Charmonium-like hybrids from lattice QCD

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Outline

I. Introduction

II. BS Wave functions and spectrum of charmonium-like hybrids

III. Mass decomposition of charmonium-like states

IV. Summary

I. Introduction

1. Hybrid mesons

As an extension of the constituent quark model, it is conjectured that gluons can be building blocks to make hadrons since they carry color charges.

Glueballs: $gg, \quad ggg, \ldots$

Hybrid mesons: $q_1\bar{q}_2 g$

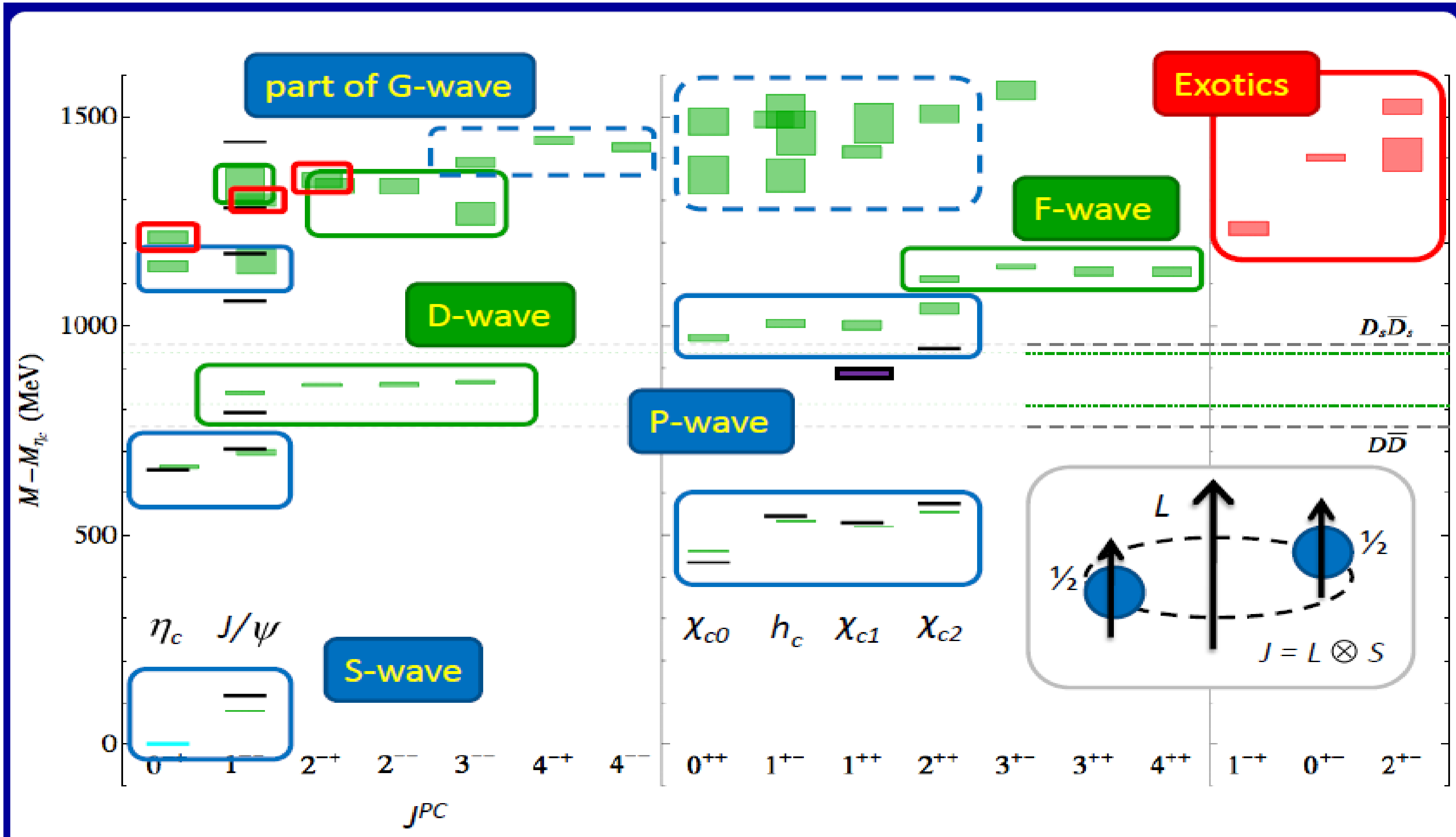
2. Phenomenological descriptions of hybrid mesons

Constituent gluon model
Flux-tube model——Born-Oppenheimer approx.

J. Juge et al., Phys. Rev. Lett. 82, 4400(1999)
E. Braaten et al., Phys. Rev. D 90, 014044 (2014)

Gluons: light and fast degrees of freedom
effect as an flux-tube along the $Q\bar{Q}$ axis

3. Latest lattice QCD results of charmonium



24^3, $M_\pi \approx 400$ MeV

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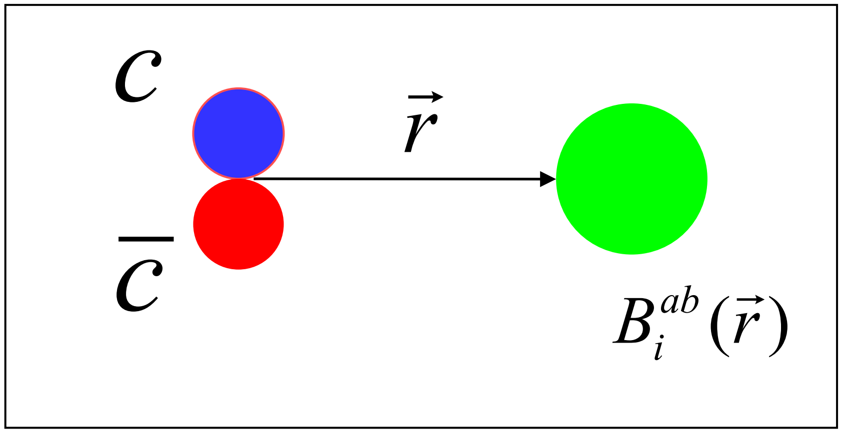
Courtesy to S. Prelovsek

Red blocks $(1^{--}, (0,1,2)^{-+})$ super-multiplet

II. BS wave functions and spectrum of charmonium-like hybrids (Y. Ma et al, arXiv:1910.09819 [hep-lat])

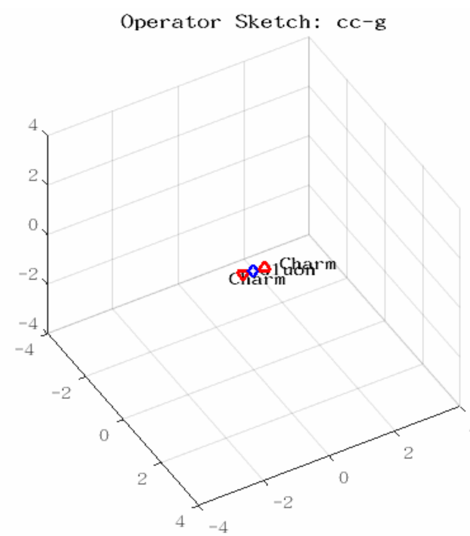
Type I operator: $O(\vec{r}) = (\bar{c}^{a}\gamma_5 c^{b})(0)B_i^{ab}(\vec{r})$ **(Coulomb gauge)**



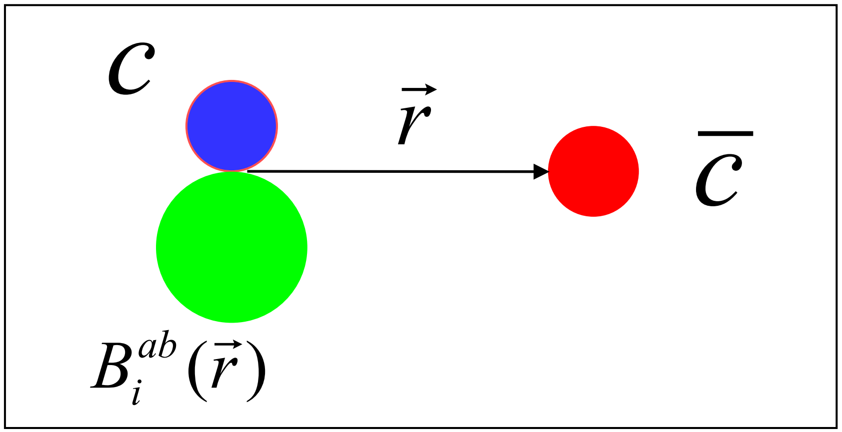
This is equivalent to give a c-cbar center of mass motion, which describes the recoil of the c-cbar against additional degrees of freedom.

