QCD Calculation for Hadronic B Decays

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Search for new particles or new phenomena is our major task in particle physics

- There are two ways to achieve that: direct search or indirect search
- Accordingly we have two directions in high energy physics experiments: high energy and high intensity ...

There are many high intensity experiments:

- Beijing electron position collider (BEPC)
- Daya Bay neutrino experiment etc.
- B-factories (two machines)
- LHCb
- There is even a super B-factory（Belle II）

Super B factory
Belle II

The discovery of direct CP violation leads to 2008 Nobel Prize

Flavor physics is important

The origin of flavour is one of the big, unsolved mysteries of fundamental physics!

While the Standard Model (SM) ***describes*** flavour physics very accurately, it does not ***explain*** its mysteries:

- ✓ Why are there 3 generations in nature?
- ✓ What determines the extreme hierarchy of fermion masses?
- ✓ What determines the elements of the CKM matrix?
- ✓ What is the origin of the matter-antimatter asymmetry (CP violation)?

The SM CP-violation is insufficient to explain the matter/antimatter asymmetry
→ progress in flavour physics may help understand open questions in cosmology

History has shown that flavour physics often gives first evidence for new discoveries:

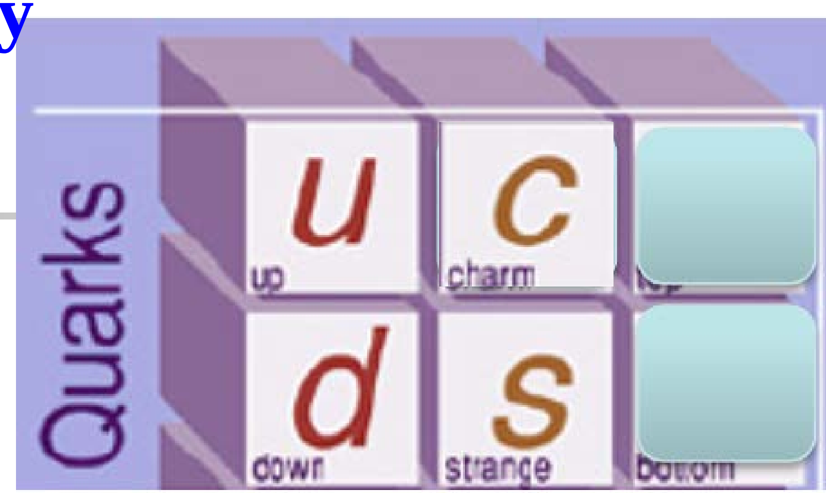
- ➢ Kaon mixing, BR($K^0_L \to \mu\mu$) & GIM → prediction of charm
- ➢ CP violation → prediction of third quark family
- ➢ B mixing → mass of top is very heavy
- ➢ rare B-decays → SUSY parameter space constrained

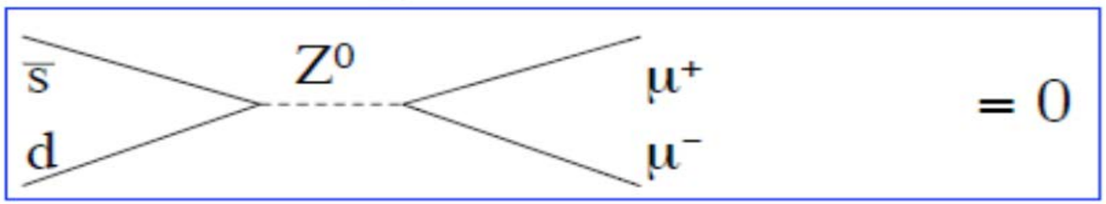
Example in the history

Long time ago, we had only 3 flavors of quarks: u,d,s.

Experimentally we found that

$$K^0 \not\to \mu^+\mu^-$$

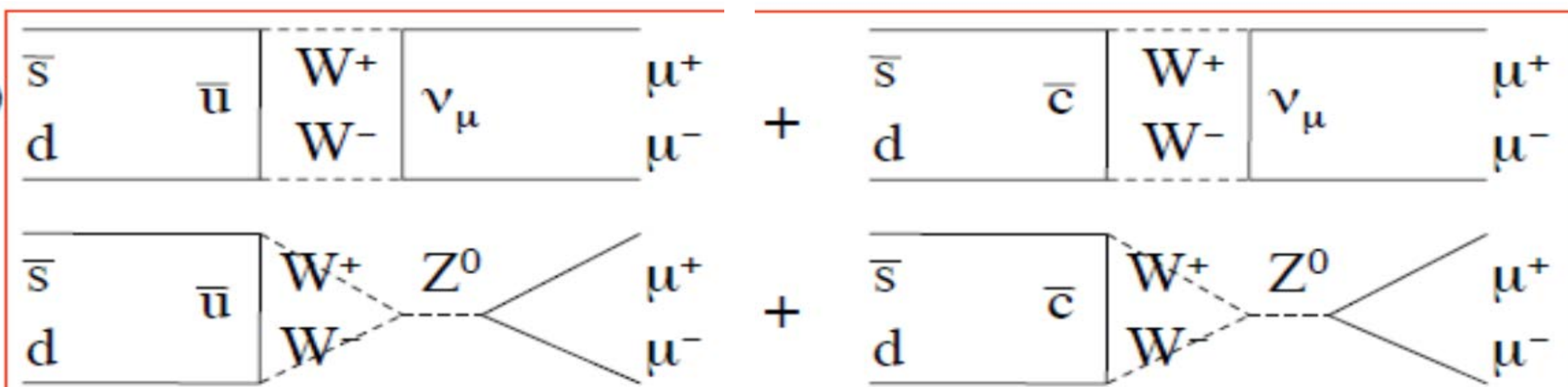




$$\begin{pmatrix} u \\ d \end{pmatrix} \begin{pmatrix} c \\ s \end{pmatrix}$$

GIM

K^0

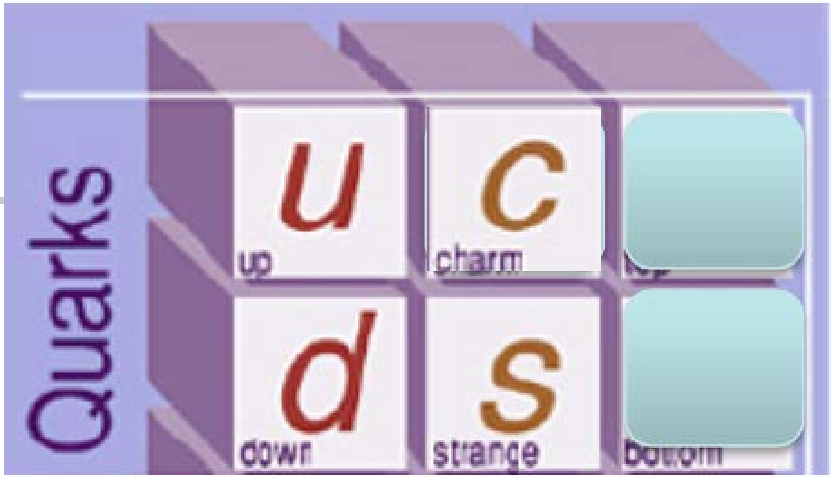


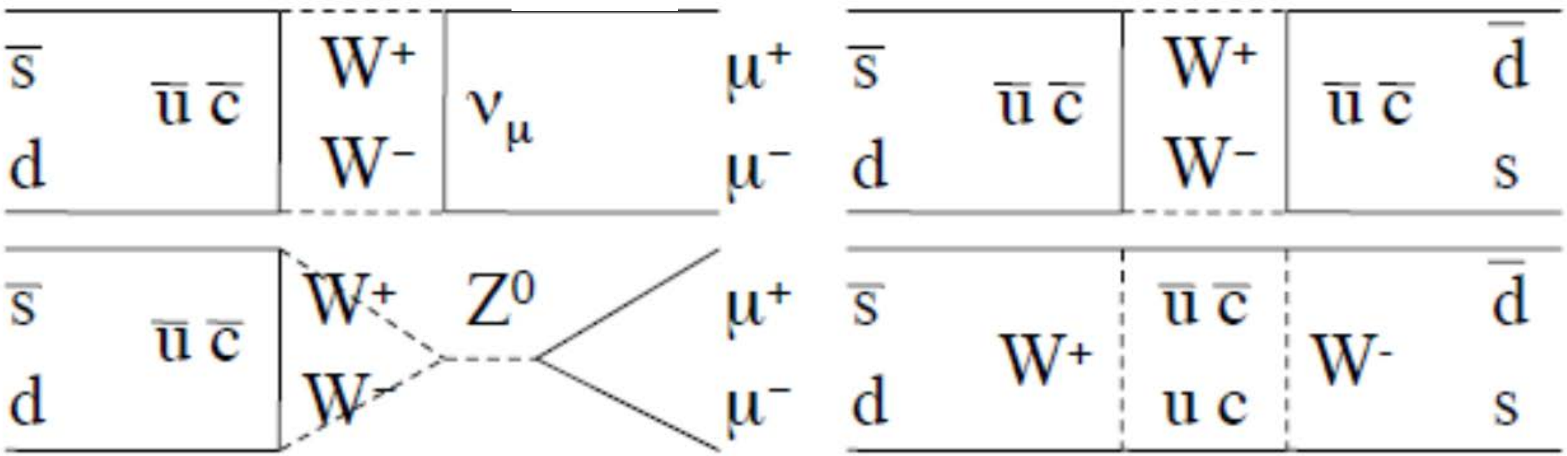
Divergent

$$= 0 \text{ if } m_u = m_c$$

Later, more precise experiments found that

$$Br(K^0 \to \mu^+\mu^-) \sim 10^{-9}$$





$$Br(K^0 \to \mu^+\mu^-) = F(m_c, \ldots) \qquad \Delta m_K = G(m_c, \ldots)$$

$$\to m_c \approx 1.5\,\mathrm{GeV}$$

Ting and Richter found that in 1974

Heavy flavor physics is a very important hot topic in particle physics recently

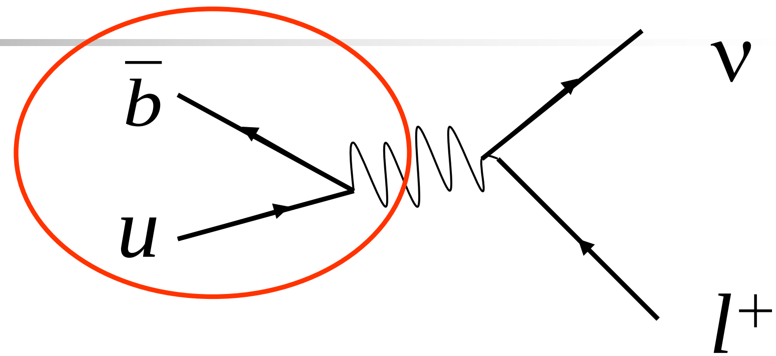
- **People expect the new physics signal from the heaviest top quark, since it is very close to the electroweak breaking scale**
- **But there are too few data of top quark production**
- **Therefore beauty quark is our best chance for new physics signals, since they both belong to the third family**

