Study of the X(6900): from partial wave decomposition

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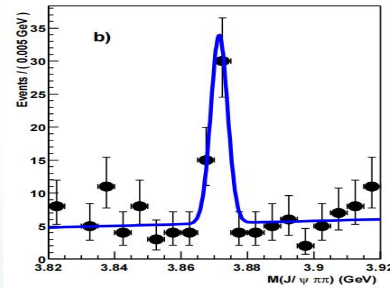
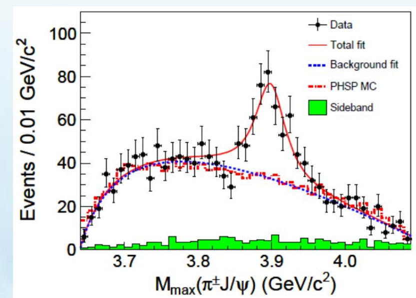
Outlines

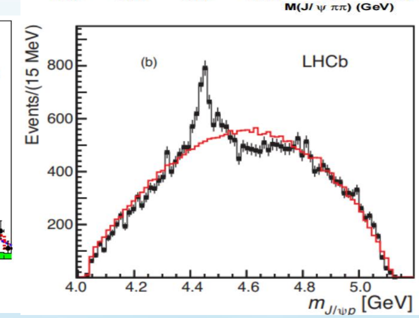
1. Introduction
2. Formalism
3. Nature of the X(6900)
4. Summary

Introduction

- X(3872) by Belle, in 2003
- Z+c(3900) by BESIII and Belle, in 2013
- Pc states by LHCb, in2015
- ……
- Their nature?
  - Quantum number?
  - The inner structure?





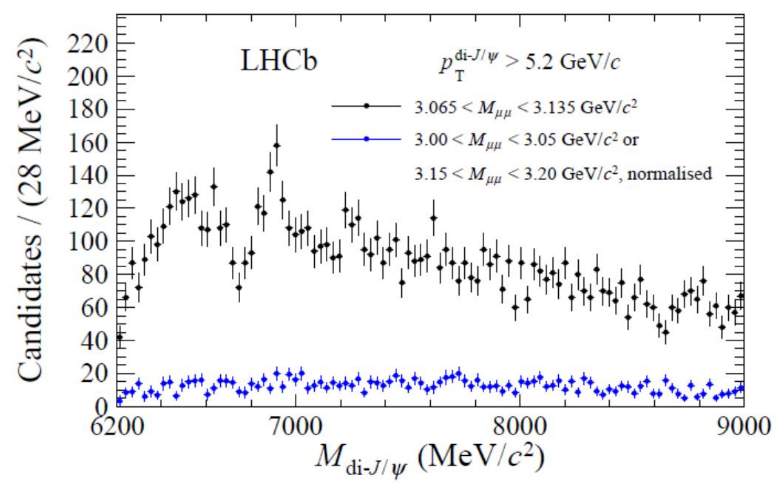


Amplitude analysis

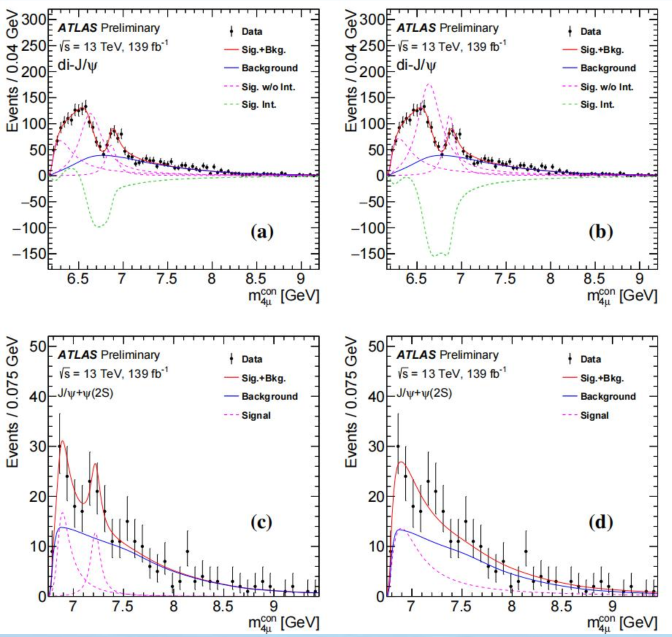
- Only one physical amplitude!
- It should satisfy the fundamental QFT principles
- It should be compatible with the data

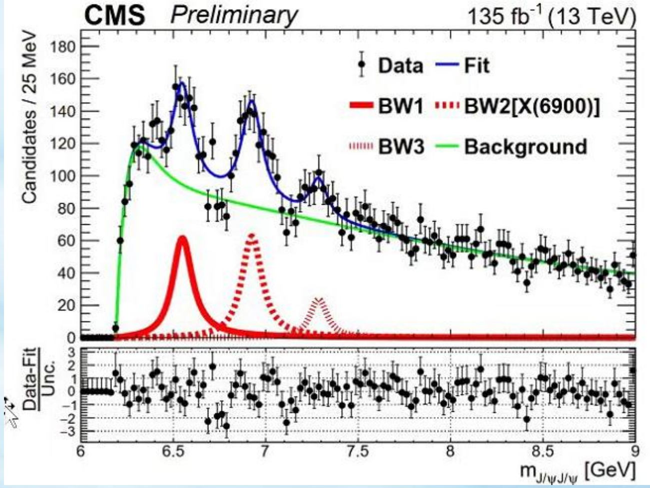
X(6900)

- Fully heavy tetra-quark state(s)?



