Light pseudoscalar meson and heavy meson scattering in chiral perturbation theory

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Chiral EFT, Nanjing, 15.10.2022

Outline

Introduction

- ChPT has been widely used to study low-energy hadronic processes and achieved many successes (N. Fettes *et al.*, Nucl. Phys. A676 (2000) 311; D. R. Entem *et al.*, Phys. Rev. C91 (2015) 014002). Similar to the HBChPT formalism in the light flavor meson-baryon and baryon-baryon interaction, one can use HMChPT to deal with the charmed mesons.
- Detailed calculations of light pseudoscalar meson-heavy meson scattering lengths up to order p^3 in SU(3) HMChPT (Y. -R. Liu *et al.*, Phys. Rev. D79 (2009) 094026). Present calculations concern the fourth order contributions, the non-perturbative results, and the partial-wave phase shifts.
- In this talk: the light pseudoscalar meson and heavy meson scattering based on papers:
  - B.-L. Huang, Z.-Y. Lin and S.-L. Zhu, Phys. Rev. D105 (2022) 036016;
  - B.-L. Huang, Z.-Y. Lin, K. Chen and S.-L. Zhu, arXiv: 2205.02619

Outline

Chiral Lagrangian

meson and heavy meson fields

Eight light meson fields in terms of the traceless hermitian 3×3 matrices Φ:

$$\Phi = \sqrt{2} \begin{pmatrix} \frac{1}{\sqrt{2}}\pi^0 + \frac{1}{\sqrt{6}}\eta & \pi^+ & K^+ \\ \pi^- & -\frac{1}{\sqrt{2}}\pi^0 + \frac{1}{\sqrt{6}}\eta & K^0 \\ K^- & \overline{K}^0 & -\frac{2}{\sqrt{6}}\eta \end{pmatrix},$$

with the SU(3) matrix $U = \xi^2 = \exp(i\Phi/f)$.

$$P = (D^0, D^+, D_s^+), \quad P_\mu^* = (D^{0*}, D^{+*}, D_s^{+*})_\mu.$$

Chiral Lagrangian

Leading order Lagrangian

The effective chiral Lagrangian:

$$\mathcal{L}_{\text{eff}} = \mathcal{L}^{(2)}_{\phi\phi} + \mathcal{L}^{(1)}_{H\phi} + \mathcal{L}^{(2)}_{H\phi} + \mathcal{L}^{(3)}_{H\phi} + \mathcal{L}^{(4)}_{H\phi}.$$

Lowest-order Lagrangians for meson-meson and meson-heavy meson interaction

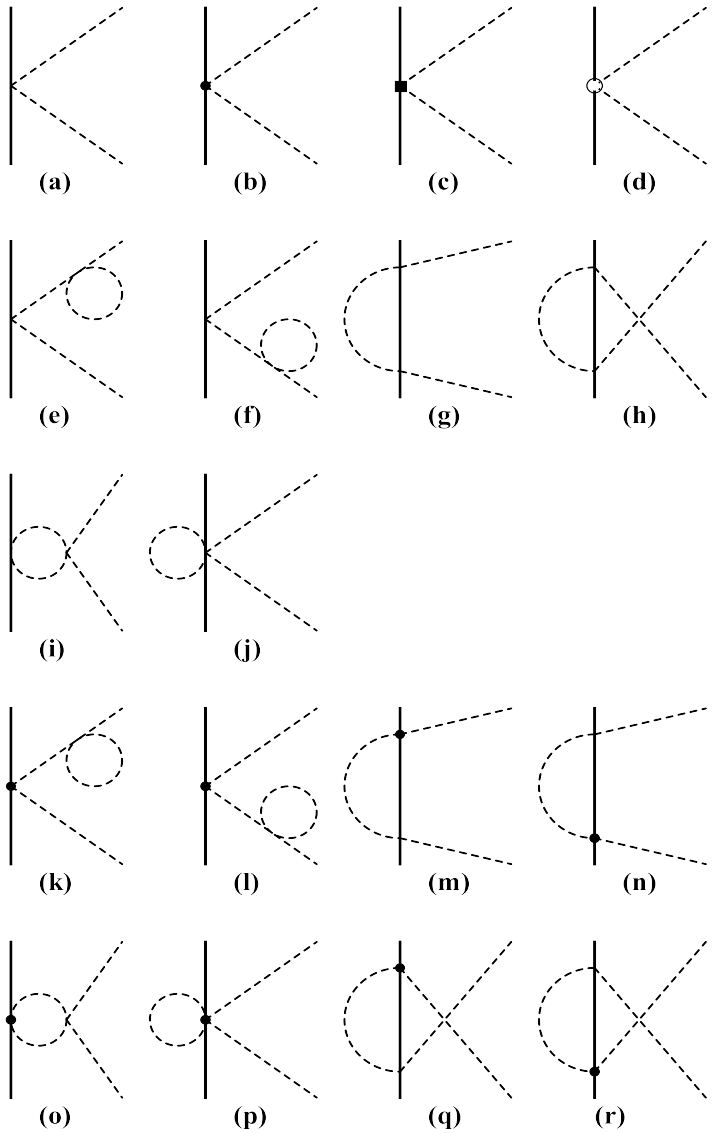
$$\mathcal{L}^{(2)}_{\phi\phi} = \frac{f^{\,2}}{4}\text{tr}(u_\mu u^\mu + \chi_+),$$

$$\mathcal{L}^{(1)}_{H\phi} = -\left\langle (iv\cdot\partial H)\bar{H}\right\rangle + \left\langle Hv\cdot\Gamma\bar{H}\right\rangle + g\left\langle Hu_\mu\gamma^\mu\gamma_5\bar{H}\right\rangle,$$

higher-order Lagrangians involving low-energy constants, see ref. (B.-L. Huang, Z.-Y. Lin and S.-L. Zhu, Phys. Rev. D105 (2022) 036016;)

