EHM and the Meson Spectrum

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- Why is its mass emergent?

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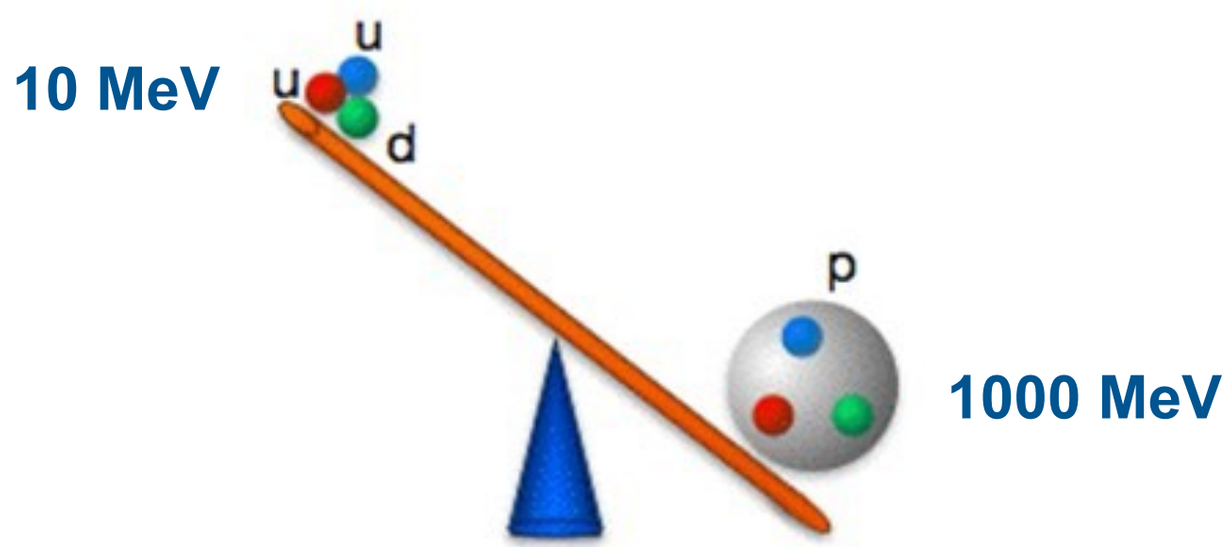
In particle physics, a **hadron** /ˈhædrɒn/ (Greek: ἁδρός, *hadrós;* "stout, thick") is a subatomic composite particle made of two or more quarks held together by the strong force in a similar way as molecules are held together by the electromagnetic force.

In philosophy, systems theory, science, and art, **emergence** occurs when an entity is observed to have properties its parts do not have on their own, properties or behaviors which emerge only when the parts interact in a wider whole.

Massive hadrons are made of (almost) massless quarks!



Can we have an intuitive and simple answer?

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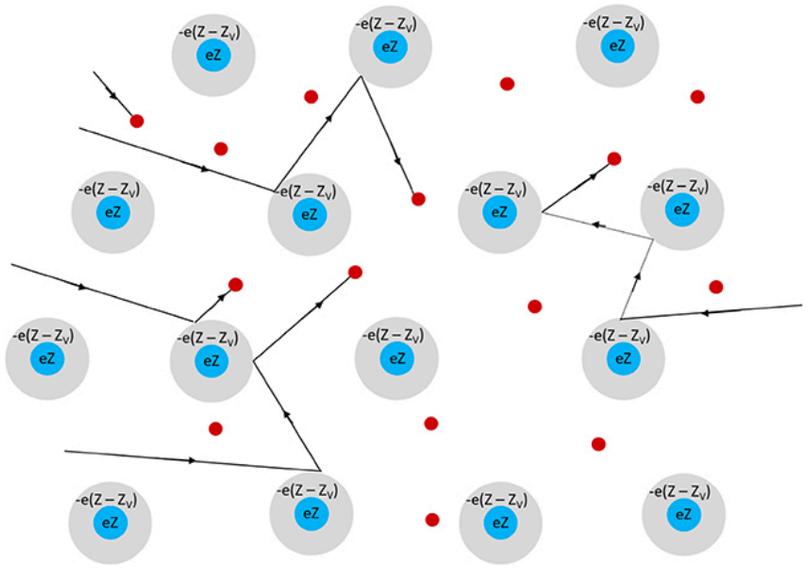
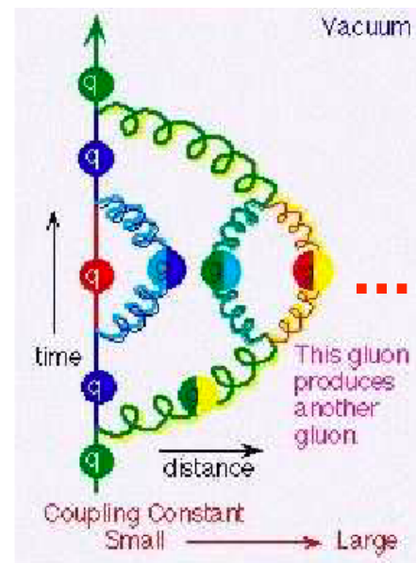
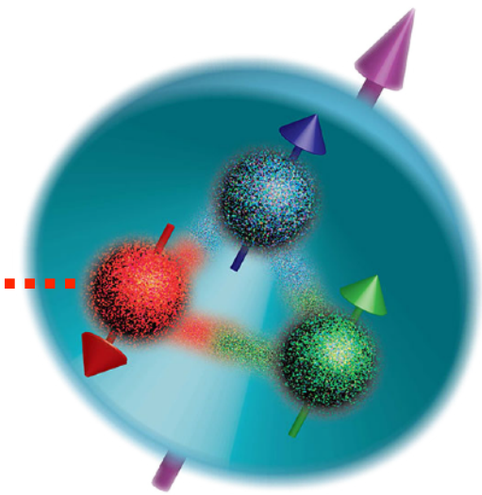