LHCb, Sci.Bull. 65 (2020) 23, 1983



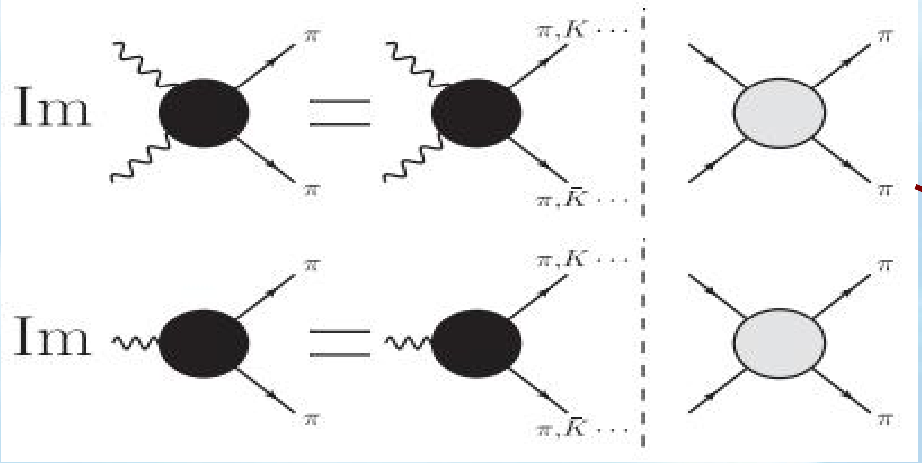


ATLAS, ATLAS-CONF-2022-040

CMS, Kai Yi’s talk at ICHEP 2022.

why FSI ?

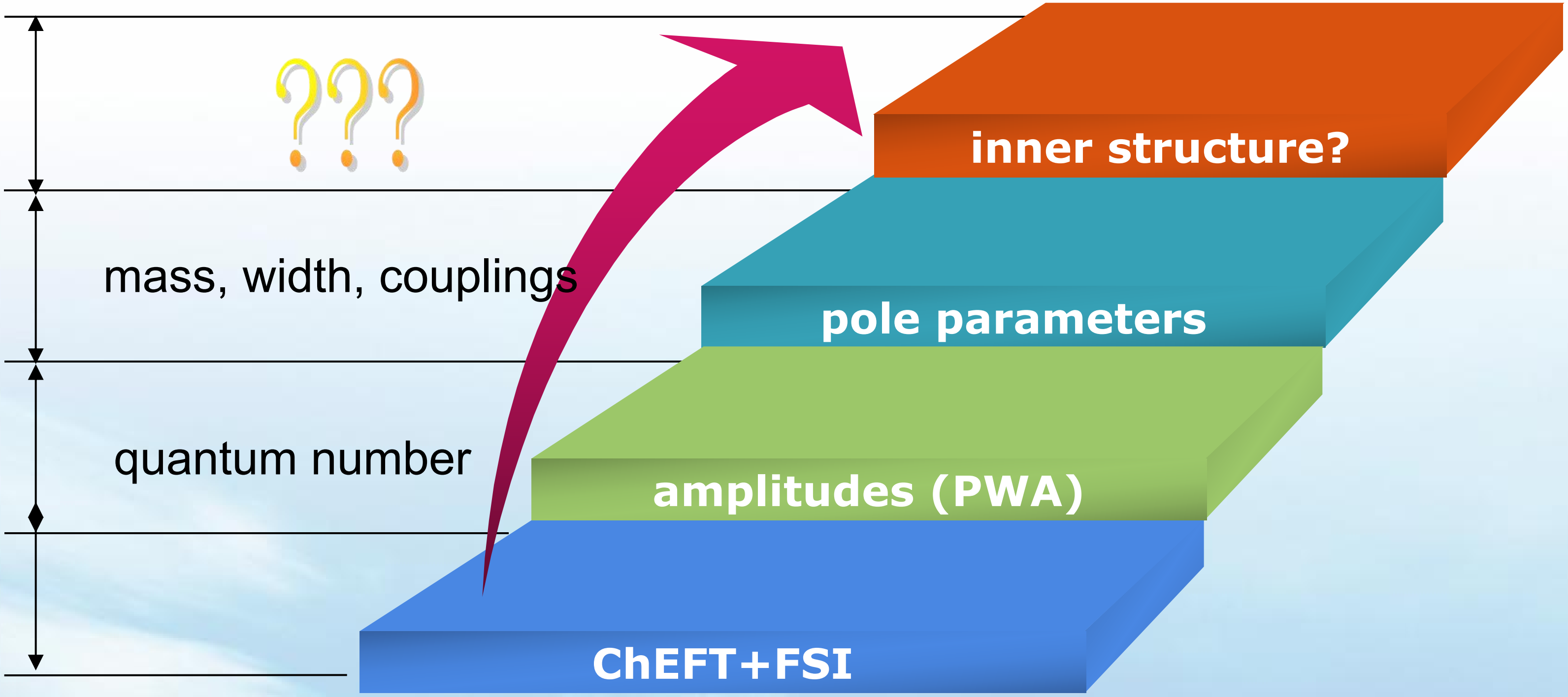
- FSI needs to be taken into account to perform an amplitude analysis
- Methods: KM, N/D, AMP, Roy equation, PKU, Pade, LSE, BSE, ChEFT, *et.al.*
- Fixed scattering, decaying amplitudes: extracting resonance information



Yao, Dai, Zheng, Zhou, RPP84(2021)076201

Inner structure?

- Interpretation these amplitudes, poles?



Pole counting rule

- Pole counting rule: distinguish molecule and BW resonance.
- At the very beginning, applied to light mesons