Current Flavor Anomalies

$\sim 3.5\sigma$	$(g-2)_\mu$ anomaly	
$\sim 3.5\sigma$	non-standard like-sign dimuon charge asymmetry	
$\sim 3.5\sigma$	enhanced $B \to D^{(*)}\tau\nu$ rates	$R_{D^{(*)}}$
$\sim 3.5\sigma$	suppressed branching ratio of $B_s \to \phi\mu^+\mu^-$	
$\sim 3\sigma$	tension between inclusive and exclusive determination of $\lvert V_{ub}\rvert$	
$\sim 3\sigma$	tension between inclusive and exclusive determination of $\lvert V_{cb}\rvert$	
$2-3\sigma$	anomaly in $B \to K^*\mu^+\mu^-$ angular distributions	P_5'
$2-3\sigma$	SM prediction for ϵ'/ϵ below experimental result	
$\sim 2.5\sigma$	lepton flavor non-universality in $B \to K\mu^+\mu^-$ vs. $B \to Ke^+e^-$	R_K
$\sim 2.5\sigma$	non-zero $h \to \tau\mu$	

Pure leptonic decays

$$\langle P(p)|\bar{q}\gamma^{\mu}Lq'|0\rangle = if_P p^{\mu}.$$



- The decay constant is

the normalization of the meson

wave function i.e. the zero point of wave function

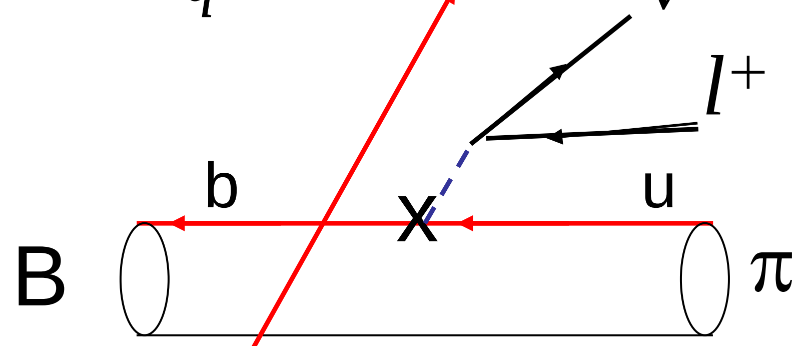
- The experimental measurement of pure leptonic decay can provide the product of decay constant and CKM matrix element.
- Theoretically decay constant can be calculated by QCD sum rule or Lattice QCD

We have two hadrons in semi-leptonic decays. It is described by form factors

$$\langle \pi | \bar{u}\gamma^{\mu} b | B \rangle = p_B{}^{\mu} f_1 + p_{\pi}{}^{\mu} f_2 \qquad q = p_B - p_{\pi}$$

$$= \left[(p_B + p_{\pi})^{\mu} - \frac{m_B{}^2 - m_{\pi}{}^2}{q^2} q^{\mu} \right] F_1(q^2) + \frac{m_B{}^2 - m_{\pi}{}^2}{q^2} q^{\mu} F_0(q^2)$$

Form factors can be calculated by lattice QCD, QCD sum rules, light cone sum rules etc.

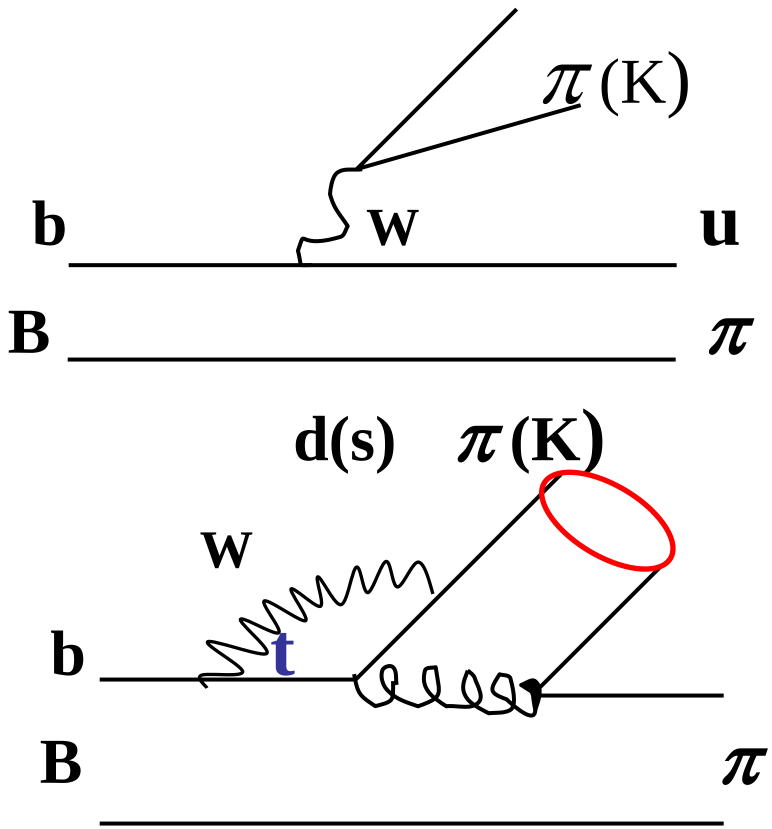


In the quark model, it is calculated by the overlap of two meson wave functions.

Not a constant but a **function**

Rich physics in hadronic B decays

CP violation, FCNC, sensitive to new physics contribution…



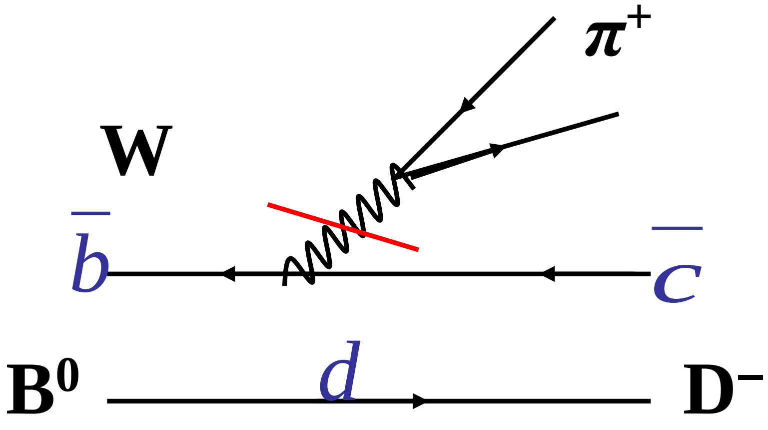
The standard model describes interactions amongst quarks and leptons

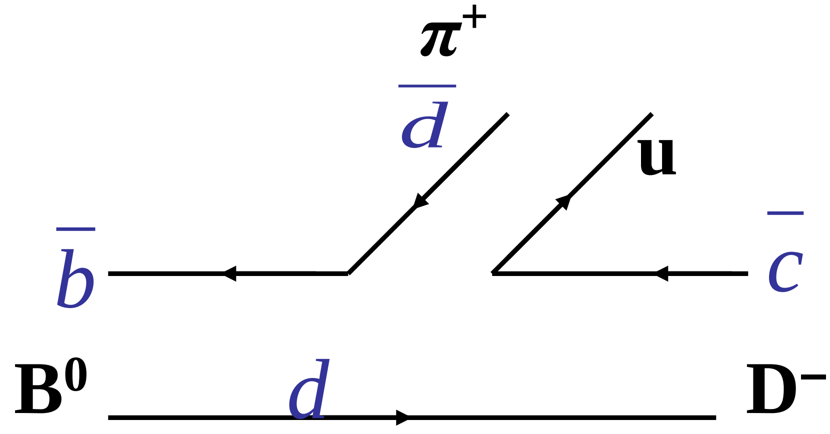
In experiments, we can only observe hadrons

pi K puzzle etc.

How can we test the standard model without solving QCD?

Naïve Factorization (BSW model)





Bauer, Stech, Wirbel, Z. Phys. C29, 637 (1985); ibid 34, 103 (1987)

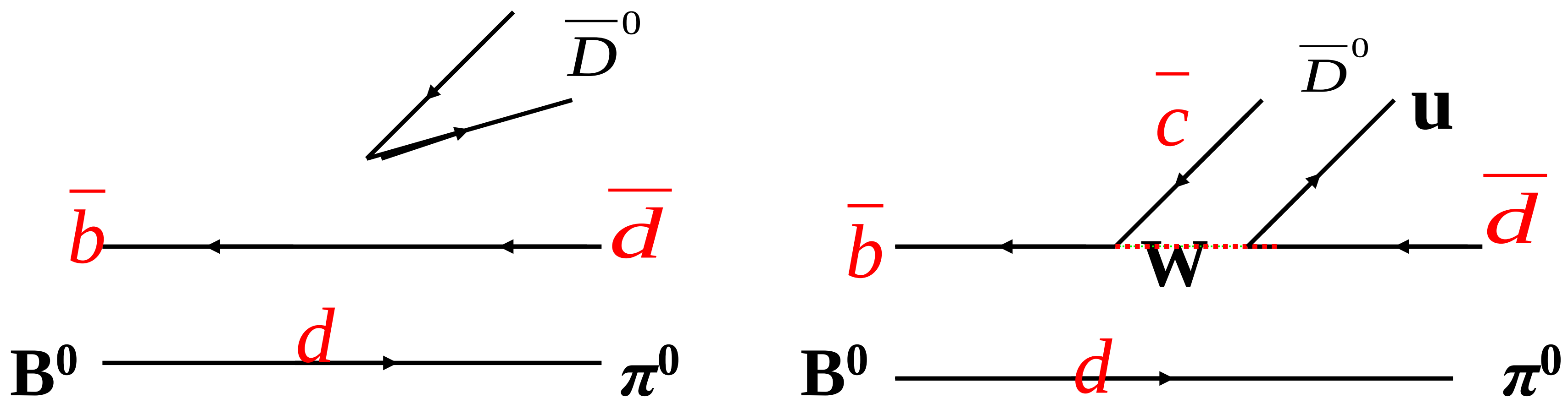
Hadronic parameters: Form factor and decay constant

$$< \pi^+ D^- | H_{eff} | B > = a_1 \; \langle \pi | \bar{u} \gamma^\mu L d | 0 \rangle \; \langle D | \bar{b} \gamma_\mu L c | B \rangle$$

Form factors calculated from quark model

Generalized Factorization Approach

Ali, Kramer, Lu, Phys. Rev. D58, 094009 (1998)



$C_1 \sim -0.2 \quad \sim \quad C_2(1/3 + s_8) \equiv C_2/N_c \sim +1/3$

$$\left\langle \pi^0 \overline{D}^0 \middle| H_{eff} \middle| B^0 \right\rangle = (C_1 + C_2/N_c)\, f_D\, F_0^{B\to\pi}$$

