Introduction of CKM Triangle

Ying Li

Yantai University

Talk given in Henan Normal University, 2021-04-21

什么是高能物理？

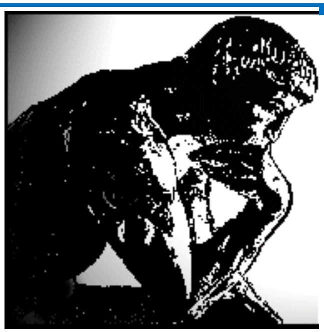
我是谁?

我从哪来?

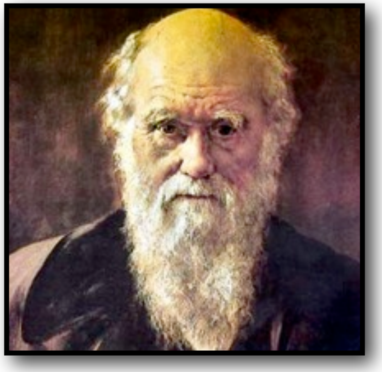
我到那去?

我是谁?

哲学家



生物学家



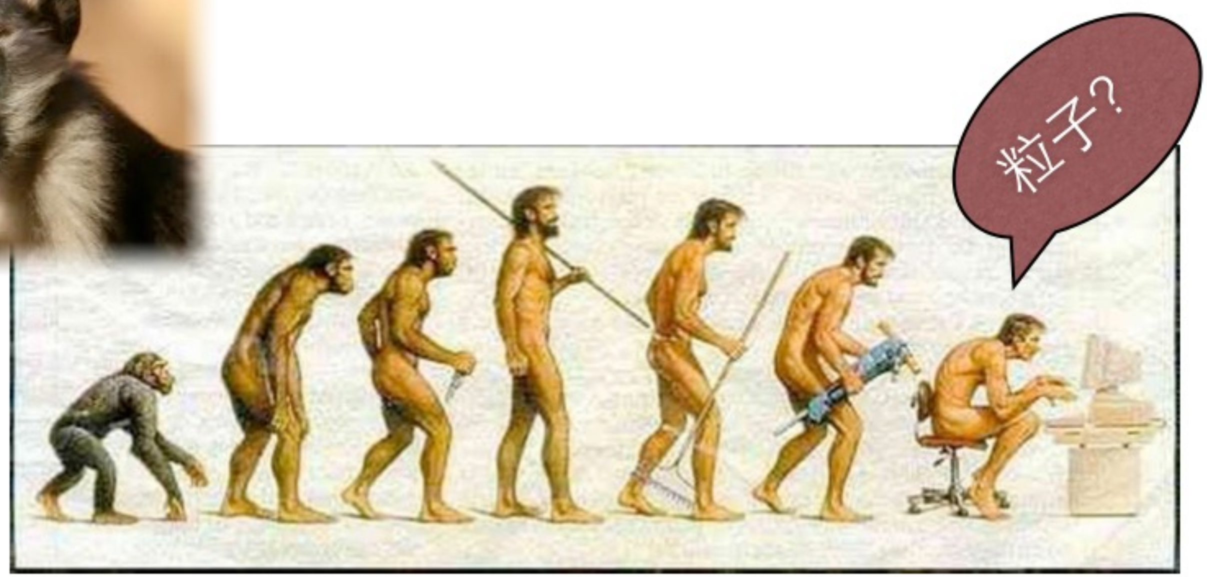
我从哪来?

我到那去?

历史学家

“世界是由什么组成的？”
“它们是怎样组成这个世界？”
“宇宙如何演化？”

物理学家



高能物理的研究对象

“世界基本组成成分为何？”

和

“它们如何相互作用？”

基本粒子物理

或

高能物理

研究自然界的

基本相互作用（力）

探测更小尺度、更高能量

厘米：
流体 （经典流体力学）

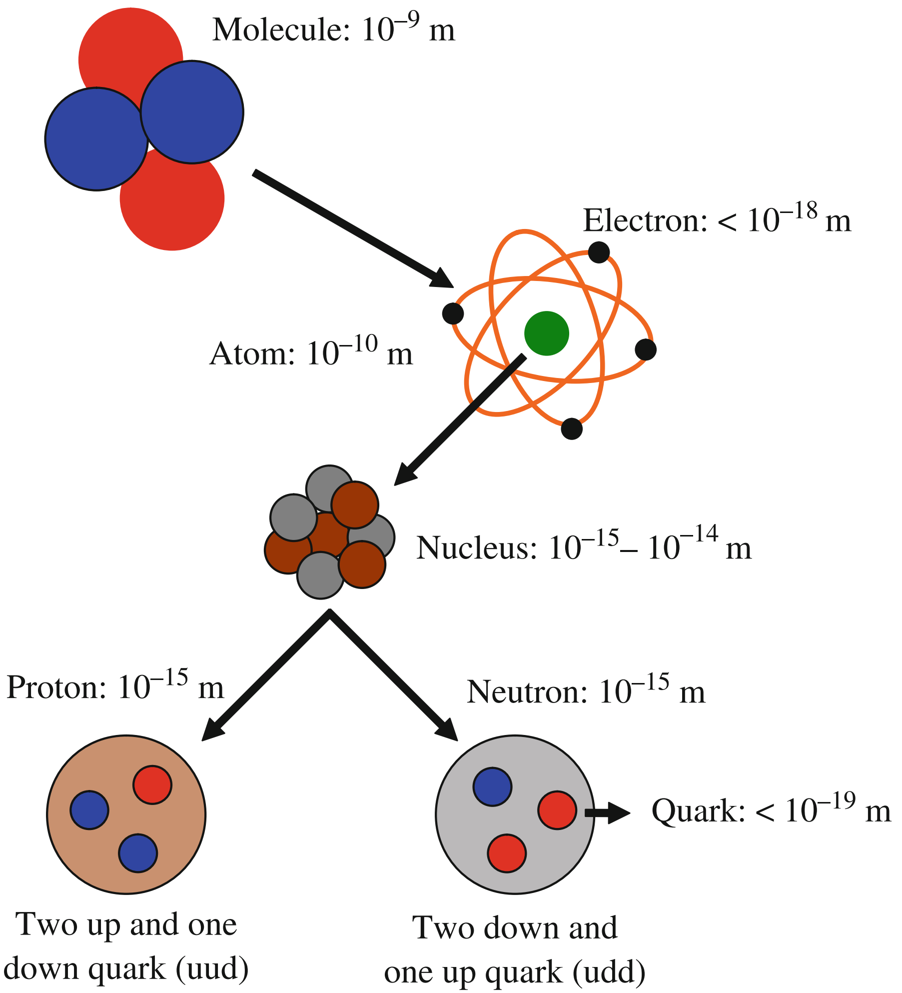
10^{-7}厘米：
分子 （分子运动学）

10^{-8}厘米：
原子 （量子力学）

10^{-13}厘米：
核子 （核物理学）

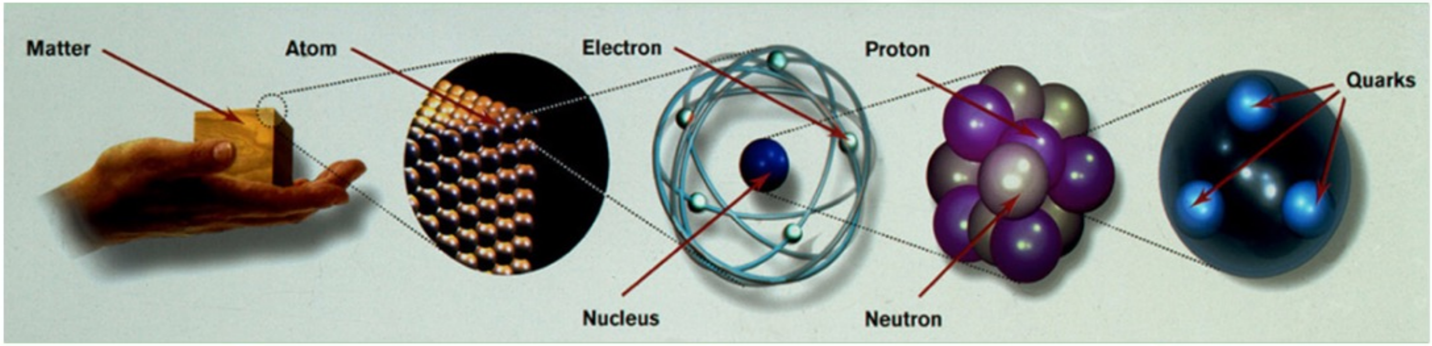
10^{-13}–10^{-18}厘米：
夸克 （量子色动力学）

10^{-33}厘米：
弦理论



探测更小尺度、更高能量

10^{-10} m | 10^{-14} m | 10^{-15} - 10^{-18} m



Motion of air atom
E = 0.04 eV

Chemical reactions/atom
E = 1 to a few eV

Nuclear reactions/atom
E = Millions of eV (MeV)

Energy $E=mc^2$ of proton
E ~ 1 Billion eV(GeV)