Morgan NPA543 (1992) 632.
Dai,Wang and Zheng
CTP57 (2012) 841, CTP58 (2012) 410

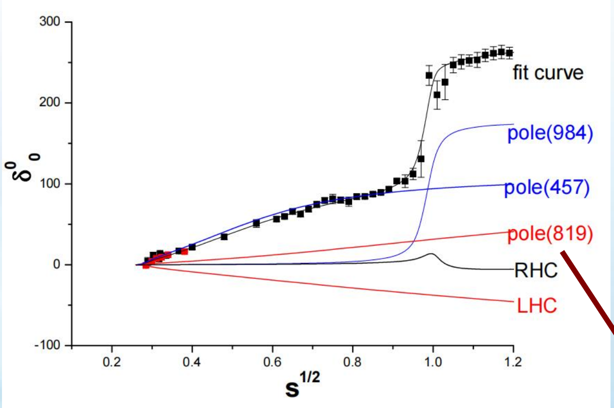
- Applied to heavier mesons

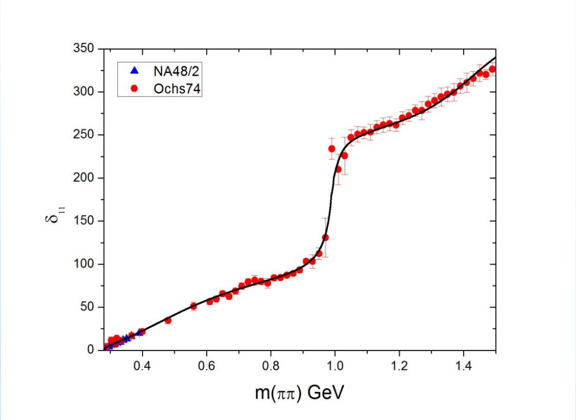
Dai, Sun, Kang, Szczepaniak, Yu,
PRD 105 (2022) 5, L051507
Kuang,**Dai**,Kang,Yao,
EPJC 80 (2020) 5, 433
Dai, Shi, Tang, Zheng,
PRD 92 (2015) 1, 014020

- Any tools else?

phase shifts?

- Phase shifts help to study hadronic scatterings and resonances therein
- Successful in study of light scalars: PKU,Roy equation, Omnes representations, etc.





Zhou, Qin, Zhang, Xiao, Zheng,JHEP 02 (2005)

Dai,Pennington, PLB736(2014)11; PRD90 (2014) 036004;

- The phase shifts of a narrow BW resonance rise from 0 to 180 degrees, crossing 90 degrees at E=M

2、Formalism

- coupled channels: J/ψJ/ψ、ψ(2s)J/ψ、ψ(3770)J/ψ

	J/ψ	$\psi(2s)$	$\psi(3770)$	η_c	h_c	χ_{c0}	χ_{c1}	χ_{c2}
J^{PC}	1^{--}	1^{--}	1^{--}	0^{-+}	1^{+-}	0^{++}	1^{++}	2^{++}
质量 (MeV)	3096.9	3686.1	3773.7	2983.9	3525.4	3414.7	3510.7	3556.2

- $\eta_c\eta_c$、h_ch_c channels are supressed by HQS

- Energy region: [6.2,7.2] GeV

Dong, Baru, Guo, Hanhart, Nefediev, PRL126, 132001(2021)
Gong, Du, Zhao, PRD, 106(5):054011, 2022

Effective Lagrangians

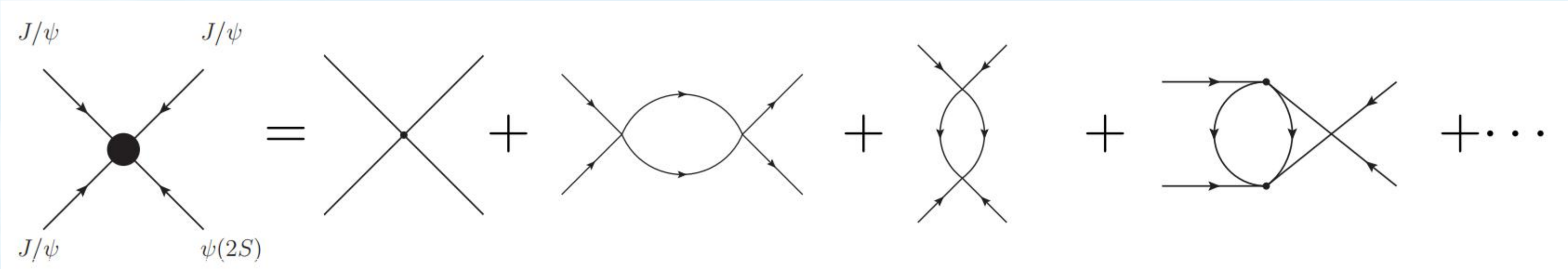
- Four vector interactions, similar to HGLS formalism

Geng, Molina, Oset. CPC41(12):124101, 2017.

$$\mathcal{L} = c_1 V_\mu V_\alpha V^\mu V^\alpha + c_2 V_\mu V_\alpha V^\mu V'^\alpha + c_3 V_\mu V'_\alpha V^\mu V'^\alpha + c_4 V_\mu V'^\mu V_\alpha V'^\alpha + c_5 V_\mu V_\alpha V^\mu V''^\alpha + c_6 V_\mu V''_\alpha V^\mu V''^\alpha + c_7 V_\mu V''^\mu V_\alpha V''^\alpha + c_8 V_\mu V'_\alpha V^\mu V''^\alpha + c_9 V_\mu V'^\mu V_\alpha V''^\alpha$$

Zhou, Guo, Kuang, Yang, Dai, arxiv:2207.07537 [hep-ph]

- Feynman diagrams up to NLO



Building amplitudes

- PWA

$$T^{ij}_{\mu_1\mu_2;\mu_3\mu_4}(s,z_s) = 16\pi N_{ij}\sum_J (2J+1) T^{J,ij}_{\mu_1\mu_2;\mu_3\mu_4}(s) d^J_{\mu\mu'}(z_s)$$

- Time reversal and parity invariance reduce the amplitudes
- Tranfer amplitudes from helicity to the JMLS representation

$$T^{ij}_{\mu_1\mu_2;\mu_3\mu_4}(s,z_s) = 16\pi N_{ij}\sum_{J=M}^{\infty}(2J+1)d^J_{\mu\mu'}(z_s)\sum_{LS,L'S'}\frac{\sqrt{(2L+1)(2L'+1)}}{2J+1}\langle LS0\mu|J\mu\rangle\langle J\mu'|L'S'0\mu'\rangle \langle s_1 s_2 \mu_1,-\mu_2|S\mu\rangle\langle S'\mu'|s_3 s_4\mu_3,-\mu_4\rangle T^{J,ij}_{LS,L'S'}\,.$$

