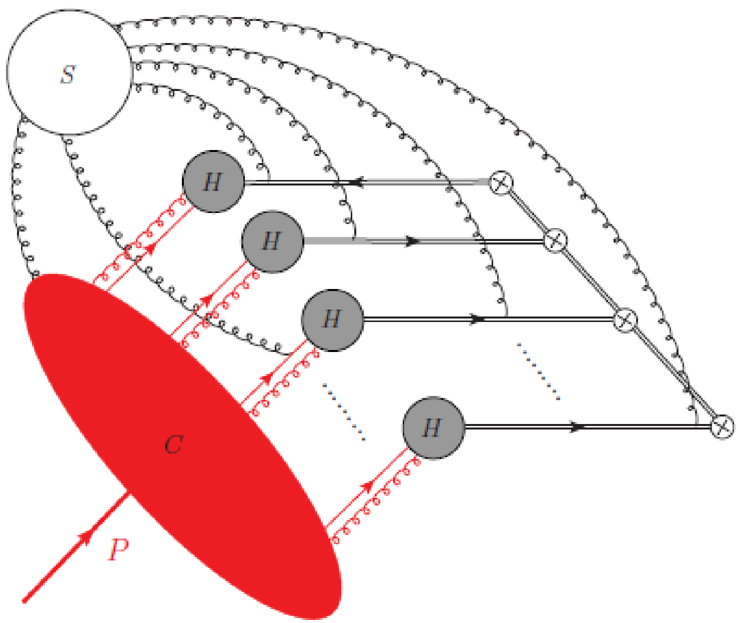
Quasi Wave Function (Euclidean)

$$\widetilde{\psi}_N^{\pm}(x_i, \vec{b}_{i\perp}, \mu, \zeta_z) = \lim_{L\to\infty} \int d\lambda_i e^{-i\lambda_i x_i - i\lambda_0 x_0}$$

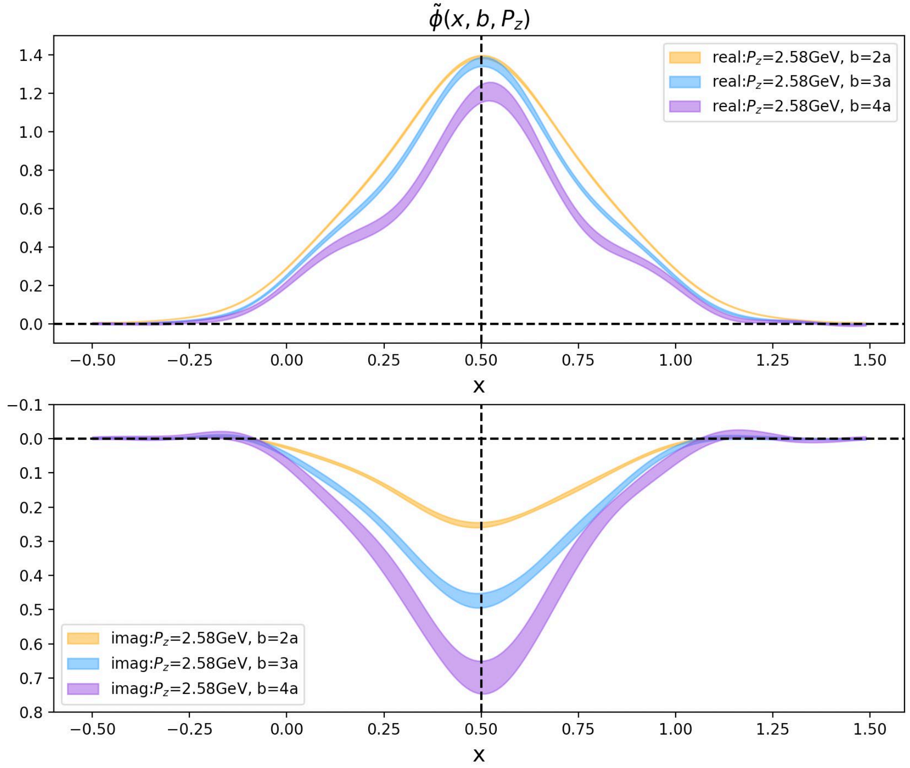
$$\frac{\langle 0|\mathcal{P}_N \prod_{i=1}^{N} \Phi_i^{\pm}(\lambda_i n_z + \vec{b}_{i\perp}; L)\Phi_0^{\pm}(\lambda_0 n_z; L)|P\rangle}{\sqrt{Z_E(2L, \vec{b}_{i\perp}, \mu)}}$$



LaMET Factorization



Meson TMD Light-Front Wave Function (LPC, preliminary)



Important for studying B-meson decays

Outlook

- GPDs & DVCS provide 2+1 D structure of the nucleon.
- LaMET is a systematic framework to calculate parton physics & 3D structure
- LaMET3.0 (~ 5% error?)
  - Improved non-pert renormalization at large z.
  - two-loop matching
  - P $\rightarrow$ 3 GeV
  - Singlet quark and gluon
- GPDs & TMDs & Wigner Functions & LFWF