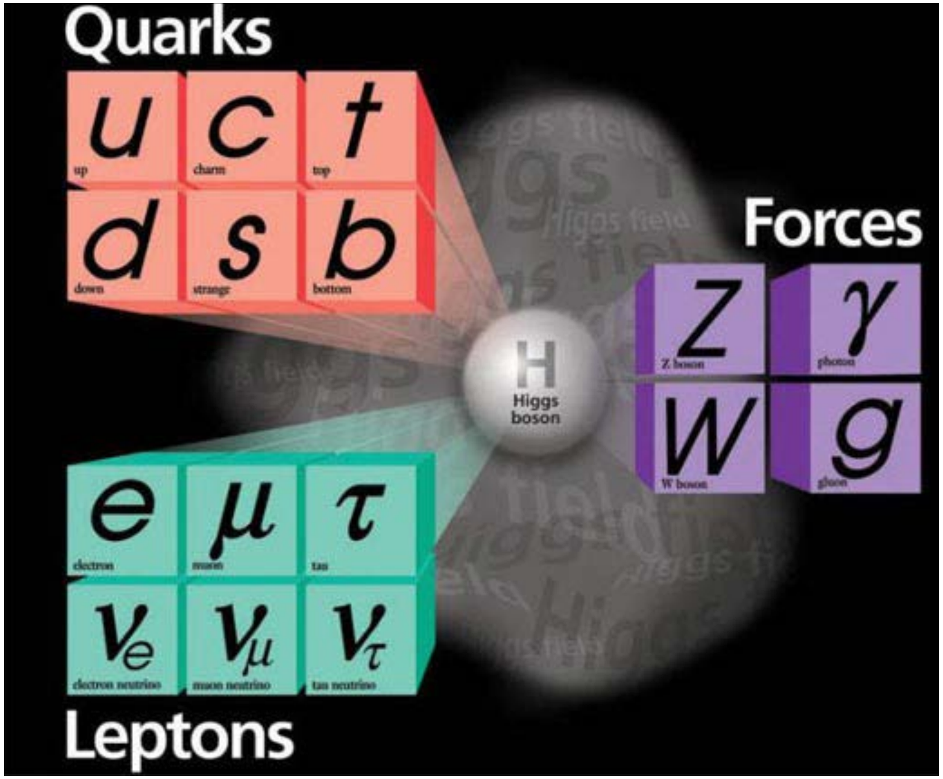
大型强子对撞机可以探测10^{-20}米

每个质子的能量是4TeV (4×10^{12} eV)
相当于宇宙大爆炸后10^{-12}到10^{-11}秒的温度

高能物理标准模型

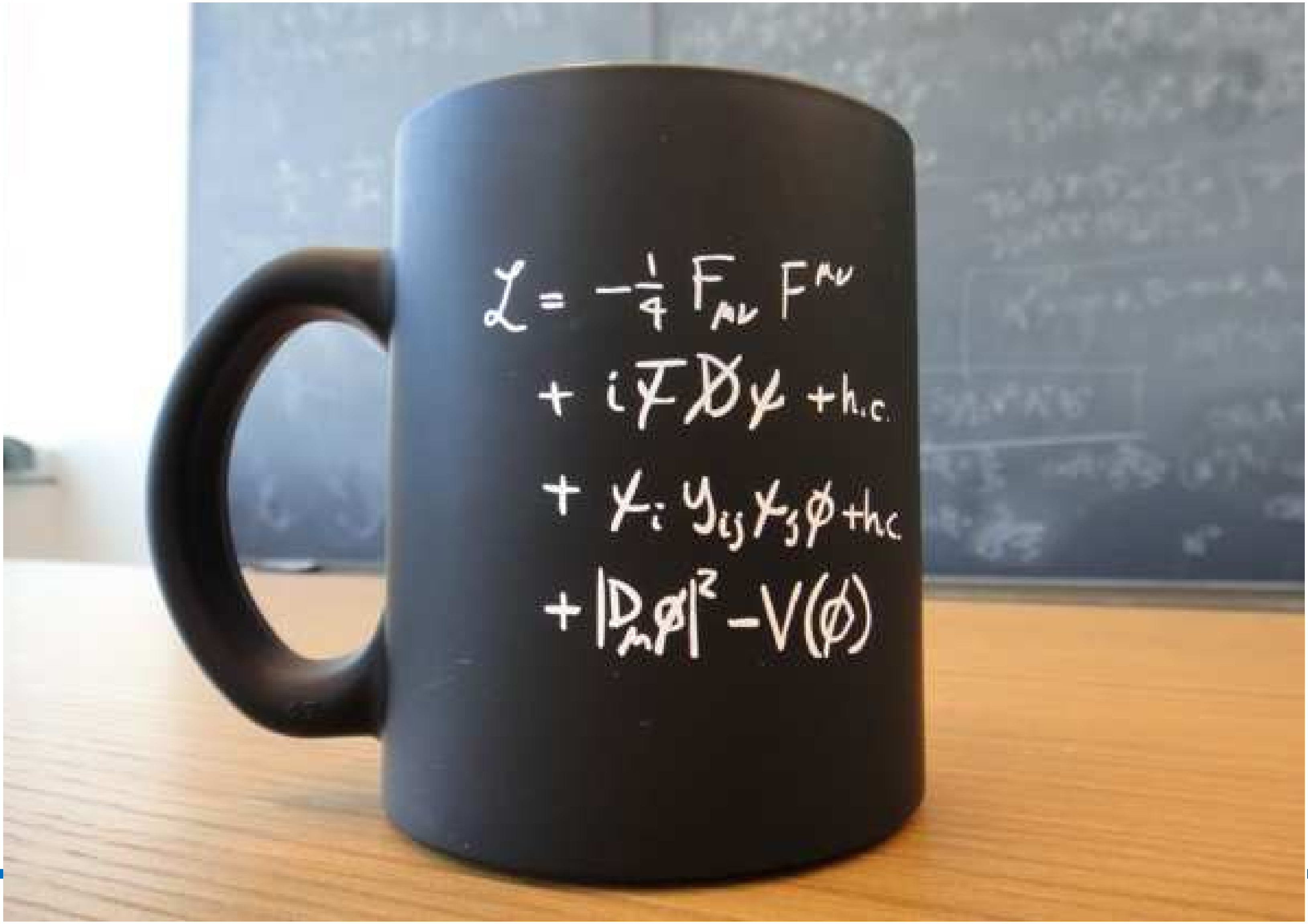
粒子物理的标准模型

- 集百年物理之大成
- 新元素周期表
- 20世纪自然科学的卓越成就之一

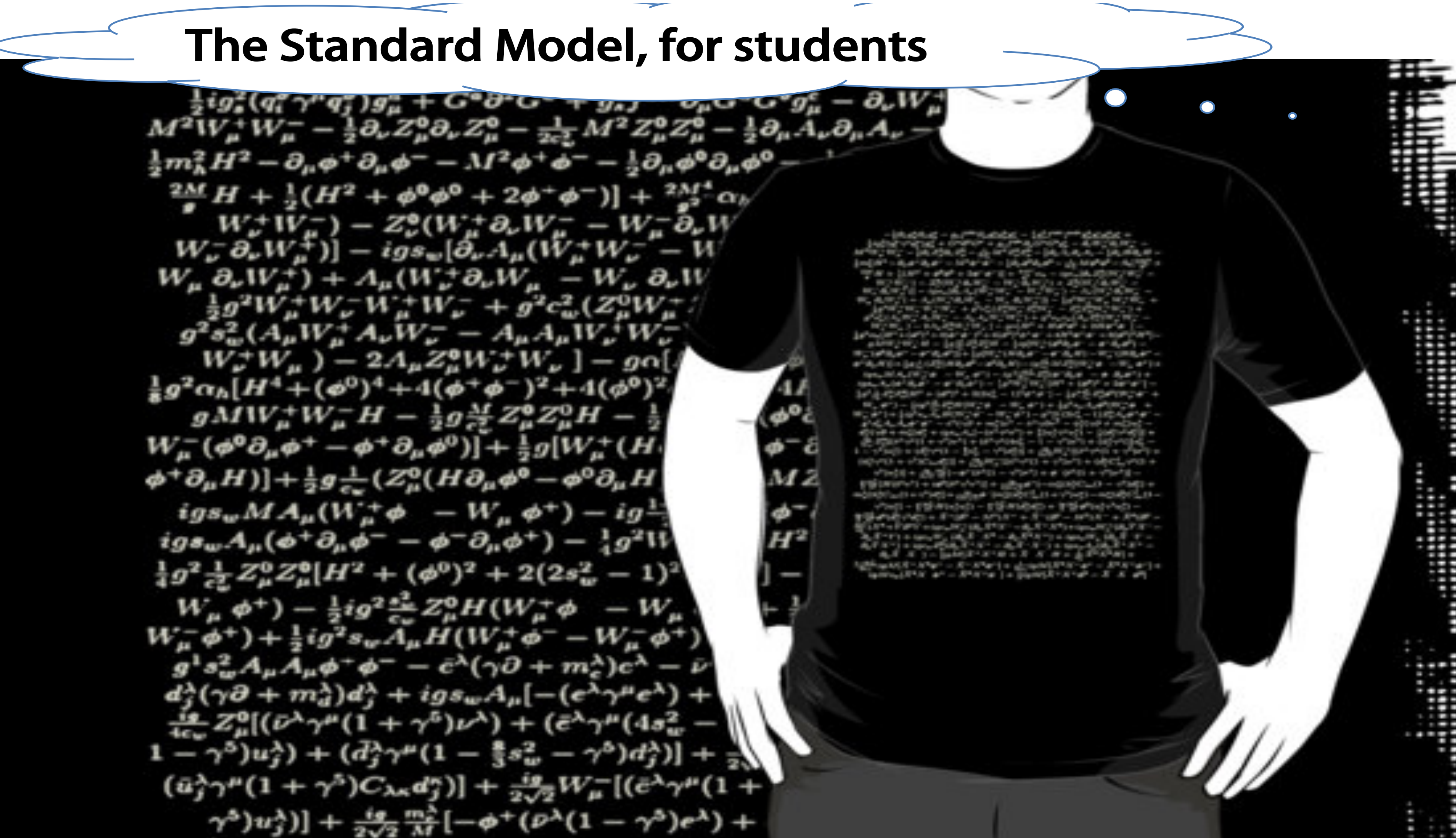


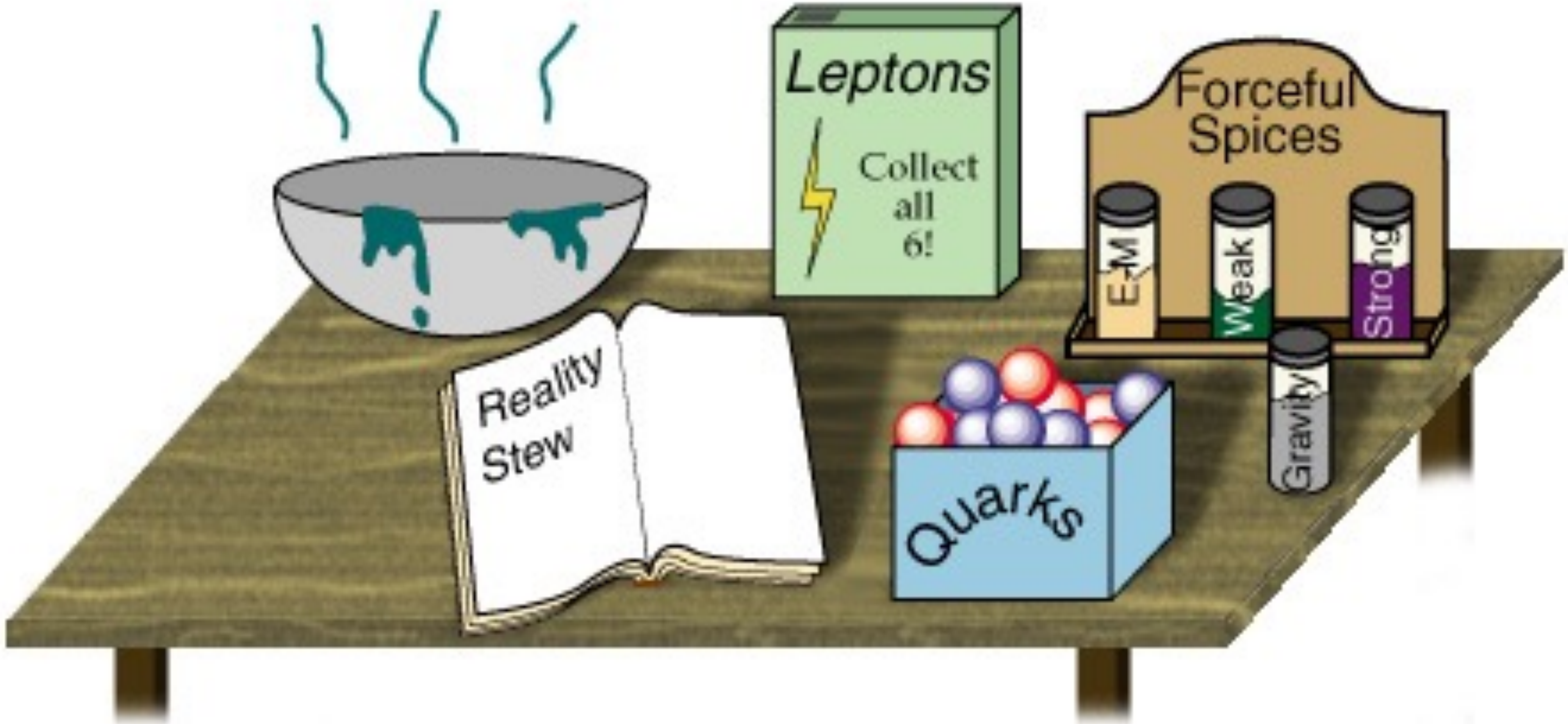
宇宙万物可以用一个“简单公式”表示出来！

粒子物理的标准模型


$\mathcal{L} = -\frac{1}{4} F_{\mu\nu} F^{\mu\nu}$
$+ i \bar{\psi} \not{D} \psi + h.c.$
$+ \psi_i y_{ij} \psi_j \phi + h.c.$
$+ |D_\mu \phi|^2 - V(\phi)$

粒子物理的标准模型




Leptons
Collect all 6!
Forceful Spices
E-M
Weak
Strong
Gravity
Reality Stew
Quarks

Contents

1

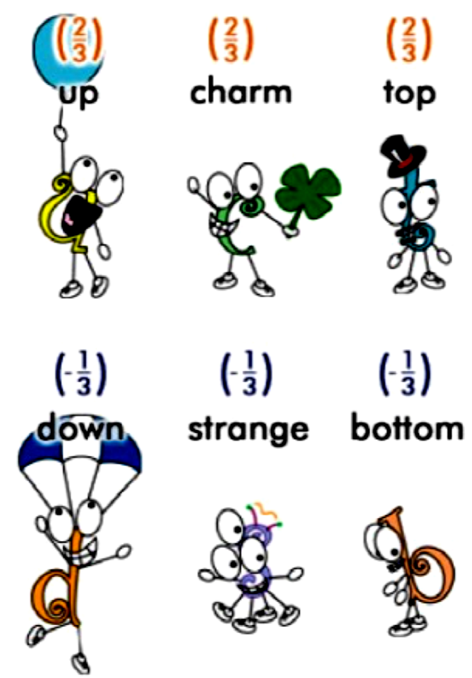
CKM

Flavour Physics

- Three generations of quarks (and leptons)
  - identical gauge quantum numbers
  - different masses

– Flavour physics describes interactions that distinguish between flavours.

