The Nucleon Energy Momentum Tensor
The pressure distribution inside the proton

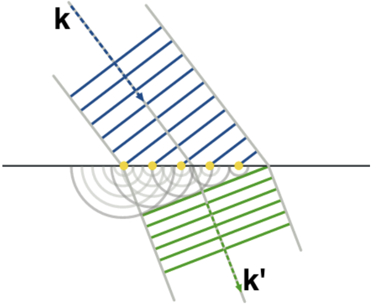
F.-X. Girod

Strong QCD from Hadron Structure Experiments

Jun 7^{th} 2021

Diffraction and Imaging

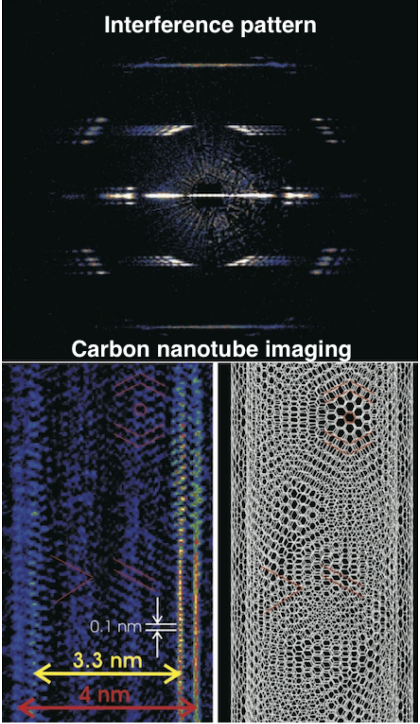
Huygens-Kirchhoff-Fresnel principle



$\vec{q} = \vec{k} - \vec{k'}$
The interference pattern is given by the superposition of spherical wavelets

$$f(\Omega_{\vec{q}}) = \int \frac{d^3\vec{r}}{(2\pi)^3} F(\vec{r}) e^{i\vec{q}\cdot\vec{r}}$$

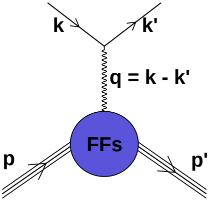
Fourier imaging



Elastic scattering

Form Factors

Probing deeper using virtual photons



$$J^\mu_{\rm EM} = F_1\gamma^\mu + \frac{\kappa}{2M}F_2 i\sigma^{\mu\nu}q_\nu$$

$$\frac{d\sigma}{d\Omega} = \frac{\sigma_{\rm Mott}}{\epsilon(1+\tau)}\left[\tau G_M^2 + \epsilon G_E^2\right]$$

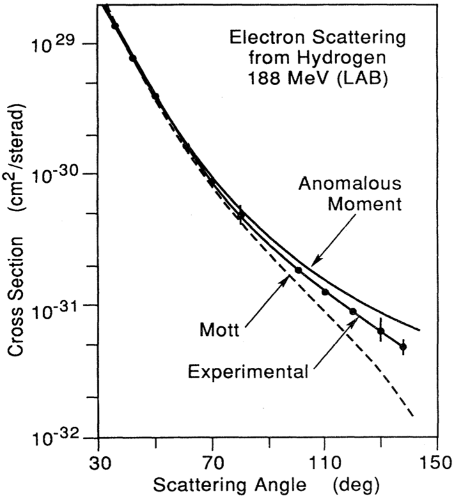
$$\tau = \frac{Q^2}{4M^2}$$

$$Q^2 = -\left(k-k'\right)^2 = -m^2_{\gamma^*}$$

$$\frac{1}{\epsilon} = 1+2(1+\tau)\tan^2\frac{\theta_e}{2}$$

$$G_E = F_1 - \tau F_2$$

$$G_M = F_1 + F_2$$



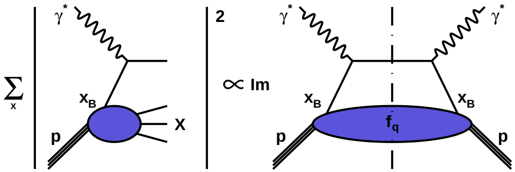
Hofstadter Nobel prize 1961

"The best fit in this figure indicates an rms radius close to $0.74 \pm 0.24 \times 10^{-13}$ cm."