Intuitively, the coupling of this kind of operators to conventional charmonia can be suppressed from two aspects:

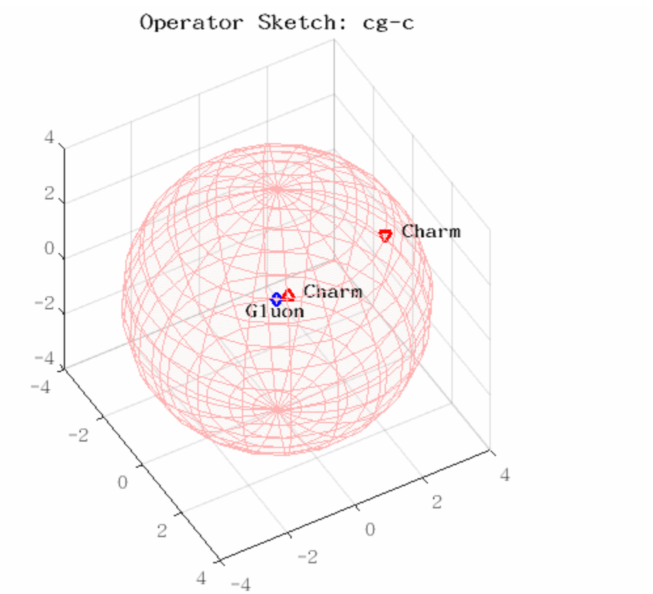
a) **spin states of the c-cbar (spin flipping is suppressed by the heavy quark mass.**
b) **center-of-mass motion (to the leading order of NR, there is no cneter-of-mass motions for conventional charmonia.)**



Type II operator: $\boldsymbol{O}(\vec{\boldsymbol{r}}) = (\bar{c}^a \Gamma B^{ab})(0)c(\vec{r})$ **(Coulomb gauge)**



We use this kind of operator to do a self-consistent check.



Intuitively, the coupling of this kind of operators to conventional charmonia can be suppressed from two aspects:

a) **spin states of the c-cbar (spin flipping is suppressed by the heavy quark mass.**
b) **the block $(\bar{c}^a \Gamma B^{ab})$ can be viewed as a dressed charm anti-quark.**

1. Lattice setup

- **Quenched approximation, anisotropic lattices;**
- **At least, the 1^{-+} hybrids are well defined in QA.**
- **$(1^{--}, (0,1,2)^{-+})$ supermultiplet is focused on**

β	ξ	$a_s(\mathrm{fm})$	$La_s(\mathrm{fm})$	$L^3 \times T$	N_{conf}
2.4	5	0.222(2)	3.55	$16^3 \times 160$	500
2.8	5	0.138(1)	3.31	$24^3 \times 192$	200

2. Numerical strategy

$$C(r,t) = \left\langle O(\vec{r},t) O^{(w)+}(0) \right\rangle = \sum_i W_i(r) e^{-m_i t}$$

Operator prototypes:

$$O_{0_H^{-+}}(t) = \sum_{\mathbf{x}} \bar{c}^a(\mathbf{x},t)\gamma_i c^b(\mathbf{x},t) B_i^{ab}(\mathbf{x},t),$$

$$O^k_{1_H^{-+}}(t) = \sum_{\mathbf{x}} \bar{c}^a(\mathbf{x},t)\gamma_i c^b(\mathbf{x},t) B_j^{ab}(\mathbf{x},t)\epsilon_{ijk},$$

$$O^k_{2_H^{-+}}(t) = \sum_{\mathbf{x}} \bar{c}^a(\mathbf{x},t)\gamma_i c^b(\mathbf{x},t) B_j^{ab}(\mathbf{x},t)|\epsilon_{ijk}|,$$

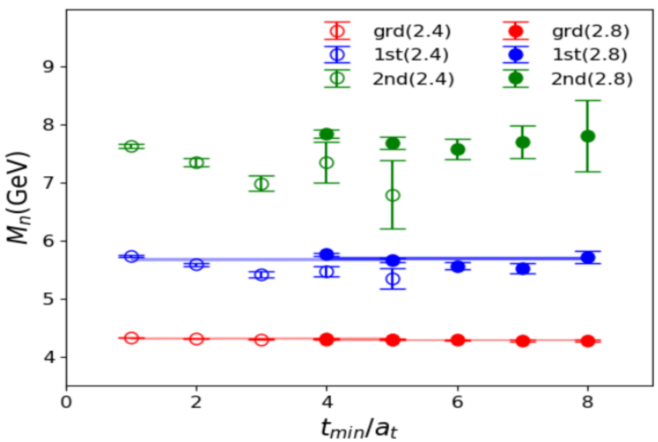
$$O^k_{1_H^{--}}(t) = \sum_{\mathbf{x}} \bar{c}^a(\mathbf{x},t)\gamma_5 c^b(\mathbf{x},t) B_k^{ab}(\mathbf{x},t),$$

$\boldsymbol{\Phi_n(r) = \langle \Omega | O(r) | i \rangle \propto W_i(r)}$: Bethe-Salpeter wave function

In practice, for the type-I and II operators, the masses and the BS functions can be derived simultaneously from a joint fit to $C(r,t)$

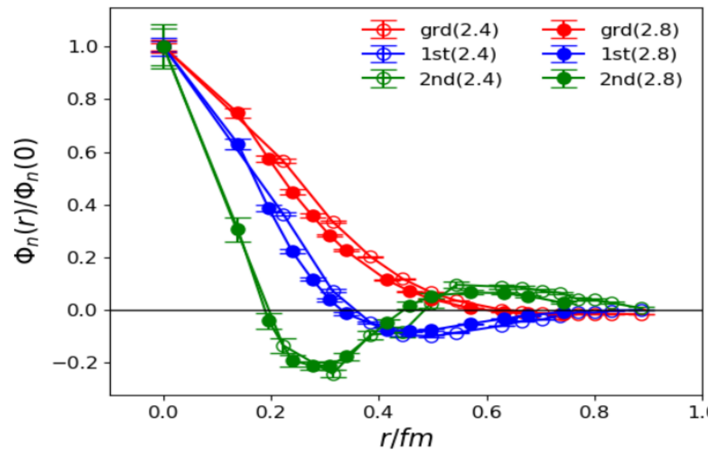
A. The 1^{-+} states

Spectrum

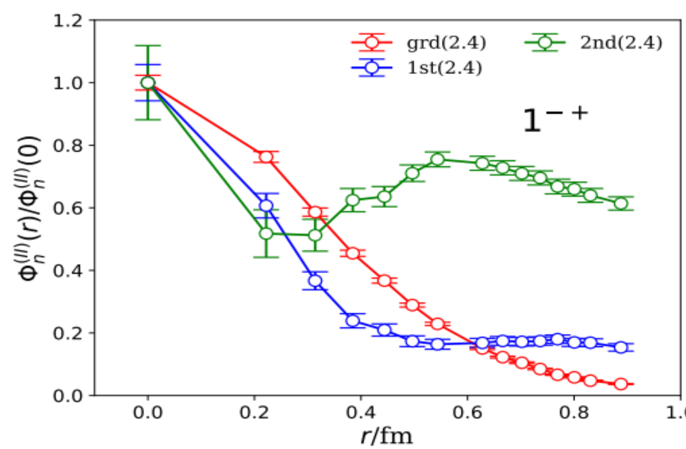


#node	$m(1^{--})$ (GeV)	$m(0^{-+})$ (GeV)	$m(1^{-+})$ (GeV)	$m(2^{-+})$ (GeV)
0	3.109(5)	3.010(4)	-	-
0	3.703(82)	3.672(76)	-	-
0	4.591(69)	4.551(63)	4.309(2)	4.419(3)
1	5.460(31)	5.393(28)	5.693(12)	5.779(12)
2	8.226(99)	8.286(109)	7.661(31)	7.708(29)