– Parity violation of electroweak interactions



- left-handed quarks :$\mathrm{SU}(2)_\mathrm{L}$ doublets

$$Q_j = \begin{pmatrix} u_L \\ d_L \end{pmatrix}, \begin{pmatrix} c_L \\ s_L \end{pmatrix}, \begin{pmatrix} t_L \\ b_L \end{pmatrix}$$

- right-handed quarks: singlets

$$U_j = u_R, c_R, t_R \qquad D_j = d_R, s_R, b_R$$

Gauge couplings of the quarks

$$\mathcal{L}_{\text{fermion}} = \sum_{j=1}^{3} \bar{Q}_j i \not{D}_Q Q_j + \bar{U}_j i \not{D}_U U_j + \bar{D}_J i \not{D}_D D_j$$

with the covariant derivatives ($Y_Q = 1/6$, $Y_U = 2/3$, $Y_D = -1/3$)

$$\begin{aligned} D_{Q,\mu} &= \partial_\mu + i g_s T^a G^a_\mu + i g \tau^a W^a_\mu + i Y_Q g' B_\mu \\ D_{U,\mu} &= \partial_\mu + i g_s T^a G^a_\mu \qquad\qquad\quad + i Y_U g' B_\mu \\ D_{D,\mu} &= \partial_\mu + i g_s T^a G^a_\mu \qquad\qquad\quad + i Y_D g' B_\mu \end{aligned}$$

- **Flavour universality:** gauge couplings are equal for all three generations

Yukawa couplings

- Flavour non-universality introduced by Yukawa couplings between the Higgs field and the quarks:

$$\mathcal{L}_{\text{Yuk}} = \sum_{i,j=1}^{3} (-Y_{U,ij}\bar{Q}_{Li}\tilde{H}U_{Rj} - Y_{D,ij}\bar{Q}_{Li}HD_{Rj} + h.c.)$$

where i, j are generation indices and $\widetilde{H} = \epsilon H^* = (H^{*0}, -H^-)^T$

- replacing H by its vacuum expectation value $\langle H \rangle = (0, v)^T$, we obtain the quark mass terms

$$\sum_{i,j=1}^{3} (-m_{U,ij}\bar{u}_{Li}u_{Rj} - m_{D,ij}\bar{d}_{Li}d_{Rj} + h.c.)$$

with the quark mass matrices given by $m_A = vY_A$ $(A = U, D, E)$.

By hand !

Diagonalizing the fermion mass matrices

- In general, the mass matrices m_U and m_D do not have to be diagonal, but they can be diagonalized with unitary transformations

$$u_L = \hat{U}_L u_L^m \qquad u_R = \hat{U}_R u_R^m \qquad d_L = \hat{D}_L d_L^m \qquad d_R = \hat{D}_R d_R^m$$

with m denoting quarks in the mass eigenstate basis.

- in this basis

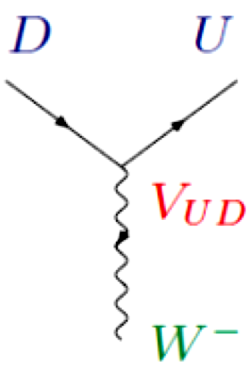
$$m_U^{\text{diag}} = \hat{U}_L^\dagger m_U \hat{U}_R \qquad m_D^{\text{diag}} = \hat{D}_L^\dagger m_D \hat{D}_R$$

  - The SM Lagrangian invariant under these field redefinitions
  - Transformations of the right-handed quarks are indeed unphysical, i. e. they leave the rest of the Lagrangian invariant

CKM Matrix

- In SM, u_{Li} and d_{Li} form the $\mathrm{SU}(2)_{\mathrm{L}}$ doublets Q_{Li}, kinetic term gives rise to the interaction

$$\frac{g}{\sqrt{2}}\bar{u}_{Li}\gamma_\mu W^{\mu+}d_{Li}$$



- transforming to the mass eigenstate basis, we obtain

$$\frac{g}{\sqrt{2}}\bar{u}_{Li}\hat{U}^\dagger_{L,ij}\hat{D}_{L,jk}\gamma_\mu W^{\mu+}d_{Lk}$$

- The combination $V_{CKM} = \hat{U}_L^+ \hat{D}_L$ is physical and is called the CKM matrix. It leads to flavour violating charged current interactions.

Parameter Counting

$$V_{\text{CKM}} = \begin{pmatrix} V_{ud} & V_{us} & V_{ub} \\ V_{cd} & V_{cs} & V_{cb} \\ V_{td} & V_{ts} & V_{tb} \end{pmatrix}$$

- Unitary 3x3 matrix V can by parameterized by 3 Euler angles and **6 phases**
- Not all phases are observable, since under phase redefinitions $q_L \rightarrow e^{i\phi} q_L$ of the quark fields:

$$V \longrightarrow \begin{pmatrix} e^{-i\varphi_u} & 0 & 0 \\ 0 & e^{-i\varphi_c} & 0 \\ 0 & 0 & e^{-i\varphi_t} \end{pmatrix} V \begin{pmatrix} e^{i\varphi_d} & 0 & 0 \\ 0 & e^{i\varphi_s} & 0 \\ 0 & 0 & e^{i\varphi_b} \end{pmatrix}, \qquad V_{ij} \longrightarrow e^{i(\varphi_d^i - \varphi_u^j)} V_{ij}$$

- 5 of 6 phases can be eliminated by suitable choices of phase differences!

CP violation and the complex CKM matrix

CP: combination of parity transformation P and charge conjugation C

- $P : \psi(r) \to \gamma^0\psi(-r)$ transforms left(right)-handed quark into right(left)-handed quark
- $C : \psi \to i(\bar{\psi}\gamma^0\gamma^2)^T$ transforms left(right)-handed quark into left(right)-handed antiquark

weak interactions neither invariant under P nor C ➢ what about CP?

$(g_W = g/\sqrt{2})$

$$
\begin{aligned}
& g_W \bar{u}_{Li} V_{\text{CKM},ik} \gamma_\mu W^{\mu+} d_{Lk} + h.c. \\
& \quad = \quad g_W \bar{u}_{Li} V_{\text{CKM},ik} \gamma_\mu W^{\mu+} d_{Lk} + g_W \bar{d}_{Lk} V^*_{\text{CKM},ik} \gamma_\mu W^{\mu-} u_{Li} \\
& \quad \xrightarrow{CP} \quad g_W \bar{d}_{Lk} V_{\text{CKM},ik} \gamma_\mu W^{\mu-} u_{Li} + g_W \bar{u}_{Li} V^*_{\text{CKM},ik} \gamma_\mu W^{\mu+} d_{Lk} \\
& \quad = \quad g_W \bar{u}_{Li} V^*_{\text{CKM},ik} \gamma_\mu W^{\mu+} d_{Lk} + h.c.
\end{aligned}
$$

➢ **CP is violated because the CKM matrix is complex** $(\delta \neq 0, \pi)$

CP violation and the complex CKM matrix

- CP violation required to explain the different abundances of matter and antimatter in the universe.

- CP violation in quark sector requires $N \geq 3$ fermion generations.

- Model for explanation of CP violation led to prediction of the third generation!

Parametrizations of the CKM Matrix

$$R_{12} = \begin{pmatrix} c_{12} & s_{12} & 0 \\ -s_{12} & c_{12} & 0 \\ 0 & 0 & 1 \end{pmatrix} \quad R_{23} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & c_{23} & s_{23} \\ 0 & -s_{23} & c_{23} \end{pmatrix} R_{13} = \begin{pmatrix} c_{13} & 0 & s_{13}e^{-i\delta} \\ 0 & 1 & 0 \\ -s_{13}e^{i\delta} & 0 & c_{13} \end{pmatrix}$$

$$V_{CKM} = R_{23} \times R_{13} \times R_{12}$$

$$s_{ij} = \sin\theta_{ij}$$
$$c_{ij} = \cos\theta_{ij}$$

$$V_{\mathrm{CKM}} = \begin{pmatrix} c_{12}c_{13} & s_{12}c_{13} & s_{13}e^{-i\delta} \\ -s_{12}c_{23}-c_{12}s_{23}s_{13}e^{i\delta} & c_{12}c_{23}-s_{12}s_{23}s_{13}e^{i\delta} & s_{23}c_{13} \\ s_{12}s_{23}-c_{12}c_{23}s_{13}e^{i\delta} & -c_{12}s_{23}-s_{12}c_{23}s_{13}e^{i\delta} & c_{23}c_{13} \end{pmatrix}$$

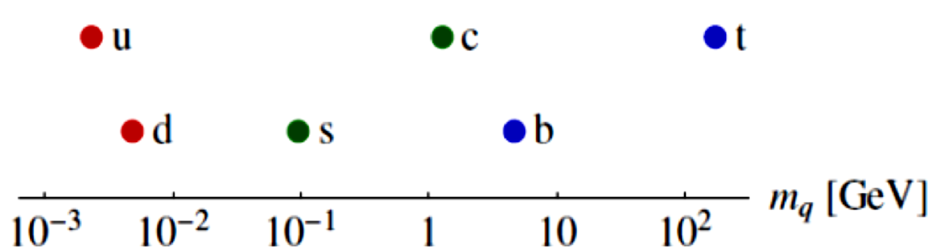
Experimentally: $s_{12} \sim 0.2$, $s_{23} \sim 0.04$, $s_{13} \sim 4 \cdot 10^{-3}$

Present status

CKM matrix is found to be close to the unit matrix

$$V_{\mathrm{CKM}} = \begin{pmatrix} 0.97427 \pm 0.00014 & 0.22536 \pm 0.00061 & 0.00355 \pm 0.00015 \\ 0.22522 \pm 0.00061 & 0.97343 \pm 0.00015 & 0.0414 \pm 0.0012 \\ 0.00886^{+0.00033}_{-0.00032} & 0.0405^{+0.0011}_{-0.0012} & 0.99914 \pm 0.00005 \end{pmatrix}$$

also quark masses exhibit strong hierarchy

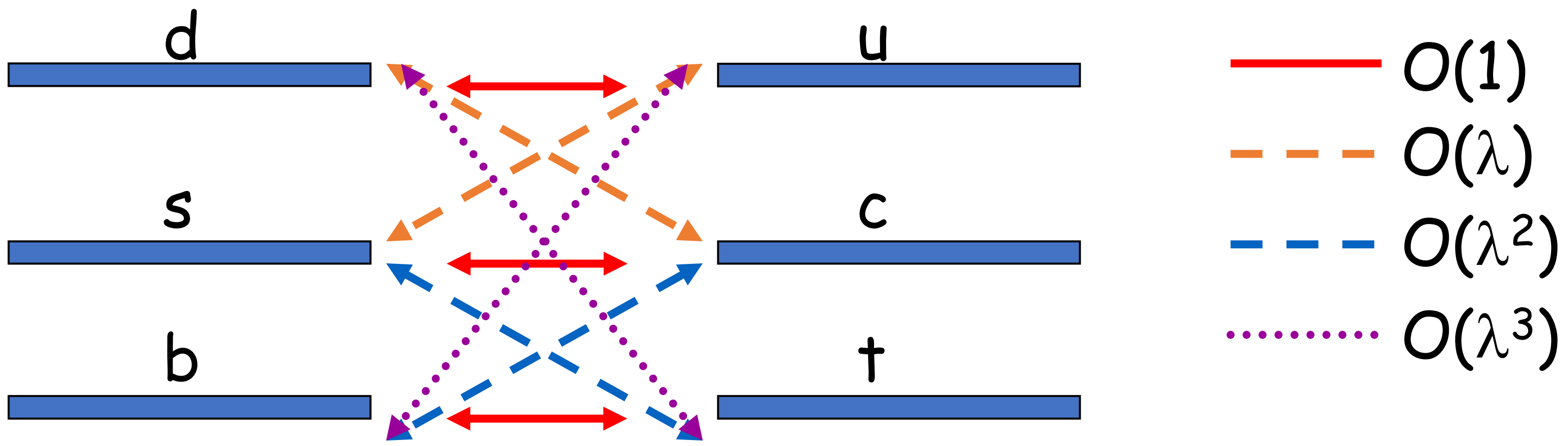


where does this hierarchical structure come from?

