

Advances in modeling neutrinoless double beta decay with nuclear forces from chiral effective field theory

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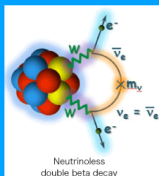
School of Physics and Astronomy, Sun Yat-sen University

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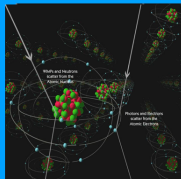
- 1 Introduction to neutrinoless double beta decay
- 2 Modeling atomic nuclei with nuclear forces from chiral EFT
- 3 Determination of the NMEs for neutrinoless double-beta decay
- 4 Summary and prospect



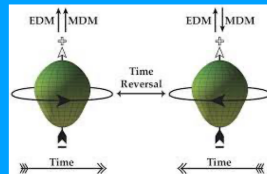
- Search for beyond standard model physics at three frontiers: the energy frontier (LHC, etc), the cosmic frontier (CMB, LHAASO, etc), and the intensity frontier ($0\nu\beta\beta$ decay, WIMP, EDM, etc)



Double beta decay



WIMP scattering



EDM and Schiff moment

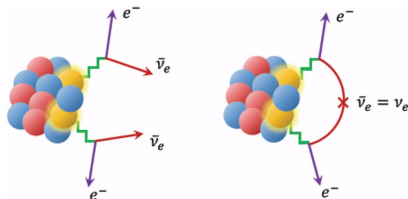
Atomic nucleus: a playground to test fundamental symmetries.

Nuclear double- β decay is a second-order weak process in which two neutrons (or protons) in a parent nucleus (A, Z) are simultaneously transforming into two protons (or neutrons) in a daughter nucleus ($A, Z \pm 2$). There are typically four types

- double-electron emission ($2\beta^-$)
- double-positron emission ($2\beta^+$)
- single-positron emission with single-electron capture ($\epsilon\beta^+$)
- double-electron capture (2ϵ)

This talk focuses on the double-beta decay with double-electron emission.

$$(A, Z) \rightarrow (A, Z + 2) + 2e^- + (2\bar{\nu}_e)$$



First prediction of $2\nu\beta\beta$ decay

Double Beta-Disintegration

M. GÖPPERT-MAYER, *The Johns Hopkins University*

(Received May 20, 1935)

From the Fermi theory of β -disintegration the probability of simultaneous emission of two electrons (and two neutrinos) has been calculated. The result is that this process occurs sufficiently rarely to allow a half-life of over 10^{21} years for a nucleus, even if its isobar of atomic number different by 2 were more stable by 20 times the electron mass.

Status of measurements

Isotope	$T_{1/2}(2\nu)$, yr	$ M_{2\nu}^{eff} $ ($G_{2\nu}$ from [24])	$ M_{2\nu}^{eff} $ ($G_{2\nu}$ from [25])	Recommended Value
$2\nu\beta\beta$:				
^{48}Ca	$5.3^{+1.2}_{-0.8} \cdot 10^{19}$	$0.0348^{+0.0030}_{-0.0024}$	$0.0348^{+0.0030}_{-0.0024}$	0.035 ± 0.003
^{76}Ge	$(1.88 \pm 0.08) \cdot 10^{21}$	$0.1051^{+0.0025}_{-0.0024}$	$0.1074^{+0.0025}_{-0.0022}$	0.106 ± 0.004
^{82}Se	$0.87^{+0.02}_{-0.01} \cdot 10^{20}$	$0.0849^{+0.0005}_{-0.0010}$	$0.0855^{+0.0005}_{-0.0010}$	0.085 ± 0.001
^{96}Zr	$(2.3 \pm 0.2) \cdot 10^{19}$	$0.0798^{+0.0037}_{-0.0032}$	$0.0804^{+0.0038}_{-0.0033}$	0.080 ± 0.004
^{100}Mo	$7.06^{+0.15}_{-0.13} \cdot 10^{18}$	$0.2071^{+0.0019}_{-0.0022}$	$0.2096^{+0.0020}_{-0.0022}$	
		$0.1852^{+0.0017}_{-0.0019}$		0.185 ± 0.002
$^{100}\text{Mo-}$	$6.7^{+0.5}_{-0.4} \cdot 10^{20}$	$0.1571^{+0.0048}_{-0.0056}$	$0.1619^{+0.0050}_{-0.0058}$	
$^{100}\text{Ru}(0^+)$		$0.1513^{+0.0053}_{-0.0047}$		0.151 ± 0.005
^{116}Cd	$(2.69 \pm 0.09) \cdot 10^{19}$	$0.1160^{+0.0020}_{-0.0019}$	$0.1176^{+0.0020}_{-0.0019}$	
		$0.1084^{+0.0024}_{-0.0019}$		0.108 ± 0.003
^{128}Te	$(2.25 \pm 0.09) \cdot 10^{24}$	$0.0406^{+0.0008}_{-0.0008}$	$0.0454^{+0.0009}_{-0.0009}$	0.043 ± 0.003
^{130}Te	$(7.91 \pm 0.21) \cdot 10^{20}$	$0.0288^{+0.0004}_{-0.0004}$	$0.0297^{+0.0004}_{-0.0004}$	0.0293 ± 0.0009
^{136}Xe	$(2.18 \pm 0.05) \cdot 10^{21}$	$0.0177^{+0.0001}_{-0.0001}$	$0.0184^{+0.0002}_{-0.0002}$	0.0181 ± 0.0006
^{150}Nd	$(9.34 \pm 0.65) \cdot 10^{18}$	$0.0543^{+0.0020}_{-0.0018}$	$0.0550^{+0.0020}_{-0.0018}$	0.055 ± 0.003
$^{150}\text{Nd-}$	$1.2^{+0.3}_{-0.2} \cdot 10^{20}$	$0.0438^{+0.0042}_{-0.0046}$	$0.0450^{+0.0043}_{-0.0048}$	0.044 ± 0.005
$^{150}\text{Sm}(0^+)$				
^{238}U	$(2.0 \pm 0.6) \cdot 10^{21}$	$0.1853^{+0.0361}_{-0.0227}$	$0.0713^{+0.0139}_{-0.0088}$	$0.13^{+0.09}_{-0.07}$

- The $2\nu\beta\beta$ decay is a second-order weak process allowed in the Standard Model.
- First discovery of $2\nu\beta\beta$ decay (geochemical method) in ^{130}Te
Inghram & Reynolds, 1950
- First direct detection of $2\nu\beta\beta$ decay in ^{82}Se
Moe & Lowenthal, 1980
- The half-life $T_{1/2}^{2\nu}$ ranges from 10^{18} to 10^{24} years
A. Barabash, 2020

- The first study of $0\nu\beta\beta$ decay (Furry, 1939), inspired by Majorana and Racah (1937) that the neutrino may coincide with its own antiparticle.
- Occur if neutrinos have mass and are Majorana particles
- Process violating lepton number by 2 units.
- Several mechanisms: light or heavy neutrino exchange, etc.

