# Lattice QCD and the description of Hadron Spectra

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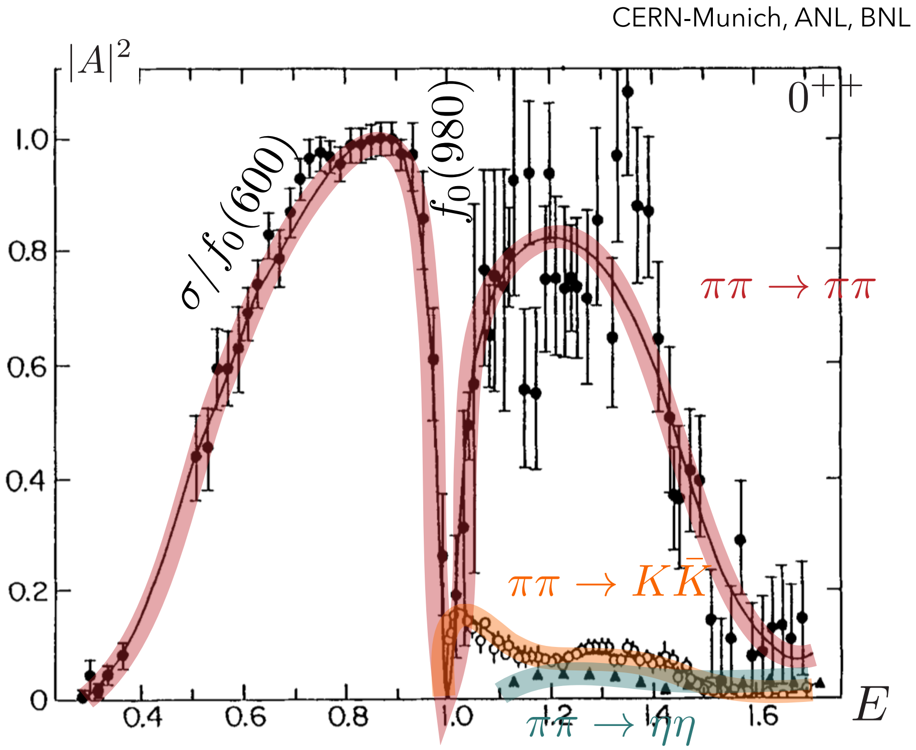
hadspec
hadspec.org

Strong QCD from hadron structure experiments
Wednesday 9th June 2021

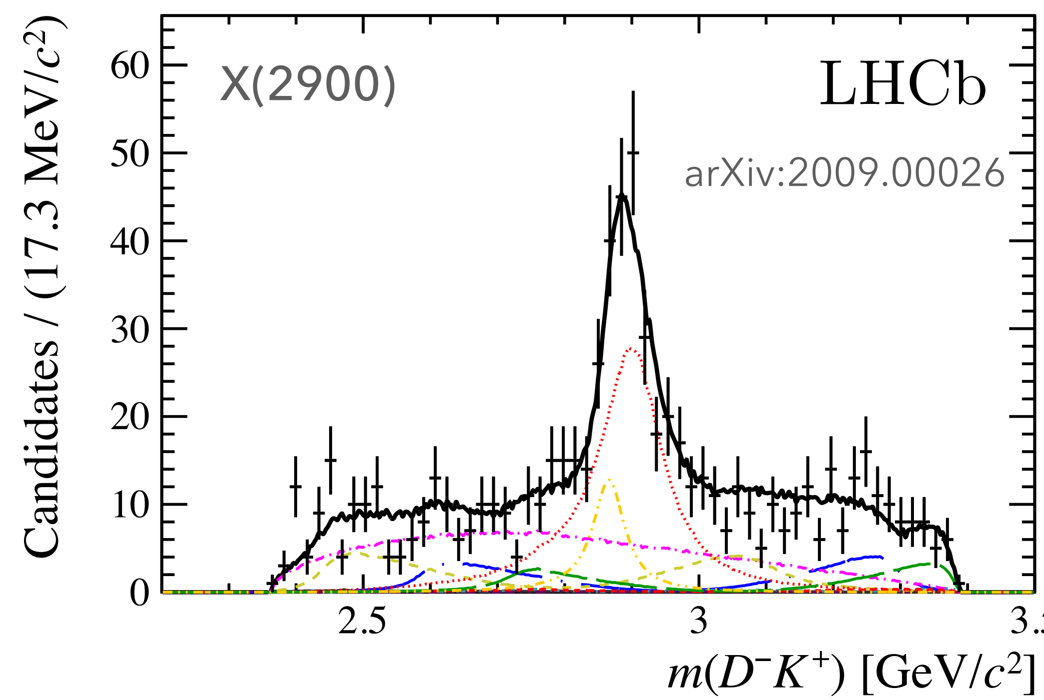
UNIVERSITY OF CAMBRIDGE

THE ROYAL SOCIETY

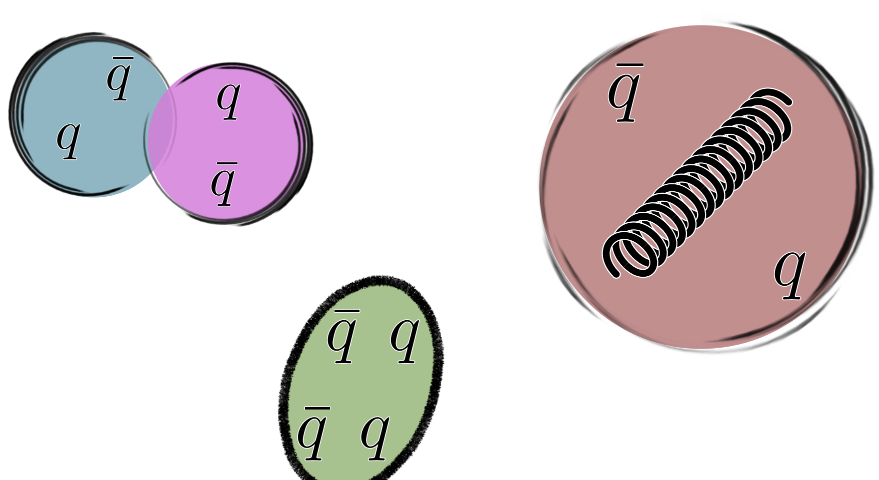
spectroscopy from first-principles is a hard problem




many puzzles - new and old




the quark model is a good guide for low-lying states




models can be useful, but what does QCD say?

provides a rigorous approach to hadron spectroscopy

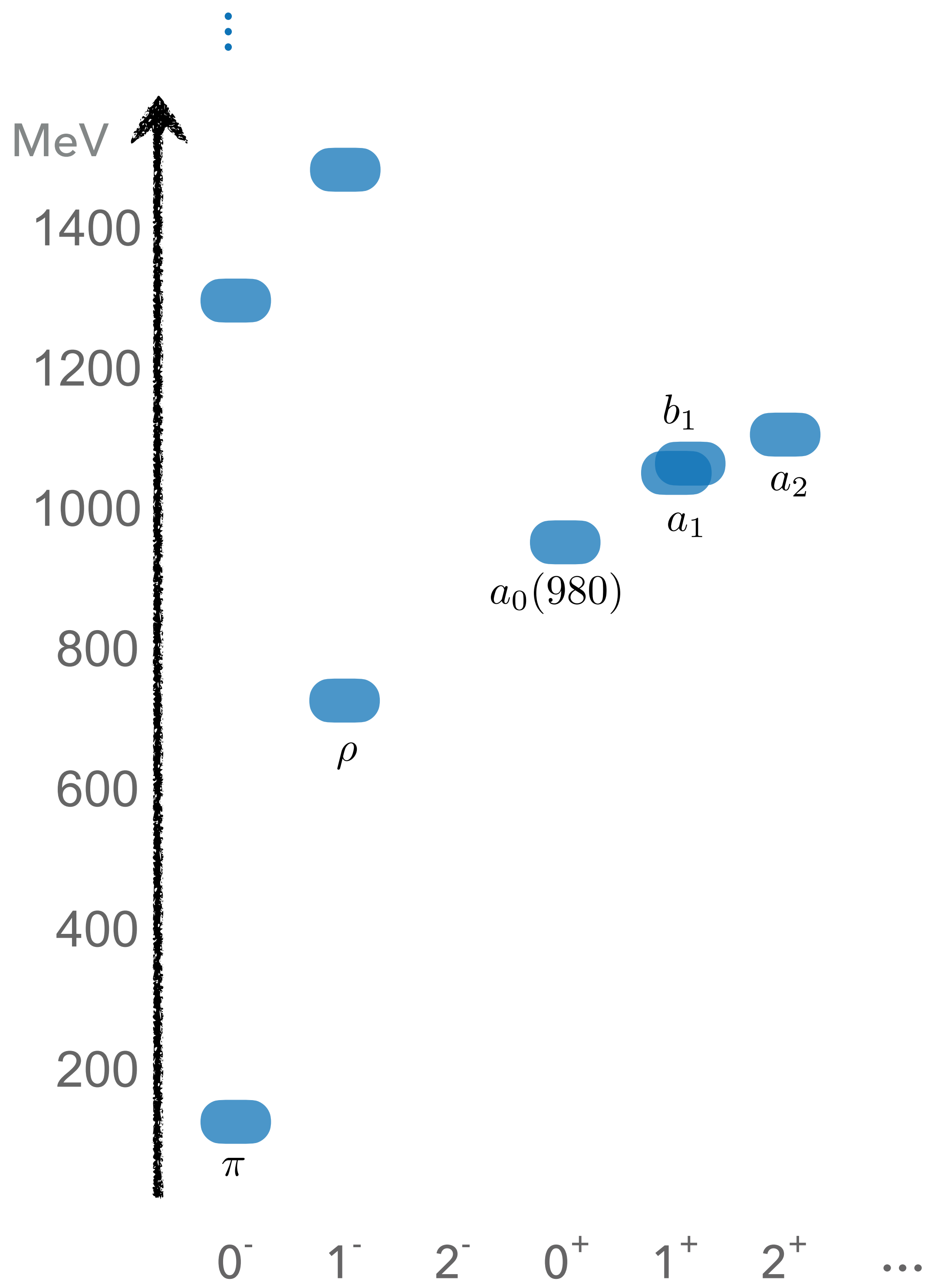
- as **rigorous** as possible
- **all** necessary **QCD** diagrams are computed
- **excited states** appear as **unstable resonances** in a scattering amplitude

tremendous progress in recent years
but not yet ready for precision comparisons

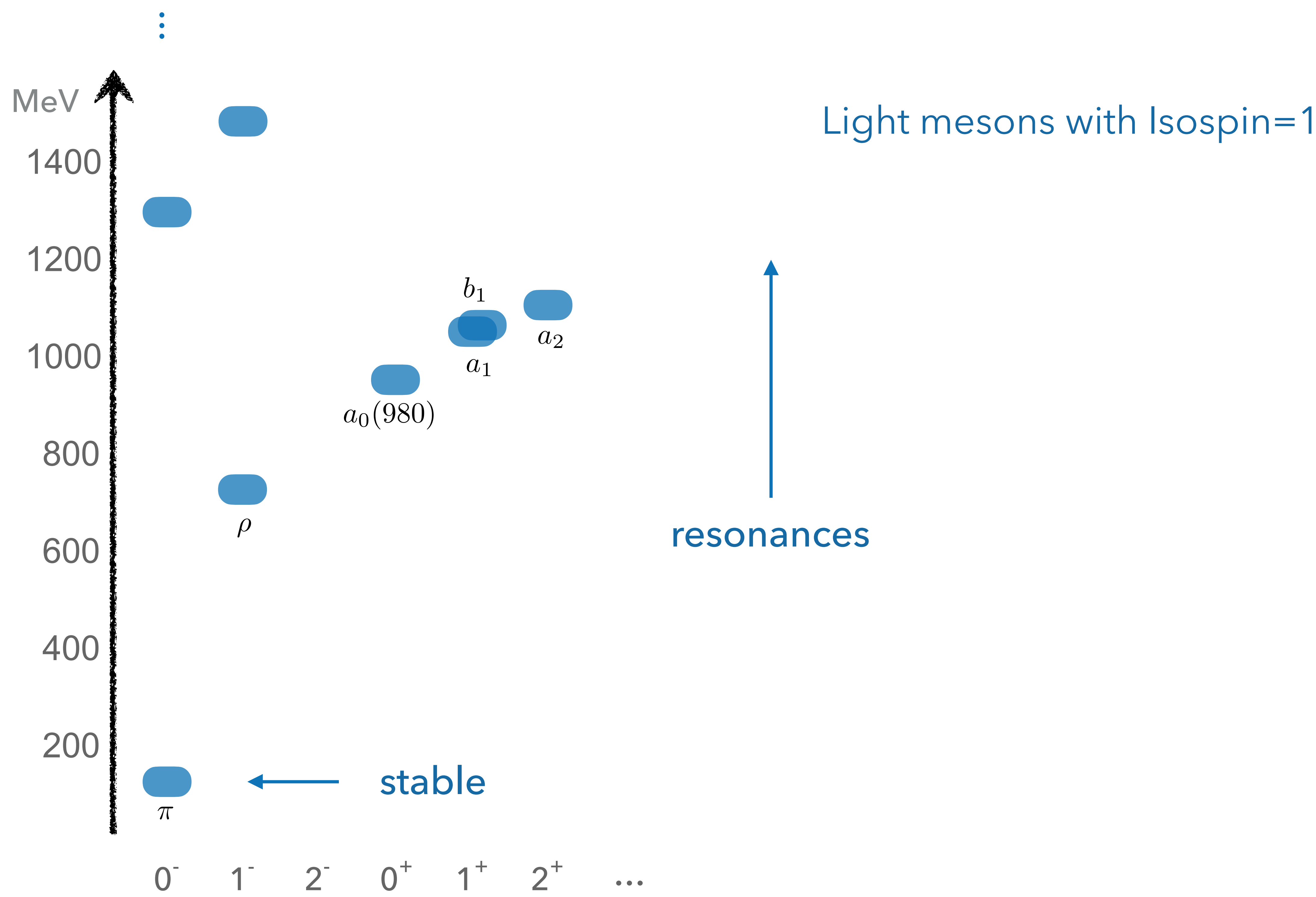
- physical pions are very light
- most interesting states can decay to **many** pions
- control of light-quark mass is a useful tool
- small effects not considered in general:

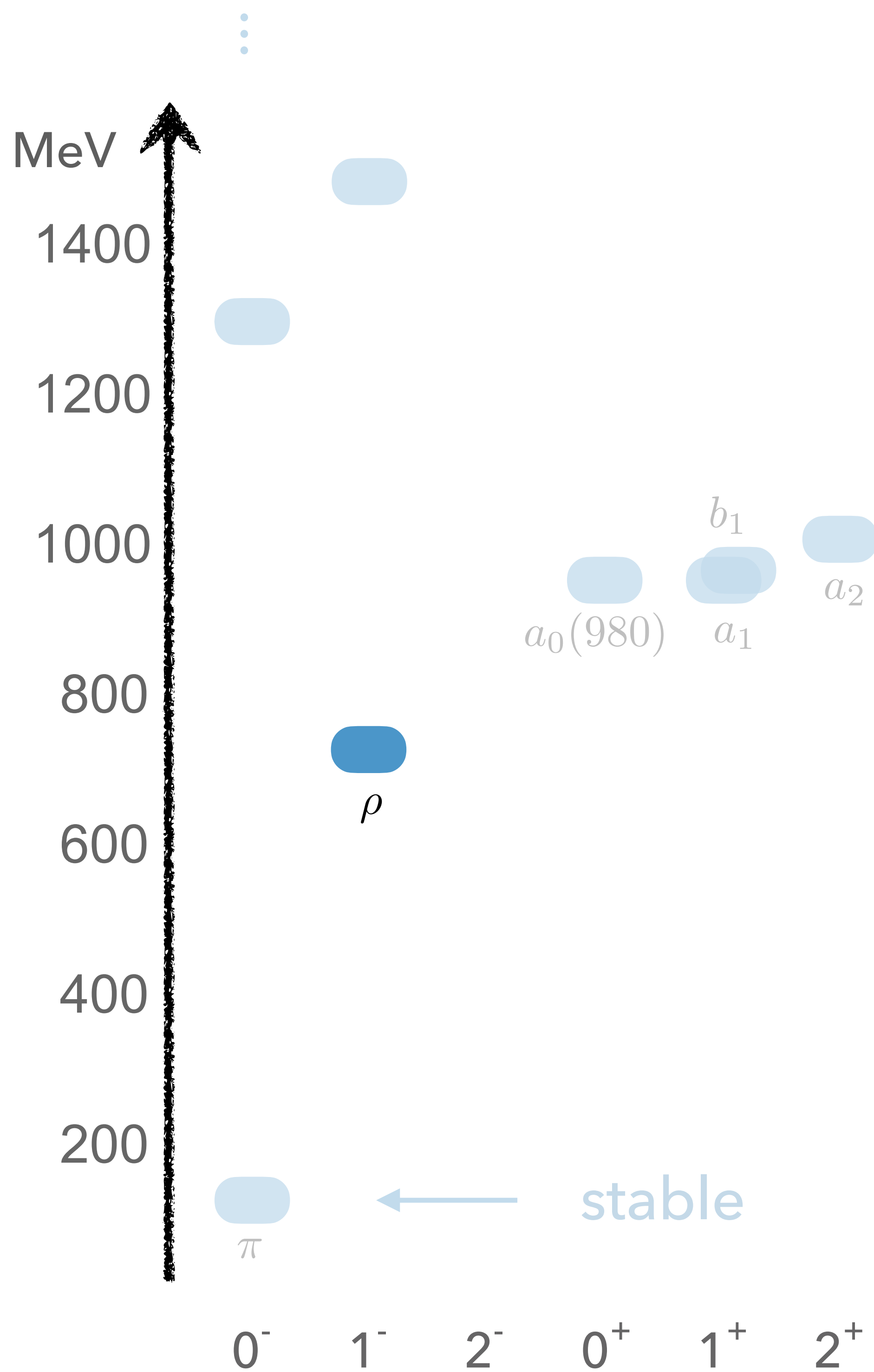
  finite lattice spacing, isospin breaking, EM interactions

