Constructing chiral effective field theory around unnatural leading-order interactions

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Outline

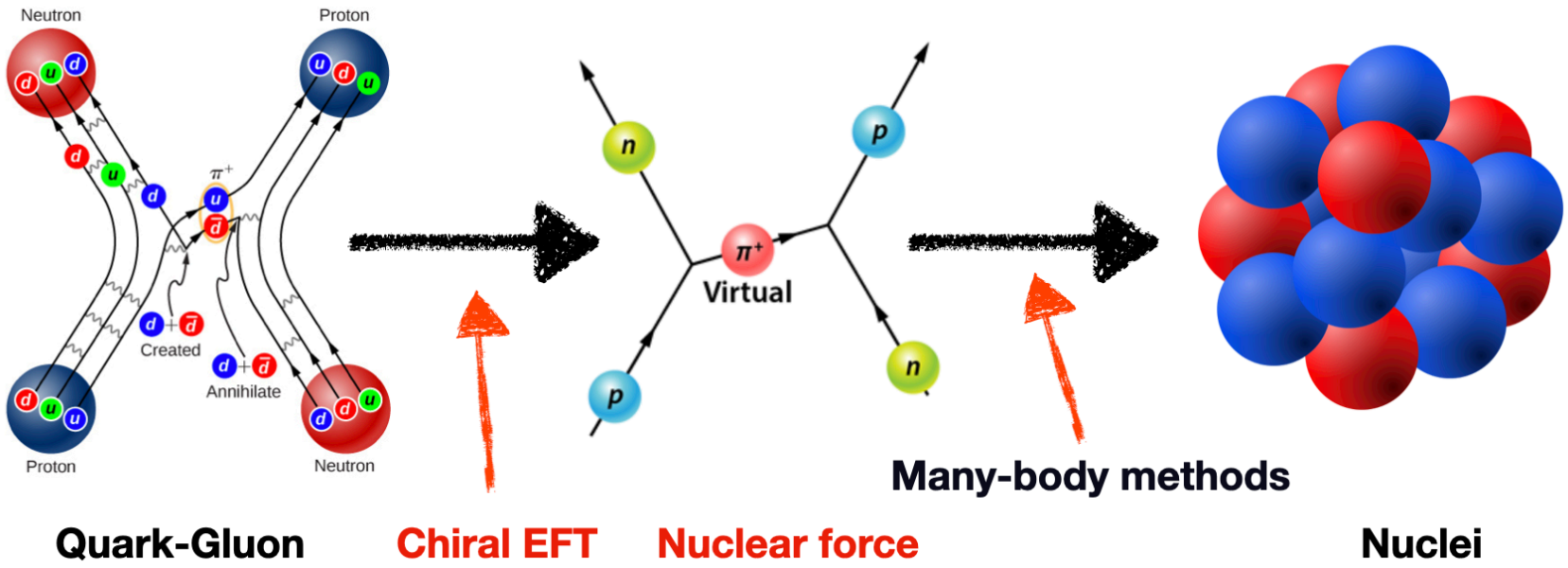
Introduction

Theoretical framework

Result

Summary

Chiral effective field theory(ChEFT)



- Nuclear force: residual quark-gluon strong interaction
- QCD is nonperturbative at low-energy region
- Chiral EFT bridge the gap between QCD and nuclear physics
- High precision Chiral nuclear force for many-body calculations

Chiral EFT

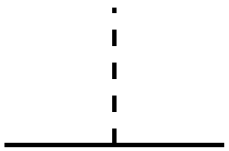
- EFT of low-energy QCD, includes all symmetries of QCD

$$\mathcal{L}_{\text{QCD}} = \sum_{f=u,d} \bar{q}_f (i\not{D} - m_f) q_f - \frac{1}{2}\text{Tr} G_{\mu\nu} G^{\mu\nu}$$

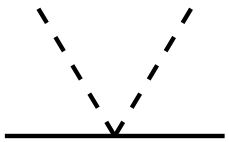
- Spontaneously breaking (approximate) chiral symmetry
  two flavors used in our work: $SU(2)_R \times SU(2)_L \to SU(2)_V$