First prediction of $0\nu\beta\beta$ decay

On Transition Probabilities in Double Beta-Disintegration

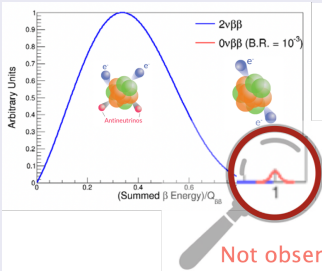
W. H. FURRY

Physics Research Laboratory, Harvard University, Cambridge, Massachusetts

(Received October 16, 1939)

The phenomenon of double β -disintegration is one for which there is a marked difference between the results of Majorana's symmetrical theory of the neutrino and those of the original Dirac-Fermi theory. In the older theory double β -disintegration involves the emission of four particles, two electrons (or positrons) and two antineutrinos (or neutrinos), and the probability of disintegration is extremely small. In the Majorana theory only two particles—the electrons or positrons—have to be emitted, and the transition probability is much larger. Approximate values of this probability are calculated on the Majorana theory for the various Fermi and Konopinski-Uhlenbeck expressions for the interaction energy. The selection rules are derived, and are found in all cases to allow transitions with $\Delta i = \pm 1, 0$. The results obtained with the Majorana theory indicate that it is not at all certain that double β -disintegration can never be observed. Indeed, if in this theory the interaction expression were of Konopinski-Uhlenbeck type this process would be quite likely to have a bearing on the abundances of isotopes and on the occurrence of observed long-lived radioactivities. If it is of Fermi type this could be so only if the mass difference were fairly large ($\epsilon \gtrsim 20$, $\Delta M \gtrsim 0.01$ unit).

Signal in the direct detection



Knowledge about neutrinos

- Neutrino mixing

$$|\nu_\alpha\rangle = \sum_{j=1}^N U_{\alpha j}^* |\nu_j\rangle.$$

- $\Delta m_{ij}^2 (\neq 0)$, and $\theta_{ij} (\neq 0)$.

What are still unknown?

- The nature of neutrinos: Dirac or Majorana
- Neutrino absolute mass m_j (ordering) and its origin.

The observation of $0\nu\beta\beta$ decay may provide answers to some.

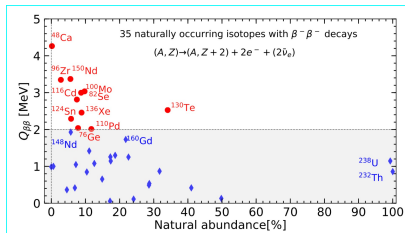
Inputs from nuclear physics

- The $Q_{\beta\beta}$ value: determining the energy region of interest and phase-space factor $G_{0\nu}$.
- The nuclear matrix elements (NMEs) $M^{0\nu}$ of $0\nu\beta\beta$ decay: determining neutrino masses

$$\left| \sum_{j=1}^3 U_{ej}^2 m_j \right| = \left[\frac{m_e^2}{g_A^4(0) G_{01} T_{1/2}^{0\nu} |M^{0\nu}|^2} \right]^{1/2}$$

What are the major challenges?

- Noise-free, large-scale experimental setups.
- Precise values of the NMEs.

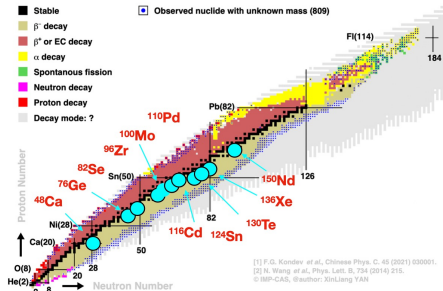


The isotopes that

- cannot decay via the single- β decay due to energy or spin forbidden
- and with large $Q_{\beta\beta} (> 2.0$ MeV) value.

are usually selected to be the candidate nuclei with experimental interest.

Nuclear Chart: decay mode of the ground state nuclide (NUBASE2020)



These candidate nuclei are located close to the β -decay stability line (classified as stable nuclei).

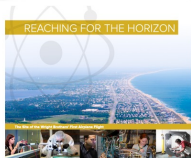
$$T_{1/2}^{0\nu} \simeq 10^{27-28} \left(\frac{0.01 \text{ eV}}{\langle m_{\beta\beta} \rangle} \right)^2 y$$

RECOMMENDATION II

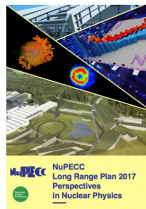
The excess of matter over antimatter in the universe is one of the most compelling mysteries in all of science. The observation of neutrinoless double beta decay in nuclei would immediately demonstrate that neutrinos are their own antiparticles and would have profound implications for our understanding of the matter-antimatter mystery.

We recommend the timely development and deployment of a U.S.-led ton-scale neutrinoless double beta decay experiment.

A ton-scale instrument designed to search for this as-yet unseen nuclear decay will provide the most powerful test of the particle-antiparticle nature of neutrinos ever performed. With recent experimental breakthroughs pioneered by U.S. physicists and the availability of deep underground laboratories, we are poised to make a major discovery.



The 2015
LONG RANGE PLAN
for NUCLEAR SCIENCE



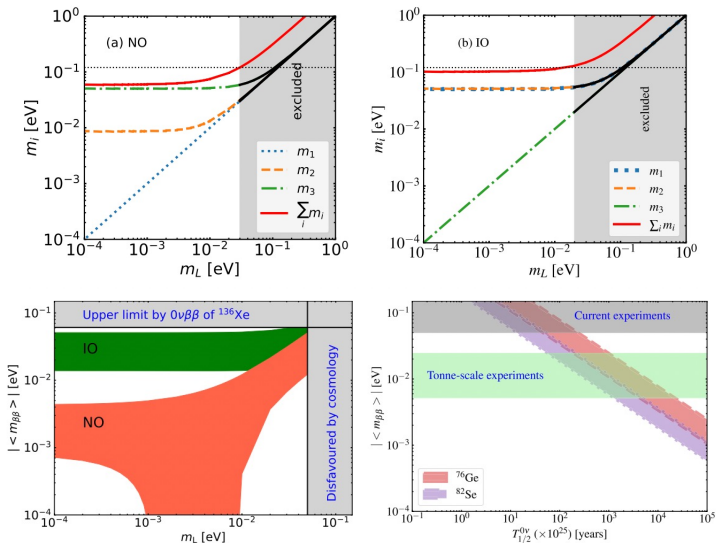
c.f. Hao Qiu's talk.

Isotopes	Experiment	Half-life sensitivity	NME	Effective mass (meV)	Reference
^{76}Ge	GERDA	1.8×10^{26}	2.66–6.34	79–180	PRL125,252502 (2020)
	MJD	4.8×10^{25}		200–433	PRC100,025501 (2019)
	LEGEND-1T	1.3×10^{28}		9–21	arXiv:2107.11462v1 (2021)
	CDEX	6.4×10^{22}		5000	Sci. China 60, 071011 (2017)
^{100}Mo	CUPID-Mo	1.5×10^{24}	3.84–6.59	310–540	PRL126, 181802(2021)
	CUPID	9.1×10^{27}		4.1–6.8	arXiv:2203.08386v1 (2022)
^{130}Te	CUORE	2.2×10^{25}	1.37–6.41	90–305	Nature, 604 (2022)
^{136}Xe	EXO-200	3.5×10^{25}	1.11–4.77	93–286	PRL123,161802(2019)
	nEXO	1.35×10^{28}		<15	JPG49, 015104 (2022)
	KamLAND-Zen	2.3×10^{26}		36–156	arXiv:2203.02139v1 (2022)
	PandaX-III	5×10^{25}		90–230	Sci. China 60, 061011 (2017)