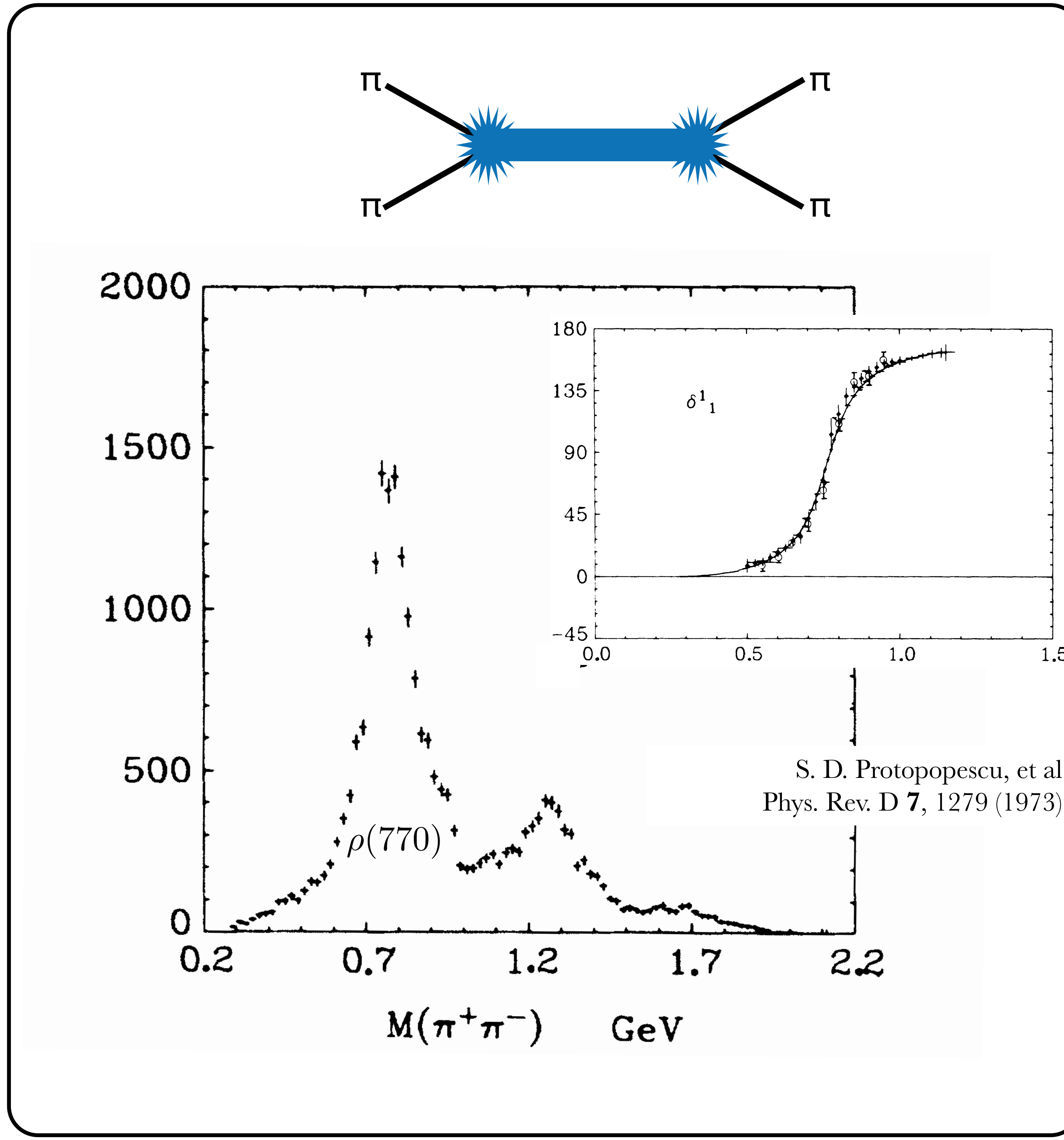
goal: what does QCD say about the excited hadron spectrum?




Light mesons with Isospin=1


MeV
1400
1200
1000
800
600
400
200
$b_1$
$a_2$
$a_1$
$a_0(980)$
$\rho$
$\pi$
stable
resonances
Light mesons with Isospin=1
0⁻ 1⁻ 2⁻ 0⁺ 1⁺ 2⁺ ...




"most rigorous" quantity for unstable states

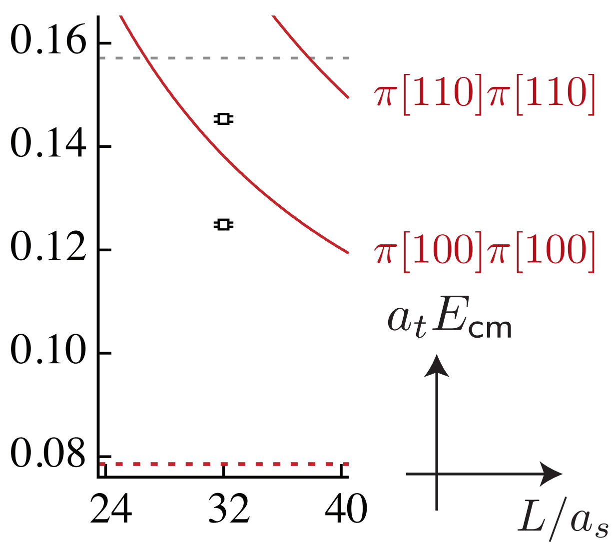
$$t \sim \frac{c^2}{s_{\rm pole} - s} \qquad \sqrt{s_{\rm pole}} = m \pm \tfrac{i}{2}\Gamma$$