Electrons in solid: $m \rightarrow m^*$

Bound-states of quasi-quarks

EHM Trilogy

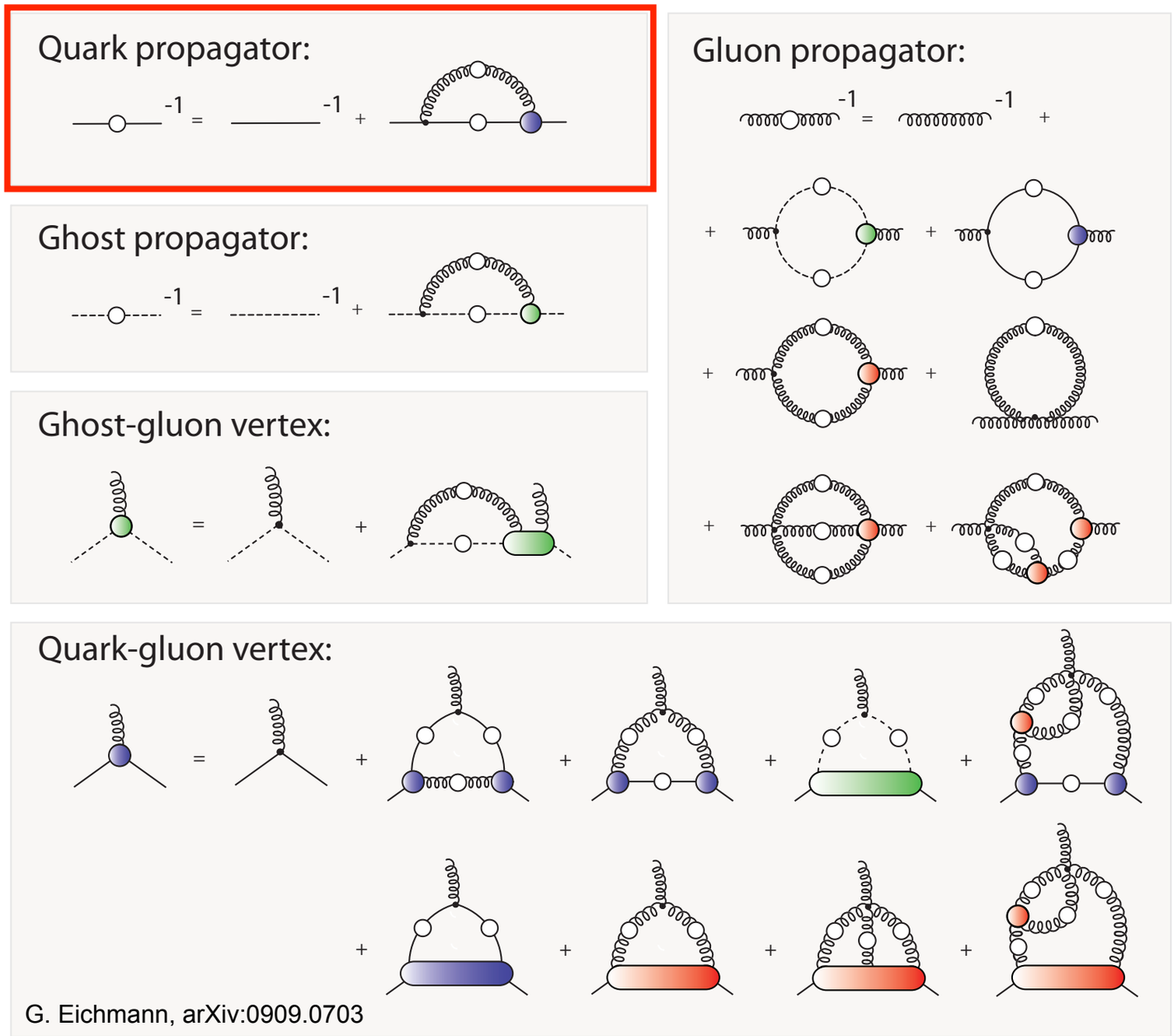
- How do quasi-quarks look like?
- How are quasi-quarks bounded?
- How is the spectrum produced?

