From the theta vacuum to the QCD axion

郭 奉 坤

中国科学院理论物理研究所

- FKG, U.-G. Meißner, Cumulants of the QCD topological charge distribution, PLB 749, 278 (2015);
- Z.-Y. Lu, M.-L. Du, FKG, U.-G. Meißner, T. Vonk, QCD θ-vacuum energy and axion properties, JHEP 05, 001 (2020);
- T. Vonk, FKG, U.-G. Meißner, Precision calculation of the axion-nucleon coupling in chiral perturbation theory, JHEP 03, 138 (2020);
- T. Vonk, FKG, U.-G. Meißner, The axion-baryon coupling in SU(3) heavy baryon chiral perturbation theory, JHEP 08, 024 (2021)
- T. Vonk, FKG, U.-G. Meißner, Pion axioproduction: The Δ resonance contribution, PRD 105, 054029 (2022)

. . . it may be that the next exciting thing to come along will be the discovery of a neutron or atomic or electron electric dipole moment. These electric dipole moments . . . seem to me to offer one of the most exciting possibilities for progress in particle physics.

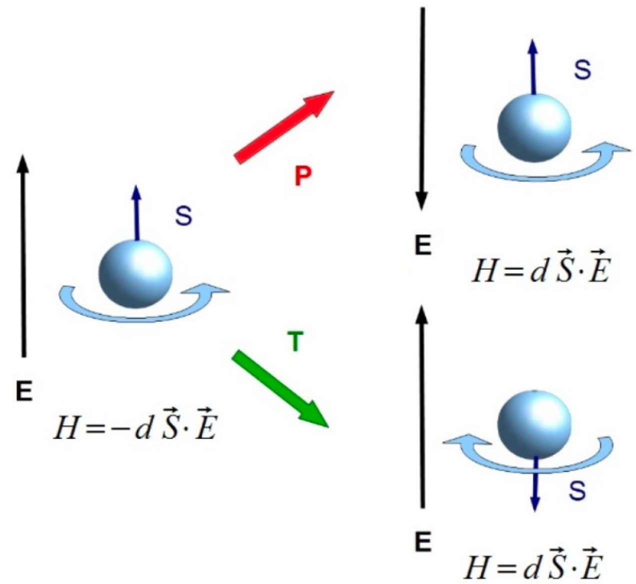
— Steven Weinberg (1992)

Electric dipole moment (EDM)

- EDM measures the polarity of a charged system, $\vec{d} = \sum_i q_i \vec{r}_i$
- For a hadron or any elementary particle at rest, $\vec{d} = d\frac{\vec{S}}{|\vec{S}|}$
- Hamiltonian for a dipole interacting with an electric field

$$\mathcal{H}_{\mathrm{edm}} = -\vec{d} \cdot \vec{E} = -d\frac{\vec{S} \cdot \vec{E}}{|\vec{S}|}$$

- EDM leads to P and T(CP) odd interactions



Nucleon EDMs

- The nucleon EDMs are highly suppressed in the Standard Model with CKM: no EDM at the first order of weak interaction; $|d_n(\mathrm{CKM})| \lesssim 10^{-31} e$ cm.

X.-G. He, McKellar, Pakvasa, IJMPA 4, 5011 (1989) [E: *ibid*, 6, 1063 (1991)]

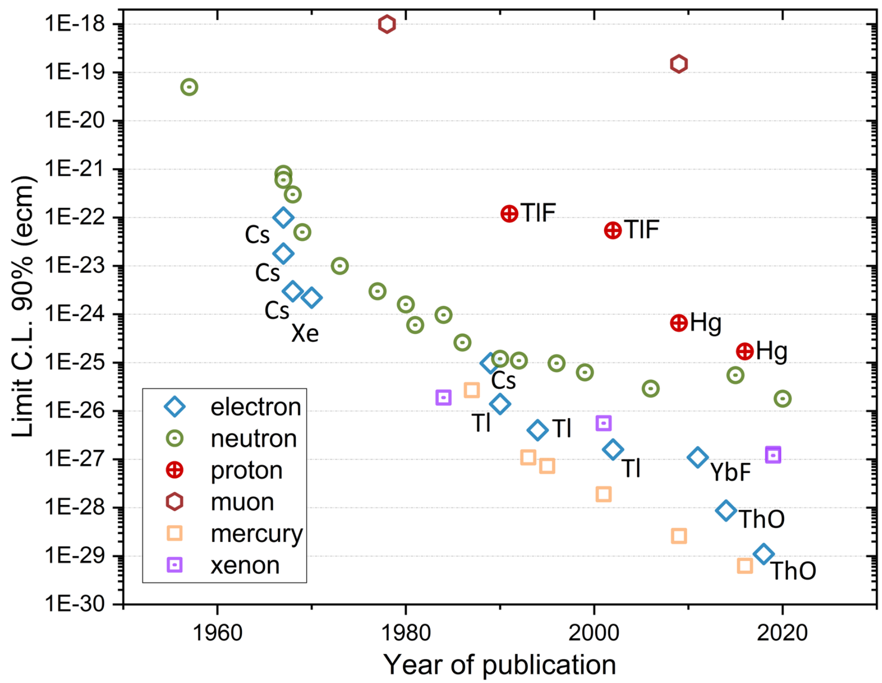
- Sensitive to the physics beyond the SM.