$m_\pi = 239$ MeV

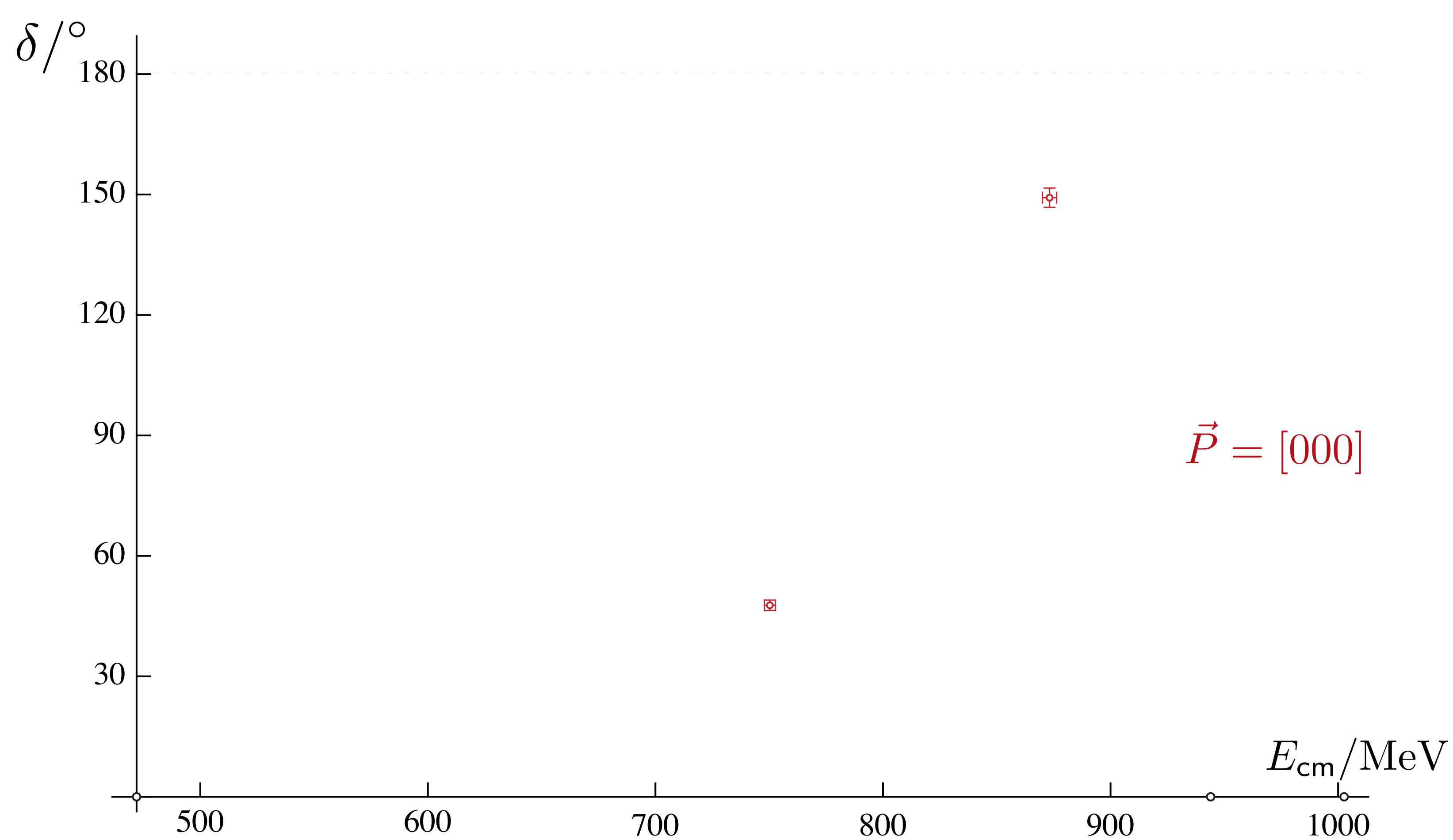
$L/a_s$=32

momentum is quantized in a finite volume $\vec{p} = \dfrac{2\pi\vec{n}}{L}$

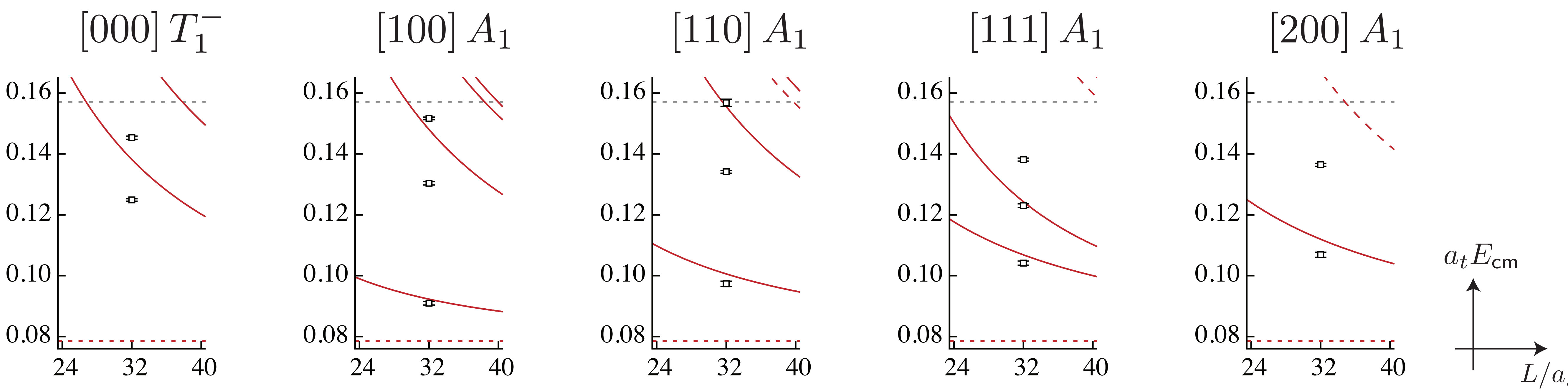
$[000]\ T_1^-$




at-rest

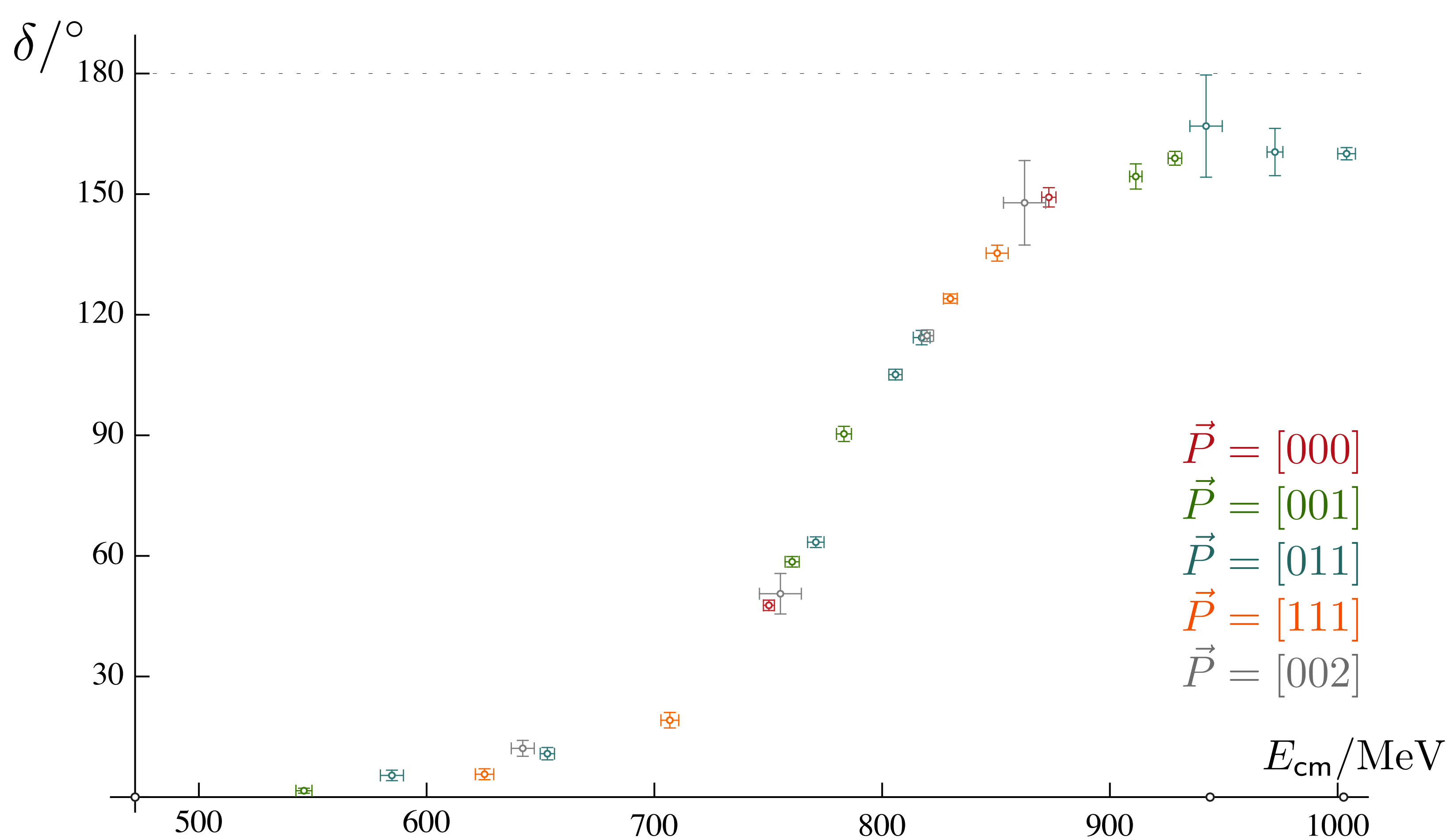


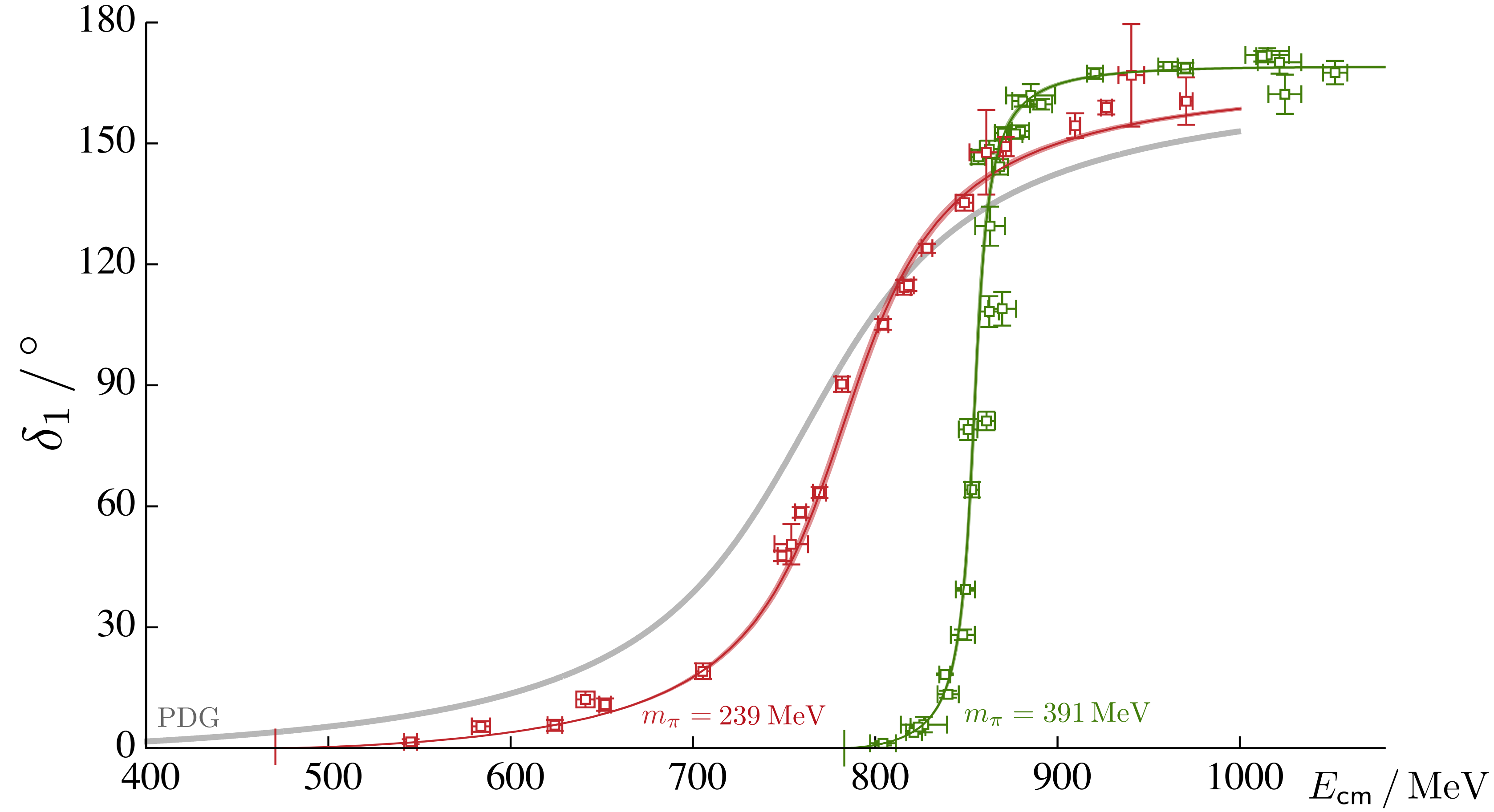
$m_\pi = 239$ MeV



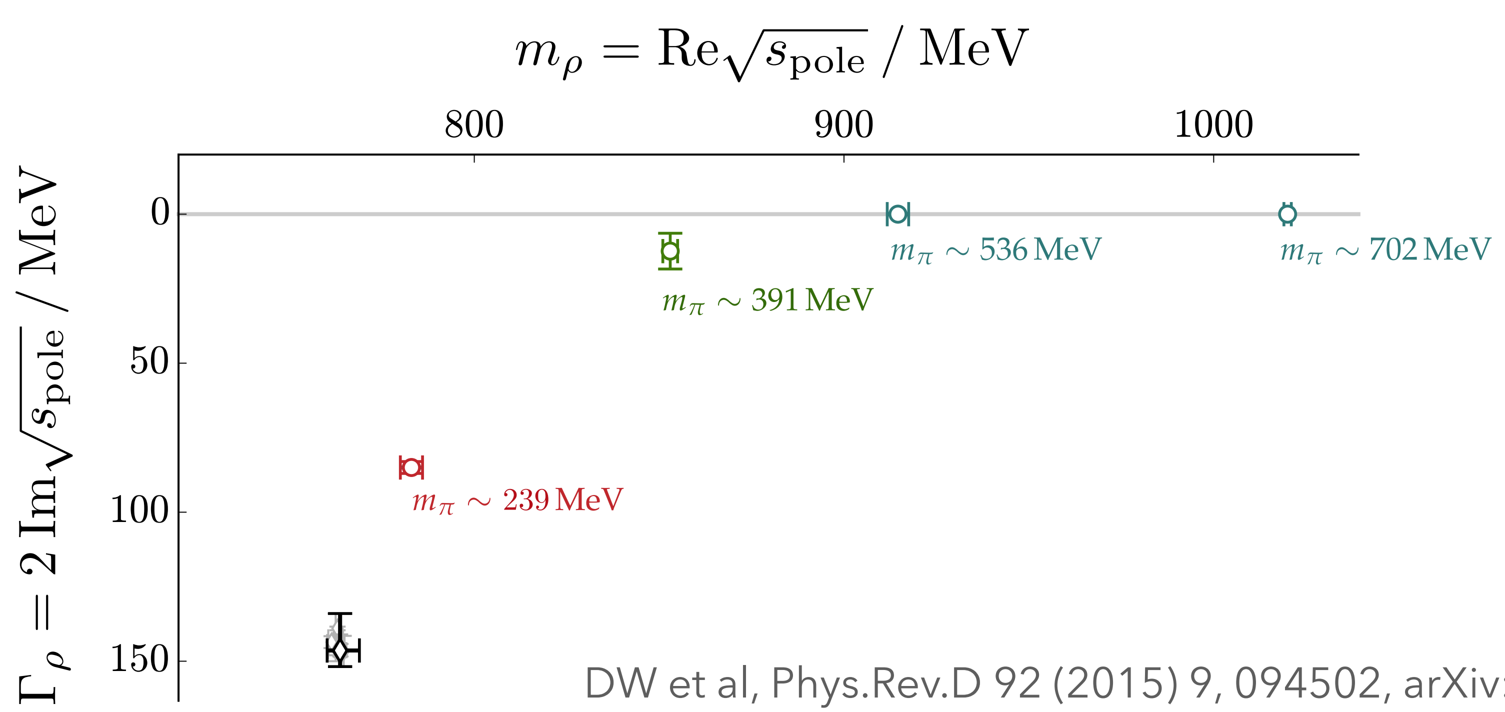

at-rest

non-zero overall momentum



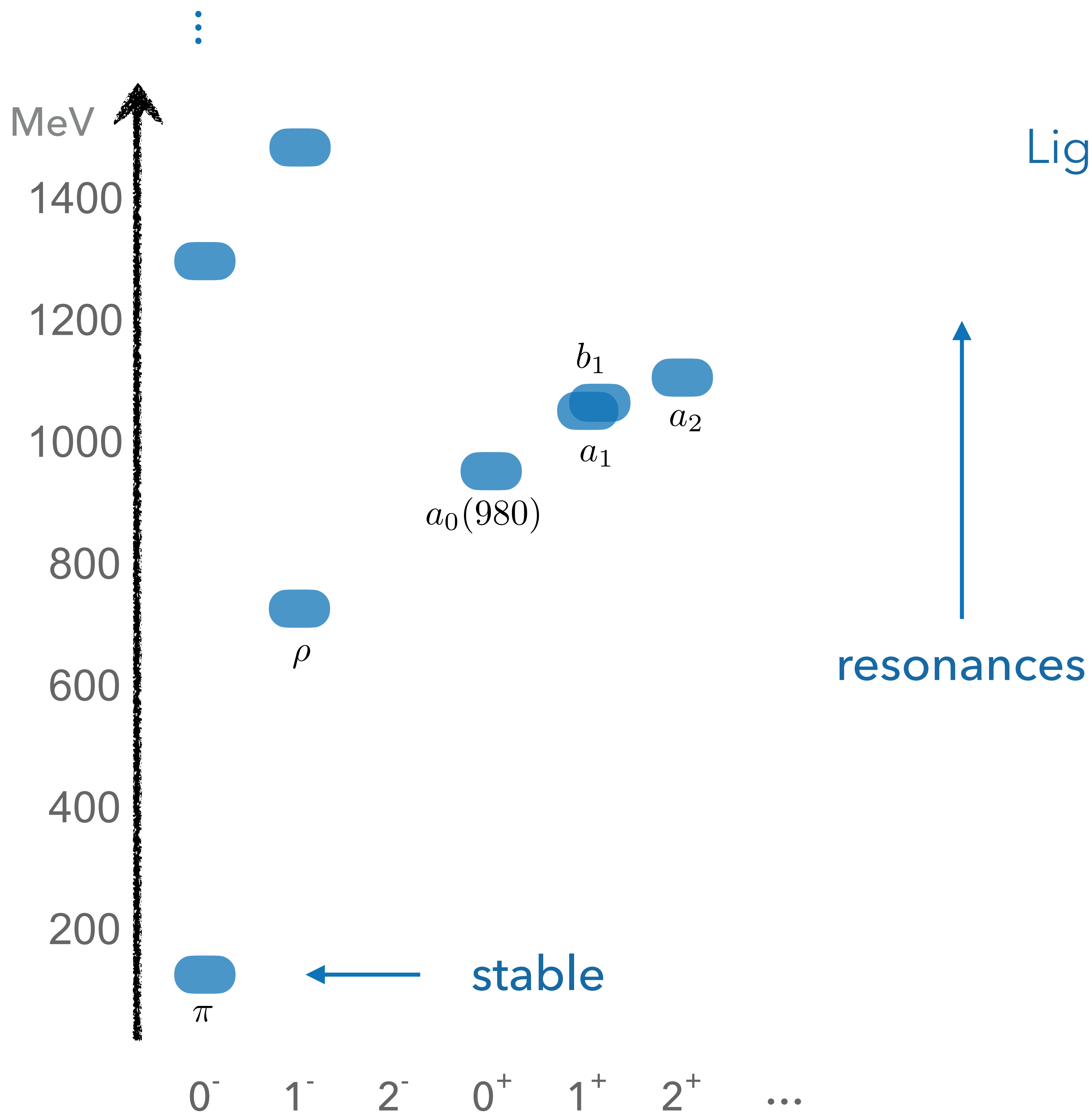


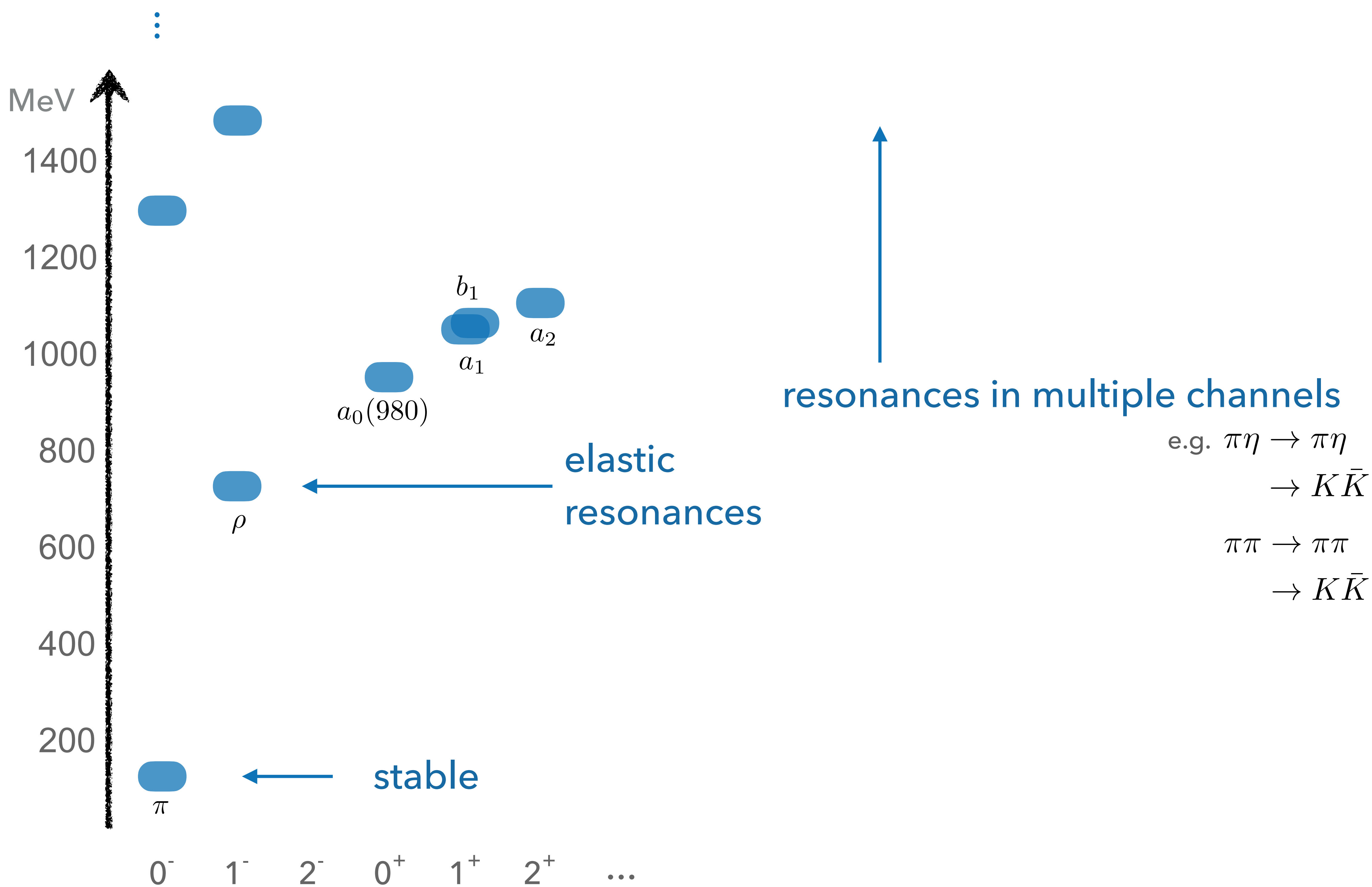

$$t \sim \frac{c^2}{s_{\text{pole}} - s}$$



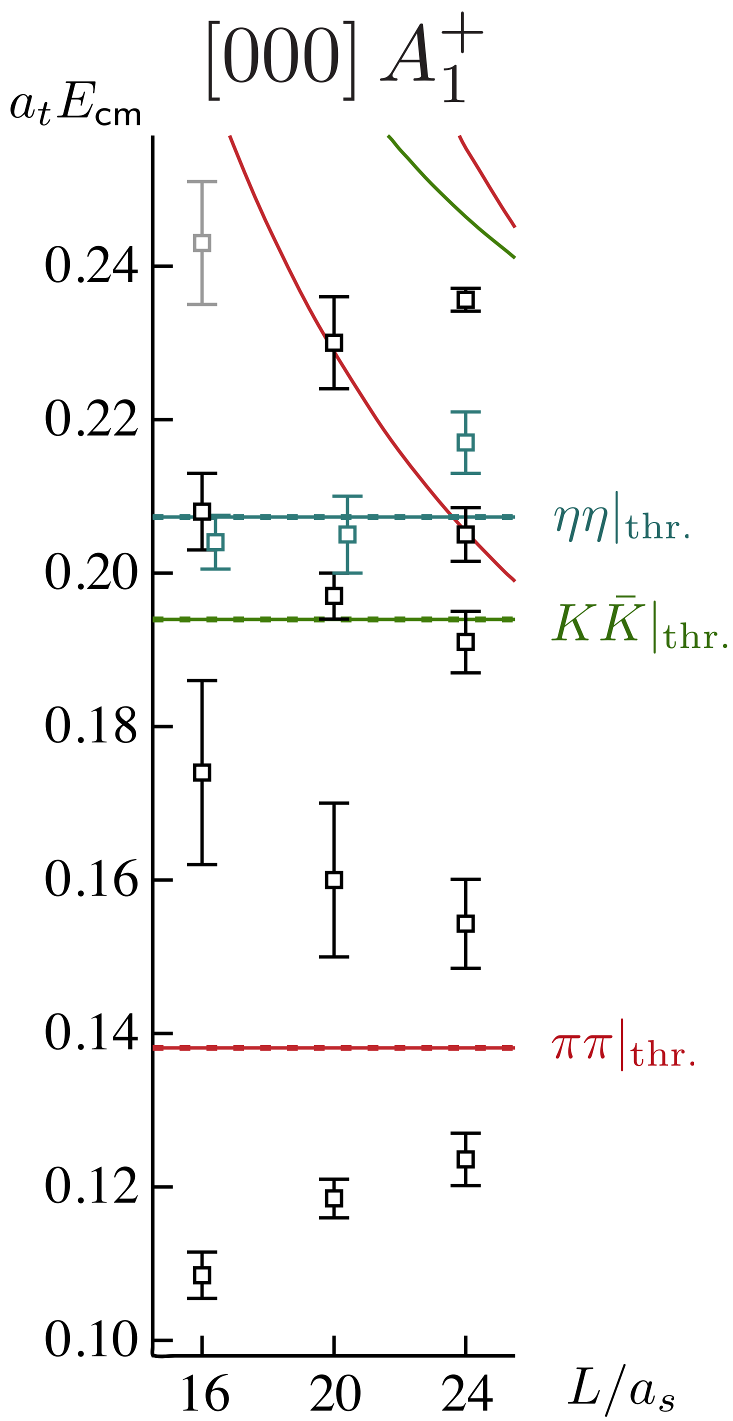

DW et al, Phys.Rev.D 92 (2015) 9, 094502, arXiv: 1507.02599
J. Dudek et al, Phys.Rev.D 87 (2013) 3, 034505, arXiv:1212.0830

Light mesons with Isospin=1

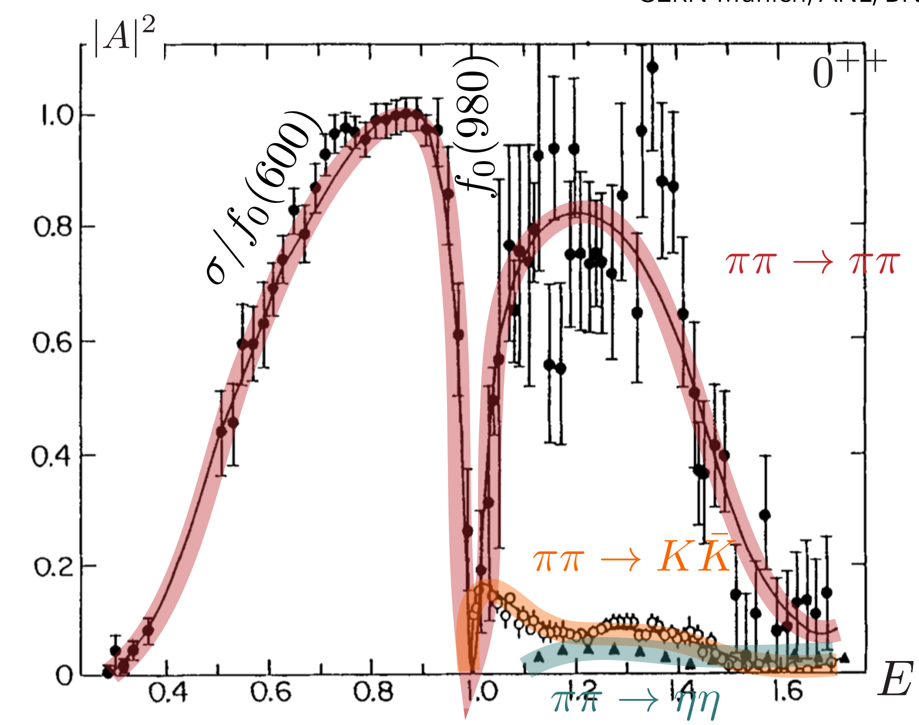



MeV
1400
1200
1000
800
600
400
200
$0^-$ $1^-$ $2^-$ $0^+$ $1^+$ $2^+$ …
$\pi$
$\rho$
$a_0(980)$
$a_1$
$b_1$
$a_2$
stable
elastic resonances
resonances in multiple channels
e.g. $\pi\eta \to \pi\eta$
$\to K\bar{K}$
$\pi\pi \to \pi\pi$
$\to K\bar{K}$

