Pion photoproduction off nucleon within Hamiltonian effective field theory

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7th workshop on chiral effective field theory 15/10/2022

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Introduction

Electromagnetic properties are important to disclose the structures of hadrons

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Photoproduction off nucleon will help to understand the properties and structures of nucleons and their resonances.

Hadron Physics

mainly focused on hadron scatterings, spectra, structures, interactions, etc.

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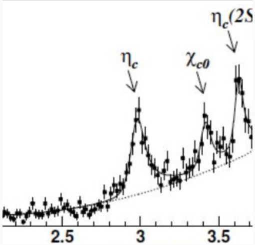

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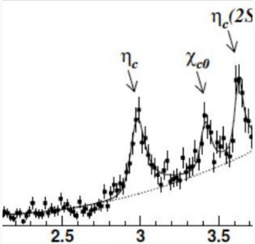

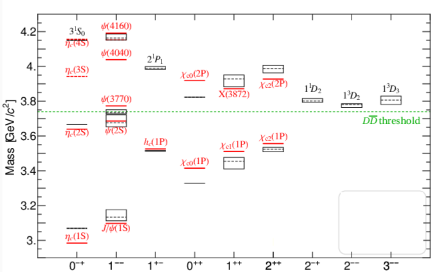

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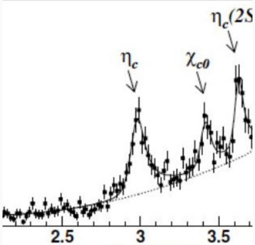

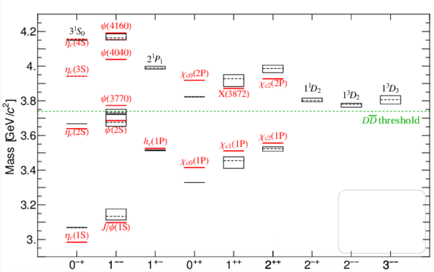

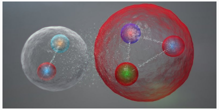

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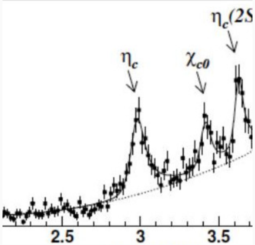

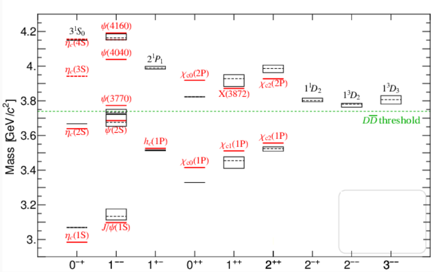

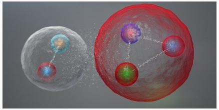

traditional perturbation expansion in series of $(\alpha_s)^n$?

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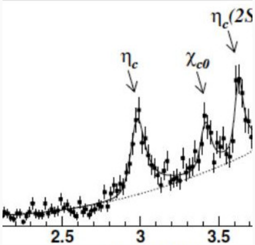

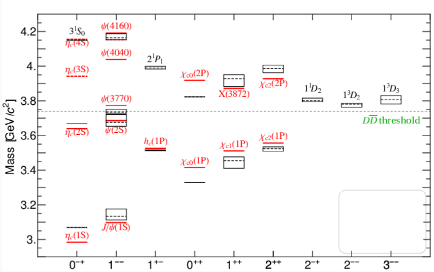

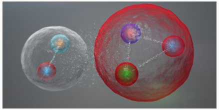

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- constituent quark model
- effective field theory
- lattice QCD
- QCD sum rule
- large Nc
- ...

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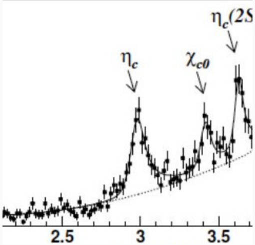

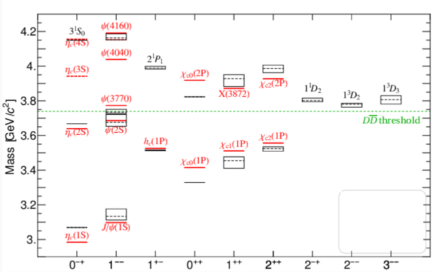

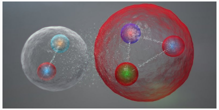

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Lattice QCD

- LQCD starts from the first principle of QCD
- model independent, reliable
- LQCD gives hadron spectra and quark distribution functions

  at finite volumes, large quark masses, discrete spaces
- not directly related to physical observables

Connection between Scattering Data and Lattice QCD Data

Lattice QCD

- large pion mass: extrapolation
- finite volume
- discrete space

Lattice QCD Data → Physical Data

- Lüscher Formalisms and extensions:
  Model independent; efficient in single-channel problems
    Spectrum → Phaseshifts;
- Effective Field Theory (EFT), Models, etc
  with low-energy constants fitted by Lattice QCD data

Physical Data → Lattice QCD Data

- EFT: discretization, analytic extension, Lagrangian modification
- various discretization: eg. discretize the momentum in the loop

Lattice QCD and Effective Field Theory

Effective field theory deals with extrapolation powerfully.