- Nonlinearly realized by hadronic degrees of freedoms (nucleons and pion) Weinberg, CCWZ

- Nucleon-pion couplings $\to$ long-range nuclear forces

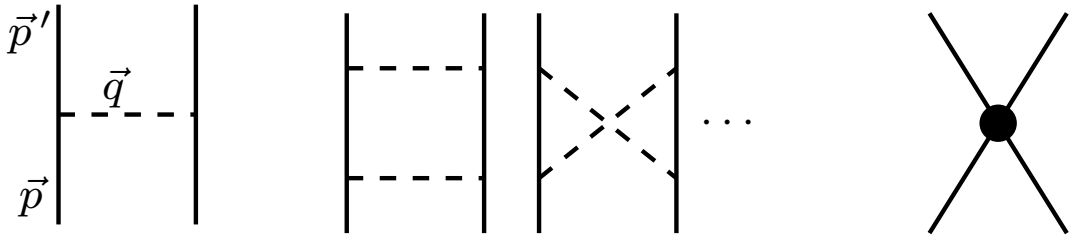


$$-\frac{g_A}{2f_\pi} N^\dagger \tau_a \vec{\sigma} \cdot \vec{\nabla} \pi_a N$$



$$-\frac{1}{4f_\pi^2} N^\dagger \epsilon_{abc} \tau_a \pi_b \dot{\pi}_c N$$

Chiral NN potential: pion exchanges & contact interactions



- Long-range part
  - one-pion-exchange(OPE)

$$V_{1\pi}(\vec{p}\,',\vec{p}\,) = -\frac{g_A^2}{4f_\pi^2}\boldsymbol{\tau}_1\boldsymbol{\tau}_2\frac{\vec{\sigma}_1\cdot\vec{q}\,\vec{\sigma}_1\cdot\vec{q}}{\vec{q}^{\,2}+m_\pi^2} \qquad \sim \mathcal{O}(Q^0)$$

  - two-pion-exchanges $\cdots$
- Short-range interactions (counterterms)

$$C_0^{(s)}(N^T P_i^{(s)} N)^\dagger (N^T P_i^{(s)} N)\,,\; C_2^{(s)}(N^T P_i^{(s)} N)^\dagger (N^T P_i^{(s)} \overleftrightarrow{\nabla}^2 N)\;\cdots$$

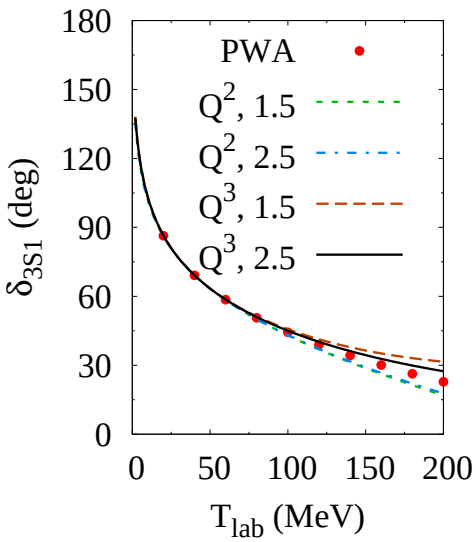
- Power counting(PC): Renormalization & PWA phase shifts