Non-factorizable contribution should be larger than expected, characterized by effective N_C

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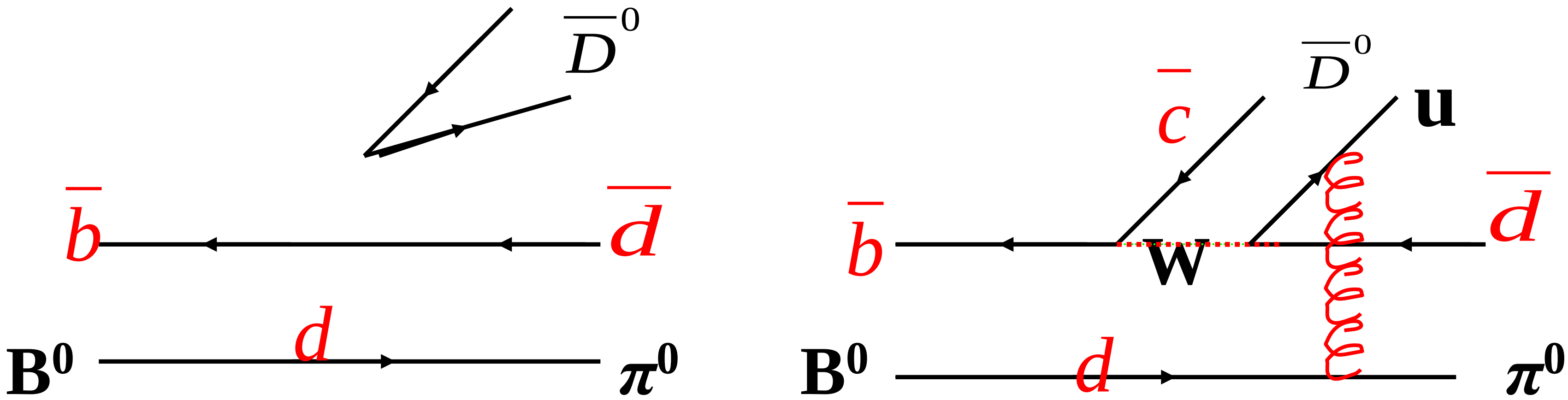

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QCD Penguin operators

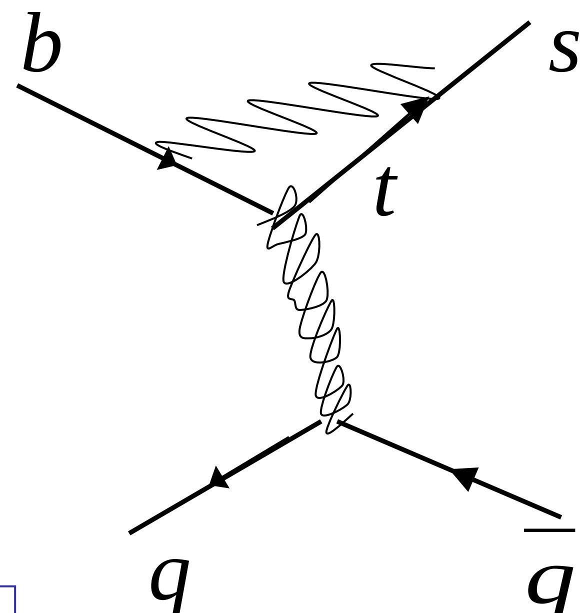
- **Wilson coefficients** $\propto \alpha_s$

$$O_3 = \bar{d}\gamma^{\mu} L b \cdot \sum_q \bar{q}\gamma_{\mu} L q$$

$$O_4 = \bar{d}_{\alpha}\gamma^{\mu} L b_{\beta} \cdot \sum_q \bar{q}_{\beta}\gamma_{\mu} L q_{\alpha}$$

$$O_5 = \bar{d}\gamma^{\mu} L b \cdot \sum_q \bar{q}\gamma_{\mu} R q$$

$$O_6 = \bar{d}_{\alpha}\gamma^{\mu} L b_{\beta} \cdot \sum_q \bar{q}_{\beta}\gamma_{\mu} R q_{\alpha}$$



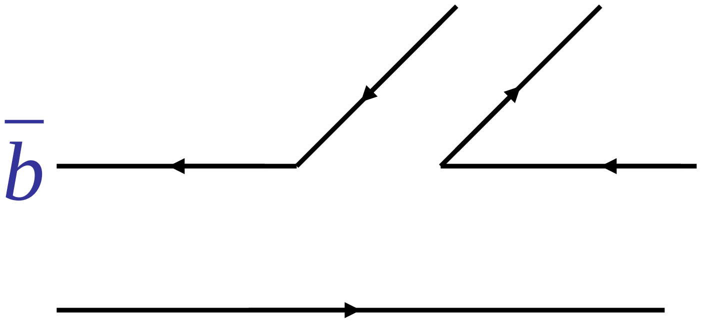
$$\boxed{R = 1 + \gamma^5}$$

Chiral enhanced penguin

Ali, Kramer, Lu, Phys. Rev. D58, 094009 (1998)

Fiertz transformation gives a

Chiral enhanced factor m_π^2/m_d



In this paper, We also found that

This makes $Br(B \to \pi^+K^-) > Br(B \to \pi^+\pi^-)$

Previously in BSW model it is the inverse case

Shortcomings of Naive FA

- Non-predictable of the non-factorizable contributions
- Form factors need input from experiments or theoretical calculations

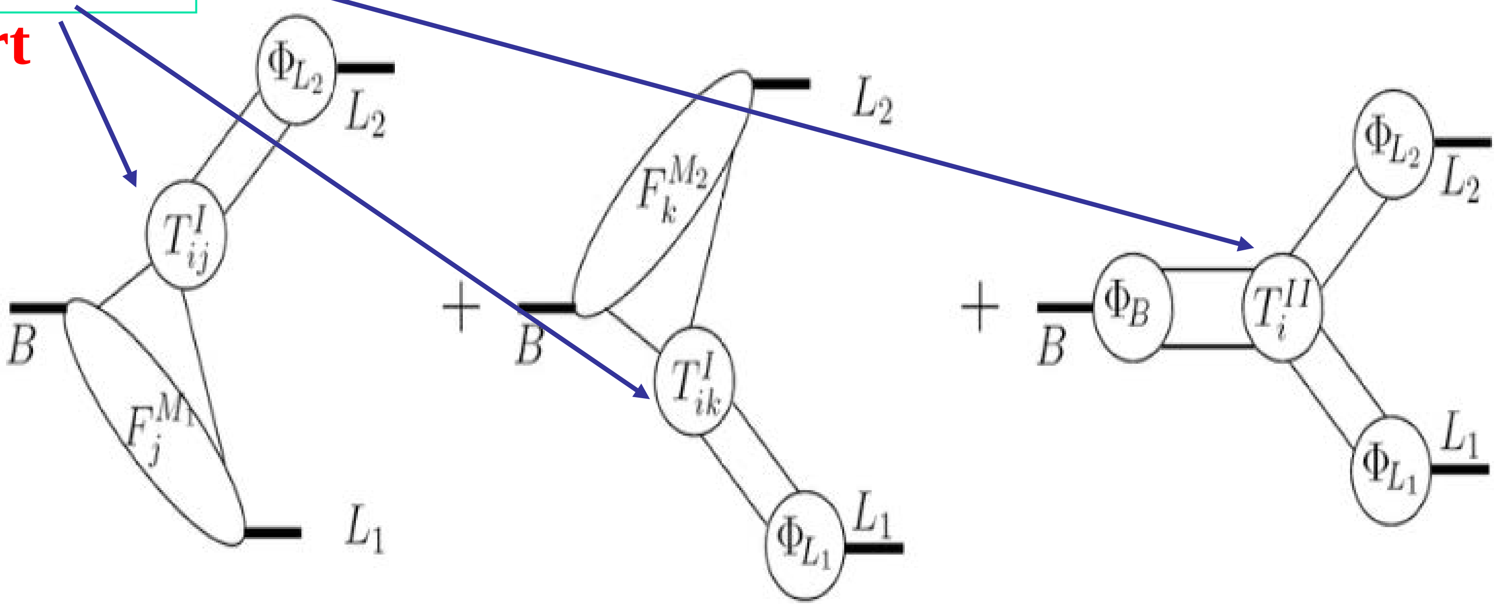
  —most important theoretical uncertainty
- Annihilation type diagram not calculable
- Strong phase too small, and not quantitatively calculable

  — Direct CP asymmetry not predictable
- Final state interaction not predictable

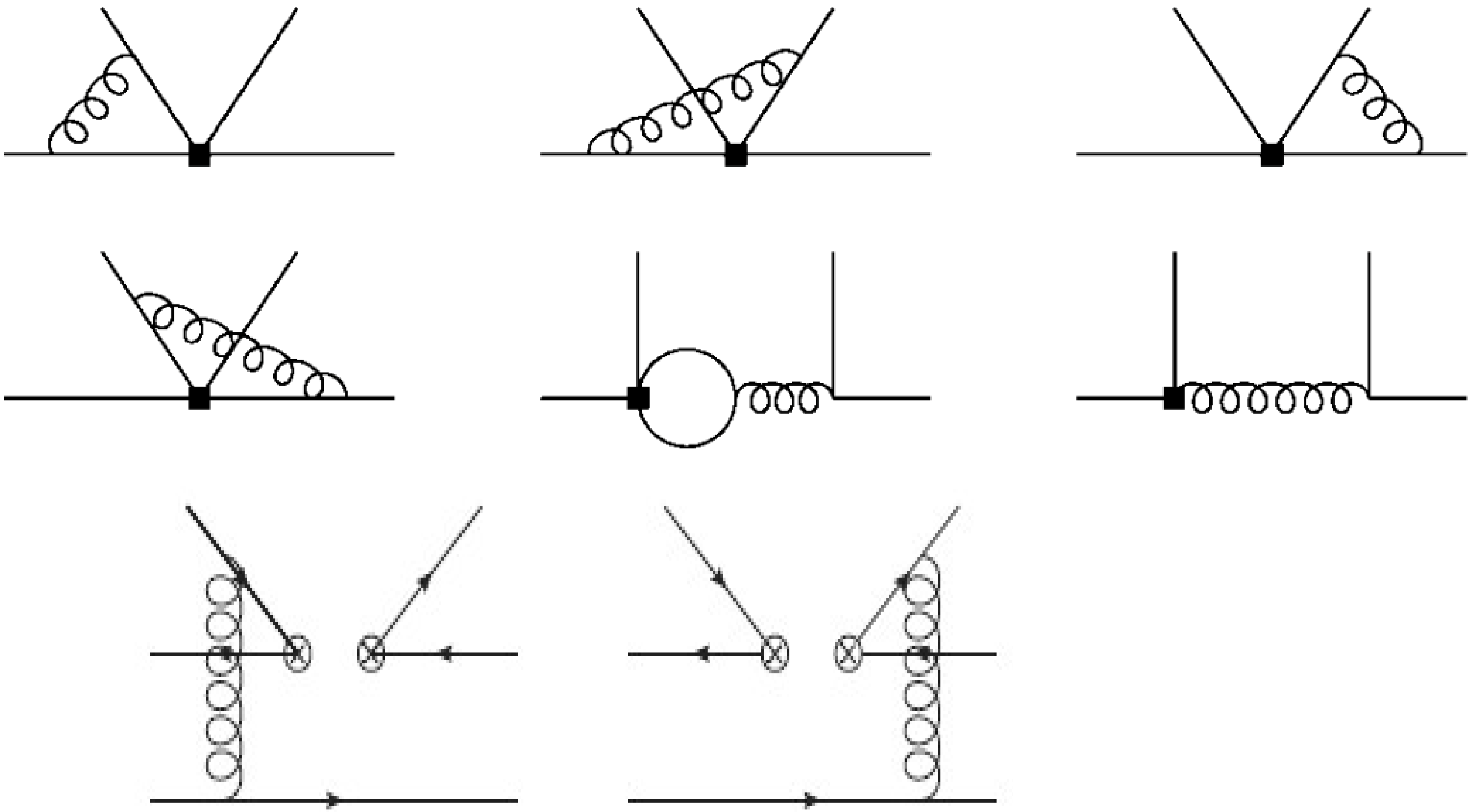
QCD factorization by BBNS: **PRL 83 (1999) 1914; NPB591 (2000) 313**