Finite-volume effect can be studied by discretizing the EFT.

Discrete spacing effects can also be studied with EFT.

Scattering Data and Lattice QCD data are two important sources for studying resonances.

We should try to analyse them both at the same time.

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Hamiltonian Effective Field Theory (HEFT)

analyses both experimental data at infinite volume

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- finite-volume and infinite-volume results are connected by the coupling constants etc.

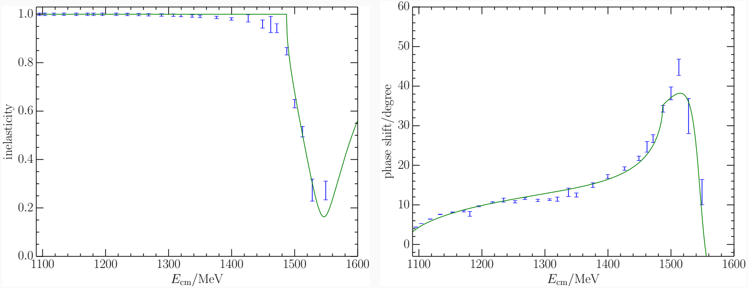
$N^*(1535)$ with πN Scattering

$N^*(1535)$ is the lowest resonance with $I(J^P) = \frac{1}{2}(\frac{1}{2}^-)$.

- One needs to consider the interactions

  among the bare baryon N_0^*, πN channel, and ηN channel.
- Phase shifts and inelasticities

  are obtained by solving Bethe-Salpeter equation with the interactions.

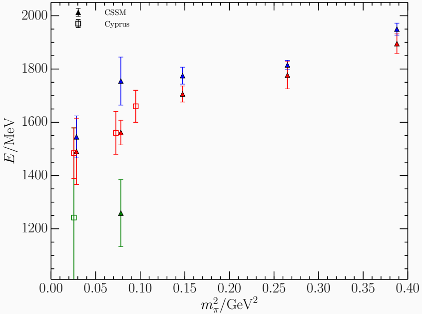


πN Scattering with $I(J^P) = \frac{1}{2}(\frac{1}{2}^-)$.

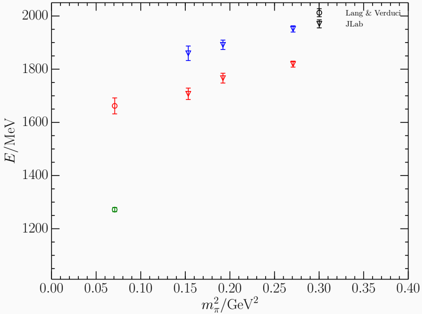
- Our Pole: $1531 \pm 29 -$ i 88 ± 2 MeV. Particle Data Group: $1510 \pm 20 -$ i 85 ± 40 MeV.

Spectra at Finite Volumes

3 sets of lattice data at different pion masses and finite volumes



$L \approx 3$ fm



$L \approx 2$ fm

Spectra with $I(J^P) = \frac{1}{2}(\frac{1}{2}^-)$ at finite volumes

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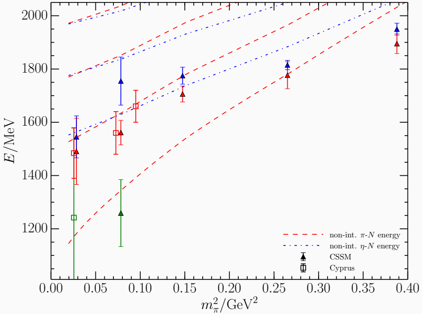
3 sets of lattice QCD data at different pion masses and finite volumes

Non-interacting energies of the two-particle channels



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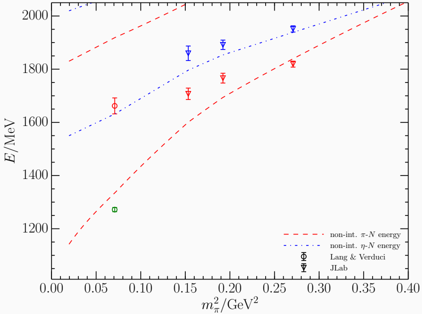

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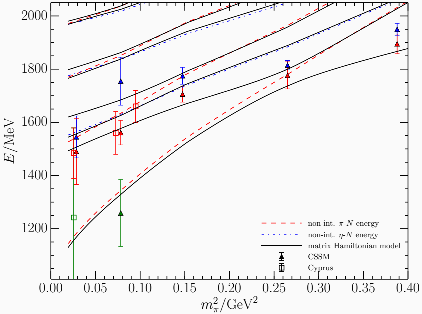
Eigenenergies of Hamiltonian effective field theory