Imaging in transverse impact parameter space

Deeply Inelastic Scattering

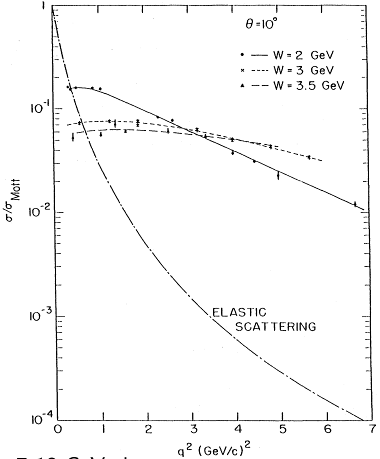
Parton Distributions



The total cross section is given by
the imaginary part of the forward amplitude

$$\nu = E_{\gamma^*} \quad , \quad x_B = \frac{Q^2}{2M\nu}$$

$\sigma_{\rm DIS}(x_B, \not{Q}^2) \rightarrow$ scaling, point-like constituents



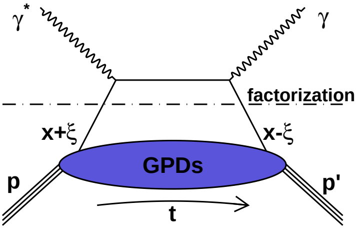
Discovery of quarks, SLAC-MIT group, 7-18 GeV electron
Friedman, Kendall, Taylor, Nobel prize 1990

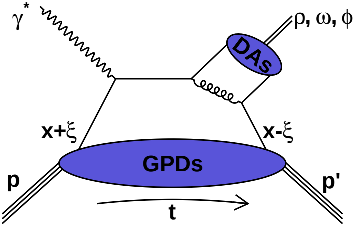
$$\lim_{Q^2\to\infty} \sigma_{\rm DIS}(x_B) = \int_{x_B}^{1} \frac{d\xi}{\xi} \sum_a f_a(\xi,\mu)\hat{\sigma}^a\left(\frac{x_B}{\xi}, \frac{Q}{\mu}\right)$$

1-D distribution in longitudinal momentum space

Deep Exclusive Scattering

Generalized Parton Distributions





$$\gamma^* p \to \gamma p', \; \gamma^* p \to \begin{cases} \rho p' \\ \omega p' \\ \phi p' \end{cases}$$

Bjorken regime :
$Q^2 \to \infty$, x_B fixed

t fixed $\ll Q^2$, $\xi \to \frac{x_B}{2-x_B}$

$$\frac{P^+}{2\pi} \int dy^- \, e^{ixP^+y^-} \langle p' | \bar{\psi}_q(0) \gamma^+ (1+\gamma^5) \psi(y) | p \rangle$$

$$= \bar{N}(p') \left[H^q(x,\xi,t)\gamma^+ + E^q(x,\xi,t) i\sigma^{+\nu} \frac{\Delta_\nu}{2M} \right.$$

$$\left. + \tilde{H}^q(x,\xi,t)\gamma^+\gamma^5 + \tilde{E}^q(x,\xi,t)\gamma^5 \frac{\Delta^+}{2M} \right] N(p)$$

spin	N no flip	N flip
q no flip	H	E
q flip	$\tilde{H}$	$\tilde{E}$

3-D Imaging conjointly in transverse impact parameter and longitudinal momentum

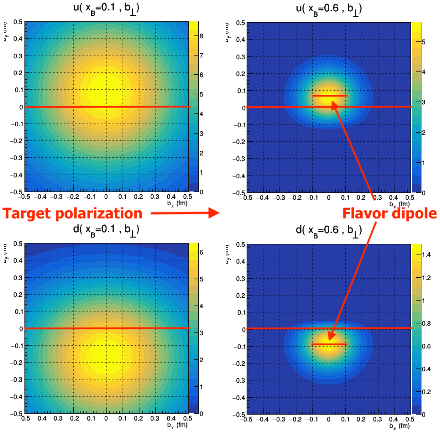
GPDs and Transverse Imaging

(x_B, t) correlations

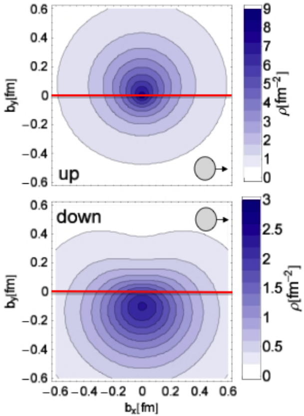
$$q_X(x, \vec{b}_\perp) = \int \frac{d^2\vec{\Delta}_\perp}{(2\pi)^2} \left[H(x,0,t) - \frac{E(x,0,t)}{2M} \frac{\partial}{\partial b_y} \right] e^{-i\vec{\Delta}_\perp \cdot \vec{b}_\perp}$$

M. Burkardt, Int. J. Mod. Phys. **A 18** 173 (2003)

QCDSF coll. PRL**98** 222001 (2007)



Lattice calculation

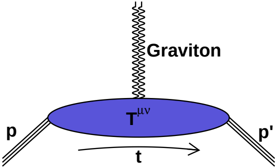


Energy Momentum Tensor

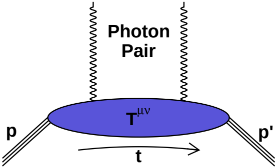
Gravitational Form Factors definition :

$$\langle p'|\hat{T}^q_{\mu\nu}|p\rangle = \bar{N}(p')\left[\; M^q_2(t)\,\frac{P_\mu P_\nu}{M} + J^q(t)\,\frac{\imath(P_\mu\sigma_{\nu\rho}+P_\nu\sigma_{\mu\rho})\Delta^\rho}{2M} + d^q_1(t)\,\frac{\Delta_\mu\Delta_\nu - g_{\mu\nu}\Delta^2}{5M}\;\right] N(p)$$

