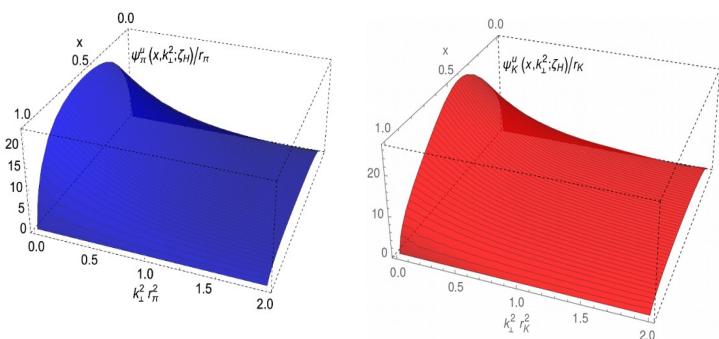




3D structure of mesons

Khépani Raya Montaño



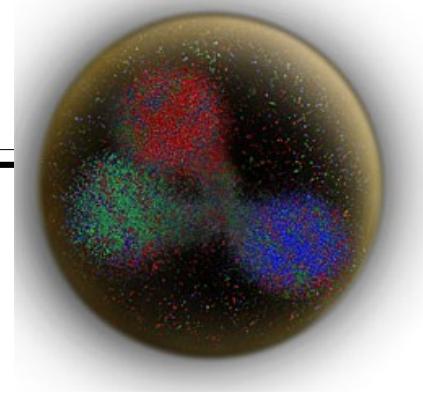
Lei Chang

Craig D. Roberts

José Rodríguez Quintero...

Strong QCD 2021
June 7 - 10, 2021. INP/NJU - China (online)

QCD and hadron physics



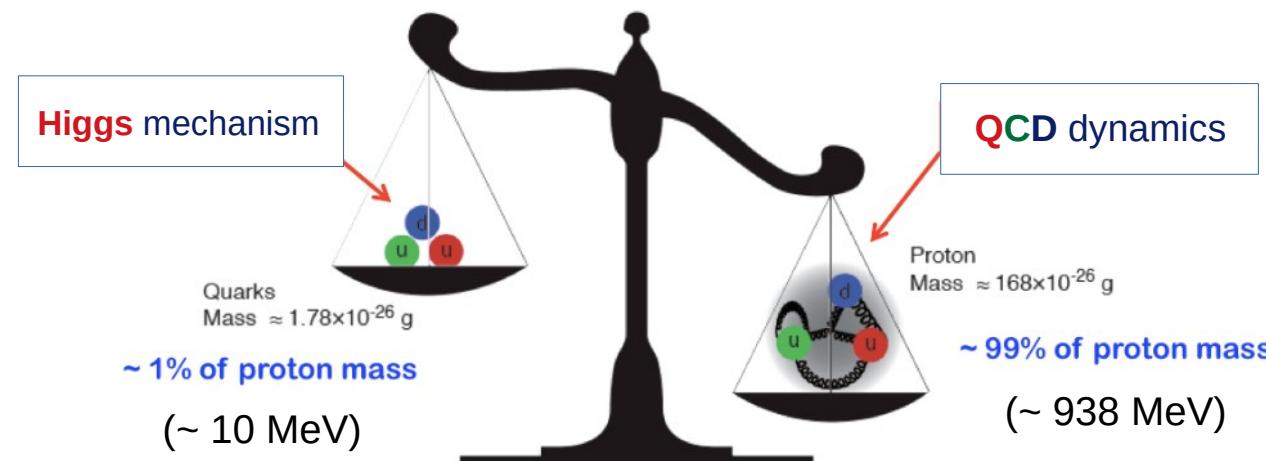
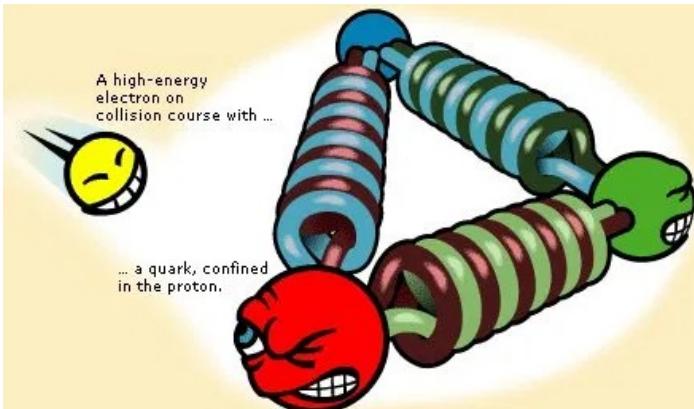
- QCD is characterized by two **emergent** phenomena:
confinement and dynamical generation of mass (**DGM**).



- ♦ Quarks and gluons not *isolated* in nature.
- Formation of colorless bound states: “**Hadrons**”

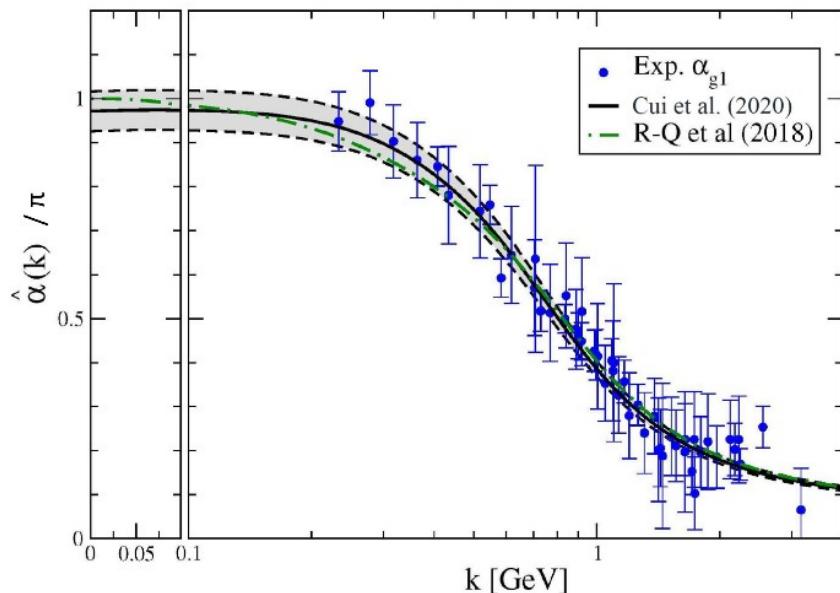


- ♦ Emergence of hadron masses (**EHM**)
from QCD **dynamics**



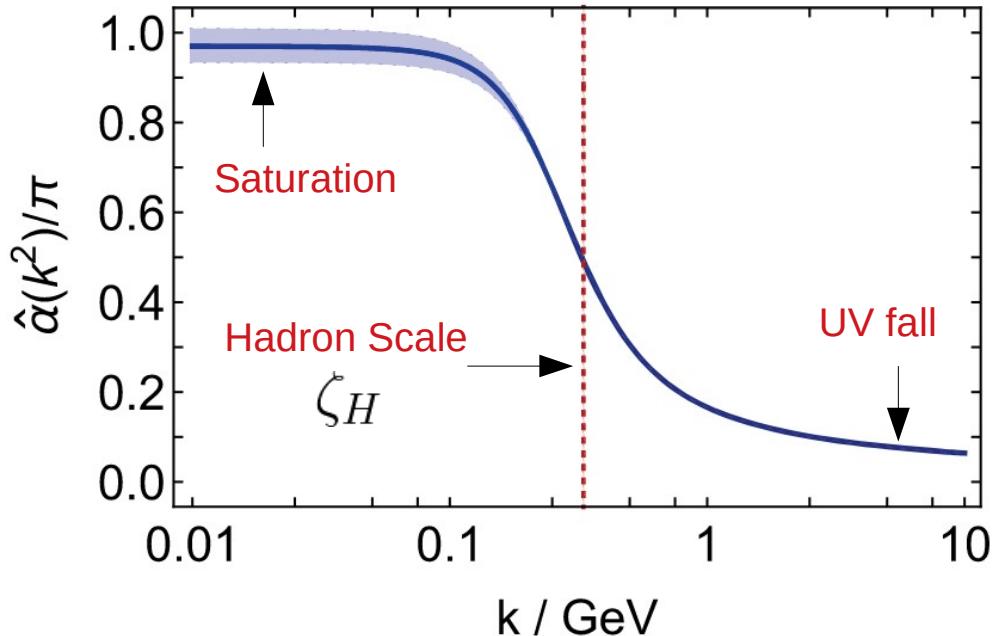
QCD and hadron physics

- These phenomena are tightly connected with QCD's peculiar **running coupling**.



Modern picture of **QCD** coupling.

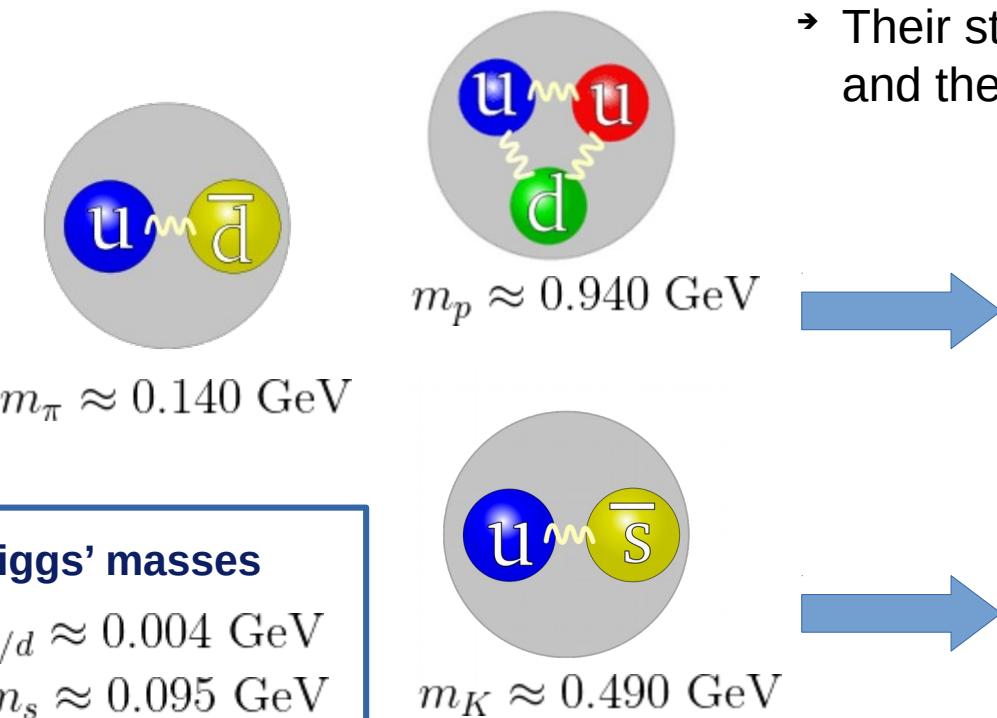
Cui: 2019dwv



ζ_H : Fully dressed **valence** quarks
express all hadron's properties

QCD and hadron physics

➤ **Pions** and **Kaons** emerge as QCD's (pseudo)-**Goldstone** bosons.



→ Their study is **crucial** to understand the **EHM** and the **hadron structure**.

- Dominated by **QCD** dynamics
Simultaneously explains the mass of the **proton** and the **masslessness** of the **pion**

- Interplay between **Higgs** and **strong** mass generating mechanisms.