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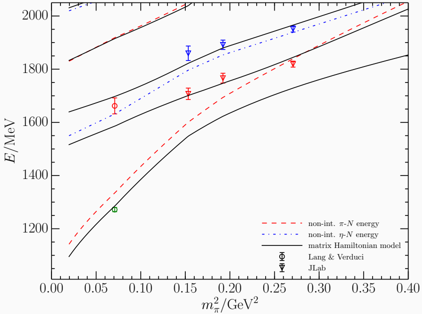

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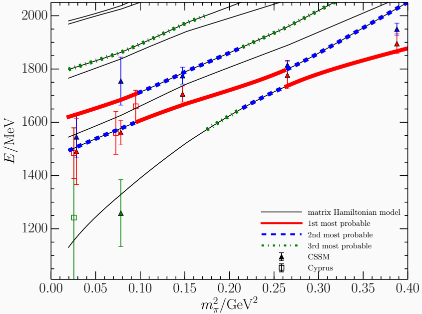
Eigenenergies of Hamiltonian effective field theory

Coloured lines indicating most probable states observed in LQCD



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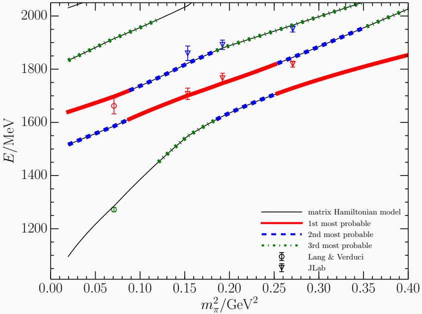


$L \approx 2$ fm

Spectra with $I(J^P) = \frac{1}{2}(\frac{1}{2}^-)$ at finite volumes

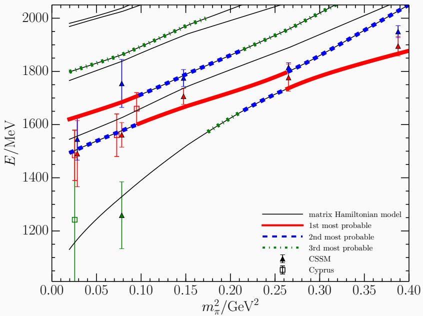
For more details, please see the following references:

Z. W. Liu, W. Kamleh, D. B. Leinweber, F. M. Stokes, A. W. Thomas and J. J. Wu, "Hamiltonian effective field theory study of the $\mathbf{N}^*(\mathbf{1535})$ resonance in lattice QCD," Phys. Rev. Lett. **116** (2016) no.8, 082004

Z. W. Liu, W. Kamleh, D. B. Leinweber, F. M. Stokes, A. W. Thomas and J. J. Wu, "Hamiltonian effective field theory study of the $\mathbf{N}^*(\mathbf{1440})$ resonance in lattice QCD," Phys. Rev. D **95** (2017) no.3, 034034

J. j. Wu, D. B. Leinweber, Z. w. Liu and A. W. Thomas, "Structure of the Roper Resonance from Lattice QCD Constraints," Phys. Rev. D **97** (2018) no.9, 094509

Components of Eigenstates with $L \approx 3$ fm

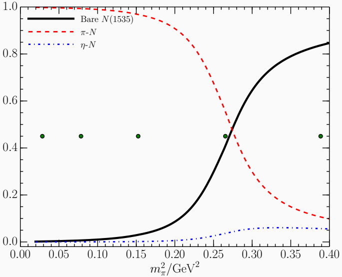


Spectra with $I(J^P) = \frac{1}{2}(\frac{1}{2}^-)$ and $L \approx 3$ fm

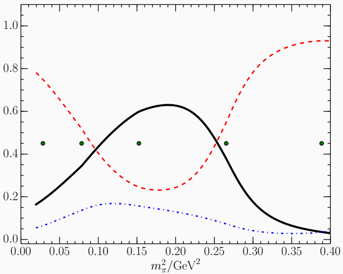
- The 1st eigenstate at light quark masses is mainly πN scattering states.
- The most probable state at physical quark mass is the 4th eigenstate.

  It contains about 60% bare $N^*(1535)$, 20% πN and 20% ηN.

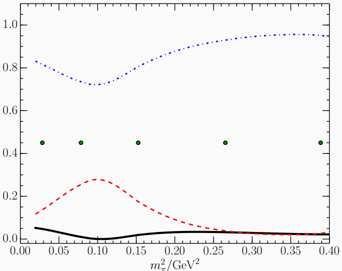
Components of Eigenstates with $L \approx 3$ fm