$m_\pi = 391$ MeV

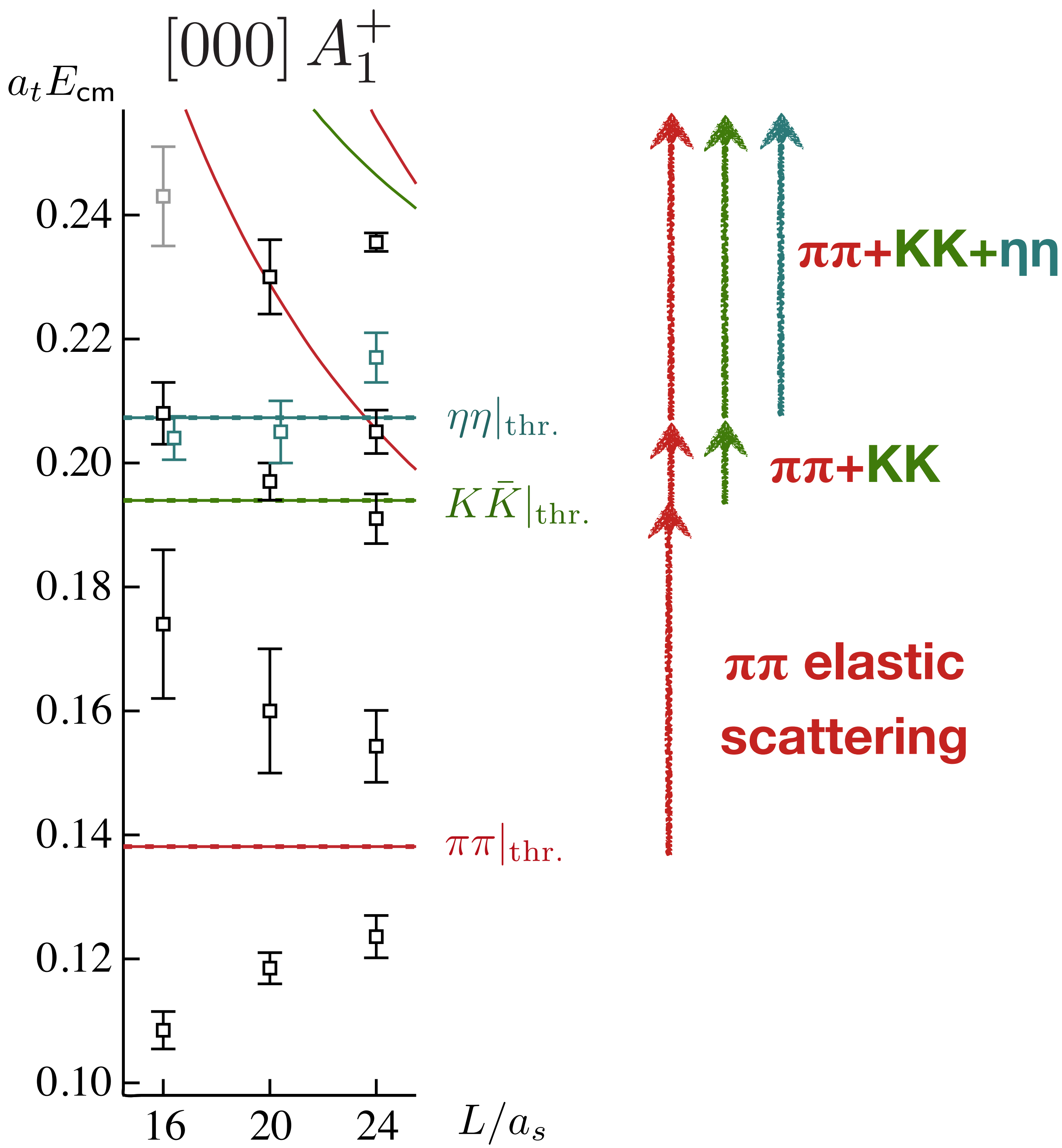



CERN-Munich, ANL, BNL




R. Briceño et al PRD 97 (2018) 5, 054513, arXiv:1708.06667

$m_\pi = 391$ MeV



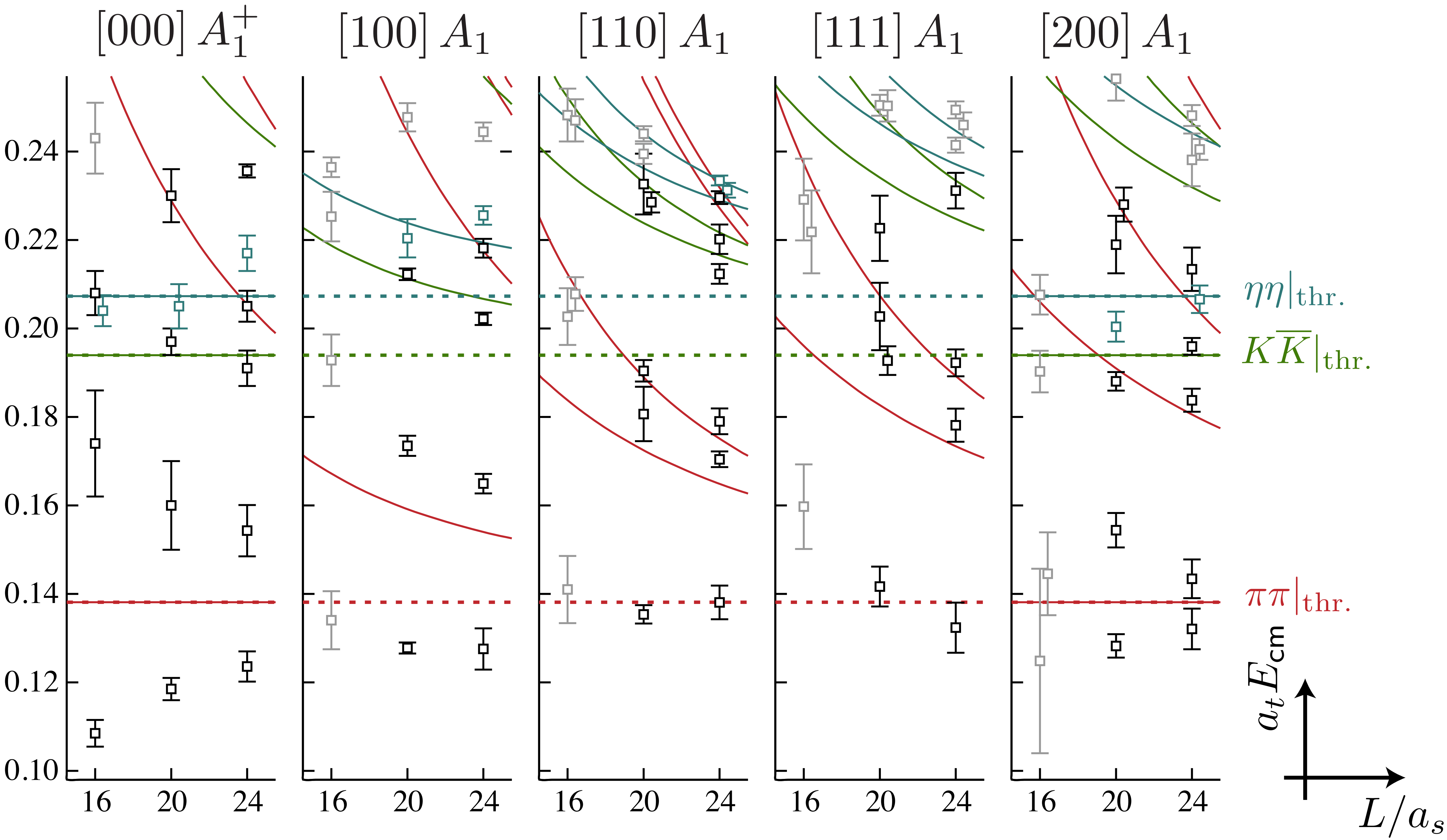

$$\mathbf{t} = \begin{pmatrix} \pi\pi \to \pi\pi & \pi\pi \to K\bar{K} & \pi\pi \to \eta\eta \\ & K\bar{K} \to K\bar{K} & K\bar{K} \to \eta\eta \\ & & \eta\eta \to \eta\eta \end{pmatrix}$$

$$\mathbf{t} = \begin{pmatrix} \pi\pi \to \pi\pi & \pi\pi \to K\bar{K} \\ & K\bar{K} \to K\bar{K} \end{pmatrix}$$

$$\mathbf{t} = \begin{pmatrix} \pi\pi \to \pi\pi \end{pmatrix}$$

R. Briceño et al PRD 97 (2018) 5, 054513, arXiv:1708.06667

local $q\overline{q}$ & 2-hadron operators
conservatively 57 energy levels
dominated by S-wave interactions




R. Briceño et al PRD 97 (2018) 5, 054513, arXiv:1708.06667

$$\det\left[\mathbf{1} + i\boldsymbol{\rho}(E)\cdot \boldsymbol{t}(E)\cdot(\mathbf{1} + i\boldsymbol{\mathcal{M}}(E,L))\right] = 0 \qquad \mathbf{t} = \begin{pmatrix} \pi\pi \to \pi\pi & \pi\pi \to K\bar{K} \\ K\bar{K} \to \pi\pi & K\bar{K} \to K\bar{K} \end{pmatrix}$$

Lüscher et al

determinant condition:

- several unknowns at each value of energy
- energy levels typically do not coincide
- underconstrained problem for a single energy

one solution: use energy dependent parameterizations

- Constrained problem when #(energy levels) > #(parameters)
- Essential amplitudes respect unitarity of the S-matrix

$$\mathbf{S}^\dagger\mathbf{S} = \mathbf{1} \quad \to \quad \mathrm{Im}\,\mathbf{t}^{-1} = -\boldsymbol{\rho} \qquad \rho_{ij} = \delta_{ij}\frac{2k_i}{E_{\mathsf{cm}}}$$

K-matrix approach:

$$\mathbf{t}^{-1} = \mathbf{K}^{-1} - i\boldsymbol{\rho}$$

Chew-Mandelstam phase space:

$$\mathbf{t}^{-1} = \mathbf{K}^{-1} + \boldsymbol{I}$$

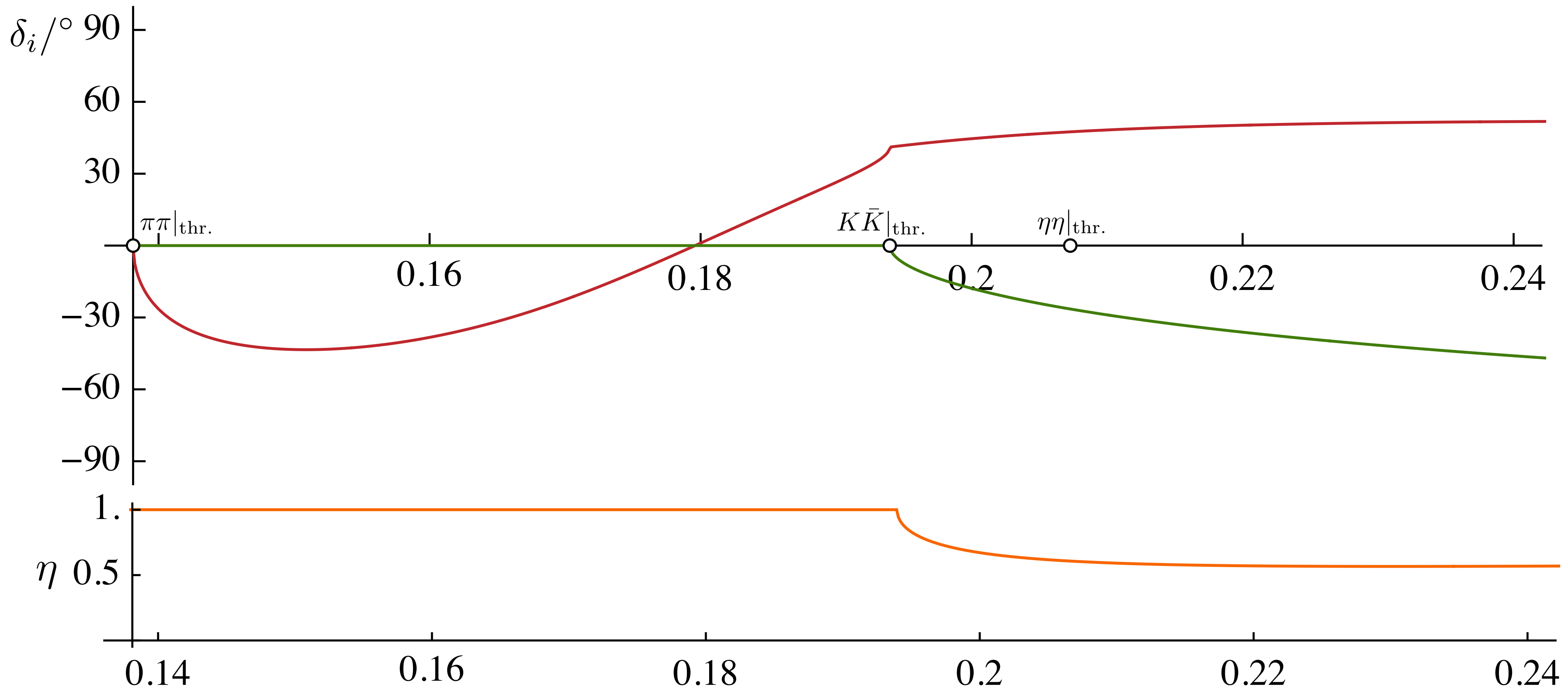
use a dispersion relation to generate a real part from iρ

- K is real for real energies
- we use a broad selection of K-matrices
- neglects left-hand cut

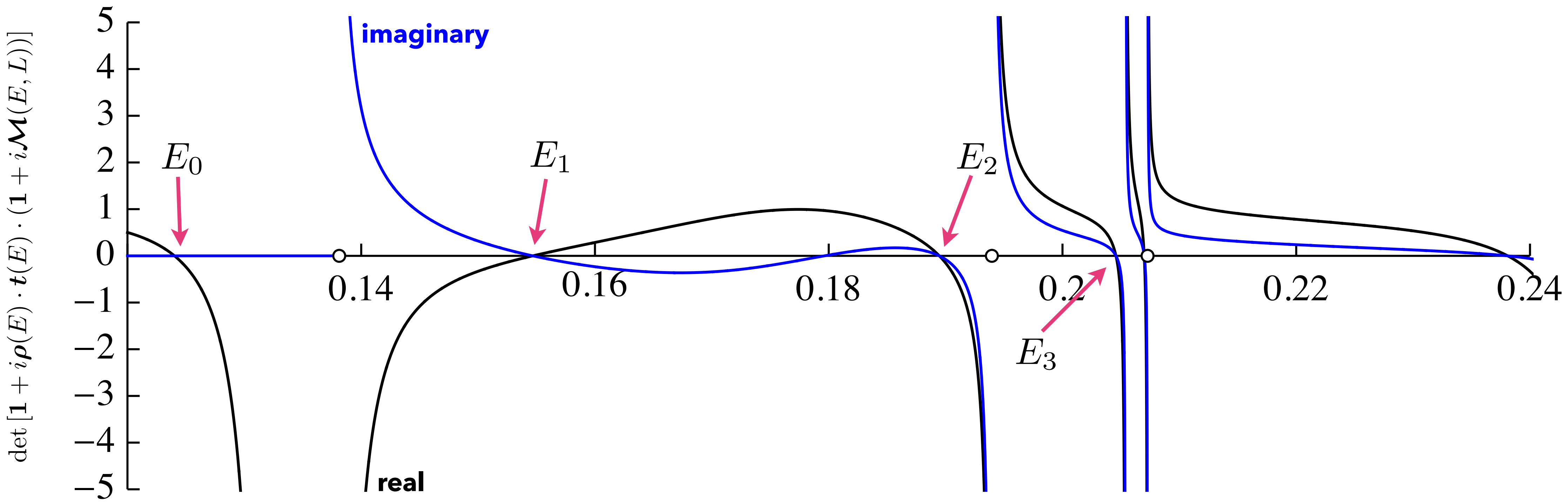
$$t_{11} = \frac{1}{2i\rho_1}\left(\eta e^{2i\delta_1} - 1\right)$$

$$t_{12} = \frac{1}{2\sqrt{\rho_1\rho_2}}(1-\eta^2)^{\frac{1}{2}} e^{i\delta_1 + i\delta_2}$$