DSE: Non-perturbative QCD Solver

Principle of Least Action

$$\left\langle \frac{\delta S[\phi(x)]}{\delta \phi(x)} \right\rangle = 0$$

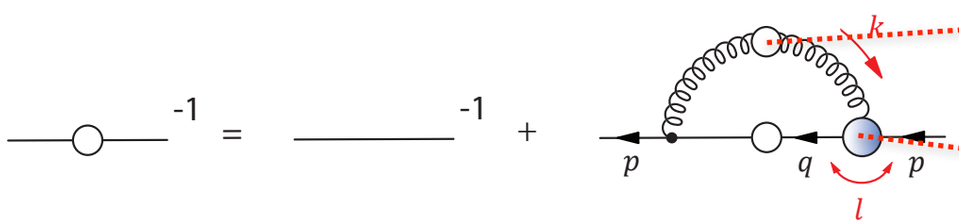
Dyson-Schwinger Equations



G. Eichmann, arXiv:0909.0703

1. How do quasi-quarks look like?

Quark gap equation

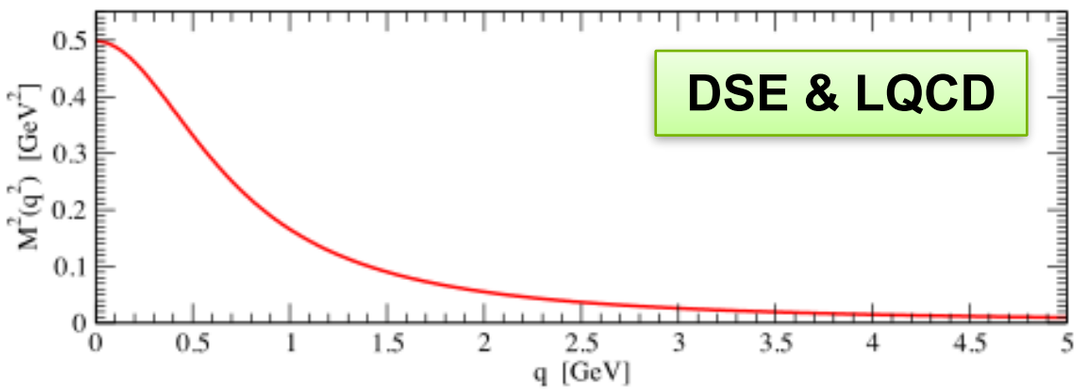


Gluon propagator

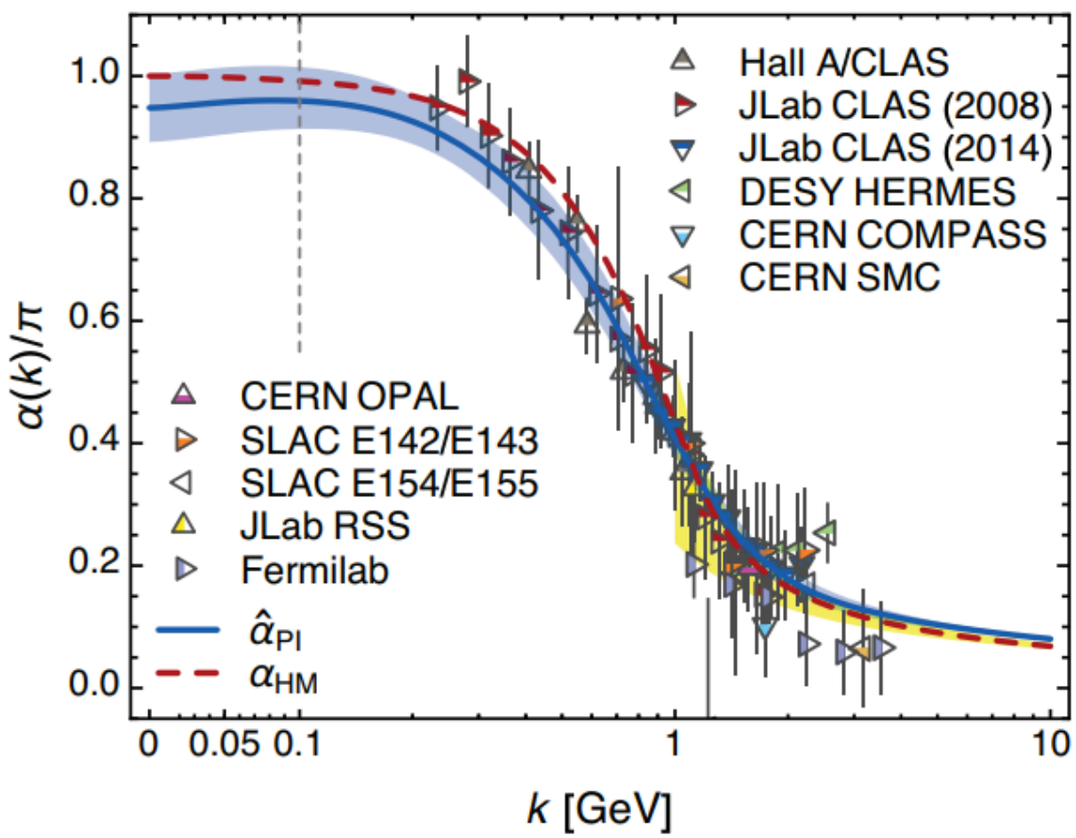
Quark-gluon vertex

1. Quasi-quark: Dynamically massive gluon

Gluon mass function: O. Oliveira et. al., J.Phys. G38, 045003 (2011)



Running coupling: Binosi, Mezrag, Papavassiliou, Roberts and Rodriguez-Quintero



The interaction can be decomposed:
gluon running mass + ***effective running coupling***

$$g^2 D_{\mu\nu}(k) = \mathcal{G}(k^2)\left(\delta_{\mu\nu} - \frac{k_\mu k_\nu}{k^2}\right)$$

$$\mathcal{G}(k^2) \approx \frac{4\pi\alpha_{RL}(k^2)}{k^2 + m_g^2(k^2)}$$

1. The dressed gluon can be well parameterized by a **mass scale**

$$m_g^2(k^2) = \frac{M_g^4}{M_g^2 + k^2}$$

2. The effective running coupling **saturates** in the infrared limit.

See, e.g., PRC 84. 042202 (2011)

1. Quasi-quarks: DCSB feedback

"Symmetry dictates interaction."

Spacetime

- ❑ **Poincaré symmetry**

Fields

- ❑ **Gauge symmetry**
- ❑ **Chiral symmetry**

❑ **Gauge symmetry:** Longitudinal WGTI

$$iq_\mu \Gamma_\mu(k,q) = S^{-1}(k) - S^{-1}(p)$$