Tonne-scale detectors with sensitivity $T_{1/2}^{0\nu} \sim 10^{28}\text{y}$

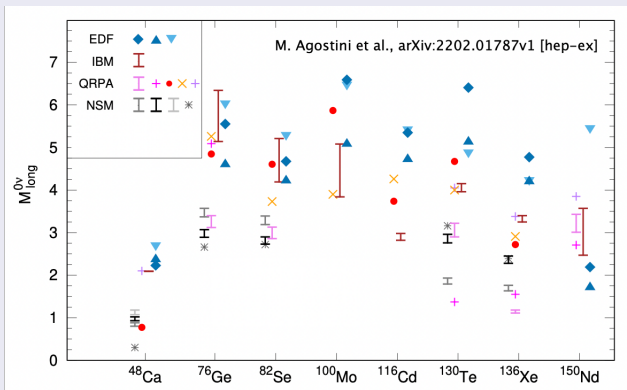
In order to have one event during one year (naive estimation),

$$N(t) = \frac{\Delta N}{\Delta t} \frac{1}{\ln 2} T_{1/2}^{0\nu} \rightarrow 1.6 \times 10^4 \text{ moles} \rightarrow 800 - 2,500 \text{ kg}$$

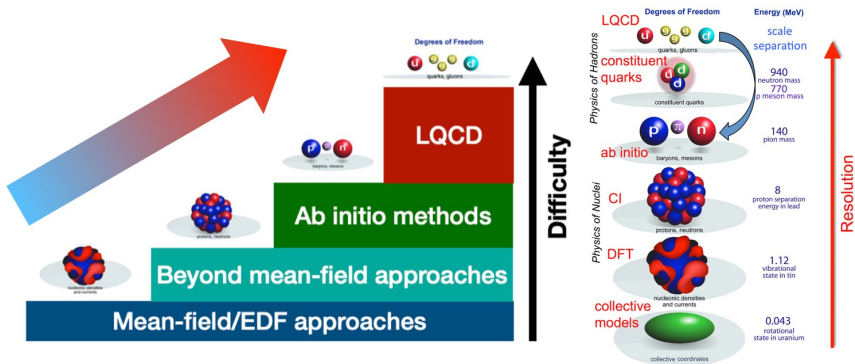


$$\langle m_{\beta\beta} \rangle = m_1 c_{12}^2 c_{13}^2 + m_2 c_{13}^2 s_{12}^2 e^{i\alpha_{21}} + m_3 s_{13}^2 e^{i(\alpha_{31}-2\delta)} \simeq 0.680 m_1 + 0.297 m_2 e^{i\alpha_{21}} + 0.022 m_3 e^{i(\alpha_{31}-2\delta)}.$$

Status of studies on the NMEs



- Variety of nuclear models based on diff. nuclear forces/EDFs
- Discrepancy by a factor of THREE or even more.
- Difficult to reduce the discrepancy.



From phenomenological to ab initio studies

- Nuclear interactions derived from chiral EFT/LQCD
- Systematically improvable many-body methods

- Non-relativistic chiral 2N+3N interactions (Weinberg power counting and others) *check with B.W. Long*

	NN	3N	4N
LO $\mathcal{O}(Q^0/\Lambda^0)$	1990 Weinberg 2 	—	—
NLO $\mathcal{O}(Q^2/\Lambda^2)$	1992 Ordonez, van Kolck 7 	1992, 1994 [166-169] Weinberg, van Kolck, Epelbaum ... —	—
N ² LO $\mathcal{O}(Q^3/\Lambda^3)$	1992 Ordonez, van Kolck 0 	1994 ... 2 	—
N ³ LO $\mathcal{O}(Q^4/\Lambda^4)$	2000–2002 Kaiser 12 	2008–2011 [183-185] 0 	2006 [186] 0
N ⁴ LO $\mathcal{O}(Q^5/\Lambda^5)$	2015 [188, 189] 0 	2011– [190-192] ? 	?

K. Hebeler, Phys. Rep. 890, 1 (2020)

- Relativistic chiral 2N interaction (N2LO)

J.-X. Lu, C.-X. Wang, Y. Xiao, L.-S. Geng, J. Meng, P. Ring, PRL128, 142002 (2022)

Similarity renormalization group (SRG) for nuclear forces

- Hard core imposes a challenge to nuclear many-body solvers.
- Soften nuclear forces with a set of continuous unitary transformations. S. K. Bogner et al., PRC75, 061001(R) (2007)

$$H_s = U_s H U_s^\dagger \equiv T_{\text{rel}} + V_s$$

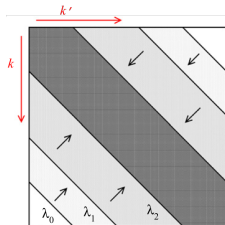
- Flow equation

$$\frac{dH_s}{ds} = [\eta_s, H_s]$$

- The generator η_s is chosen to diagonalize $H(s)$ in the eigenbasis of T_{rel} ,

$$\eta_s = [T_{\text{rel}}, H_s]$$

$$\lambda = s^{-1/4} [\text{fm}^{-1}]$$



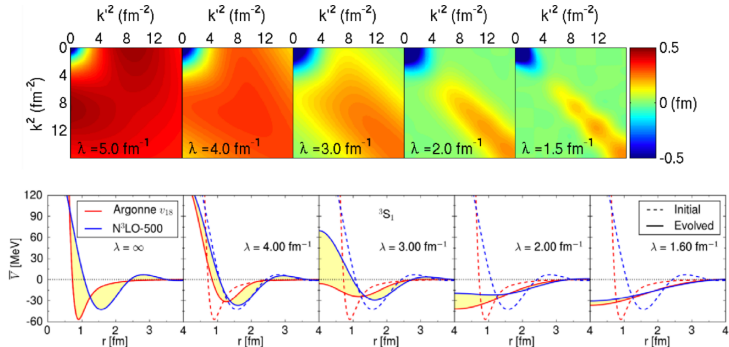


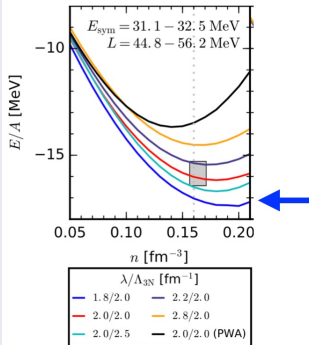
Figure: Local projection of AV18 and N³LO(500 MeV) potentials $V(r)$ in 3S_1 channel.

- The repulsive core *disappears* in the low- λ nuclear potentials.
- The SRG reformulates the original 2N force in terms of a softened 2N force + induced many-body forces.

- Nuclear potentials on lattice (check with B.N. Lu)
- Nuclear potentials in HO basis

The magic interaction "EM1.8/2.0"

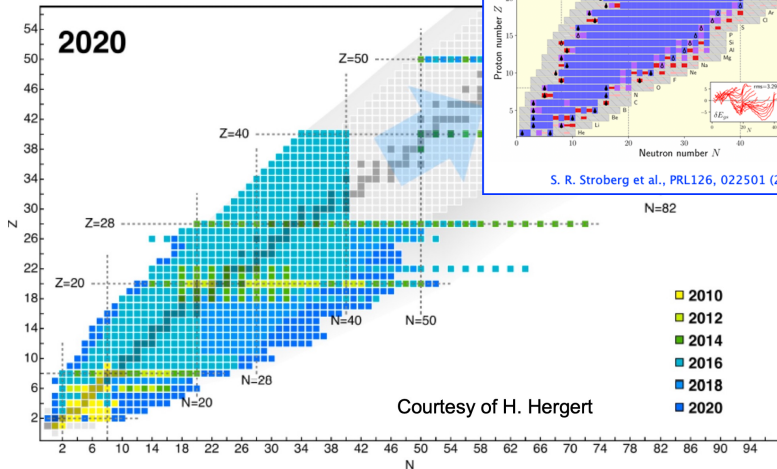
- The N3LO two-body NN interaction : D.R. Entem, R. Machleidt (2003)
- The N2LO local 3N interactions: K. Hebeler et al. (2011).
- Overestimates somewhat the binding energy and saturation density \rightarrow underestimates nuclear radius.