- Current experimental upper limits on hadron EDMs:
  - $|d_n| < 1.8 \times 10^{-26} e$ cm C. Abel, et al., PRL 124, 081803 (2020)
  - $|d_p| < 2.1 \times 10^{-25} e$ cm B.K. Sahoo, PRD 95, 013002 (2017), based on the measurement in W.C. Griffith, et al., PRL 102, 101601 (2009)
  - $|d_\Lambda| < 1.5\times10^{-16} e$ cm, based on 3×10^6 $\Lambda \to p\pi^-$ events

    L. Pondrom, et al., PRD 23, 814 (1981)

  ➢ $J/\psi \to \Lambda\bar{\Lambda} \to [p\pi^-][\bar{p}\pi^+]$ at BESIII with 10^{10} J/ψ: 2.6×10^6

  BESIII, CPC 44, 040001 (2020)



From talk by P. Schmidt-Wellenburg (2022)

Theta term and strong CP problem

- Another source of CP violation in SM: the θ term of QCD

$$\mathcal{L}_{\mathrm{QCD},0} = -\frac{1}{2}\mathrm{Tr}\left[G_{\mu\nu}G^{\mu\nu}\right] + \mathcal{L}_{\mathrm{quarks}}$$

  - Instanton solutions to the classical EoM Belavin et al., PLB 59, 85 (1975)
  - Non-trivial vacuum structure Callan, Dashen, Gross, PLB 63, 334 (1976); see e.g, Donoghue et al., Dynamics of the Standard Model

    $|n\rangle \to |n+Q\rangle$, gauge invariant vac. must be superposition of all topological classes

    $$|\theta\rangle = \sum_n e^{-in\theta}|n\rangle$$

    Topological charge (integer): $Q = \frac{1}{16\pi^2}\int d^4x\, \mathrm{Tr}[\tilde{G}_{\mu\nu}(x)G^{\mu\nu}(x)]$, $\tilde{G}_{\mu\nu} = \frac{1}{2}\epsilon_{\mu\nu\rho\sigma}G^{\rho\sigma}$
  - QCD Lagrangian with a θ term

$$\mathcal{L}_{\mathrm{QCD}} = \mathcal{L}_{\mathrm{QCD},0} + \frac{\theta}{16\pi^2}\mathrm{Tr}[\tilde{G}_{\mu\nu}G^{\mu\nu}]$$

- The theta term violates P and CP:

$$G_{\mu\nu}\tilde{G}^{\mu\nu} \propto \mathbf{E}\cdot\mathbf{B} \xrightarrow{\mathrm{CP}} -\mathbf{E}\cdot\mathbf{B}$$

Theta term and strong CP problem

- Generally, M_q is complex and non-diagonal in the Standard Model. We will encounter a $\mathrm{U}_A(1)$ transformation $\exp(i\alpha) \equiv \exp[i(\varphi_R - \varphi_L)/2]$

$$M_q \to e^{-i\varphi_L} M_q e^{i\varphi_R}, \quad q_L \to e^{-i\varphi_L} q_L, \quad q_R \to e^{-i\varphi_R} q_R$$

$\Rightarrow \arg(\det M_q) \to \arg(\det M_q) + 2N_f \alpha$

- For massless quarks
  - ☞ Classically, $\partial_\mu J_5^\mu = 0$
  - ☞ Quantum $\mathrm{U}_A(1)$ anomaly: $\partial_\mu J_5^\mu = \frac{N_f g^2}{16\pi^2} G^a_{\mu\nu} \tilde{G}^{a,\mu\nu}$
    $\Rightarrow \theta \to \theta - 2N_f \alpha$
  - ☞ One cannot solve the strong CP problem by simply imposing CP on QCD

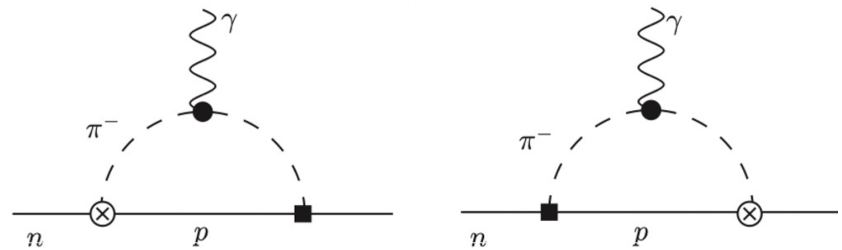
$\boxed{\text{The measurable quantity is not } \theta \text{ but } \theta_0 = \theta + \arg(\det M_q)}$

Theta term and strong CP problem

- The contribution from the θ term to the neutron EDM

$$d_n \sim 10^{-16}\theta_0 \; e \text{ cm}$$

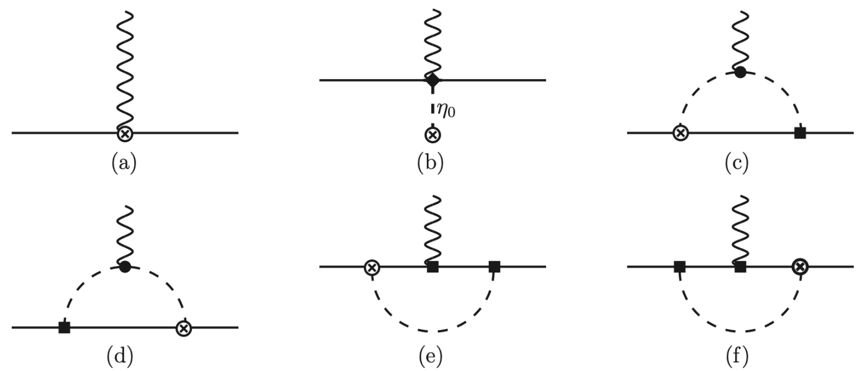
Crewther et al., (1979)