Outline

Feynman diagrams



Outline

Threshold T matrices

The threshold T matrix with the four orders read

$$
\begin{aligned}
T_{DK}^{(1)} = & \Big\{0\Big\}_{\text{LO}} + \Big\{\frac{2(4c_0 + c_2 + c_4)m_K^2}{f_K^2}\Big\}_{\text{NLO}} + \Big\{\frac{m_K^2}{8\pi^2 f_K^4} L_i(m_K, m_\pi)\Big\}_{\text{N2LO}} \\
& + \Big\{ - \frac{2\bar{e}_1 m_K^4}{f_K^2} + \frac{1}{6912\pi^2 f_K^4}\{192c_0 M(40,-5,7) - 96c_1[M(48,5,-5 \\
& + 36m_K(m_\pi^2 + m_K^2)L_i(m_K, m_\pi)] - 6c_2 M(1,-8,7) \\
& - c_3[M(1753,-8,19) + 1728m_K^3 L_i(m_K, m_\pi)] + 48c_4 M(40,-5,7) \\
& + 8c_5[M(-170,7,19) - 216m_K^3 L_i(m_K, m_\pi)]\}\Big\}_{\text{N3LO}},
\end{aligned}
$$

.....omitting many more equations.....

T matrices with q

The T matrix with the three orders read

$$
\begin{aligned}
T_{\pi D}^{(1/2)} =& \frac{2w_\pi}{f_\pi^2} + \frac{1}{f_\pi^2}[8c_0 m_\pi^2 + 4c_1 m_\pi^2 + 2c_2 w_\pi^2 + c_3 w_\pi^2 + 2c_4(w_\pi^2 - q^2 z) \\
&+ c_5(w_\pi^2 - q^2 z)] + \frac{1}{f_\pi^2}[16\bar{\kappa}_1 m_\pi^2 w_\pi + 4\bar{\kappa}_2 w_\pi^3 + 4\bar{\kappa}_3 w_\pi(w_\pi^2 - q^2 z)] \\
&- \frac{w_\pi}{12 f_\pi^4}\{3w_\pi[3J_0(-w_\pi, m_K) + 4J_0(-w_\pi, m_\pi) - 9J_0(w_\pi, m_K) \\
&- 17J_0(w_\pi, m_\pi)] + 12I_2(t, m_K) + 16I_2(t, m_\pi)\},
\end{aligned}
$$

.....omitting many more equations.....

Outline

Phase shifts

The partial-wave amplitudes $f_l^{(I)}(q)$ are obtained by a projection:

$$f_l^{(I)}(q) = \frac{M_H}{16\pi\sqrt{s}} \int_{-1}^{+1} dz\, [T_{\phi H}^{(I)} P_l(z)],$$

The phase shifts $\delta_l^{(I)}(q)$ are calculated as (N. Fettes *et al.*, Nucl. Phys. A640 (1998) 199)

$$\delta_l^{(I)}(q) = \arctan[q \mathrm{Re} f_l^{(I)}(q)].$$

The scattering lengths for the S waves and the scattering volumes for P waves are obtained by

$$a_l^{(I)} = \lim_{q\to 0} q^{-2l} f_l^{(I)}(q).$$

Scattering lengths

The S-wave scattering length is defined through:

$$a = \frac{M}{8\pi(M+m)} T_{\text{th}},$$

For a single channel separable potential, the scattering lengths is given by (N. Kaiser *et al.*, Nucl. Phys. A594 (1995) 325)

$$a = a_{\text{Born}}(1 - \frac{1}{2}\mu\, a_{\text{Born}})^{-1}.$$

Here a_{Born} includes the contributions from all the diagrams except for the iterated diagrams.