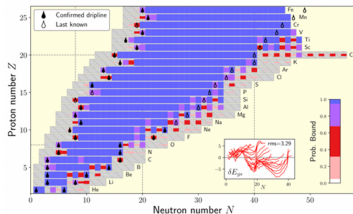
C. Drischler et al., PRL122, 042501 (2019)

Other chiral interactions (NR)

- **The NNLO_{sat} interaction:** both the NN and 3N are truncated up to N²LO with the 16 LECs fitted simultaneously to NN data (< 35 MeV), binding energies and charge radii of $A = 3, 4$ systems, carbon and oxygen isotopes. A. Ekström et al., PRC91, 051301(R) (2015)
- **The NN+3N(1n1) interaction**, an improved version of the NN+3N(400) interaction, adjusted solely on $A = 2, 3, 4$ systems. V. Somà et al., PRC101, 014318 (2020)
- **A consistent EMN family of NN+3N interaction:** construct 3N interactions using the same chiral order (N³LO), the same non-local regulator scheme, and the same regulator scale as in the NN interaction. Constrained by energies of $A = 3, 16$ systems. T. Hüther et al., PLB808, 135651 (2020)
- **The Delta-full NNLO_{go} interaction:** similar to NNLO_{sat} , but with Δ degree of freedom explicitly. W. G. Jiang et al., PRC102, 054301 (2020)



First-principles calculations predict the properties of nearly 700 isotopes between helium and iron



S. R. Stroberg et al., PRL126, 022501 (2021)

- The total NME of $0\nu\beta\beta$ decay for the ground-state to ground-state transition

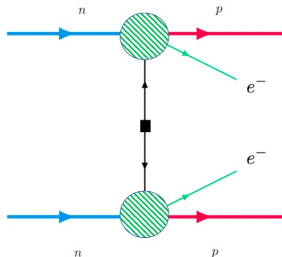
$$M^{0\nu\beta\beta} = \sum_{\alpha=F,GT, \text{Tensor}} \left\langle 0_f^+ \left| \sum_{1,2} h_{\alpha,K}(r_{12}) C_{\alpha}^K \cdot S_{\alpha}^K \tau_1^+ \tau_2^+ \right| 0_i^+ \right\rangle.$$

where the spin-spatial part

$$C_F^0 = 1, \quad S_F^0 = 1, .$$

$$C_{GT}^0 = 1, \quad S_{GT}^0 = \boldsymbol{\sigma}_1 \cdot \boldsymbol{\sigma}_2, .$$

$$C_T^2 = \sqrt{\frac{24\pi}{5}} Y_2(\hat{\mathbf{r}}_{12}), \quad S_T^2 = [\boldsymbol{\sigma}_1 \otimes \boldsymbol{\sigma}_2]^2.$$



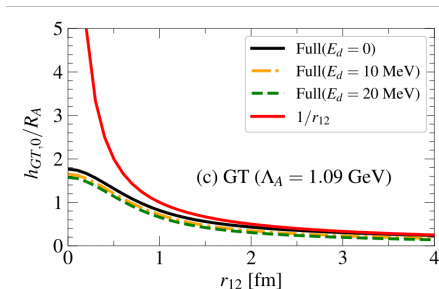
- The coordinate-space neutrino potential ($K = 0, 2$)

$$h_{\alpha,K}(r_{12}) = \frac{2R_A}{\pi g_A^2} \int_0^\infty dq q^2 \frac{h_{\alpha,K}(q^2)}{q(q + E_d)} j_K(qr_{12}).$$

$$h_{F,0}(q^2) = -g_V^2(q^2)$$

$$h_{GT,0}(q^2) = g_A^2(q^2) - \frac{2}{3} \frac{q^2}{2m_p} g_A(q^2) g_P(q^2) + \frac{1}{3} \frac{q^4}{4m_p^2} g_P^2(q^2) + \frac{2}{3} \frac{q^2}{4m_p^2} g_M^2(q^2),$$

$$h_{T,2}(q^2) = \frac{2}{3} \frac{q^2}{2m_p} g_A(q^2) g_P(q^2) - \frac{1}{3} \frac{q^4}{4m_p^2} g_P^2(q^2) + \frac{1}{3} \frac{q^2}{4m_p^2} g_M^2(q^2)$$



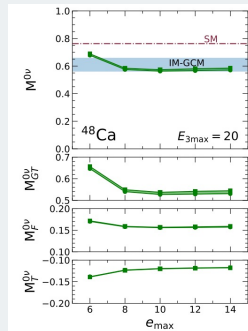
Ab initio methods for $0\nu\beta\beta$ decay starting with chiral potentials

For light nuclei: benchmark studies

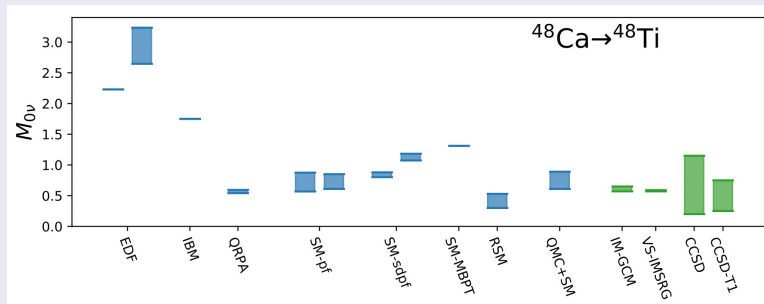
- Quantum Monte-Carlo (QMC) A. Baroni et al., PRC98, 044003 (2018)
- No-core shell model (NCSM) R. A. M. Basili et al., PRC102, 014302 (2020); S. Novario et al., PRL126, 182502 (2021); JMY et al., PRC103, 014315 (2021)

For candidate nuclei (EM1.8/2.0)

- Multi-reference in-medium similarity renormalization group (IMSRG)+GCM (IM-GCM)
JMY et al., PRL124, 232501 (2020)
- IMSRG+Shell Model (VS-IMSRG)
A. Belley et al., PRL126, 042502 (2021)
- Coupled-cluster with singlets, doublets, and partial triplets (CCSDT1) .
S. Novario et al., PRL126, 182502 (2021)



A comprehensive comparison

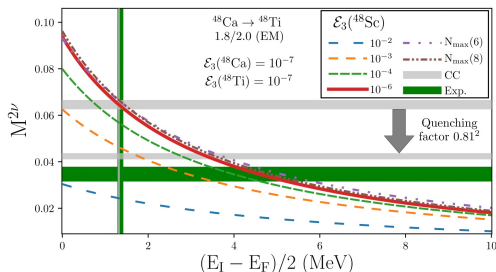


White paper/report for the Topical Collaboration on neutrinoless double-beta decay, funded by DOE (US)

V. Cirigliano et al., arXiv:2207.01085v1 (2022)

- All NMEs are calculated based on the same long-range transition operator in the standard mechanism.