Confinement forces from space-space components of EMT
The graviton with spin 2 couples directly to EMT
But gravity is too weak to produce count rates in the detector



We can construct a spin 2 operator using two spin 1 operators
→ use a process with two photons to measure the EMT?
X. Ji *PRL* **78** 610 (1997) ; M. Polyakov *PLB* **555** 57 (2003)



GPDs and Energy Momentum Tensor

(x, ξ) correlations

Form Factors accessed *via* second x-moments :

$$\langle p'|\hat{T}^q_{\mu\nu}|p\rangle = \bar{N}(p')\left[\ M^q_2(t)\,\frac{P_\mu P_\nu}{M} + J^q(t)\,\frac{\imath(P_\mu\sigma_{\nu\rho}+P_\nu\sigma_{\mu\rho})\Delta^\rho}{2M} + d^q_1(t)\,\frac{\Delta_\mu\Delta_\nu - g_{\mu\nu}\Delta^2}{5M}\ \right] N(p)$$

Angular momentum distribution

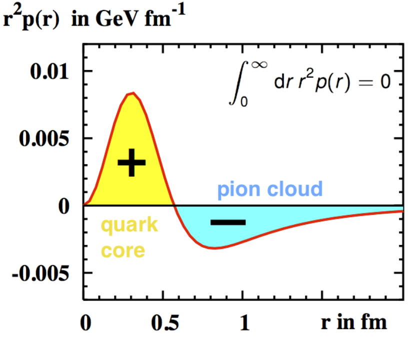
$$J^q(t) = \frac{1}{2}\int_{-1}^{1} \mathrm{d}x\, x\,[H^q(x,\xi,t) + E^q(x,\xi,t)]$$

Mass and force/pressure distributions

$$M^q_2(t) + \frac{4}{5}d_1(t)\xi^2 = \frac{1}{2}\int_{-1}^{1} \mathrm{d}x\, xH^q(x,\xi,t)$$

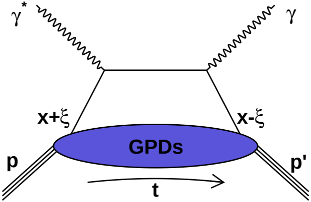
$$d_1(t) = 15M\int \mathrm{d}^3\vec{r}\,\frac{j_0(r\sqrt{-t})}{2t}\,p(r)$$

Distribution of pressure

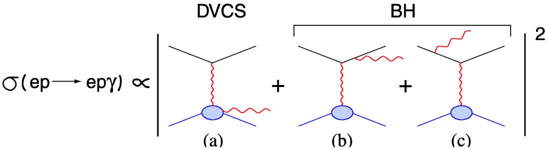


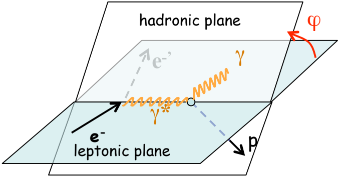
Deeply Virtual Compton Scattering

The cleanest GPD probe at low and medium energies



$$ep \to ep\gamma$$





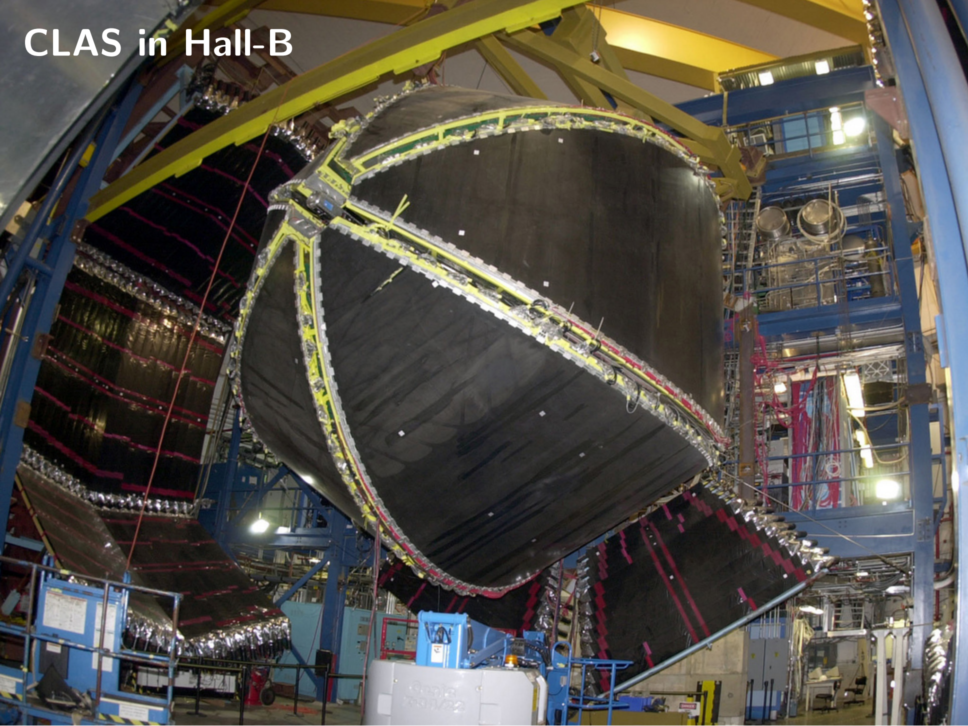
$$A_{\mathrm{LU}} = \frac{d^4\sigma^{\rightarrow} - d^4\sigma^{\leftarrow}}{d^4\sigma^{\rightarrow} + d^4\sigma^{\leftarrow}} \overset{\text{twist-2}}{\approx} \frac{\alpha \sin\phi}{1 + \beta\cos\phi}$$