- A complete leading one-loop calculation of baryon EDMs in large N_c ChPT, with lattice QCD inputs (in units of $10^{-16}\theta_0 \; e \text{ cm}$)

FKG, U.-G. Meißner, JHEP 07, 097 (2012)

$$\begin{array}{llll} d_n & = -2.9 \pm 0.9, & d_p & = 1.1 \pm 1.1 \\ d_\Lambda & = -2.5 \pm 0.4, & d_{\Sigma^+} & = -0.7 \pm 1.1 \\ d_{\Sigma^0} & = 0.7 \pm 0.4, & d_{\Sigma^-} & = 2.2 \pm 0.5 \\ d_{\Xi^0} & = -3.4 \pm 0.9, & d_{\Xi^-} & = 0.6 \pm 0.5 \end{array}$$



$$|\theta_0| \lesssim 10^{-10}$$

- Strong CP problem: why is θ_0 so tiny?

In the following, I will write θ_0 as θ for simplicity

Cumulants of the QCD topological distribution

- The QCD partition function in a θ vacuum

$$Z(\theta) = \int [DG][Dq][D\bar{q}]e^{-S_{\mathrm{QCD},0}[G,q,\bar{q}]-i\theta Q}$$

- For large Euclidean time τ, dominated by the ground state (vacuum) energy

$$Z(\theta) = e^{-\tau E_{\mathrm{vac}}(\theta)} = e^{-V e_{\mathrm{vac}}(\theta)}$$

V: space-time volume; $e_{\mathrm{vac}}(\theta)$: vacuum energy density

- In terminology of statistics
  - $Z(\theta)$: moment-generating function for the distribution of Q, $Z(\theta) = \sum_n m_n \frac{\theta^n}{n!}$
  - $e_{\mathrm{vac}}(\theta)$: cumulant-generating function, $e_{\mathrm{vac}}(\theta) = \sum_n c_n \frac{\theta^n}{n!}$
  - Cumulants

$$c_2 = \chi_t = \frac{1}{V}\langle Q^2\rangle_{\theta=0}, \quad c_4 = -\frac{1}{V}\left(\langle Q^4\rangle - 3\langle Q^2\rangle^2\right)_{\theta=0}$$

topological susceptibility

For a general relation between cumulants and moments, see FKG, U.-G. Meißner, PLB 749, 278 (2015)

Theta vacuum energy in ChPT

- The θ term can be built into chiral perturbation theory (ChPT) with a complex quark mass matrix: $\chi_\theta = 2B_N\mathcal{M}_q\exp(i\theta/N)$
- Leading order (LO) vacuum energy density in SU(N) ChPT

$$e_{\text{vac}}^{(2)}(\theta) = -\frac{F_N^2}{4}\langle\chi_\theta U_0^\dagger + \chi_\theta^\dagger U_0\rangle$$

- Vacuum alignment for 2-flavor ChPT: $U_0 = \text{diag}\{e^{i\varphi}, e^{-i\varphi}\}$, minimizing $e_{\text{vac}}^{(2)}(\theta)$ gives

$$\tan\varphi = -\epsilon\tan\frac{\theta}{2}, \quad \epsilon = \frac{m_d-m_u}{m_d+m_u}$$

LO results: R. Brower et al., PLB 560, 64 (2003)

- Vacuum energy density: $e_{\text{vac}}^{(2)}(\theta) = -F^2\mathring{M}^2(\theta)$
- Pion mass:

$$\mathring{M}^2(\theta) = 2B\bar{m}\cos\frac{\theta}{2}\sqrt{1+\epsilon^2\tan^2\frac{\theta}{2}}$$

- Topological susceptibility and the 4th cumulant

$$\chi_t^{(2)} = \frac{1}{2}F^2B\hat{m}(1-\epsilon^2)$$

$$c_4^{(2)} = -\frac{1}{8}F^2B\hat{m}(1+2\epsilon^2-3\epsilon^4) \qquad \hat{m} = \frac{m_d+m_u}{2}$$

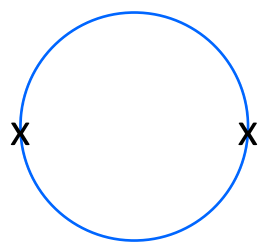
See also H. Leutwyler, A.V. Smilga, PRD 46, 5607 (1992); S. Aoki, H. Fukaya, PRD 81, 034022 (2010); Y.Y. Mao, T.W. Chiu, PRD 80, 034502 (2009)

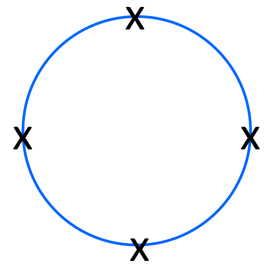
Theta vacuum energy at NLO

FKG, U.-G. Meißner, PLB 749, 278 (2015)

- χ_t, c_4 were calculated up to the next-to-leading order (NLO) after complicated steps

Y.Y. Mao, T.W. Chiu, PRD 80, 034502 (2009); V. Bernard et al., JHEP 12, 080 (2012)





- They can be computed much easier, and at once for all cumulants, by observing

$$\frac{1}{8}\langle\{\lambda_P, \lambda_Q^\dagger\}\big(\chi_\theta^\dagger U_0 + \chi_\theta U_0^\dagger\big)\rangle = \delta_{PQ}\mathring{M}_P^2(\theta)$$