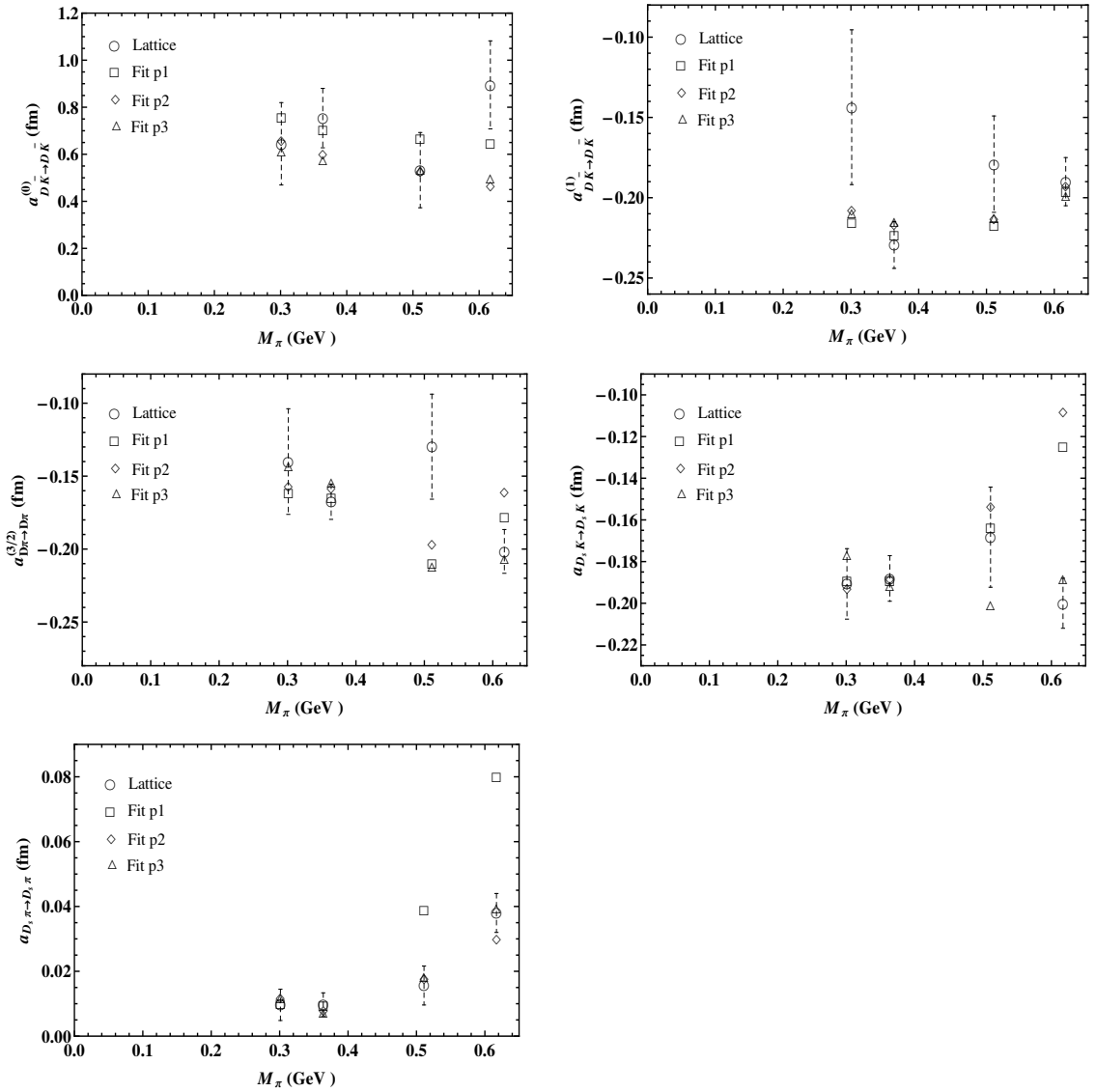
Outline

Fits through the perturbative formula

	Fit p1	c_2	c_3	c_4	c_5	$\bar{\kappa}$	$\bar{e}_1$	$\bar{e}_2$
c_2 (GeV^{-1})	0.53 ± 3.12	1.00	-0.83	-0.99	0.83	0.90	-0.25	-0.73
c_3 (GeV^{-1})	-2.82 ± 3.59		1.00	0.90	-0.99	-0.86	0.67	0.49
c_4 (GeV^{-1})	-0.26 ± 2.71			1.00	-0.90	-0.91	0.38	0.69
c_5 (GeV^{-1})	3.22 ± 4.44				1.00	0.86	-0.67	-0.49
$\bar{\kappa}$ (GeV^{-2})	0.87 ± 0.51					1.00	-0.33	-0.85
$\bar{e}_1$ (GeV^{-3})	-0.44 ± 1.30						1.00	-0.18
$\bar{e}_2$ (GeV^{-3})	-6.15 ± 4.54							1.00
χ^2/d.o.f.	$\frac{3.41}{10-7} = 1.14$							

.....omitting the other two fits.....

Scattering lengths through the perturbative formula



Scattering lengths through the perturbative formula

Fit p1	$\mathcal{O}(p)$	$\mathcal{O}(p^2)$	$\mathcal{O}(p^3)$	$\mathcal{O}(p^4)$	Total
$a_{DK}^{(1)}$	0.00	0.07	$-0.12+0.19i$	$0.09-0.13i$	$0.04(3)+0.07(16)i$
$a_{DK}^{(0)}$	0.51	0.29	0.77	0.41	1.98(93)
$a_{D\bar{K}}^{(1)}$	-0.25	0.18	-0.49	0.33	$-0.23(3)$
$a_{D\bar{K}}^{(0)}$	0.25	-0.04	0.73	-0.16	0.80(6)
a_{D_sK}	-0.26	0.18	-0.38	0.24	$-0.21(7)$
$a_{D_s\bar{K}}$	0.26	0.18	$0.21+0.29i$	$0.33+0.19i$	$0.97(47)+0.49(22)i$
$a_{D\pi}^{(3/2)}$	-0.12	0.02	-0.04	0.04	$-0.09(3)$
$a_{D\pi}^{(1/2)}$	0.24	0.02	0.02	-0.01	0.27(2)
$a_{D_s\pi}$	0.00	0.01	-0.04	0.05	0.02(2)