❑ **Lorentz symmetry + :** Transverse WGTIs

$$\begin{aligned} q_\mu \Gamma_\nu(k,p) - q_\nu \Gamma_\mu(k,p) &= S^{-1}(p)\sigma_{\mu\nu} + \sigma_{\mu\nu} S^{-1}(k) \\ &\quad + 2im\Gamma_{\mu\nu}(k,p) + t_\lambda \epsilon_{\lambda\mu\nu\rho}\Gamma^A_\rho(k,p) \\ &\quad + A^V_{\mu\nu}(k,p)\,, \end{aligned}$$

$$\begin{aligned} q_\mu \Gamma^A_\nu(k,p) - q_\nu \Gamma^A_\mu(k,p) &= S^{-1}(p)\sigma^5_{\mu\nu} - \sigma^5_{\mu\nu} S^{-1}(k) \\ &\quad + t_\lambda \epsilon_{\lambda\mu\nu\rho}\Gamma_\rho(k,p) \\ &\quad + V^A_{\mu\nu}(k,p)\,, \qquad \sigma^5_{\mu\nu} = \sigma_{\mu\nu}\gamma_5 \end{aligned}$$

✦ The WGTIs express the curls and divergences of the vertices. $\nabla \cdot \Phi$ $\nabla \times \Phi$

✦ The WGTIs of the vertices can be decoupled and (partially) solved.

See, e.g., PLB722, 384 (2013)

1. Quasi-quarks: DCSB feedback

1. The **appearance** of the vertex is dramatically modified by the **dynamics**.

2. There is a dynamic chiral symmetry breaking (**DCSB**) **feedback**:

Gap equation (EoM)

$$S = \frac{1}{i\gamma \cdot pA(p^2) \; + \; B(p^2)}$$

$$\gamma^\mu \Delta_A \; + \; i\sigma_{\mu\nu} Q^\nu \Delta_B \; + \; \text{etc.} \sim \Gamma^\mu$$

WGTIs

The **Dirac** and **Pauli** terms: for an on-shell fermion, the vertex can be decomposed by two form factors:

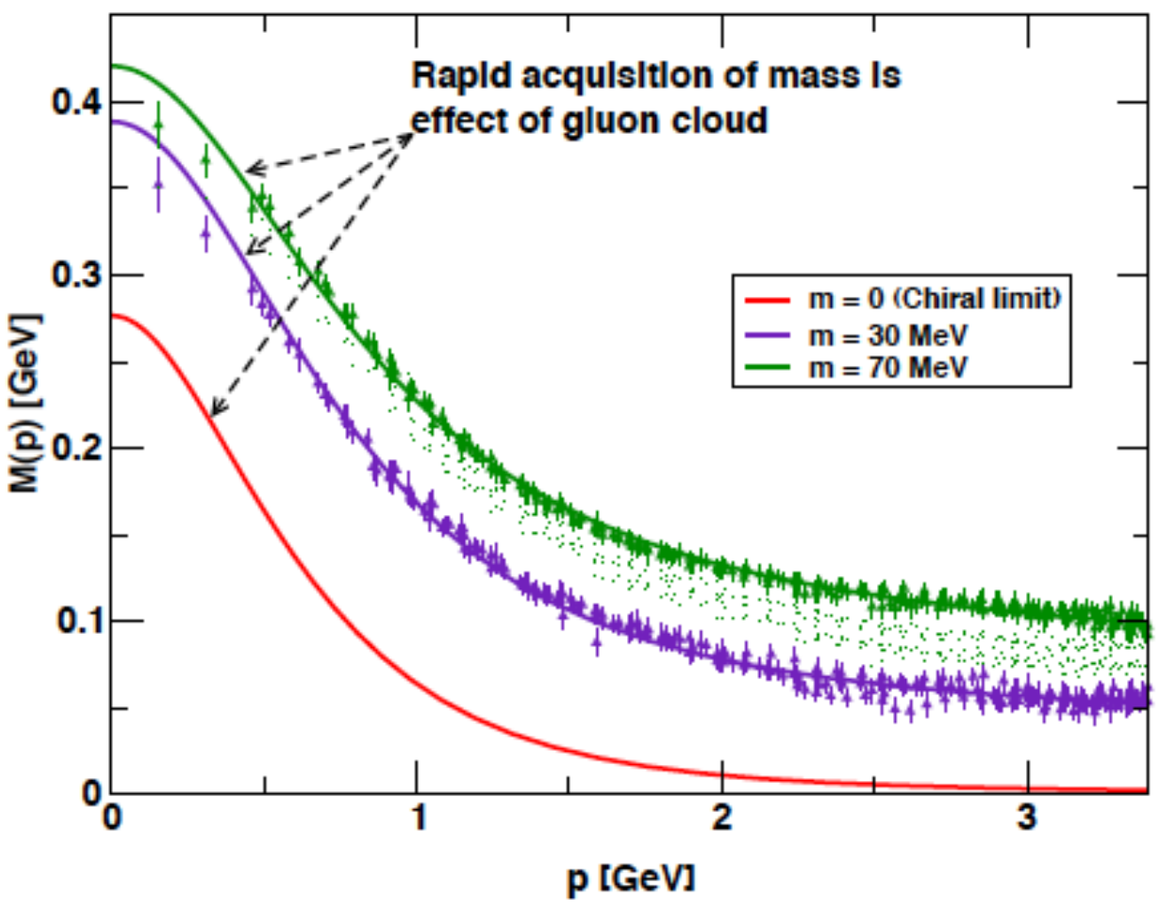
$$\Gamma^\mu(P', P) = \gamma^\mu F_1(Q^2) + \frac{i\sigma_{\mu\nu}}{2M_f} Q^\nu F_2(Q^2)$$

which express **charge** and **magnetization** densities. A so-called **anomalous magnetic moment** is proportional to the Pauli term.

See, e.g., PLB722, 384 (2013)

1. **Quasi-quarks:** Running mass function

$$S(p) = \frac{1}{i\gamma \cdot pA(p^2) + B(p^2)} = \frac{Z(p^2)}{i\gamma \cdot p + M(p^2)}$$



✦ **Now:**

1. The quark's **effective mass** runs with its momentum.

2. The most **constituent mass** of a light quark comes from a cloud of gluons.

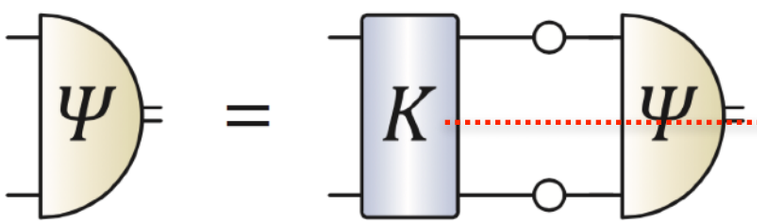
✦ **Next:**

1. What is the **infrared scale** of quark mass function?

2. How does the **transition** connect the non-perturbative and perturbative regions?

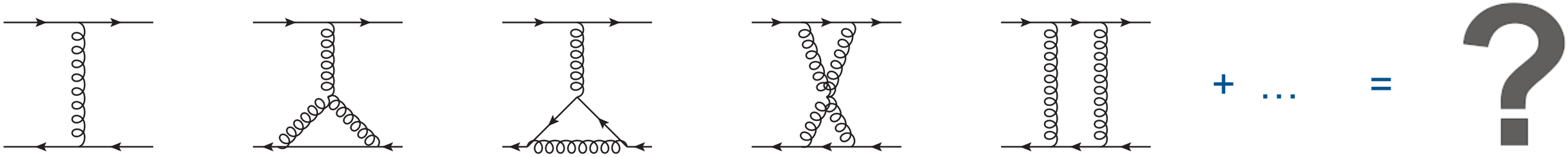
2. How are quasi-quarks bounded?