➢ **flavour hierarchy problem**

Parametrizations of the CKM Matrix

- Hierarchy of the quark transitions mediated through charged currents



- This hierarchy is reflected in the standard parametrization as follows:

$$s_{12} = 0.22 \gg s_{23} = O(10^{-2}) \gg s_{13} = O(10^{-3})$$

- New parameters: $s_{12} \equiv \lambda = 0.22\,, \quad s_{23} = A\lambda^2 \quad s_{13} = A\lambda^2(\rho - i\eta)$

$$\hat{V}_{\mathrm{CKM}} = \begin{pmatrix} 1-\frac{1}{2}\lambda^2 & \lambda & A\lambda^3(\rho - i\,\eta) \\ -\lambda & 1-\frac{1}{2}\lambda^2 & A\lambda^2 \\ A\lambda^3(1-\rho-i\,\eta) & -A\lambda^2 & 1 \end{pmatrix}$$

The unitarity triangle

CKM matrix is unitary

9 conditions (6 normalisation, 3 orthoganality)

$$V_{ud}V_{ub}^{*} + V_{cd}V_{cb}^{*} + V_{td}V_{tb}^{*} = 0 \qquad \text{(db)}$$

$$V_{us}V_{ub}^{*} + V_{cs}V_{cb}^{*} + V_{ts}V_{tb}^{*} = 0 \qquad \text{(sb)}$$

$$V_{ud}V_{us}^{*} + V_{cd}V_{cs}^{*} + V_{td}V_{ts}^{*} = 0 \qquad \text{(ds)}$$

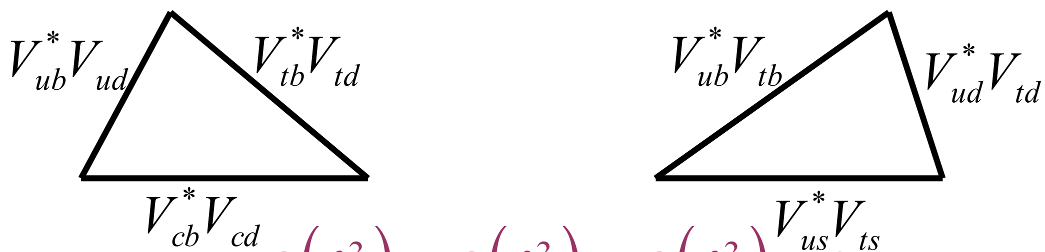
$$V_{ud}V_{td}^{*} + V_{us}V_{ts}^{*} + V_{ub}V_{tb}^{*} = 0 \qquad \text{(ut)}$$

$$V_{cd}V_{td}^{*} + V_{cs}V_{ts}^{*} + V_{cb}V_{tb}^{*} = 0 \qquad \text{(ct)}$$

$$V_{ud}V_{cd}^{*} + V_{us}V_{cs}^{*} + V_{ub}V_{cb}^{*} = 0 \qquad \text{(uc)}$$

The unitarity triangle

(db) (ut)

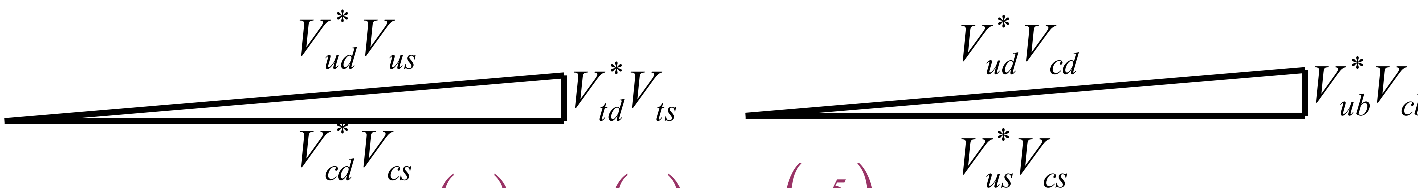


$$O(\lambda^3)+O(\lambda^3)+O(\lambda^3)=0$$

(sb) (ct)

$$O(\lambda^4)+O(\lambda^2)+O(\lambda^2)=0$$

(ds) (uc)



$$O(\lambda)+O(\lambda)+O(\lambda^5)=0$$

The unitarity triangle

- CKM matrix parametrises charged current interactions

$$V_{\mathsf{CKM}} = \begin{pmatrix} V_{ud} & V_{us} & V_{ub} \\ V_{cd} & V_{cs} & V_{cb} \\ V_{td} & V_{ts} & V_{tb} \end{pmatrix}$$

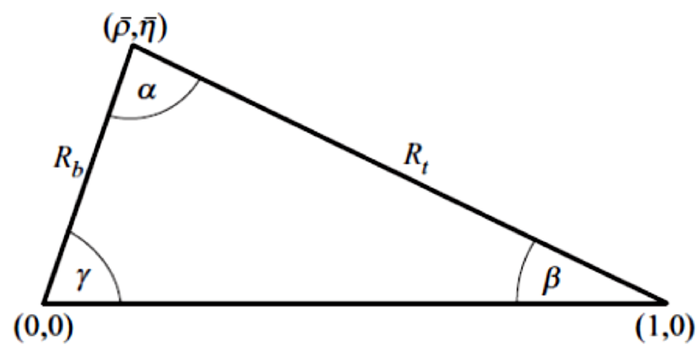
- its unitarity implies various relations among its elements, e. g.

$$V_{ud}V_{ub}^* + V_{cd}V_{cb}^* + V_{td}V_{tb}^* = 0$$

➢ **unitarity triangle** in the complex plane $\bar{\rho} = (1 - \frac{\lambda^2}{2})\rho,\ \bar{\eta} = (1 - \frac{\lambda^2}{2})\eta$

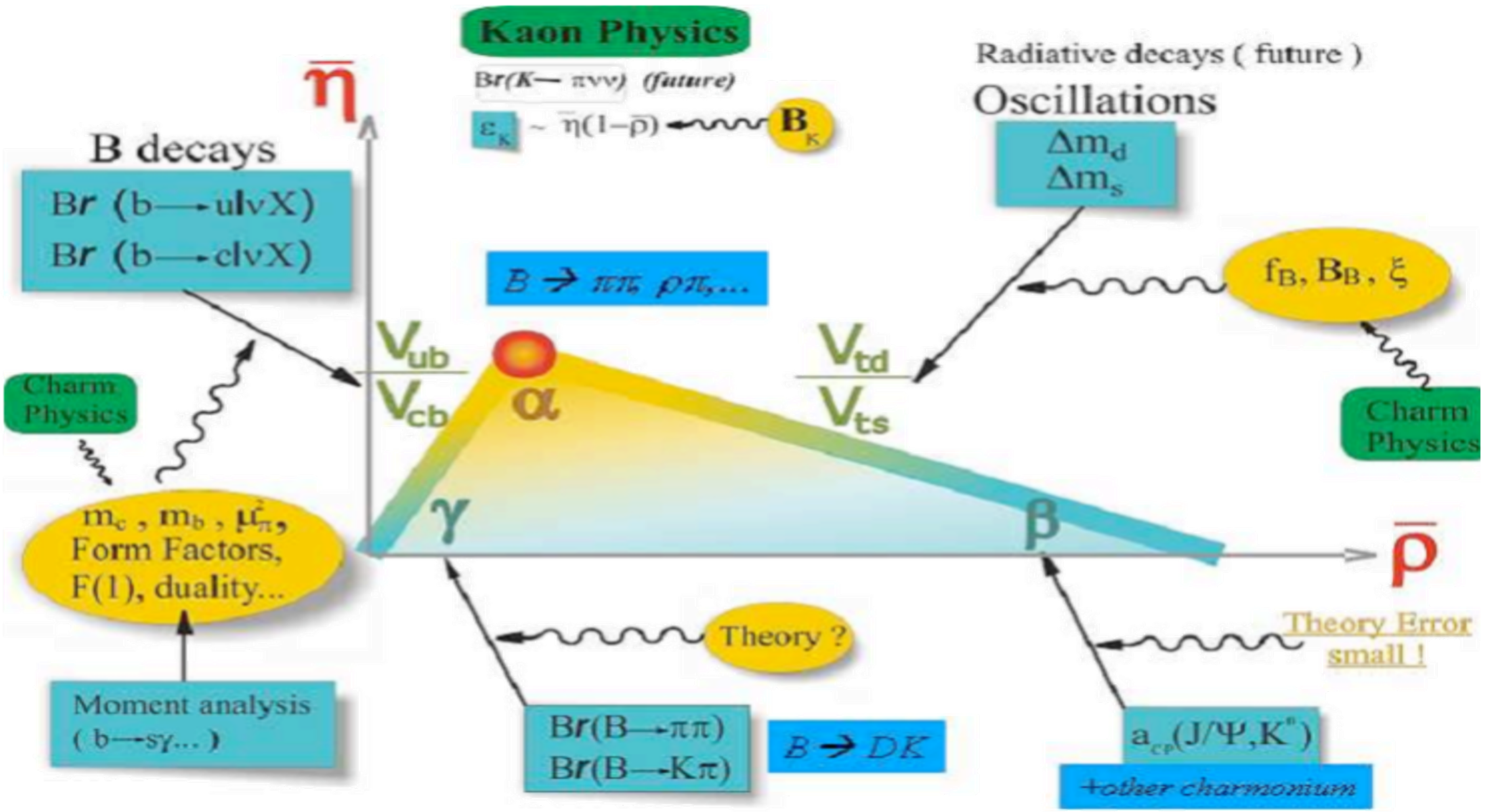
$$R_b = \left|\frac{V_{ud}V_{ub}^*}{V_{cd}V_{cb}^*}\right|$$

$$R_t = \left|\frac{V_{td}V_{tb}^*}{V_{cd}V_{cb}^*}\right|$$



$$\alpha = \arg\left(-\frac{V_{td}V_{tb}^*}{V_{ud}V_{ub}^*}\right), \quad \beta = \arg\left(-\frac{V_{cd}V_{cb}^*}{V_{td}V_{tb}^*}\right), \quad \gamma = \arg\left(-\frac{V_{ud}V_{ub}^*}{V_{cd}V_{cb}^*}\right).$$

The unitarity triangle



Some Details on V_{CKM}

$$V_{CKM} = \begin{pmatrix} V_{ud} & V_{us} & V_{ub} \\ V_{cd} & V_{cs} & V_{cb} \\ V_{td} & V_{ts} & V_{tb} \end{pmatrix} \approx \begin{pmatrix} 1 & \lambda & V_{ub} \\ -\lambda & 1 & V_{cb} \\ V_{td} & V_{ts} & V_{tb} \end{pmatrix}$$

- First 2×2 sub-matrix: four $|V_{ij}|$ are measured by nucleus, pion, kaon and charm hadron decays.
- It is "almost" unitary with one single parameter.
- $\lambda\ (\equiv\ \sin\theta)\ =\ |V_{us}|\ =\ 0.2243\ \pm\ 0.0005$ [PDG2021]

Some Details on V_{CKM}

$$V_{\mathrm{CKM}} = \begin{pmatrix} V_{\mathrm{ud}} & V_{\mathrm{us}} & V_{\mathrm{ub}} \\ V_{\mathrm{cd}} & V_{\mathrm{cs}} & V_{\mathrm{cb}} \\ V_{\mathrm{td}} & V_{\mathrm{ts}} & V_{\mathrm{tb}} \end{pmatrix} \approx \begin{pmatrix} 1 & \lambda & V_{\mathrm{ub}} \\ -\lambda & 1 & V_{\mathrm{cb}} \\ V_{\mathrm{td}} & V_{\mathrm{ts}} & V_{\mathrm{tb}} \end{pmatrix}$$

- |Vcb| and |Vub| measured by semi-leptonic Bu and Bd decays

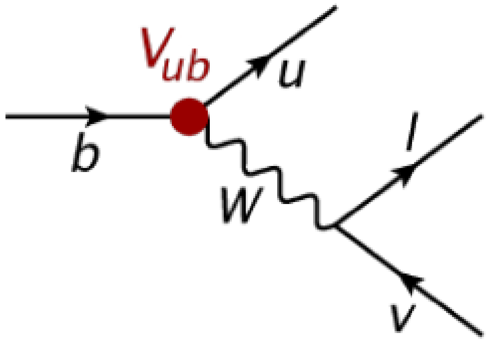
$$|V_{cb}| = \begin{cases} (42.2 \pm 0.8) \times 10^{-3} & \text{Inclusive} \\ (41.9 \pm 2.0) \times 10^{-3} & \text{exclusive} \end{cases}$$

$$|V_{ub}| = \begin{cases} (4.49 \pm 0.16 \pm 0.17 \pm 0.17) \times 10^{-3} & \text{Inclusive} \\ (3.70 \pm 0.10 \pm 0.12) \times 10^{-3} & \text{exclusive (Lattice)} \\ (3.67 \pm 0.09 \pm 0.12) \times 10^{-3} & \text{exclusive (LCQCDSR)} \end{cases}$$

Exclusives systematically smaller than inclusive?
Better QCD calculations needed.