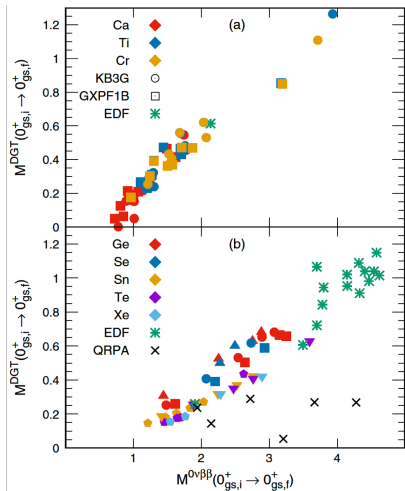
$$M^{2\nu} = \sum_{\mu} \frac{\langle 0_F^+ | \sigma\tau^- | 1_{\mu}^+ \rangle \langle 1_{\mu}^+ | \sigma\tau^- | 0_I^+ \rangle}{E_{\mu} - E_I + (E_I - E_F)/2}.$$



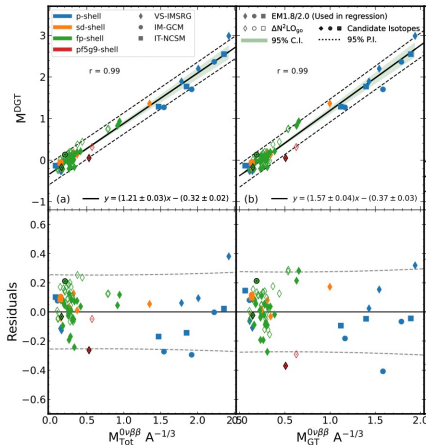
- CCSDT1: $M^{2\nu} = 0.042$ with the quenching factor $q^2 = 0.81^2$ deduced from two-body currents, somewhat larger than the data $M^{2\nu} = 0.035$. S. Novario et al., PRL126, 182502 (2021)
- VS-IMSRG: $M^{2\nu} = 0.030$ without the quenching factor.

Charlie G. Payne, B.S. thesis of the University of Waterloo, 2015

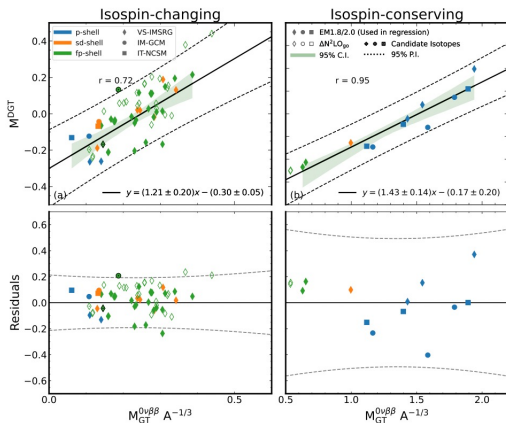
Determining the NME with correlation relation?



N. Shimizu et al., PRL120, 142502 (2018)

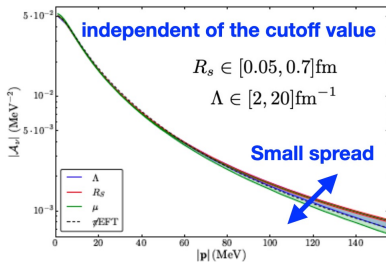
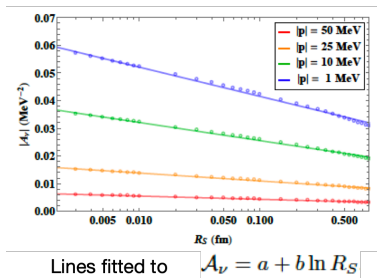


JMY et al., PRC106, 014315 (2022)



- Might not be able to calibrate the $M^{0\nu}$ for the candidate $0\nu\beta\beta$ decay with the data of DGT.
- Other observables: $2\nu\beta\beta$ decay?

The $nn \rightarrow ppe^-e^-$ transition amplitude A_ν by the long-range transition operator at the LO



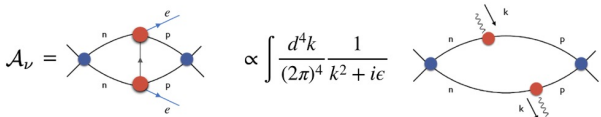
- The transition amplitude is regulator-dependent (left panel)!
- With the following contact term at LO, the A_ν becomes regulator independent (right panel)

$$V_{\nu,S} = -2g_{\nu}^{NN}\tau^{(1)+}\tau^{(2)+} +$$

Determination of the LEC for the contact operator

The LO contribution to $nn \rightarrow ppe^-e^-$ transition amplitude \mathcal{A}_ν

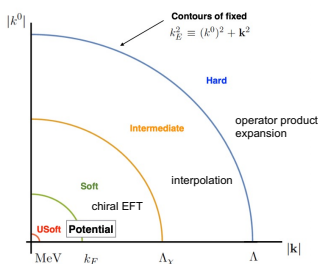
- represents the \mathcal{A}_ν as the momentum integral of a known kernel (proportional to the neutrino propagator) times the generalized forward Compton scattering amplitude $n(p_1)n(p_2)W^+(k) \rightarrow p(p'_1)p(p'_2)W^-(k)$, in analogy to the Cottingham formula [W.N. Cottingham, Ann. Phys. 25, 424 (1963)] for the electromagnetic contribution to hadron masses.

$$\mathcal{A}_\nu \propto \int \frac{d^4k}{(2\pi)^4} \frac{g_{\mu\nu}}{k^2 + i\epsilon} \int d^4x e^{ik \cdot x} \langle pp | T \{ j_W^\mu(x) j_W^\nu(0) \} | nn \rangle$$


The diagram on the left shows a contact interaction between two neutrons (n) and two protons (p) via a virtual photon (e). The diagram on the right shows a contact interaction between two neutrons (n) and two protons (p) via a virtual photon (k).

V. Cirigliano et al., PRL126, 172002 (2021); JHEP05, 289 (2021)

- model-independent representations of the integrand in the low- and high-momentum regions, through chiral EFT and the operator product expansion, respectively.
- Construct a model for the full amplitude by interpolating between the two regions.



$$\mathcal{A}_\nu^{\text{full}} = \int_0^\infty d|k| a^{\text{full}}(|k|) = \mathcal{A}^< + \mathcal{A}^>, \\ \mathcal{A}^< = \int_0^\Lambda d|k| a_<(|k|), \\ \mathcal{A}^> = \int_\Lambda^\infty d|k| a_>(|k|),$$

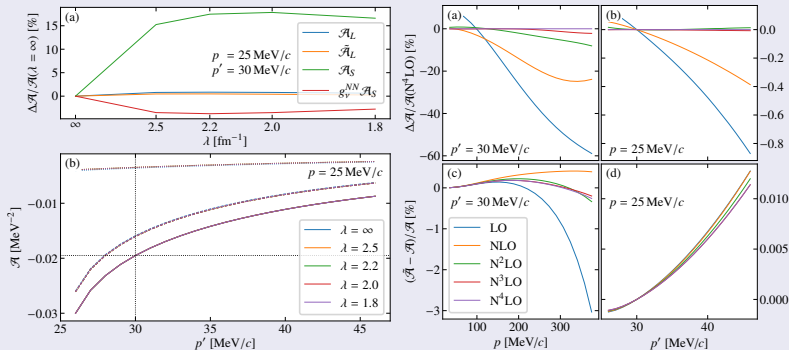
The LEC of the contact term is determined by the following matching

$$\mathcal{A}_\nu^{\chi\text{EFT}} = \mathcal{A}^< + \mathcal{A}^>$$

- The final (scheme-independent) amplitude

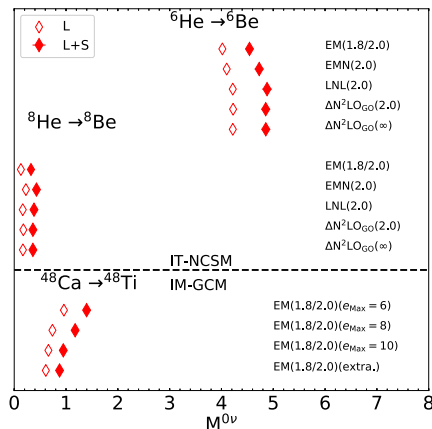
$$\mathcal{A}_\nu(|\mathbf{p}| = 25\text{MeV}, |\mathbf{p}'| = 30\text{MeV}) = -0.0195(5)\text{MeV}^{-2}$$

Determination of the LEC g_{ν}^{NN} for diff. chiral potentials



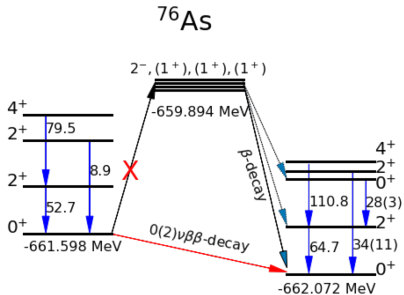
- Dependence of the short- and long-range parts of the transition amplitude on the SRG scale λ for the NN potential.
- Converges w.r.t. the power of chiral NN expansion.