nonperturbative renormalization, not simple cancellation

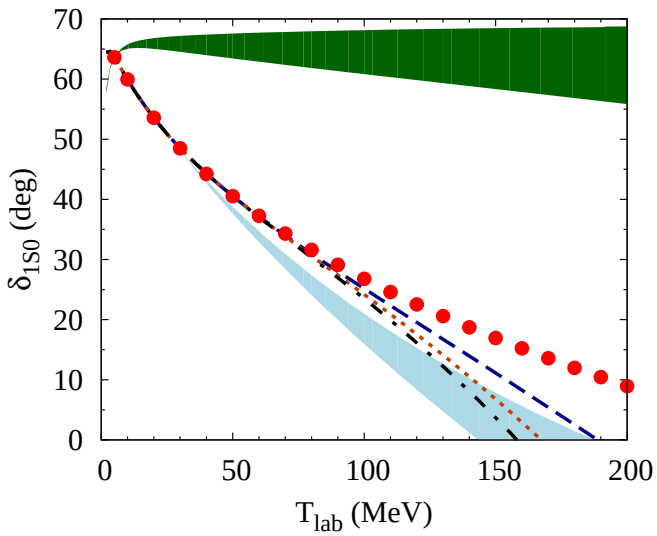
Nucleon-Nucleon(NN) elastic scattering

- LECs $(C_0^{(s)}, C_2^{(s)}, \cdots)$: fit to NN scattering data
- Organized by partial-wave quantum numbers: $s = {}^1S_0, {}^3S_1, {}^3P_0 \cdots$
- NN elastic scattering phase shift in each partial-wave

C_{3S1}



C_{1S0}

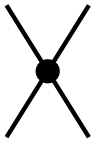


${}^3S_1$ and ${}^1S_0$

$^1S_0$ NN scattering

- Weinberg's power counting

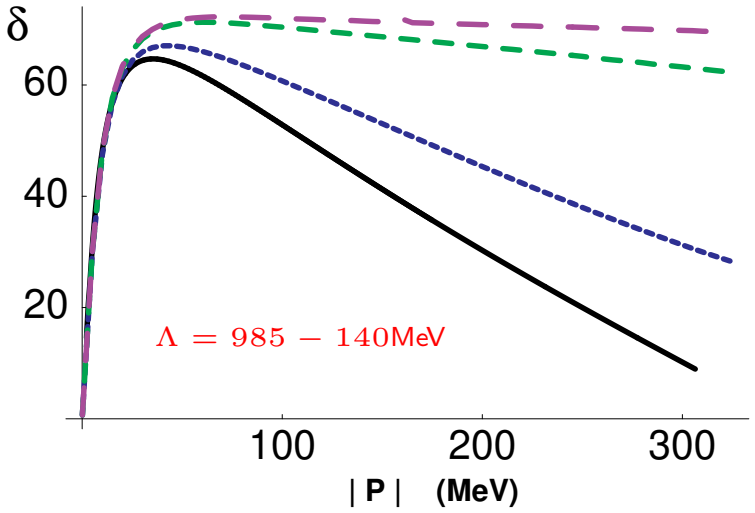
$$V^{(0)} = V_{1\pi} + C_0$$



$$V^{(1)} = 0$$

strong deviation from PWA

Beane et al., NPA **700** 377–402 (2002)

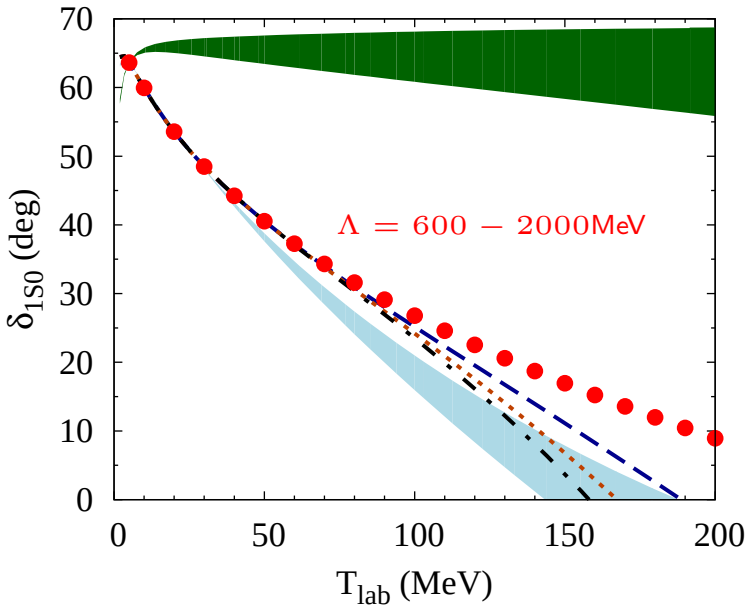


- Minimal Modified WPC(MMW)

$$V^{(1)} = \frac{C_2^{(0)}}{2}(p^2 + p'^2)$$

slow convergence

B. Long, C-J. Yang, PRC **86** 02400 (2012)



$^1S_0$ NN scattering

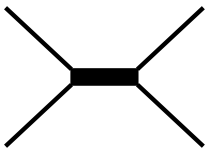
- OPE is weak in $^1S_0$ channel
- C_0 is independent of k
- Large scattering length and effective range