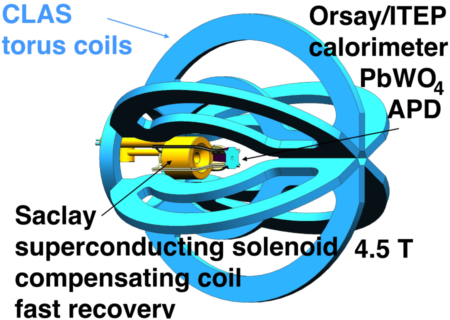
$$\alpha \propto \mathcal{I}\mathrm{m}\left(F_1\mathcal{H} + \xi G_M\tilde{\mathcal{H}} - \frac{t}{4M^2}F_2\mathcal{E}\right)$$

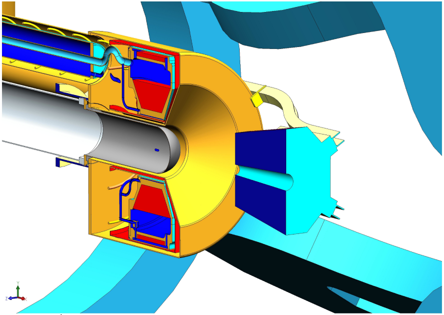
$$\mathcal{H}(\xi,t) = i\pi H(\xi,\xi,t) + \mathcal{P}\int_{-1}^{1} \mathrm{d}x\, \frac{H(x,\xi,t)}{x-\xi}$$

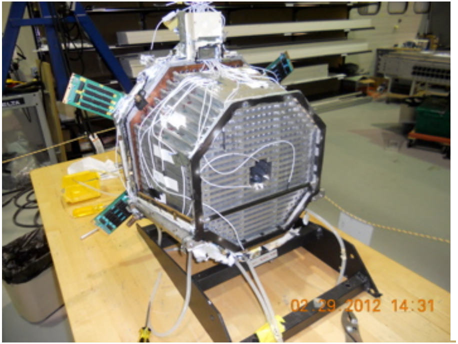
$$A_{\mathrm{UL}} \propto \mathcal{I}\mathrm{m}\left(F_1\tilde{\mathcal{H}} + \xi G_M\mathcal{H} + G_M\frac{\xi}{1+\xi}\mathcal{E} + \cdots\right)\sin\phi$$

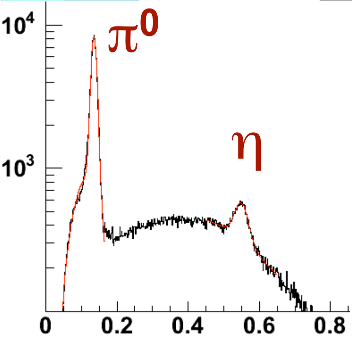
CLAS in Hall-B





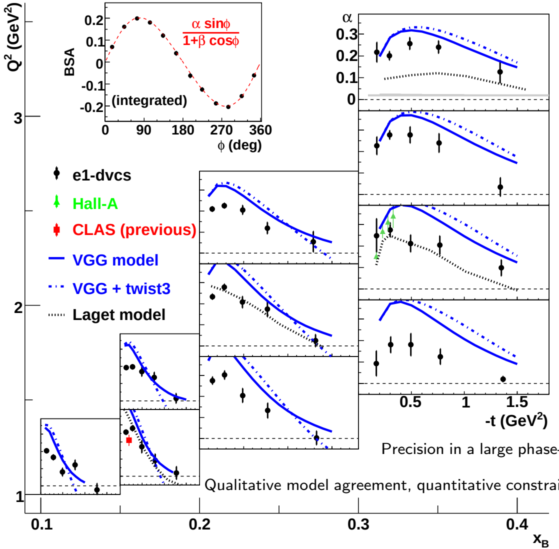






$$M_{\gamma\gamma} = \sqrt{2E_{\gamma_1}E_{\gamma_2}(1-\cos\theta_{\gamma\gamma})}$$