'Higgs' masses

$$m_{u/d} \approx 0.004 \text{ GeV}$$
$$m_s \approx 0.095 \text{ GeV}$$

Light-front wave function (LFWF)



$$\psi_M^q(x, k_\perp^2) = \text{tr} \int dk_\parallel \delta_n^x(k_M) \gamma_5 \gamma \cdot n \chi_M(k_-, P)$$



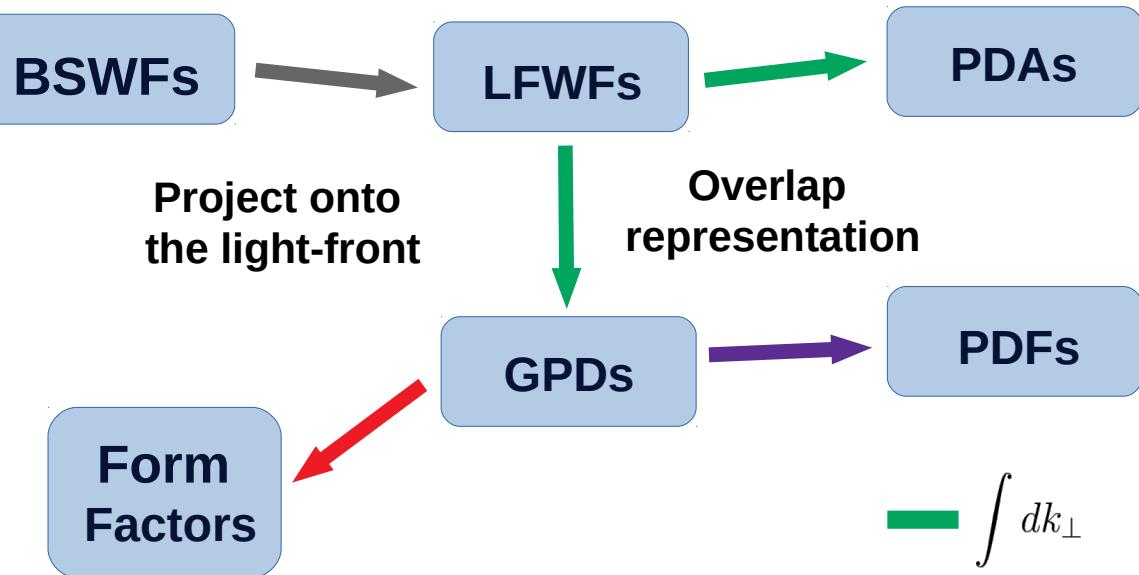
Bethe-Salpeter wave function

- Yields a **variety** of **distributions**.

"One ring to rule them all"

Light-front wave function approach

- Goal: get a **broad picture** of the pion and kaon structure.



The idea:

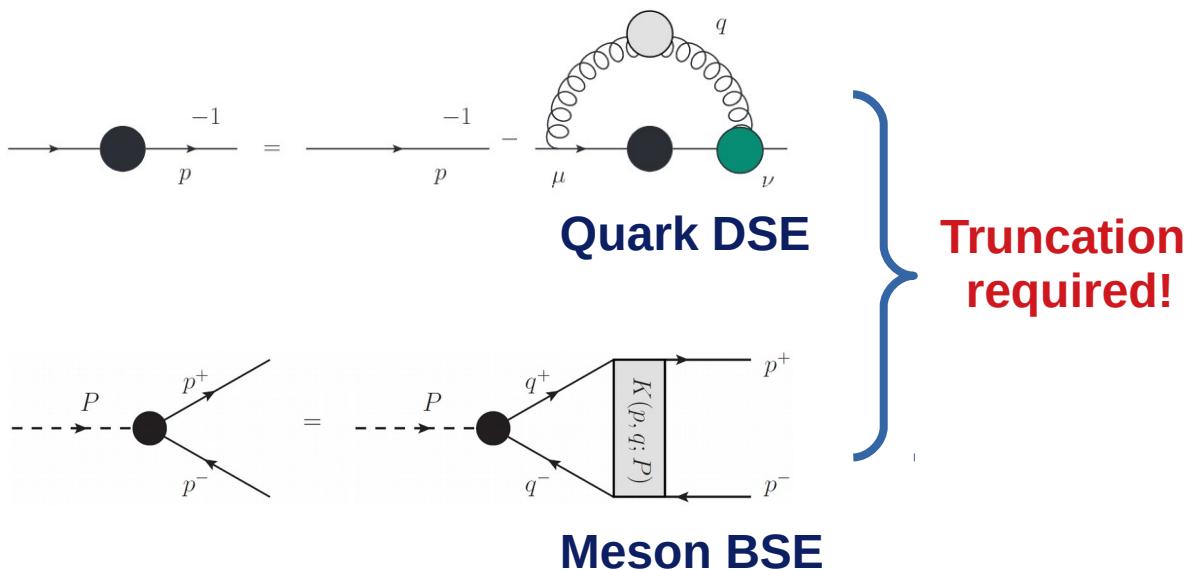
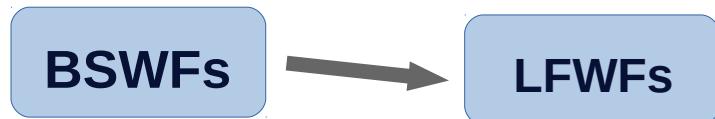
Compute ***everything*** from the **LFWF**.

- $\int dk_{\perp}$
- $\int dx$
- $t = 0, \xi = 0$

LFWF approach

$$\psi_M^q(x, k_\perp^2) = \text{tr} \int_{dk_\parallel} \delta_n^x(k_M) \gamma_5 \gamma \cdot n \chi_M(k_-, P)$$

› Goal: get a **broad picture** of the pion and kaon structure.



The idea:

Compute **everything** from the **LFWF**.

The inputs:

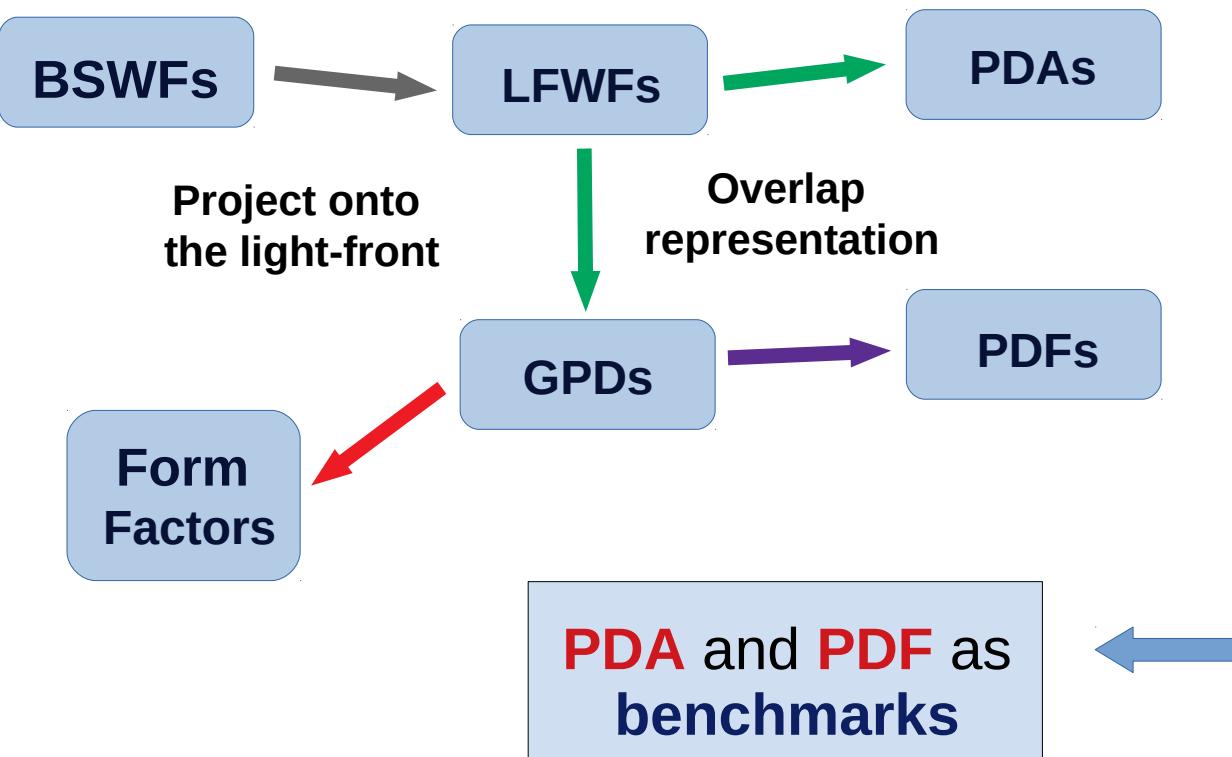
Solutions from quark **DSE** and meson **BSE**.

- ✓ Numerically **challenging**, but doable
- ✓ Already on the market: PDAs, PDFs, Form factors...

K. Raya *et al.*,
arXiv: 1911.12941 [nucl-th]

Light-front wave function approach

- Goal: get a **broad picture** of the pion and kaon structure.



The idea:

Compute **everything** from the **LFWF**.

The inputs:

Solutions from quark DSE and meson BSE.

The alternative inputs:

Model BSWF from realistic DSE *predictions*.

LFWF: Nakanishi model

- A Nakanishi-like representation for the **BSWF**:

$$n_K \chi_K^{(2)}(k_-^K; P_K) = \mathcal{M}(k; P_K) \int_{-1}^1 d\omega \rho_K(\omega) \mathcal{D}(k; P_K),$$


1 2 3

1: Matrix structure:

$$\mathcal{M}(k; P_K) = -\gamma_5 [\gamma \cdot P_K M_u + \gamma \cdot k (M_u - M_s) + \sigma_{\mu\nu} k_\mu P_{K\nu}],$$

Equivalent to considering the **leading** Bethe-Salpeter amplitude:

(from a total of 4)

$$\Gamma_M(q; P) = i\gamma_5 E_M(q; P)$$

LFWF: Nakanishi model

- A Nakanishi-like representation for the BSWF:

$$n_K \chi_K^{(2)}(k_-^K; P_K) = \mathcal{M}(k; P_K) \int_{-1}^1 d\omega \rho_K(\omega) \mathcal{D}(k; P_K) ,$$


1: Matrix structure (leading BSA):

$$\mathcal{M}(k; P_K) = -\gamma_5 [\gamma \cdot P_K M_u + \gamma \cdot k (M_u - M_s) + \sigma_{\mu\nu} k_\mu P_{K\nu}] ,$$

2: Spectral weight: Tightly connected with the meson properties.

3: Denominators: $\mathcal{D}(k; P_K) = \Delta(k^2, M_u^2) \Delta((k - P_K)^2, M_s^2) \hat{\Delta}(k_{\omega-1}^2, \Lambda_K^2) ,$

where: $\Delta(s, t) = [s + t]^{-1}$, $\hat{\Delta}(s, t) = t \Delta(s, t)$.

LFWF: Nakanishi model

- Recall the expression for the **LFWF**:

$$\psi_M^q(x, k_\perp^2) = \text{tr} \int dk_{\parallel} \delta_n^x(k_M) \gamma_5 \gamma \cdot n \chi_M(k_-, P) \quad \langle x \rangle_M^q := \int_0^1 dx x^m \psi_M^q(x, k_\perp^2)$$

- Algebraic manipulations yield:

+ Uniqueness of
Mellin moments



$$\Rightarrow \psi_M^q(x, k_\perp) \sim \int dw \rho_M(w) \dots$$

- Compactness of this result is a merit of the AM.
- Thus, $\rho_M(w)$ determines the profiles of, e.g. **PDA** and **PDF**: (it also works the **other way around**)

$$f_M \phi_M^q(x; \zeta_H) = \int \frac{d^2 k_\perp}{16\pi^3} \psi_M^q(x, k_\perp; \zeta_H)$$

$$q_M(x; \zeta_H) = \int \frac{d^2 k_\perp}{16\pi^3} |\psi_M^q(x, k_\perp; \zeta_H)|^2$$

LFWF: Nakanishi model

› More **explicitly**:

$$\psi_M^q(x, k_\perp^2; \zeta_H) = 12 [M_q(1-x) + M_{\bar{h}}x] X_P(x; \sigma_\perp^2)$$

$$\sigma_\perp = k_\perp^2 + \Omega_P^2$$

$$X_M(x; \sigma_\perp^2) = \left[\int_{-1}^{1-2x} dw \int_{1+\frac{2x}{w-1}}^1 dv + \int_{1-2x}^1 dw \int_{\frac{w-1+2x}{w+1}}^1 dv \right] \frac{\rho_M(w)}{n_M} \frac{\Lambda_M^2}{\sigma_\perp^2}$$

$$\begin{aligned} \Omega_M^2 &= v M_q^2 + (1-v) \Lambda_P^2 \\ &+ (M_{\bar{h}}^2 - M_q^2) (x - \tfrac{1}{2}[1-w][1-v]) \\ &+ (x[x-1] + \tfrac{1}{4}[1-v][1-w^2]) m_M^2 \end{aligned}$$

› Model **parameters**:

P	m_P	M_u	M_h	Λ_P	b_0^P	ω_0^P	v_P
π	0.14	0.31	M_u	M_u	0.275	1.23	0
K	0.49	0.31	$1.2M_u$	$3M_s$	0.1	0.625	0.41

$$\rho_P(\omega) = \frac{1 + \omega v_P}{2a_P b_0^P} \left[\operatorname{sech}^2 \left(\frac{\omega - \omega_0^P}{2b_0^P} \right) + \operatorname{sech}^2 \left(\frac{\omega + \omega_0^P}{2b_0^P} \right) \right]$$

Chiral limit / Factorized model

- In the **chiral limit**, the **Nakanishi model** reduces to:

$$\psi_M^q(x, k_\perp^2; \zeta_H) \sim \tilde{f}(k_\perp) \phi_M^q(x; \zeta_H) \sim f(k_\perp) [q_M(x; \zeta_H)]^{1/2}$$

“Factorized model”

$$[\phi_M^q(x; \zeta_H)]^2 \sim q_M(x; \zeta_H)$$

↗ Sensible assumption as long as:

$$m_M^2 \approx 0 \quad \text{(meson mass)}$$

$$M_{\bar{q}}^2 - M_q^2 \approx 0 \quad \text{(antiquark – quark masses)}$$

↗ Produces **identical** results
as Nakanishi model for **pion**

- Therefore:

$$\psi_M^q(x, k_\perp^2; \zeta_H) = [q_M(x; \zeta_H)]^{1/2} \left[4\sqrt{3}\pi \frac{M_q^3}{(k_\perp^2 + M_q^2)^2} \right]$$

Single parameter!