$$\begin{aligned} T^{ij}_{^3P_2}(s) = {} & \frac{2}{5}T^2_{++++}(s) + \frac{\sqrt{3}}{5}T^2_{+++0}(s) + \frac{\sqrt{3}}{5}T^2_{+0++}(s) + \frac{\sqrt{3}}{5}T^2_{++0+}(s) + \frac{\sqrt{3}}{5}T^2_{0+++}(s) - \frac{2}{5}T^2_{++--}(s) \\ & - \frac{\sqrt{3}}{5}T^2_{++-0}(s) - \frac{\sqrt{3}}{5}T^2_{-0++}(s) - \frac{\sqrt{3}}{5}T^2_{++0-}(s) - \frac{\sqrt{3}}{5}T^2_{0-++}(s) + \frac{3}{10}T^2_{+0+0}(s) + \frac{3}{10}T^2_{+00+}(s) \\ & + \frac{3}{10}T^2_{0++0}(s) + \frac{3}{10}T^2_{0+0+}(s) - \frac{3}{10}T^2_{0+0-}(s) - \frac{3}{10}T^2_{+00-}(s) - \frac{3}{10}T^2_{0-+0}(s) - \frac{3}{10}T^2_{-0+0}(s)\,. \end{aligned}$$

PW amplitudes

- Five PWs at last

L	$S=0$	$S=1$	$S=2$
0	$\mathbf{0^{++}}$ $(\mathbf{^1S_0})$	1^{+-}	$\mathbf{2^{++}}$ $(\mathbf{^5S_2})$
1	1^{--}	$\mathbf{0^{-+}}$ $(\mathbf{^3P_0})$ $\mathbf{1^{-+}}$ $(\mathbf{^3P_1})$ $\mathbf{2^{-+}}$ $(\mathbf{^3P_2})$	1^{--} 2^{--} 3^{--}
2	2^{++}	1^{+-} 2^{+-} 3^{+-}	0^{++} 1^{++} 2^{++} 3^{++} 4^{++}
$\vdots$	$\vdots$	$\vdots$	$\vdots$

- Ignoring PWs with L>=2

- No S-D coupled channel scatterings

Unitarization

- unitarity: ignore the S-D coupled waves

$$\langle L'S'|T^J - T^{J\dagger}|LS\rangle = i\frac{4|\vec{p}\,''|}{E''_{cm}} \sum_{L''S''} \langle L'S'|T^{J\dagger}|L''S''\rangle \langle L''S''|T^J|LS\rangle$$

$$\mathrm{Im}T^{ij}_{JLS} = \sum_{k=1}^{a} T^{ik}_{JLS}\,\rho_k\,T^{kj\,*}_{JLS}$$

- Pade approximation: successful in confirming existence of light scalars, restore unitarity and perturbative amplitudes up to NLO

$$T = T^{LO}\cdot[T^{LO} - T^{NLO}]^{-1}\cdot T^{LO}$$

Dai,Wang and Zheng
CTP57 (2012) 841, CTP58 (2012) 410

Invariant mass spectra

- Each channel contributes a ratio α_i

$$\frac{d\ \text{Events}^{1,2}}{d\sqrt{s}} = \tilde{N}_{1,2}\ p_{cm}(s) \sum_{\mu_1\mu_2\mu_3\mu_4} \int_{-1}^{1} dz_s \Big|\sum_{i=1}^{a} \alpha_i T^{i1,i2}_{\mu_1\mu_2\mu_3\mu_4}(s,z_s)\Big|^2$$

- Into the forms of PWs

$$\sum_{\mu_1\mu_2\mu_3\mu_4} \int_{-1}^{1} \Big|\sum_{i=1}^{a} \alpha_i T^{i1,i2}_{\mu_1\mu_2\mu_3\mu_4}(s,z_s)\Big|^2 dz_s = 512\pi^2 \left[\ |F^{1,2}_{{}^1S_0}(s)|^2 + 5|F^{1,2}_{{}^5S_2}(s)|^2 + |F^{1,2}_{{}^3P_0}(s)|^2 + 3|F^{1,2}_{{}^3P_1}(s)|^2 + 5|F^{1,2}_{{}^3P_2}(s)|^2\ \right]$$

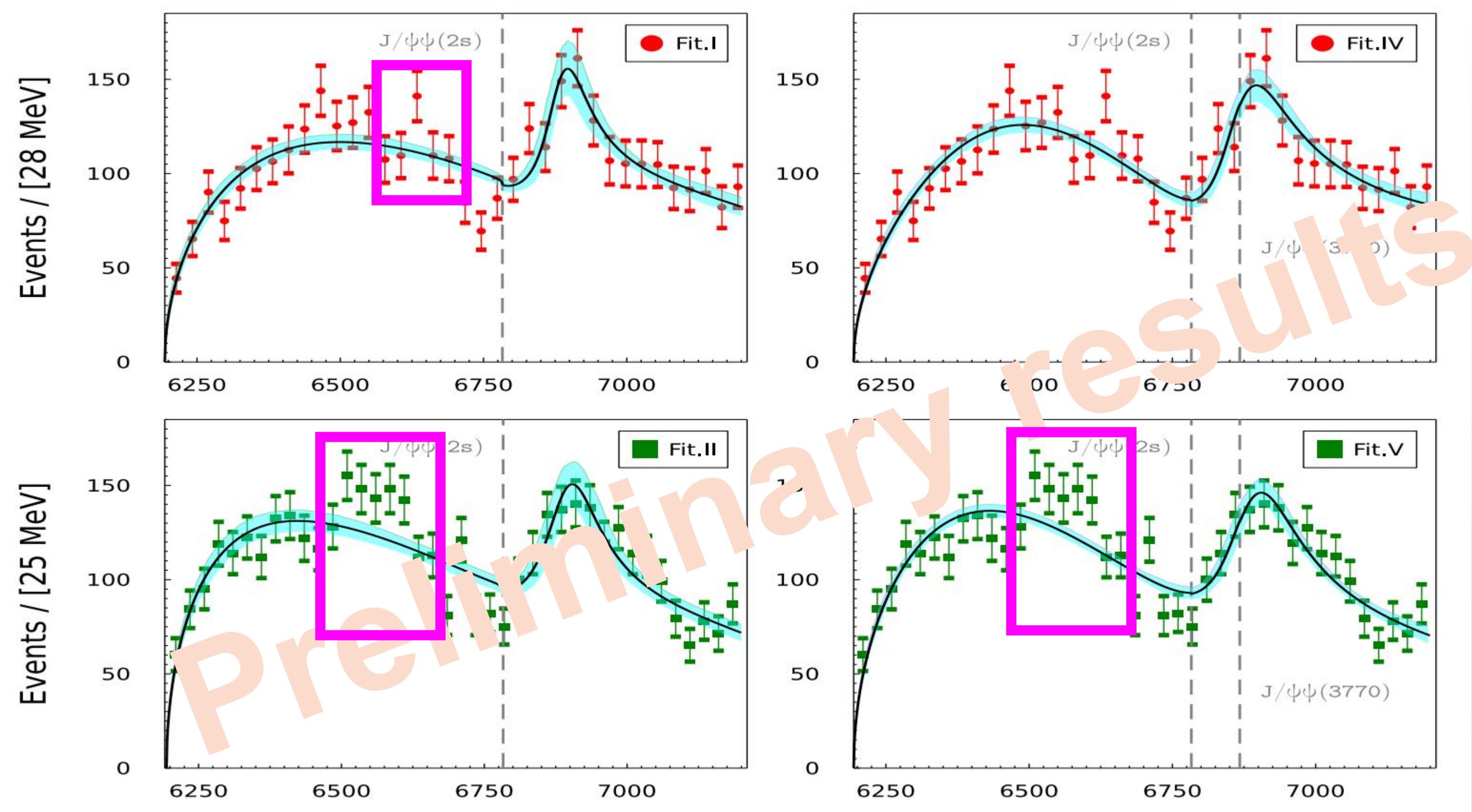
- Compatible with AMP method
  - absorbing l.h.c. and distant r.h.c.
  - Fulfill FSI theorem