CLAS proton Beam Spin Asymmetry



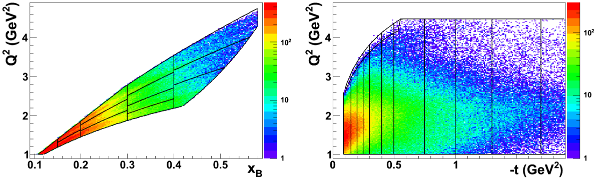
Precision in a large phase-space (x_B, Q^2, t)

Qualitative model agreement, quantitative constraints on parameters

F.-X. G. *et al.*, PRL **100** (2008) 162002

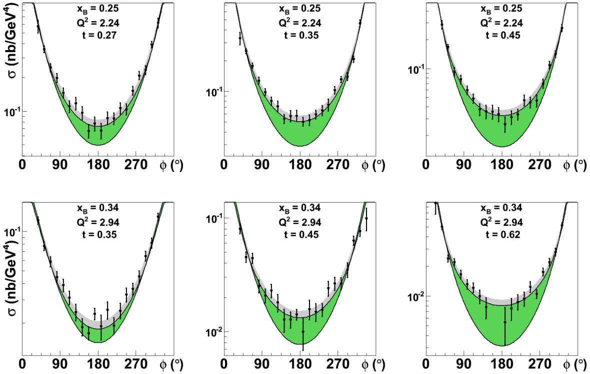
CLAS proton cross-section

More than 3k bins



Dispersion relation :

$$\mathcal{R}\mathrm{e}\ \mathcal{H} = \left[\int \mathcal{I}\mathrm{m}\ \mathcal{H}\right] + \Delta$$

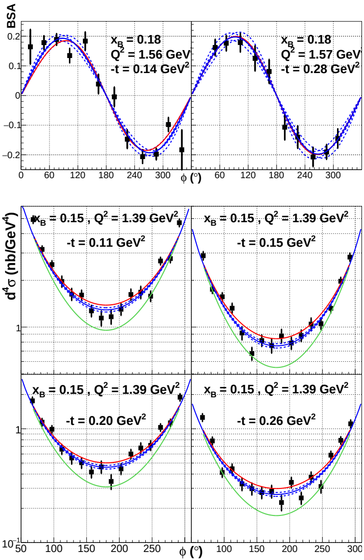


green band shows difference with BH

→ sensitivity to d_1

H.-S. Jo *et al.* PRL **115** (2015) 212003

Global Fits to extract the D-term



Beam Spin Asymmetries

$$\mathrm{Im}\mathcal{H}(\xi,t) = \frac{N}{1+x}\left(\frac{2\xi}{1+\xi}\right)^{-\alpha_R(t)}\left(\frac{1-\xi}{1+\xi}\right)^{b}\left(\frac{1-\xi}{1+\xi}\,\frac{t}{M^2}\right)^{-1}$$

Unpolarized cross-sections

Use dispersion relation:

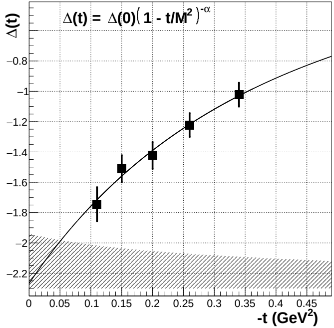
$$\mathrm{Re}\mathcal{H}(\xi,t) = \Delta + \mathcal{P}\int dx\left(\frac{1}{\xi-x}-\frac{1}{\xi+x}\right)\mathrm{Im}\mathcal{H}(\xi,t)$$

pure Bethe-Heitler

local fit + uncertainty range

resulting global fit

DVCS Dispersion: subtraction constant results



$$\mathrm{Im}\mathcal{H}(\xi, t) = \frac{N}{1+x}\left(\frac{2\xi}{1+\xi}\right)^{-\alpha_R(t)}\left(\frac{1-\xi}{1+\xi}\right)^{b}\left(\frac{1-\xi}{1+\xi}\frac{t}{M^2}\right)^{-1}$$

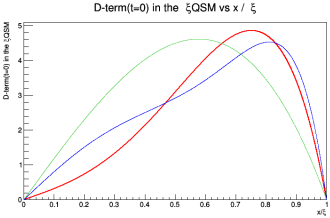
α_R is fixed from small-x Regge phenomenology
b is a free parameter for the large x behavior
p is fixed to 1 for the valence
M is a free parameter controlling the t dependence

$$\Delta(t) = \Delta(0)\left(1 - \frac{t}{M^2}\right)^{-\alpha} = 2\int_{-1}^{1} dz\,\frac{D(z,t)}{1-z}$$

$$D(z,t) = (1-z^2)\sum_{k=1}^{\infty}\left[e_u^2\ d^u_{2k-1}(t) + e_d^2\ d^d_{2k-1}(t)\right]\ C^{3/2}_{2k-1}(z)$$