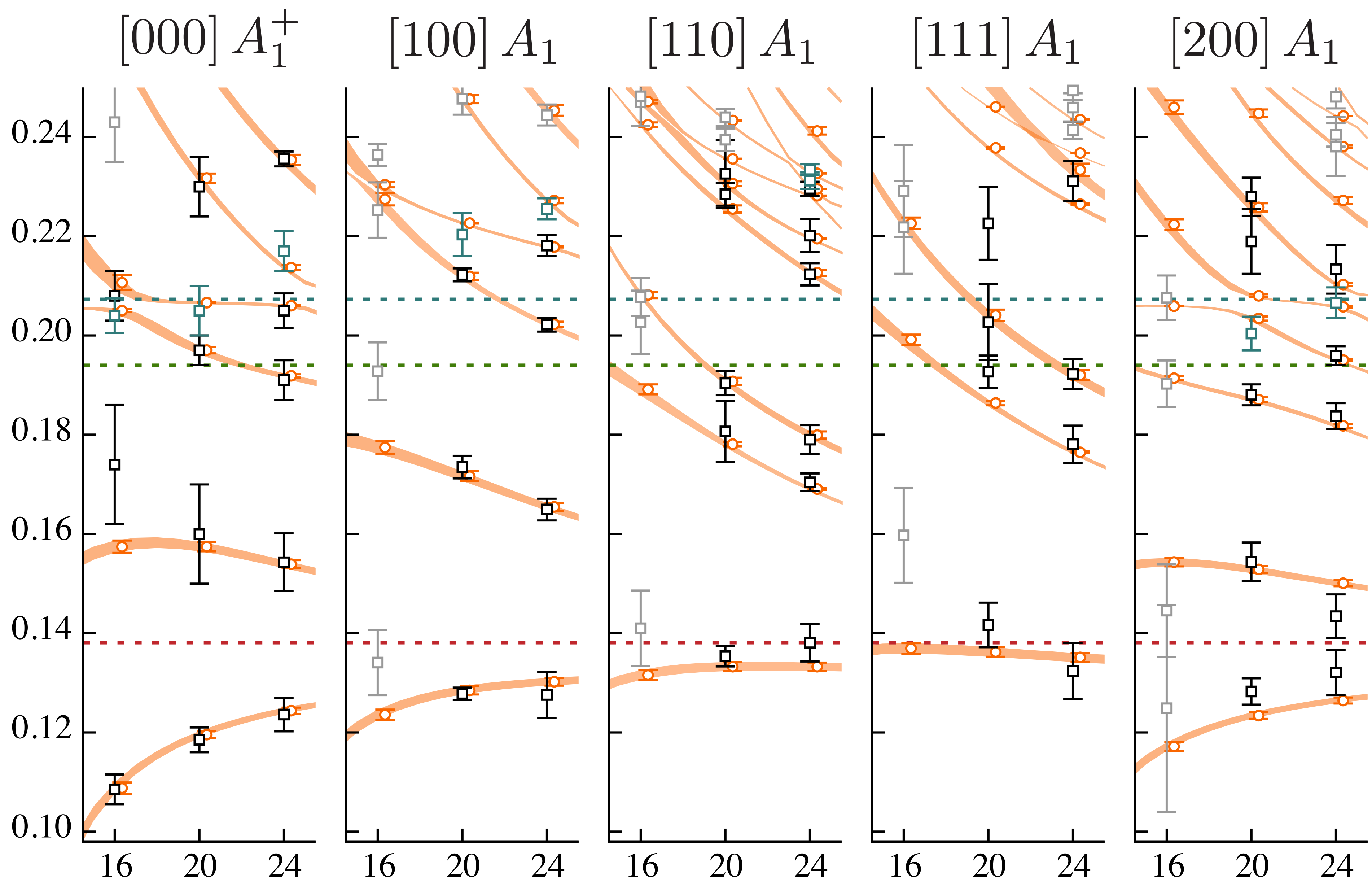
$$S_{ii} = \eta e^{2i\delta_i}$$




we can identify the solutions



$$\det\left[\mathbf{1} + i\boldsymbol{\rho}(E)\cdot\boldsymbol{t}(E)\cdot\left(\mathbf{1}+i\boldsymbol{\mathcal{M}}(E,L)\right)\right] = 0$$




$$\chi^2/N_{\mathrm{dof}} = \frac{44.0}{57-8} = 0.90$$

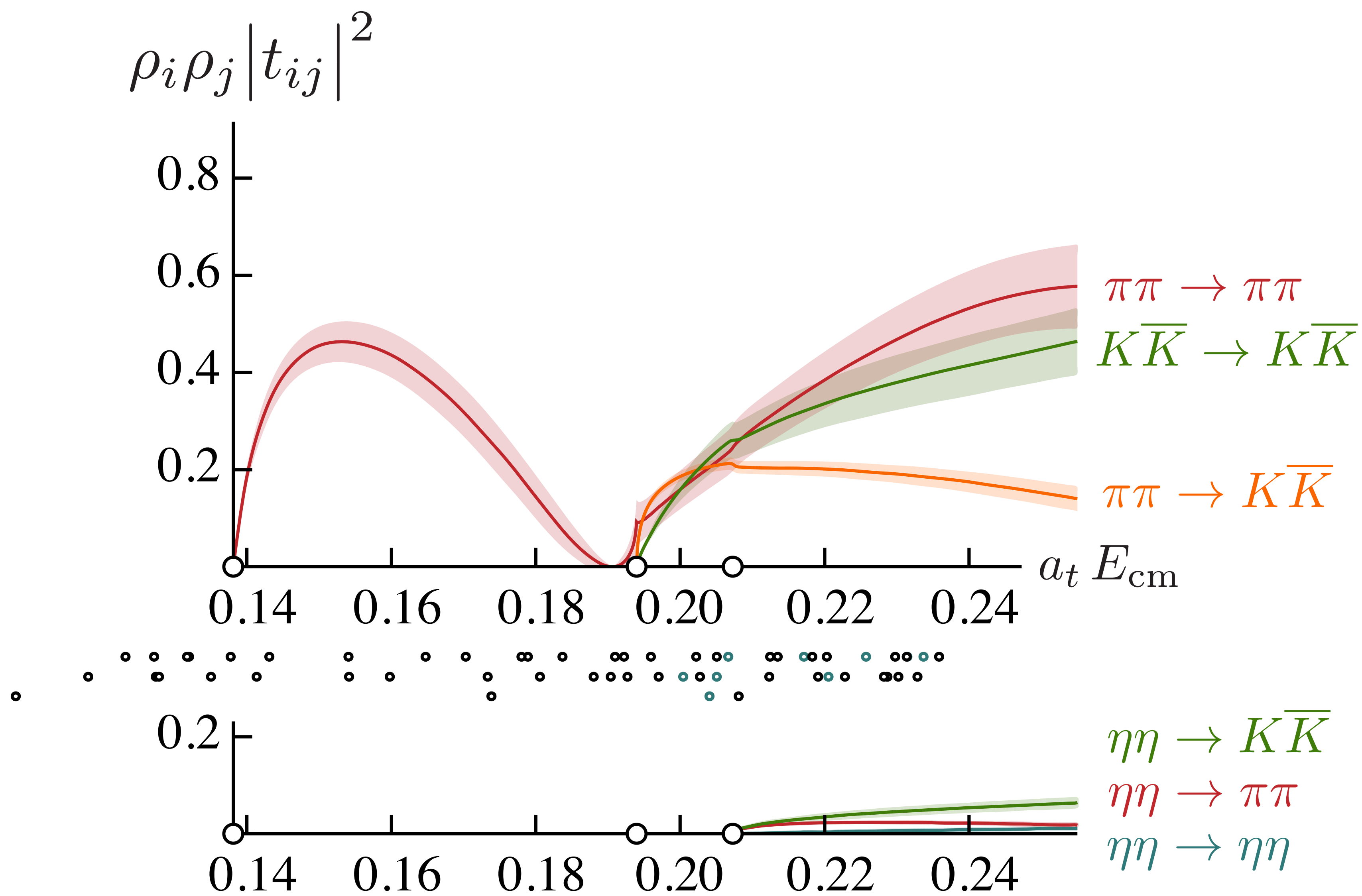
An example S-wave spectrum fit

$$\mathbf{t}^{-1} = \mathbf{K}^{-1} + \mathbf{I}$$

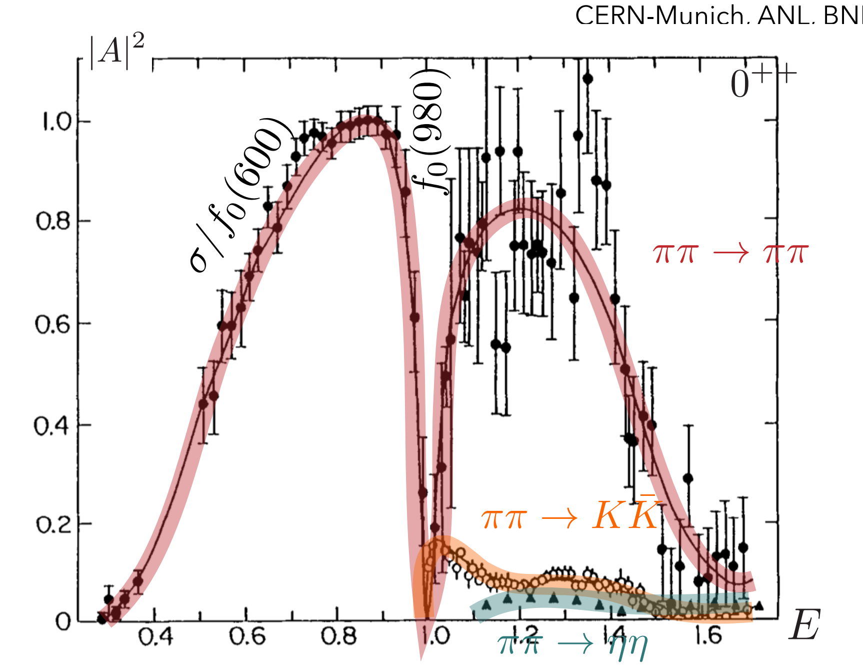
$$\mathbf{K}(s) = \begin{pmatrix} a+bs & c+ds & e \\ c+ds & f & g \\ e & g & h \end{pmatrix}$$

$$\chi^2/N_{\mathrm{dof}} = \frac{44.0}{57-8} = 0.90$$

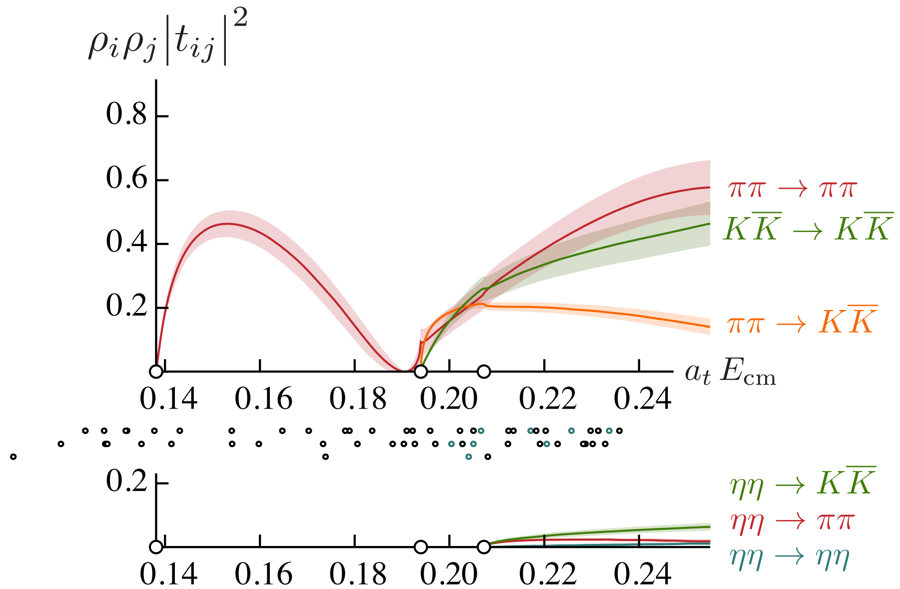
57 energy levels

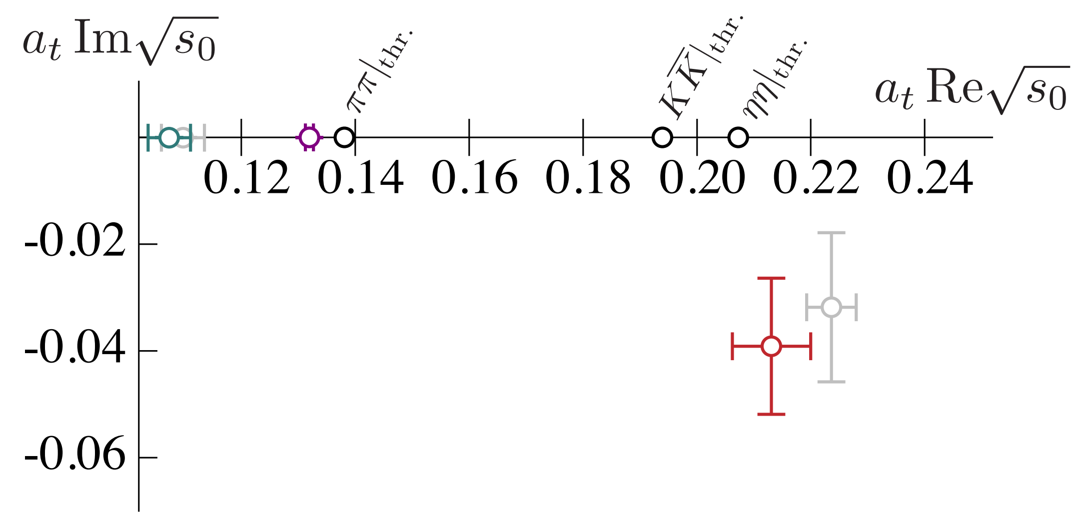


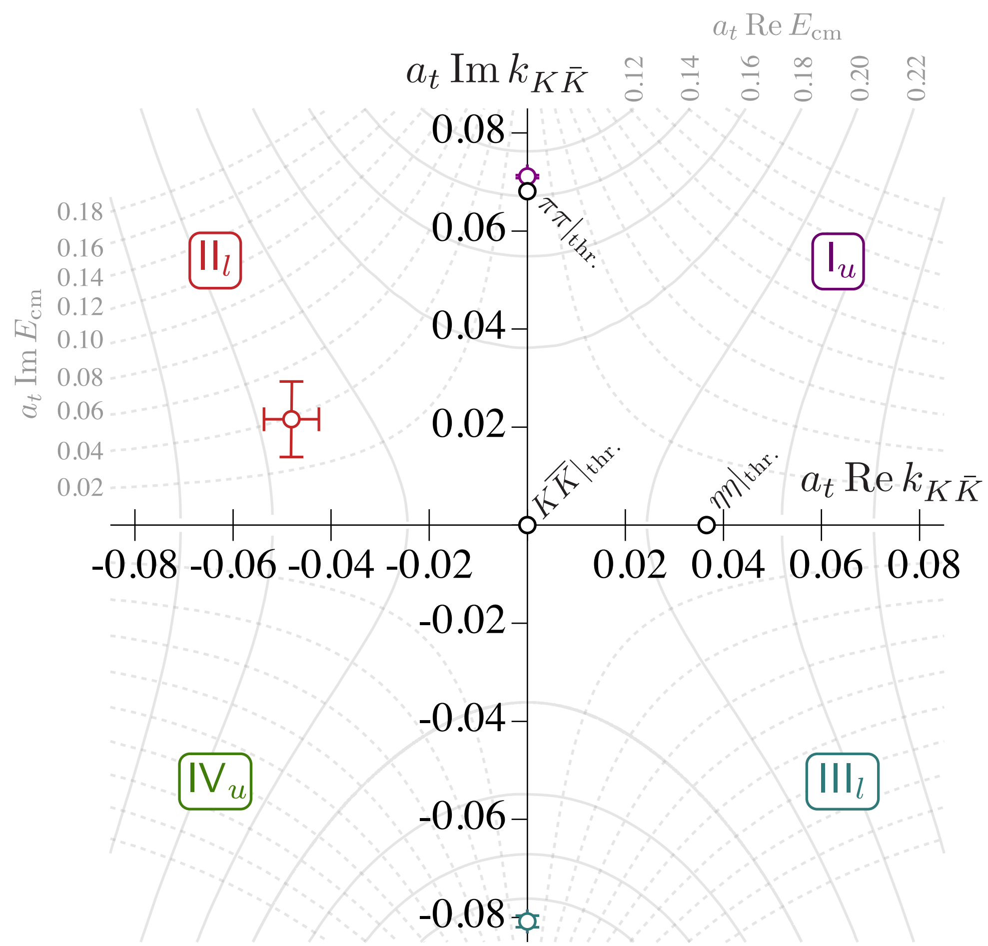

CERN-Munich. ANL. BNL

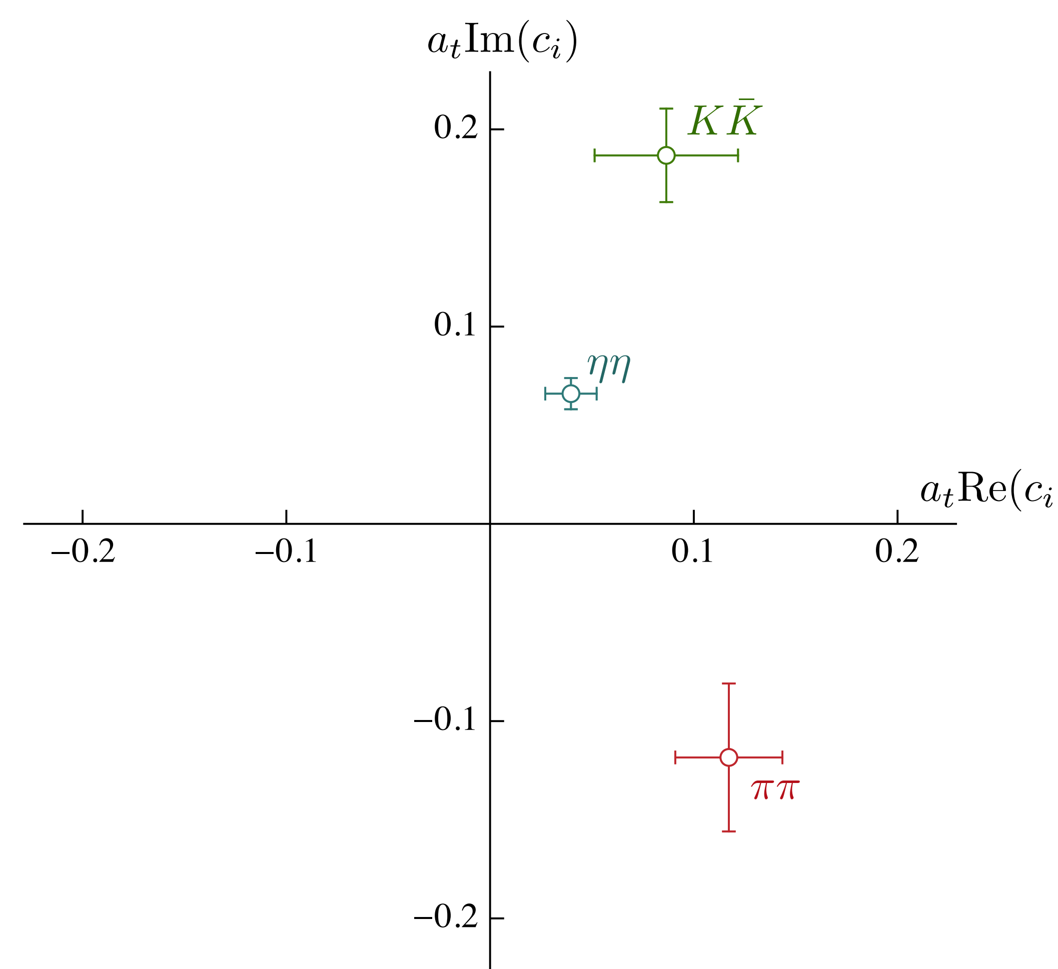


a pole in multiple channels: $t_{ij} \sim \dfrac{c_i c_j}{s_{\text{pole}} - s}$ $\quad \sqrt{s_{\text{pole}}} = m \pm \frac{i}{2}\Gamma$