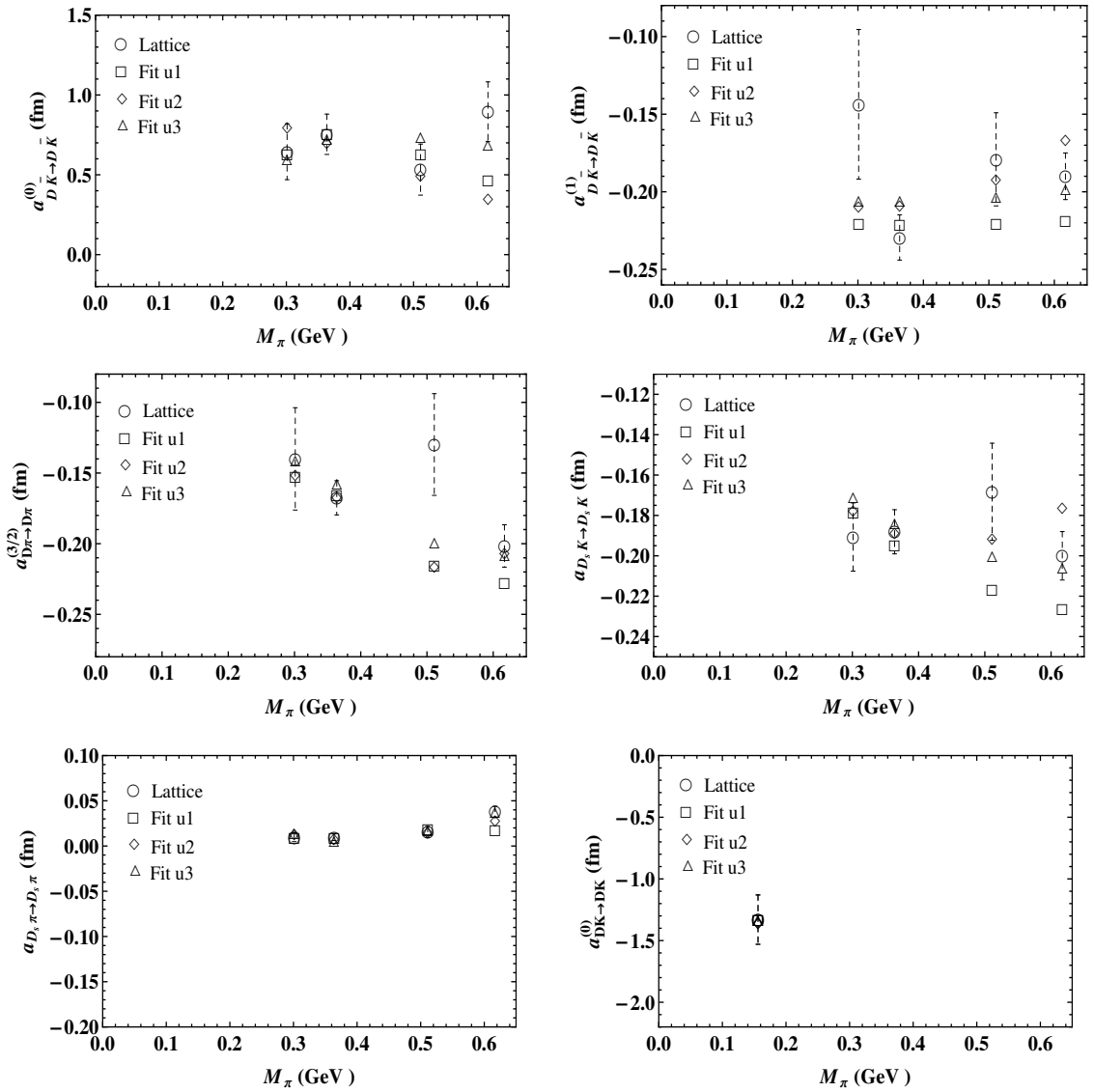
Outline

Fits through the iterated method

	Fit u1	c_2	c_3	c_4	c_5	$\bar{\kappa}$	$\bar{e}_1$	$\bar{e}_2$	μ
c_2 (GeV^{-1})	-4.60 ± 4.58	1.00	-0.92	-0.99	0.89	0.92	0.70	0.75	0.19
c_3 (GeV^{-1})	3.32 ± 3.42		1.00	0.94	-0.98	-0.87	-0.57	-0.71	-0.10
c_4 (GeV^{-1})	4.10 ± 3.47			1.00	-0.91	-0.92	-0.68	-0.75	-0.17
c_5 (GeV^{-1})	-5.11 ± 3.73				1.00	0.92	0.44	0.78	-0.07
$\bar{\kappa}$ (GeV^{-2})	-0.06 ± 0.17					1.00	0.41	0.77	-0.19
$\bar{e}_1$ (GeV^{-3})	2.38 ± 1.65						1.00	0.39	0.80
$\bar{e}_2$ (GeV^{-3})	-5.78 ± 1.83							1.00	-0.11
μ (GeV)	1.03 ± 0.36								1.00
χ^2/d.o.f.	$\frac{3.90}{11-8} = 1.30$								

.....omitting the other two fits.....

Scattering lengths through the iterated formula



Scattering lengths through the iterated formula

	Fit u1	Fit u2	Fit u3	Liu2013	Guo2019
$a_{DK}^{(1)}$	$-0.03(4)$	$-0.01(3)$	$-0.06(1)$	$0.07^{+0.03}_{-0.03} + i0.17^{+0.02}_{-0.01}$	$-0.01^{+0.05}_{-0.03} + i0.39^{+0.04}_{-0.04}$
$a_{DK}^{(0)}$	$-1.55(39)$	$-1.42(28)$	$-1.67(45)$	$-0.84^{+0.17}_{-0.22}$	$-1.51^{+0.72}_{-2.35}$
$a_{D\bar{K}}^{(1)}$	$-0.23(2)$	$-0.24(2)$	$-0.21(1)$	$-0.20(1)$	$-0.20^{+0.01}_{-0.01}$
$a_{D\bar{K}}^{(0)}$	8.76^*	$1.81(48)$	8.95^*	$0.84(15)$	21.9^*
a_{D_sK}	$-0.14(3)$	$-0.17(3)$	$-0.14(1)$	$-0.18(1)$	$-0.20^{+0.01}_{-0.01}$
$a_{D_s\bar{K}}$	$0.14(36)$	371.58^*	$0.05(18)$	$-0.09^{+0.06}_{-0.05} + i0.44^{+0.05}_{-0.05}$	$-0.57^{+0.06}_{-0.04} + i0.35^{+0.08}_{-0.07}$
$a_{D\pi}^{(3/2)}$	$-0.07(4)$	$-0.06(2)$	$-0.05(1)$	$-0.100(2)$	$-0.103^{+0.003}_{-0.003}$
$a_{D\pi}^{(1/2)}$	$1.45(169)$	$0.61(11)$	6.00^*	$0.37^{+0.03}_{-0.02}$	$0.40^{+0.03}_{-0.02}$
$a_{D_s\pi}$	$0.02(3)$	$0.03(2)$	$0.05(2)$	$-0.002(1)$	$0.012^{+0.003}_{-0.003}$
$a_{D\eta}$	$-0.04(14)$	$0.03(8)$	$-0.09(2)$		$0.29^{+0.15}_{-0.22} + i0.61^{+0.30}_{-0.26}$
$a_{D_s\eta}$	$-0.18(8)$	$0.03(16)$	$-0.19(2)$		$-0.39^{+0.05}_{-0.03} + i0.06^{+0.02}_{-0.02}$

Outline

Mesons and doubly charmed baryons scattering

Meson and doubly charmed baryon (Ξ_{cc} and Ω_{cc}) scattering lengths were estimated based on heavy diquark-antiquark (HDA) symmetry.