$$F^{1,2}_{JLS}(s) = \sum_{i=1}^{a} \alpha_i N_i T^{i1,i2}_{JLS}(s)$$

Dai,Pennington, PLB736(2014)11; PRD90 (2014) 036004;
Dai, Shi, Tang, Zheng,PRD 92 (2015) 1, 014020

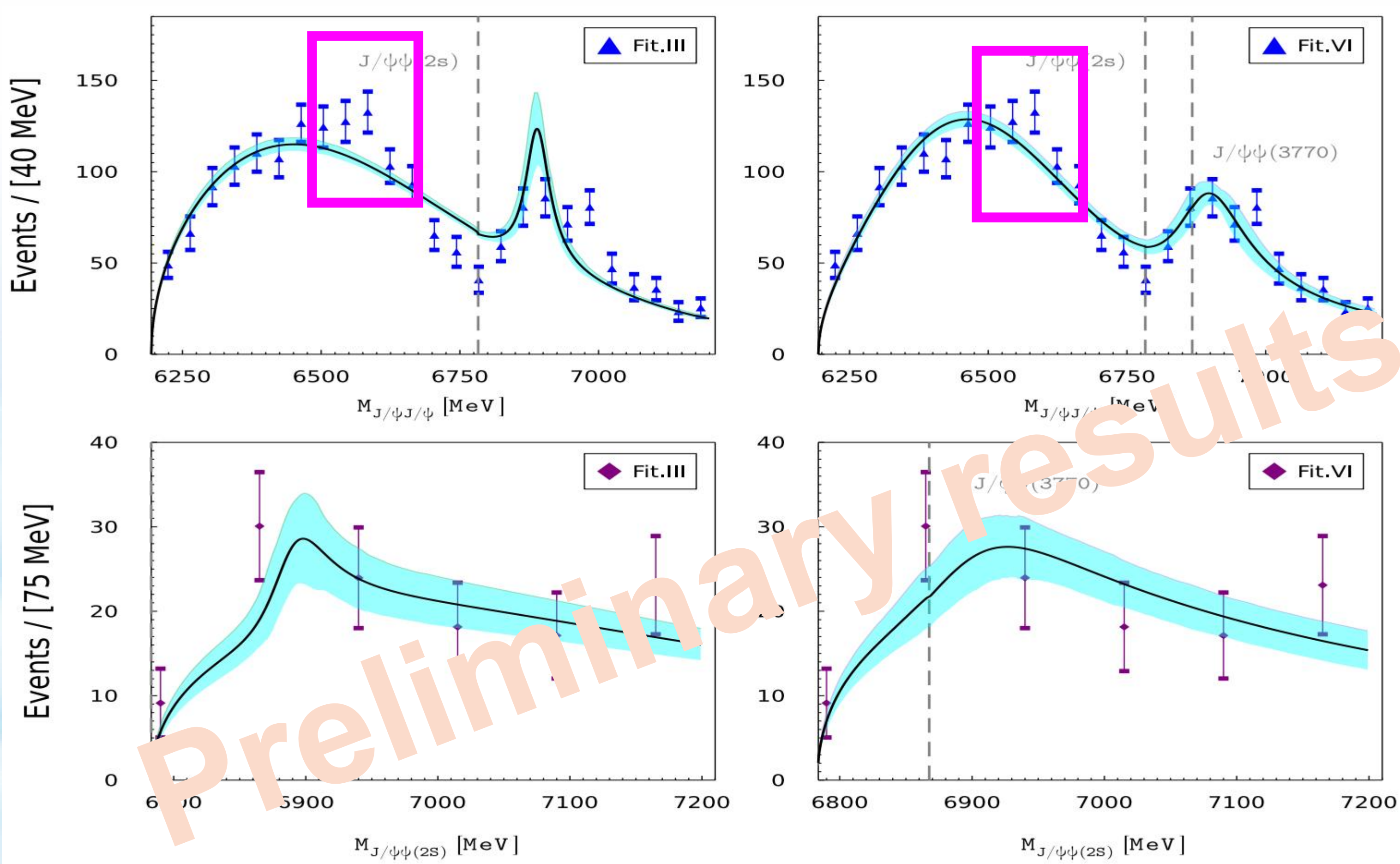
3. Nature of the X(6900)

- High quality Fits for all the data-sets



Fits

- Coupled channe fits on the data-sets of ATLAS



Couplings

- Couplings are reasonable
- So far, so good....