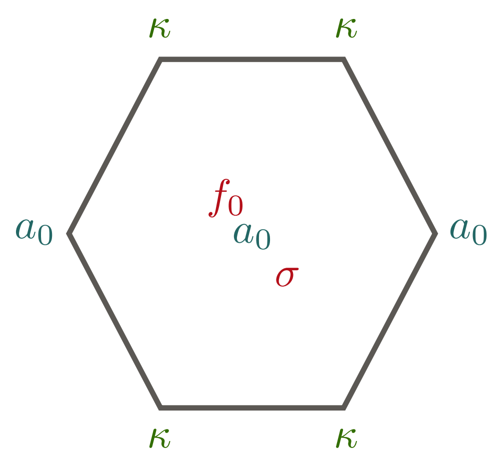


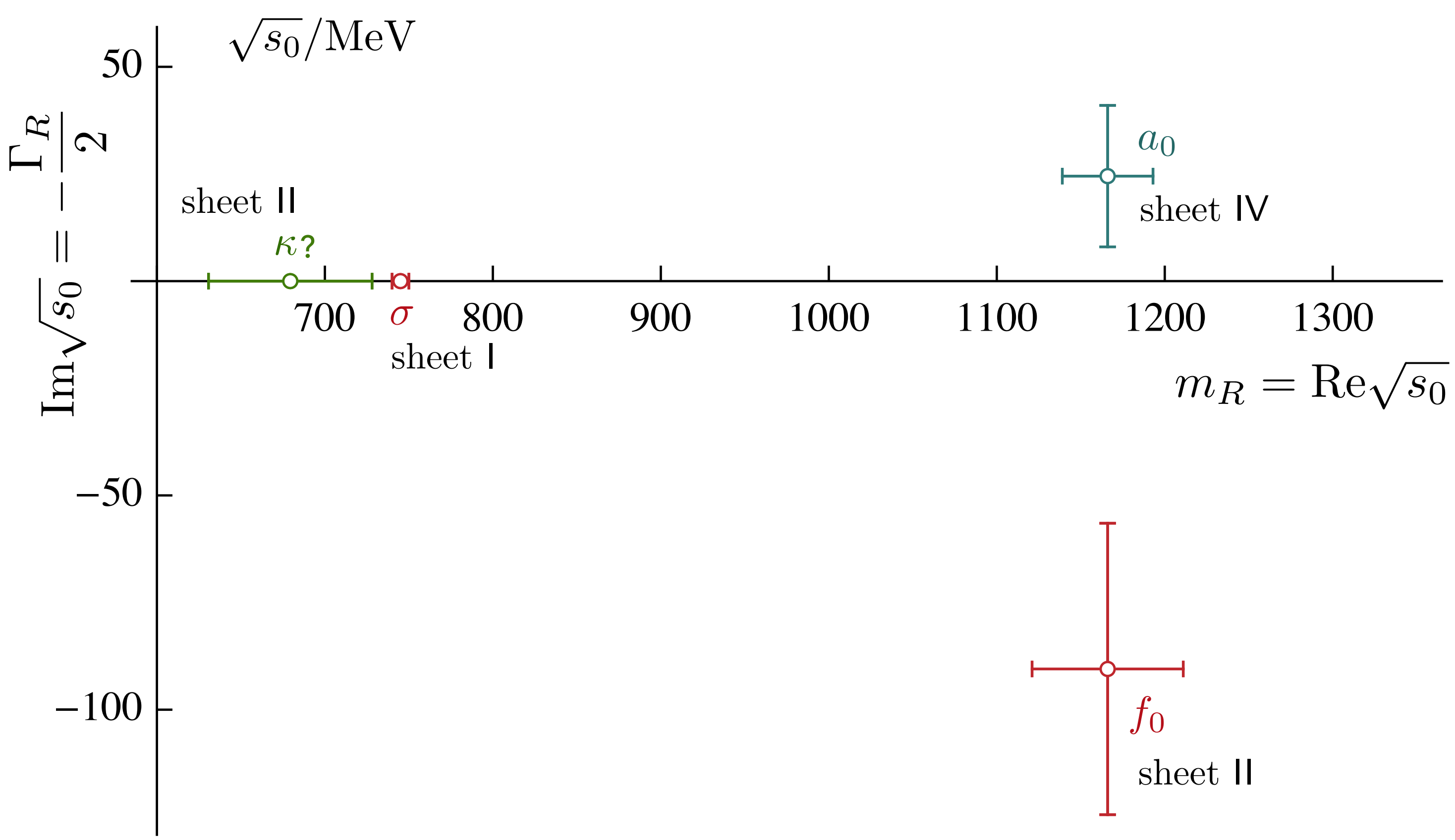



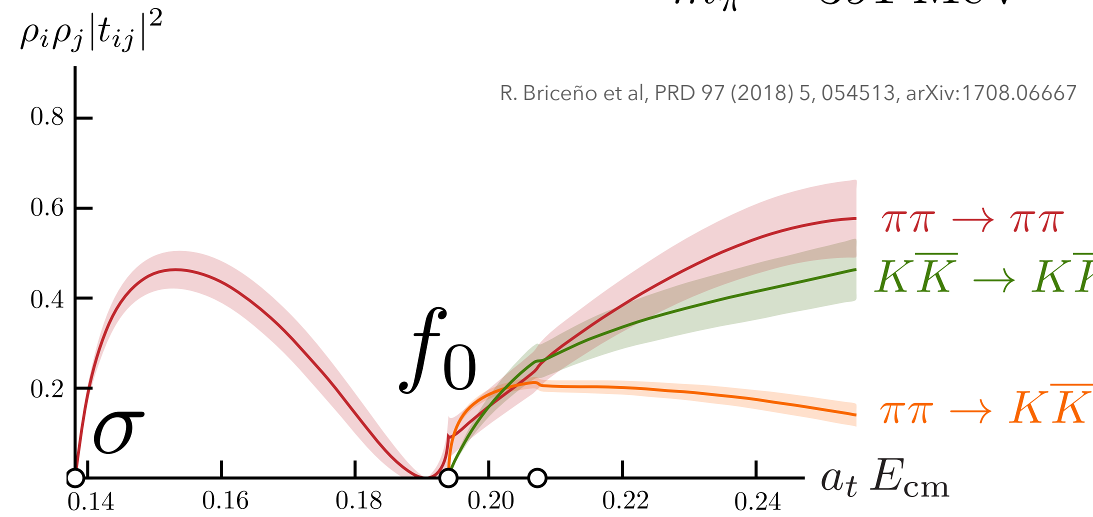



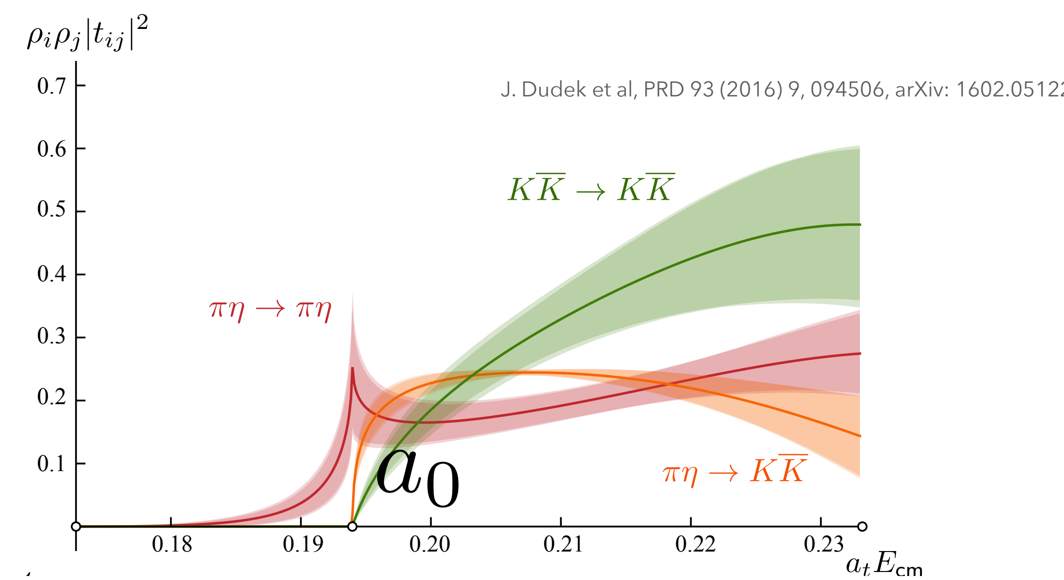



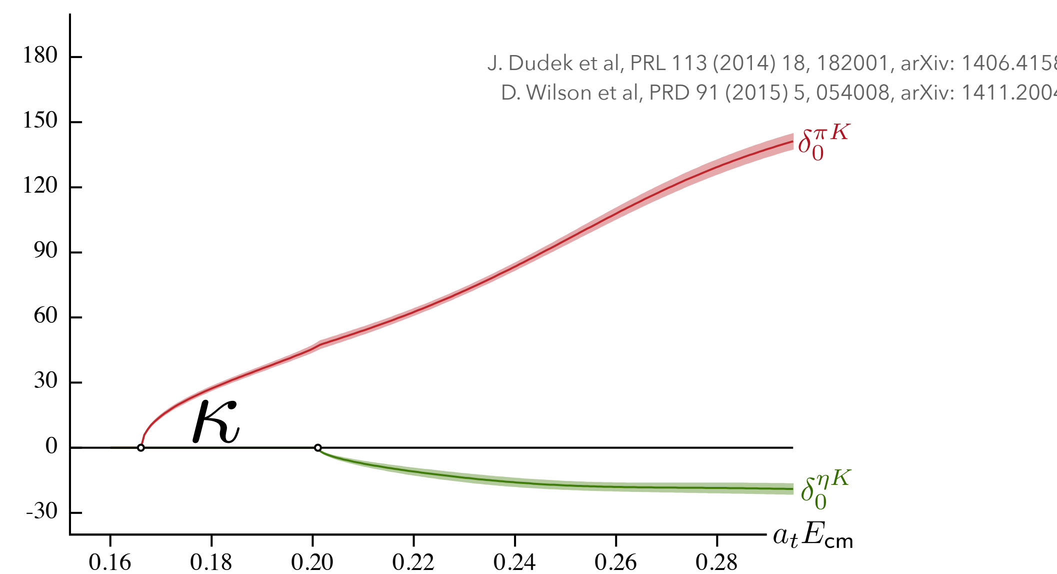
$m_\pi = 391$ MeV


$\kappa$ $\kappa$
$a_0$ $f_0$ $a_0$ $\sigma$ $a_0$
$\kappa$ $\kappa$


$\sqrt{s_0}$/MeV
$\mathrm{Im}\sqrt{s_0} = -\frac{\Gamma_R}{2}$
50
−50
−100
700
800
900
1000
1100
1200
1300
$m_R = \mathrm{Re}\sqrt{s_0}$
sheet II
$\kappa$?
$\sigma$
sheet I
$a_0$
sheet IV
$f_0$
sheet II


$\rho_i\rho_j|t_{ij}|^2$
R. Briceño et al, PRD 97 (2018) 5, 054513, arXiv:1708.06667
0.8
0.6
0.4
0.2
0.14
0.16
0.18
0.20
0.22
0.24
$a_t\,E_{\mathrm{cm}}$
$\pi\pi \to \pi\pi$
$K\overline{K} \to K\overline{K}$
$\pi\pi \to K\overline{K}$
$\sigma$
$f_0$

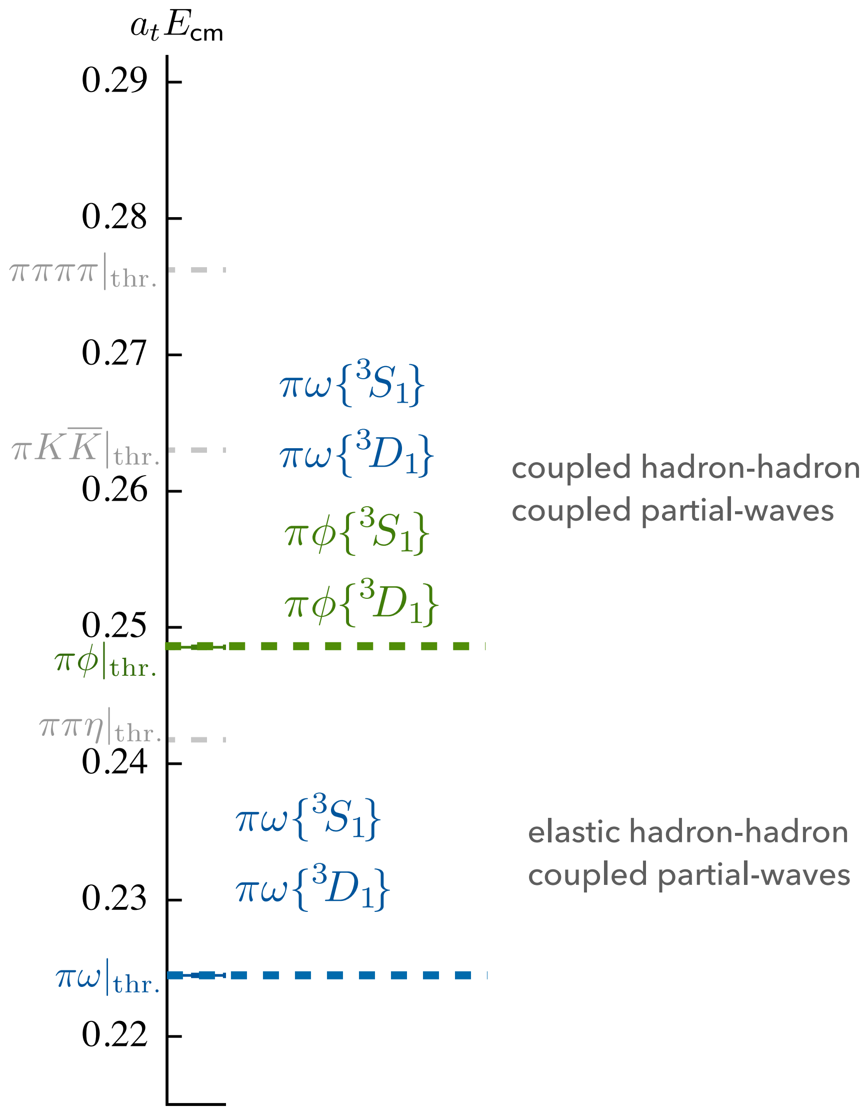

$\rho_i\rho_j|t_{ij}|^2$
J. Dudek et al, PRD 93 (2016) 9, 094506, arXiv: 1602.05122
0.7
0.6
0.5
0.4
0.3
0.2
0.1
0.18
0.19
0.20
0.21
0.22
0.23
$a_t E_{\mathrm{cm}}$
$K\overline{K} \to K\overline{K}$
$\pi\eta \to \pi\eta$
$\pi\eta \to K\overline{K}$
$a_0$