Scattering lengths in the perturbative formula

Fit p1	$\mathcal{O}(p)$	$\mathcal{O}(p^2)$	$\mathcal{O}(p^3)$	$\mathcal{O}(p^4)$	Total
$a^{(1)}_{K\Xi_{cc}}$	-0.28	0.20	-0.55	0.37	$-0.26(3)$
$a^{(0)}_{K\Xi_{cc}}$	0.28	-0.04	0.82	-0.17	$0.89(7)$
$a^{(1)}_{\bar{K}\Xi_{cc}}$	0.00	0.08	$-0.13+0.22i$	$0.10-0.14i$	$0.05(3)+0.08(18)i$
$a^{(0)}_{\bar{K}\Xi_{cc}}$	0.56	0.32	0.86	0.46	$2.20(103)$
$a_{K\Omega_{cc}}$	0.28	0.20	$0.23+0.33i$	$0.36+0.21i$	$1.07(53)+0.54(24)i$
$a_{\bar{K}\Omega_{cc}}$	-0.28	0.20	-0.42	0.27	$-0.23(8)$
$a^{(3/2)}_{\pi\Xi_{cc}}$	-0.12	0.03	-0.04	0.04	$-0.10(4)$
$a^{(1/2)}_{\pi\Xi_{cc}}$	0.25	0.03	0.02	-0.01	$0.28(2)$

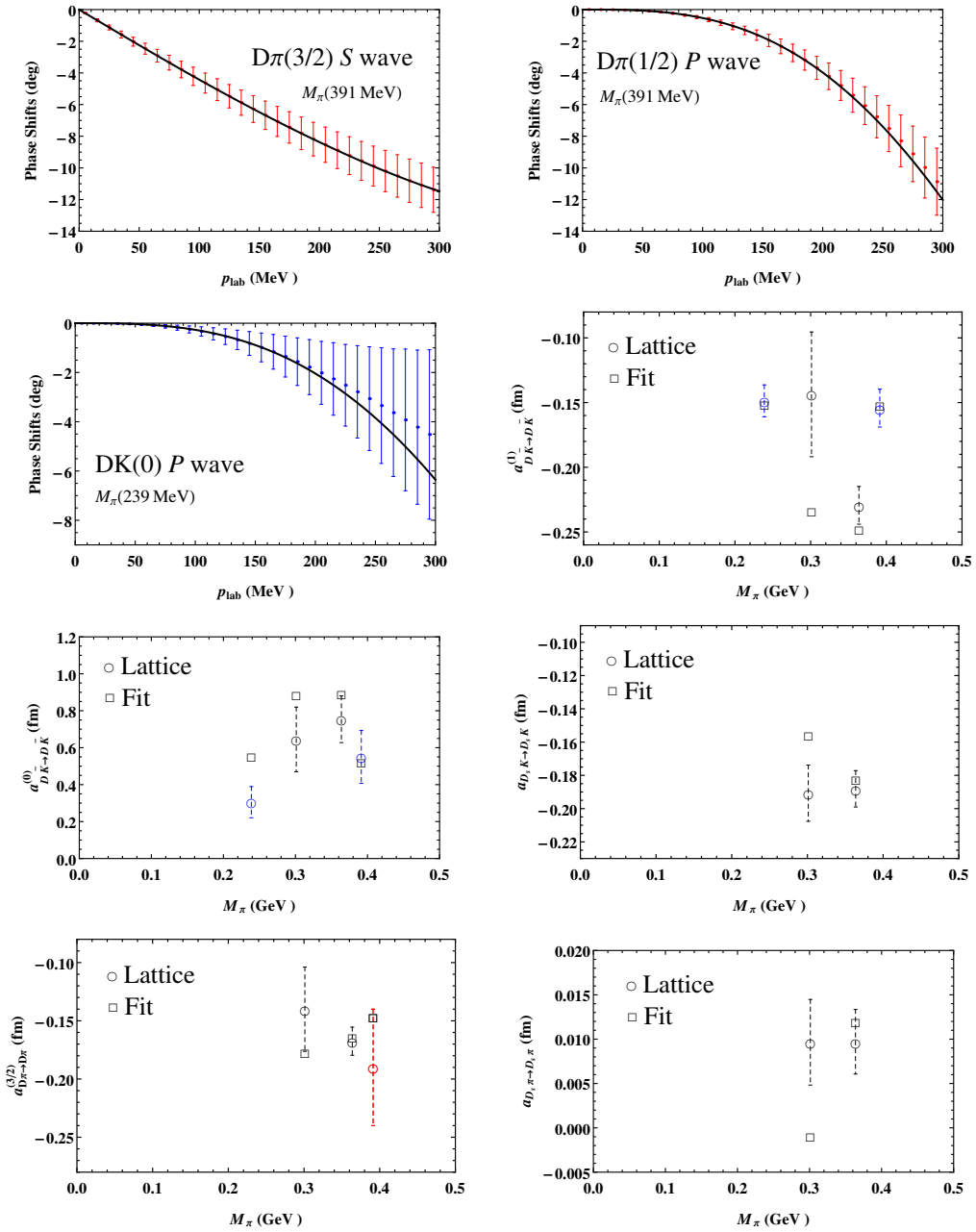
Scattering lengths in the iterated formula

	Fit u1	Fit u2	Fit u3	Guo2017
$a^{(1)}_{K\Xi_{cc}}$	$-0.24(3)$	$-0.25(2)$	$-0.22(2)$	$-0.19^{+0.02}_{-0.02}$
$a^{(0)}_{K\Xi_{cc}}$	-6.15^*	2.81^*	-5.04^*	$5.2, -3.6, -1.4$
$a^{(1)}_{\bar{K}\Xi_{cc}}$	$-0.04(4)$	$-0.01(3)$	$-0.06(2)$	$-0.22^{+0.14}_{-0.14} + i0.45^{+0.00}_{-0.09}$
$a^{(0)}_{\bar{K}\Xi_{cc}}$	$-1.19(21)$	$-1.29(24)$	$-1.20(19)$	$-0.49^{+0.10}_{-0.19}$
$a_{K\Omega_{cc}}$	$0.16(43)$	-7.80^*	$0.06(20)$	$-0.55^{+0.11}_{-0.16} + i0.13^{+0.19}_{-0.07}$
$a_{\bar{K}\Omega_{cc}}$	$-0.15(3)$	$-0.18(3)$	$-0.15(1)$	$-0.19^{+0.02}_{-0.02}$
$a^{(3/2)}_{\pi\Xi_{cc}}$	$-0.07(3)$	$-0.07(2)$	$-0.05(1)$	$-0.095^{+0.003}_{-0.004}$
$a^{(1/2)}_{\pi\Xi_{cc}}$	1.73^*	$0.65(12)$	16.26^*	$0.55^{+0.16}_{-0.10}$

Outline

Fit to the phase shifts and scattering lengths.