$$
\begin{aligned}
-\langle L_1 L_2 | Q_i | \bar{B} \rangle \;=\; & \sum_j F_j^{B\to L_1}(m_2^2) \int_0^1 du\, T_{ij}^I(u)\, \Phi_{L_2}(u) \\
& + \sum_k F_k^{B\to L_2}(m_1^2) \int_0^1 dv\, T_{ik}^I(v)\, \Phi_{L_1}(v), \\
& + \int_0^1 d\xi du dv\, T_i^{II}(\xi, u, v)\, \Phi_B(\xi)\, \Phi_{L_1}(v)\, \Phi_{L_2}(u)
\end{aligned}
$$

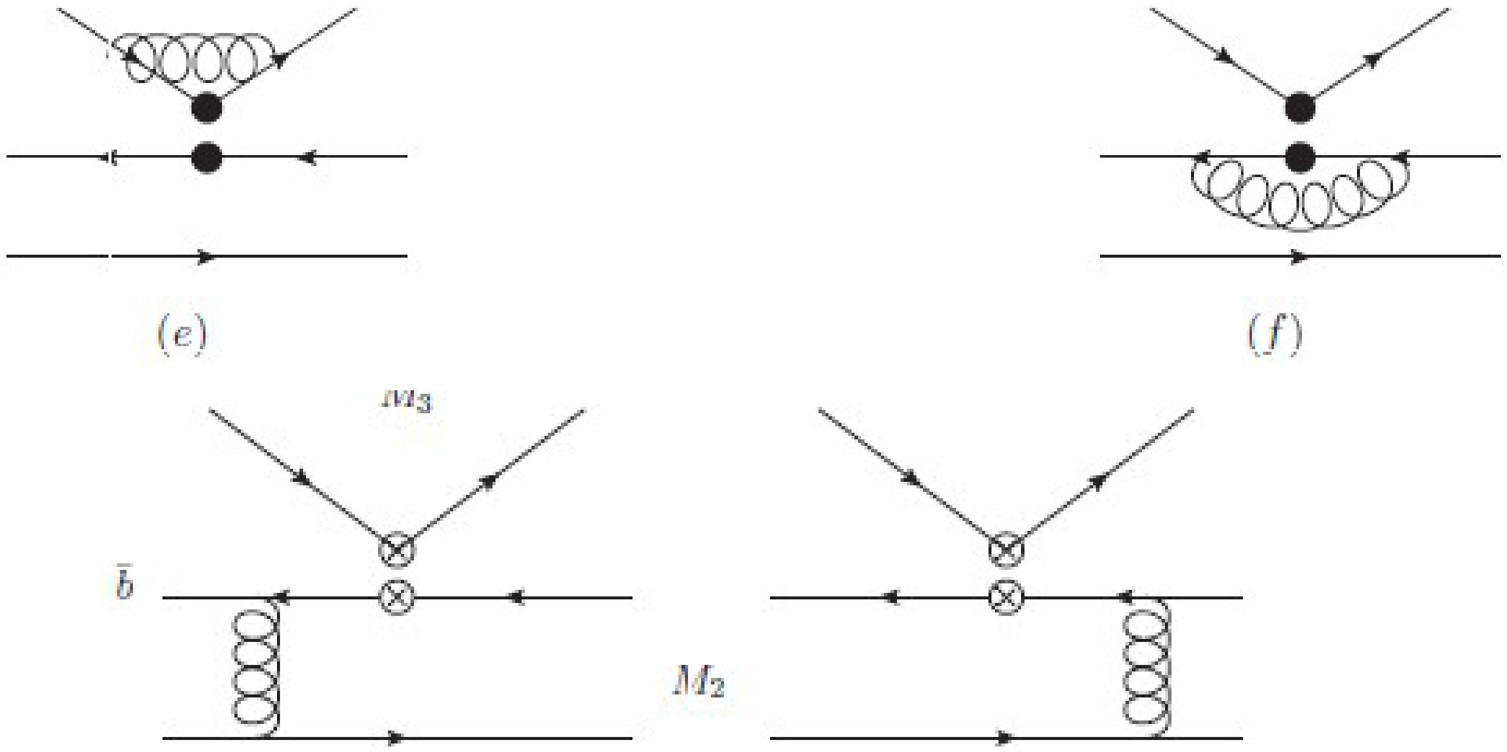
hard part



α_s corrections to the hard part T

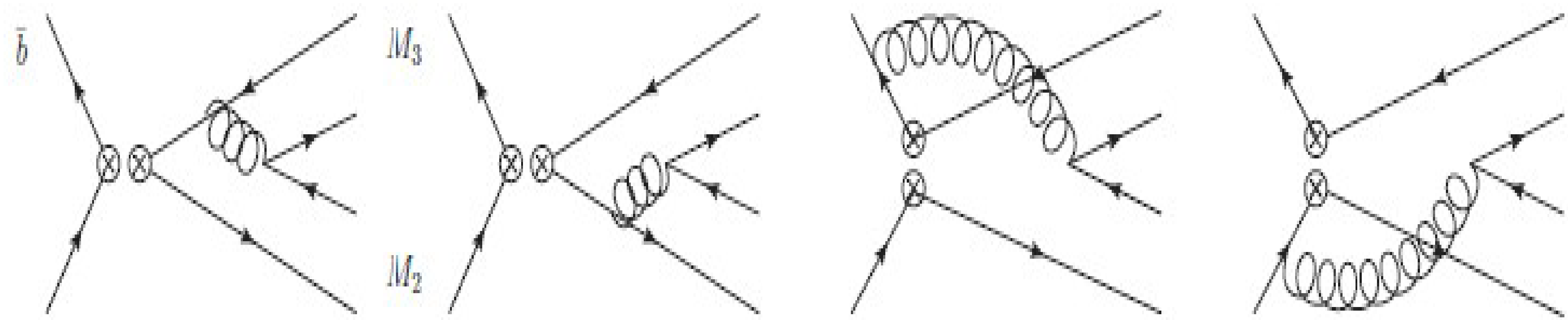


The missing diagrams, which contribute to the renormalization of decay constant or form factors



Endpoint divergence appears in these calculations

The annihilation type diagrams are important to the source of strong phases



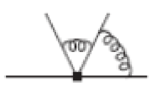
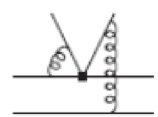
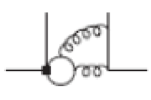
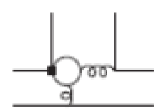
- **However, these diagrams are similar to the form factor diagrams, which have endpoint singularity, not perturbatively calculable.**
- **These divergences are not physical, can only be treated in QCDF as free parameters, which makes CP asymmetry not predictable:**

$$\int_0^1 \frac{dy}{y} \to X_A^{M_1}, \qquad \int_0^1 dy \frac{\ln y}{y} \to -\frac{1}{2}(X_A^{M_1})^2$$

Status of NNLO QCD factorization calculations

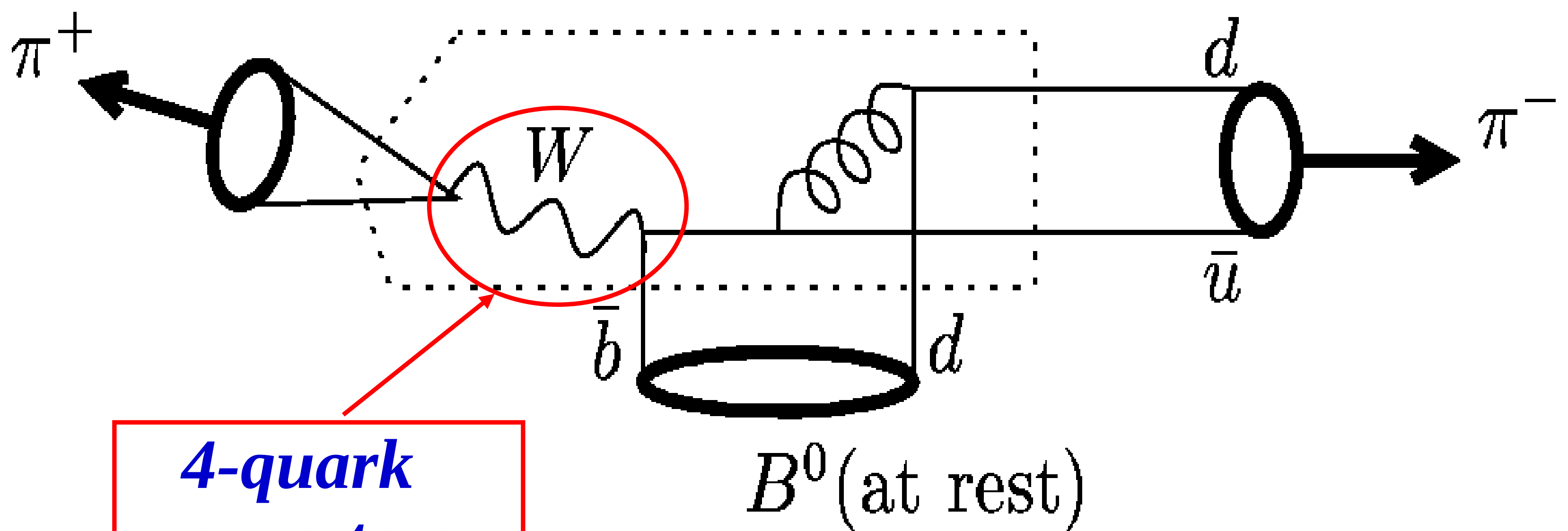
$$\langle M_1 M_2 | C_i O_i | \bar{B} \rangle_{\mathcal{L}_{\rm eff}} = \sum_{\rm terms} C(\mu_h) \times \Big\{ F_{B\to M_1} \times \underbrace{T^{\rm I}(\mu_h,\mu_s)}_{1+\alpha_s+\dots} \star f_{M_2}\Phi_{M_2}(\mu_s)$$

$$+ f_B \Phi_B(\mu_s) \star \Big[\underbrace{T^{\rm II}(\mu_h,\mu_I)}_{1+\dots} \star \underbrace{J^{\rm II}(\mu_I,\mu_s)}_{\alpha_s+\dots} \Big] \star f_{M_1}\Phi_{M_1}(\mu_s) \star f_{M_2}\Phi_{M_2}(\mu_s) \Big\}$$