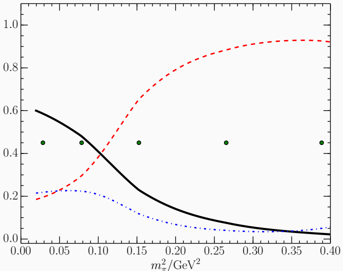


1st eigenstate



2nd eigenstate





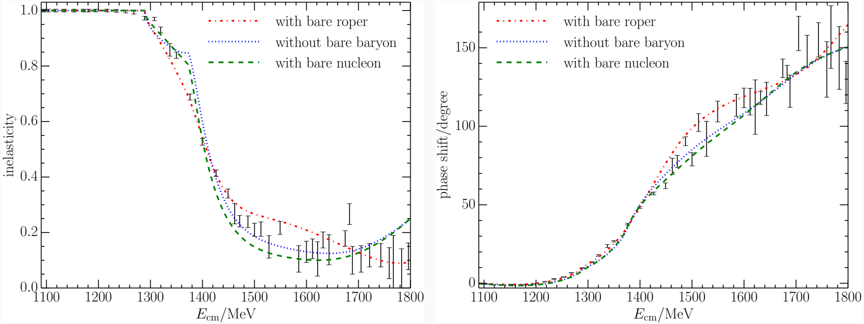
$N^*(1440)$ Resonance

- $N^*(1440)$, usually called Roper, is the excited state $I(J^P) = \frac{1}{2}(\frac{1}{2}^+)$
- Naive quark model predicts $m_{N^*(1440)} > m_{N^*(1535)}$
  if they are both dominated by 3-quark core. But contrary to experiment.

To check whether a 3-quark core largely exists in Roper, we consider models

- with a bare Roper
- without any bare baryons
- including the effect of the bare nucleon

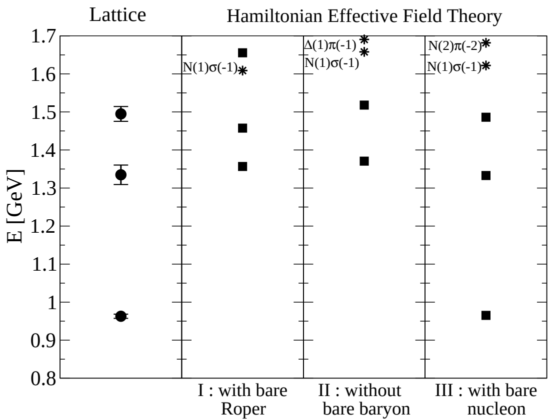
$N^*(1440)$ Resonance



πN scattering with $I(J^P) = \frac{1}{2}(\frac{1}{2}^+)$

- with a bare Roper
- without any bare baryons
- including the effect of the bare nucleon

An original figure from later lattice QCD work



interpolating operators: $N(0)$, $N(0)\sigma(0)$, $N(p)\pi(-p)$, $\Delta(p)\pi(-p)$.

from Lang, Leskovec, Padmanath, Prelovsek, PRD95 (2017) no.1, 014510.

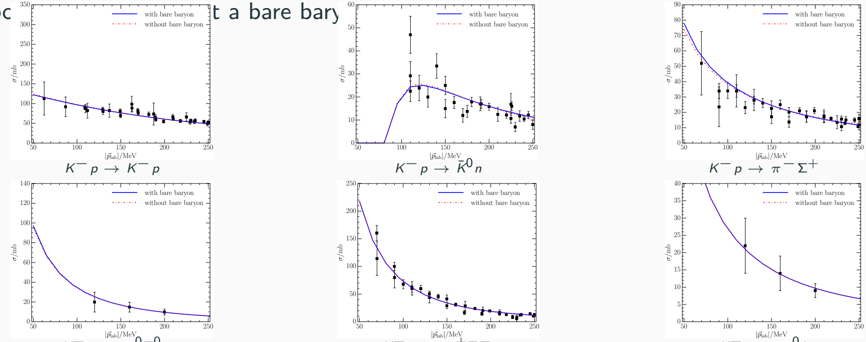
$\Lambda(1405)$ with $K^- p$ scattering

- The well-known Weinberg-Tomozawa potentials are used.

  momentum-dependent, non-separable

- We can fit the cross sections of $K^- p$ well

  bc … t a bare bary



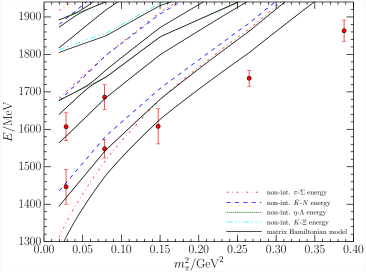
$K^- p \to K^- p$ $\quad$ $K^- p \to \bar{K}^0 n$ $\quad$ $K^- p \to \pi^- \Sigma^+$

$K^- p \to \pi^0 \Sigma^0$ $\quad$ $K^- p \to \pi^+ \Sigma^-$ $\quad$ $K^- p \to \pi^0 \Lambda$

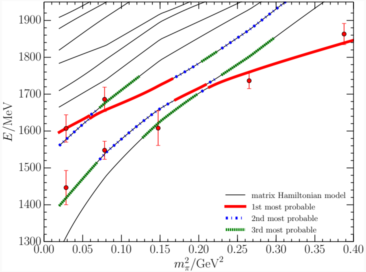
- Two-pole structure of $\Lambda(1405)$

  $1430 - i22$ MeV, $\quad 1338 - i89$ MeV

Spectrum on the Lattice



without a bare baryon



with a bare baryon

Spectra with $S = -1, I(J^P) = 0(\frac{1}{2}^-)$ in the finite volume.