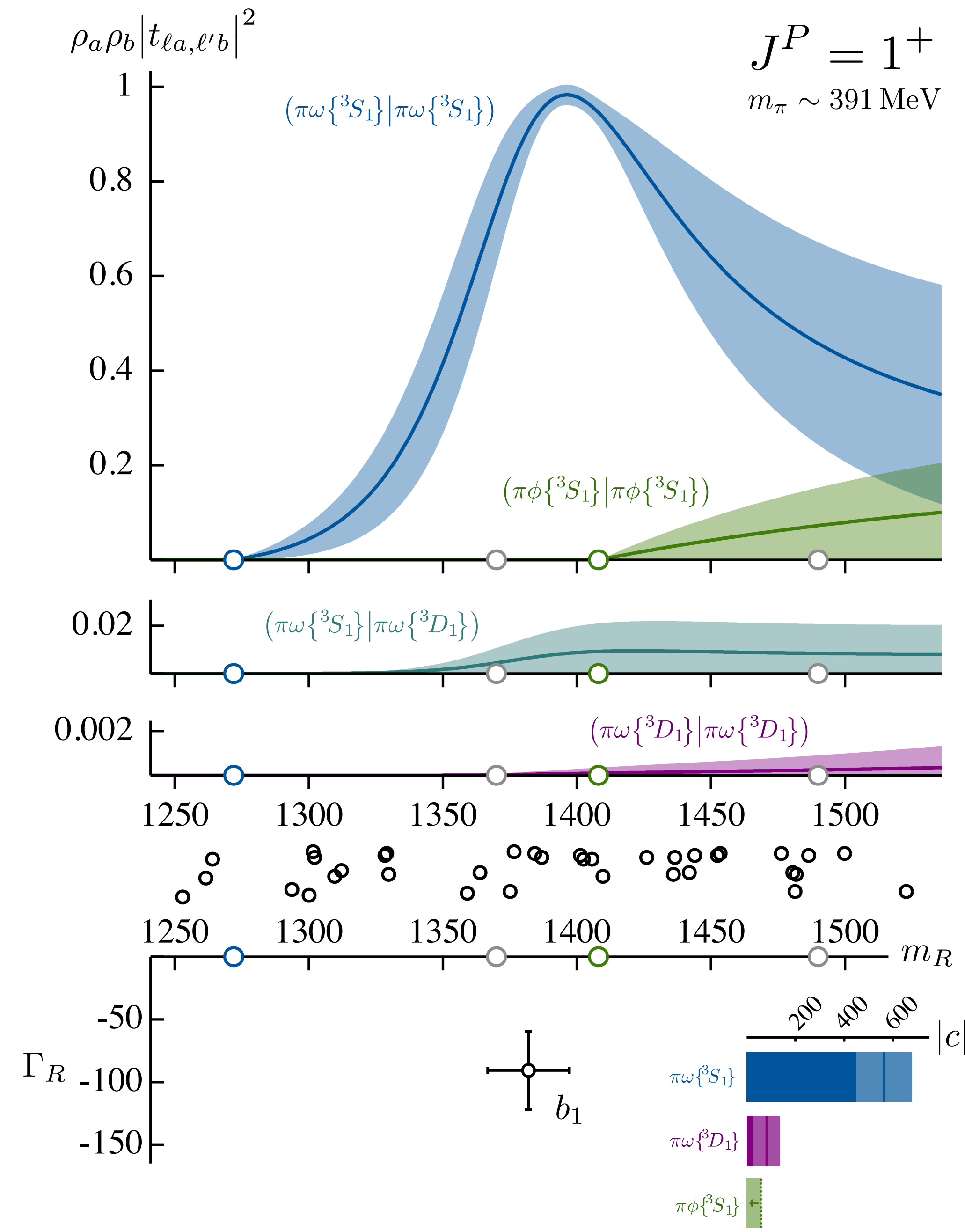

J. Dudek et al, PRL 113 (2014) 18, 182001, arXiv: 1406.4158
D. Wilson et al, PRD 91 (2015) 5, 054008, arXiv: 1411.2004
180
150
120
90
60
30
0
-30
0.16
0.18
0.20
0.22
0.24
0.26
0.28
$a_t E_{\mathrm{cm}}$
$\delta_0^{\pi K}$
$\delta_0^{\eta K}$
$\kappa$

## scattering with spinning particles

A. J. Woss, et al, PRD100 (2019) 5, 054506, arXiv: 1904.04136

stable vector ω scattering



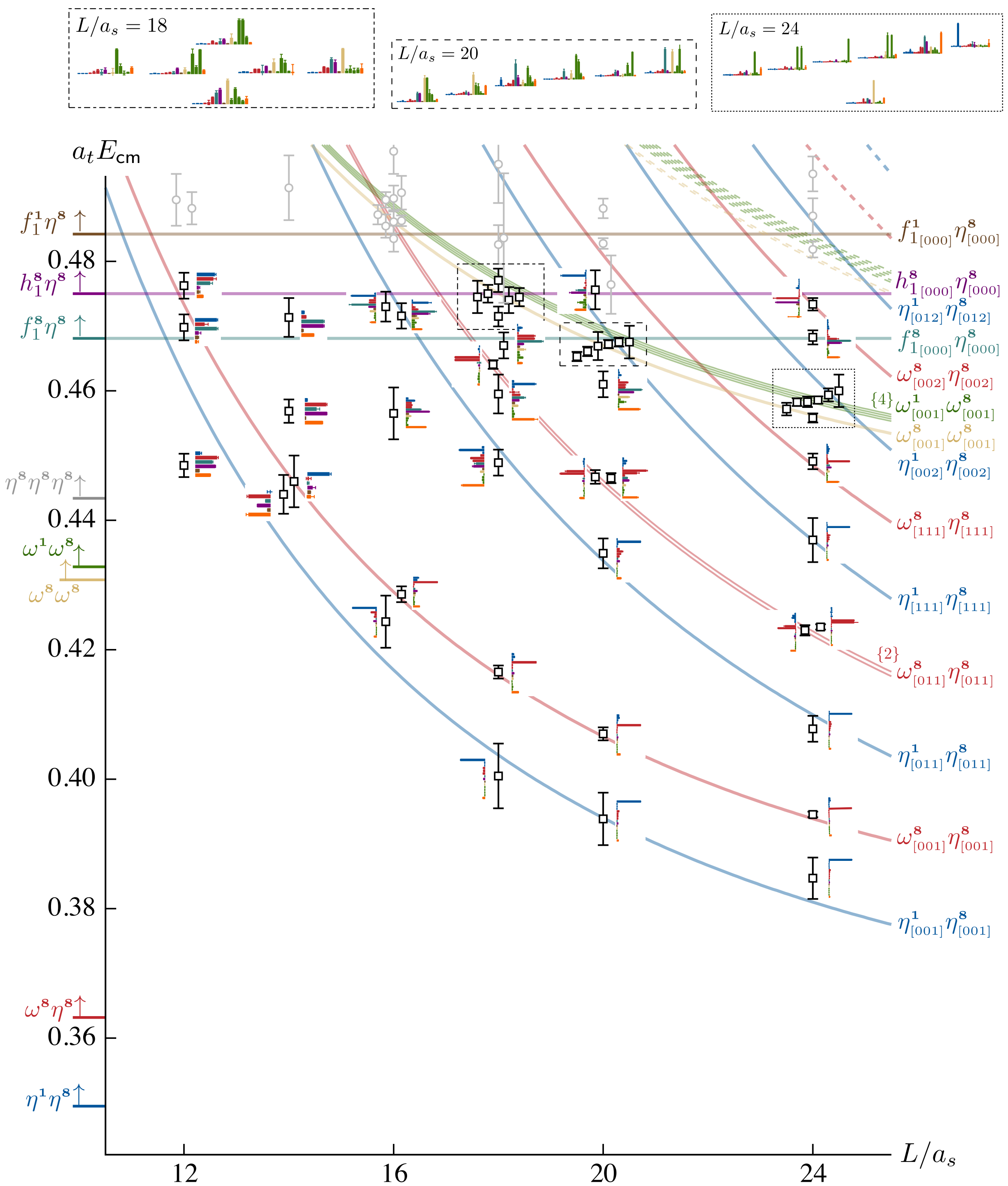

**$b_1(1235)$** $I^G(J^{PC}) = 1^+(1^{+-})$

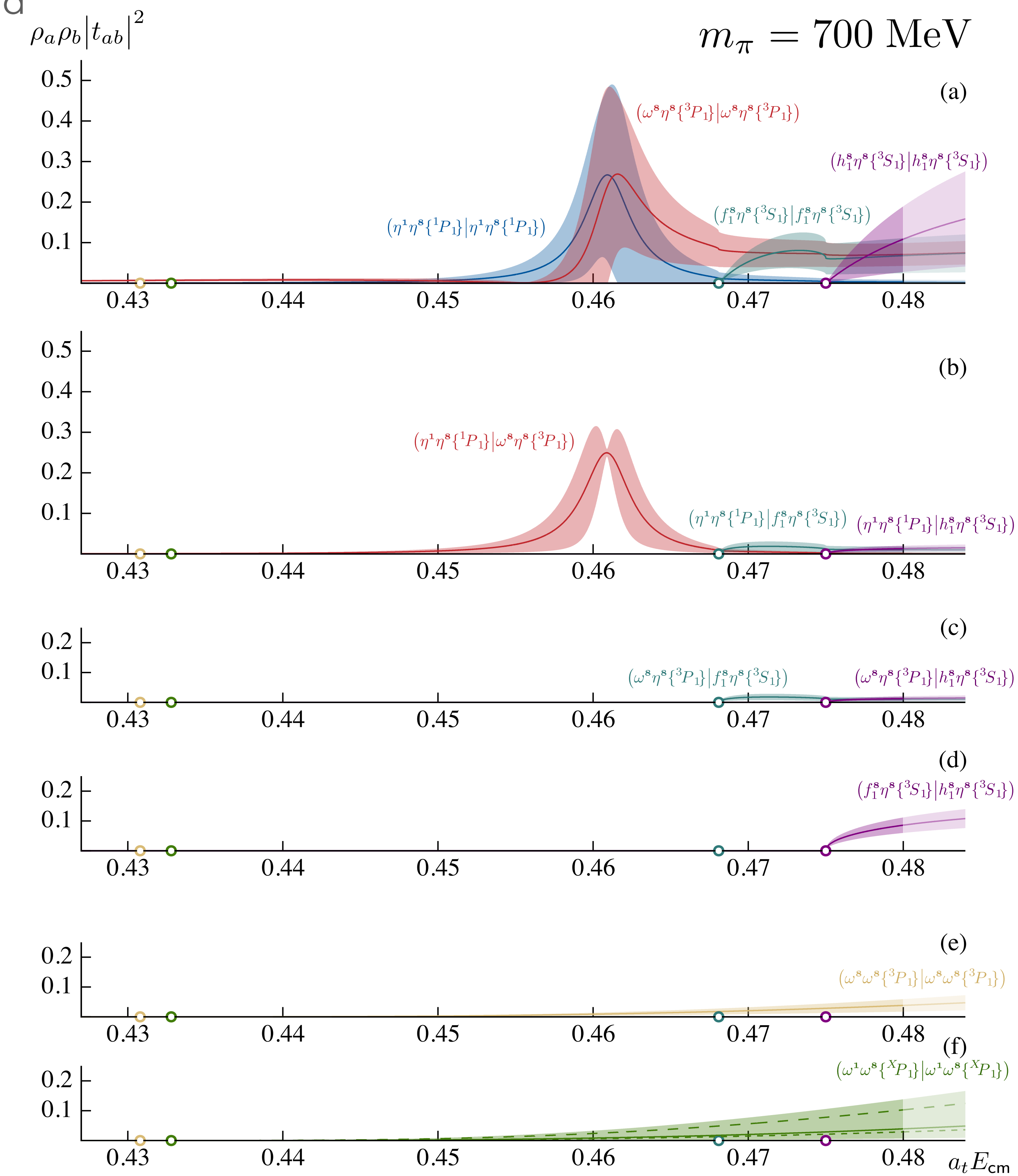
Mass $m = 1229.5 \pm 3.2$ MeV (S = 1.6)
Full width $\Gamma = 142 \pm 9$ MeV (S = 1.2)

| $b_1(1235)$ DECAY MODES | Fraction ($\Gamma_i/\Gamma$) | Confidence level | $p$ (MeV/c) |
|---|---|---|---|
| $\omega\pi$ | dominant | | 348 |
| [$D/S$ amplitude ratio = $0.277 \pm 0.027$] | | | |

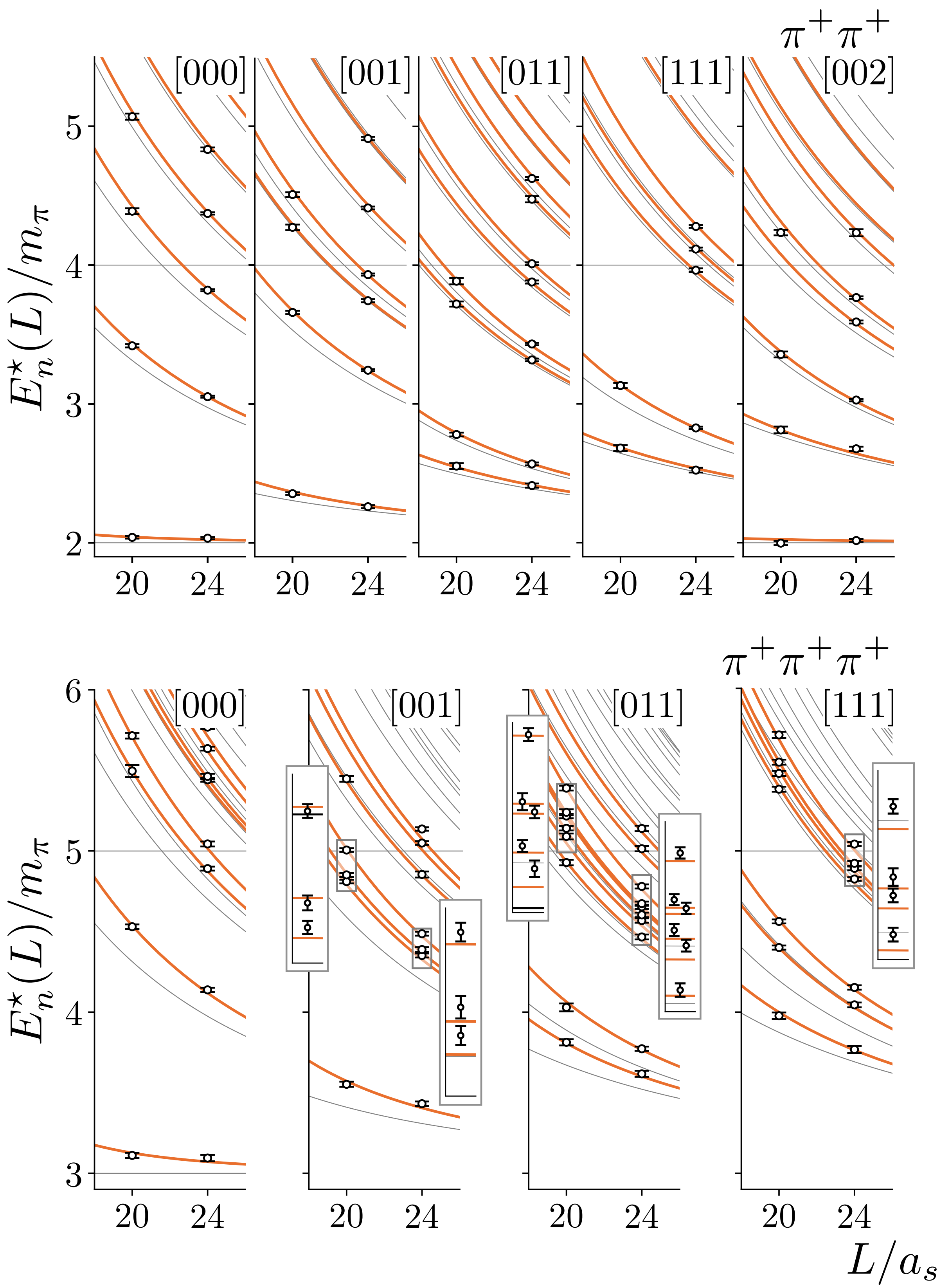
A. J. Woss et al, PRD103 (2021) 5, 054502 arXiv:2009.10034

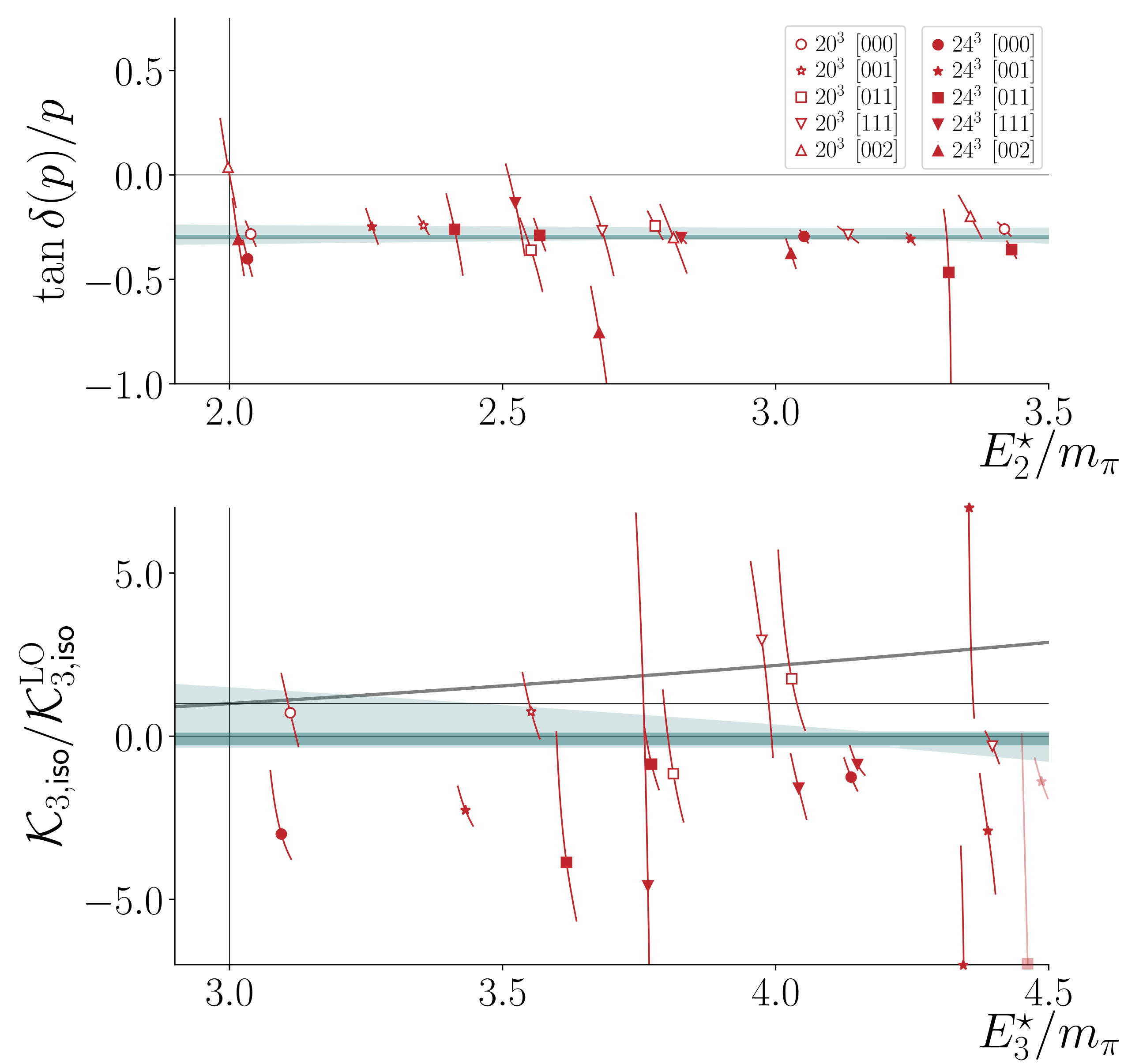
- first computation of the decays of an exotic hybrid