D. Kaplan, PRC **60** 064002 (1999)

Beane et al., NPA **700** 377–402 (2002)

- Dibaryon formalism

$$V^{(0)} = V_{1\pi} + \frac{\sigma y^2}{E + \Delta}$$

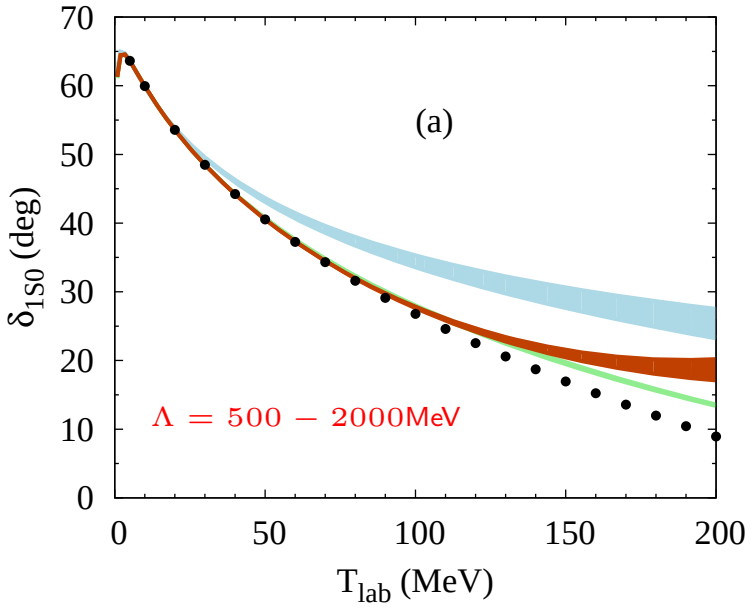


B. Long, PRC **88** 014002 (2013)

  - less attractive with higher k

energy dependent

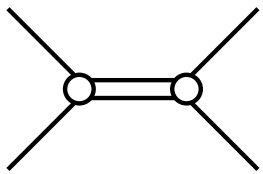
short-range repulsion and long-range attraction



Unnatural dibaryon potential

- Potential

$$V_{\text{spr}}^{(0)} = -\frac{4\pi}{m_N} \frac{\lambda}{\sqrt{p^2 + m_N \Delta}\sqrt{p'^2 + m_N \Delta}}$$

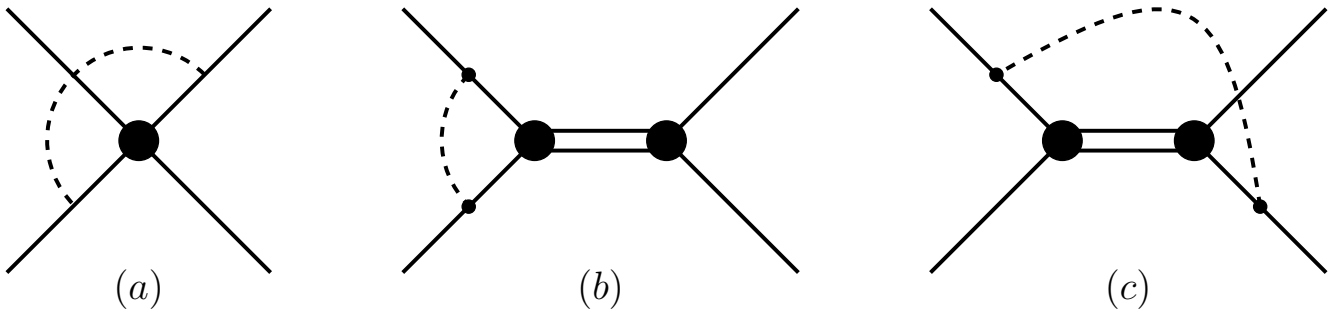


  - stationary dibaryon: momentum dependent
  - two parameters: λ and $\sqrt{m_N \Delta} \sim m_\pi$
  - tree level: two Dibaryon potentials have same form when nucleons are on-shell