BS wave functions from type-I operator $(c\bar{c} - g)$

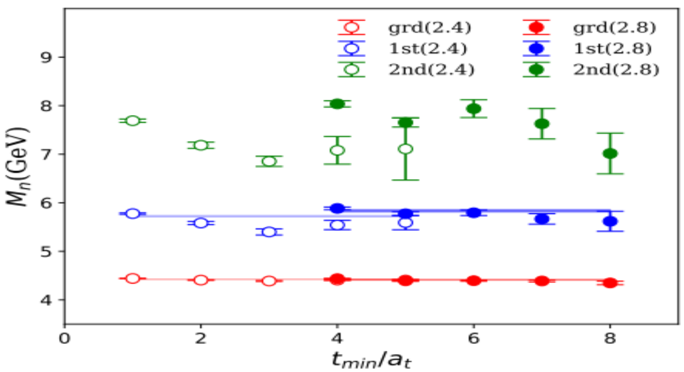


BS wave functions from type-II operator $(\bar{c}g - c)$



B. The 2^{-+} states

Spectrum



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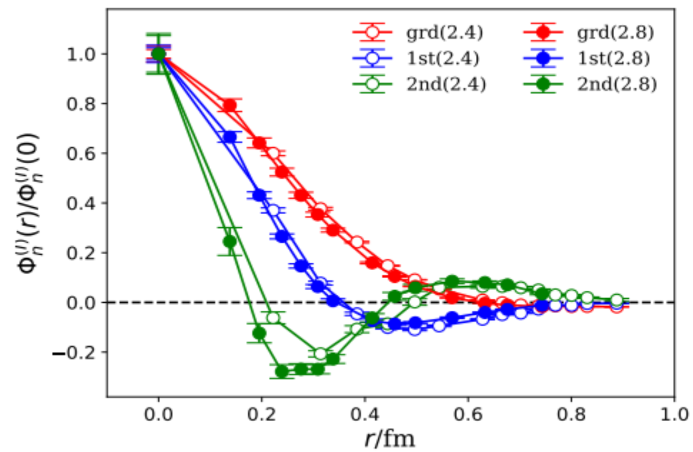
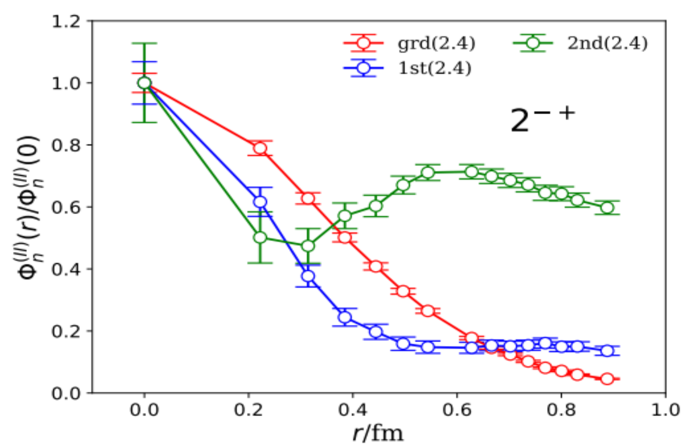
BS wave functions from type-I operator $(c\bar{c}-g)$



BS wave functions from type-II operator $(\bar{c}g-c)$



- 2^{-+} $c\bar{c}$ $(1^1D_2, \eta_{c2})$ mass ~3.83 GeV
- $c\bar{c}g$ operator couples almost exclusively with states higher than 4 GeV.
- The states observed might be members of $\{1^{--}, (0,1,2)^{-+}\}$ supermultiplets.

C. The 0^{-+} states

Spectrum

#node	$m(1^{--})$ (GeV)	$m(0^{-+})$ (GeV)	$m(1^{-+})$ (GeV)	$m(2^{-+})$ (GeV)
0	3.109(5)	3.010(4)	-	-
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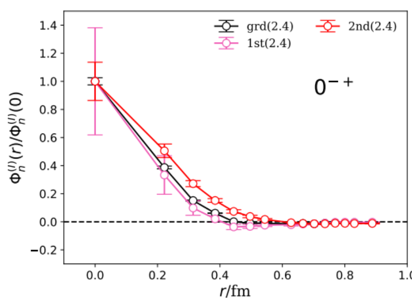
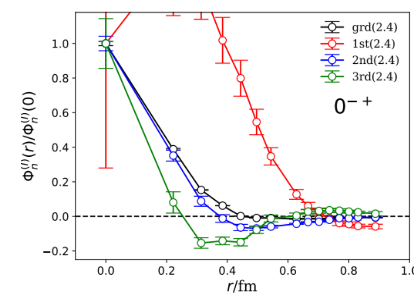
- 0^{-+} is conventional for quarkonium
- Conventional charmonia appear in this channel.
- The lowest two states can be identified to be $\eta_c(1S)$ and $\eta_c(2S)$. This is supported by the masses and w.f. from type-II operators.
- The 4-5 th states have almost degenerate masses and similar w.f. to their $(1,2)^{-+}$ counterparts. They may be members of of $\{1^{--},(0,1,2)^{-+}\}$ supermultiplets.
- These are tentative interpretations.

$$C(r,t)=\sum_i W_i(r)e^{-m_i t}$$

BS w. f. from type-I operator $(c\bar{c}-g)$

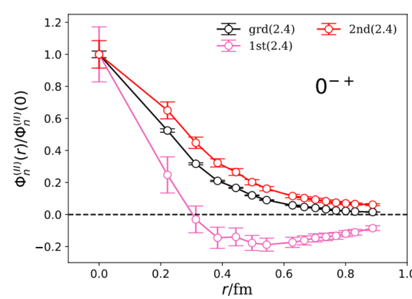


3-mass-term fit

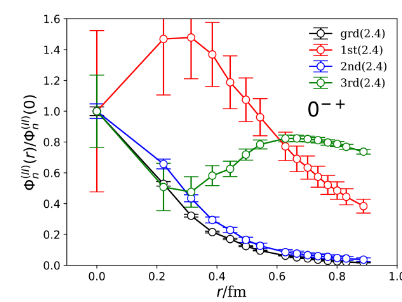


4-mass-term fit

BS w. f. from type-II operator $(\bar{c}g-c)$



3-mass-term fit



4-mass-term fit

B. The 0^{-+} states

$$C(r,t)=\sum_i W_i(r)e^{-m_i t}$$

Spectrum

#node	$m(1^{--})$ (GeV)	$m(0^{-+})$ (GeV)	$m(1^{-+})$ (GeV)	$m(2^{-+})$ (GeV)
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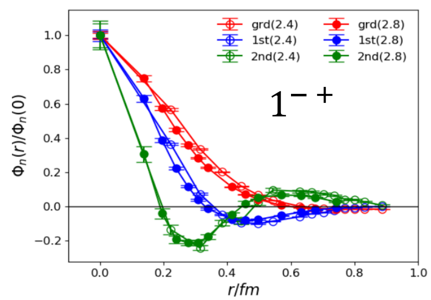
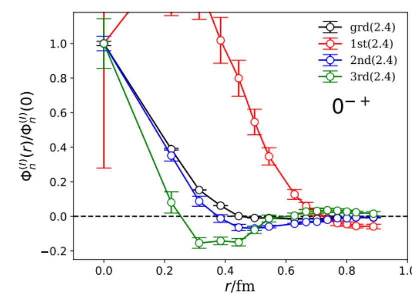
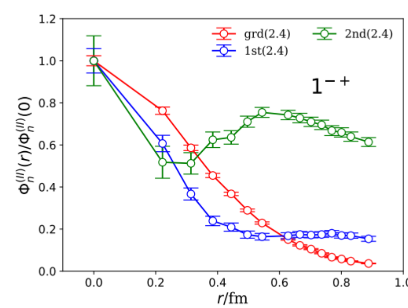
BS w. f. from type-I operator $(c\bar{c}-g)$