	Values	c_0	c_1	c_2	c_3	c_4	c_5	$\bar{\kappa}_1$	$\bar{\kappa}_2$	$\bar{\kappa}_3$
c_0 (GeV^{-1})	-0.77 ± 0.39	1.00	0.99	-0.86	-0.88	0.00	0.00	0.02	-0.92	0.00
c_1 (GeV^{-1})	-0.64 ± 0.35		1.00	-0.86	-0.89	0.00	0.00	0.01	-0.93	0.00
c_2 (GeV^{-1})	-5.04 ± 1.83			1.00	0.95	-0.51	-0.41	-0.01	0.61	0.50
c_3 (GeV^{-1})	5.47 ± 1.57				1.00	-0.36	-0.45	0.10	0.67	0.40
c_4 (GeV^{-1})	8.99 ± 0.93					1.00	0.80	0.00	0.37	-0.98
c_5 (GeV^{-1})	-3.08 ± 0.70						1.00	0.00	0.34	-0.90
$\bar{\kappa}_1$ (GeV^{-2})	0.21 ± 0.04							1.00	0.01	0.00
$\bar{\kappa}_2$ (GeV^{-2})	7.81 ± 3.88								1.00	-0.38
$\bar{\kappa}_3$ (GeV^{-2})	-1.87 ± 1.47									1.00
χ^2/d.o.f.	$\frac{34.85}{195-9} = 0.19$									