Status	2-loop vertex corrections (T_i^I)	1-loop spectator scattering (T_i^{II})
Trees	 [GB 07, 09] [Beneke, Huber, Li 09]	 [Beneke, Jäger 05] [Kivel 06] [Pilipp 07]
Penguins	 in progress	 [Beneke, Jäger 06] [Jain, Rothstein, Stewart 07]

With more and more precise data, power corrections are urgently needed

Picture of PQCD Approach

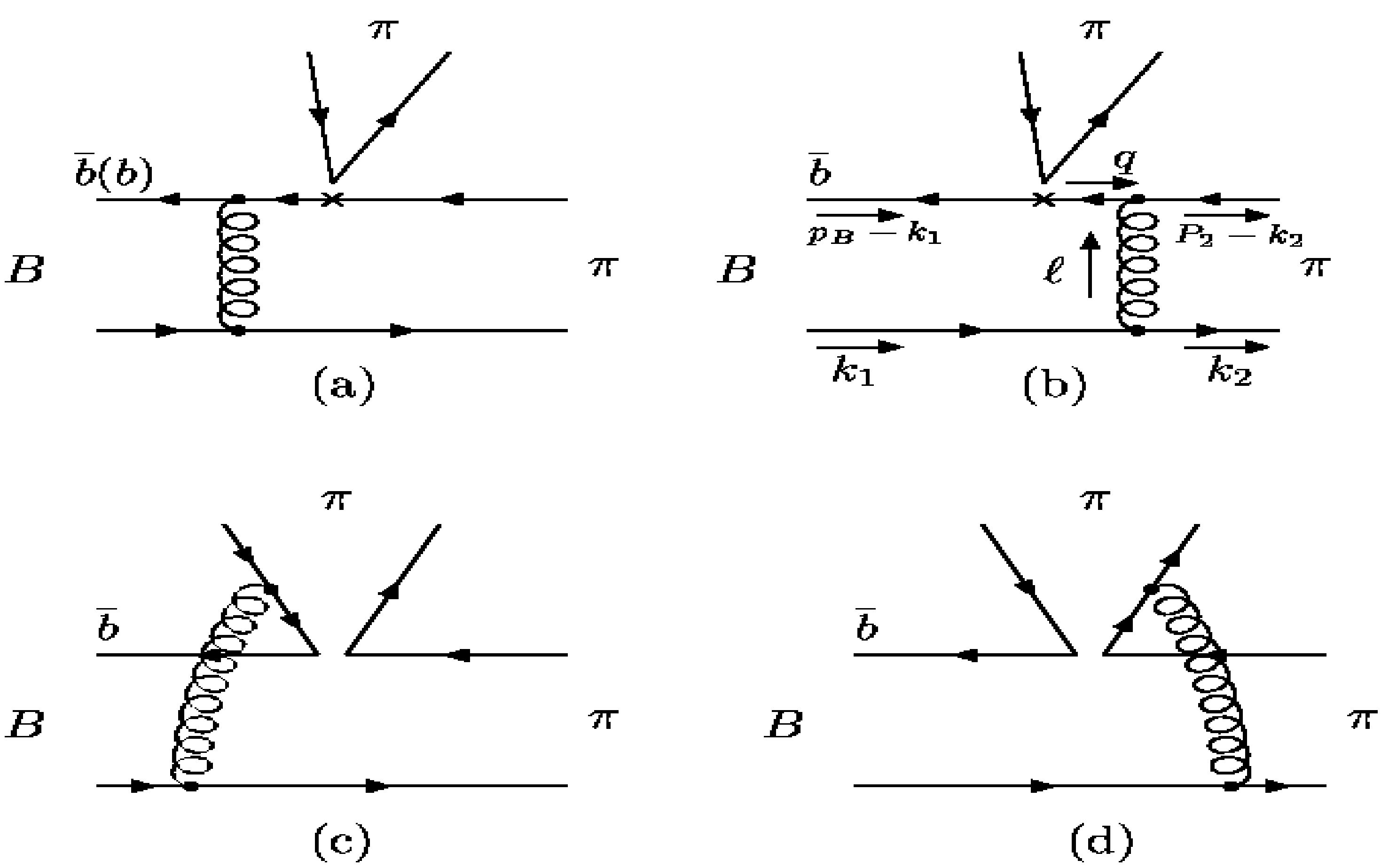


4-quark operators

Keum, Li, Sanda, Phys.Rev. D63 (2001) 054008;

Lu, Ukai, Yang, Phys.Rev. D63 (2001) 074009

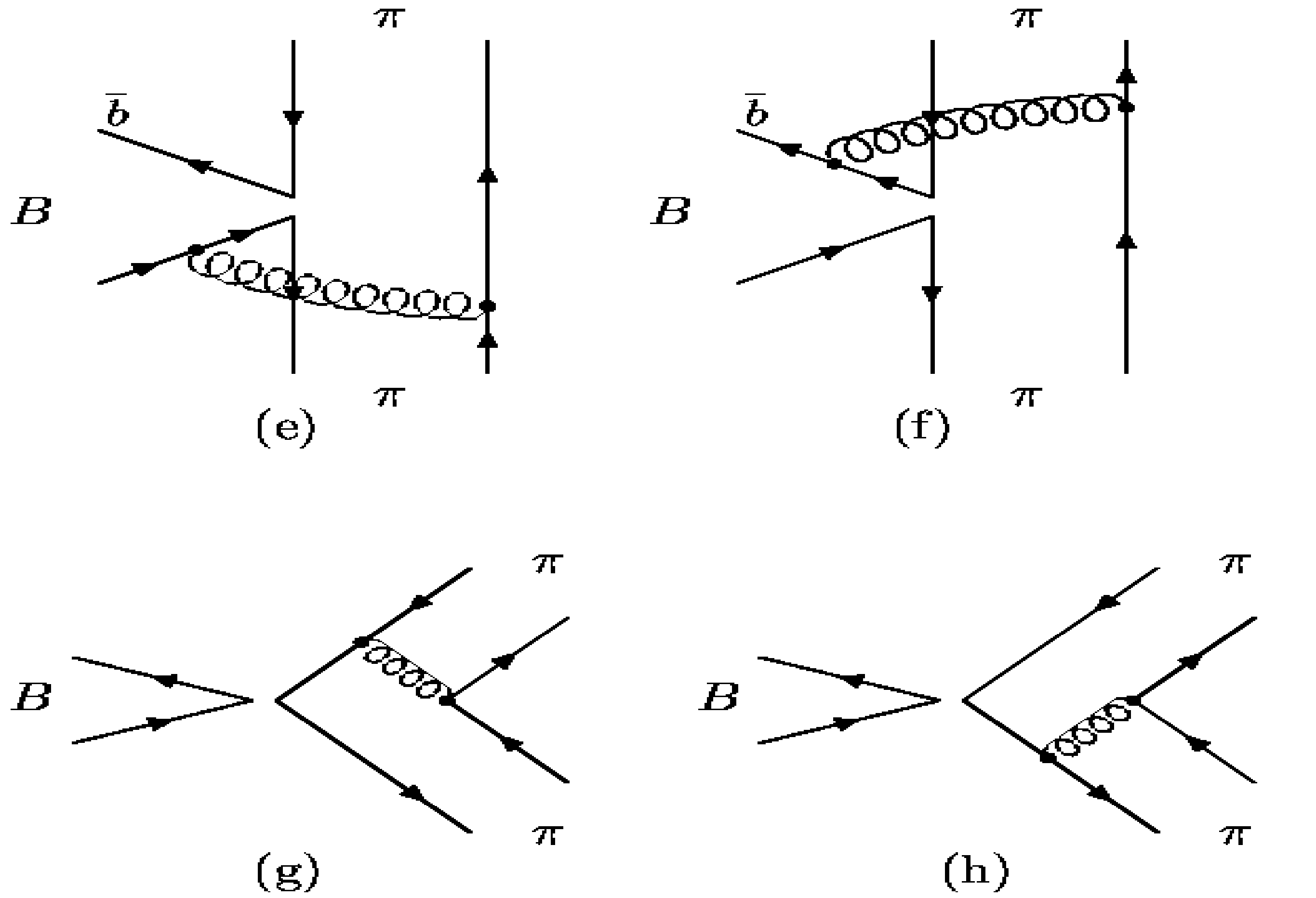
The leading order emission Feynman diagram in PQCD approach



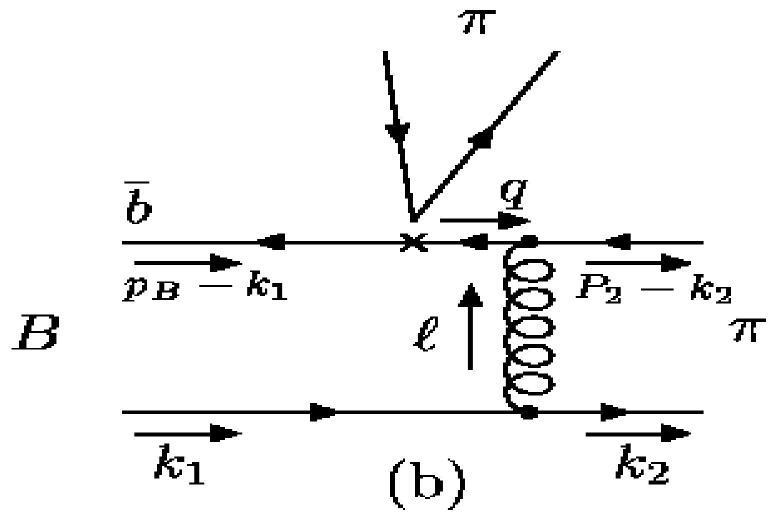
Form factor diagram

Hard scattering diagram

The leading order Annihilation type Feynman diagram in PQCD approach



Endpoint singularity



- **Gluon propagator**

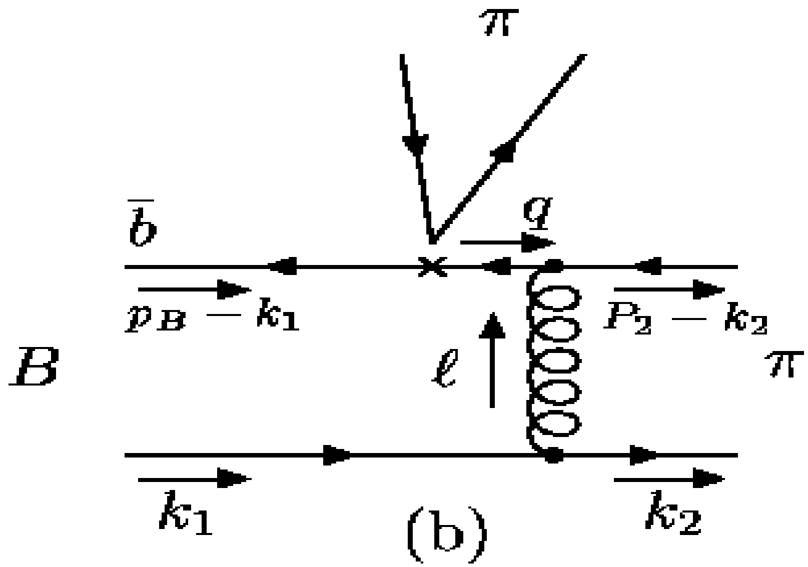
$$\frac{i}{(k_1-k_2)^2}=\frac{i}{-2xym_B^2}$$