Beane S.R., Farrell R.C., Few-Body Syst **63** 45 (2022)

- Not just another model
  - order-by-order convergence (NLO, NNLO)
  - compatible with chiral Lagrangian
  - renormalization-group (RG) invariance at each order

Radiative corrections at N^{2}LO



- Non-trivial radiative corrections at N^{2}LO
  - long range forces
  - contribute to triplet channels
- a: contact vertices: trivial result, polynomials in momenta
- b: $^1S_0$ channel, c: triplet channels
- nontrivial radiation-pion: virtual pion as third particle, test the potential

Power counting

- Subleading orders calculated perturbatively

$$T^{(0)} = V^{(0)} + V^{(0)} G_0 T^{(0)} \,,$$
$$T^{(1)} = (1 + T^{(0)} G_0) V^{(1)} (G_0 T^{(0)} + 1)$$

- ${}^1S_0$ channel

$$V^{(0)} = V^{(0)}_{\text{spr}} + V_{1\pi}$$
$$V^{(1)} = C^{(0)}_0 + V^{(1)}_{\text{spr}}$$
$$V^{(2)} = C^{(2)}_0 + \frac{C^{(0)}_2}{2}(p^2 + p'^2) + V^{(2)}_{\text{spr}} + V_{\text{TPE0}} + V^{\text{1S0}}_{\text{rad}}$$

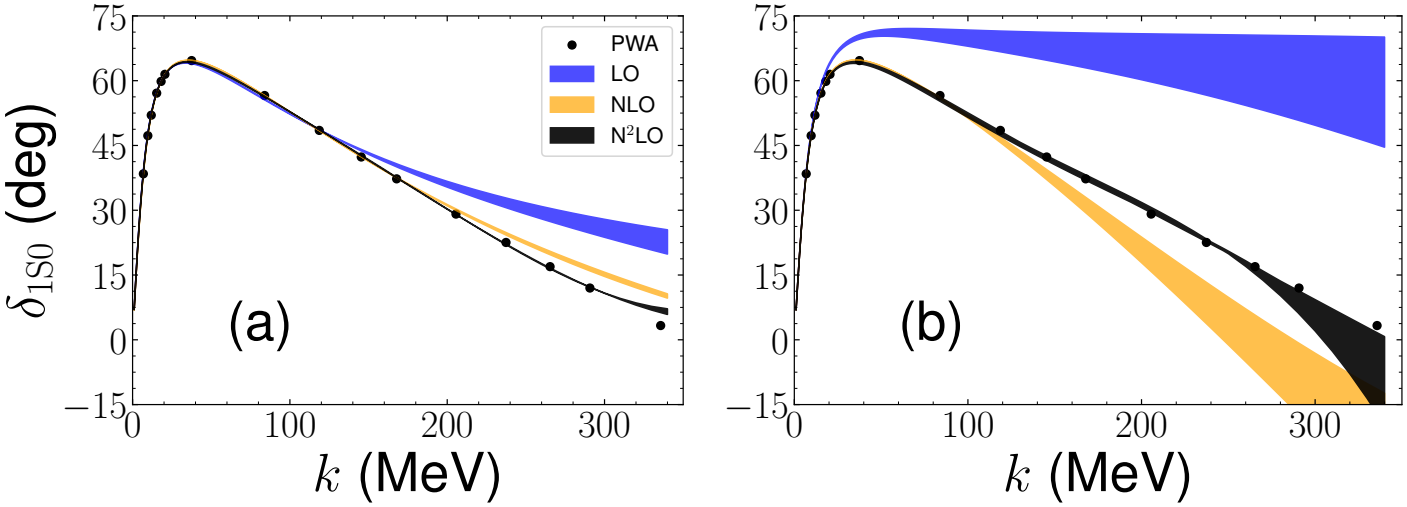
- ${}^3S_1 - {}^3D_1$ channel

$$V^{(0)} = C^{(0)}_0 + V_{1\pi}$$
$$V^{(2)} = C^{(2)}_0 + \frac{D^{(0)}}{2}(p^2 + p'^2) + E^{(0)} p^2 (p'^2) + V_{\text{TPE0}} + V^{\text{3S1}}_{\text{rad}}$$