No need to determine the spectral weight !

Factorized/chiral limit models

$$\psi_M^q(x, k_\perp^2; \zeta_H) = [q^M(x; \zeta_H)]^{1/2} \left[4\sqrt{3}\pi \frac{M_q^3}{(k_\perp^2 + M_q^2)^2} \right]$$

“Chiral M1”

- Can be improved as follows:

Implies an **extra** power of $1/k_\perp^2$

$$\psi_M^q(x, k_\perp^2; \zeta_H) = [q^M(x; \zeta_H)]^{1/2} \left[\frac{4\pi}{1 + \gamma\sqrt{3} + \gamma^2} \left(\frac{\sqrt{3}M_q^3}{(k_\perp^2 + M_q^2)^2} + \gamma \frac{M_q}{k_\perp^2 + M_q^2} \right) \right]$$

“Chiral M2”

Equivalent to considering the two
most dominant BSAs:

$$\Gamma_M(q; P) = \gamma_5 [iE_M(q; P) + \gamma \cdot P F_M(q; P)]$$

Factorized/chiral limit models

$$\psi_M^q(x, k_\perp^2; \zeta_H) = [q^M(x; \zeta_H)]^{1/2} \left[4\sqrt{3}\pi \frac{M_q^3}{(k_\perp^2 + M_q^2)^2} \right]$$

“Chiral M1”

- >We can also consider a “Gaussian model”:

$$\psi_M^q(x, k_\perp^2; \zeta_H) = [q^M(x; \zeta_H)]^{1/2} \left(\frac{32\pi^2 r_M^2}{\chi_M^2(\zeta_H)} \right)^{1/2} \exp \left[-\frac{r_M^2 k_\perp^2}{2\chi_M^2(\zeta_H)} \right]$$

$$\chi_M^2(\zeta_H) = \langle x^2 \rangle_{\zeta_H}^{\bar{h}} + \frac{1}{2}(1-c)\langle x^2 \rangle_{\zeta_H}^{\bar{h}}$$


Asymmetry factor $\sim M_{\bar{h}}^2 - M_u^2$

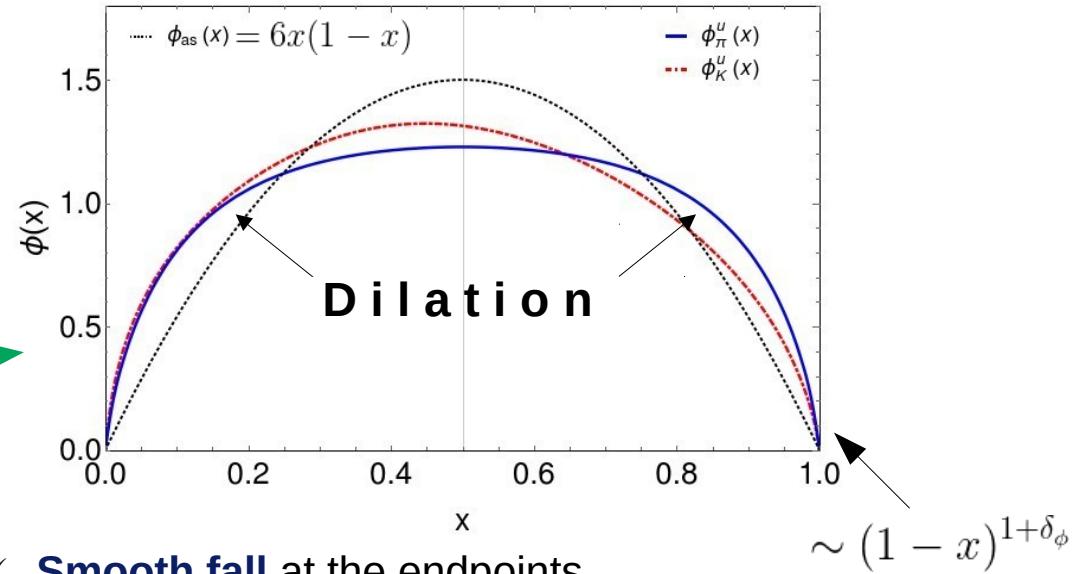
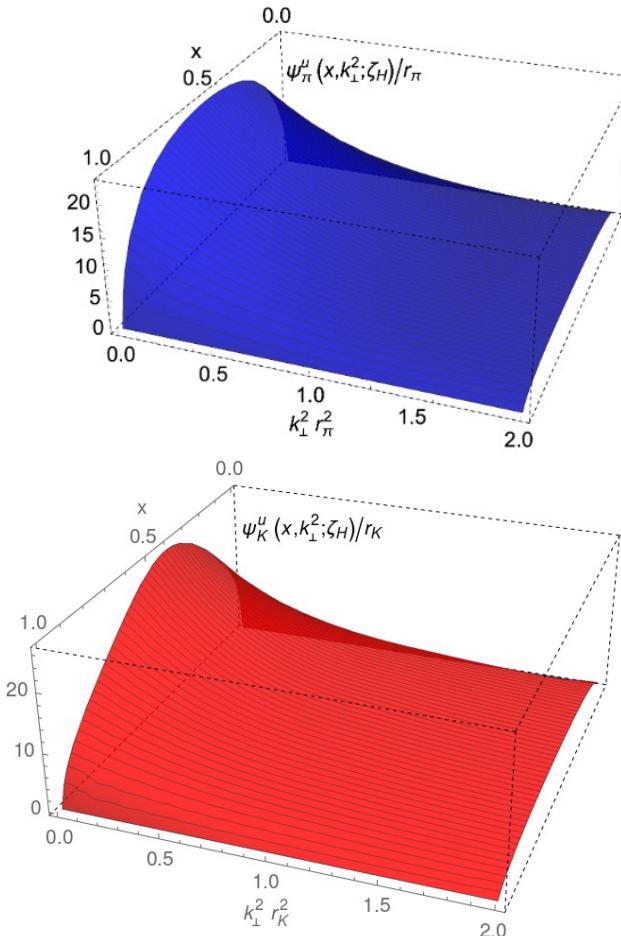
- **One parameter** to determine **both** models:
 - Either M_q or r_M
(charge radius)



- Unless specified otherwise, **Nakanishi model** results will be shown.
- By construction, **PDA** and **PDF** are the **same** in any presented model.
- In general, **Chiral M1 \approx Nakanishi** (for pion)

LFWFs and PDAs

$$f_M \phi_M^q(x; \zeta_H) = \int \frac{d^2 k_\perp}{16\pi^3} \psi_M^q(x, k_\perp; \zeta_H)$$



- ✓ **Smooth fall** at the endpoints
- ✓ **Broad** and concave functions of x
 - Consequence of **DCSB**
- ✓ **Higgs induced asymmetry for Kaon:**
 - Modulated by the difference $M_s - M_u$

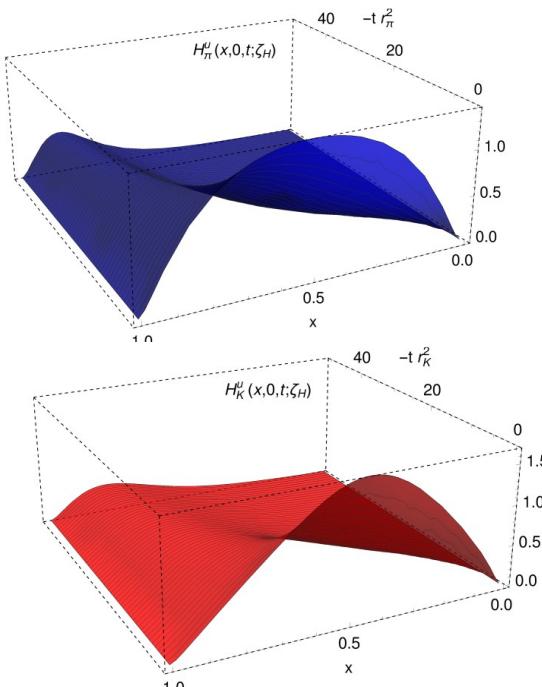
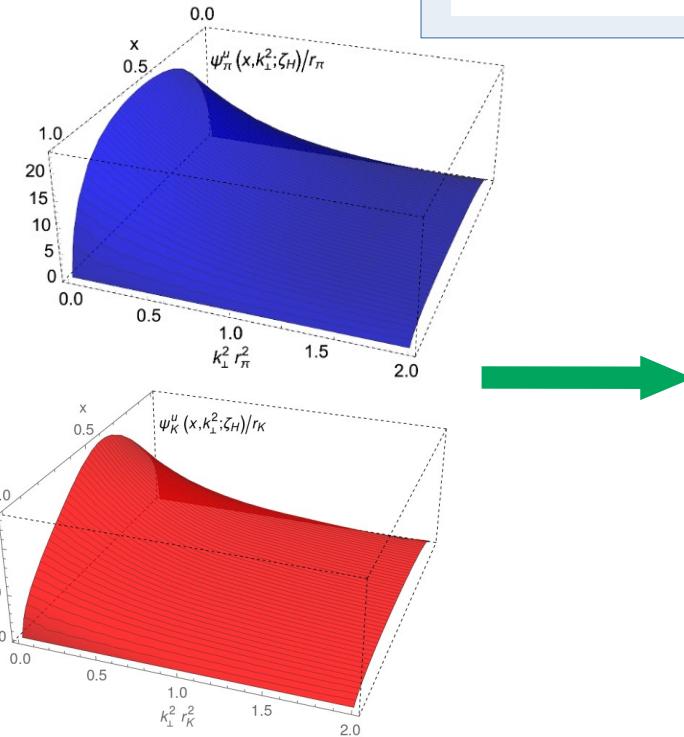
LFWFs and GPDs

LFWFs

GPDs

- In the **overlap representation**, the valence-quark **GPD** reads as:

$$H_M^q(x, \xi, t) = \int \frac{d^2 k_\perp}{16\pi^3} \psi_M^{q*}(x^-, (k_\perp^-)^2) \psi_M^q(x^+, (k_\perp^+)^2)$$



- ✓ **Valid** in the **DGLAP** region
- ✓ **Positivity** fulfilled
- ✓ Can be **extended** to the **ERBL** region $|x| \leq \xi$
- ✓ **Analytic** in our factorized models.

