Importance of Approximate Symmetries in Nuclear Physics

Legacy Shell Models ... (LSM)

Low-Energy (QED) Discoveries!

High-Energy (QCD) Challenges?

Strong QCD Teleworkshop NJU-INP

21st Century Cross Over

... Femto-Science World (FSW)

Two forces – strong color force and the electromagnetic force – are responsible for holding the fundamental pieces – quarks & gluons plus electrons together ...

Where LSM (QED) & FSW (QCD) meet!

PIs: Jerry P. Draayer, Kristina D. Launey, Tomas Dytrych, Feng Pan
GAs: David S. Kekejian & Grigor Sargsyan

U.S. NSF & DOE & LSU & SURA Sponsored Research
Overarching Historical Perspectives

Nuclear (LE-QED) Discoveries!

Maria Goeppert Mayer & Hans Jansen
Won Nobel Prize (1963) “…for their discoveries (late 1940s) concerning nuclear shell structure”

Numerous Follow-on Developments:
- Single-particle Models
- SP plus Pairing Modes
- Many-body Approaches
- Collective Models (BH)
- Geometrical Theories
- Quasi-particle Models
- Algebraic Approaches
- Effective Interactions

1990s – HPC – 2000s
- No-Core Shell Model (NCSM)
- Non-compact Symplectic Model

Symplectic Symmetry & EFT Roots*

*Sp(3,R) -> SU(3) is the Dynamical Symmetry Group of the 3D Oscillator!

Particle (HE-QCD) Challenges?

WHERE DOES YOUR MASS COME FROM?

Eugene Wigner

Numerous Follow-on Developments:
- Confinement – Long & Short Range
- What is Mass, or a Quantum?

1990s – HPC – 2000s
- Dynamic Mass Generation
- Lattice or Continuum (Strong) QCD

“Where’s the Mass, Man?”

Standard Model?

Higgs?

Show me Something Mister!

Numerous Follow-on Developments:
- ? Simplicity within Complexity ?

Overarching Historical Perspectives

Importance of Approximate Symmetries in Nuclear Physics

(2 of 15)

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Importance of Approximate Symmetries in Nuclear Physics

**No-Core (Regular & Symplectic) Shell Model**

**No-Core Shell Model (NCSM)**
(Vary, Navratil, & Barrett, ~2000 to present)

- "No Big Deal" to "Major Action"

**Symplectic Shell Model (Sp-NCSM)**
(Rowe & Rosensteel, ~1980s to present)

- Typical Vertical Symplectic Excitation
  (Multiple 1p-1h /+2Ω)

**Reorganize Shell-Model Space**

- Cluster & Collective (Natural Subspaces)

**Symmetry Adapted Basis**
(Special Features)
- Canonical & unitary
- Organized by shapes
- Quadratic in x's & p's
- Band-heads \( np-nh \)
- Spurious free bands
- Captures collectivity
- No effective charges
- Deformation dominates
- Algebraic framework
- EFT known & natural

**EFT: Symplectic Symmetry is the Dynamical Extension of the Harmonic Oscillator!**

- Simple logical algebraic underpinning
  - Elliott SU(3) if no symplectic modes
  - Sp(3,R) add monopole & quadrupole


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**No-Core (Regular & Symplectic) Shell Model**

- Realistic interaction (local or not; NN, NNN, ...)
  - In principle, exact solutions, up to \( N_{\text{max}} \) cutoff
  - Successful descriptions to date through \( ^{16}\text{O} \)


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**Importance of Approximate Symmetries in Nuclear Physics**

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Symplectic Symmetry / Spectrum Generating Algebra

Realized by 21 generators of the Sp(3,R) symplectic algebra – all distinct quadratic forms in coordinates \((q_i)\) and momenta \((p_i)\)

\[
Q_{ij} = \sum_n q_{in} q_{jn} \quad K_{ij} = \sum_n p_{in} p_{jn} \quad L_{ij} = \sum_n (q_{in} p_{jn} - q_{jn} p_{in}) \quad S_{ij} = \sum_n (q_{in} p_{jn} + p_{in} q_{jn})
\]

Quadrupole (Shape) “Tensor” (6)  Kinetic Energy “Tensor” (6)  Orbital Angular Momentum Operator (3)  Moment of Inertia “Circulation / Vorticity” (6)

Symplectic Symmetry is the Dynamical Symmetry Group of the Harmonic Oscillator ... !

Every configuration is labeled by the SU(3) \((\lambda, \mu)\) quantum numbers of the Elliott Model (1958), which in turn defines the shape of the so-called “band head” configuration.)

David Kekejian – Graduate Student (JPD#20 PhD Expected 12/2021) from Armenia
Symmetry Adapted NCSM (SA-NCSM) Campaign
(Going on 20 years ... 2001 - 2021)

Goal –
Reproduce and predict properties of heavy as well as light nuclei, starting with and building upon QCD/EFT informed and inspired interactions …

Plan –
✓ Exploit existing capabilities to evaluate probability of success and level of effort required to develop a full-blown symmetry adapted NCSM
✓ Develop a symmetry adapted no-core shell model code that capitalizes on exact and approximate (partial) symmetries of nuclei (SA-NCSM)
  • Exploit existing NCSM technology to prove efficacy of method, revealing (or not) any inherent limitations
  • Explore need (or not) for renormalization, winnowing space to physically relevant and tractable subspaces
  • Evaluate extensibility of theory and its characteristics vis-à-vis current/emerging computational resources
✓ Study the emergence of collective phenomena, tracking their evolution to and from fundamental (ab initio) features of the interaction
  • Apply the theory to study of extreme processes known to be important to understanding nuclei and nuclear systems
  • Develop a user-friendly desktop version of code for simple applications as well as educational and training purposes
  • Extend theory to include coupling to the continuum, and apply to the result to the study of nuclear reactions