Hereafter assume $d^u_{2k-1} \approx d^d_{2k-1}$

Separation of the GFF d_1



In the χQSM:
$d_1(0) \approx -4.0$; $d_3(0) \approx -1.2$; $d_5(0) \approx -0.4$

$$H(x,\xi,t) = \cdots + \theta\left[1 - \frac{x^2}{\xi^2}\right] D(\frac{x}{\xi}, t)$$

$$D(z,t) = \left(1 - z^2\right)\left[d_1(t) C_1^{3/2}(z) + d_3(t) C_3^{3/2}(z) + \cdots\right]$$

$$C_1^{3/2}(z) = 3z$$

$$C_3^{3/2}(z) = \frac{5}{2}\left(7z^3 - 3z\right)$$

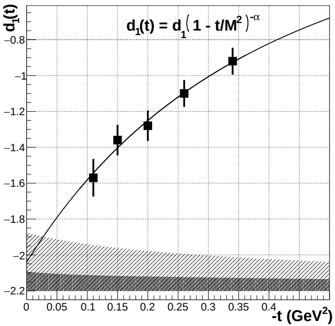
$$C_5^{3/2}(z) = \frac{21}{8}\left(33z^5 - 30z^3 + 5z\right)$$

To separate orthogonal Gegenbauer polynomials:
requires measurement of (x,ξ) dependence (or at least $z = x/\xi$ dependence)
different reaction such as DDVCS, will require higher luminosity

For now, to make progress it is necessary to make assumptions
Use guidance from models such as χQSM or lattice results
Also implement constraints from theory into phenomenological fits

Separation of the GFF d_1

$$D^q(\tfrac{x}{\xi}, t) = \left(1 - \tfrac{x^2}{\xi^2}\right) \left[d_1^q(t) C_1^{3/2}(\tfrac{x}{\xi}) + d_3^q(t) C_3^{3/2}(\tfrac{x}{\xi}) + \cdots\right]$$



t-dependence of the D-term :

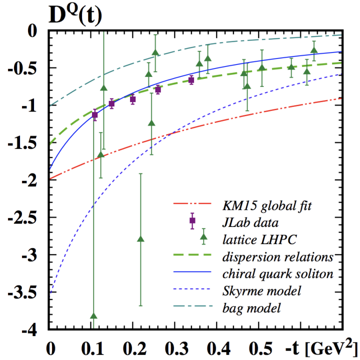
- Dipole gives singular pressure at $r = 0$
- Power law implied by counting rules?
- Exponential?
- ...

$d_1(0) < 0$ dynamical stability of bound state

$d_1(0) = -2.04 \pm 0.14 \pm 0.33$

First Measurement of new fundamental quantity

D-term comparison with theory



Dispersion Relation Analysis

Chiral quark soliton model

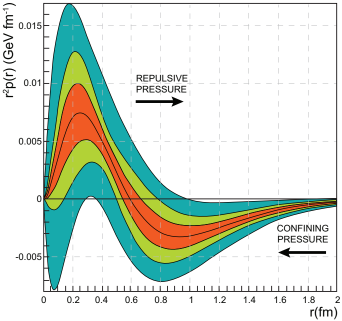
Lattice results LHPC

Global fit

M. V. Polyakov, P. Schweitzer Int.J.Mod.Phys. **A33** (2018)

em:	$\partial_\mu J^\mu_{\text{em}} = 0$	$\langle N'\|J^\mu_{\text{em}}\|N\rangle$	$\longrightarrow$	$Q = 1.602176487(40) \times 10^{-19}\text{C}$ $\mu = 2.792847356(23)\mu_N$
weak:	PCAC	$\langle N'\|J^\mu_{\text{weak}}\|N\rangle$	$\longrightarrow$	$g_A = 1.2694(28)$ $g_p = 8.06(55)$
gravity:	$\partial_\mu T^{\mu\nu}_{\text{grav}} = 0$	$\langle N'\|T^{\mu\nu}_{\text{grav}}\|N\rangle$	$\longrightarrow$	$m = 938.272013(23)\,\text{MeV}/c^2$ $J = \frac{1}{2}$ $\boldsymbol{D} = \boldsymbol{?}$

Proton Pressure distribution results



The pressure at the core of the proton is $\sim 10^{35}$ Pa
About 10 times the pressure at the core of a neutron star

Positive pressure in the core (repulsive force)
Negative pressure at the periphery: pion cloud
Pressure node around $r \approx 0.6$ fm