Chouika:2017rzs

GPD: factorized model

$$H_M^q(x, \xi, t) = \int \frac{d^2 k_\perp}{16\pi^3} \psi_M^{q*}(x^-, (\mathbf{k}_\perp^-)^2) \psi_M^q(x^+, (\mathbf{k}_\perp^+)^2)$$

Overlap
representation

Factorized
LFWF

$$\psi_M^q(x, k_\perp^2; \zeta_H) = [q^M(x; \zeta_H)]^{1/2} \tilde{\psi}_M(k_\perp^2; \zeta_H)$$

PDF controls (mostly) the x -dependence

$$H_M^q(x, \xi, t; \zeta_H) = \theta(x_-) [q^M(x_-; \zeta_H) q^M(x_+; \zeta_H)]^{1/2} \Phi_M(z; \zeta_H)$$

$$\Phi_M(z; \zeta_H) = \int \frac{d^2 k_\perp}{16\pi^3} \tilde{\psi}_M(k_\perp^2; \zeta_H) \tilde{\psi}_M((k_\perp - s_\perp)^2; \zeta_H)$$

$$x_\pm = \frac{x \pm \xi}{1 \pm \xi}$$

t -dependence of GPD is contained herein

$$z = s_\perp^2 = \frac{-t(1-x)^2}{1-\xi^2}$$

GPD: factorized model

$$\psi_M^q(x, k_\perp^2; \zeta_H) = [q^M(x; \zeta_H)]^{1/2} \left[\frac{4\pi}{1 + \gamma\sqrt{3} + \gamma^2} \left(\frac{\sqrt{3}M_q^3}{(k_\perp^2 + M_q^2)^2} + \gamma \frac{M_q}{k_\perp^2 + M_q^2} \right) \right]$$

“Chiral M2”

Chiral M2 **factorized** model produces:

$$\Phi_M(z; \zeta_H) = \frac{1}{1 + \gamma\sqrt{3} + \gamma^2} \left(\Phi_M^{(A)}(z; \zeta_H) + \gamma\sqrt{3}\Phi_M^{(AB)}(z; \zeta_H) + \gamma^2\Phi_M^{(B)}(z; \zeta_H) \right)$$

↑

$$\Phi_M^{(A)}(z; \zeta_H) = \frac{6M^6}{(z + 4M^2)^3} \left(10 + \frac{z}{M^2} + \frac{8(z + M^2)}{z} \left[\sqrt{\frac{z + 4M^2}{z}} \operatorname{atanh} \left(\sqrt{\frac{z}{z + 4M^2}} \right) - 1 \right] \right)$$

GPD: factorized model

$$\psi_M^q(x, k_\perp^2; \zeta_H) = [q^M(x; \zeta_H)]^{1/2} \left[\frac{4\pi}{1 + \gamma\sqrt{3} + \gamma^2} \left(\frac{\sqrt{3}M_q^3}{(k_\perp^2 + M_q^2)^2} + \gamma \frac{M_q}{k_\perp^2 + M_q^2} \right) \right]$$

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↑

$$\Phi_M^{(AB)}(z; \zeta_H) = \frac{8M^4}{(z + 4M^2)^2} \left(1 + \frac{z}{4M^2} + \sqrt{\frac{z + 4M^2}{z}} \operatorname{atanh} \left(\sqrt{\frac{z}{z + 4M^2}} \right) \right)$$

GPD: factorized model

$$\psi_M^q(x, k_\perp^2; \zeta_H) = [q^M(x; \zeta_H)]^{1/2} \left[\frac{4\pi}{1 + \gamma\sqrt{3} + \gamma^2} \left(\frac{\sqrt{3}M_q^3}{(k_\perp^2 + M_q^2)^2} + \gamma \frac{M_q}{k_\perp^2 + M_q^2} \right) \right]$$

“Chiral M2”

Chiral M2 **factorized** model produces:

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Observations:

- $\gamma=0$ recovers Chiral M1
- Chiral M1 \approx Nakanishi (for pion)
- Gaussian model also gives an algebraic GPD

Zhang:2021mtn

$$\Phi_M^{(B)}(z; \zeta_H) = \frac{4M^2}{z + 4M^2} \frac{\operatorname{atanh}\left(\sqrt{\frac{z}{z + 4M^2}}\right)}{\sqrt{\frac{z}{z + 4M^2}}}$$

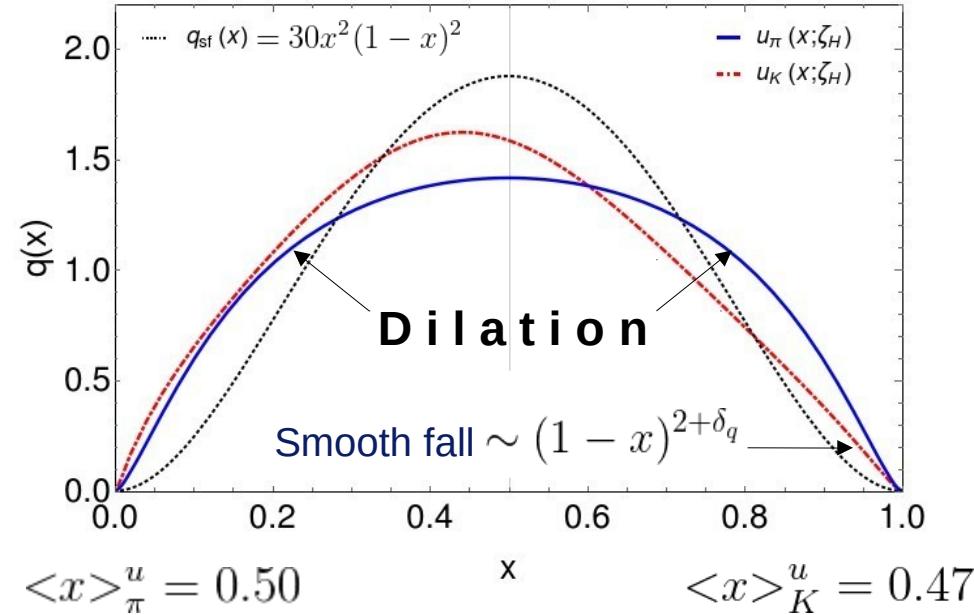
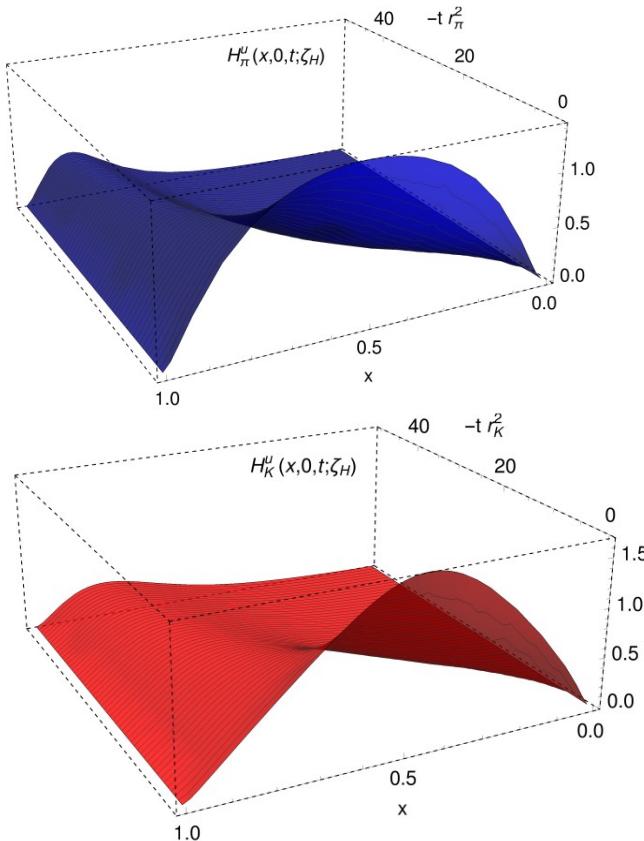
LFWFs and PDFs

GPD

PDF

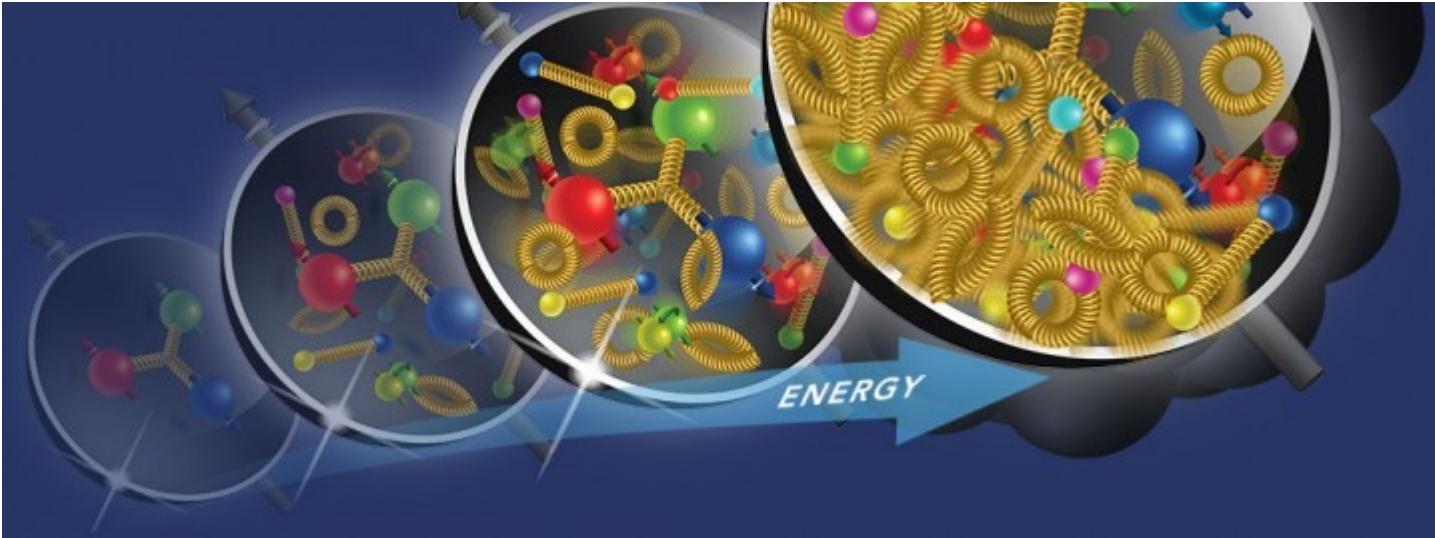
- The PDF is obtained from the **forward limit** of the GPD.

$$q(x) = H(x, 0, 0)$$



- ζ_H : meson properties determined by the fully-dressed valence-quarks.
- Broad + Higgs**-induced asymmetry

Some words on evolution...



DGLAP + Effective Coupling

Idea. Define an effective coupling such that:

Starting from fully-dressed
quasiparticles, at ζ_H

(at which **valence quarks**
carry **all** meson's **properties**)

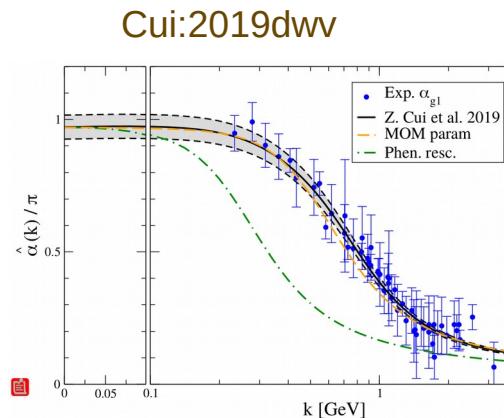
Unveil **Sea** and **Gluon** content,
as prescribed by **QCD**

$$\left\{ \zeta^2 \frac{d}{d\zeta^2} \int_0^1 dy \delta(y-x) - \boxed{\frac{\alpha(\zeta^2)}{4\pi}} \int_x^1 \frac{dy}{y} \begin{pmatrix} P_{qq}^{\text{NS}}\left(\frac{x}{y}\right) & 0 \\ 0 & \mathbf{P}^S\left(\frac{x}{y}\right) \end{pmatrix} \right\} \begin{pmatrix} H_\pi^{\text{NS},+}(y, t; \zeta) \\ \mathbf{H}_\pi^S(y, t; \zeta) \end{pmatrix} = 0$$

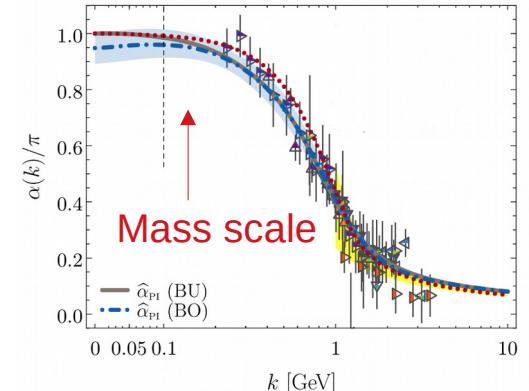
"All orders hypothesis"

- Features of **QCD effective charge** lead to the **answer**.
- And ζ_H can be **properly defined**.

Not tuned!