- The bare baryon is important for interpreting the lattice QCD data at large pion masses.
- $\Lambda(1405)$ is mainly a $\bar{K}N$ molecular state

  containing very little of bare baryon at physical pion mass.

Z. W. Liu, J. M. M. Hall, D. B. Leinweber, A. W. Thomas and J. J. Wu, Phys. Rev. D **95** 014506

ALICE Collaboration @ LHC have verified our $K^- p$ scattering length

Physics Letters B 822 (2021) 136708

Contents lists available at ScienceDirect

Physics Letters B

2366–2382.

[77] Z.-W. Liu, J.-J. Wu, D.B. Leinweber, A.W. Thomas, Kaonic hydrogen and deuterium in Hamiltonian effective field theory, Phys. Lett. B 808 (2020) 135652, arXiv:2003.09181.

Kaon–proton strong interaction at low relative momentum via femtoscopy in Pb–Pb collisions at the LHC

ALICE Collaboration *

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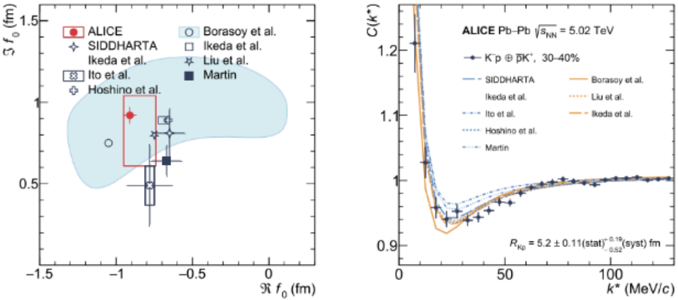
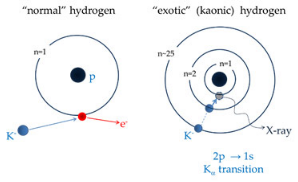


Fig. 3. Left: scattering parameters obtained from the Lednický–Lyuboshitz fit compared with available world data and theoretical calculations. Statistical uncertainties are represented as bars and systematic uncertainties, if provided, as boxes. Right: experimental femtoscopic correlation function for $K^-p \oplus K^+\bar{p}$ pairs in the 30–40% centrality interval, together with various Lednický–Lyuboshitz calculations obtained using the scattering length parameters from Refs. [17,18,75–79] and the source radius from this analysis. The statistical and systematic uncertainties of the measured data points are added in quadrature and shown as vertical bars.

Mesonic Atoms

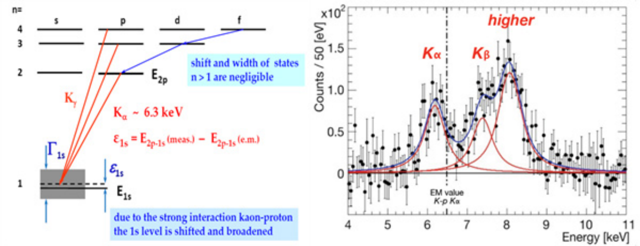


Experimental progresses

- pionic hydrogen and deuterium,
  the Paul Scherrer Institute (PSI), Ref[Hauser:1998yd]
- kaonic hydrogen, SIDDHARTA-2, Ref[Curceanu:2013bxa]
- kaonic deuterium, proposed by SIDDHARTA-2 and the J-PARC E57

Kaonic Hydrogen

energy shift and width of 1s level were measured at SIDDHARTA-2



$$
\begin{aligned}
\epsilon_{1s}^{p} &= 283 \pm 36(\mathrm{stat}) \pm 6(\mathrm{sys})\ \mathrm{eV}, \\
\Gamma_{1s}^{p} &= 541 \pm 89(\mathrm{stat}) \pm 22(\mathrm{sys})\ \mathrm{eV}\,,
\end{aligned}
$$

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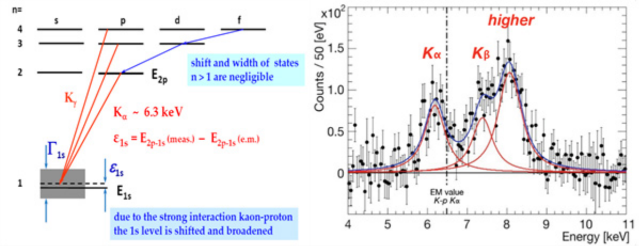

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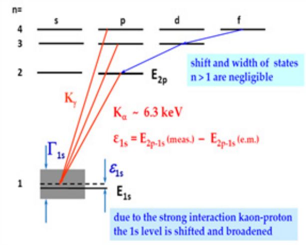
\end{aligned}
$$

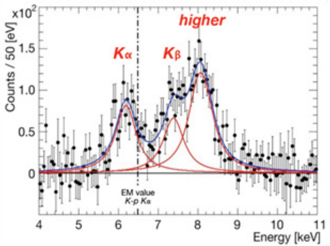
- they are related to the scattering length of $K^- p$

$$
\begin{aligned}
&\epsilon^p_{1S} - \frac{i}{2}\Gamma^p_{1S} \\
&= \frac{-2\alpha_e^3\,\mu^2_{K^-p}\,a_{K^-p}}{1 + 2\alpha_e\,\mu_{K^-p}\,(\ln\alpha_e - 1)\,a_{K^-p}}\,,
\end{aligned}
$$