- One-loop generating functional for free GB fields

$$Z_0(\theta) = \frac{i}{2}\mathrm{Tr}\ln D_0(\theta)\,, \qquad D_{0PQ}(\theta) = \delta_{PQ}\left[\partial_\mu\,\partial^\mu + \mathring{M}_P^2(\theta)\right]$$

- 1-loop:

$$e_{\rm vac}^{(4,\rm loop)}(\theta) = -\frac{i}{2}(N^2-1)\int\frac{d^dp}{(2\pi)^d}\ln\left[-p^2+\mathring{M}^2(\theta)\right]$$

$$= \frac{i}{2}(N^2-1)\int\frac{d^dp}{(2\pi)^d}\int_0^\infty\frac{d\tau}{\tau}\,e^{-\tau[-p^2+\mathring{M}^2(\theta)]}$$

$$= (N^2-1)\,\mathring{M}^4(\theta)\left\{\frac{\lambda}{2} - \frac{1}{128\pi^2}\left[1-2\ln\frac{\mathring{M}^2(\theta)}{\mu^2}\right]\right\}$$

UV divergence

Theta vacuum energy at NLO

FKG, U.-G. Meißner, PLB 749, 278 (2015)

- Tree-level contribution from the counterterms

$$
\begin{aligned}
e_{\text{vac}}^{(4,\text{tree})}(\theta) =& -\frac{l_3}{16}\left\langle \chi_\theta^\dagger U_0 + \chi_\theta U_0^\dagger \right\rangle^2 + \frac{l_7}{16}\left\langle \chi_\theta^\dagger U_0 - \chi_\theta U_0^\dagger \right\rangle^2 \\
& - \frac{h_1+h_3}{4}\left\langle \chi_\theta^\dagger \chi_\theta \right\rangle - \frac{h_1-h_3}{2}\,\mathrm{Re}\,(\det \chi_\theta) \\
=& -\mathring{M}^4(\theta)\left\{ l_3 + l_7\left[\frac{\left(1-\epsilon^2\right)\tan(\theta/2)}{1+\epsilon^2\tan^2(\theta/2)}\right]^2 \right\} \\
& - 2B^2\bar{m}^2\left[(h_1+h_3)\left(1+\epsilon^2\right) + (h_1-h_3)\left(1-\epsilon^2\right)\cos\theta\right]
\end{aligned}
$$

- UV divergences from 1-loop and from counterterms cancel, we get the NLO vacuum energy density in a closed, simple form:

$$
\begin{aligned}
e_{\text{vac}}(\theta) =& -F^2\mathring{M}^2(\theta) - \mathring{M}^4(\theta)\left\{ \frac{3}{128\pi^2}\left[1 - 2\ln\frac{\mathring{M}^2(\theta)}{\mu^2}\right] \right. \\
& \left. + l_3^r + h_1^r - h_3 + l_7\left[\frac{\left(1-\epsilon^2\right)\tan(\theta/2)}{1+\epsilon^2\tan^2(\theta/2)}\right]^2 \right\}
\end{aligned}
$$

- A similar expression can be derived for SU(N) in the symmetric limit
- Any topological cumulants can now be easily computed!

Theta vacuum energy at NLO

Z.-Y. Lu, M.-L. Du, FKG, U.-G. Meißner, T. Vonk, JHEP 05, 001 (2020)

- SU(N) ($N \geq 3$) with symmetry breaking

$$e_{\text{vac}} = -F_0^2 B_0 \sum_i m_i \cos\phi_i - \sum_P \frac{\mathring{M}_P^4(\theta)}{128\pi^2}\left[1 - 2\ln\frac{\mathring{M}_P^2(\theta)}{\mu^2}\right] - 16B_0^2\left[L_6^r\left(\sum_i m_i\cos\phi_i\right)^2 + N\left(NL_7^r + L_8^r\right)m_1^2\cos^2\phi_1\right]$$

- No closed expression is possible; relation between the vacuum alignment angle ϕ_f and θ is complicated

$$\phi_f = \sum_{n=0}^{\infty} C_{f,2n+1}\theta^{2n+1}$$

- Recursion relation:

$$C_{f,2n+1} = \sum_{t=1}^{n}\sum_{(k_1,\ldots,k_t)} s_{K_t}\binom{K_t}{k_1,\ldots,k_t}\left[\frac{\bar{m}}{m_f}\sum_{i=1}^{N}\prod_{j=1}^{t}C_{i,2j-1}^{k_j} - \prod_{j=1}^{t}C_{f,2j-1}^{k_j}\right]$$

and $C_{f,1} = \bar{m}/m_f$ $\qquad \frac{1}{\bar{m}} = \sum_i \frac{1}{m_i}$

- The θ vacuum energy density ($\theta \to a/f_a$) is the QCD axion potential!