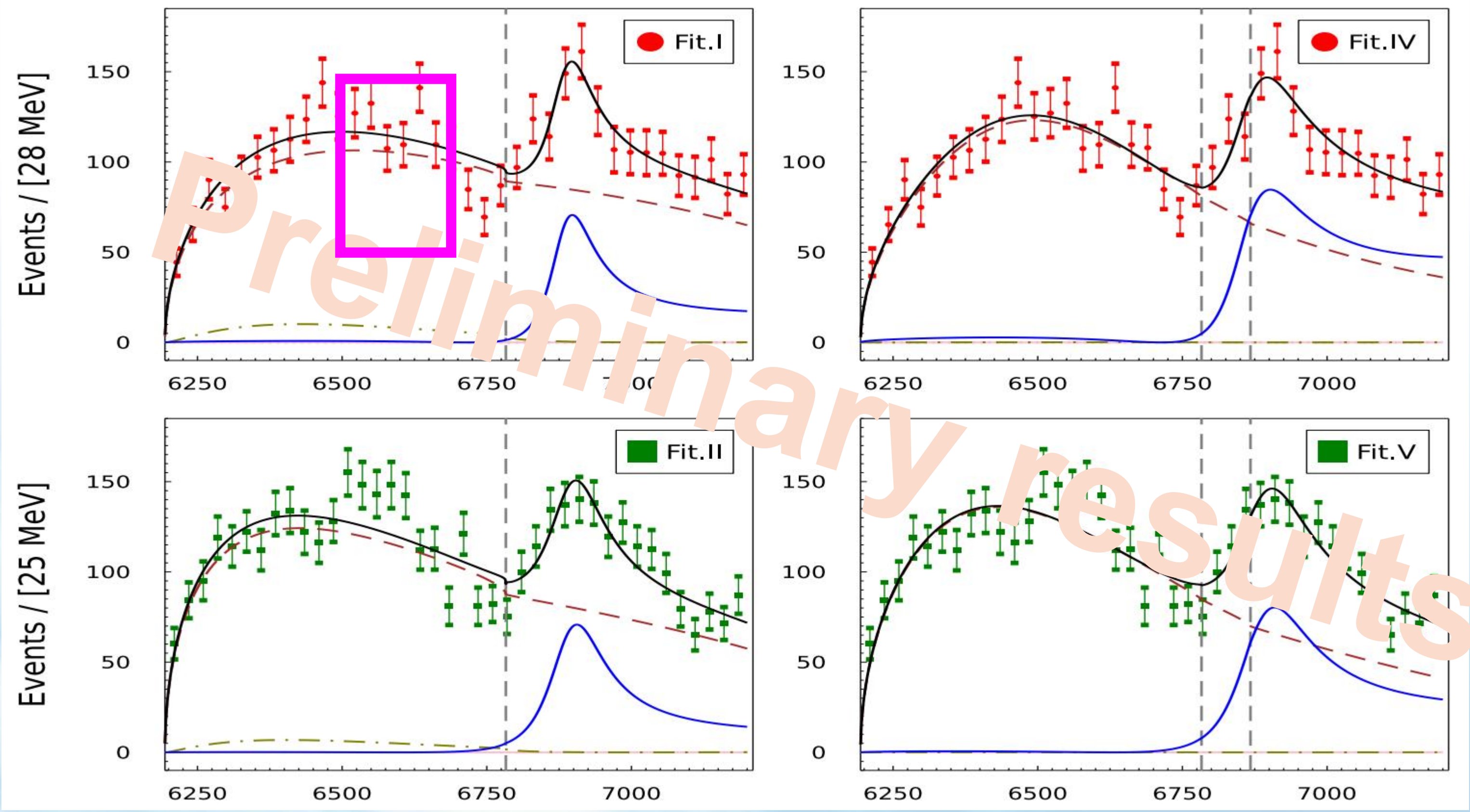
parameter	**Fit.I(LHCb)**	Fit.II(CMS)	Fit.III(ATLAS)
c_1	$-0.1232^{+0.0001}_{-0.0001}$	$-0.1504^{+0.0001}_{-0.0002}$	$-0.0617^{+0.0001}_{-0.0001}$
c_2	$-0.5359^{+0.0021}_{-0.0001}$	$-0.6203^{+0.0001}_{-0.0001}$	$-0.3370^{+0.0001}_{-0.0001}$
c_3	$-0.3250^{+0.0171}_{-0.0001}$	$-0.3492^{+0.0004}_{-0.0001}$	$-0.3230^{+0.0005}_{-0.0003}$
c_4	-0.52[illegible] $^{+0.0234}_{-0.0202}$	$-0.6835^{+0.0004}_{-0.0001}$	$-0.5426^{+0.0009}_{-0.0001}$
$\tilde{N}_1$	1.258[illegible] $^{+[illegible].628[illegible]}_{-0.8[illegible]0}$	$0.5336^{+0.1404}_{-0.0193}$	$0.0490^{+0.0020}_{-0.0092}$
$\tilde{N}_2$	$\cdots$	$\cdots$	$0.0746^{+0.0021}_{-0.0098}$
α_1	$0.3691^{+0.0104}_{-0.0052}$	0.3510 [illegible]	[illegible]$0424^{+0.0001}_{-0.0001}$
α_2	$-0.9294^{+0.0089}_{-0.0022}$	$-0.9364^{+0.0001}_{-0.0009}$	[illegible]
$\chi^2_{d.o.f.}$	1.29	1.77	2.73

Preliminary results

- Lack of angular distributions!