Rodriguez-Quintero:2018wma



DGLAP + Effective Coupling

Cui:2020tdf

Idea. Define an effective coupling such that:

$$\frac{d}{dt}q(x; t) = -\frac{\alpha(t)}{4\pi} \int_x^1 \frac{dy}{y} q(y; t) P\left(\frac{x}{y}\right)$$

“All orders hypothesis”

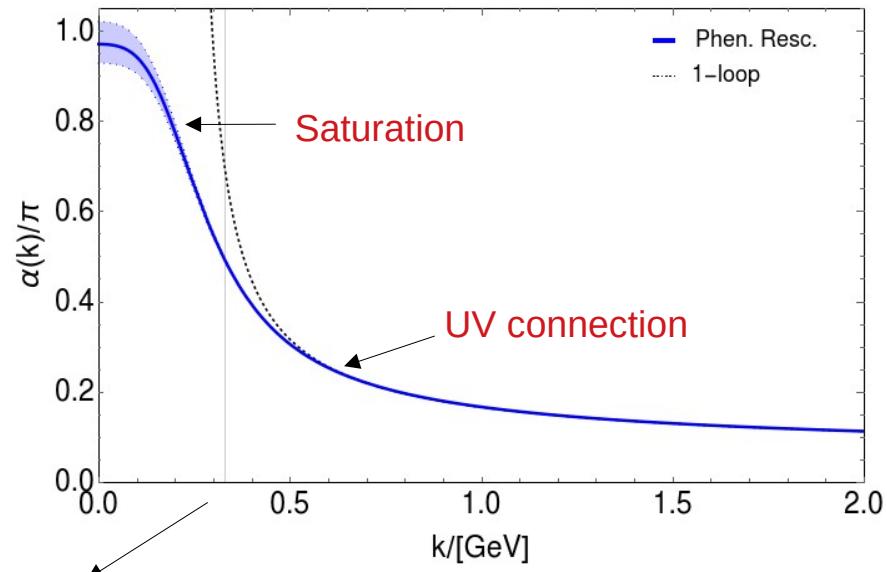
› The ***coupling***:

$$\alpha(k^2) = \frac{\gamma_m \pi}{\ln \left[\frac{\mathcal{M}^2(k^2)}{\Lambda_{\text{QCD}}^2} \right]} ; \quad \alpha(0) = 0.97(4)$$

› Where $\mathcal{M}(k^2 = \Lambda_{\text{QCD}}^2) := m_G = 0.331(2) \text{ GeV}$ defines a screening mass.

› We identify: $\zeta_H := m_G(1 \pm 0.1)$  **10% uncertainty**

(fully dressed quasiparticles are the correct degrees of freedom)



'All orders' hypothesis

Implication 1: valence-quark PDF

$$\langle x^n(\zeta_f) \rangle_q = \exp\left(-\frac{\gamma_{qq}^{(n)}}{4\pi} S(\zeta_0, \zeta_f)\right) \langle x^n(\zeta_0) \rangle_q = \langle x^n(\zeta_H) \rangle_q \underbrace{\left(\frac{\langle x(\zeta_f) \rangle_q}{\langle x(\zeta_H) \rangle_q}\right)^{\gamma_{qq}^{(n)} / \gamma_{qq}^{(1)}}}_{\text{This ratio encodes the information of the charge}}$$

$$q = u, \bar{d}$$

$$S(\zeta_0, \zeta_f) = \int_{2 \ln(\zeta_0 / \Lambda_{\text{QCD}})}^{2 \ln(\zeta_f / \Lambda_{\text{QCD}})} dt \alpha(t)$$

Implication 2: glue and sea-quark DFs ($n_f=4$)

$$\langle 2x(\zeta_f) \rangle_q = \exp\left(-\frac{8}{9\pi} S(\zeta_H, \zeta_f)\right), \quad q = u, \bar{d};$$

$$\langle x(\zeta_f) \rangle_{\text{sea}} = \langle x(\zeta_f) \rangle_{\sum_q q + \bar{q}} - (\langle x(\zeta_f) \rangle_u + \langle x(\zeta_f) \rangle_{\bar{d}}),$$

$$= \frac{3}{7} + \frac{4}{7} \langle 2x(\zeta_f) \rangle_u^{1/4} - \langle 2x(\zeta_f) \rangle_u$$

$$\langle x(\zeta_f) \rangle_g = \frac{4}{7} \left(1 - \langle 2x(\zeta_f) \rangle_u^{1/4}\right);$$

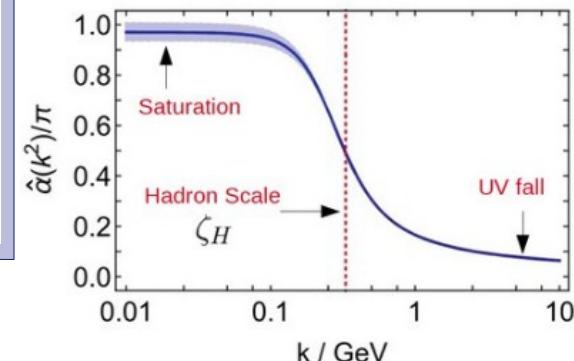
Momentum sum rule:

$$\langle 2x(\zeta_f) \rangle_q + \langle x(\zeta_f) \rangle_{\text{sea}} + \langle x(\zeta_f) \rangle_g = 1$$

$\zeta_f / \zeta_H \rightarrow \infty$

A textbook result:
G. Altarelli, Phys. Rep. 81, 1 (1982)

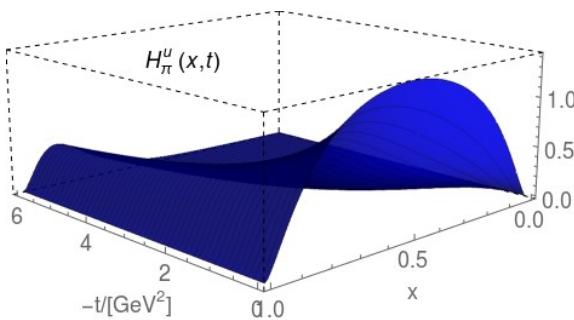
- Closed **algebraic** relations between momentum fractions
- **Recovery** of sum rule and asymptotic limits
- Clear connection with the **hadron scale**.
- Therefore, the scale is **unambiguously** defined (**not** fine tuned)



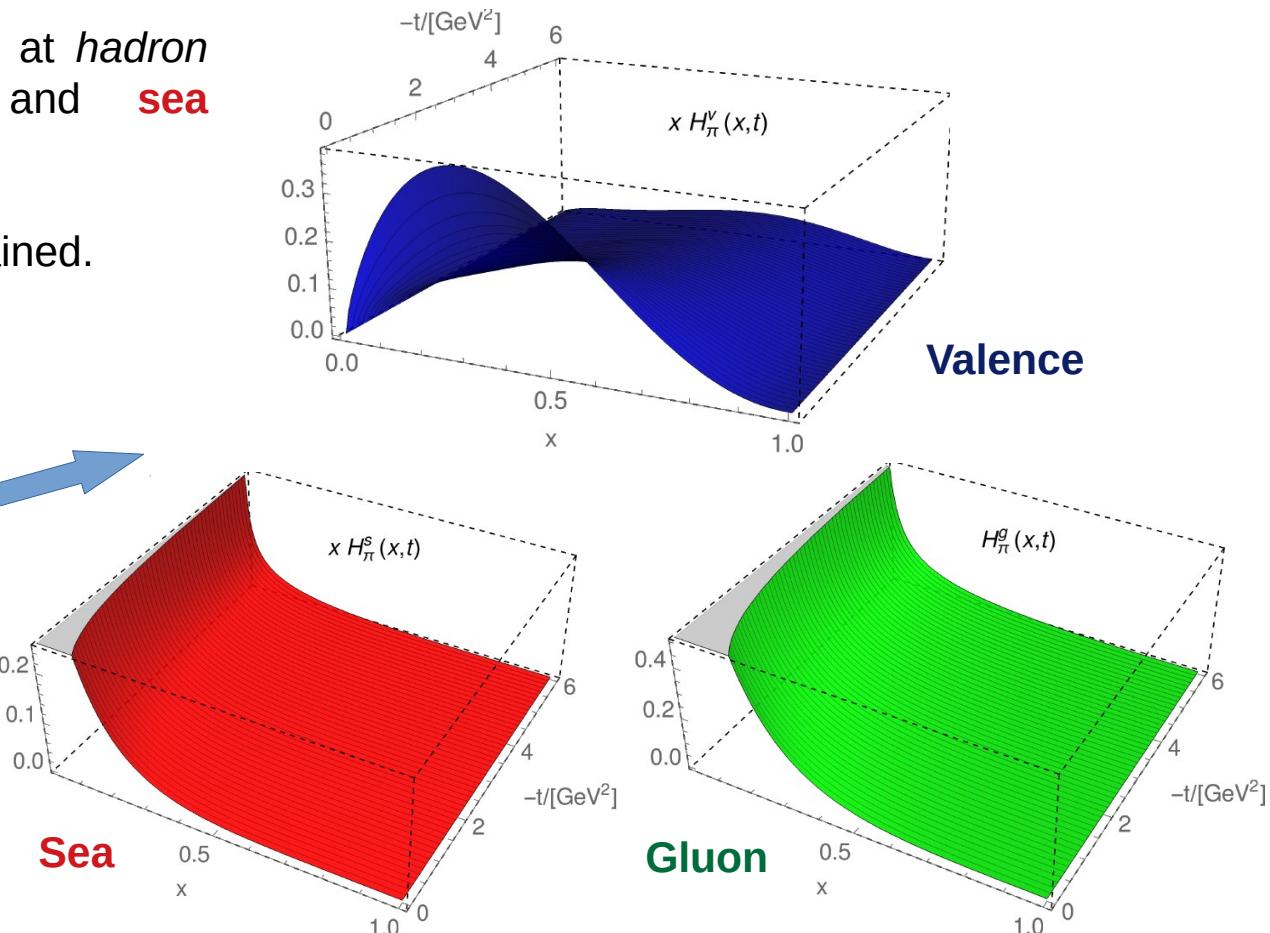
Evolved distributions: GPDs

$\zeta = 5.2 \text{ GeV}$

- Starting with **valence** distributions, at *hadron scale*, and generate **gluon** and **sea** distributions via evolution equations.
- Thus **gluon** and **sea** GPDs are obtained.



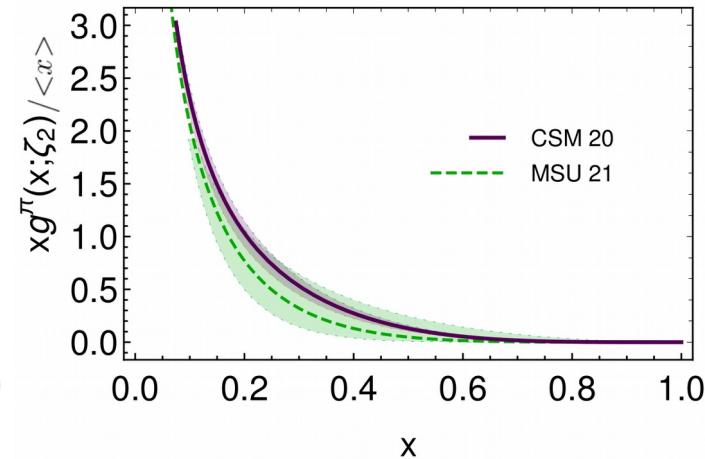
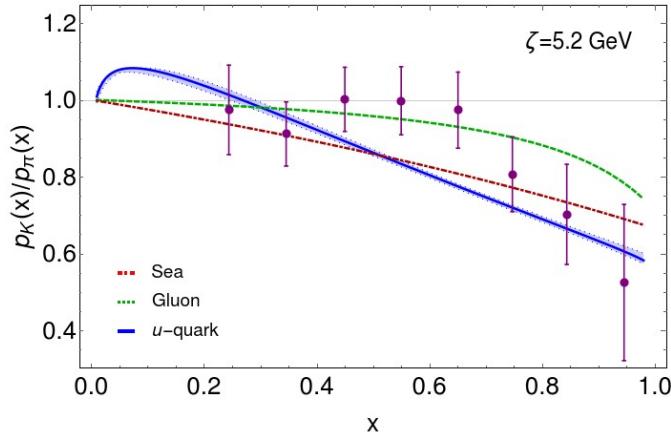
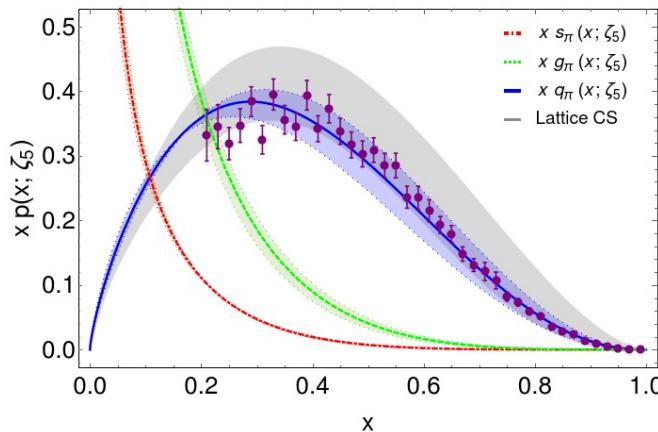
$$\zeta_H = 0.331 \text{ GeV}$$