QCD axion

For a recent review, see L. Di Luzio et al., Phys.Rept. 870, 1 (2020)

- A possible solution to the strong CP problem
  - Peccei-Quinn mechanism, hidden $\mathrm{U}(1)_A$ symmetry — Peccei, Quinn (1977)
  - Nambu-Goldstone boson: pseudoscalar axion — Weinberg (1978); Wilczek (1978)

$$\mathcal{L}_a = \frac{1}{2}(\partial_\mu a)^2 + \mathcal{L}(\partial_\mu a, \psi) + \frac{g_s^2}{32\pi^2}\frac{a}{f_a} G^c_{\mu\nu}\tilde{G}^{c,\mu\nu}$$

  - Its VEV cancels the theta term $\theta + \frac{\langle a \rangle}{f_a} = 0$, thus solves the strong CP problem

$$\mathcal{L}_{G\tilde{G}} = \left(\theta + \frac{a}{f_a}\right)\frac{g_s^2}{32\pi^2} G^c_{\mu\nu}\tilde{G}^{c,\mu\nu}$$

    - Expanding around $\langle a \rangle \Rightarrow$ the θ vacuum energy gives the axion potential, $\theta \to a/f_a$
    - $\chi_t \Rightarrow m_a, c_4 \Rightarrow$ axion self-interaction

$$m^2_{a,\mathrm{LO}} = \frac{F_\pi^2 M_{\pi^+}^2 \bar{m}}{2 f_a^2 \hat{m}};$$

isospin limit: $m_a^2 f_a^2 = \frac{1}{4} F_\pi^2 M_\pi^2$

- Dark matter candidate — Preskill, Wise, Wilczek (1983); Abbott, Sikivie (1983); Dine, Fischler (1983); …
- Axion decay constant window from astrophysical and cosmological data:

$$10^9\,\mathrm{GeV} \lesssim f_a \lesssim 10^{12}\,\mathrm{GeV}$$

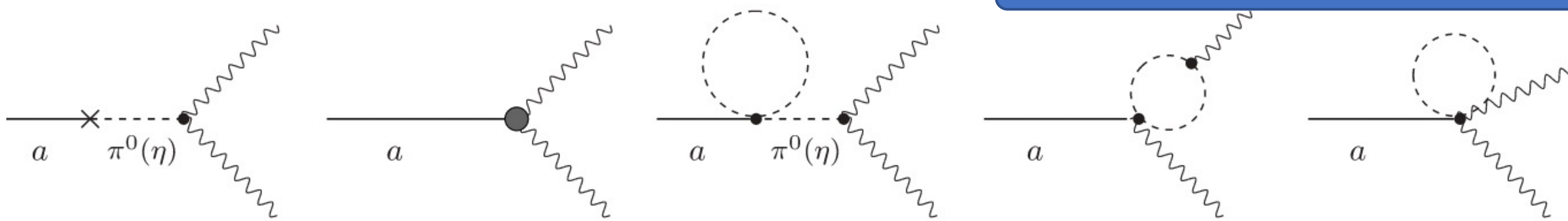
e.g., J.E. Kim, G. Carosi, RMP 82, 557 (2010)

Axion mass and photon coupling at NLO

Z.-Y. Lu, M.-L. Du, FKG, U.-G. Meißner, T. Vonk, JHEP 05, 001 (2020)

- Axion mass and self-coupling at the NLO
- Axion-photon coupling $\mathcal{L}_{a\gamma\gamma} = \frac{1}{4} g_{a\gamma\gamma} a F^{\mu\nu}\tilde{F}_{\mu\nu}$, $\quad g_{a\gamma\gamma} = \frac{\alpha_{\rm em}}{2\pi f_a}\frac{\mathcal{E}}{\mathcal{C}} + g_{a\gamma\gamma}^{\rm QCD}$

depending on high-energy model



$$g_{a\gamma\gamma} = \frac{\alpha_{\rm em}}{2\pi f_a}\Bigg\{\frac{\mathcal{E}}{\mathcal{C}} - \frac{2}{3}\frac{m_u+3\bar{m}}{m_u} - \frac{1024\pi^2}{3\hat{m}}\bar{m}M_{\pi^0}^2\underbrace{\left(C_7^W+3C_8^W\right)}_{\mathcal{O}(p^6)\text{ ano. term, }\eta\to\gamma\gamma} + \frac{2\bar{m}M_{\pi^0}^2}{3\hat{m}}\overbrace{\left[\frac{f_+(\cos,\sin)}{\sqrt{3}M_\eta^2} + \frac{f_-(\sin,\cos)}{M_{\pi^0}^2}\right]}^{\text{Loop corrections}}\Bigg\},$$

N	$m_a\ \left[\mu\text{eV}\cdot\frac{10^{12}\,\text{GeV}}{f_a}\right]$	$(-\lambda_4)^{1/4}\ [10^{-2}\text{GeV}/f_a]$	$g_{a\gamma\gamma}^{\rm QCD}\ \left[\frac{\alpha_{\rm em}}{2\pi f_a}\right]$	$\chi_t^{1/4}$ [MeV]	b_2
2 [53]	5.70(7)	5.79(10)	−1.92(4)	75.5(5)	−0.029(2)
3	5.71(9)	5.77(18)	−2.05(3)	75.6(6)	−0.028(3)

[53]: G. Grilli di Cortona et al., JHEP 01, 034 (2016)

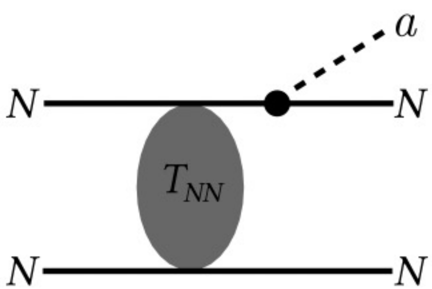
For models with $\frac{\mathcal{E}}{\mathcal{C}} \simeq 2$, $g_{a\gamma\gamma}$ is extremely small

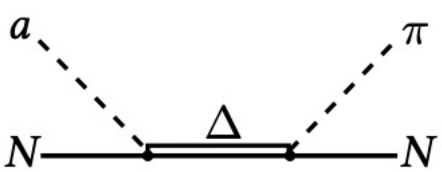
Axion-baryon couplings

- Why?
  - ☐ nuclear bremsstrahlung processes $NN \to NNa$ in massive stellar objects

    e.g., Turner, Phys.Rept. 197, 67 (1990); Raffelt, Phys.Rept. 197, 67 (1990); …
  - ☐ Impact of hyperons in neutron stars   Tolos, Fabbietti, PPNP 112, 103770 (2020)
  - ☐ novel perspectives in experimental axion searches?