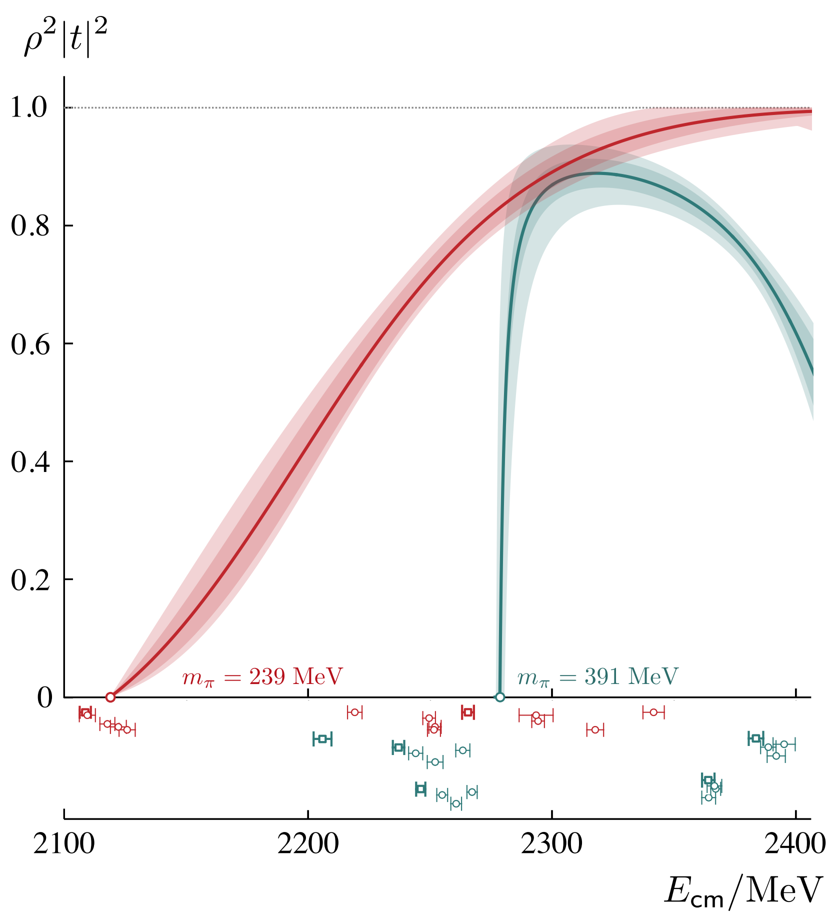



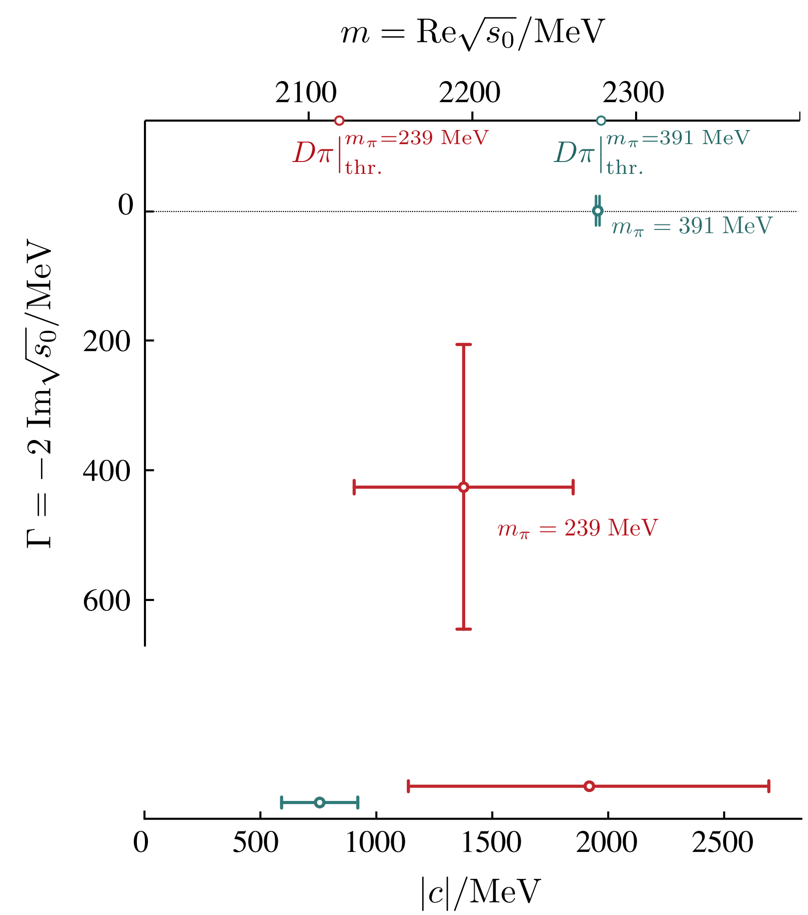
M. T. Hansen et al, PRL 126 (2021) 012001, arXiv:2009.04931



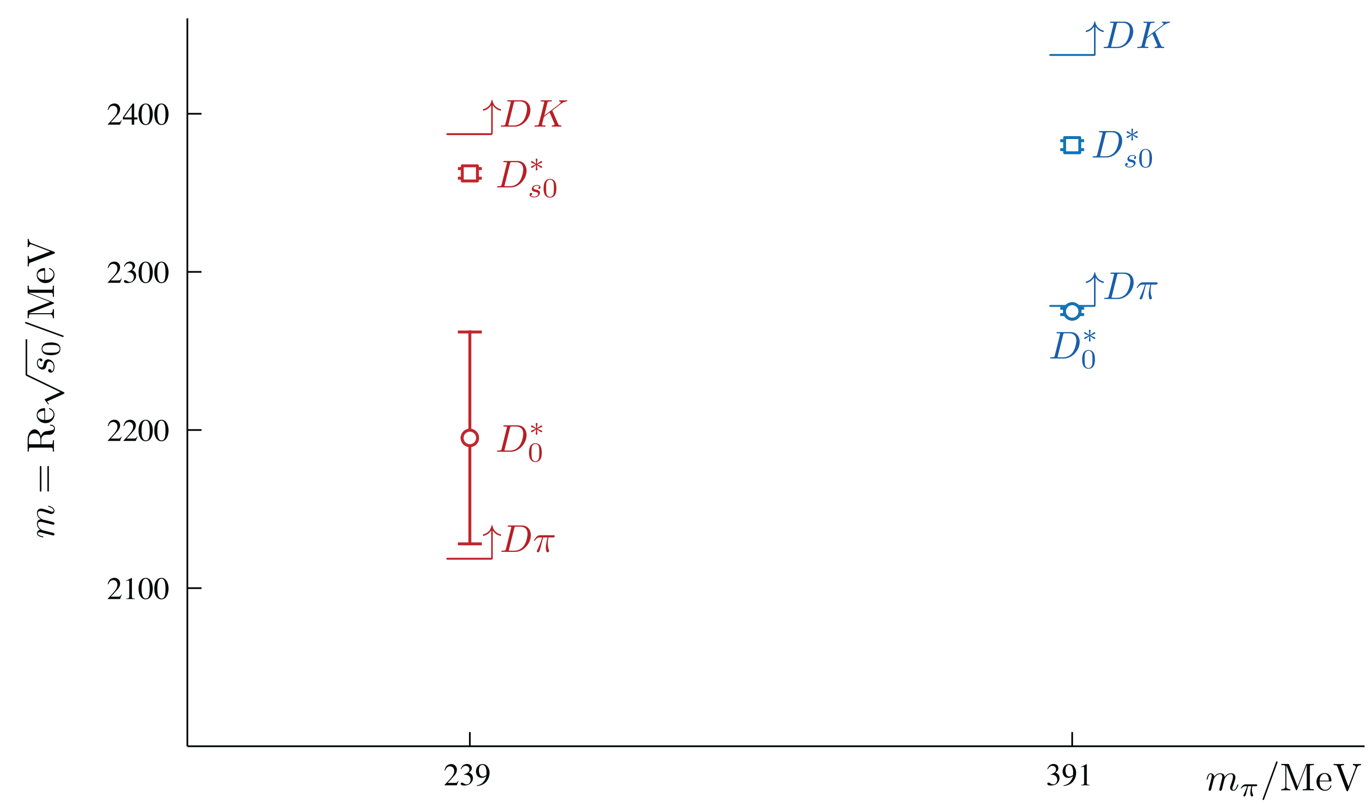



G. Cheung et al, JHEP 02 (2021) 100 arXiv: 2008.06432
L. Gayer, N. Lang et al, arXiv:2102.04973

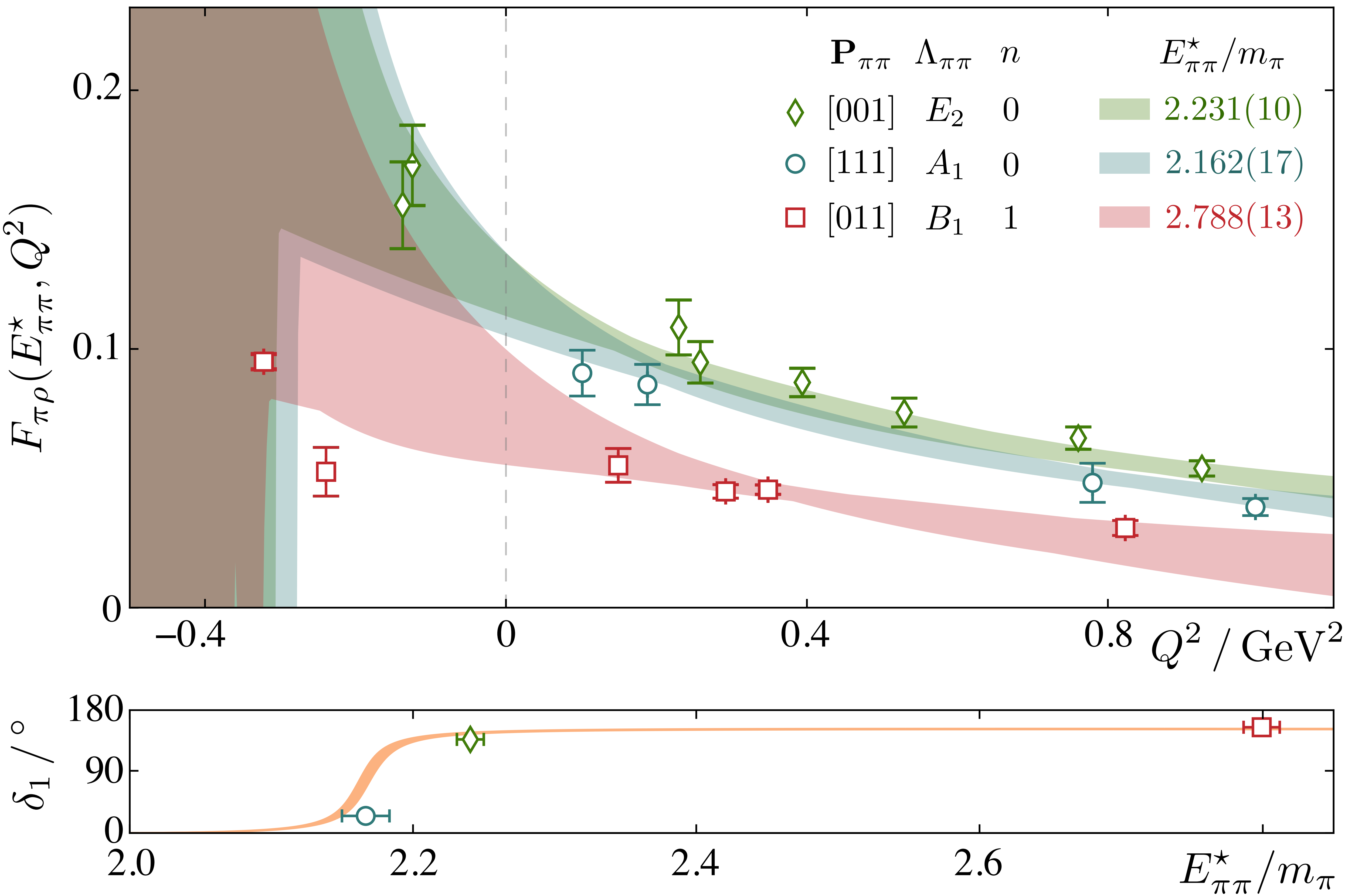


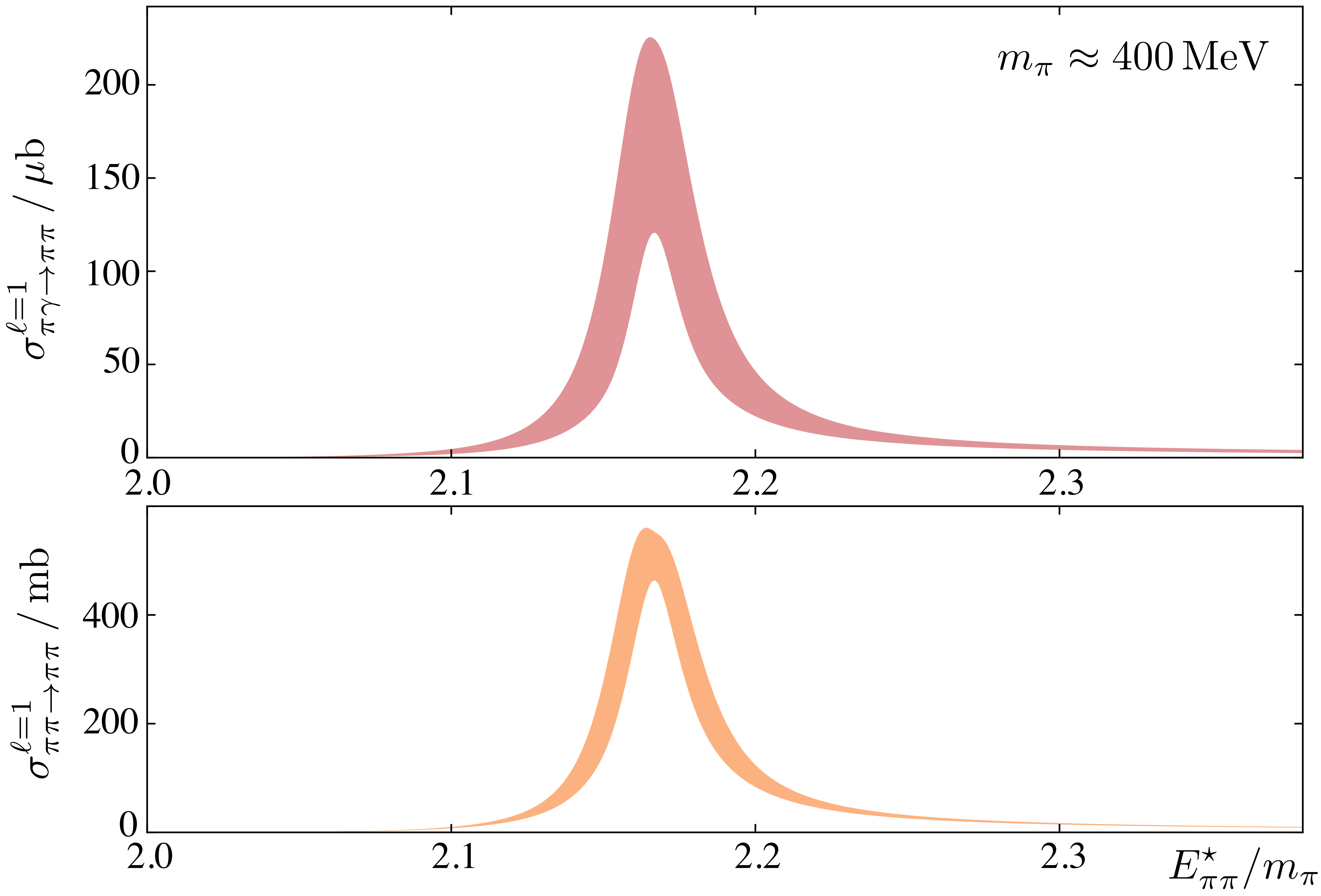

suggestive of a much lighter $D_0^*$ compared with the $D_{s0}^*$



Briceno et al, PRL 115 (2015) 242001

Briceno et al, PRL 115 (2015) 242001

Lattice QCD provides a first-principles tool
to do hadron spectroscopy

These methods are widely applicable
- coupled-channel scattering
- baryons
- charm quarks, beauty quarks
- form factors, radiative transitions (incl. resonances)

…

Control of 3+ body effects needed for
- lighter pion masses
- higher resonances