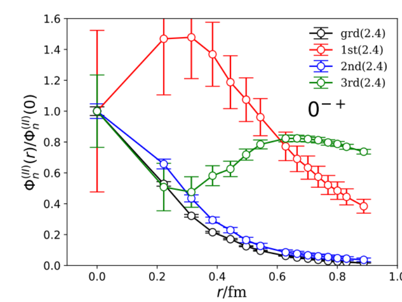




Comp. with 1^{-+} **4-mass-term fit**

BS w. f. from type-II operator $(\bar{c}g-c)$





Comp. with 1^{-+} **4-mass-term fit**

D. The 1^{--} states

Spectrum

#node	$m(1^{--})$ (GeV)	$m(0^{-+})$ (GeV)	$m(1^{-+})$ (GeV)	$m(2^{-+})$ (GeV)
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- 1^{--} is conventional for quarkonium
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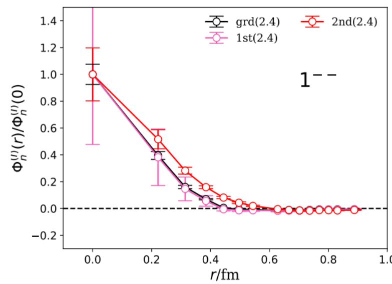
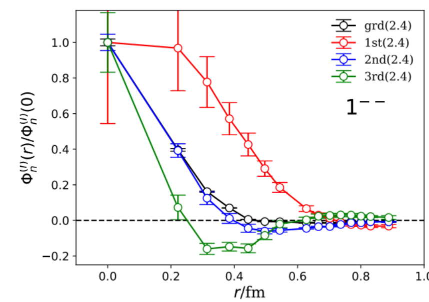
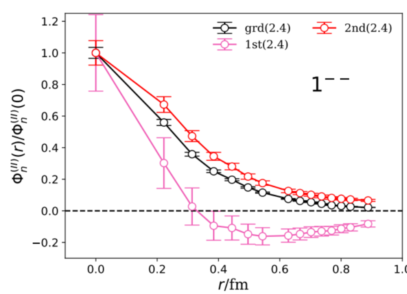


3-mass-term fit

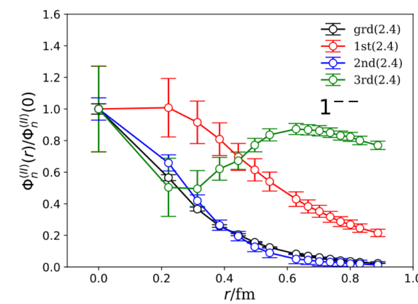


4-mass-term fit

BS w. f. from type-II operator $(\bar{c}g - c)$



3-mass-term fit



4-mass-term fit

D. The 1^{--} states

Spectrum

#node	$m(1^{--})$ (GeV)	$m(0^{-+})$ (GeV)	$m(1^{-+})$ (GeV)	$m(2^{-+})$ (GeV)
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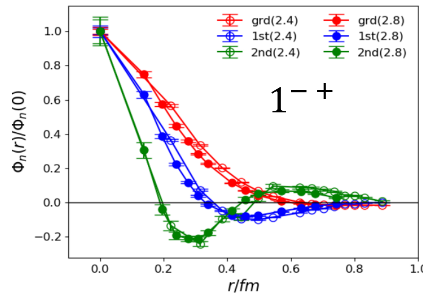
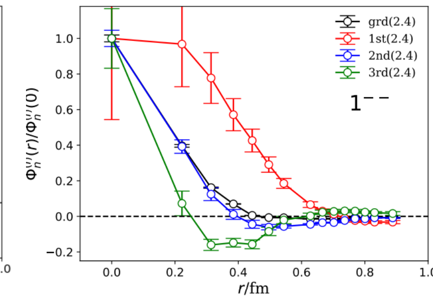
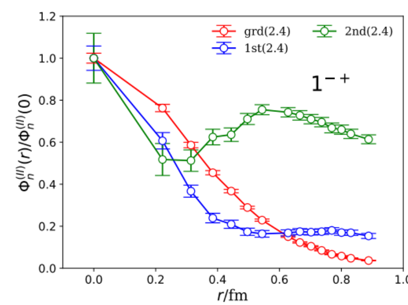
BS w. f. from type-I operator $(c\bar{c}-g)$

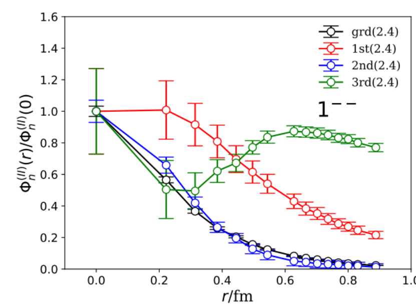




Comp. with 1^{-+} **4-mass-term fit**

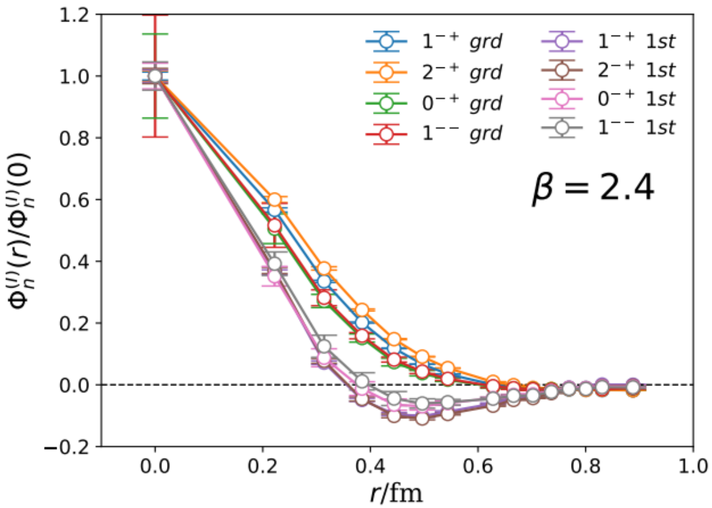
BS w. f. from type-II operator $(\bar{c}g-c)$

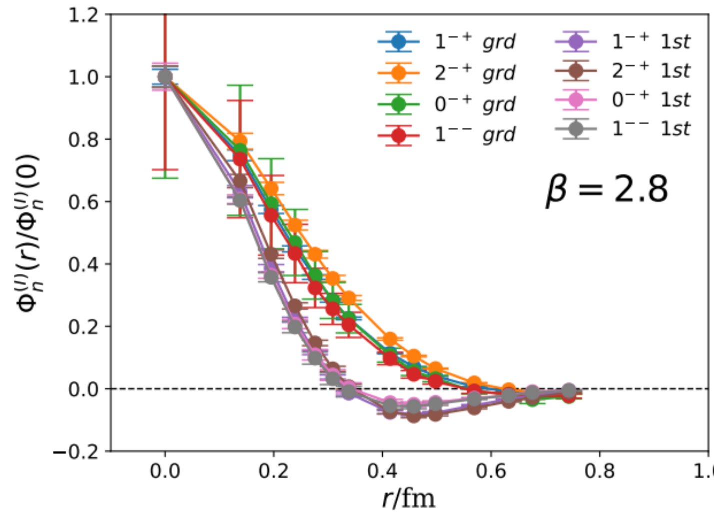




Comp. with 1^{-+} **4-mass-term fit**

3. Discussion





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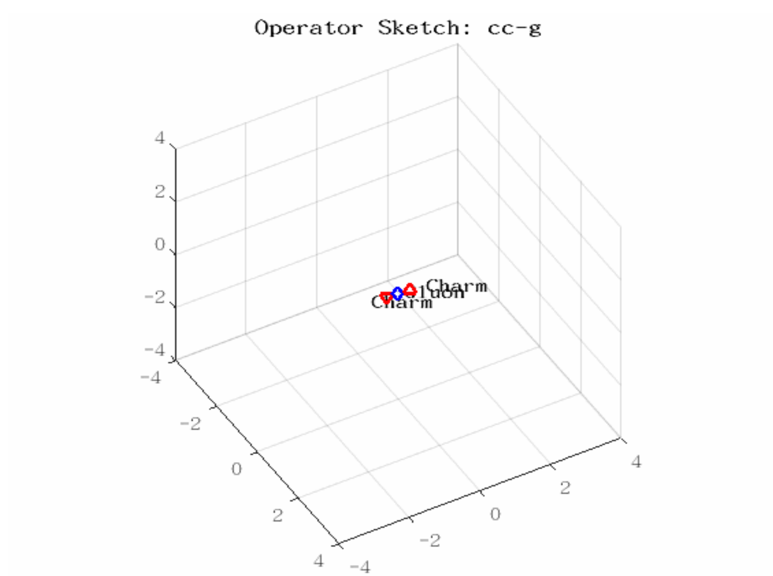


A. Phenomenological picture of the hybrid-like charmonium

"Halo charmonium":

The spatial displacement between the $c\bar{c}$ and the gluonic degree of freedom seems a physically relevant dynamical variable in the description of the internal structure of hybrid-like charmonia.