Fits

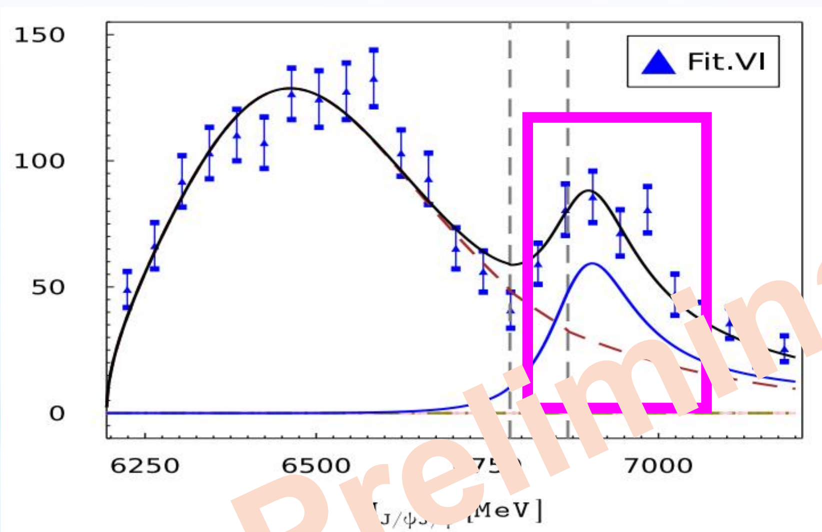
- Individual contributions of each PW

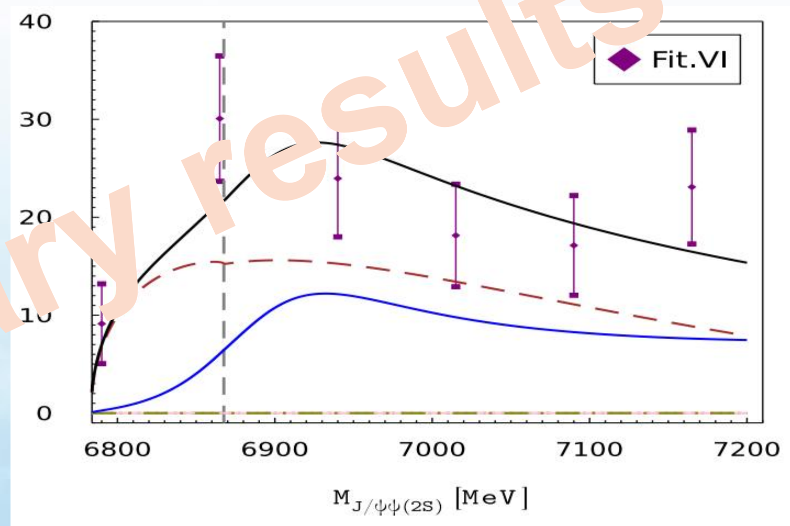


- $^1S_0$ dominates around 6900 MeV!

Fits

- $^1S_0$ dominates around 6900 MeV for both J/ψJ/ψ、ψ(2S)J/ψ channels





- $^5S_2$ Contributes a large background

Pole analysis

- One resonace found in $^1S_0$ wave: X(6900)----0^{++}
- Couple channels case: a pair of accompanying poles
- Triple channels case: Four poles in unphysical sheets, implying the BW origin.

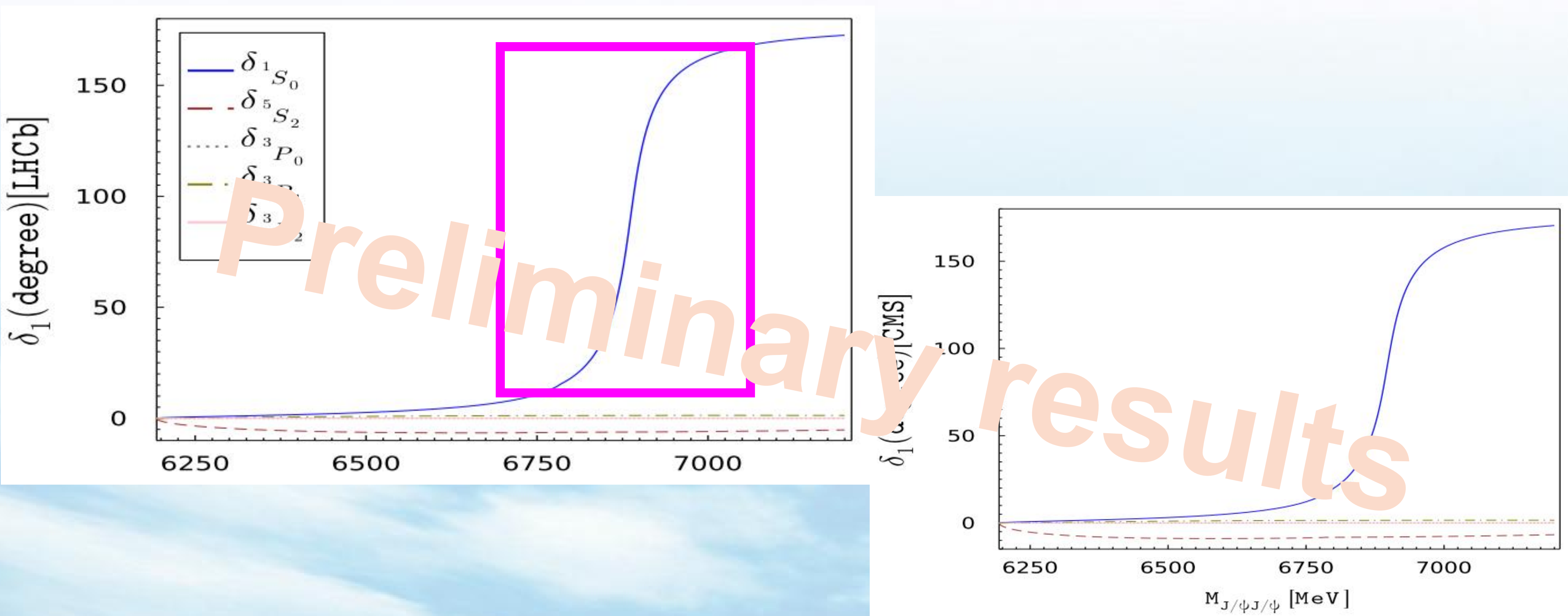
pole counting rule:
Morgan,
NPA543 (1992) 632.
Dai,Wang and Zheng
CTP57 (2012) 841