Stability condition : $\int_0^\infty \mathrm{d}t\, r^2 p(r) = 0$

Rooted into Chiral Symmetry Breaking

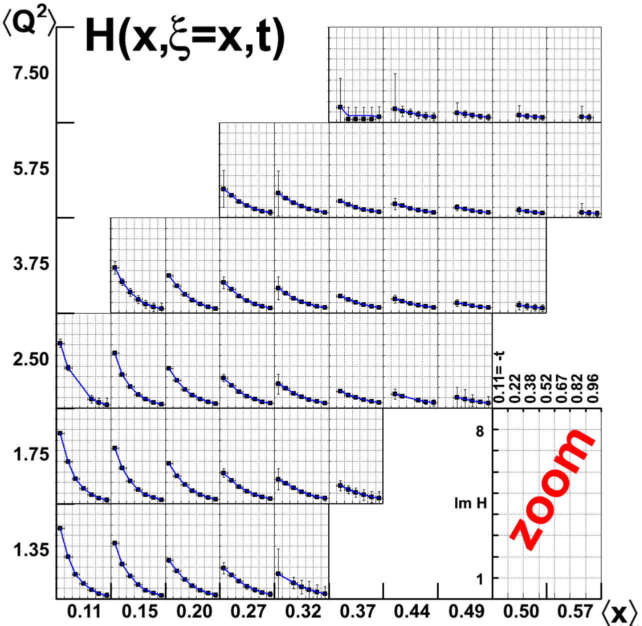
World data fit

CLAS 6 GeV data

Projected CLAS12 data E12-16-010B

Projected impact on GPD extraction methods

Using simulated data based on VGG model. Input GPD H extracted with good accuracy



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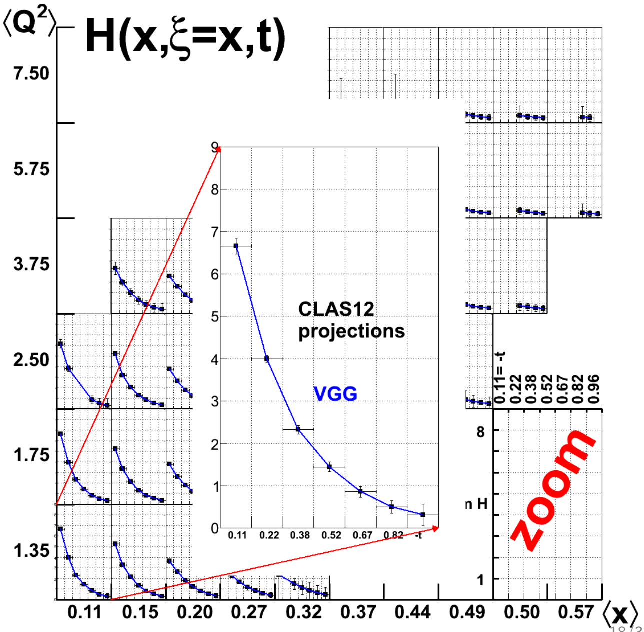

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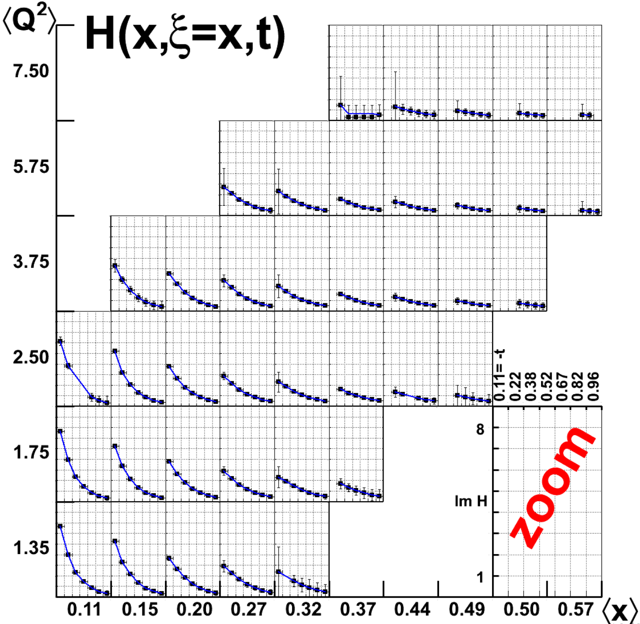