A relatively localized kernel of color octet $c\bar{c}$ surrounded by a gluonic cloud.

The gluonic cloud can be easily hadronized into light hadrons by emitting or absorbing a soft gluon.

Consequently halo charmonium has large branching ratios to decay into a conventional charmoium by emitting light hadrons.

B. Comparison with the Born-Oppenheimer picture

K. Juge, J. Kuti and C Morningstar, Nucl. Phys. B Proc. Suppl. 63, 326 (1998), Phys. Rev. Lett. 82, 4400 (1999)

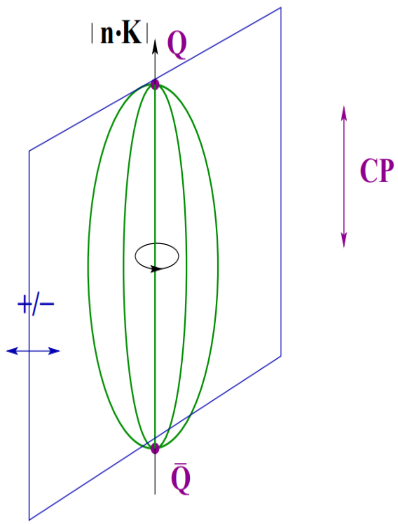


Figure 1: Quarkonium hybrid symmetries.

In analogy with bi-atomic molecule:

Λ_η^ϵ ——cylinder symmetry along the $Q\bar{Q}$ axis

$$|\vec{n}\cdot K| = 0, 1, 2, \cdots \Longrightarrow \Lambda = \Sigma, \Pi, \Delta, \cdots$$

η: CP of the flux tube, $CP = +, \ - \ \ (g, u)$

ϵ: reflection symmetry with respect to the plane including the $Q\bar{Q}$ axis

Spin of the meson : $J = L + S$

$$L = L_{Q\bar{Q}} + K$$

$$L_{Q\bar{Q}}^2 = L^2 - 2\Lambda^2 + K^2 + \cdots$$

PC of the meson : $P = \epsilon(-1)^{L+\Lambda+1}, C = \eta\epsilon(-1)^{L+\Lambda+S}$

Radial Schoedinger Equation : $\left\langle J^2_{light} \right\rangle \approx 2$

$$\boxed{\left[-\frac{1}{m_Q}\left(\frac{d}{dr}\right)^2 + \frac{L(L+1) - 2\Lambda^2 + \langle \boldsymbol{J}^2_{\text{light}}\rangle_{\Gamma,\boldsymbol{r}}}{m_Q r^2} + V_\Gamma(r)\right] rR(r) = E\,rR(r)}$$

quarkonia and hybrids

$\Gamma(nL)$	$S=0$	$S=1$
$\Sigma_g^+(1S)$	0^{-+}	1^{--}
$\Sigma_g^+(1P)$	1^{+-}	$(0,1,2)^{++}$
$\Sigma_g^+(1D)$	2^{-+}	$(1,2,3)^{--}$
$\Pi_u^+(1P)$	1^{--}	$(0,\mathbf{1},2)^{-+}$
$\Pi_u^+(1D)$	2^{++}	$(1,\mathbf{2},3)^{+-}$
$\Pi_u^+(1F)$	3^{--}	$(2,\mathbf{3},4)^{-+}$
$\Pi_u^-(1P)$	1^{++}	$(\mathbf{0},1,\mathbf{2})^{+-}$
$\Pi_u^-(1D)$	2^{--}	$(\mathbf{1},2,\mathbf{3})^{-+}$
$\Pi_u^-(1F)$	3^{++}	$(\mathbf{2},3,\mathbf{4})^{+-}$
$\Sigma_u^-(1S)$	0^{++}	1^{+-}
$\Sigma_u^-(1P)$	1^{--}	$(0,\mathbf{1},2)^{-+}$
$\Sigma_u^-(1D)$	2^{++}	$(1,\mathbf{2},3)^{+-}$

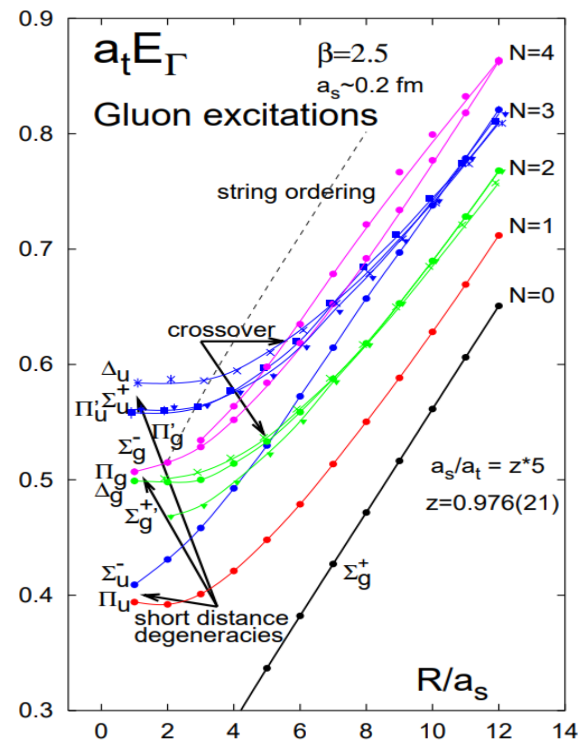
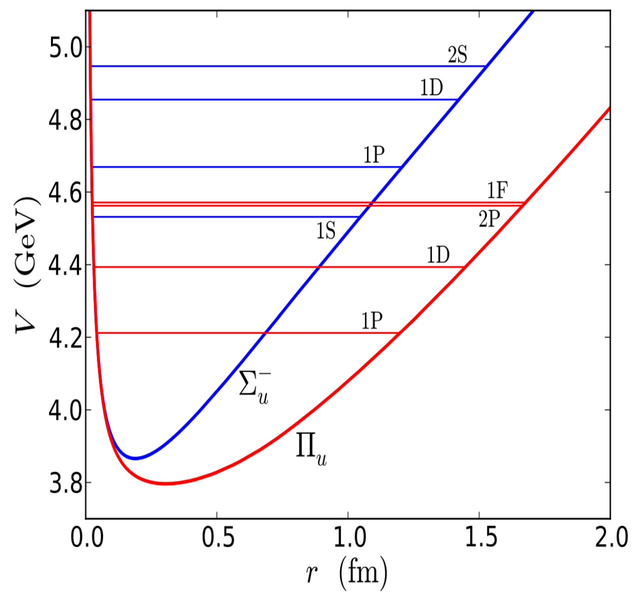
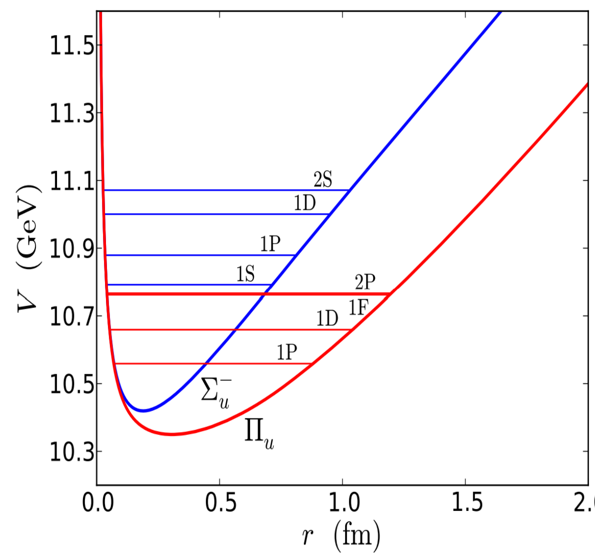


Figure 2: Hybrid static energies, E_Γ, in lattice units, from [14].