    Carenza et al., PRL 126, 071102 (2021)





QCD with axion

- QCD Lagrangian with axion

$$\mathcal{L}_{QCD} = \mathcal{L}_{\rm QCD,0} - \bar{q}\mathcal{M}q + \frac{a}{f_a}\left(\frac{g}{4\pi}\right)^2 \mathrm{Tr}\left[G_{\mu\nu}\tilde{G}^{\mu\nu}\right] + \bar{q}\gamma^\mu\gamma_5 \frac{\partial_\mu a}{2f_a}\mathcal{X}_q q$$

last term in some axion models;

here assuming diagonal axion-quark coupling matrix: $\mathcal{X}_q = \mathrm{diag}\{X_q\}$

$$X_q^{\rm KSVZ} = 0$$
$$X_{u,d,s}^{\rm DFSZ} = \frac{1}{3}\sin^2\beta$$
$$X_{c,b,t}^{\rm DFSZ} = \frac{1}{3}\cos^2\beta = \frac{1}{3} - X_{u,d,s}^{\rm DFSZ}$$

typical invisible axion models

$\cot\beta$: ratio of VEVs of the two Higgs doublets in the DFSZ model

KSVZ: Kim, PRL 43, 103 (1979); Shifman, Vainshtein, Zakharov, NPB 166, 493 (1980)
DFSZ: Dine, Fischler, Srednicki, PLB 104, 199 (1981); Zhitnitsky, SJNP 31, 260 (1980)

QCD with axion

T. Vonk, FKG, U.-G. Meißner, JHEP 03, 138 (2020); JHEP 08, 024 (2021)

- With an axial rotation, so that

$$\mathcal{L}_{aq} = -\left(\bar{q}_L \mathcal{M}_a q_R + \text{ h.c. }\right) + \bar{q}\gamma^\mu\gamma_5 \frac{\partial_\mu a}{2f_a}\left(\mathcal{X}_q - \mathcal{Q}_a\right) q$$

$$\mathcal{M}_a = \exp\left(i\frac{a}{f_a}\mathcal{Q}_a\right)\mathcal{M}_q, \quad \mathcal{Q}_a \approx \frac{1}{1+\underbrace{z}_{m_u/m_d}+\underbrace{w}_{m_u/m_s}} \operatorname{diag}(1,z,w,0,0,0)$$

- Rewrite in the form of coupling to external currents $s, p, a_\mu, a_{\mu,i}^{(s)}$:

$$\left(\bar{q}\gamma^\mu\gamma_5\frac{\partial_\mu a}{2f_a}\left(c^{(1)} + c^{(3)}\lambda_3 + c^{(8)}\lambda_8\right) q\right)_{q=(u,d,s)^{\mathrm{T}}} + \sum_{q=\{c,b,t\}}\left(\bar{q}\gamma^\mu\gamma_5\frac{\partial_\mu a}{2f_a}X_q q\right)$$

$$c^{(1)} = \frac{1}{3}\left(X_u + X_d + X_s - 1\right)$$

$$c^{(3)} = \frac{1}{2}\left(X_u - X_d - \frac{1-z}{1+z+w}\right)$$

$$c^{(8)} = \frac{1}{2\sqrt{3}}\left(X_u + X_d - 2X_s - \frac{1+z-2w}{1+z+w}\right)$$

$$s + ip = \mathcal{M}_a$$

$$a_\mu = \frac{\partial_\mu a}{2f_a}\left(c^{(3)}\lambda_3 + c^{(8)}\lambda_8\right)$$

$$a_{\mu,i}^{(s)} = c_i\frac{\partial_\mu a}{2f_a}, \quad i = 1,\dots,4$$

$$c_1 = c^{(1)},\ c_2 = X_c,\ c_3 = X_b,\ c_4 = X_t$$

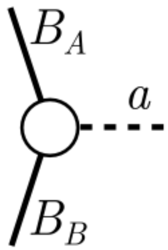
- Building blocks with axion:

$$\boxed{\begin{aligned} u_\mu &= i\left[u^\dagger\partial_\mu u - u\partial_\mu u^\dagger - iu^\dagger a_\mu u - iua_\mu u^\dagger\right] \\ u_{\mu,i} &= i\left[-iu^\dagger a_{\mu,i}^{(s)} u - iua_{\mu,i}^{(s)} u^\dagger\right] = 2a_{\mu,i}^{(s)} \\ \chi_\pm &= 2B_0\left[u^\dagger(s+ip)u^\dagger \pm u(s+ip)^\dagger u\right] \end{aligned}}$$