- **x,y Integrate from 0→1, that is endpoint singularity**
- **The reason is that, one neglects the transverse momentum of quarks, which is not applicable at endpoint.**
- **If we pick back the transverse momentum, the divergence disappears**

$$\frac{i}{(k_1-k_2)^2}=\frac{i}{-2xym_B^2-(k_1^T-k_2^T)^2}$$

Endpoint singularity

- **It is similar for the quark propagator**

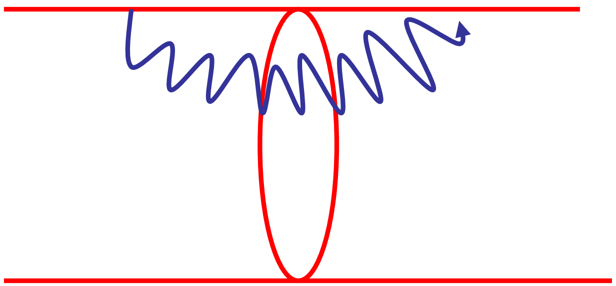


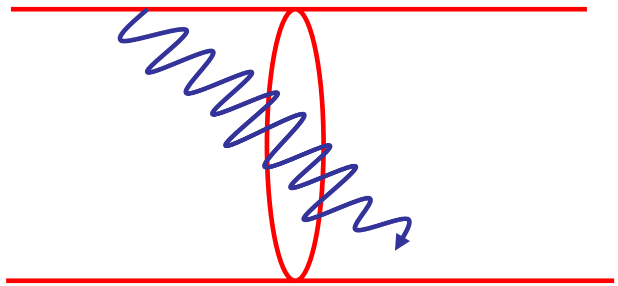
$$\int_0^1 \frac{1}{x} dx = \ln \frac{1}{\varepsilon}$$

$$\int_0^1 \frac{1}{x+k} dx dk = \int dk \left[\ln(x+k)\right]_0^1 = \int dk \left[\ln(1+k) - \ln k\right]$$

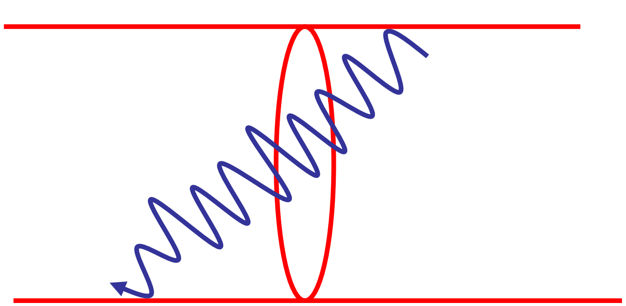
The logarithm divergence disappear if one has an extra dimension

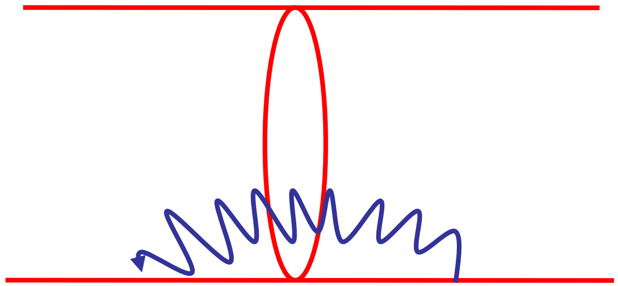
However, with transverse momentum, means one extra energy scale





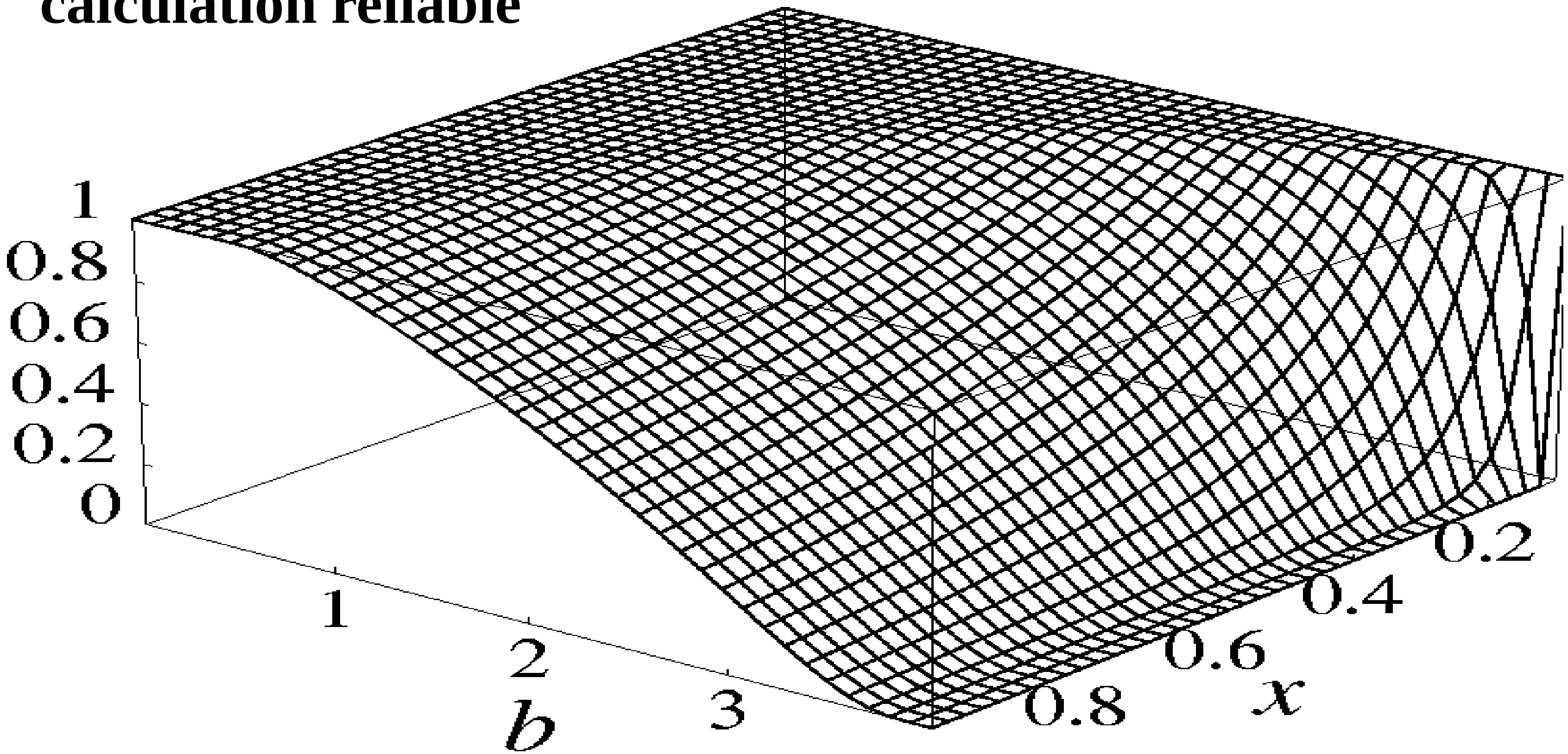
The overlap of Soft and collinear divergence will give double logarithm $ln^2 Pb$, which is too big to spoil the perturbative expansion. We have to use renormalization group equation to resum all of the logs to give the so called Sudakov Form factor





Sudakov Form factor exp{−S(x,b)}

This factor exponentially suppresses the contribution at the endpoint (small k_T), makes our perturbative calculation reliable



CP Violation in $B \rightarrow \pi\pi(K)$ (real prediction before exp.)

CP(%)	FA	BBNS	PQCD (2001)	Exp (2004)
π^+K^-	$+9 \pm 3$	$+5 \pm 9$	-17 ± 5	-11.5 ± 1.8
π^0K^+	$+8 \pm 2$	7 ± 9	-13 ± 4	$+4 \pm 4$
π^+K^0	1.7 ± 0.1	1 ± 1	-1.0 ± 0.5	-2 ± 4
$\pi^+\pi^-$	-5 ± 3	-6 ± 12	$+30 \pm 10$	$+37 \pm 10$

CP Violation in $B \to \pi\pi(K)$

Including large annihilation fixed from exp.

CP(%)	FA	Cheng,HY	PQCD (2001)	Exp
$\pi^+ K^-$	$+9 \pm 3$	-7.4 ± 5.0	-17 ± 5	-9.7 ± 1.2
$\pi^0 K^+$	$+8 \pm 2$	0.28 ± 0.10	-13 ± 4	4.7 ± 2.6
$\pi^+ K^0$	1.7 ± 0.1	4.9 ± 5.9	-1.0 ± 0.5	0.9 ± 2.5
$\pi^+ \pi^-$	-5 ± 3	17 ± 1.3	$+30 \pm 10$	$+38 \pm 7$

Perturbative calculations

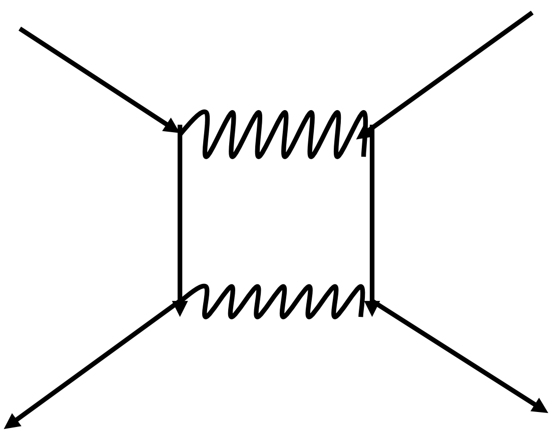
- In principle, all hadronic physics should be calculated by QCD
- In fact, you can always use QCD to calculate any process,

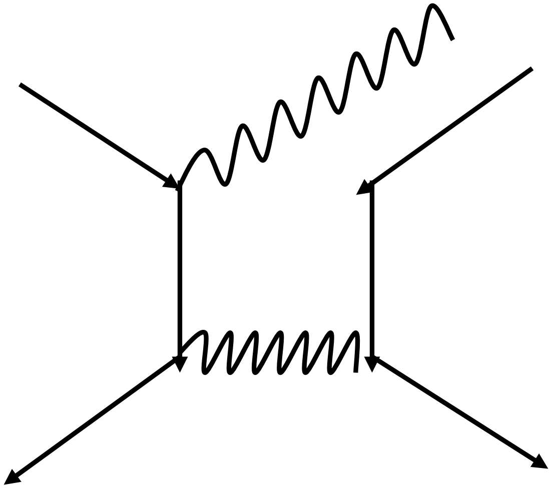
provided you can renormalize the infinities and do all order calculations.