Bethe-Salpeter equation

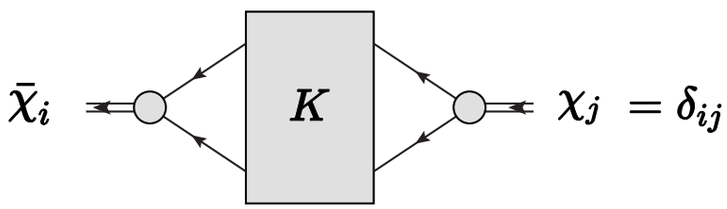


Scattering Kernel

2. **Binding:** Symmetries of the kernel



✦ Discrete symmetries (C,P,T):



$$K = \mathbf{1} \otimes \mathbf{1} + \gamma_5 \otimes \gamma_5 + \cancel{\mathbf{1} \otimes \gamma_5 + \gamma_5 \otimes \mathbf{1}}$$

Lorentz covariance guarantees CPT-symmetry; T-symmetry is obtained for free.

✦ Continuous symmetries:

$$\int_q \mathcal{K}_{\alpha\alpha',\beta'\beta}\{S(q_+)[S^{-1}(q_+) - S^{-1}(q_-)]S(q_-)\}_{\alpha'\beta'} = \int_q D_{\mu\nu}(k-q)\gamma_\mu[S(q_+)\Gamma_\nu(q_+,k_+) - S(q_-)\Gamma_\nu(q_-,k_-)],$$

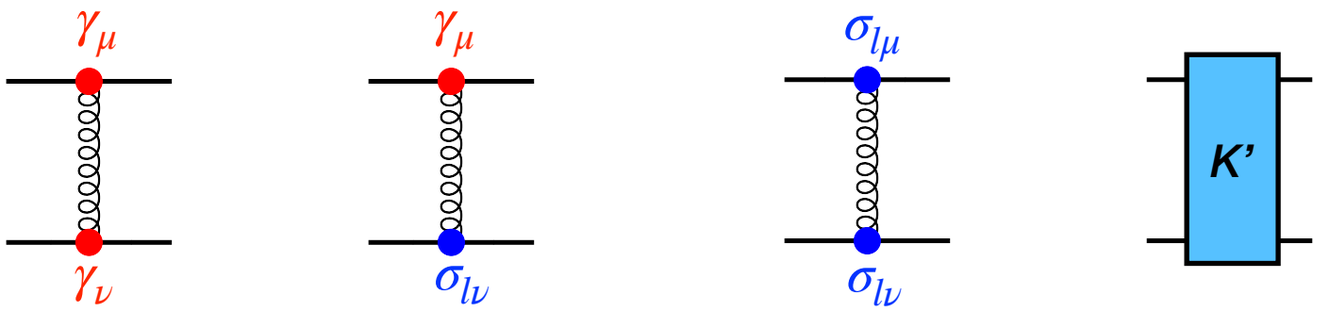
$$\int_q \mathcal{K}_{\alpha\alpha',\beta'\beta}\{S(q_+)[S^{-1}(q_+)\gamma_5 + \gamma_5 S^{-1}(q_-)]S(q_-)\}_{\alpha'\beta'} = \int_q D_{\mu\nu}(k-q)\gamma_\mu[S(q_+)\Gamma_\nu(q_+,k_+)\gamma_5 - \gamma_5 S(q_-)\Gamma_\nu(q_-,k_-)].$$

The color-singlet WGTIs for **vector** and **axial-vector** currents conserve.

See, e.g., CPL 38, 071201 (2021)