R. Wirth, JMY, H. Hergert, PRL127, 242502 (2021)



- The contact term (S) enhances the NME for ${}^{48}\text{Ca}$ by 43(7)%, the uncertainty is propagated only from the synthetic datum.
- More accurate predictions require the LEC from lattice QCD.

R. Wirth, JMY, H. Hergert, PRL127, 242502 (2021)

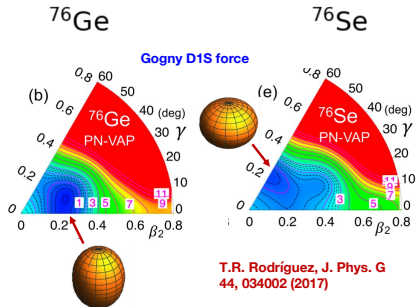


Challenges

- Both ^{76}Ge and ^{76}Se are medium-mass deformed nuclei with a shape-coexistence phenomenon.
- Triaxial deformation turns out to be essential in their low-lying states.

Imposing a computational challenge for nuclear ab initio methods.

The IM-GCM is well suited for the low-lying states of shape-coexistence nuclei.



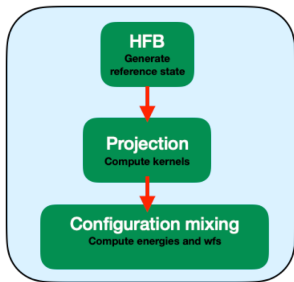
- The trial wave function of a GCM state

$$|\Psi^{JNZ\dots}\rangle = \sum_Q F_Q^{JNZ} \hat{P}^J \hat{P}^N \hat{P}^Z \dots |\Phi_Q\rangle$$

$|\Phi_Q\rangle$ are a set of HFB wave functions from constraint calculations, Q is the so-called generator coordinate.

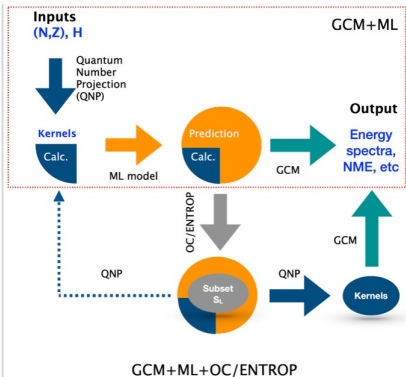
- The Hill-Wheeler-Griffin equation:

$$\sum_{Q'} \left[\mathcal{H}^{JNZ}(Q, Q') - E^J \mathcal{N}^{JNZ}(Q, Q') \right] F_{Q'}^{JNZ} = 0$$



- The Hilbert space is controlled by the Q .
- The Q is usually chosen as deformation parameters.
- The computation complexity is generally smaller than full CI calculations, but also grows with the number of Q . *more details? c.f. Changfeng Jiao's talk*

Flowchart for our algorithm



The noises by ML models may impact the results, but this impact can be avoided by our algorithm.

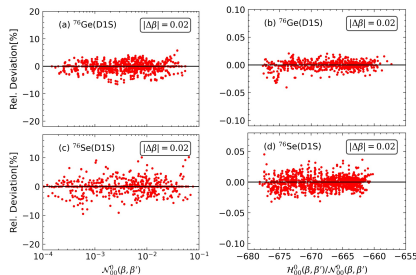
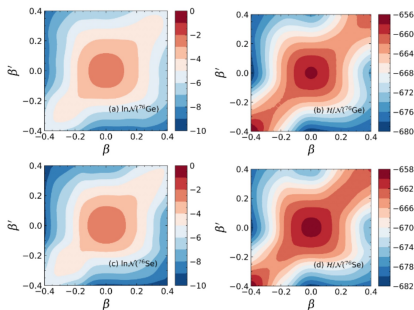
A space-reduction algorithm

- Computing partial of the kernels (\mathcal{N}, \mathcal{H}) exactly
- Training ML models for the kernels and predicting the rest of kernels
- Selection of a subspace based on orthogonality condition (OC)
- Computing all the elements of the kernels (\mathcal{N}, \mathcal{H}) within the subspace
- Determination of observables (energy spectra, $M^{0\nu}$)

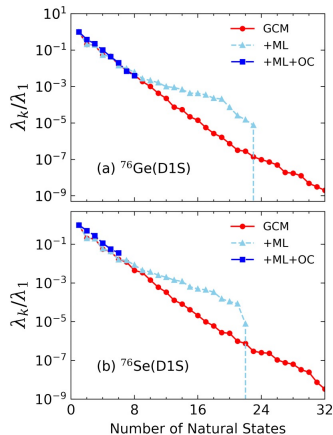
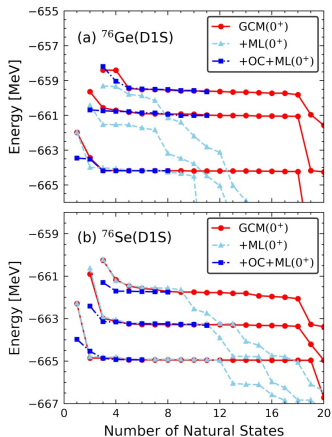
X. Zhang et al. in preparation (2022)

Learning $\ln \mathcal{N}$ and \mathcal{H}/\mathcal{N} with polynomial regression

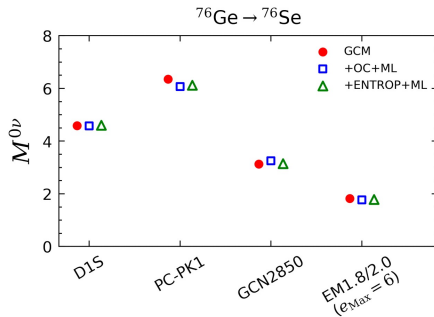
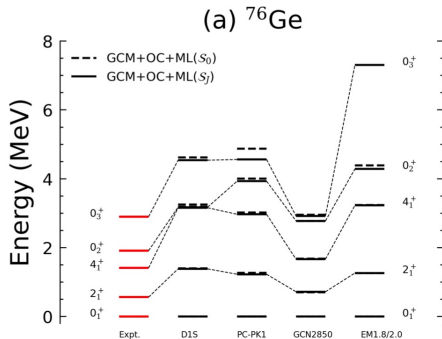
$$\hat{y}^{(i)}(\theta; N) = \sum_{n=0}^N \theta_n \cdot \left(\mathbf{x}^{(i)} \right)^n$$