Phase shifts

- $^1S_0$ channel fit to PWA empirical phase shifts

$\Lambda = 600 - 2400$ MeV

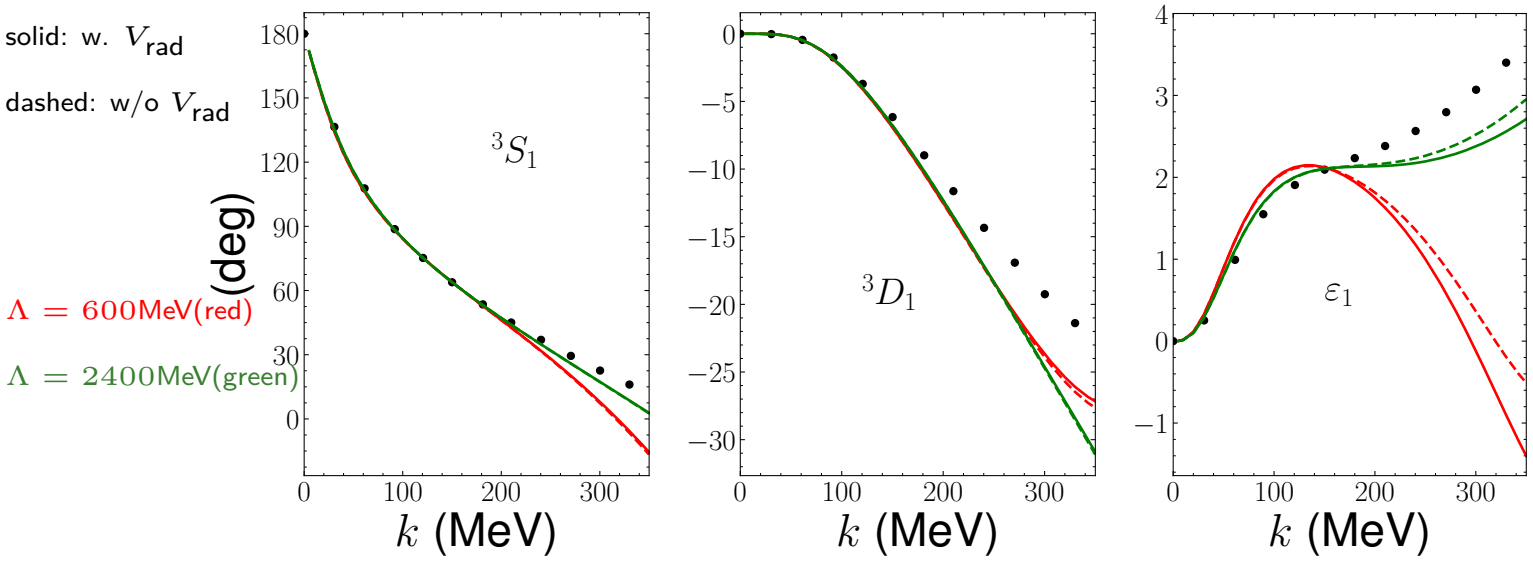


- Rapid order-by-order convergence: comparable to energy-dependent dibaryon potential

- Radiative correction has no 'bad' effects

- Mild sensitivity to the cutoff Λ: UV suppression from $(p^2 + m_N\Delta)^{-\frac{1}{2}}$

Phase shift in $^3S_1 - {}^3D_1$ channel



- Radiative corrections didn't ruin MMW results
- Insensitive to the cutoff when the cutoff is sufficiently large
- $^3P_0, {}^3P_1, {}^3D_2$: negligible within numerical precision

Triton binding energy

$B_{3\mathrm{H}}^{\mathrm{exp}} = -8.48\mathrm{MeV}$

	SEP		MMW	
Λ (MeV)	LO	NLO	LO	NLO
400	-8.67	-8.50	-11.02	-7.68
600	-6.10	-6.10	-6.46	-6.90
800	-5.57	-5.58	-5.30	-6.64
1600	-5.37	-5.51	-4.89	-6.49