Results for $^6$Li with $N_{\text{max}} = 10$
(Proof of Principle)

*Bon, Argonne, Idaho, N3LO (optimized), etc. ... yield similar results ...
Creation of $^{12}\text{C}$ in Hot Stars / Nucleosyntheses

... The Elusive Hoyle State ...
Three Primary “Slices” in NCSpM Description

- $^{12}$C
  - $2p-2h$
    - $(6\ 2)$ Bandhead (Prolate)
  - $4p-4h$
    - $(12\ 0)$ Bandhead (More Prolate)
  - $0p-0h$
    - $(0\ 4)$ Bandhead (Oblate)

Intertwining Shell & Alpha Cluster Pictures
$^{12}$C Systematics as a Function of $N_{\text{max}}$

($N_{\text{max}}$ = Total Number of $2\hbar\Omega$ Excitations above Ground State)
$^{12}\text{C} - \text{Cluster Formations}$

(Now with mixing at the band-head level turned on ... !)

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Standard ab initio NCSM *(NCSpM & Band Mixing)
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Ne-20: grey arrow is gamma used

Almost "Laptop" Level Calculations!
Further sd-shell Results (Robert Baker - GS)

Selected (pre-thesis) Examples (Now onto Beta Decay)

Ab initio description (converged selected spaces) (NNLO$_{opt}$, $\hbar \omega=15$ MeV, 13 HO shells)

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**Plus fp-shell Results (Grigor Sargsyan - GS)**

8 shells, N2LOopt

$^48$Ca

0$^+$

SA-NCSM (selected): ................ 966,152

Complete model space: ...... 3,162,511,819

2$^+$

SA-NCSM (selected): ................ 3,055,554

Complete model space: ...... 14,522,234,982

48 Ti, Q(2$^+$) [e fm$^2$]

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Experiment .................... -17.7

8 shells ........................... -19.3

(no effective charges)

8 shells, N2LOopt

$^48$Sc

$^48$Ti

Neutrinoless $\beta\beta$ decay

48 Ti

0$^+$

SA-NCSM (selected): ................ 602,493

Complete model space: ...... 24,694,678,414

2$^+$

SA-NCSM (selected): ................ 1,178,834

Complete model space: ...... 113,920,316,658

$^{48}$Ti, $^{48}$Ca, $^{48}$Sc, $^{48}$Ti

48 Ti, Q(2$^+$) [e fm$^2$]

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Experiment .................... -17.7

8 shells ........................... -19.3

(no effective charges)
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Constructing an Effective Field Theory (David Kekejian – GS)

Generic (Scalar) Field Theory

\[ \mathcal{L} = \frac{1}{2} (\partial_\mu \varphi)(\partial^\mu \varphi) + \frac{1}{2} m^2 \varphi^2 \]

Quantum (Scalar) Field Theory

\[ H = \sum_k E_k (b_k^+ b_k + \frac{1}{2}) \]

\[ \varphi(r, t) = \frac{1}{\sqrt{V}} \sum_k b_k^- \frac{1}{\sqrt{|2k^0|}} e^{-ik \mu x_\mu} + \frac{1}{\sqrt{V}} \sum_k b_k^+ \frac{1}{\sqrt{|2k^0|}} e^{ik \mu x_\mu} \]

\[ \mathcal{L}^{(n)} = \frac{\alpha^n}{2(n+1)!} (\partial_\mu \varphi \partial^\mu \varphi^* + m^2 \varphi \varphi^*)^n \]

\[ \mathcal{H}^{(n)} = \frac{\alpha^n}{2(n+1)!} (\varphi \varphi^* - \varphi' \cdot \varphi'^* + m^2 \varphi \varphi^*)^n ((2n+1)\varphi \varphi^* + \varphi' \cdot \varphi'^* - m^2 \varphi \varphi^*) \]

\[ H^{(n)} \sim \sqrt{\alpha}(gQ \cdot Q)^n, (K \cdot K)^n, (gQ \cdot K)^n, (gK \cdot Q)^n \]

\[ \frac{\alpha}{V} \hbar \Omega = \frac{\beta^2}{8 N_\sigma} \]

\[ g = \frac{m^2}{\hbar^2 \omega^2} \]

Symplectic symmetry emerges naturally from quantum effective field theory!

Paper in Preparation

David Kekejian (Thesis - LSU 4Q2021)
... Here's the Deal ...

Nuclear (LE-QED) Discoveries!

Particle (HE-QCD) Challenges?

20\textsuperscript{th} Century Subatomic Physics

Great Minds and Creative Models
Numerous Prestigious Awards Won
Major New Facilities Commissioned
Simplicity within Complexity Good Analytic Solutions Well Regarded

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Physics Changed Technology
1990s – HPC/ERA – 2010s
Technology Changed Physics

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21\textsuperscript{st} Century Subatomic Physics

Bigger Computers! $\rightarrow$ Better Results?

$\langle$---------- NCSM
Lattice QCD $\rightarrow$$\rangle$

$\langle$----------SpNCSM
Continuum QCD $\rightarrow$$\rangle$

Partial Symmetries Expose Coherent Features
Simpler Picture! $\rightarrow$ Cleaner Results?

Lunch (5 years back) at JLab: “Is the nucleon deformed (round)?”

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