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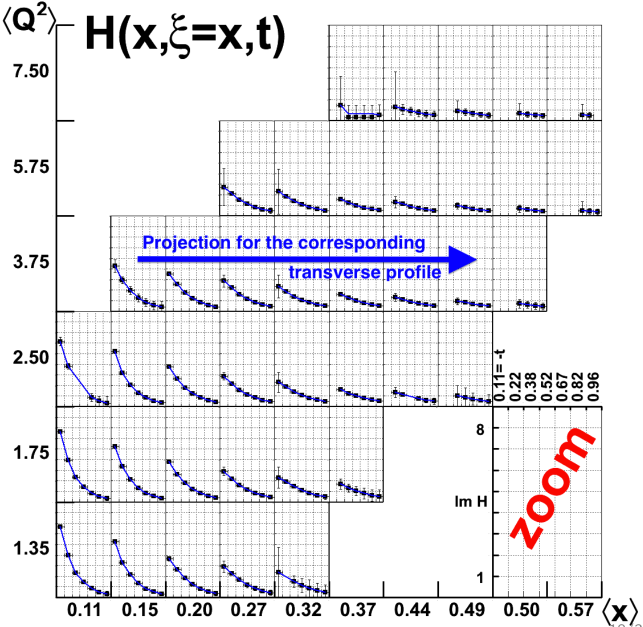
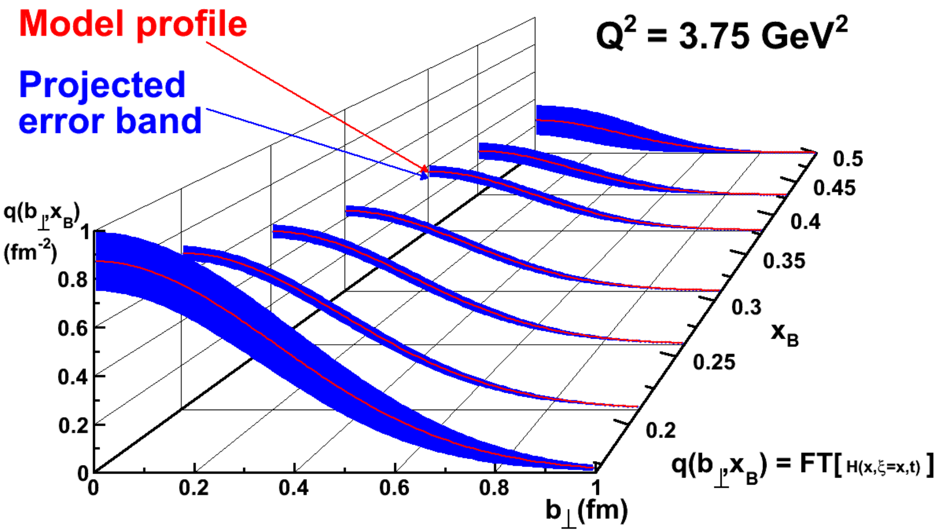


Projection for the Nucleon transverse profile



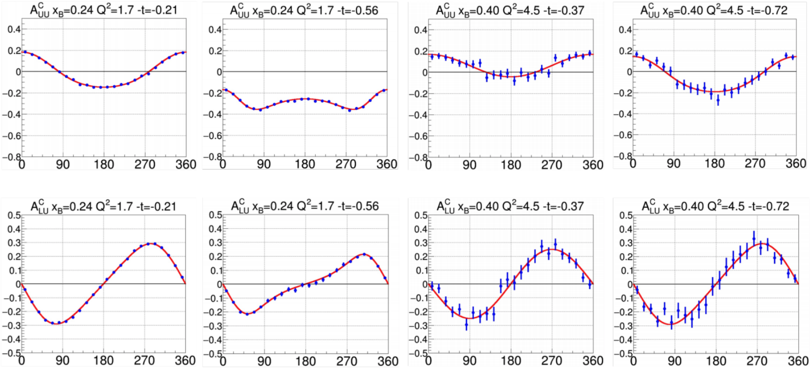
Precision tomography in the valence region

DVCS with a Polarized Positron beam

PEPPo production injecting 60 MeV **100 nA positron polarized at 60%**

(PEPPo Collaboration) D. Abbott *et al.* , **PRL**116 (2016) 214801 ; L. Cardman *et al.* AIP CP 1970 (2018) 050001

Proposal 100 days (80+20) at $\mathcal{L} = 0.6 \times 10^{35}$ cm^{-2}s^{-1}

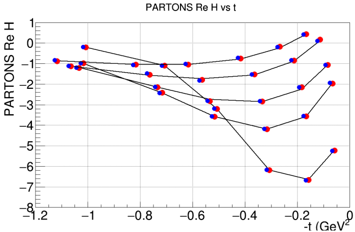


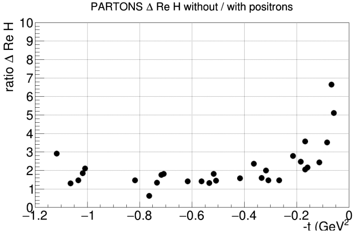
Impact of the CLAS12 Positron data

Global analysis of CLAS12 program observables $\{\sigma_{UU}, A_{LU}, A_{UL}, A_{LL}, A^C_{UU}, A^C_{LU}\}$

unpolarized beam charge asymmetry $\mathbf{A^C_{UU}}$ sensitive to the amplitude **real part**
polarized beam charge asymmetry $\mathbf{A^C_{UU}}$ sensitive to the amplitude **imaginary part**

Fitting $\{\mathcal{H}, \tilde{\mathcal{H}}\}$ assuming model values for $\{\mathcal{E}, \tilde{\mathcal{E}}\}$





Improvement of the **statistical** and **systematical** uncertainties

Model independent separation of the Interference with BH and DVCS^2

Summary and Outlook

- Generalized Parton Distributions and Imaging
- New perspective on Exclusive Reactions Physics: Mechanical Properties!
- **First Measurement of Gravitational Form Factors**
- Opens a new avenue to test confinement mechanism
- Partonic Energy Momentum Tensor
- Essential part of the EIC program