Axion-baryon couplings

T. Vonk, FKG, U.-G. Meißner, JHEP 03, 138 (2020); JHEP 08, 024 (2021)

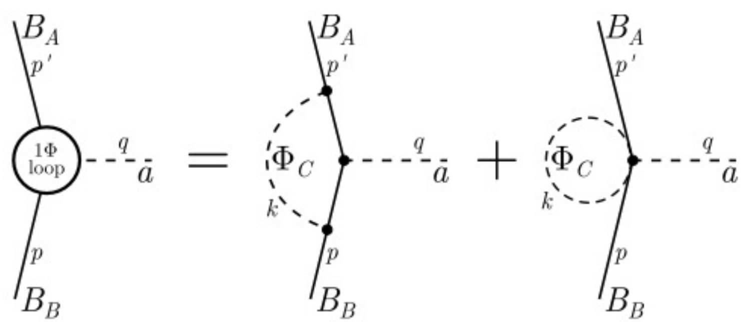
- General form of axion-baryon couplings:



$$= G_{aAB}\,(S\cdot q) \quad \text{with } G_{aAB} = -\frac{1}{f_a} g_{aAB} + \mathcal{O}\left(\frac{1}{f_a^2}\right)$$

- Expansion in the chiral power counting

$$g_{aAB} = \underbrace{g_{aAB}^{(1)}}_{\text{LO,tree}} + \underbrace{g_{aAB}^{(2)}}_{\text{NLO},1/m_B} + \underbrace{g_{aAB}^{(3)}}_{\text{NNLO},1/m_B^2,\text{one-loop}} + \ldots$$



Axion-baryon couplings

T. Vonk, FKG, U.-G. Meißner, JHEP 08, 024 (2021)

- Precision calculation w/ naturalness assumption for the unknown LECs at $\mathcal{O}(p^3)$ (dominant uncertainty)

Process	KSVZ	DFSZ
$\Sigma^+ \to \Sigma^+ + a$	$-0.547(84)$	$-0.709(94) + 0.446(54)\sin^2\beta$
$\Sigma^- \to \Sigma^- + a$	$-0.245(80)$	$-0.113(92) - 0.142(54)\sin^2\beta$
$\Sigma^0 \to \Sigma^0 + a$	$-0.399(78)$	$-0.417(87) + 0.158(43)\sin^2\beta$
$p \to p + a$	$-0.432(86)$	$-0.589(96) + 0.436(53)\sin^2\beta$
$\Xi^- \to \Xi^- + a$	$0.166(79)$	$0.299(91) - 0.161(52)\sin^2\beta$
$n \to n + a$	$0.003(83)$	$0.271(94) - 0.400(53)\sin^2\beta$
$\Xi^0 \to \Xi^0 + a$	$0.303(81)$	$0.570(92) - 0.409(52)\sin^2\beta$
$\Lambda \to \Lambda + a$	$0.138(87)$	$0.314(96) - 0.228(47)\sin^2\beta$
$\Sigma^0 \to \Lambda + a$, $\Lambda \to \Sigma^0 + a$	$-0.161(24)$	$-0.323(33) + 0.309(32)\sin^2\beta$

Pion axioproduction

- Effects of pions in supernova, $\pi^- p \to na$, found to be important
  - surpasses bremsstrahlung $NN \to NNa$
  - leads to harder axions
  - $aN \to \pi N$ was estimated as

$$\sigma(aN \to \pi N) \approx \frac{F_\pi^2}{f_a^2} \boxed{\sigma(\pi N \to \pi N)}$$

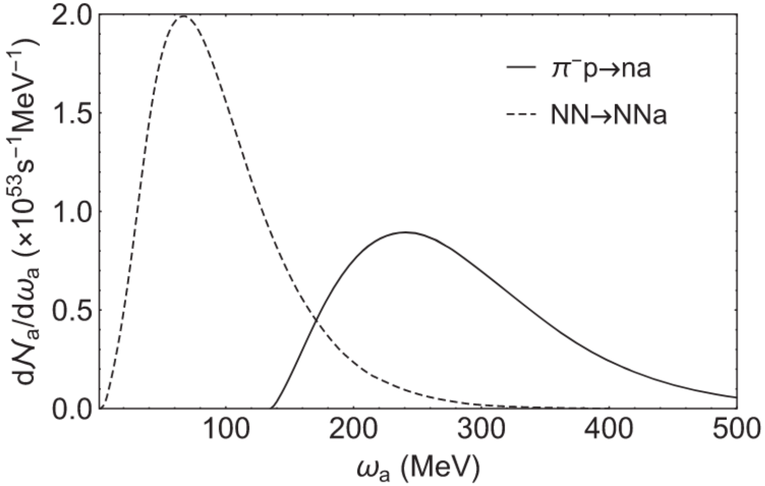
assumed to be ~ 100 mb

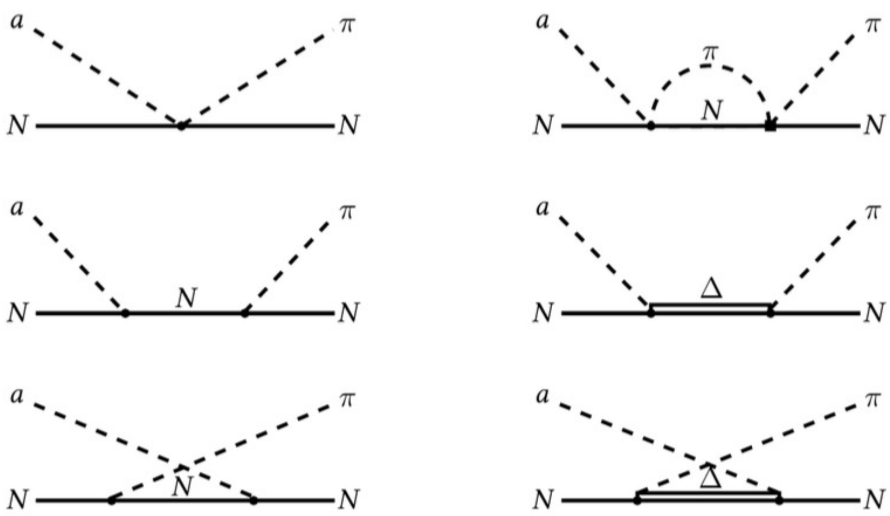
and Δ was argued to be important

- However, $aN \to \Delta \to \pi N$ breaks isospin!