- Solving Faddeev equations
- Power counting: partially-perturbative pions B. Long, PRC **99** 024003 (2019)
- Separable NLO correction is much smaller

  triton is relatively shallow: $\sqrt{2m_N B_{3\mathrm{H}}/3} \sim 73\mathrm{MeV}$

- Light nuclei results: M. Sanchez Sanchez et al., PRC **102** 024324 (2020)
  C. J. Yang et al., PRC **103** 054304 (2021)

Chiral symmetry

- Lagrangian

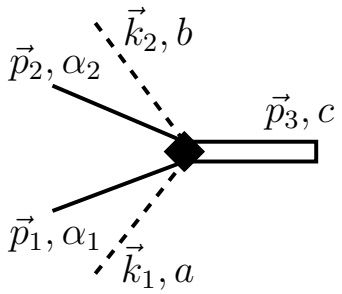
$$\frac{g_{2n}}{g} = \binom{-\frac{1}{2}}{n}, \ \lambda \equiv \sigma m_N g^2/4$$

$$\mathcal{L} = \sigma \Delta \phi_a^\dagger \phi_a - \sqrt{\frac{4\pi}{m_N}} \sum_{n=0}^{\infty} \frac{g_{2n}}{2} \left[\phi_a^\dagger N^T P_a \left(\frac{-\overleftrightarrow{\nabla}^2}{4 m_N \Delta} \right)^n N + h.c. \right] + \cdots$$

- Chiral-covariant derivative

$$\mathscr{D}_i N \equiv \left(\nabla_i + \frac{\boldsymbol{\tau}}{2} \cdot \boldsymbol{E}_i \right) N, \quad \boldsymbol{E}_i \equiv i \frac{\boldsymbol{\pi}}{f_\pi} \times \boldsymbol{D}_i, \quad \boldsymbol{D}_i \equiv (1 + \frac{\boldsymbol{\pi}^2}{4 f_\pi^2})^{-1} \frac{\nabla_i \boldsymbol{\pi}}{2 f_\pi}$$

- Non-trivial chiral connections: $NN\pi\pi \to \phi$ vertex



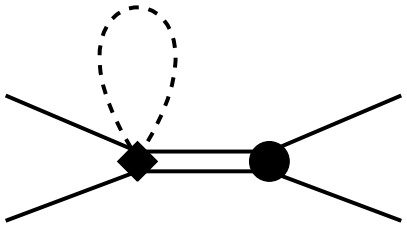
$$\mathcal{A}_{\phi N N \pi\pi} = \frac{ig}{4 f_\pi^2} \sqrt{\frac{4\pi}{m_N}} \Big\{ i \left(\delta_{bc} \mathcal{P}_a - \delta_{ac} \mathcal{P}_b \right)_{\alpha_2 \alpha_1} \mathcal{B}_+ + \frac{1}{\sqrt{8}} \epsilon_{abc} (\sigma_2 \tau_2)_{\alpha_2 \alpha_1} \mathcal{B}_- \Big\}$$

$$\mathcal{B}_\pm \equiv u \left(\left| \vec{p} + \vec{k}_1/2 \right|, \left| \vec{p} + \vec{k}_2/2 \right| \right) \pm u \left(\left| \vec{p} - \vec{k}_1/2 \right|, \left| \vec{p} - \vec{k}_2/2 \right| \right),$$

$$u(x, y) \equiv \left(1 + \frac{x^2}{m_N \Delta} \right)^{-\frac{1}{2}} - \left(1 + \frac{y^2}{m_N \Delta} \right)^{-\frac{1}{2}}.$$