Dπ(3/2) S wave
Mπ(391 MeV)
Phase Shifts (deg)
plab (MeV)
Dπ(1/2) P wave
Mπ(391 MeV)
Phase Shifts (deg)
plab (MeV)
DK(0) P wave
Mπ(239 MeV)
Phase Shifts (deg)
plab (MeV)
Lattice
Fit
Mπ (GeV)
Lattice
Fit
Mπ (GeV)
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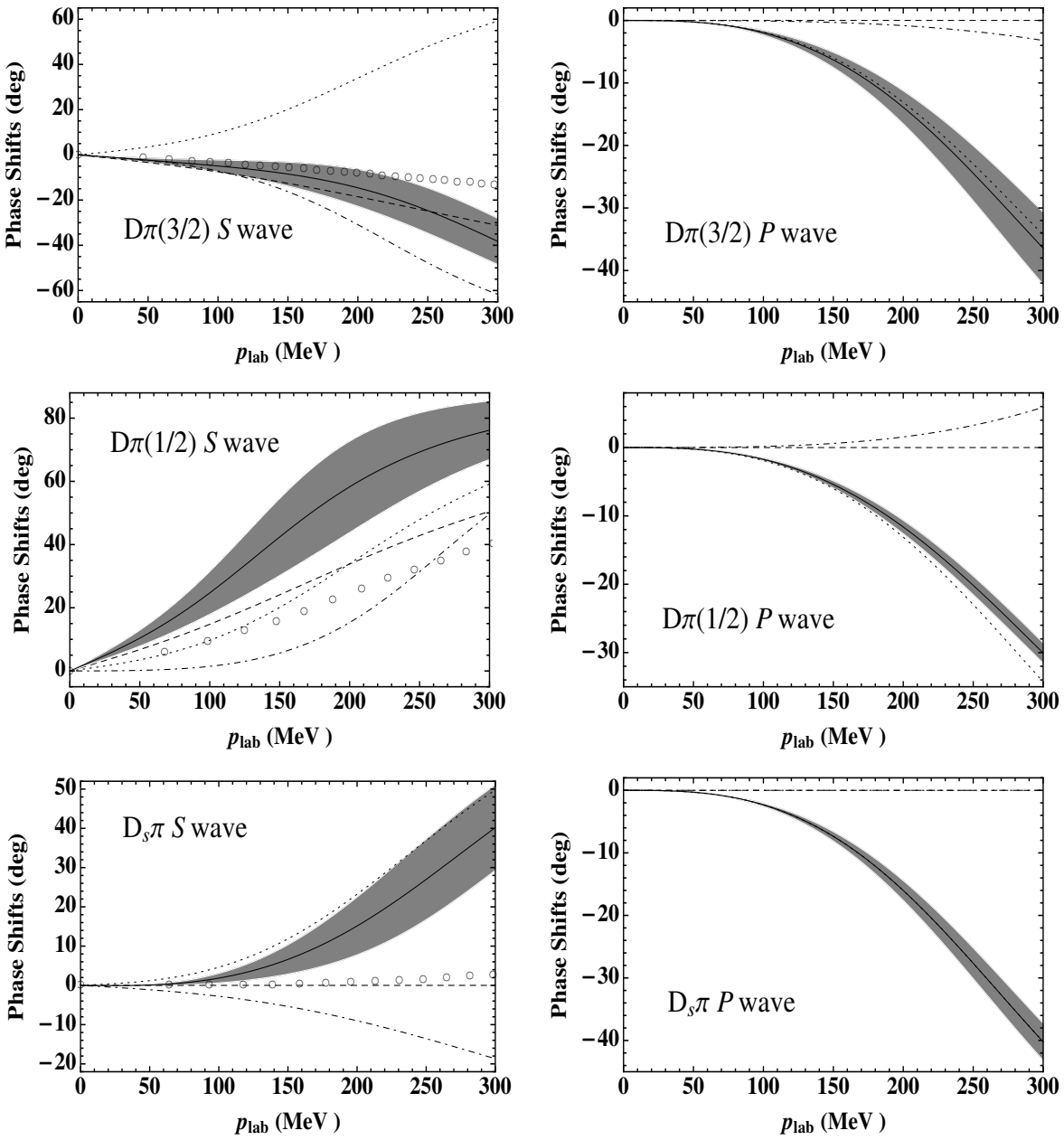
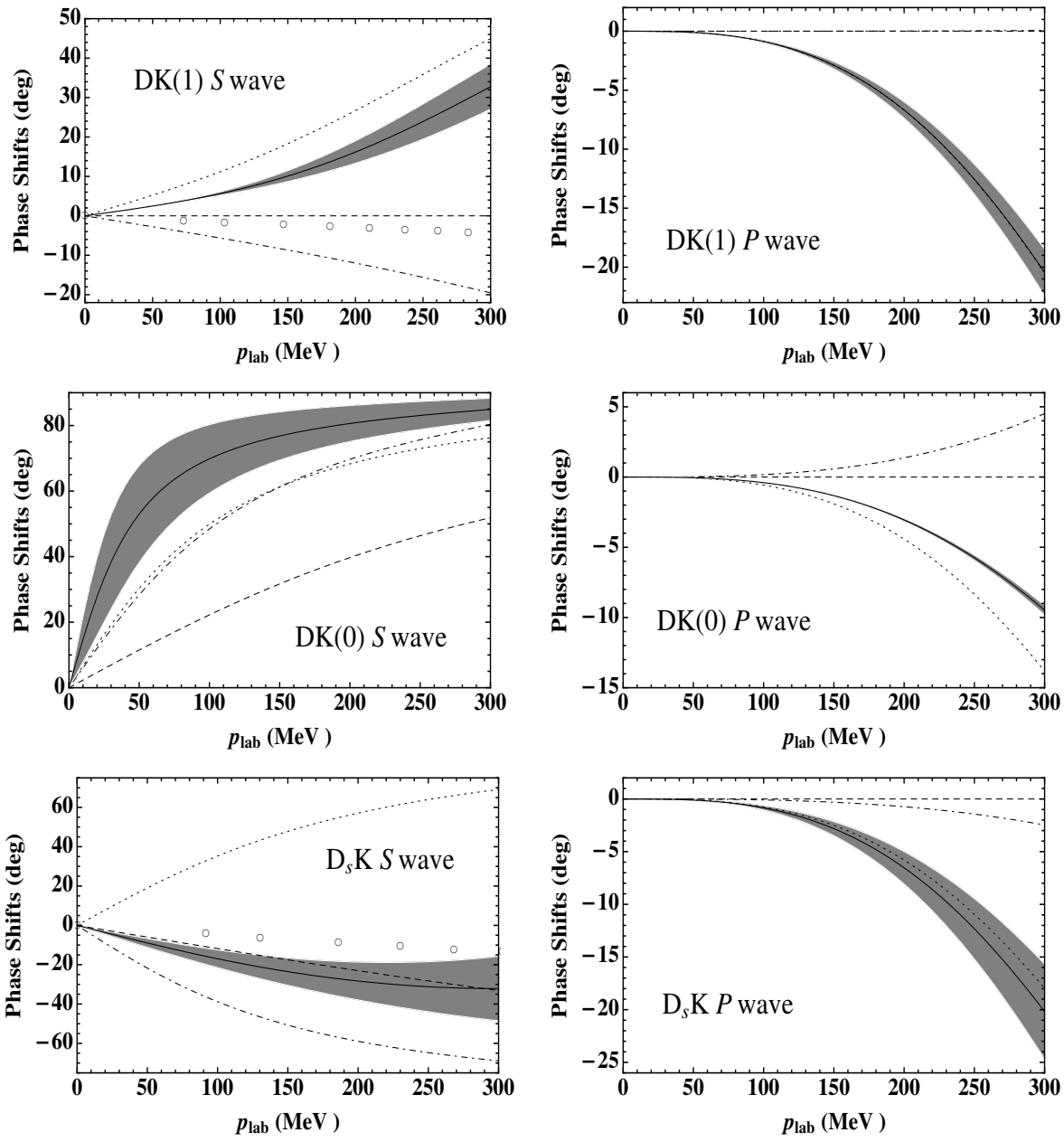
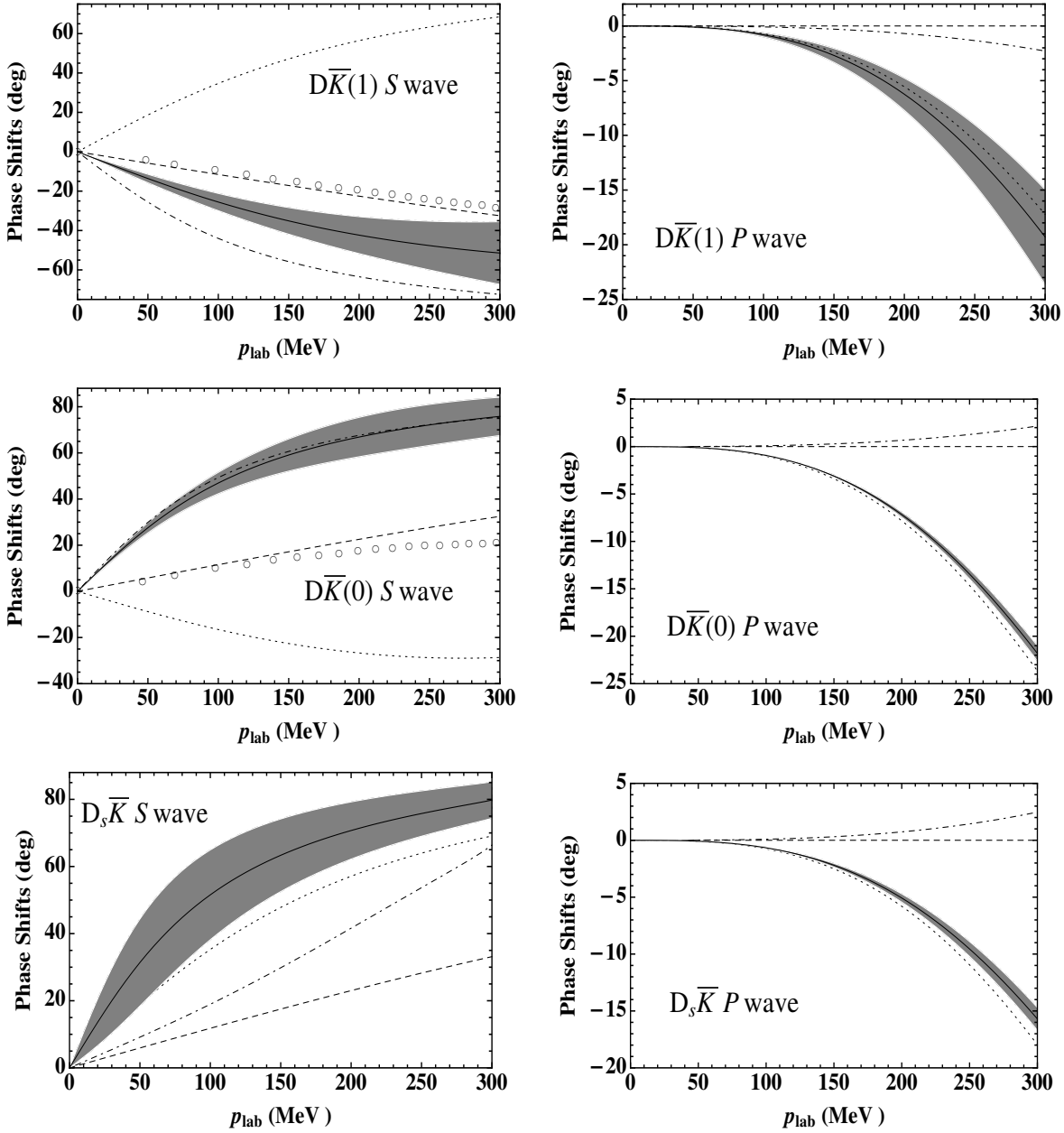
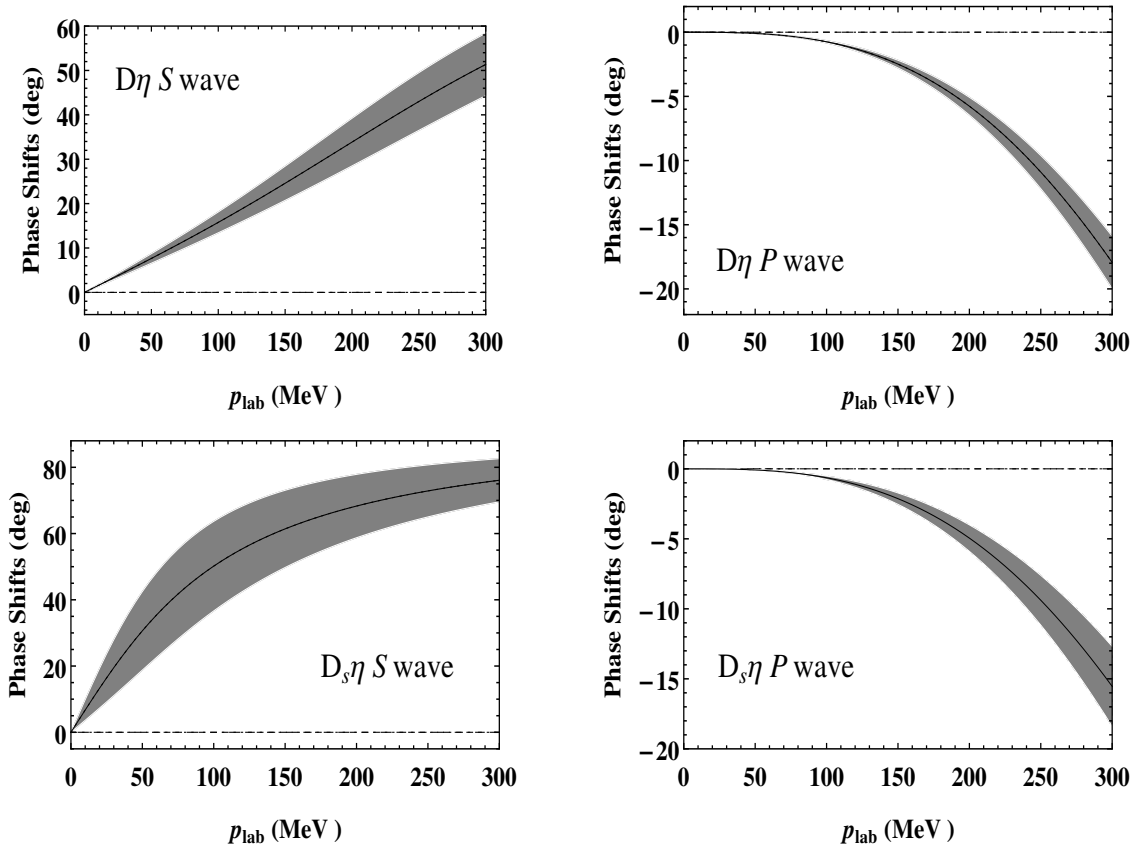