Example-1: Semileptonic charged currents

Consider $b \to u l \bar{\nu}_l$ decay that is relevant for $|V_{ub}|$ measurements



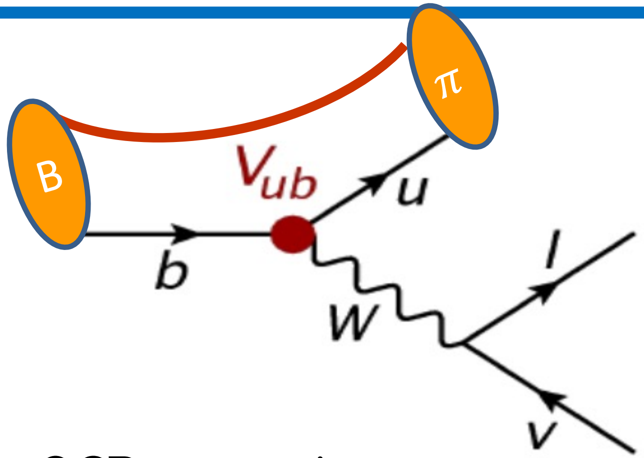
- momentum of W boson propagator can be neglected:

$$\frac{g}{\sqrt{2}} V_{ub} (\bar{u}\gamma^\nu P_L b) \frac{g_{\mu\nu}}{p^2 - M_W^2} \frac{g}{\sqrt{2}} (\bar{\ell}\gamma^\mu P_L \nu) \xrightarrow{p^2 \ll M_W^2} -\frac{g^2}{2M_W^2} V_{ub} (\bar{u}b)_{V-A} (\bar{\ell}\nu)_{V-A}$$

➢ effective Hamiltonian with the well-known Fermi constant

$$\mathcal{H}_{\text{eff}} = \frac{4G_F}{\sqrt{2}} V_{ub} (\bar{u}b)_{V-A} (\bar{\ell}\nu)_{V-A} \qquad \text{with } \frac{G_F}{\sqrt{2}} = \frac{g^2}{8M_W^2}$$

Example: Semileptonic charged currents



- include known QCD corrections
- evaluate the form factors of $\langle\pi|(\bar{u}b)_{V-A}|B\rangle$ from Nonperturbative QCD approach.

➢ measuring the $B \to \pi l v$ branching ratio determines $|V_{ub}|$

$$|V_{ub}|^{\pi\ell\nu} = (3.72 \pm 0.16) \cdot 10^{-3}$$

- Note: it is a tree level transition, therefore highly unlikely affected by new physics!

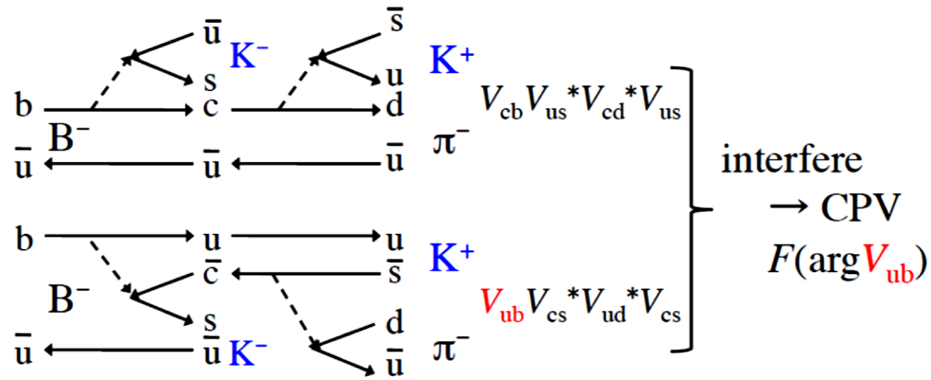
Some Details on V_{CKM}

$$V_{CKM} = \begin{pmatrix} V_{ud} & V_{us} & V_{ub} \\ V_{cd} & V_{cs} & V_{cb} \\ V_{td} & V_{ts} & V_{tb} \end{pmatrix} \approx \begin{pmatrix} 1 & \lambda & V_{ub} \\ -\lambda & 1 & V_{cb} \\ V_{td} & V_{ts} & V_{tb} \end{pmatrix}$$

- |Vcb| and |Vub| measured by semi-leptonic Bu and Bd decays

Arg Vcb =0 by a phase convention

Arg Vub= γ by CP Violation in B-->DK



$$Br\,(B^-\to[K^+\pi^-]_{D\text{-mass}}\,K^-) \neq Br\,(B^+\to[K^-\pi^+]_{D\text{-mass}}\,K^+)$$

$$Br\,(B^-\to[K^-\pi^+]_{D\text{-mass}}\,K^-) \neq Br\,(B^+\to[K^+\pi^-]_{D\text{-mass}}\,K^+)$$

$$\gamma = (73.5^{+4.2}_{-5.1})^\circ.$$

Some Details on V_{CKM}

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- |Vcb| and |Vub| measured by semi-leptonic Bu and Bd decays

Arg Vcb =0 by a phase convention

Arg Vub= γ by CP Violation in B-->DK

$V_{tb} \approx 1$ if we assume V_{CKM} to be unitary

$|V_{td}| \times |V_{tb}|$ by B_0-B_0 oscillation frequency (Δm_d)

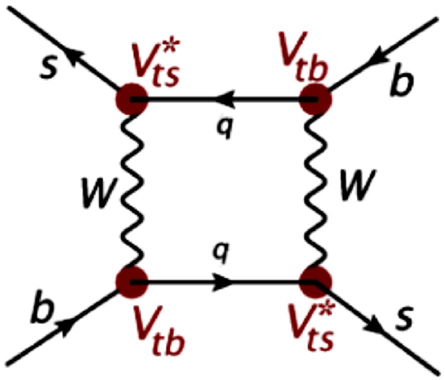
$|V_{ts}| \times |V_{tb}|$ by B_s-B_s oscillation frequency (Δm_s)

$$\Delta m_d = (0.5064 \pm 0.0019)\,\mathrm{ps}^{-1}$$
$$\Delta m_s = (17.757 \pm 0.021)\,\mathrm{ps}^{-1}$$

$|V_{td}|=(8.1 \pm 0.5)\times 10^{-3}$
$|V_{ts}|=(39.4 \pm 2.3)\times 10^{-3}$
errors are totally theoretical.

$$|V_{td}/V_{ts}| = 0.210 \pm 0.001 \pm 0.008.$$

Example-2: $Bs - \bar{B}s$ mixing



box diagram mediating $B_s - \bar{B}_s$ mixing:

$$\frac{G_F^2}{16\pi^2} M_W^2 \sum_{i,j=u,c,t} V_{ib}^* V_{is} V_{jb}^* V_{js} F(x_i, x_j)$$

Simplifications:

- external quark momenta negligible
- GIM mechanism: mass-independent piece of $F(x_i, x_j)$ drops out
- $m_i \ll m_t$ and $|V_{ib}^* V_{is}| \ll |V_{tb}^* V_{ts}|$ $(i = u, c)$ ➢ only top quark contribution relevant
- perturbative QCD corrections can be included by adding a factor η_B (known from tedious calculations)

Example-2: $Bs - \bar{B}s$ mixing

➢ effective Hamiltonian for $B_s - \bar{B}_s$ mixing:

$$\mathcal{H}_{\rm eff}^{B_s-\bar{B}_s} = \frac{G_F^2}{16\pi^2} M_W^2 \eta_B (V_{tb}^* V_{ts})^2 S_0(x_t) (\bar{b}s)_{V-A} (\bar{b}s)_{V-A} + h.c.$$

$$\text{with } S_0(x_t) = \frac{4x_t - 11x_t^2 + x_t^3}{4(1-x_t)^2} - \frac{3x_t^3 \ln x_t}{2(1-x_t)^3}$$

now sandwich $\mathcal{H}_{\rm eff}^{B_s-\bar{B}_s}$ between initial and final state meson to obtain mixing matrix element:

$$M_{12} = \frac{1}{2m_{B_s}} \left\langle \bar{B}_s \left| \mathcal{H}_{\rm eff}^{B_s-\bar{B}_s} \right| B_s \right\rangle^*$$

with the hadronic matrix element $\langle \bar{B}_s \left| (\bar{b}s)_{V-A} (\bar{b}s)_{V-A} \right| B_s \rangle$ calculated on the lattice

$$\equiv \frac{8}{3} B_{Bs} f_{Bs}^2\, m_{Bs}^2$$

$$\Delta m = \frac{G_F^2 m_W^2}{16\pi^2} |V_{tb} V_{ts}^*|^2 S_0(x_t) \eta_{QCD} \frac{1}{m_{Bs}} \langle B_s^0 | (\bar{s}b)_{V-A} (\bar{s}b)_{V-A} | \bar{B}_s^0 \rangle$$

Some Details on V_{CKM}

$$V_{CKM} = \begin{pmatrix} V_{ud} & V_{us} & V_{ub} \\ V_{cd} & V_{cs} & V_{cb} \\ V_{td} & V_{ts} & V_{tb} \end{pmatrix} \approx \begin{pmatrix} 1 & \lambda & V_{ub} \\ -\lambda & 1 & V_{cb} \\ V_{td} & V_{ts} & V_{tb} \end{pmatrix}$$

- |Vcb| and |Vub| measured by semi-leptonic Bu and Bd decays

Arg Vcb =0 by a phase convention

Arg Vub= γ by CP Violation in B-->DK

$V_{tb} \approx 1$ if we assume V_{CKM} to be unitary

$|V_{td}| \times |V_{tb}|$ by B_0-B_0 oscillation frequency (Δm_d)

$|V_{ts}| \times |V_{tb}|$ by B_s-B_s oscillation frequency (Δm_s)

arg V_{td} by CP violation in $B_d \rightarrow J/\psi K_S$

arg V_{ts} by CP violation in $B_s \rightarrow J/\psi\phi$ **See later!**

2 CP VIOLATION

CP Violation

1. CPV in mixing

$$|B^0\rangle \quad \ldots\ldots t \ldots\ldots \quad \beta\,|\bar{B}^0\rangle$$
$$|\bar{B}^0\rangle \quad \ldots\ldots t \ldots\ldots \quad \bar{\beta}\,|B^0\rangle$$

if $|\beta| \neq |\bar{\beta}|$, it is possible to measure CPV in decays to final states accessible only to B^0or $\bar{B}^0$

2. CPV in decay

$$\Gamma(B \to f) \neq \Gamma(\bar{B} \to \bar{f})$$

$$A(B \to f) = \sum A_k e^{i(\delta_k + \phi_k)} \longrightarrow A(\bar{B} \to \bar{f}) = \sum A_k e^{i(\delta_k - \phi_k)}$$

from QCD from CKM

3. CPV in the interference of decays with and without mixing

$$\begin{tikzcd} B^0 \arrow[rr] \arrow[dr] & & f_{CP} \\ & \bar{B}^0 \arrow[ur] & \end{tikzcd}$$

CP Violation

- There are three quantities that drive CPV effects:

$$\left|\frac{q}{p}\right|, \quad \left|\frac{\bar{A}_{\bar{f}}}{A_f}\right|, \quad \lambda_f = \frac{q}{p}\frac{\bar{A}_f}{A_f}$$

- CPV in mixing: $|q/p| \neq 1$
- CPV in decay: $|\bar{A}_{\bar{f}}/A_f| \neq 1$
- CPV in the interference between decay and mixing: $\lambda_f \neq \pm 1$

CP Violation

- Given a final state f, let us introduce the two amplitudes $A_f = \langle f|\mathcal{H}_d|B^0\rangle$ and $\bar{A}_f = \langle f|\mathcal{H}_d|\bar{B}^0\rangle$, and the quantity:

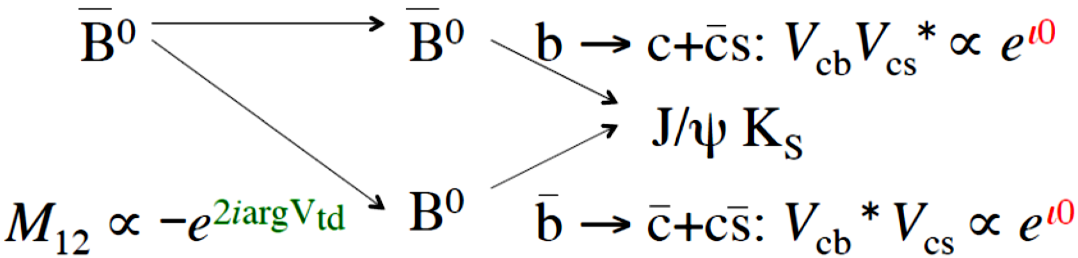
$$\lambda_f = \frac{q}{p}\frac{\bar{A}_f}{A_f} = -\frac{M_{12}^*}{|M_{12}|}\frac{\bar{A}_f}{A_f}\left[1 - \underbrace{\frac{1}{2}\mathrm{Im}\frac{\Gamma_{12}}{M_{12}}}_{a}\right]$$