Data	RS	pole location (MeV)	$g_{J/\psi J/\psi} = \lvert g\rvert e^{i\varphi}$		$g_{J/\psi\psi(2S)} = \lvert g\rvert e^{i\varphi}$		$g_{J/\psi\psi(3770)} = \lvert g\rvert e^{i\varphi}$	
			$\lvert g_1\rvert$(MeV)	$\varphi_1(^\circ)$	$\lvert g_2\rvert$(MeV)	$\varphi_2(^\circ)$	$\lvert g_3\rvert$(MeV)	$\varphi_3(^\circ)$
LHCb(Fit.IV)	II(- + +)	$6872.7^{+6.0}_{-8.6}$-$i46.5^{+2.1}_{-1.0}$	$1352.7^{+28.1}_{-11.7}$	$86.2^{+0.1}_{-0.1}$	$946.5^{+18.8}_{-8.0}$	$84.9^{+0.1}_{-0.1}$	$14.4^{+0.9}_{-0.3}$	[illegible]$3.3^{+1.7}_{-0.6}$
	III(- - +)	$6861.0^{+6.3}_{-8.8}$ -$i64.5^{+2.8}_{-1.7}$	$1326.2^{+24.7}_{-11.3}$	$81.0^{+0.2}_{-0.1}$	$917.9^{+15.1}_{-7.9}$	$78.0^{+0.3}_{-0.}$[illegible]	[illegible]	[illegible]$.0^{+1.2}_{-0.7}$
	IV(- - -)	$6861.0^{+6.3}_{-8.8}$ -$i64.5^{+2.8}_{-1.7}$	$1322.9^{+26.2}_{-14.6}$	$81.0^{+0.3}_{-0.1}$	915.6[illegible]	[illegible]	[illegible]$6.1^{+0.8}_{-0.4}$	$-70.7^{+0.9}_{-0.8}$
	VII(- + -)	$6872.7^{+6.0}_{-8.6}$ -$i46.5^{+2.1}_{-1.0}$	$1349.7^{+29.1}_{-10.8}$	$86.2^{+0.1}$[illegible]	[illegible]	[illegible]$.0^{+0.1}_{-0.1}$	$14.5^{+0.9}_{-0.3}$	$-72.6^{+1.5}_{-0.4}$
CMS(Fit.V)	II(- + +)	$6888.6^{+11.3}_{-7.2}$-$i58.5^{+1.7}_{-0.5}$	$1450.0^{+23.1}$[illegible]	[illegible]	$796.3^{+12.2}_{-4.3}$	$83.4^{+0.1}_{-0.1}$	$40.9^{+2.1}_{-0.1}$	$81.9^{+0.3}_{-0.1}$
	III(- - +)	$6879.2^{+11.3}_{-7.4}$ -$i72.3$[illegible]	[illegible]	[illegible]$2.1^{+0.1}_{-0.1}$	$775.1^{+15.5}_{-4.2}$	$77.9^{+0.2}_{-0.1}$	$38.5^{+2.2}_{-0.1}$	$65.1^{+1.6}_{-0.4}$
	IV(- - -)	6879.2[illegible]	[illegible]$424.0^{+18.8}_{-5.0}$	$82.1^{+0.1}_{-0.1}$	$772.4^{+8.7}_{-3.1}$	$77.9^{+0.2}_{-0.1}$	$38.7^{+2.1}_{-0.1}$	$65.1^{+1.6}_{-0.4}$
	[illegible]	[illegible] -$i58.6^{+1.7}_{-0.5}$	$1450.1^{+24.4}_{-5.6}$	$85.6^{+0.1}_{-0.1}$	$796.1^{+13.6}_{-3.4}$	$83.3^{+0.1}_{-0.1}$	$41.6^{+2.2}_{-0.1}$	$83.1^{+0.4}_{-0.2}$
ATLAS(Fit.VI)	[illegible]	$6896.9^{+19.1}_{-4.3}$-$i50.8^{+0.9}_{-0.2}$	$1410.1^{+12.0}_{-1.9}$	$86.3^{+0.1}_{-0.1}$	$997.4^{+8.8}_{-1.8}$	$85.1^{+0.1}_{-0.1}$	$9.2^{+0.1}_{-0.1}$	$70.0^{+0.8}_{-0.3}$
	III(- - +)	$6883.1^{+18.3}_{-4.0}$ -$i73.2^{+2.8}_{-0.7}$	$1379.8^{+7.3}_{-2.7}$	$80.9^{+0.1}_{-0.1}$	$964.5^{+5.6}_{-1.3}$	$77.6^{+0.1}_{-0.2}$	$9.6^{+0.1}_{-0.1}$	$41.3^{+1.1}_{-1.0}$
	IV(- - -)	[illegible]$883.1^{+18.3}_{-4.0}$ -$i73.2^{+2.8}_{-0.7}$	[illegible]$377.3^{+10.0}_{-2.0}$	$80.8^{+0.1}_{-0.1}$	$962.8^{+7.1}_{-1.2}$	$77.6^{+0.1}_{-0.1}$	$9.7^{+0.1}_{-0.1}$	$41.7^{+1.1}_{-1.0}$
	VII(- + -)	$6896.9^{+19.1}_{-4.3}$ -$i50.8^{+0.9}_{-0.2}$	$1412.1^{+10.4}_{-2.0}$	$86.2^{+0.1}_{-0.1}$	$998.9^{+7.4}_{-2.3}$	$85.1^{+0.1}_{-0.1}$	$9.3^{+0.2}_{-0.1}$	$70.9^{+0.9}_{-0.2}$

Preliminary results

Phase shifts

- Phase shifts of $^1S_0$ is very likely to the one generated by a narrow BW resonacne: tetra-quark
- Other waves should contribute backgrounds



4、Summary

PWA

One needs PWA to extract pole parameters for the resonance: quantum number, mass, width, residues...

FSI

Amplitude analsysis connects QFT principles and Exp. FSI needs to be considered when performing amplitude analysis.

X6900

It is very likely to be a compact tetraquark, and has quantum number 0++

Next?

We lack enough data on the angular distribution, which would be essential to separate the partial waves.Our framework can be applied to other scatterings.

Thank You For your patience !