Lightest hybrid super-multiplet: $\boldsymbol{L=1, \Lambda=1, \eta=u, \epsilon=+}$

The non-relativistic potential model solutions for heavy quarkonium-like hybrids in BO approximation (E. Braaten et al., PRD 90 (2014)014044)





Spectrum of $c\bar{c}$ hybrids

$\Delta\mathrm{M}(2\mathrm{P}-1\mathrm{P}) \sim 380 MeV$

Spectrum of $b\bar{b}$ hybrids

$\Delta\mathrm{M}(2\mathrm{P}-1\mathrm{P}) \sim 200\ MeV$

In comparison with our results:

$$M\big(\psi_H(2S)-\psi_H(1S)\big) \approx 1200\ MeV \approx 2\ M\big(\psi(2S)-\psi(1S)\big)$$

C. Static potential from lattice QCD

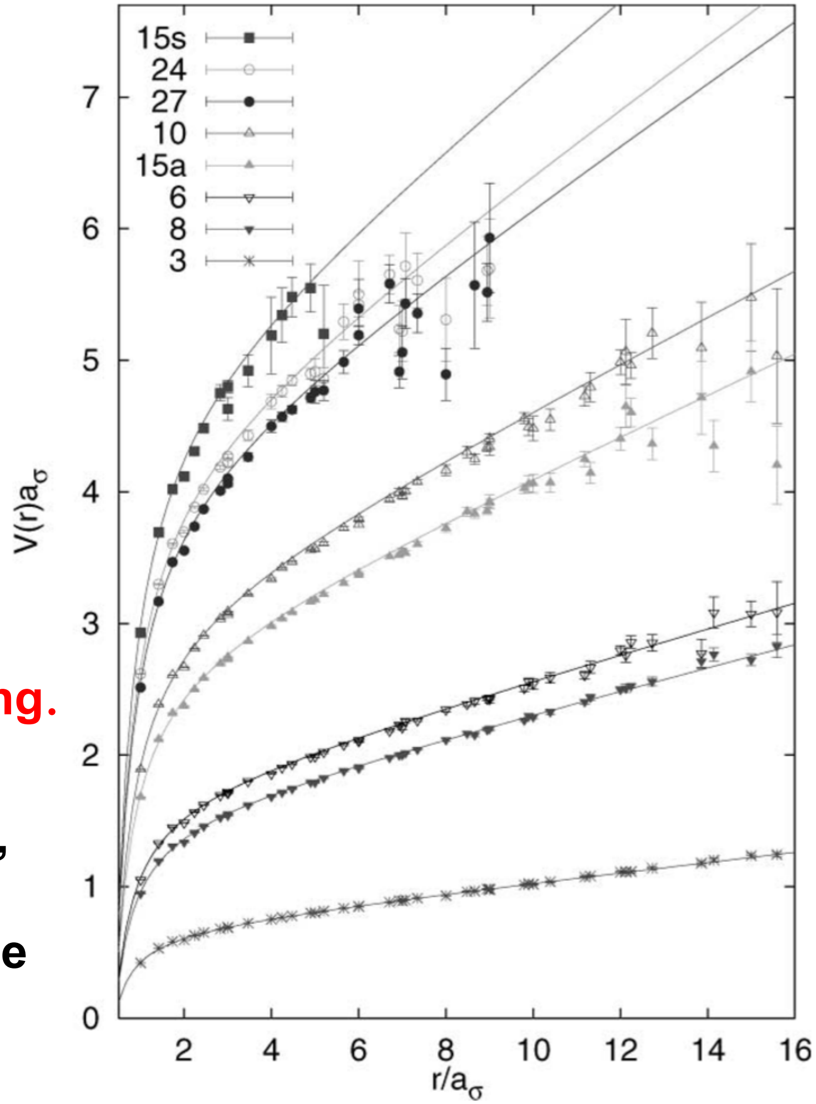
(G. S. Bali, Phys. Rev. D62, 114503 (2000))

- **The color singlet static potential between two color charge in D and D^* representation can be parameterized as**

$$V_D(r) = V_{D,0} - C_D \frac{\alpha_s}{r} + \sigma_D r,$$

where C_D is the Casimir operator of D.

- **In the plot, the filled lines are plotted assuming the relation $\sigma_D = \frac{C_D}{C_F}\sigma$.**
- **This behavior is called Casimir Scaling.**
- **Even in the quenched approximation, the flux tube between adjoint color charge is expected to break easily due to the excitation of two soft gluons.**



If the energy splitting is insensitive to the constituent masses, **for example,**

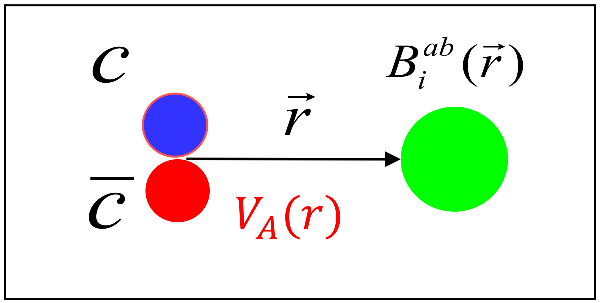
$$M\big(\Upsilon(2S) - \Upsilon(1S)\big) = 563\ MeV$$
$$M\big(\psi(2S) - \psi(1S)\big) = 589\ MeV$$

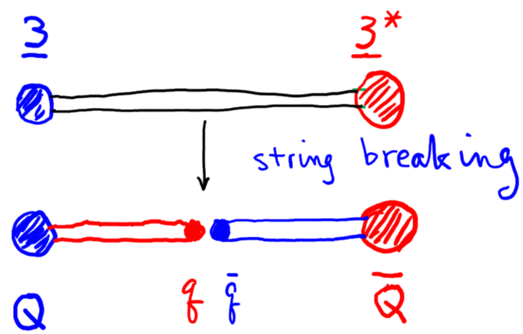
Then the mass splitting is possibly dependent only of the string tension, say, $\boldsymbol{\sigma_D = \frac{C_D}{C_F}\sigma}$**.**

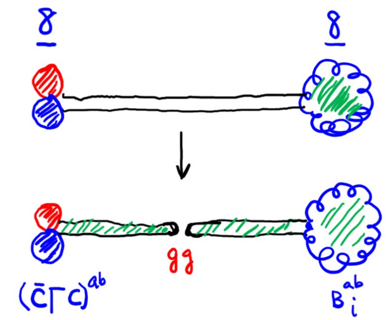
$$M\big(\psi_H(2S) - \psi_H(1S)\big) \approx 1200\ MeV\ \approx 2\ M\big(\psi(2S) - \psi(1S)\big)$$

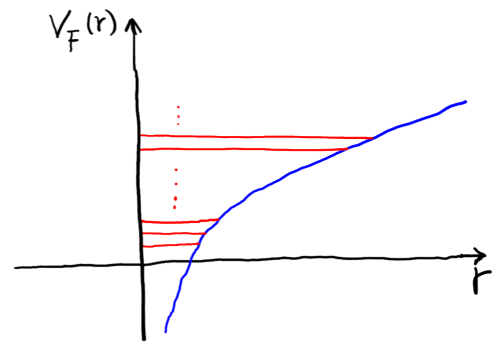
$$\boldsymbol{\sigma_A = \frac{C_A}{C_F}\sigma = 2.25\sigma}$$

D. Implication to the decay of charmonium-like hybrids



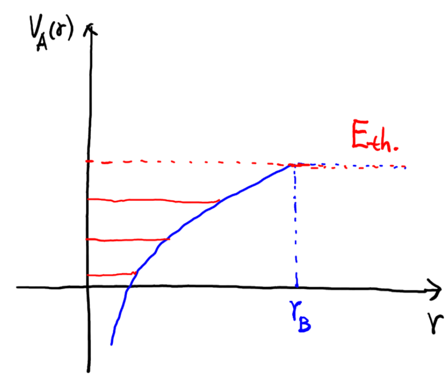




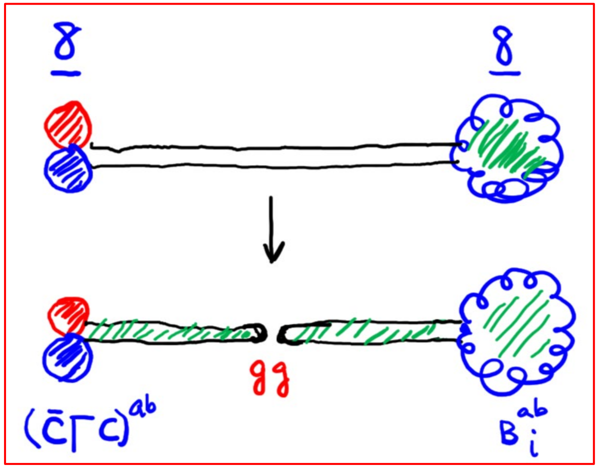


If the excitation of hybrid charmonium can be understood as the bound state energy levels in the adjoint potential $V_A(r)$,

then the number of the energy levels is much less than that of the fundamental potential.