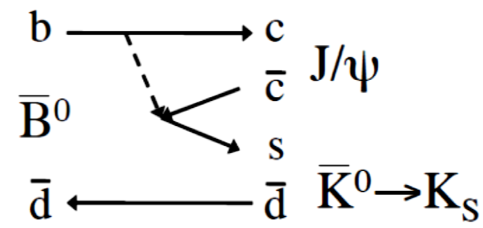
- Master equations:

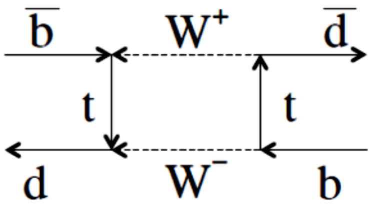
$$\begin{aligned}\Gamma(B^0(t)\to f) &= \mathcal{N}_f|A_f|^2 e^{-\Gamma t}\Big\{\frac{1+|\lambda_f|^2}{2}\cosh\frac{\Delta\Gamma t}{2} + \frac{1-|\lambda_f|^2}{2}\cos(\Delta m t) \\ &\quad -\mathrm{Re}\lambda_f \sinh\frac{\Delta\Gamma t}{2} - \mathrm{Im}\lambda_f \sin(\Delta m t)\Big\}\end{aligned}$$

$$\begin{aligned}\Gamma(\bar{B}^0(t)\to f) &= \mathcal{N}_f|A_f|^2(1+a) e^{-\Gamma t}\Big\{\frac{1+|\lambda_f|^2}{2}\cosh\frac{\Delta\Gamma t}{2} - \frac{1-|\lambda_f|^2}{2}\cos(\Delta m t) \\ &\quad -\mathrm{Re}\lambda_f \sinh\frac{\Delta\Gamma t}{2} + \mathrm{Im}\lambda_f \sin(\Delta m t)\Big\}\end{aligned}$$

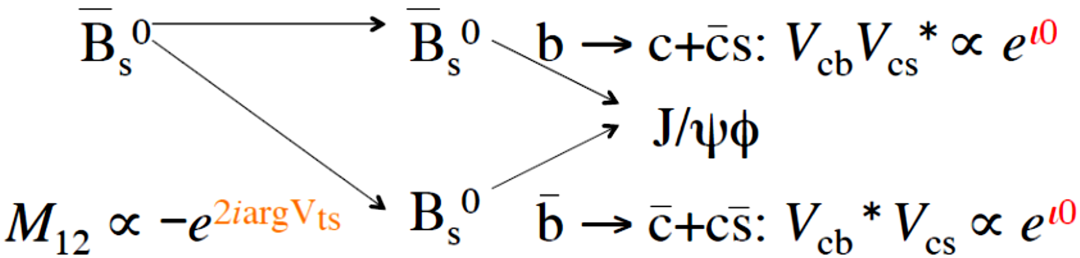
Sin2β in $B \to J/\psi K_s$

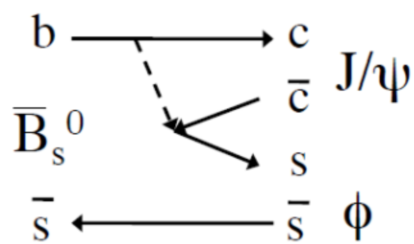


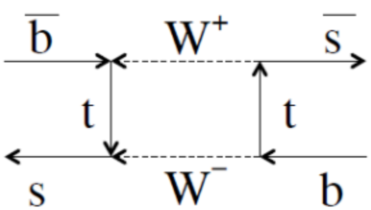




two processes interfere → CPV ∝ sin $2\arg V_{td}$







two processes interfere → CPV ∝ sin $2\arg V_{ts}$

Sin2β in $B \to J/\psi K_s$

- The final state is a CP eigenstate (with eigenvalue -1)
- This process is mediated by both tree and penguin operators:

$$\begin{aligned} A &= T_{c\bar{c}s}\, V_{cb}V_{cs}^* + P_s^u\, V_{ub}V_{us}^* + P_s^c\, V_{cb}V_{cs}^* + P_s^t\, V_{tb}V_{ts}^* \\ &= (T_{c\bar{c}s} + P_s^c - P_s^t)\, V_{cb}V_{cs}^* + \underbrace{(P_s^u - P_s^t)\, V_{ub}V_{us}^*}_{\text{penguin pollution}} \end{aligned}$$

H-N Li, 2004

- The last term is doubly suppressed:

$$\frac{V_{ub}V_{us}^*}{V_{cb}V_{cs}^*} \simeq 10^{-2}$$

$$P/T \sim O(0.1)$$

$$\Rightarrow \quad \frac{A(B^0 \to J/\psi K^0)}{A(\bar{B}^0 \to J/\psi \bar{K}^0)} \simeq \frac{V_{cb}V_{cs}^*}{V_{cb}^*V_{cs}}$$

Sin2β in $B \to J/\psi K_s$

- There is a problem: $B^0 \to K^0$ and $\bar{B}^0 \to \bar{K}^0$
- K_s is the lighter mass eigenstate: $|K_s\rangle = p_K|K^0\rangle + q_K|\bar{K}^0\rangle$
- Interference between $B^0 \to J/\psi K_s$ and $\bar{B}^0 \to J/\psi K_s$ is only possible through K mixing:

$$\frac{A(B^0 \to J/\psi K_s)}{A(\bar{B}^0 \to J/\psi K_s)} = \frac{A(B^0 \to J/\psi K^0)}{A(\bar{B}^0 \to J/\psi \bar{K}^0)} \frac{p_K}{q_K} = -\eta_{\psi K_s} \frac{V_{cb} V_{cs}^*}{V_{cb}^* V_{cs}} \frac{V_{cs} V_{cd}^*}{V_{cs}^* V_{cd}}$$

- Putting everything together:

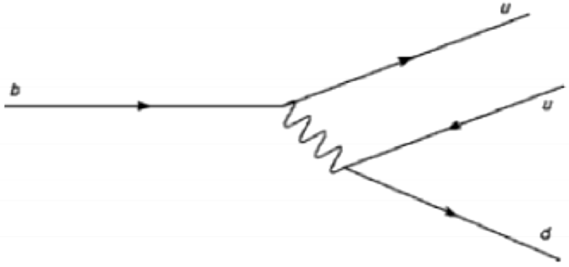
$$\lambda_{\psi K_s} = \frac{q}{p} \frac{A_{\psi K^0}}{\bar{A}_{\psi \bar{K}^0}} \frac{p_K}{q_K} = \eta_{\psi K_s} \frac{V_{tb}^* V_{td}}{V_{tb} V_{td}^*} \frac{V_{cb} V_{cs}^*}{V_{cb}^* V_{cs}} \frac{V_{cs} V_{cd}^*}{V_{cs}^* V_{cd}} = \eta_{\psi K_s} e^{-2i\beta}$$

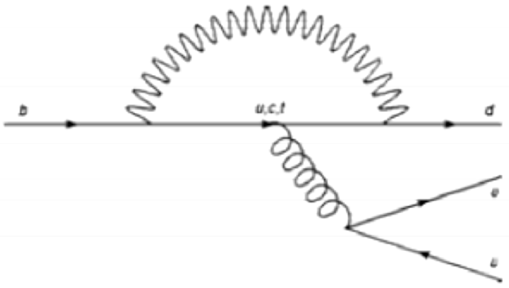
- and the time dependent CP asymmetry is

$$\begin{aligned} \mathcal{A}_{\psi K_s} &= \frac{\Gamma(\bar{B}^0(t) \to J/\psi K_s) - \Gamma(B^0(t) \to J/\psi K_s)}{\Gamma(\bar{B}^0(t) \to J/\psi K_s) + \Gamma(B^0(t) \to J/\psi K_s)} \\ &= -\eta_{\psi K_s} \sin(2\beta) \sin(\Delta m_{B_d} t) \end{aligned}$$

Sin2α in $B \rightarrow \pi\pi$

- This process is mediated by both tree and penguin operators:





$$
\begin{aligned}
A &= T_{u\bar{u}d}\, V_{ub}V_{ud}^{*} + P_d^u\, V_{ub}V_{ud}^{*} + P_d^c\, V_{cb}V_{cd}^{*} + P_d^t\, V_{tb}V_{td}^{*} \\
&= (T_{u\bar{u}d} + P_d^u - P_d^c)\, V_{ub}V_{ud}^{*} \;+\; (P_d^t - P_d^c)\, V_{tb}V_{td}^{*} \\
&= T_{\pi\pi}\, V_{ub}V_{ud}^{*} + P_{\pi\pi}\, V_{tb}V_{td}^{*} \\
&= T_{\pi\pi}\, V_{ub}V_{ud}^{*}\left(1 + r_{\pi\pi}\frac{V_{tb}V_{td}^{*}}{V_{ub}V_{ud}^{*}}\right) = T_{\pi\pi}\, V_{ub}V_{ud}^{*}\,\underbrace{(1 + r_{\pi\pi}\,\kappa)}_{\text{penguin pollution}}
\end{aligned}
$$

- Is the last term suppressed?

$$\left|\frac{V_{tb}V_{td}^{*}}{V_{ub}V_{ud}^{*}}\right| \sim 2$$

$$r_{\pi\pi} \sim O(0.1)$$

$$\frac{A(B^0 \rightarrow \pi\pi)}{A(\bar{B}^0 \rightarrow \pi\pi)} \simeq \frac{V_{ub}V_{ud}^{*}}{V_{ub}^{*}V_{ud}}\,\frac{1 + r_{\pi\pi}\,\kappa}{1 + r_{\pi\pi}\,\kappa^{*}}$$

Sin2α in $B \to \pi\pi$

- $$\lambda_{\pi\pi} = \frac{q}{p}\frac{A_{\pi\pi}}{\bar{A}_{\pi\pi}} = \underbrace{\frac{V_{tb}^* V_{td}}{V_{tb} V_{td}^*}}_{e^{-2i\beta}} \underbrace{\frac{V_{ub} V_{ud}^*}{V_{ub}^* V_{ud}}}_{e^{-2i\gamma}} \frac{1 + r_{\pi\pi}\,\kappa}{1 + r_{\pi\pi}\,\kappa^*} = e^{2i\alpha}\frac{1 + r_{\pi\pi}\,\kappa}{1 + r_{\pi\pi}\,\kappa^*}$$

- The time-dependent CP asymmetry is:

$$\begin{aligned} \mathcal{A}_{\pi\pi} &= -C_{\pi\pi}\ \cos(\Delta m_{B_d} t) + S_{\pi\pi}\ \sin(\Delta m_{B_d} t) \\ S_{\pi\pi} &= \sin(2\alpha) + O(r_{\pi\pi}) \\ C_{\pi\pi} &= O(r_{\pi\pi}) \end{aligned}$$

- What do we get from experiments

$$\begin{aligned} S_{\pi\pi}^{\exp} &= -0.59 \pm 0.09 \\ C_{\pi\pi}^{\exp} &= -0.39 \pm 0.07 \end{aligned}$$

Sin2α in $B \to \pi\pi$

- Solutions:
  - use effective theories to calculate $r_{\pi\pi}$
  - use SU(3) flavor symmetry to relate B→Kπ and B→ππ
  - isospin analysis