- Perturbation calculation means order by order
- Involving loop diagrams
- Therefore divergences unavoidable

Divergences

- Ultraviolet divergences → renormalization
- Infrared divergences ? **Infrared divergence in virtual corrections should be canceled by real emission**
- In exclusive QCD processes → factorization





Factorization can only be proved in power expansion by operator product expansion. To achieve that, we need a hard scale Q

- In the certain order of 1/Q expansion, the hard dynamics characterized by Q factorize from the soft dynamics
- Hard dynamics is process-dependent, but calculable
- Soft dynamics are universal (process-independent) ⇨ predictive power of factorization theorem
- Factorization theorem holds up to all orders in α_s, but to certain power in 1/Q
- In B decays the hard scale Q is just the b quark mass

QCD-methods based on factorization work well for the leading power of $1/m_b$ expansion

collinear QCD Factorization approach
[Beneke, Buchalla, Neubert, Sachrajda, 99']

Perturbative QCD approach based on k_T factorization
[Keum, Li, Sanda, 00'; Lu, Ukai, Yang, 00']

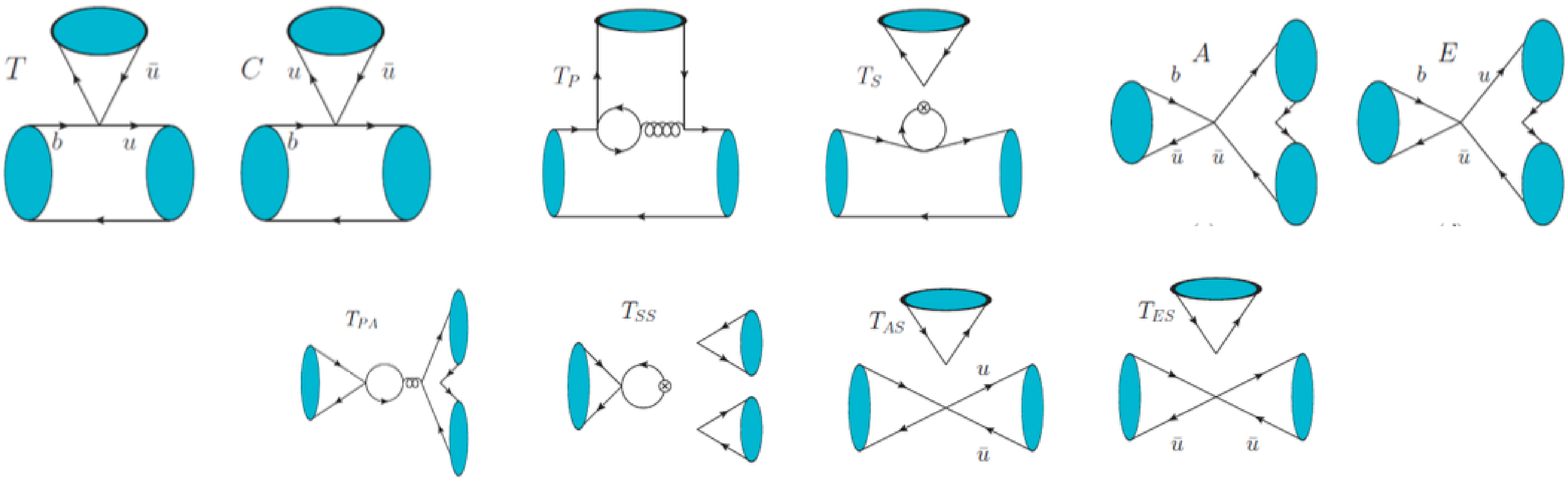
Soft-Collinear Effective Theory
Bauer, Fleming, **Pirjol, Stewart, Phys.Rev. D63 (2001) 114020**

- **Work well for most of charmless B decays, except for ππ, πK puzzle etc.**

The prove of factorization of QCD from electroweak is not needed

- Flavour SU(3) irreducible matrix elements
- Topological amplitudes (often with flavour SU(3) or SU(2))

$$T, C, P, P_{\mathrm{EW}}, S, E, A, \ldots$$



SU(3) breaking effect was lost. Limited precision!

Factorization assisted topological diagram approach first applied in hadronic D decays

[Li, Lu, Yu, PRD86 (2012) 036012]   **[FAT]**

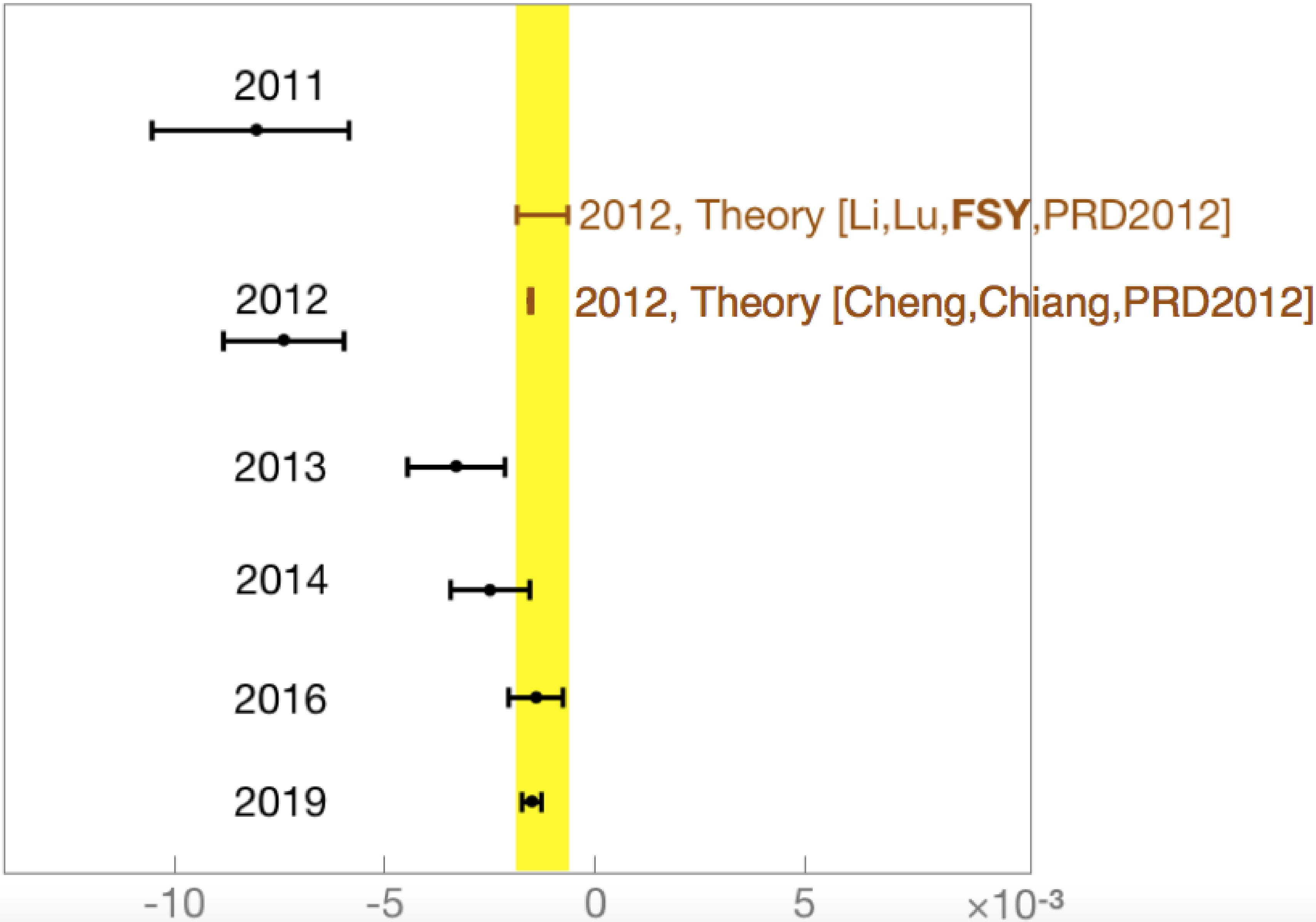
Predictions of Direct CP asymmetries

Modes	A_{CP}(FSI)	A_{CP}(diagram)	A_{CP}^{tree}	A_{CP}^{tot}
$D^0 \to \pi^+\pi^-$	0.02 ± 0.01	0.86	0	0.58
$D^0 \to K^+K^-$	0.13 ± 0.8	−0.48	0	−0.42
$D^0 \to \pi^0\pi^0$	-0.54 ± 0.31	0.85	0	0.05
$D^0 \to K^0\bar{K}^0$	-0.28 ± 0.16	0	1.11	1.38
$D^0 \to \pi^0\eta$	1.43 ± 0.83	−0.16	−0.33	−0.29
$D^0 \to \pi^0\eta'$	-0.98 ± 0.47	−0.01	0.53	1.53
$D^0 \to \eta\eta$	0.50 ± 0.29	−0.71	0.29	0.18
$D^0 \to \eta\eta'$	0.28 ± 0.16	0.25	−0.30	−0.94

Δ_{CP} =

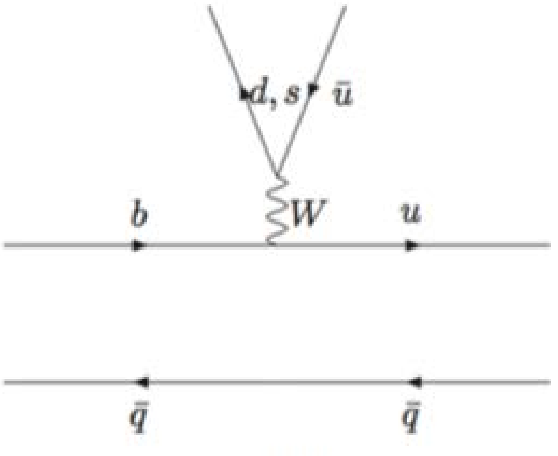
-1×10^{-3}

Exp Averages



Tree topology diagram contributing to Charmless B decays

For the color favored diagram (T), it is proved factorization to all order of α_s expansion in soft-collinear effective theory,