Dπ(3/2) S wave
Dπ(3/2) P wave
Dπ(1/2) S wave
Dπ(1/2) P wave
D_sπ S wave
D_sπ P wave
Phase Shifts (deg)
p_lab (MeV)


DK(1) S wave
DK(1) P wave
DK(0) S wave
DK(0) P wave
D$_s$K S wave
D$_s$K P wave
Phase Shifts (deg)
p_{lab} (MeV)


$D\overline{K}(1)$ S wave
$D\overline{K}(1)$ P wave
$D\overline{K}(0)$ S wave
$D\overline{K}(0)$ P wave
$D_s\overline{K}$ S wave
$D_s\overline{K}$ P wave
Phase Shifts (deg)
$p_{\rm lab}$ (MeV)


Dη S wave
Dη P wave
Ds η S wave
Ds η P wave
Phase Shifts (deg)
plab (MeV)
0
50
100
150
200
250
300
60
40
30
20
10
80
−5
−10
−15
−20

Summary and Outlook

- Threshold T-matrices to the fourth order for light meson and heavy meson scattering in SU(3) HMChPT.
- Scattering lengths for light meson and heavy meson scattering using perturbative and nonperturbative methods.
- Mesons and doubly charmed baryons scattering based on the HDA symmetry.
- Phase shifts to the third order for light meson and heavy meson scattering in SU(3) HMChPT.
- Outlook
  - Higher-order calculations.
  - Including the $D_{s0}^*(2317)$ resonance.
  - Nonperturbative calculations.
  - Combining with more Lattice QCD data.

Thank you for your attention!