Kaonic Hydrogen

energy shift and width of 1s level were measured at SIDDHARTA-2





$$
\begin{aligned}
\epsilon_{1S}^{p} &= 283 \pm 36(\mathrm{stat}) \pm 6(\mathrm{sys})\ \mathrm{eV}, \\
\Gamma_{1S}^{p} &= 541 \pm 89(\mathrm{stat}) \pm 22(\mathrm{sys})\ \mathrm{eV}\,,
\end{aligned}
$$

- they are related to the scattering length of $K^- p$

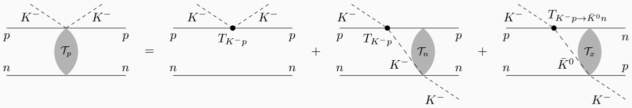
$$
\epsilon_{1S}^{p} - \frac{i}{2}\Gamma_{1S}^{p} = \frac{-2\alpha_e^3\,\mu_{K^-p}^2\,a_{K^-p}}{1 + 2\alpha_e\,\mu_{K^-p}\,(\ln\alpha_e - 1)\,a_{K^-p}}\,,
$$

- With $\bar{K}N$ interactions not fine tuned, HEFT provides

$$
\epsilon_{1S}^{p} = 307\ \mathrm{eV}, \qquad \Gamma_{1S}^{p} = 533\ \mathrm{eV}\,.
$$

Kaonic Deuteron without Recoil Effect

$\bar{K}NN$ scattering amplitude can be solved by the Faddeev equation



$, \dots$

With the static approximation,

$$a_{K^-d} = \frac{m_d}{m_K + m_d} \int d^3\vec{r}\, |\psi_d(\vec{r})|^2 \; \hat{A}_{K^-d}(r)\,,$$

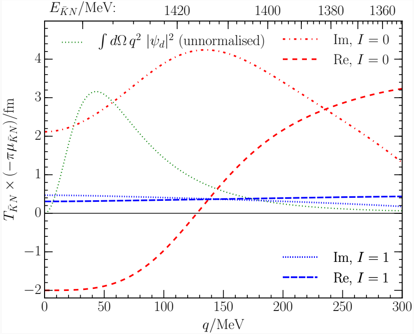
where

$$\hat{A}_{K^-d}(r) = \frac{\tilde{a}_{K^-p} + \tilde{a}_{K^-n} + (2\tilde{a}_{K^-p}\tilde{a}_{K^-n} - b_x^2)/r - 2b_x^2\tilde{a}_{K^-n}/r^2}{1 - \tilde{a}_{K^-p}\tilde{a}_{K^-n}/r^2 + b_x^2\tilde{a}_{K^-n}/r^3}\,.$$

Our results without recoil effect are similar to others

$$\epsilon_{1S}^d\big|_{\mathrm{StaticApprox}} = 855\ \mathrm{eV}, \quad \Gamma_{1S}^d\big|_{\mathrm{StaticApprox}} = 1127\ \mathrm{eV}\,.$$

Recoil Effect



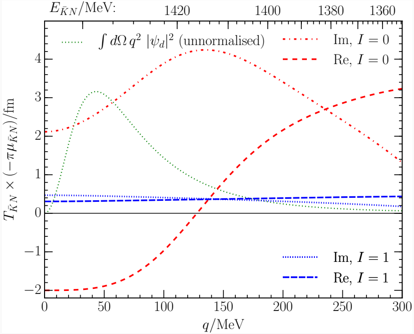
- The recoil effect is mainly from the single scattering process

$$\langle\, T^d_{\bar{K}N} \,\rangle \equiv \int d^3\vec{q}\,|\psi_d(\vec{q})|^2 \;\; T_{\bar{K}N}(\vec{q}).$$

- If no $\Lambda(1405)$ exists,

  this kind of recoil effect can be totally neglected.

Recoil Effect



Z. W. Liu, J. J. Wu, D. B. Leinweber and A. W. Thomas, Phys. Lett. B **808** (2020), 135652

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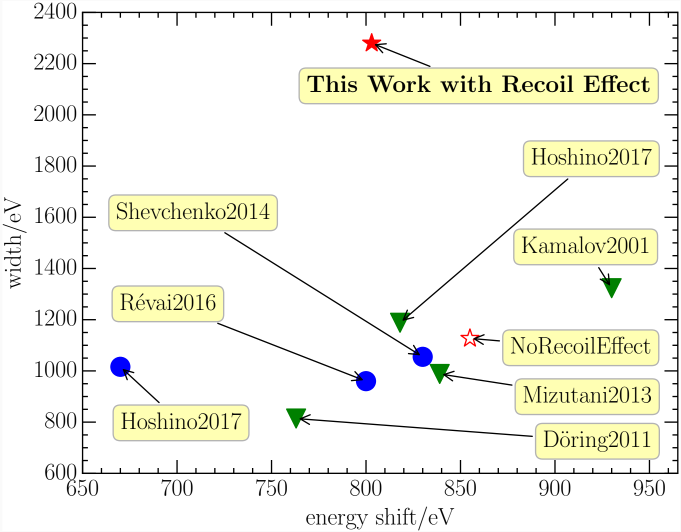
Comparison