2. Binding: Twofold role of the pion

1. A minimal kernel involves the Dirac terms and the Pauli terms:

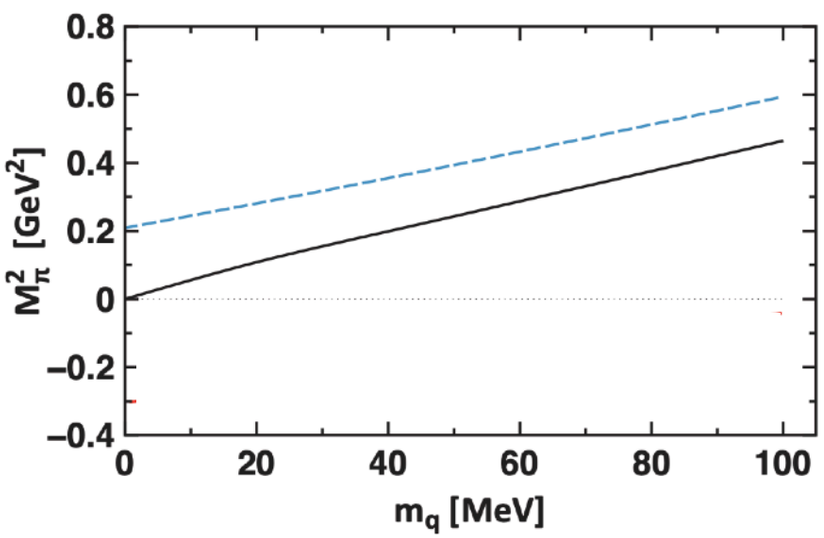


2. A deep connection between one-body and two-body problem:

$$f_\pi E_\pi(k^2) = B(k^2)$$

Pion exists if, and only if, the **quark mass** is dynamically generated.

Two-body problem solved, almost completely, once solution of **one-body** problem is known.



Bound state of quark and anti-quark, but abnormally light:

$$M_\pi \ll M_u + M_{\bar{d}}$$

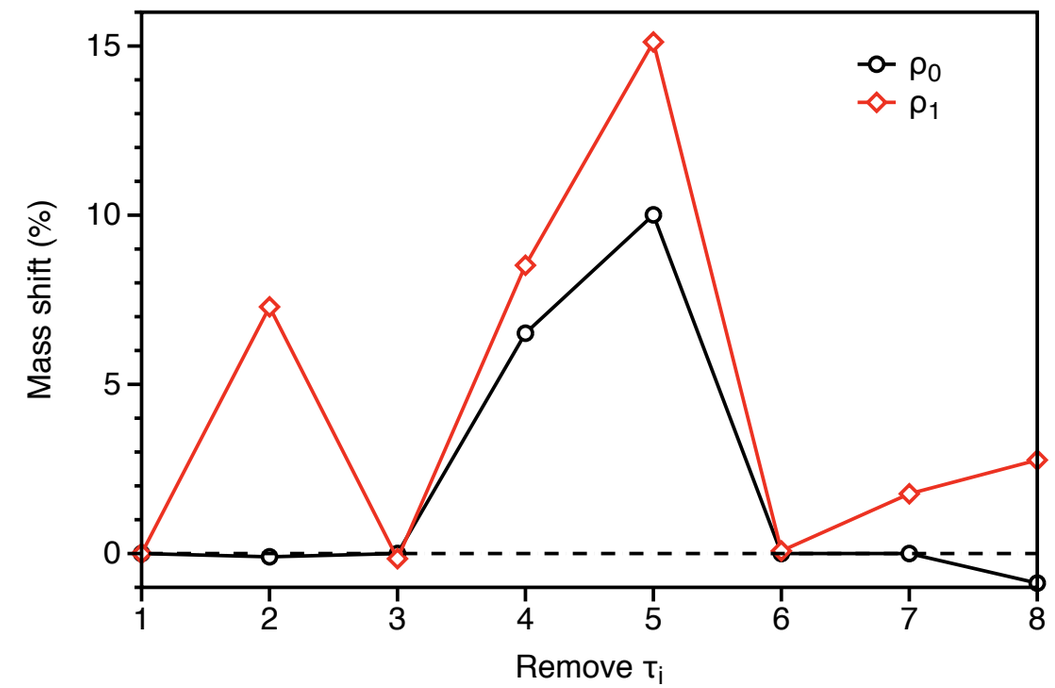
Goldstone's theorem: If a generic continuous symmetry is spontaneously broken, then new massless scalar particles appear in the spectrum of possible excitations.

3. How is the spectrum produced?

3. Spectrum: Partial-wave structure

✦ Structure of wave function, e.g. ρ meson:

$$
\begin{aligned}
\tau^1_{1^-} &= i\gamma^T_\mu, \\
\tau^2_{1^-} &= i\left[3k^T_\mu \gamma\cdot k^T - \gamma^T_\mu k^T\cdot k^T\right], \\
\tau^3_{1^-} &= i k^T_\mu\, k\cdot P\, \gamma\cdot P, \\
\tau^4_{1^-} &= i\left[\gamma^T_\mu \gamma\cdot P\, \gamma\cdot k^T + k^T_\mu \gamma\cdot P\right], \\
\tau^5_{1^-} &= k^T_\mu, \\
\tau^6_{1^-} &= k\cdot P\left[\gamma^T_\mu \gamma^T\cdot k - \gamma\cdot k^T \gamma^T_\mu\right], \\
\tau^7_{1^-} &= (k^T)^2\left(\gamma^T_\mu \gamma\cdot P - \gamma\cdot P \gamma^T_\mu\right) - 2k^T_\mu \gamma\cdot k^T \gamma\cdot P, \\
\tau^8_{1^-} &= k^T_\mu \gamma\cdot k^T \gamma\cdot P.
\end{aligned}
$$



$$
J = 1 \begin{cases} S = 0,\ L = 1 \\ S = 1,\ L = 0 \\ S = 1,\ L = 2 \end{cases}
$$