- Up to isospin breaking corrections we can describe

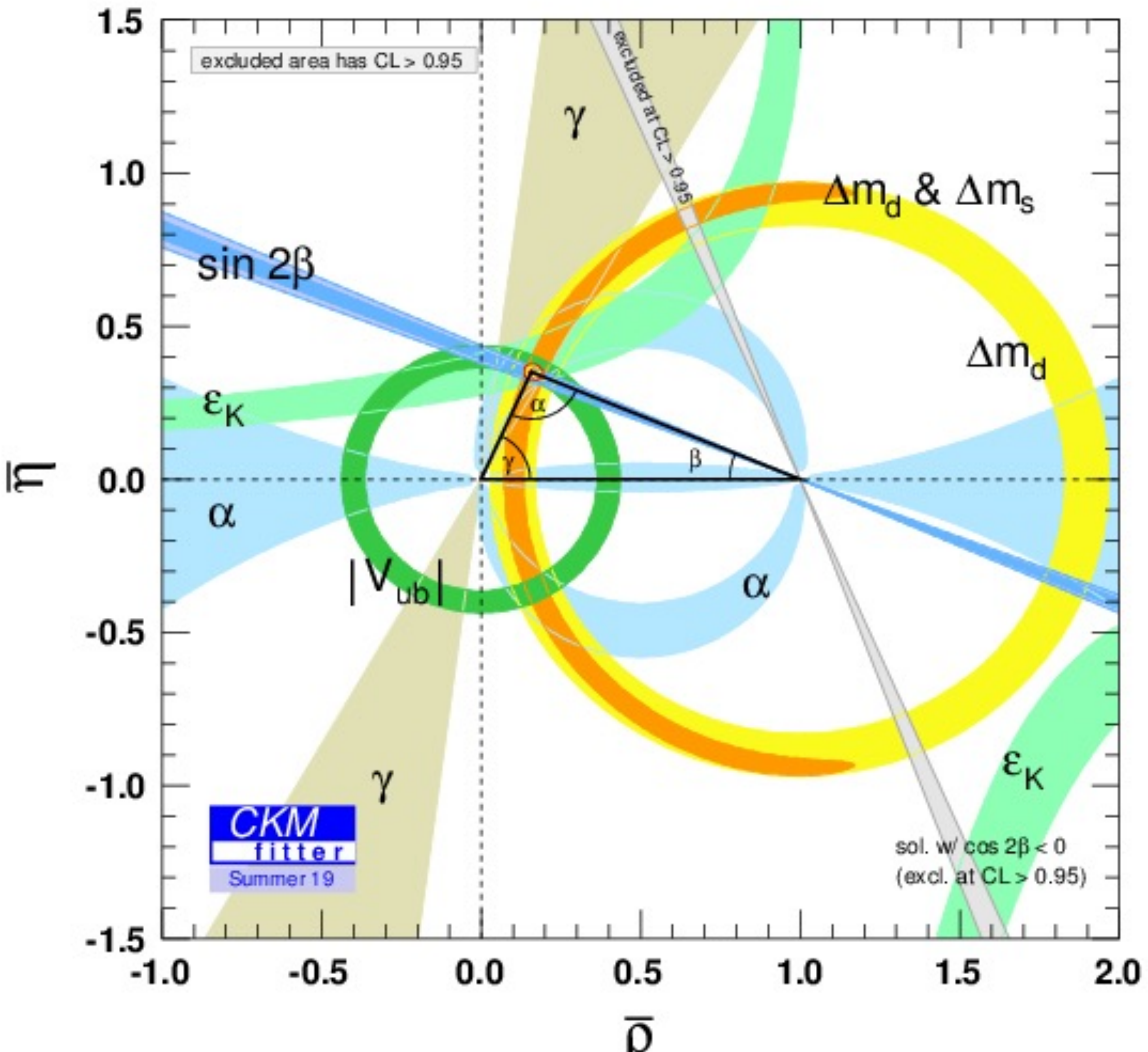
$$A(B^+ \to \pi^+\pi^0),\ A(B^0 \to \pi^+\pi^-),\ A(B^0 \to \pi^0\pi^0)$$

in terms of two isospin amplitudes

$$A(B \to [\pi\pi]_0),\ A(B \to [\pi\pi]_2)$$

- At the end of the day we are able to eliminate $r_{\pi\pi}$ and extract sin(2α)

The unitarity triangle

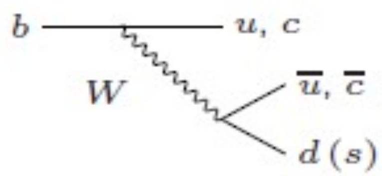


3

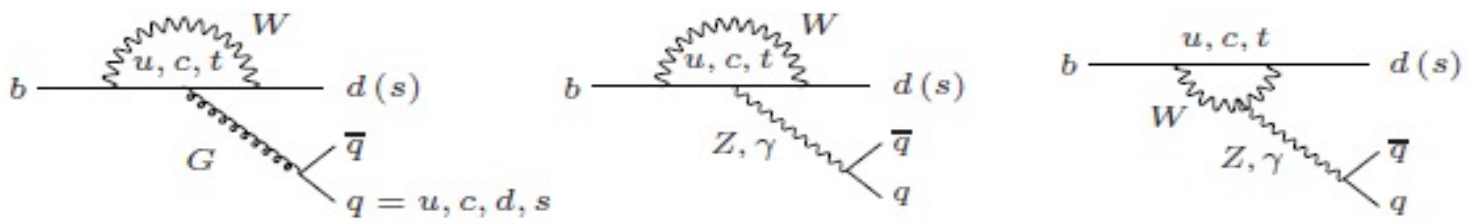
Rare Decays

Topologies & Classification

- Tree diagrams:



- Penguin diagrams:
  - ◇ QCD penguins:
  - ◇ Electroweak (EW) penguins:



- Classification (depends on the flavour content of the final state):
  - – Only tree diagrams.
  - – Tree and penguin diagrams.
  - – Only penguin diagrams.

Low-Energy Effective Hamiltonians

- Operator product expansion (OPE): $\Rightarrow$

$$\boxed{\langle f|\mathcal{H}_{\text{eff}}|i\rangle = \frac{G_{\rm F}}{\sqrt{2}} V_{\rm CKM} \sum_k C_k(\mu)\, \langle f|Q_k(\mu)|i\rangle}$$

[$G_{\rm F}$: Fermi constant, $V_{\rm CKM}$: CKM factor, μ: renormalization scale]

- The operator product expansion allows a separation of the short-distance from the long-distance contributions:
  - *Perturbative* Wilson coefficients $C_k(\mu)$ $\rightarrow$ short-distance physics.
  - *Non-perturbative* hadronic MEs $\langle f|Q_k(\mu)|i\rangle$ $\rightarrow$ long-distance physics.

- The Q_k are local operators, which are generated through the electroweak interactions and QCD, and govern "effectively" the considered decay.

- The Wilson coefficients $C_k(\mu)$ describe the scale-dependent "couplings" of the interaction vertices that are associated with the operators Q_k.

- Four-quark operators Q_k^{jr} ($j \in \{u, c\}$, $r \in \{d, s\}$):
  - Current–current operators (tree-like processes):

$$\begin{aligned} Q_1^{jr} &= (\bar{r}_\alpha j_\beta)_{\text{V-A}} (\bar{j}_\beta b_\alpha)_{\text{V-A}} \\ Q_2^{jr} &= (\bar{r}_\alpha j_\alpha)_{\text{V-A}} (\bar{j}_\beta b_\beta)_{\text{V-A}} \end{aligned}$$

  - QCD penguin operators:

$$\begin{aligned} Q_3^r &= (\bar{r}_\alpha b_\alpha)_{\text{V-A}} \sum_{q'} (\bar{q}'_\beta q'_\beta)_{\text{V-A}} \\ Q_4^r &= (\bar{r}_\alpha b_\beta)_{\text{V-A}} \sum_{q'} (\bar{q}'_\beta q'_\alpha)_{\text{V-A}} \\ Q_5^r &= (\bar{r}_\alpha b_\alpha)_{\text{V-A}} \sum_{q'} (\bar{q}'_\beta q'_\beta)_{\text{V+A}} \\ Q_6^r &= (\bar{r}_\alpha b_\beta)_{\text{V-A}} \sum_{q'} (\bar{q}'_\beta q'_\alpha)_{\text{V+A}} \end{aligned}$$

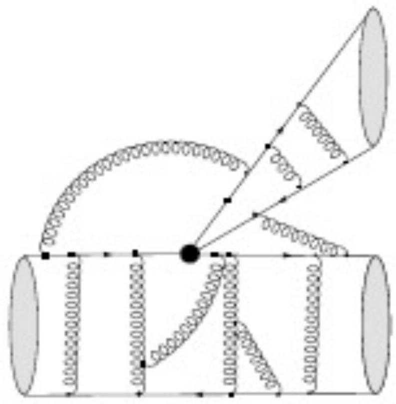
  - EW penguin operators:

$$\begin{aligned} Q_7^r &= \tfrac{3}{2} (\bar{r}_\alpha b_\alpha)_{\text{V-A}} \sum_{q'} e_{q'} (\bar{q}'_\beta q'_\beta)_{\text{V+A}} \\ Q_8^r &= \tfrac{3}{2} (\bar{r}_\alpha b_\beta)_{\text{V-A}} \sum_{q'} e_{q'} (\bar{q}'_\beta q'_\alpha)_{\text{V+A}} \\ Q_9^r &= \tfrac{3}{2} (\bar{r}_\alpha b_\alpha)_{\text{V-A}} \sum_{q'} e_{q'} (\bar{q}'_\beta q'_\beta)_{\text{V-A}} \\ Q_{10}^r &= \tfrac{3}{2} (\bar{r}_\alpha b_\beta)_{\text{V-A}} \sum_{q'} e_{q'} (\bar{q}'_\beta q'_\alpha)_{\text{V-A}} \end{aligned}$$

[Here α, β are $SU(3)_C$ indices, V$\pm$A refers to $\gamma_\mu(1 \pm \gamma_5)$, $q' \in \{u, d, c, s, b\}$ runs over the active quark flavours at $\mu = \mathcal{O}(m_b)$, and the $e_{q'}$ are the electrical charges]

Factorization

$$|A_j|e^{i\delta_j} \propto \sum_k \underbrace{C_k(\mu)}_{\text{pert. QCD}} \times \boxed{\langle \bar{f}|Q_k^j(\mu)|\bar{B}\rangle}$$



- Recent developments:
  - *QCD factorization (QCDF)*:
    Beneke, Buchalla, Neubert & Sachrajda (1999–2001); ...
  - *Perturbative Hard-Scattering (PQCD) Approach*:
    Li & Yu ('95); Cheng, Li & Yang ('99); Keum, Li & Sanda ('00); Ali, Lü, *et al.* ('07); ...
  - *Soft Collinear Effective Theory (SCET)*:
    Bauer, Pirjol & Stewart (2001); Bauer, Grinstein, Pirjol & Stewart (2003); ...
  - QCD *sum rules*:
    Khodjamirian (2001); Khodjamirian, Mannel & Melic (2003); ...

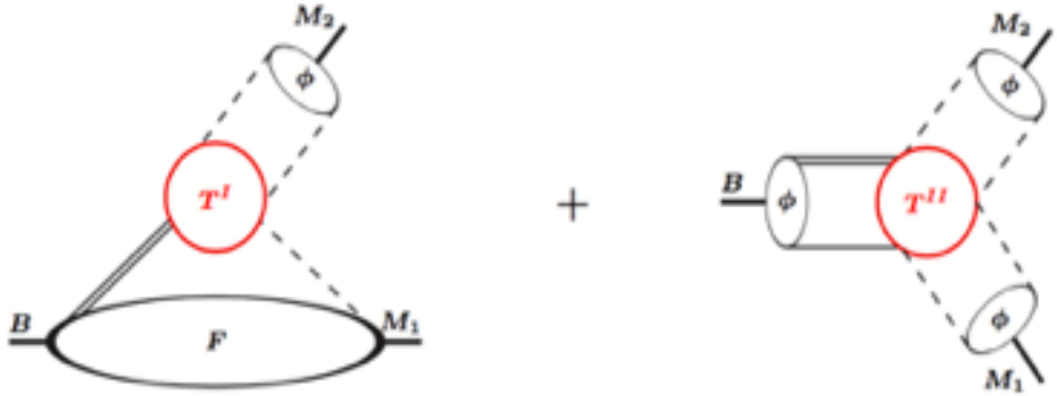
$$\boxed{\text{Data} \Rightarrow \text{theoretical challenge remains ...}}$$

Impact of New Physics

- Theoretical description through effective low-energy Hamiltonians:
  - The NP particles (such as the charginos, squarks in the MSSM) are "integrated out" as the W boson and the top quark in the SM:
    - Initial conditions for RG evolution: $C_k(\mu = M_W) \to C_k^{\rm SM} + C_k^{\rm NP}$
  - Operators, which are absent or strongly suppressed in the SM, may actually play an important rôle:
    - Operator basis: $\{Q_k\} \to \{Q_k^{\rm SM}, Q_l^{\rm NP}\}$
- Popular NP scenario: *Minimal Flavour Violation (MFV)*
  - Flavour and CP violation still governed by the SM Yukawa matrices.
  - Essentially no effects in CP asymmetries, but various interesting correlations between rare decay observables, mixing parameters, etc.