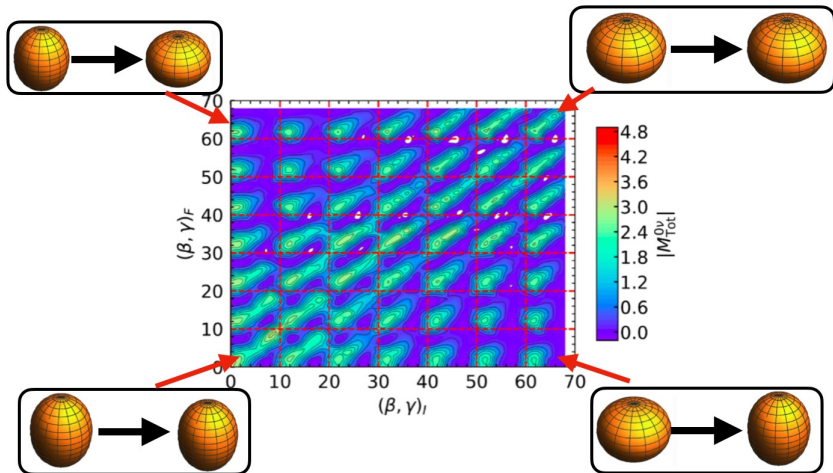
The norm kernels \mathcal{N} are highly non-local and vary by several orders of magnitude. Thus, the performance of the ML model for \mathcal{N} is much worse than that for \mathcal{H}/\mathcal{N} .



The noises introduced by the ML models spoil the energy plateau condition in the GCM+ML, but this effect is avoided in the GCM+ML+OC calculation.

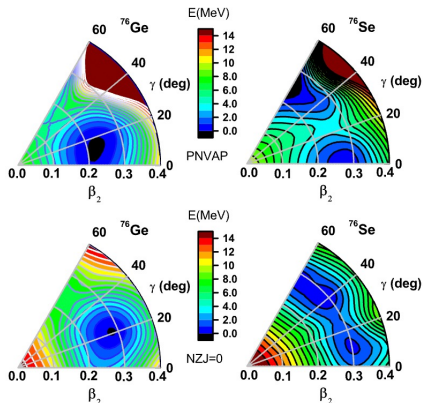


- Both the energy spectra and $M^{0\nu}$ of GCM reproduced by GCM+OC/ENTRO+ML.
- The ML algorithm speeds up the GCM calculation by a factor of 3-9 (axial case) for the energy spectra and NME.
- Expected to be more efficient with multi-coordinates.

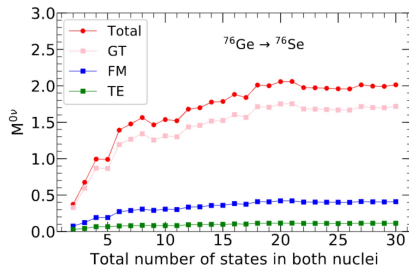


beta₂=0.1, 0.2, 0.3, 0.4; gamma=0, 10, 20,...,60

Energy surfaces by the chiral interaction EM1.8/2.0

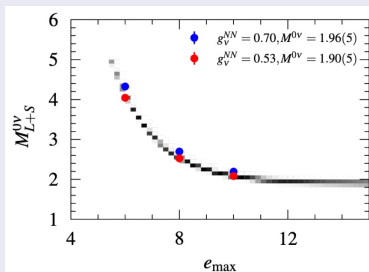


Convergence of the $M_L^{0\nu}$ ($e_{\text{Max}} = 08, \hbar\omega = 12$ MeV)

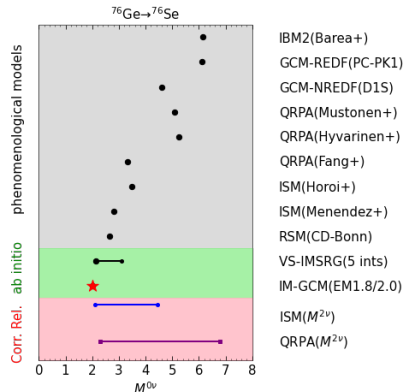


- The contribution of only the long-range operators is shown.

Extrapolation of the $M^{0\nu}$



- Uncertainty quantifications (chiral interactions, many-body approximation)



JMY et al. in preparation (2022)

- $0\nu\beta\beta$ decay provides a complementary way (oscillation exp, direct measurements, cosmological observations) to determine the absolute mass scale of neutrinos. Experimental searches of $0\nu\beta\beta$ decay are pushing up to tonne-scale detectors with a half-life sensitivity of up to 10^{28} years.
- The precision of the extracted neutrino mass depends on the NMEs which have the model uncertainty of a factor of up to three. A lot of efforts are devoted to reducing the discrepancy.
- Remarkable progress achieved in ab initio calculation of the NMEs of candidate nuclei. The contribution of the short-range operators turns out to be significant.
- The first-principle calculations of the NMEs for heavier candidate nuclei (^{76}Ge , ^{82}Se , ^{100}Mo , ^{130}Te , ^{136}Xe) starting from nuclear chiral potentials are in progress. Stay tuned!

Uncertainty quantification

- Uncertainty from chiral interactions (statistic and systematic errors, many-body currents, relativistic effect) and many-body truncations.
- Emulate the complicated nuclear model for the NMEs of $0\nu\beta\beta$ decay.

Contributions from other mechanisms

- The "master formula" for the $0\nu\beta\beta$ decay in EFT

$$\begin{aligned}
 [T_{1/2}^{0\nu}]^{-1} = & g_A^4 \left\{ G_{01} \left(|\mathcal{A}_\nu|^2 + |\mathcal{A}_R|^2 \right) \right. \\
 & + 2G_{04} |\mathcal{A}_{m_e}|^2 + 4G_{02} |\mathcal{A}_E|^2 + G_{09} |\mathcal{A}_M|^2 \\
 & - 2(G_{01} - G_{04}) \operatorname{Re} [\mathcal{A}_\nu^* \mathcal{A}_R] + 2G_{04} \operatorname{Re} [\mathcal{A}_{m_e}^* (\mathcal{A}_\nu + \mathcal{A}_R)] \\
 & - G_{03} \operatorname{Re} [(\mathcal{A}_\nu + \mathcal{A}_R) \mathcal{A}_E^* + 2\mathcal{A}_{m_e} \mathcal{A}_E^*] \\
 & \left. + G_{06} \operatorname{Re} [(\mathcal{A}_\nu - \mathcal{A}_R) \mathcal{A}_M^*] \right\}
 \end{aligned}$$

Collaborators

- SYSU: Chenrong Ding, Changfeng Jiao, Gang Li, Wei Lin, Xin Zhang
- PKU: Lingshuang Song, Jie Meng, Peter Ring
- LZU: Yifei Niu
- MSU: Scott Bogner, Heiko Hergert, Roland Wirth
- UNC: Jonathan Engel
- TRIUMF: Antonie Belly, Jason Holt
- TUD: Takayuki Miyagi
- NDU: Ragnar Stroberg
- UAM: Benjamin Bally, Tomas Rodriguez

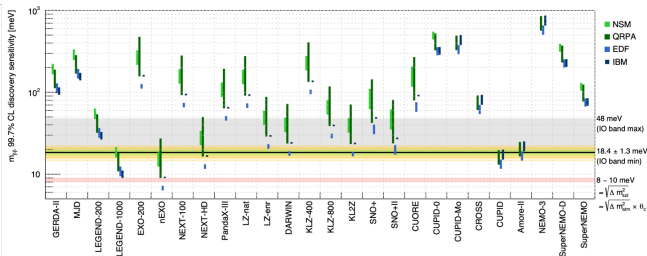
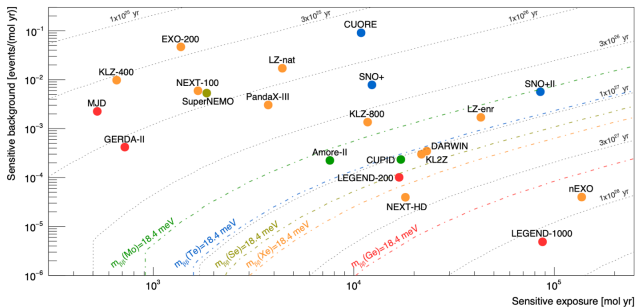
This work is partially supported by the National Natural Science Foundation of China and the Fundamental Research Funds for the Central Universities, Sun Yat-sen University.