Evolved PDFs

GPD

PDF



- **Not tuned**, initial scale for evolution $\zeta_H = 0.331 \text{ GeV}$
- Valence at **2 GeV**

	$\langle x \rangle_u^\pi$	$\langle x^2 \rangle_u^\pi$	$\langle x^3 \rangle_u^\pi$
IQCD [53]	0.21(1)	0.16(3)	
IQCD [54]	0.254(03)	0.094(12)	0.057(04)
Ref. [102]	0.24	0.098	0.049
Refs. [39, 40]	0.24(2)	0.098(10)	0.049(07)
Herein	0.24(2)	0.094(13)	0.047(08)

- In **agreement** with:
 - ✓ **ASV analysis** Aicher:2010ocb
 - ✓ **Lattice CS** Sufian:2020vzb
Sufian:2019bol
 - ✓ **DSEs** Cui:2020tdf

- Valence at **5.2 GeV**



$$\langle x \rangle_\pi^{\text{val}} = 0.41(4)$$

$$\langle x \rangle_K^{\text{val}} = 0.43(4)$$

**Moving on to form factors
and so on...**

Electromagnetic FFs

GPD



FFs

- Electromagnetic form factor is obtained from the **t-dependence** of the **0-th moment**:

$$F_M^q(-t = \Delta^2) = \int_{-1}^1 dx H_M^q(x, \xi, t)$$

Can safely take $\xi = 0$

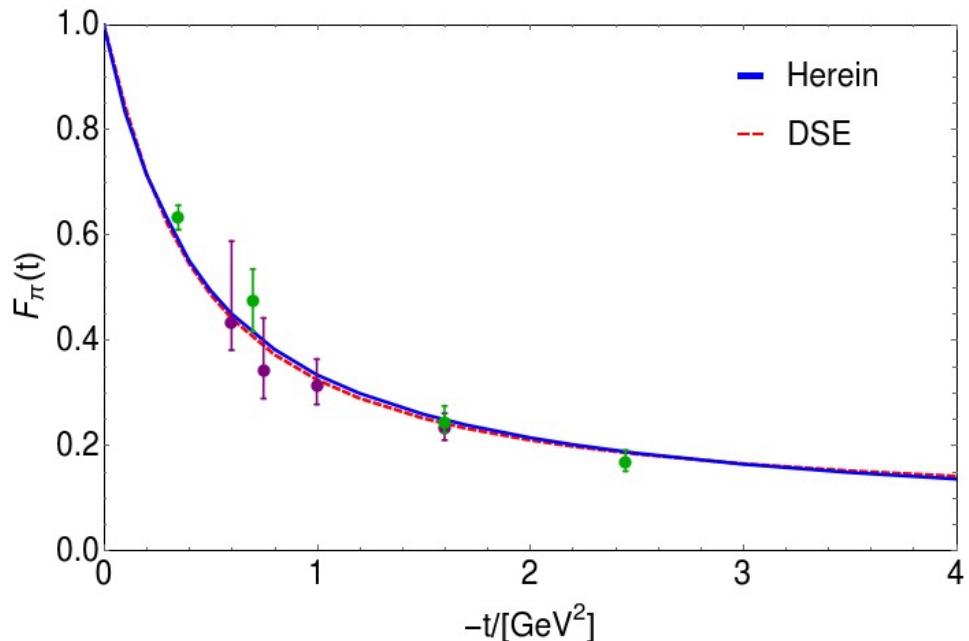
“Polynomiality”

$$F_M(\Delta^2) = e_u F_M^u(\Delta^2) + e_{\bar{f}} F_M^{\bar{f}}(\Delta^2)$$

Weighed by electric charges

- Isospin symmetry

$$\rightarrow F_{\pi^+}(-t) = F_{\pi^+}^u(-t)$$



Data: G.M. Huber *et al.* PRC 78 (2008) 045202

DSE: L. Chang *et al.* PRL 111 (2013) 14, 141802

Electromagnetic FFs

GPD

FFs

- Electromagnetic form factor: pion models

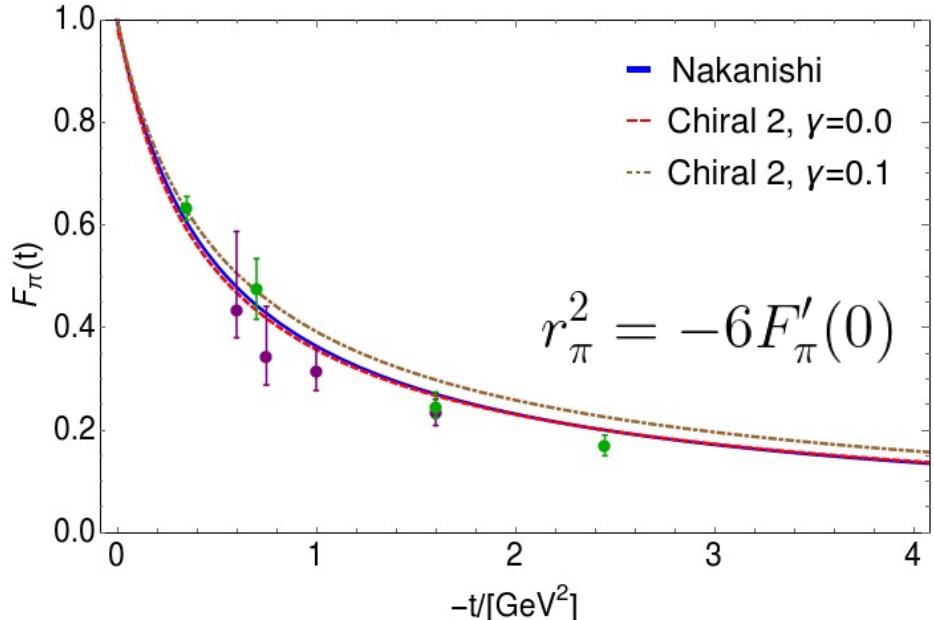
$$F_M^q(-t = \Delta^2) = \int_{-1}^1 dx H_M^q(x, \xi, t)$$

- In the **chiral limit M2**:

$$\frac{1 + \frac{5\sqrt{3}}{9}\gamma + \frac{5}{18}\gamma^2}{1 + \gamma\sqrt{3} + \gamma^2} = \frac{5}{18} \frac{M^2 r_\pi^2}{\langle x^2 \rangle_u^{\zeta_H}}$$

- For $M_q \simeq 0.3$ GeV and $\gamma \simeq 0.1$

→ $r_\pi \simeq 0.66$ fm



“Chiral M2”

$i\gamma_5 E_\pi(k)$ $\gamma_5 \gamma \cdot P F_\pi(k)$

$$\psi_M^q(x, k_\perp^2; \zeta_H) = [q^M(x; \zeta_H)]^{1/2} \left[\frac{4\pi}{1 + \gamma\sqrt{3} + \gamma^2} \left(\frac{\sqrt{3}M_q^3}{(k_\perp^2 + M_q^2)^2} + \gamma \frac{M_q}{k_\perp^2 + M_q^2} \right) \right]$$

Gravitational FFs

GPD

FFs

- Gravitational form factors are obtained from the **t-dependence** of the **1-st moment**:

$$J_M(t, \xi) = \int_{-1}^1 dx x H_M(x, \xi, t) = \Theta_2^M(t) - \xi^2 \Theta_1^M(t)$$

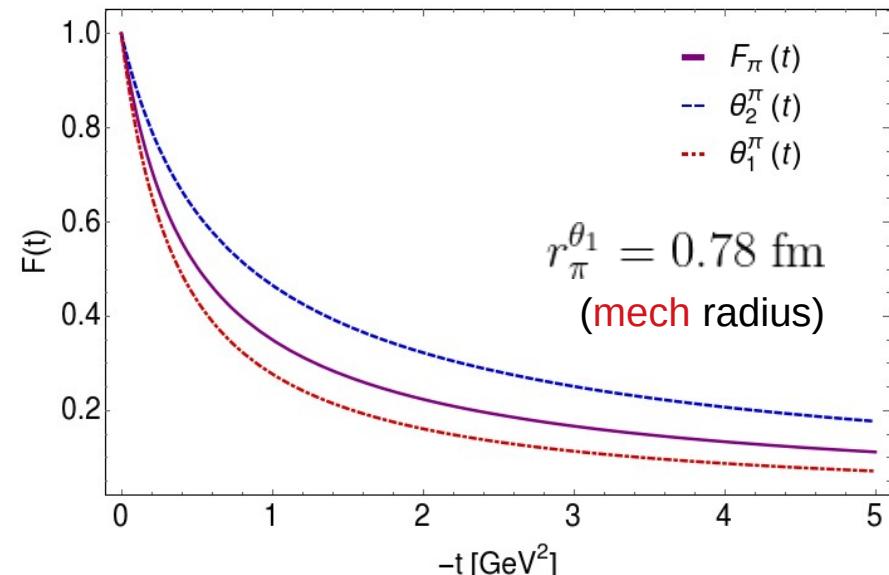
- ✓ Directly obtained if $\xi = 0$
- ✓ Only **DGLAP** GPD is required
 - ✗ **ERBL** GPD needed

- Sophisticated techniques exist. Chouika:2017dhe
- But a sound expression can be constructed:

$$\theta_1^{P_q}(\Delta^2) = c_1^{P_q} \theta_2^{P_q}(\Delta^2)$$

“Soft pion theorem”

$$+ \int_{-1}^1 dx x \left[H_P^q(x, 1, 0) P_{M_q}(\Delta^2) - H_P^q(x, 1, -\Delta^2) \right]$$



$r_\pi^E = 0.68 \text{ fm}$, $r_\pi^{\theta_2} = 0.56 \text{ fm}$
 (charge radius) (mass radius)