Understanding the Y(4260) decays in the "halo-charmonium" picture



$J/\psi\pi^+\pi^-$ mode:
relative S-wave between J/ψ and $\pi^+\pi^-$

$\chi_{c0}\omega$ mode:
relative S-wave between χ_{c0} and ω

$h_c\pi^+\pi^-$ mode: relative P-wave between h_c and $\pi^+\pi^-$

The $c\bar{c}$ in the halo charmonium is spin singlet (S=0),

$J/\psi\pi^+\pi^-$ mode: J/ψ (S=1), spin flipping, m_c suppressed,
no centrifugal barrier

$\chi_{c0}\omega$ mode: χ_{c0} (S=1), spin flipping, m_c suppressed,
no refugal barrier

$h_c\pi^+\pi^-$ mode: h_c (S=0), no spin flipping,
but suppressed by the centrifugal barrier .

In this picture, it is understandable that the above three modes have similar cross section at $\sqrt{s} \sim 4.22\ GeV$

Possible decay modes of is charmonium-like $(0,1,2)^{-+}$ hybrids,

$\chi_{cJ}\eta(\eta')$ mode: χ_{cJ} (S=1), no spin flipping, no centrifugal barrier

$J/\psi\,\omega(\phi)$ mode: J/ψ (S=1), no spin flipping, $P-$ wave centrifugal barrier

These decay modes can be searched by LHCb and Belle II.

III. Mass decomposition of charmonium-like states

W. Sun et al. (χQCD Collab., PRD 103 (2021)094503

- **Mass structure:** Another aspect to explore the difference of internal structure of conventional charmonia and exotic charmonia

- Conventional charmonia involved in this study

  $1S$ states: η_c , J/ψ

  $1P$ states: χ_{cJ}, h_c $1D$ state: η_{c2}

- Charmonium-like state with exotic quantum number

  $J^{PC} = 1^{-+}$: $\bar{c}c - g$ or $\bar{c}c - \bar{q}_1 q_2$

c) Lattice setup: $N_f = 2 + 1$ configurations generated by RBC/UKQCD Collaboration

ensemble	$L^3 \times T$	a (fm)	m_π (MeV)	$N_{\rm cfg}$
32I	$32^3 \times 64$	0.0828(3)	300	200
48IF	$48^3 \times 96$	0.0711(3)	278	100

1. Theoretical framework

- **Hadron mass components can be investigated through the energy-momentum tensor of QCD (X. Ji, Phys. Rev. Lett. 74, 1071 (1995)).**

- **Euclidean space:** $T_{\mu\nu} = \frac{1}{4}\bar{\psi}\gamma_{(\mu}\overleftrightarrow{D}_{\nu)}\psi + F_{\mu\alpha}F_{\nu}^{\alpha} - \frac{1}{4}\delta_{\mu\nu}F^2$

$$H_{QCD} \equiv -\int d^3\vec{x}\, T_{44}(\vec{x}) = H_q + H_g + \frac{1}{4}\left(H_g^a + H_m^\gamma\right)$$

$$= H_m + H_E + H_g + \frac{1}{4}H_a$$

$$H_q = -\sum_{u,d,s,\cdots}\int d^3\vec{x}\;\bar{\psi}(D_4\gamma_4)\psi = \sum_{u,d,s,\cdots}\int d^3\vec{x}\;\left(\bar{\psi}\left(\vec{D}\cdot\gamma\right) + m\right)\psi$$

$$H_m = \sum_{u,d,s,\cdots}\int d^3\vec{x}\; m\bar{\psi}\psi \qquad H_E = \sum_{u,d,s,\cdots}\int d^3\vec{x}\;\bar{\psi}\left(\vec{D}\cdot\gamma\right)\psi$$

$$H_g = \int d^3\vec{x}\frac{1}{2}\left(B^2 - E^2\right) \qquad H_a = H_g^a + H_m^\gamma$$

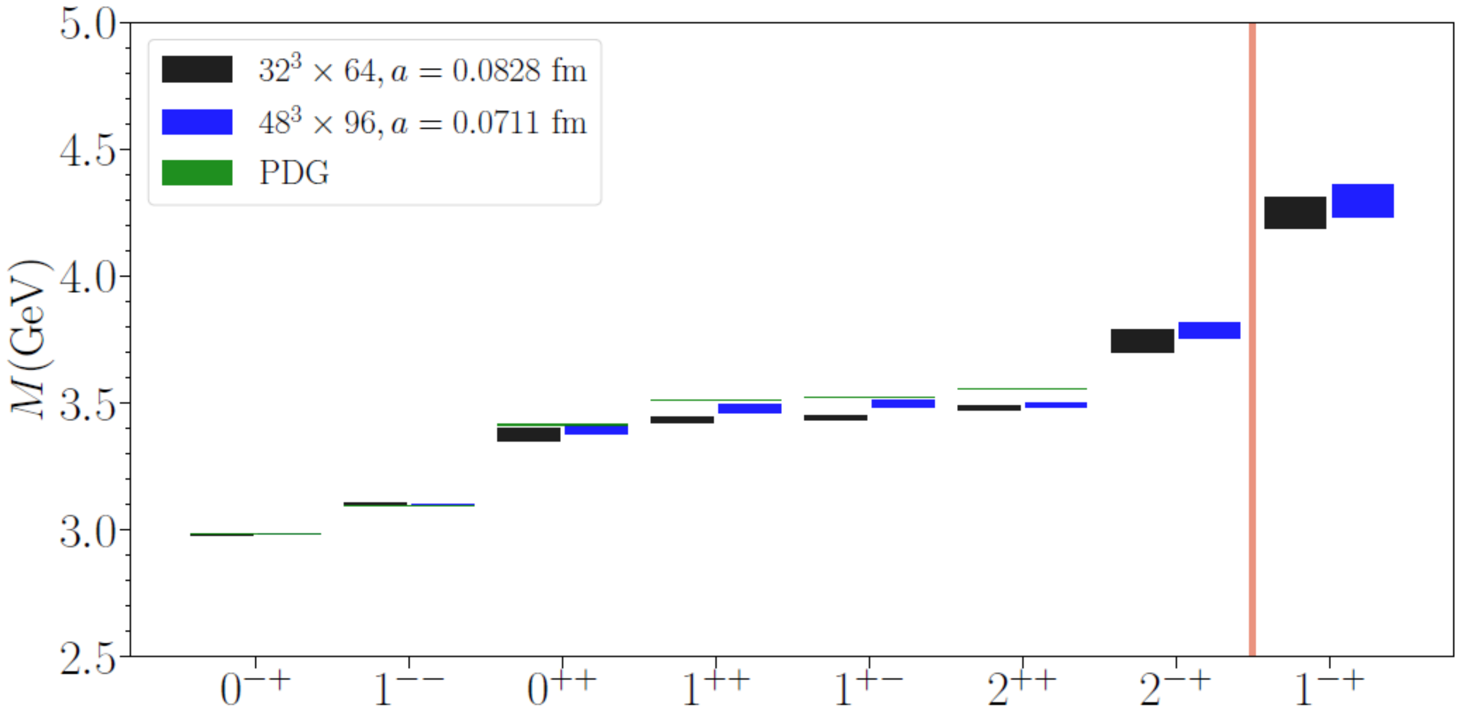
$$H_g^a = -\int d^3\vec{x}\frac{\beta(g)}{g}\left(E^2 + B^2\right) \qquad H_m^\gamma = \sum_{u,d,s,\cdots}\int d^x\vec{x}\;\gamma_m m\bar{\psi}\psi$$

- **Hadron mass:** $$M = \frac{\left\langle P \middle| -\int d^3\vec{x}\; T_{44}(0,\vec{x}) \middle| P \right\rangle}{\langle P|P\rangle} \equiv \langle T^{00}\rangle$$ $$\boxed{P = (M, \vec{0})}$$