$(a)\,T$

The decay amplitudes is just the decay constants and form factors times Wilson coeficients of four quark operators. The SU(3) breaking effect is automatically kept

$$T^{P_1P_2} = i\frac{G_F}{\sqrt{2}} V_{ub}V_{uq'} a_1(\mu) f_{p_2}(m_B^2 - m_{p_1}^2) F_0^{BP_1}(m_{p_2}^2),$$

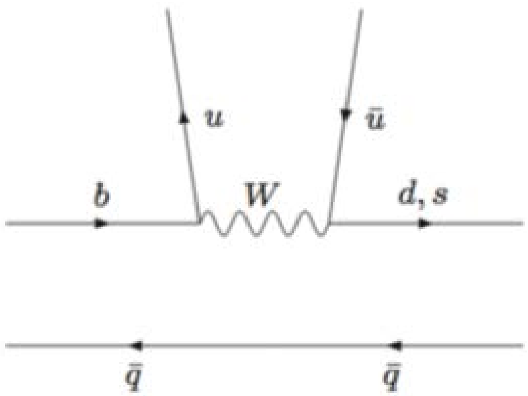
$$T^{PV} = \sqrt{2} G_F V_{ub} V_{uq'} a_1(\mu) f_V m_V F_1^{B-P}(m_V^2)(\varepsilon_V^* \cdot p_B),$$

$$T^{VP} = \sqrt{2} G_F V_{ub} V_{uq'} a_1(\mu) f_P m_V A_0^{B-V}(m_P^2)(\varepsilon_V^* \cdot p_B),$$

No free parameter

For other diagrams, we extract the amplitude and strong phase from experimental data by χ^2 fit

We factorize out the decay constants and form factor to keep the SU(3) breaking effect



(b) C

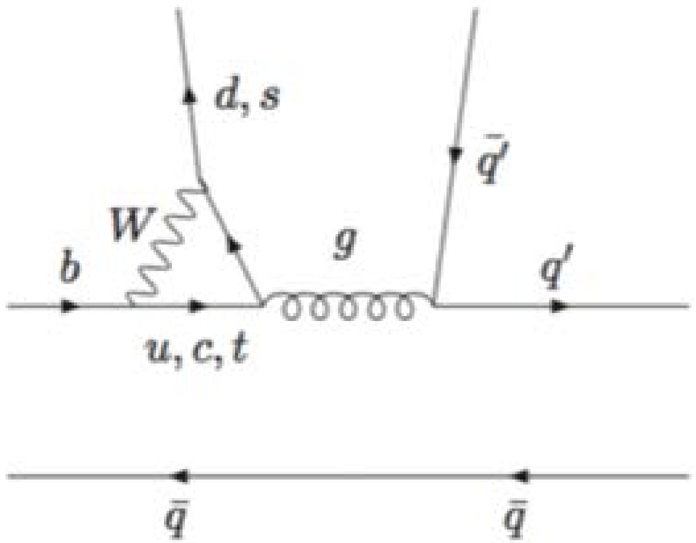
For the color suppressed tree diagram (C), we have two kinds of contributions

$$C^{P_1P_2} = i\frac{G_F}{\sqrt{2}}V_{ub}V_{uq'}\chi^C e^{i\phi^C} f_{P_2}(m_B^2 - m_{p_1}^2)F_0^{BP_1}(m_{p_2}^2),$$

$$C^{PV} = \sqrt{2}G_F V_{ub}V_{uq'}\chi^{C'} e^{i\phi^{C'}} f_V m_V F_1^{B-P}(m_V^2)(\varepsilon_V^* \cdot p_B),$$

$$C^{VP} = \sqrt{2}G_F V_{ub}V_{uq'}\chi^{C} e^{i\phi^{C}} f_P m_V A_0^{B-V}(m_P^2)(\varepsilon_V^* \cdot p_B),$$

The penguin emission diagram(P) is the dominant diagram comparable with color favored tree (T).



(a) P

It is approved factorization in SCET, we can calculate without ambiguity. The additional chiral enhanced penguin of this diagram need to be fitted

$$P^{PP} = -i\frac{G_F}{\sqrt{2}}V_{tb}V^*_{tq'}[a_4(\mu) + \chi^P e^{i\phi^P} r_\chi] f_{p_2}(m_B^2 - m_{p_1}^2)F_0^{BP_1}(m_{p_2}^2),$$

$$P^{PV} = -\sqrt{2}G_F V_{tb}V^*_{tq'} a_4(\mu) f_V m_V F_1^{B-P} m_V^2(\varepsilon_V^* \cdot p_B),$$

$$P^{VP} = -\sqrt{2}G_F V_{tb}V^*_{tq'}[a_4(\mu) - \chi^P e^{i\phi^P} r_\chi] f_P m_V A_0^{B-V}(m_P^2)(\varepsilon_V^* \cdot p_B).$$

Global Fit for all B→PP, VP and PV decays

with $\chi^2/\text{d.o.f} = 45.2/34 = 1.3.$

35 branching Ratios and 11 CP violation observations data are used for the fit

$$
\begin{aligned}
\chi^C &= 0.48 \pm 0.06, & \phi^C &= -1.58 \pm 0.08,\\
\chi^{C'} &= 0.42 \pm 0.16, & \phi^{C'} &= 1.59 \pm 0.17,\\
\chi^E &= 0.057 \pm 0.005, & \phi^E &= 2.71 \pm 0.13,\\
\chi^P &= 0.10 \pm 0.02, & \phi^P &= -0.61 \pm 0.02.\\
\chi^{P_C} &= 0.048 \pm 0.003, & \phi^{P_C} &= 1.56 \pm 0.08,\\
\chi^{P'_C} &= 0.039 \pm 0.003, & \phi^{P'_C} &= 0.68 \pm 0.08,\\
\chi^{P_A} &= 0.0059 \pm 0.0008, & \phi^{P_A} &= 1.51 \pm 0.09,
\end{aligned}
$$

$$\chi^2 = \sum_{i=1}^{n} \left(\frac{x_i^{\text{th}} - x_i}{\Delta x_i} \right)^2.$$

Large strong phase

Zhou, Zhang, Lyu and Lü, EPJC (2017) 77: 125

CKM angle gamma extraction

All the tree amplitudes in charmless B decays are proportional to $V_{ub}V_{ud,s}^*$; while the penguin amplitudes are proportional to $V_{tb}V_{td,s}^* = -(V_{ub}V_{ud,s}^* + V_{cb}V_{cd,s}^*)$.

Except $V_{ub} \equiv |V_{ub}|e^{-i\gamma}$, all other CKM matrix elements are approximately real numbers without electroweak phase.

So after input the magnitudes of the following CKM matrix elements,

$$|V_{ud}| = 0.97420 \pm 0.00021\,, \quad |V_{us}| = 0.2243 \pm 0.0005\,, \quad |V_{ub}| = 0.00394 \pm 0.00036\,,$$

$$|V_{cd}| = 0.218 \pm 0.004\,, \quad |V_{cs}| = 0.997 \pm 0.017\,, \quad |V_{cb}| = 0.0422 \pm 0.0008\,.$$

We can extract the CKM angle gamma by global fit all the charmless B decays

Global Fit for all B→PP, VP and PV decays with gamma as free parameter

with $\chi^2/\mathrm{d.o.f} = 45.4/33 = 1.4.$

We use 37 branching ratios and 11 CP violation observations of all $B \to P\,P,\ P\,V$ decays from the current experimental data

$$\gamma = (69.8 \pm 2.1)^\circ$$

$$\chi^C = 0.41 \pm 0.06, \quad \phi^C = -1.74 \pm 0.09,$$

$$\chi^{C'} = 0.40 \pm 0.17, \quad \phi^{C'} = 1.78 \pm 0.10,$$

$$\chi^E = 0.06 \pm 0.006, \quad \phi^E = 2.76 \pm 0.13,$$

$$\chi^P = 0.09 \pm 0.003, \quad \phi^P = 2.55 \pm 0.03$$

$$\chi^{P_C} = 0.045 \pm 0.003, \quad \phi^{P_C} = 1.53 \pm 0.08,$$

$$\chi^{P'_C} = 0.037 \pm 0.003, \quad \phi^{P'_C} = 0.67 \pm 0.08,$$

$$\chi^{P_A} = 0.006 \pm 0.0008, \quad \phi^{P_A} = 1.49 \pm 0.09,$$

Global Fit for all B→PP, VP and PV decays with gamma as free parameter

with $\chi^2/\mathrm{d.o.f} = 45.4/33 = 1.4.$

$$\gamma = (69.8 \pm 2.1 \pm 0.9)^\circ$$

Uncertainty from input parameters

We use 37 branching ratios and 11 CP violation observations of all B → P P, P V decays from the current experimental data

$$\gamma = (69.8 \pm 2.1)^\circ$$

$$\begin{aligned}
\chi^C &= 0.41 \pm 0.06, & \phi^C &= -1.74 \pm 0.09, \\
\chi^{C'} &= 0.40 \pm 0.17, & \phi^{C'} &= 1.78 \pm 0.10, \\
\chi^E &= 0.06 \pm 0.006, & \phi^E &= 2.76 \pm 0.13, \\
\chi^P &= 0.09 \pm 0.003, & \phi^P &= 2.55 \pm 0.03 \\
\chi^{P_C} &= 0.045 \pm 0.003, & \phi^{P_C} &= 1.53 \pm 0.08, \\
\chi^{P'_C} &= 0.037 \pm 0.003, & \phi^{P'_C} &= 0.67 \pm 0.08, \\
\chi^{P_A} &= 0.006 \pm 0.0008, & \phi^{P_A} &= 1.49 \pm 0.09,
\end{aligned}$$

Comparison of gamma measurement

$$\gamma = (69.8 \pm 2.1 \pm 0.9)^\circ$$

HFLAV Collaboration $\gamma = (71.1^{+4.6}_{-5.3})^\circ$

CKMfit Collaboration $\gamma = (73.5^{+4.2}_{-5.1})^\circ$

Less uncertainty than others

UTfit Collaboration $\gamma = (70.0 \pm 4.2)^\circ$

Zhou and Lu
CPC 44 (2020) 063101

Recent LHCb result $\gamma = (74.0^{+5.0}_{-5.8})^\circ$

Summary/Challenges

- Hadronic B Decays are important in the test of standard model and search for signals of new physics.
- Power corrections in QCDF are very important that need to be calculated precisely
- Such as The annihilation type diagrams are the key point in explaining the K pi puzzle and large direct CP asymmetry found in B decays
- Next-to-leading order perturbative calculations is needed to explain the more and more precise experimental data