This Work with Recoil Effect
Hoshino2017
Shevchenko2014
Kamalov2001
Révai2016
NoRecoilEffect
Mizutani2013
Hoshino2017
Döring2011
width/eV
energy shift/eV
2400
2200
2000
1800
1600
1400
1200
1000
800
600
650
700
750
800
850
900
950

Pion Photoproduction off Nucleon with Hamiltonian EFT

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- combining
  - $\pi N \to \pi N$
  - lattice QCD data
  - $\gamma + N \to \pi + N$

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$$\begin{aligned}\mathcal{M}(\gamma N \to \pi N) \quad \sim \quad & \mathcal{M}^{\mathrm{EM}}(\gamma N \to \pi N) \\ & + \mathcal{M}^{\mathrm{EM}}(\gamma N \to \pi N) \otimes \mathcal{M}^{\mathrm{FSI}}(\pi N \to \pi N) \\ & + \mathcal{M}^{\mathrm{EM}}(\gamma N \to \eta N) \otimes \mathcal{M}^{\mathrm{FSI}}(\eta N \to \pi N)\end{aligned}$$

Pion Photoproduction off Nucleon with Hamiltonian EFT

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  - understand the structure of $N(1535)$ and the interactions of $\pi N/\eta N$ at low energies and near the resonance

Pion Photoproduction off Nucleon with Hamiltonian EFT

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  - understand the structure of $N(1535)$ and the interactions of $\pi N/\eta N$ at low energies and near the resonance
  - necessities for the photon-nucleus investigation

Electromagnetic Multipoles

- $|\gamma N\rangle \to |\phi\,(\vec{k}),\ N(-\vec{k}, s_z'^N)\rangle,$ $k_x, k_y, k_z, s_z'^N$
- $|\gamma N\rangle \to |\phi\, N; k, J, J_z, L\rangle$, $k,\ \ J,\ \ J_z,\ \ L$
- $|\gamma N\rangle \to |\phi\, N; k, J, J_z, \lambda_N'\rangle$, $k,\ \ J,\ \ J_z, \lambda_N'$

Electromagnetic Multipoles

- $|\gamma N\rangle \to |\phi\,(\vec{k}),\ N(-\vec{k}, s_z'^N)\rangle$, $\quad k_x, k_y, k_z, s_z'^N$
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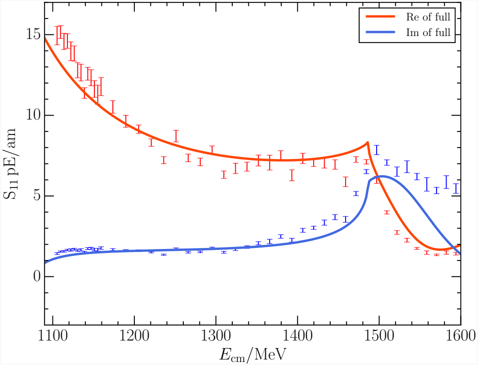
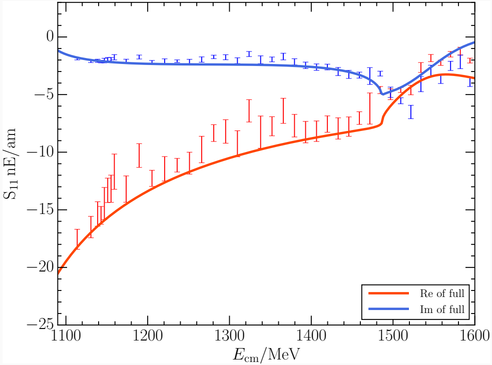
$$V_{\alpha,\gamma N}(J, \lambda_N', \lambda_\gamma, \lambda_N; k, q) = 2\pi \int_{-1}^{1} \mathrm{d}(\cos\theta) \sum_{s_z'^N} d^J_{\lambda_\gamma - \lambda_N, -\lambda_N'}(\theta) d^{1/2}_{s_z'^N, -\lambda_N'}(\theta)^* \mathcal{M}_{\alpha,\gamma N}(s_z'^N, \lambda_N, \lambda_\gamma; \vec{k}, \vec{q}),$$

$$V^{JLS;\lambda_\gamma\lambda_N}_{\alpha,\gamma N}(k,q) = \sqrt{\frac{2L+1}{2J+1}} \sum_{\lambda_N'} \langle L, S, 0, -\lambda_N' | J, -\lambda_N'\rangle \times V_{\alpha,\gamma N}(J, \lambda_N', \lambda_\gamma, \lambda_N; k, q).$$