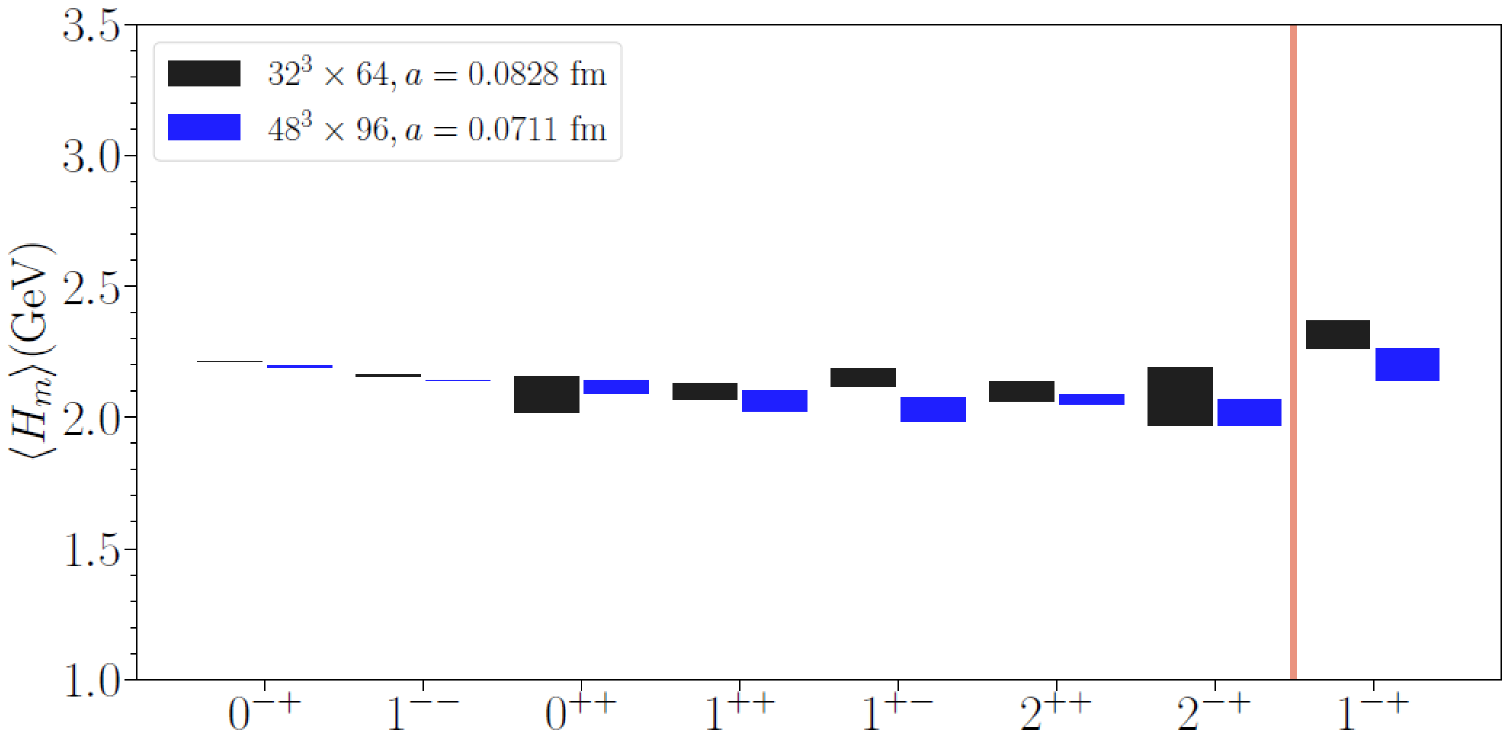
- **Charm quark momentum fraction of charmonium(-like) states :**

$$\langle x\rangle_c = \left(\frac{4}{3}\langle H|\bar{T}^c_{44}|H\rangle\right) / (M_H\langle H|H\rangle)$$
$$\bar{T}^c_{44} = \frac{1}{2}\int d^3\vec{x}\;\bar{c}(x)\left(\gamma_4\overleftrightarrow{D}_4 - \frac{1}{4}\gamma_\mu\overleftrightarrow{D}_\mu\right)c(x)$$

- **Mass spectrum (from two-point functions)**



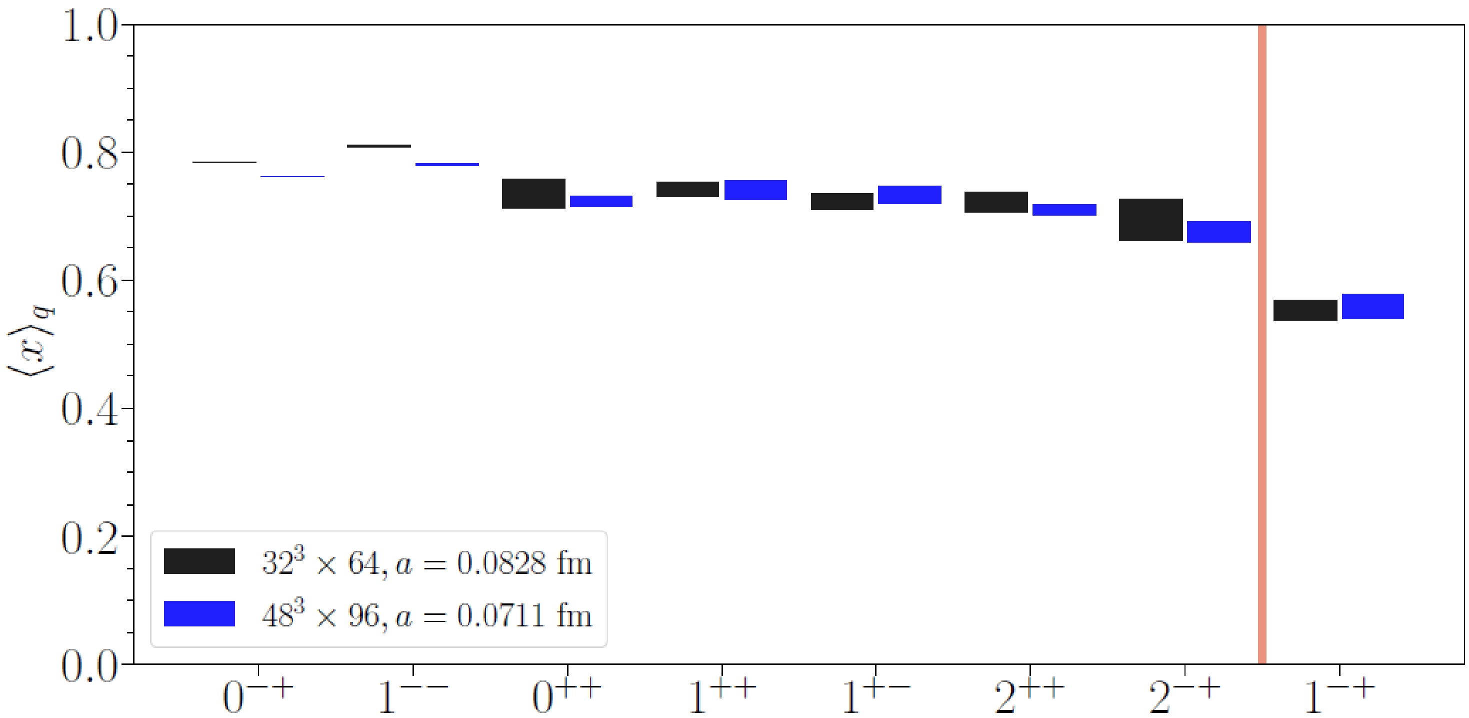
- **Quark mass contribution to charmonium(-like) states $\langle H_m \rangle$;**



For all these states,

$$\langle H_m \rangle \approx 2\left(\bar{m}_c(3GeV) \sim \bar{m}_c(\bar{m}_c)\right)$$

- **Charm quark momentum fraction $\langle x \rangle_c^R$**



- $\langle x \rangle_c^R$ is **almost the same for conventional charmonia**;
- $\langle x \rangle_c^R$ of 1^{-+} charmonium-like state is obviously **smaller** .
- **Additional degrees of freedom in the 1^{-+} state?**

  gluons or light quarks?

V. Summary

- The internal structure of hybrid charmonia can be reflected by the Bethe-Salpeter wave function with respect to the spatial displacement between the gluonic degree of freedom and the $c\bar{c}$

- "Halo-charmonium" picture for charmonium-like hybrids

- Lattice results do not support the Born-Oppenheimer flux-tube model for heavy quarkonium-like hybrids.

- Present results are obtained from Quenched LQCD. Full-QCD lattice studies are desired and decay properties should be considered.

- The mass structure of charmonium-like states are investigated by 2+1 full lattice QCD. The charm quark momentum fraction of the 1^{-+} charmonium-like state is obviously smaller than that of conventional charmonia.

Thanks!