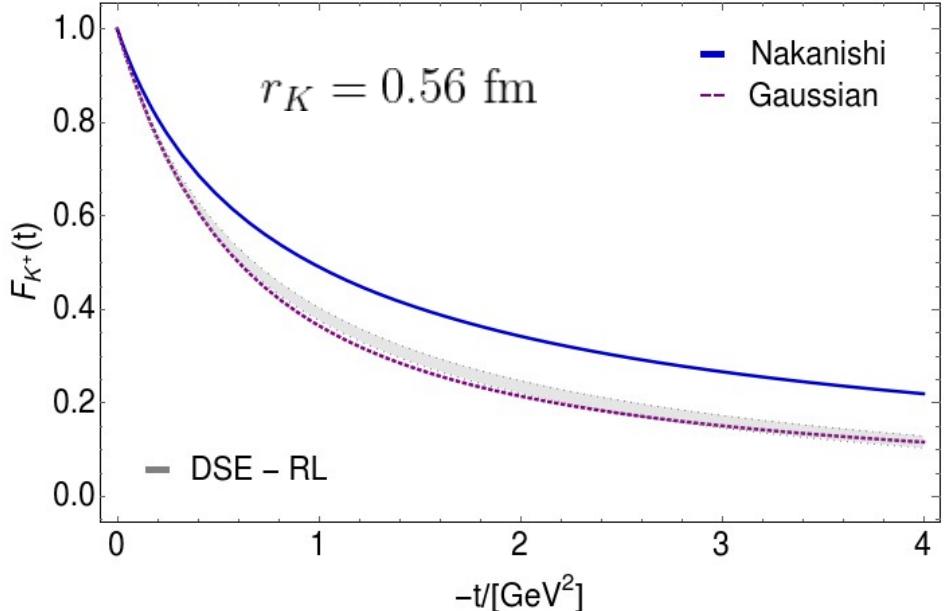
Electromagnetic FFs

GPD



FFs

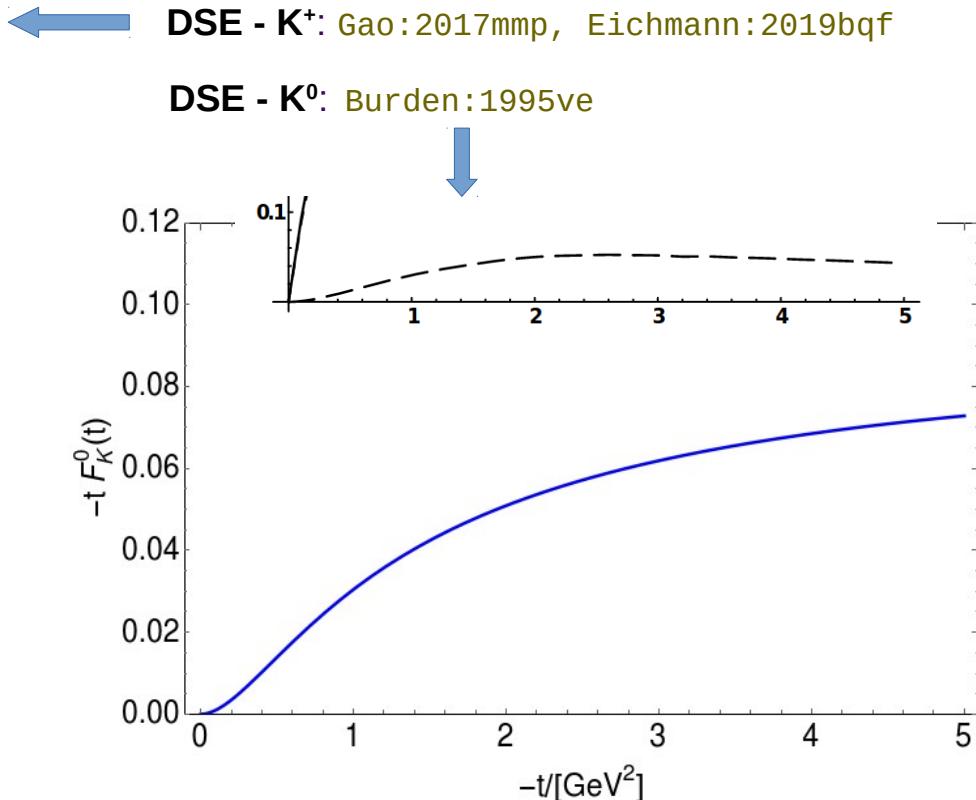
- Electromagnetic form factor: **charged** and **neutral** kaon



Kaon is more
compressed

$$r_K^j \approx 0.85 r_\pi^j$$

$j = \text{mech, charge, mass}$



On the Radii

GPD

FFs

$$H_M^q(x, \xi, t; \zeta_H) = \theta(x_-) [q^M(x_-; \zeta_H) q^M(x_+; \zeta_H)]^{1/2} \Phi_M(z; \zeta_H)$$

- In the **factorized** models:

$$\frac{\partial^n}{\partial z^n} \Phi_P^u(z; \zeta_H) \Big|_{z=0} = \frac{1}{\langle x^{2n} \rangle_{\bar{h}}^{\zeta_H}} \left. \frac{d^n F_P^u(\Delta^2)}{d(\Delta^2)^n} \right|_{\Delta^2=0}$$

↑
Derivatives of EFF

PDF moments

$$\begin{aligned} \frac{\partial}{\partial z} \Phi_P^u(z; \zeta_H) \Big|_{z=0} &= -\frac{r_P^2}{4\chi_P^2(\zeta_H)}, \\ \frac{\partial}{\partial z} \Phi_{\bar{P}}^{\bar{h}}(z; \zeta_H) \Big|_{z=0} &= (1 - d_P) \frac{\partial}{\partial z} \Phi_P^u(z; \zeta_H) \Big|_{z=0} \end{aligned}$$

↑
Asymmetry term = 0 for pion

- Therefore, the **mass radius**:

$$r_{P_u}^{\theta_2} = \frac{3r_P^2}{2\chi_P^2} \langle x^2(1-x) \rangle_{P_{\bar{h}}},$$

$$r_{P_{\bar{h}}}^{\theta_2} = \frac{3r_P^2}{2\chi_P^2} (1 - d_P) \langle x^2(1-x) \rangle_{P_u}$$

$$\left(\frac{r_{\pi}^{\theta_2}}{r_{\pi}^E} \right)^2 = \frac{\langle x^2(1-x) \rangle_{\zeta_H}^q}{\langle x^2 \rangle_{\zeta_H}^q} \approx \left(\frac{4}{5} \right)^2$$

Determined from **PDF moments!**

On the Radii

GPD

FFs

$$J_M(t, \xi) = \int_{-1}^1 dx x H_M(x, \xi, t) = \Theta_2^M(t) - \xi^2 \Theta_1^M(t)$$

$$F_M^q(-t = \Delta^2) = \int_{-1}^1 dx H_M^q(x, \xi, t)$$

- The **ordering** of **radii**: (in fm)

$$\begin{array}{ccc} r_\pi^{\theta_1} = 0.78 & > r_\pi^E = 0.68 & > r_\pi^{\theta_1} = 0.56 \\ (\text{mech}) & & (\text{charge}) & & (\text{mass}) \end{array}$$

$$r_K^j \approx 0.85 r_\pi^j$$

j = charge, mass, mech.

- Mean-squared **transverse extent**:

$$\langle b_\perp^2(\zeta_{\mathcal{H}}) \rangle_u^K = 0.71 r_K^2, \langle b_\perp^2(\zeta_{\mathcal{H}}) \rangle_{\bar{s}}^K = 0.58 r_K^2$$

$$\langle b_\perp^2(\zeta_{\mathcal{H}}) \rangle_u^\pi = \frac{2}{3} r_\pi^2 = \langle b_\perp^2(\zeta_{\mathcal{H}}) \rangle_{\bar{d}}^\pi$$

Algebraic derivation!

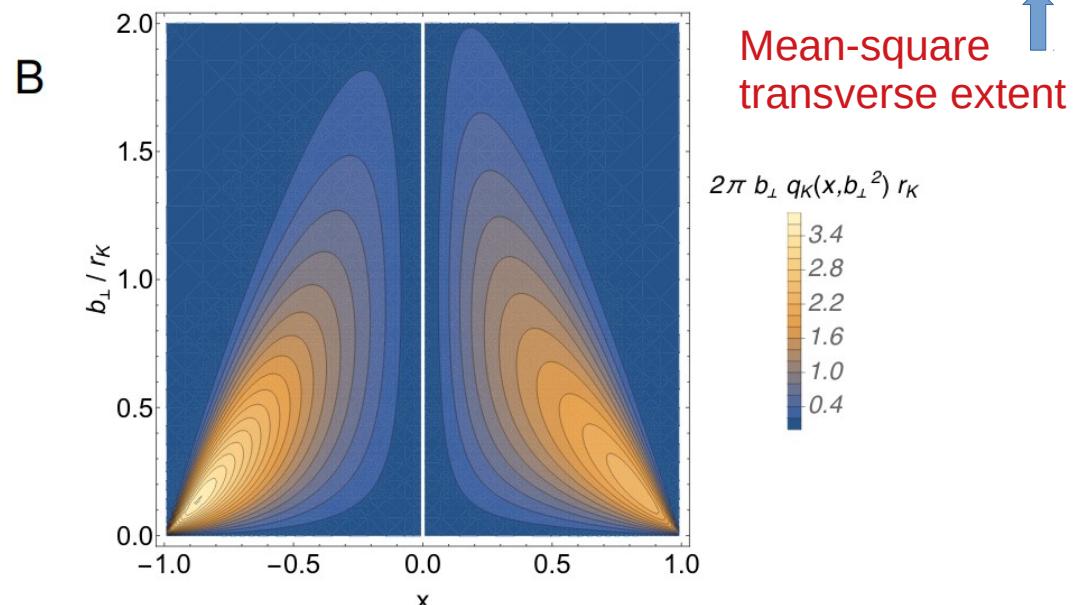
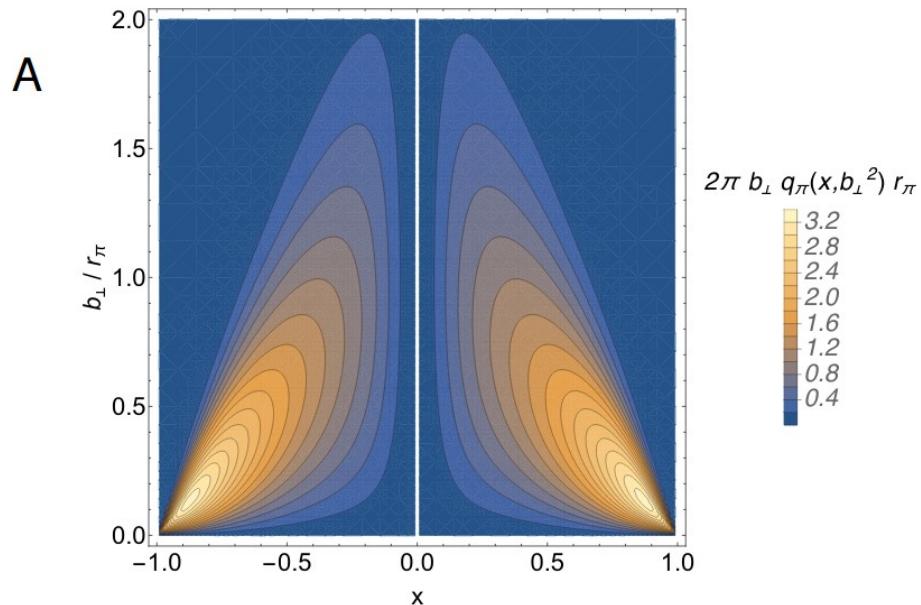
Merely from the definitions of charge radius and Impact Parameter Space **GPD**:

$$u^P(x, b_\perp^2; \zeta_{\mathcal{H}}) = \int_0^\infty \frac{d\Delta}{2\pi} \Delta J_0(b_\perp \Delta) H_P^u(x, 0, -\Delta^2; \zeta_{\mathcal{H}})$$

Impact parameter space GPDs

$$u^P(x, b_\perp^2; \zeta_{\mathcal{H}}) = \int_0^\infty \frac{d\Delta}{2\pi} \Delta J_0(b_\perp \Delta) H_P^u(x, 0, -\Delta^2; \zeta_{\mathcal{H}})$$

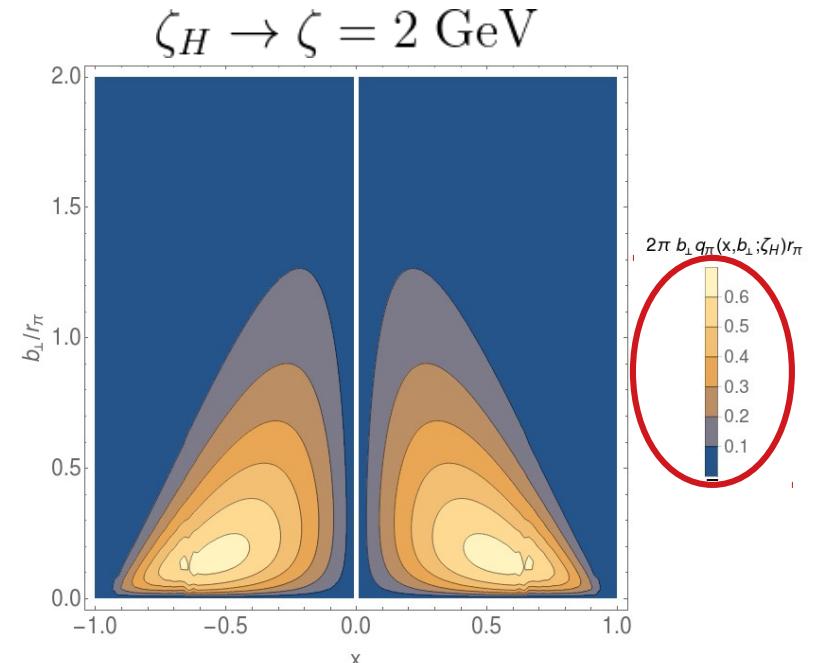
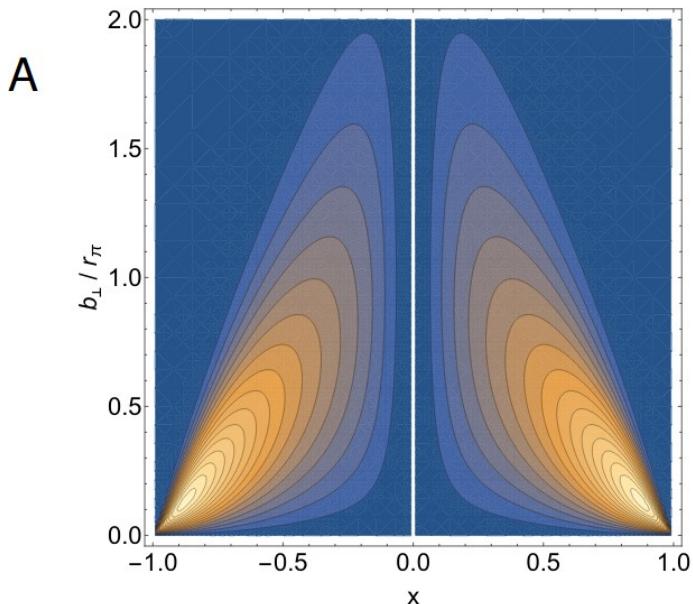
$$\langle b_\perp^2(\zeta_{\mathcal{H}}) \rangle_u^\pi = \frac{2}{3} r_\pi^2 = \langle b_\perp^2(\zeta_{\mathcal{H}}) \rangle_{\bar{d}}^\pi,$$
$$\langle b_\perp^2(\zeta_{\mathcal{H}}) \rangle_u^K = 0.71 r_K^2, \langle b_\perp^2(\zeta_{\mathcal{H}}) \rangle_{\bar{s}}^K = 0.58 r_K^2.$$



- › Likelihood of finding a valence-**quark** with **momentum fraction** x , at **position** b .