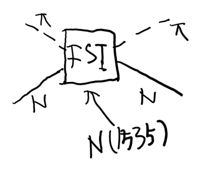
D. Guo and Z. W. Liu, Phys. Rev. D **105** (2022) no.11, 11

Numerical results

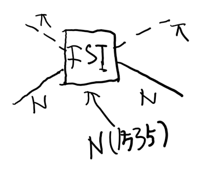


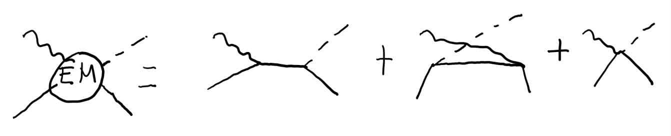


The bare triquark core in $N^*(1535)$



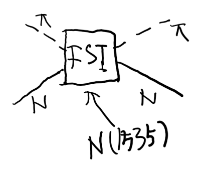
The bare triquark core in $N^*(1535)$

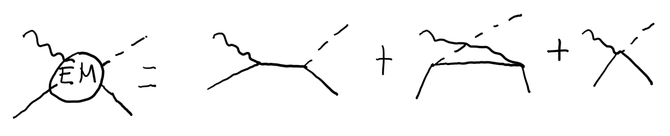




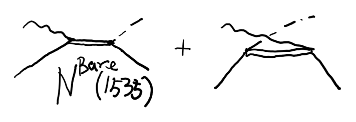
$+\dots$

The bare triquark core in $N^*(1535)$

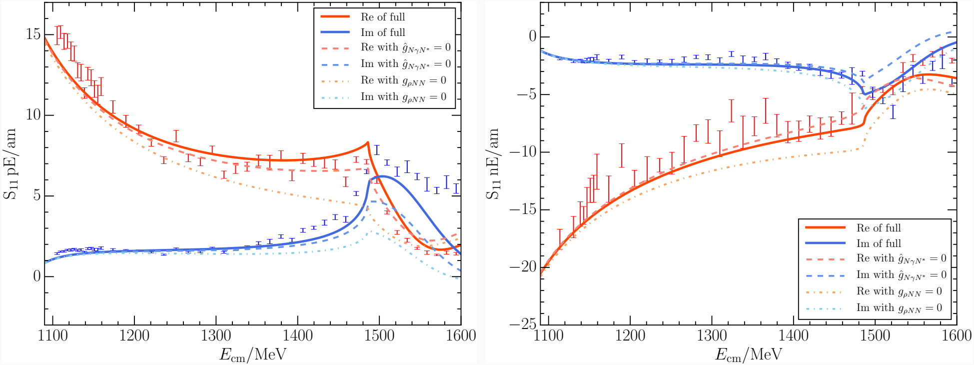




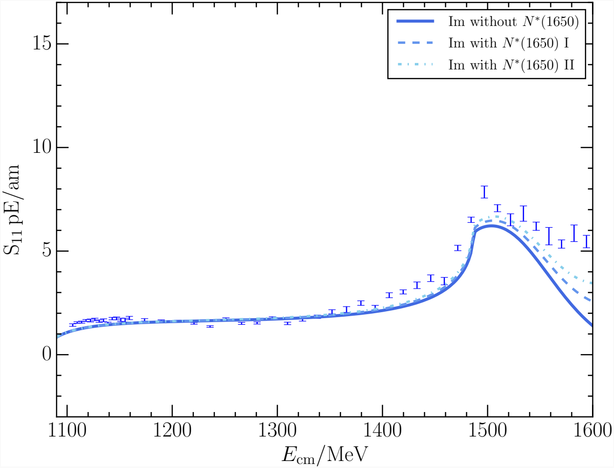
$+\ldots$



The bare triquark core in $N^*(1535)$ cannot be absent in pion photoproduction



Estimation of the $N^*(1650)$ contribution



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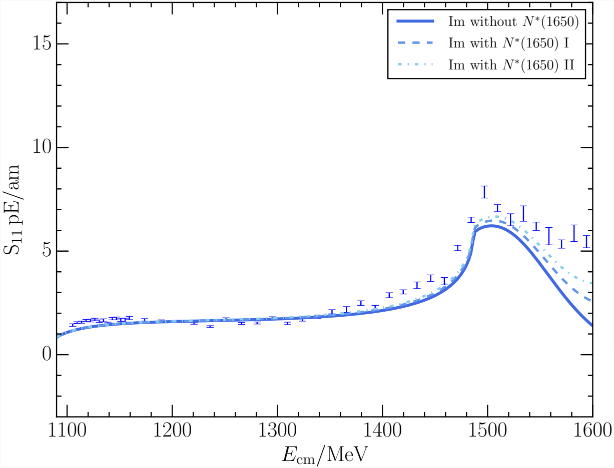

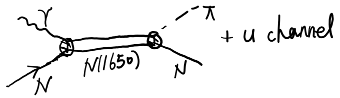

Summary

Summary

In this report, I have briefly discussed

- the low-lying baryons and Kaonic Atoms with Hamiltonian EFT
- Pion Photoproduction off Nucleon with Hamiltonian EFT

Thanks for your attentions!