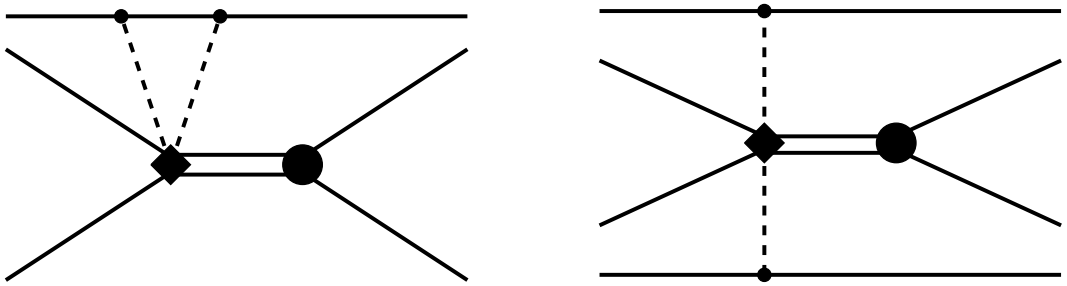
Chiral symmetry

- Contribution to NN potential at N^2LO



vanish

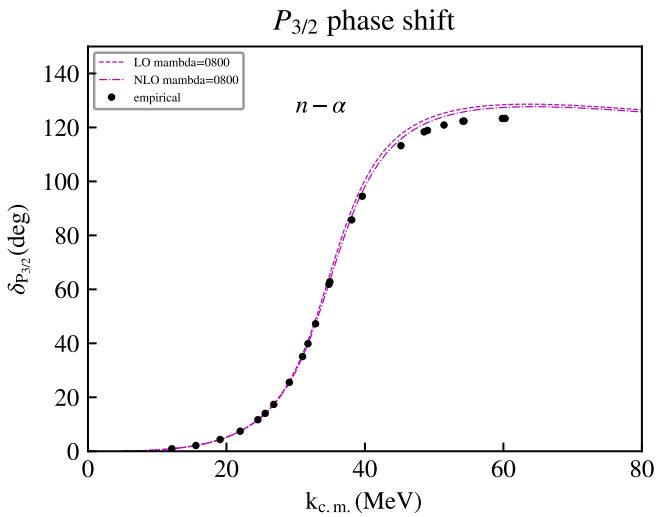
- Contributions to 3- and 4-body forces

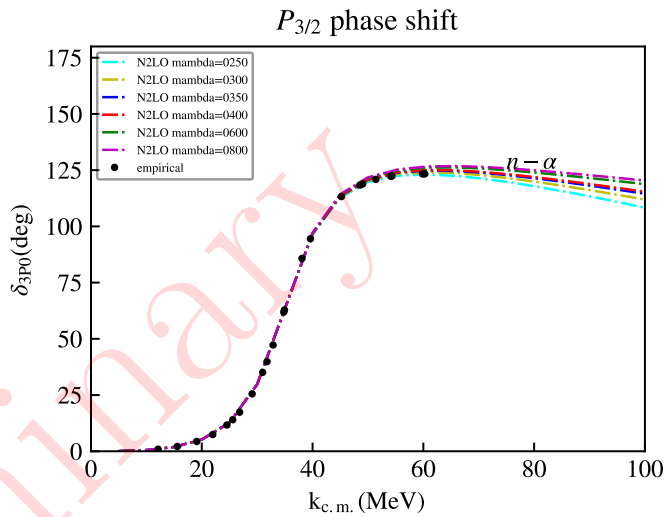


Both enter at N^4LO

- The unnatural LO potential doesn't affects the existing convergence

Shallow P-wave resonance





- P-wave potential

$$V_p^{(0)}(p',p) = -\frac{4\pi}{m}\frac{\lambda p' p}{\sqrt{p'^2+m\Delta}\sqrt{p^2+m\Delta}}$$

- neutron-α scattering phase shift

- More P-wave interacted neutrons: ^{6}He $\cdots$

Summary

- A new formulation of chiral EFT to improve the slow convergence of the $^1S_0$ NN phase shifts and facilitate many-body calculation
- Chiral symmetry is checked: pionic radiative corrections and chiral connection terms
- Extension to P-wave Halo nuclei
- Other approaches

  Covariant ChPT: Jun-Xu Lu, et al., PRL **128** 142002(2022)

  TPE as LO: Mishra, et al., PRC **106** 024004(2022)

Thanks ...

... to my collaborators:

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... and for your attention!