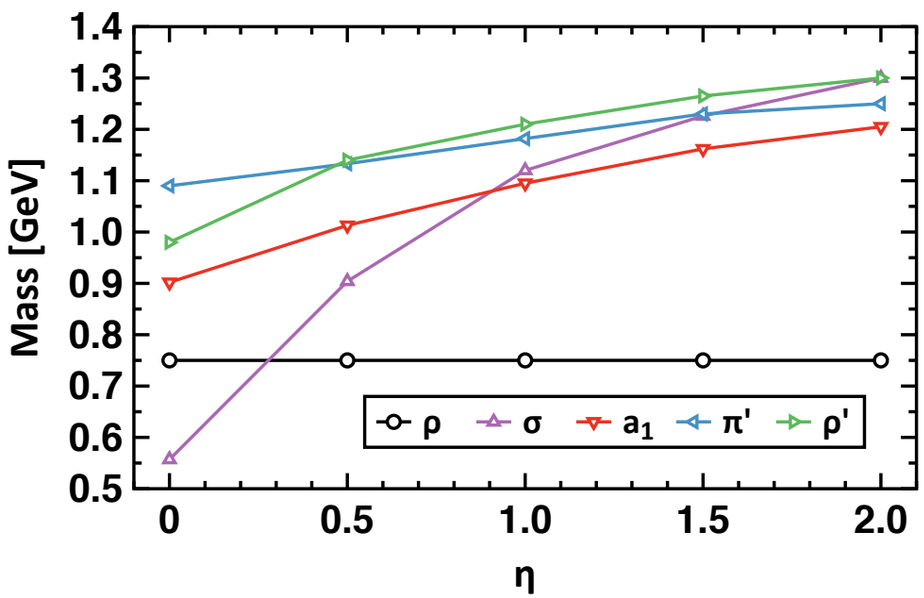
✦ Total angular momentum **J** is a good quantum number, but **S** and **L** are not. The partial waves mix together.

✦ Missing some partial waves could **remarkably** affect the mass, especially of **radial excitation** states.

See, e.g., PRC 85, 035202 (2012)

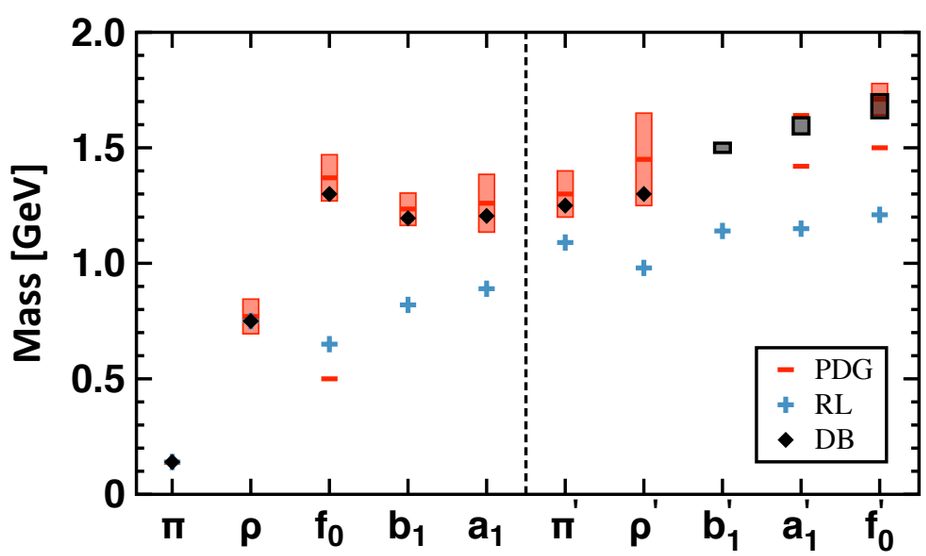
3. Spectrum: Spin-orbit interaction

✦ Impact of the Pauli term (AM):



✦ With increasing the AM strength, the $a_1-\rho$ mass-splitting rises very rapidly. From a quark model perspective, the DCSB-enhanced kernel increases spin-orbit repulsion.

✦ Light-flavor meson spectrum:



✦ The spin-orbit boosted quark-core mass of the f_0 is greater than the empirical value, and matches an estimate the result obtained using chiral perturbation theory.

✦ The magnitude and ordering of radial excitation states can be fixed with the DCSB-enhanced kernel.

See, e.g., CPL 38, 071201 (2021)

Summary

- ✦ (Almost) massless quarks become massive quasi-quarks which bond into hadrons.

- ✦ The quasi-quark picture is intuitive and simple and can fit into a general framework.