QCD Factorization

$$\langle M_1 M_2|\mathcal{O}|B\rangle = F^{BM_1}\int du\, T^{I}(u)\phi_{M_2}(u) + \int d\omega\, du\, dv\, T^{II}(\omega,u,v)\phi_B(\omega)\phi_{M_1}(u)\phi_{M_2}(v)$$



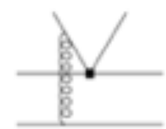
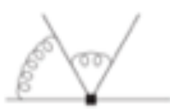
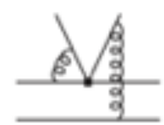
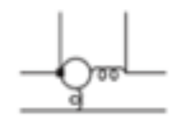
- Vertex corrections: $T^{I}(u) = 1 + \mathcal{O}(\alpha_s)$
- Spectator scattering: $T^{II}(\omega,u,v) = \mathcal{O}(\alpha_s)$ – (power suppressed if M_1 is heavy)
- Strong phases are perturbative $[\mathcal{O}(\alpha_s)]$ or power suppressed $[\mathcal{O}(\Lambda/m_b)]$.

QCD Factorization

Two hard-scattering kernels for each operator insertion: T^I (vertex), T^{II} (spectator)

$$\langle M_1 M_2 | \mathcal{O}_i | B \rangle \simeq F^{BM_1}\, T_i^I \otimes \phi_{M_2} + T_i^{II} \otimes \phi_B \otimes \phi_{M_1} \otimes \phi_{M_2}$$

and two classes of topological amplitudes: "Tree", "Penguin".

	T^I, tree	T^I, penguin	T^{II}, tree	T^{II}, penguin
LO: $\mathcal{O}(1)$				
NLO: $\mathcal{O}(\alpha_s)$ BBNS '99-'04				
NNLO: $\mathcal{O}(\alpha_s^2)$	 Bell '07,'09 Beneke, Huber, Li '09	 Kim, Yoon '11, Bell Beneke, Huber, Li '15	 Beneke, Jager '05 Kivel '06, Pilipp '07	 Beneke, Jager '06 Jain, Rothstein, Stewart '07

QCD Factorization

$$T \equiv a_1(\pi\pi) = 1.009 + [0.023 + 0.010i]_{\text{NLO}} + [0.026 + 0.028i]_{\text{NNLO}}$$
$$- \left[\frac{r_{\text{sp}}}{0.485}\right]\left\{[0.015]_{\text{LOsp}} + [0.037 + 0.029i]_{\text{NLOsp}} + [0.009]_{\text{tw3}}\right\}$$
$$= 1.00 + 0.01i \quad \to \quad 0.93 - 0.02i \quad (\text{if } 2 \times r_{\text{sp}})$$

$$C \equiv a_2(\pi\pi) = 0.220 - [0.179 + 0.077i]_{\text{NLO}} - [0.031 + 0.050i]_{\text{NNLO}}$$
$$+ \left[\frac{r_{\text{sp}}}{0.485}\right]\left\{[0.123]_{\text{LOsp}} + [0.053 + 0.054i]_{\text{NLOsp}} + [0.072]_{\text{tw3}}\right\}$$
$$= 0.26 - 0.07i \quad \to \quad 0.51 - 0.02i \quad (\text{if } 2 \times r_{\text{sp}})$$

	Theory I	Theory II	Experiment
$B^- \to \pi^-\pi^0$	$5.43\,^{+0.06}_{-0.06}\,^{+1.45}_{-0.84}$ $(\star)$	$5.82\,^{+0.07}_{-0.06}\,^{+1.42}_{-1.35}$ $(\star)$	$5.59^{+0.41}_{-0.40}$
$\bar{B}^0_d \to \pi^+\pi^-$	$7.37\,^{+0.86}_{-0.69}\,^{+1.22}_{-0.97}$ $(\star)$	$5.70\,^{+0.70}_{-0.55}\,^{+1.16}_{-0.97}$ $(\star)$	5.16 ± 0.22
$\bar{B}^0_d \to \pi^0\pi^0$	$0.33\,^{+0.11}_{-0.08}\,^{+0.42}_{-0.17}$	$0.63\,^{+0.12}_{-0.10}\,^{+0.64}_{-0.42}$	1.55 ± 0.19
		BELLE CKM 14:	0.90 ± 0.16

QCD Factorization

Main limitation of QCDF approach, e.g. weak annihilation

 $\sim \quad \int d\omega\, du\, dv\; T(\omega, u, v)\; \phi_B(\omega)\; \phi_{M_1}(v)\; \phi_{M_2}(u) \quad ?$

- convolutions diverge at endpoints $\Rightarrow$ non-factorisation in SCET-2
- currently modelled with arbitrary soft rescattering phase

Pure annihilation decays

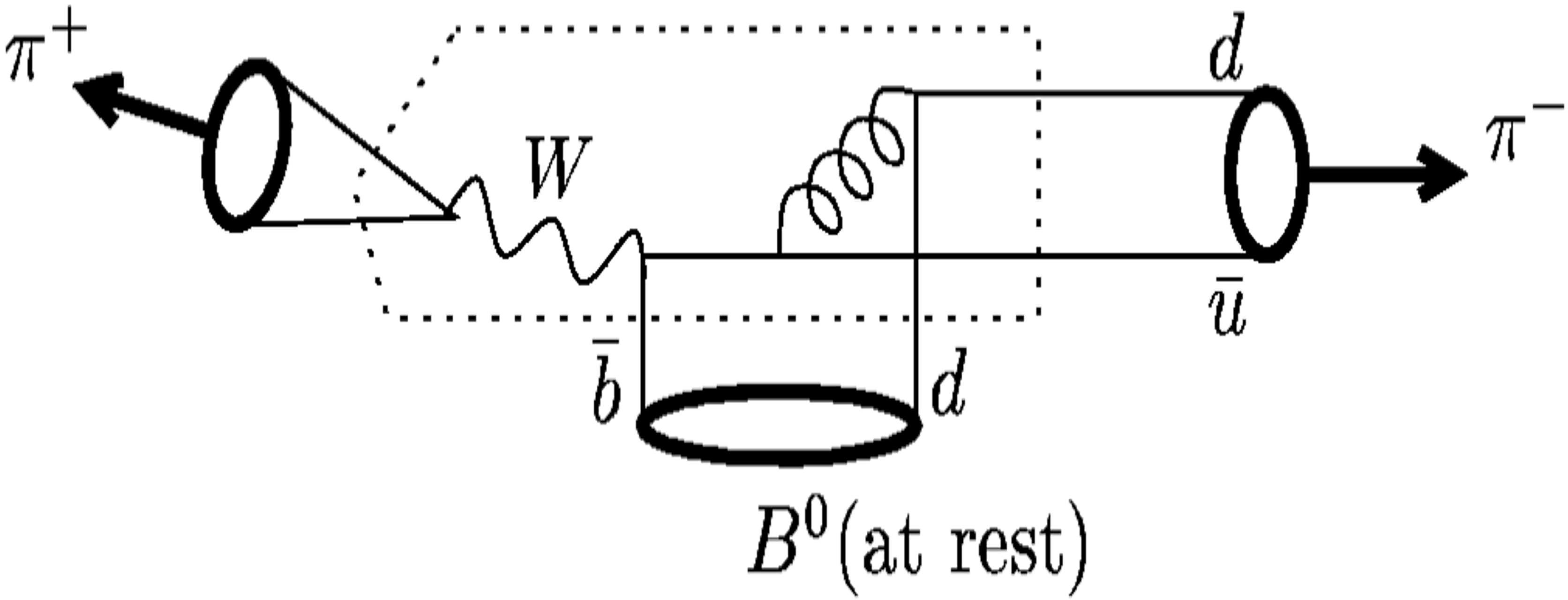
$10^6\,\mathrm{Br}(B_d \to K^+K^-) = 0.13 \pm 0.05$ $\quad (\Delta D = 1$, exchange topology)

$10^6\,\mathrm{Br}(B_s \to \pi^+\pi^-) = 0.76 \pm 0.13$ $\quad (\Delta S = 1$, penguin annihilation)

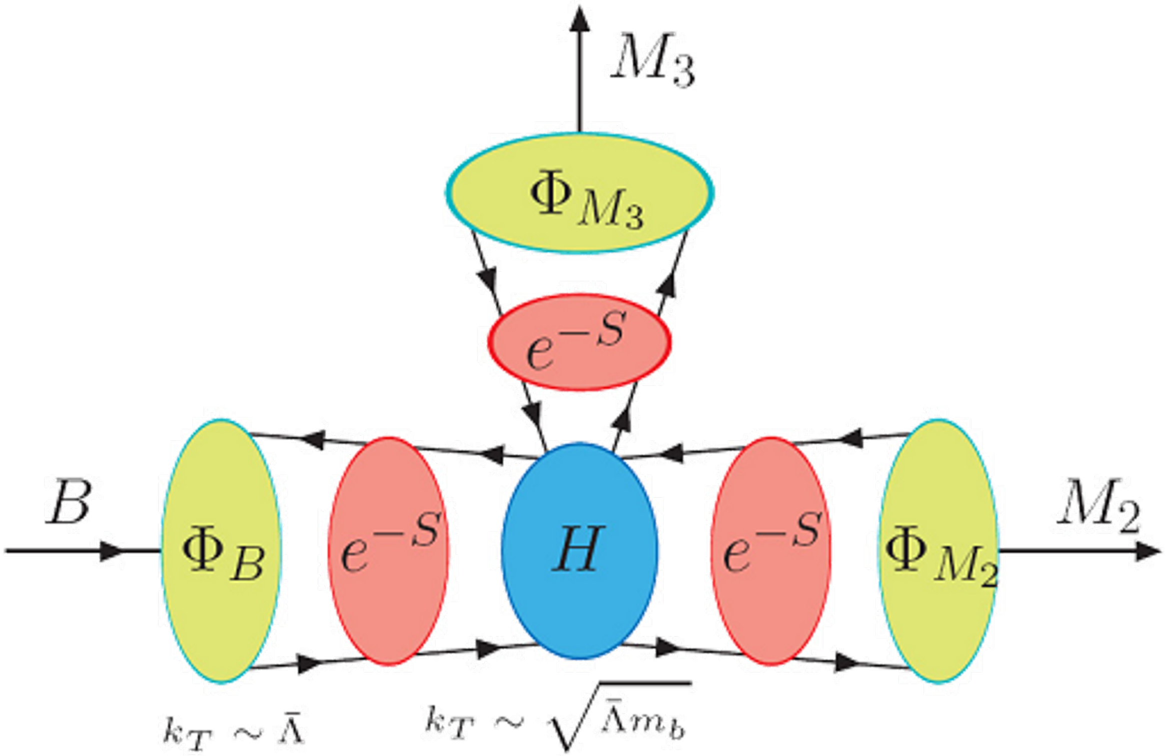
$\Rightarrow$ extract weak annihilation amplitudes from data [Wang, Zhu 13; Bobeth, Gorbahn, Vickers 14; Chang, Sun, Yang, Li 14]

▹ Or use "clean" combinations, *e.g.* $\Delta = T - P$ in penguin mediated decays

Perturbative QCD Approach



Perturbative QCD Approach



Perturbative QCD Approach

$$A \sim \int d^4k_1\, d^4k_2\, d^4k_3\, C(t)\Phi_B(k_1)\, \Phi_1(k_2)\, \Phi_2(k_3)\, H(k_1, k_2, k_3, t)\, \exp\{-S(t)\}$$

- $\Phi(k)$ is wave function in the light cone, which is universal.
- $C(t)$: Wilson coefficients of corresponding four quark operators
- $\exp\{-S(t)\}$ are Sudukov form factor (double log resummation)， which relate the long distance contribution and short one. And the long distance effects have been suppressed.
- $H(k_1, k_2, k_3, t)$ is six quark interaction, and it can be calculated perturbatively, and it is process depended.

表 2.1 推广的因子化方法 (GFA)， QCD 因子化方法 (BBNS) 和微扰 QCD 方法 (PQCD) 的比较。

	GFA	BBNS	PQCD
可计算		重 → 轻衰变的 不可因子化贡献	形状因子 不可因子化贡献 湮灭图贡献
不可计算	形状因子 不可因子化贡献 末态相互专用	形状因子 末态相互专用	末态相互专用
忽略部分	湮灭图贡献 末态相互专用	湮灭图贡献 末态相互专用	末态相互专用
输入参数	形状因子	形状因子 波函数	波函数
强相位产生机制	BSS	BSS 非因子化相应	湮灭图 非因子化效应

4

SUMMARY

- In my view , flavor physics remains one of the most promising windows search for CP Violation and new physics.
- Many new results require better statistics and further measurements: potential for multiple 5-sigma effects.
- Conversely if nothing is found in LHC the new colliders will significantly push up the effective scale of flavor .
- More interplays between experimentalists and theorists are needed.

Thanks for your attentions!