Thank you for your attention!

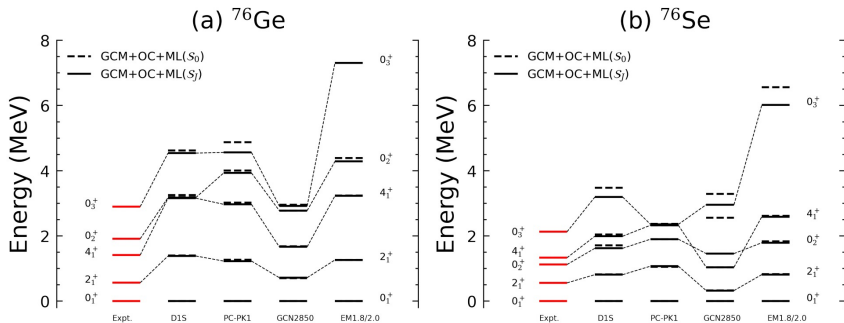


Backup slides

Current and next-generation experiments

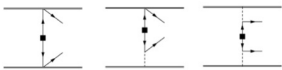
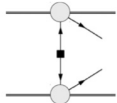
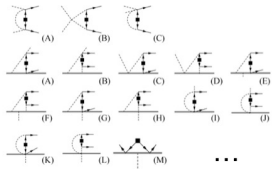
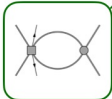
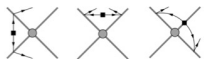


M. Agostini et al., arXiv:2202.01787v1



X. Zhang et al., in preparation (2022).

Standard mechanism for the $0\nu\beta\beta$ decay in (chiral) EFT

	Chiral EFT	Nuclear EFT (with pion integrated)
LO $\nu = 0$		
N ² LO $\nu = 2$		  <p>V. Cirigliano et al., PRL120, 202001 (2018) V. Cirigliano et al., PRC97, 065501 (2018)</p>

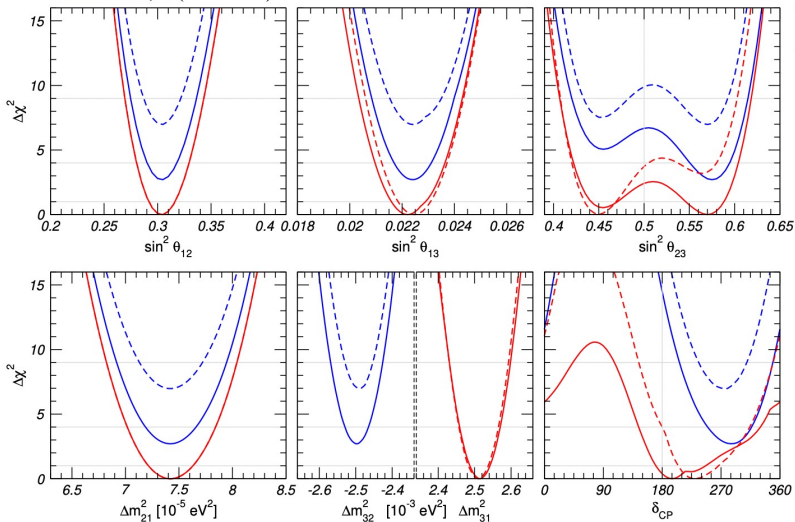
- Strategy: promoting the contact transition operator to the LO term.

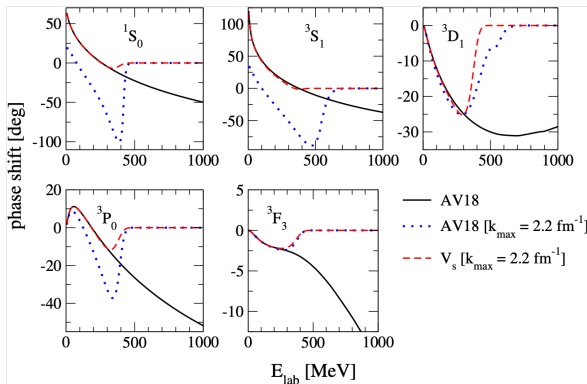
Global 6-parameter fit <http://www.nu-fit.org>

Esteban, Gonzalez-Garcia, Maltoni, Schwetz, Zhou, JHEP'20 [2007.14792]

— NO, IO (w/o SK-atm)
 - - NO, IO (with SK-atm)

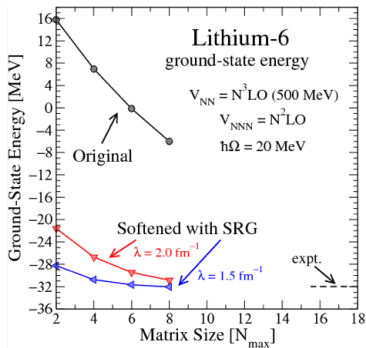
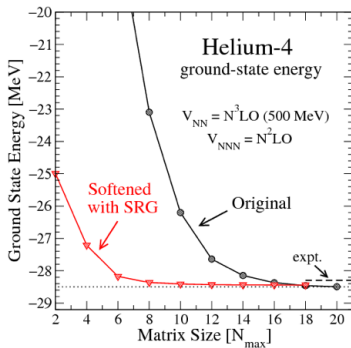
NuFIT 5.1 (2021)





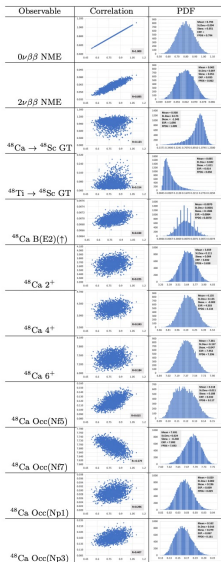
- The phase shifts from different treated AV18 interactions.
- The phase shifts are preserved in the SRG (on top of the black curves).

S. K. Bogner et al. (2007); D. Jurgenson et al. (2008)

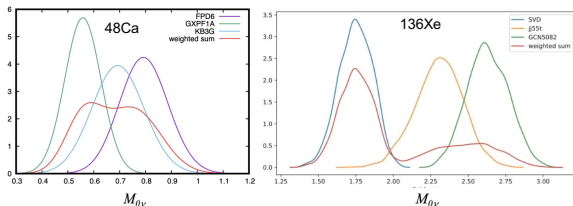


- The convergence of many-body (NCSM) calculations becomes faster using the SRG-softened nuclear force with a lower resolution parameter λ .

D. Jurgenson et al. (2009)



Statistic analysis within the shell models



M. Horoi, A. Neacsu, S. Stoica, arXiv:2203.10577 [nucl-th]

- Starting from three different shell-model Hamiltonians.
- Each of the two-body interaction matrix elements of the original shell-model Hamiltonian varies by 10%.
- Correlation between $M^{0\nu}$ and $M^{2\nu}$.

The End