Impact-parameter space GPD

$$u^P(x, b_\perp^2; \zeta_H) = \int_0^\infty \frac{d\Delta}{2\pi} \Delta J_0(b_\perp \Delta) H_P^u(x, 0, -\Delta^2; \zeta_H)$$



- **Likelihood** of finding a parton with LF momentum x at transverse position \mathbf{b}

- Peaks **broaden** and **maximum drifts**:

$$\max = 3.49 \rightarrow \max = 0.63$$

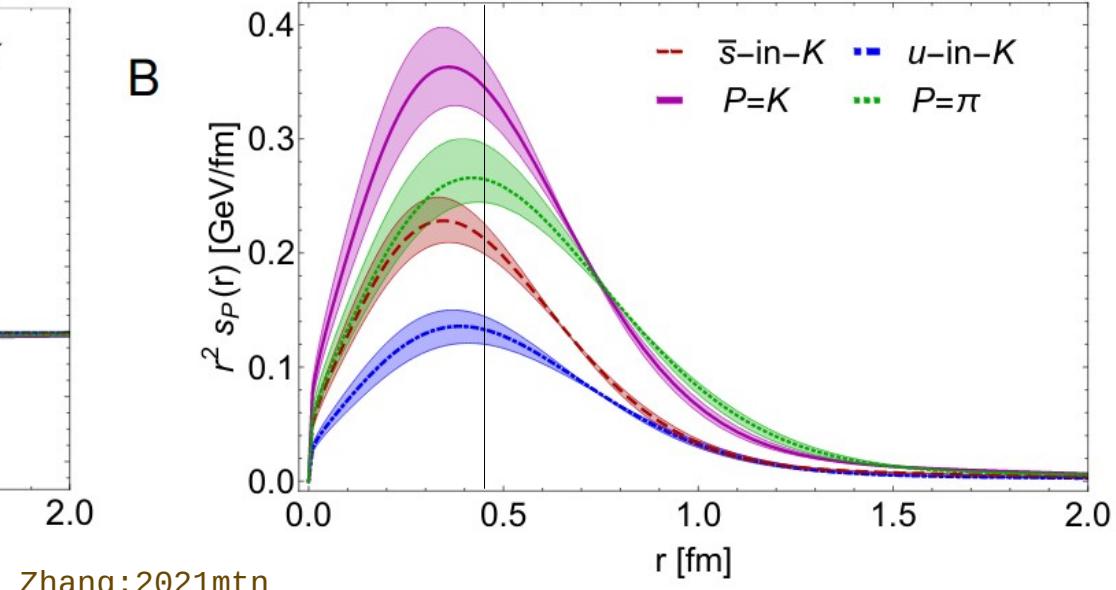
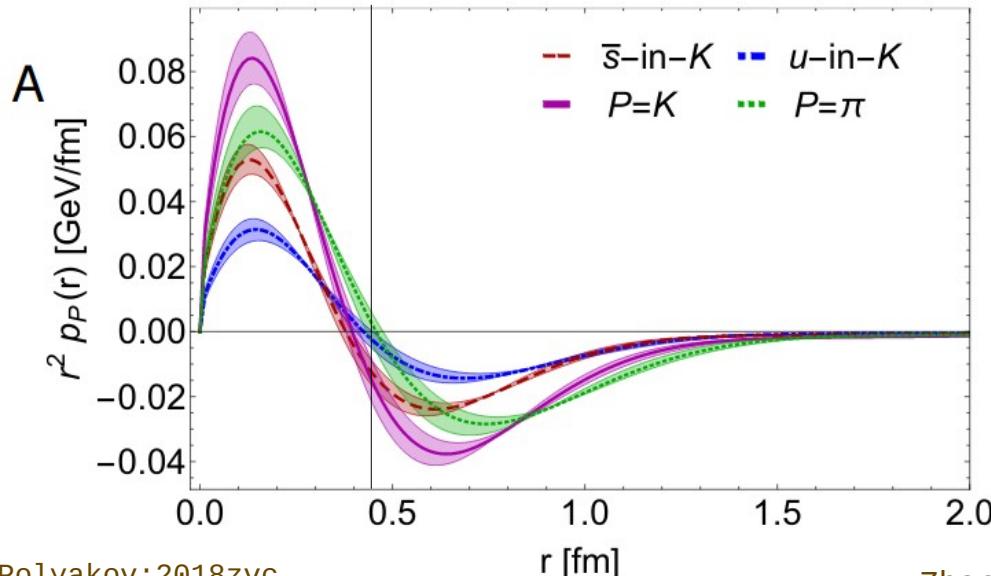
$$|x| \approx 0.91 \rightarrow |x| \approx 0.53$$

Pressure distributions

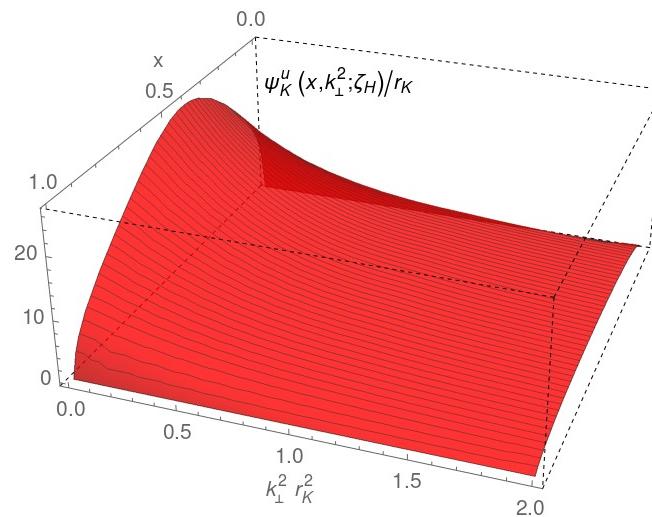
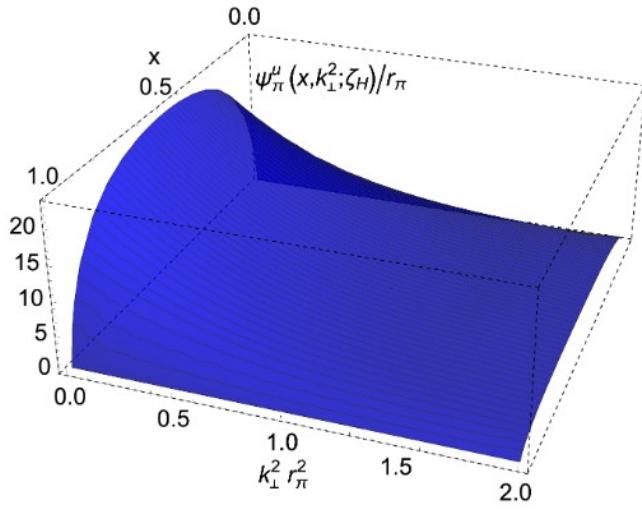
$$p_K^u(r) = \frac{1}{6\pi^2 r} \int_0^\infty d\Delta \frac{\Delta}{2E(\Delta)} \sin(\Delta r) [\Delta^2 \theta_1^{K_u}(\Delta^2)],$$

$$s_K^u(r) = \frac{3}{8\pi^2} \int_0^\infty d\Delta \frac{\Delta^2}{2E(\Delta)} j_2(\Delta r) [\Delta^2 \theta_1^{K_u}(\Delta^2)],$$

“Pressure” Quark attraction/repulsion
CONFINEMENT
 “Shear” Deformation QCD forces



Summary and Highlights

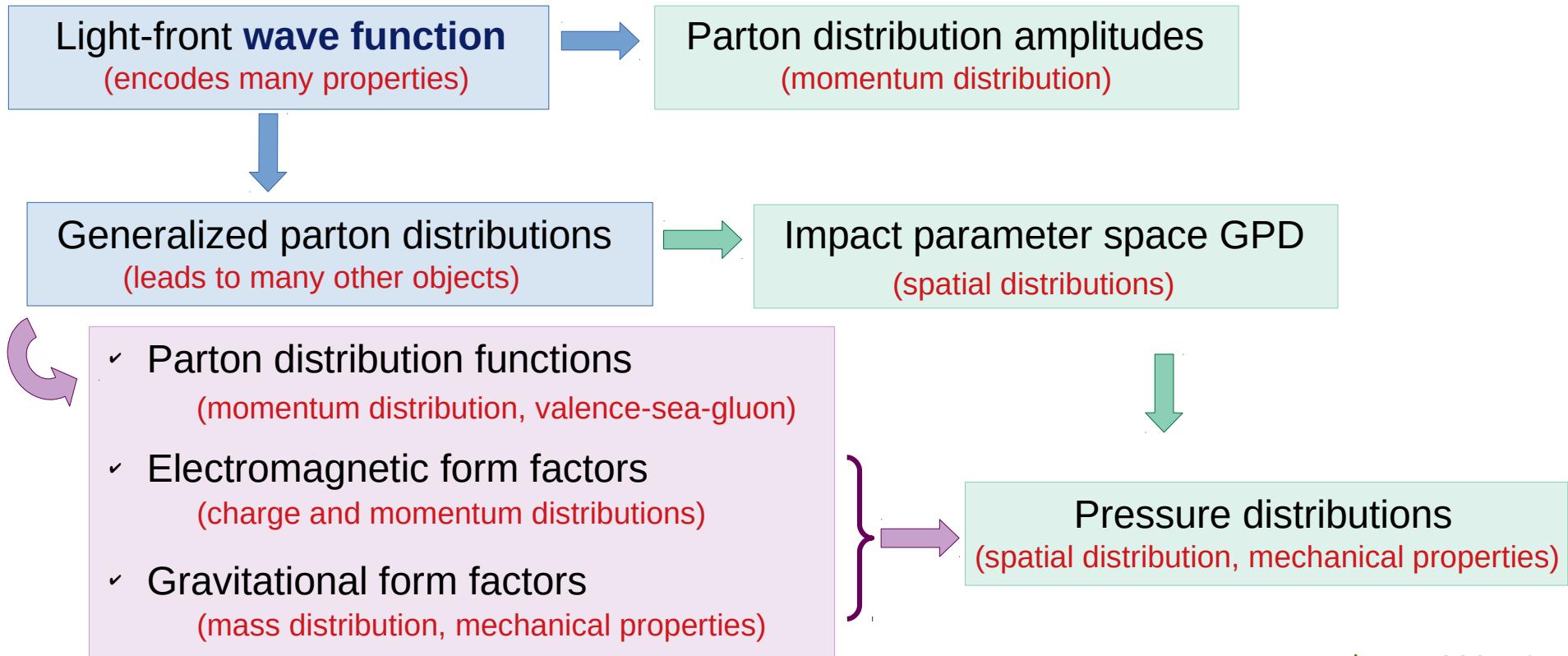


I just need
the main ideas



Summary

- Focusing on the **pion** and **kaon**, we discussed a variety of **parton distributions**:



Highlights

- QCD's EHM produce **broad π -K** distributions.
- Interplay between **QCD** and **Higgs** mass generation:
 - Slightly *skewed* kaon distributions.
- The ordering of **radii**:
$$r_\pi^{\theta_1} > r_\pi^E > r_\pi^{\theta_2}$$
- Kaon is more compressed:
$$r_K^j \approx 0.85 r_\pi^j$$
- **Gluon** and **sea** generated through **evolution**.

- **Factorized** models:
 - ✓ Insightful
 - ✓ Analytical
 - ✓ Adequate for pion

‘All orders’ hypothesis
Realistic **PDF** + r_M^E is all we need.