T. Vonk, FKG, U.-G. Meißner, PRD 105, 054029 (2022)

Carenza et al., PRL 126, 071102 (2021)

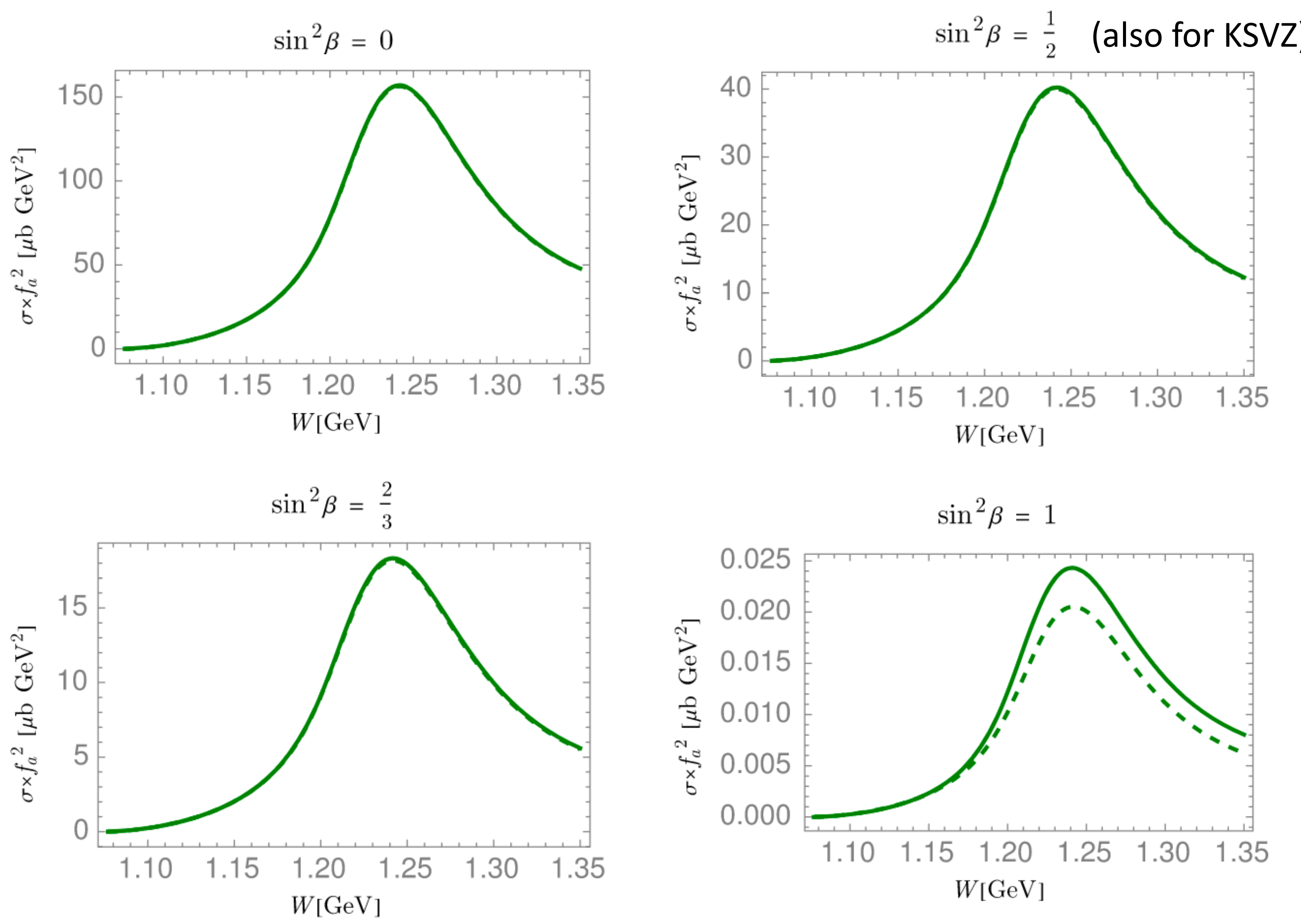




Pion axioproduction

T. Vonk, FKG, U.-G. Meißner, PRD 105, 054029 (2022)

- Suppression of 10^{-1} to 10^{-5} depending on $\sin^2\beta$ compared to the naïve estimate



- Here isospin breaking provides a suppression: $(m_d - m_u)/(m_d + m_u) \approx 0.34$
  - Much milder than usual isospin breaking in hadron physics, which is given by $(m_d - m_u)/m_s$ or $(m_d - m_u)/\Lambda_{\rm QCD}$

Summary

- Derived a closed form of the θ-vacuum energy density (QCD axion potential) at NLO
- Precision calculation of the axion properties
  - mass, self-couplings, $a\gamma\gamma$ coupling
  - axion-baryon couplings
    - g_{ann} could be extremely tiny
    - $g_{a\Lambda\Lambda}$ could be much larger than g_{ann}, impact on the axion emissivity of dense stellar objects such as neutron start?

Thank you for your attention!