第七届手征有效场论研讨会

2022 年 09 月 15 日 — 17 日， 南京（线上）

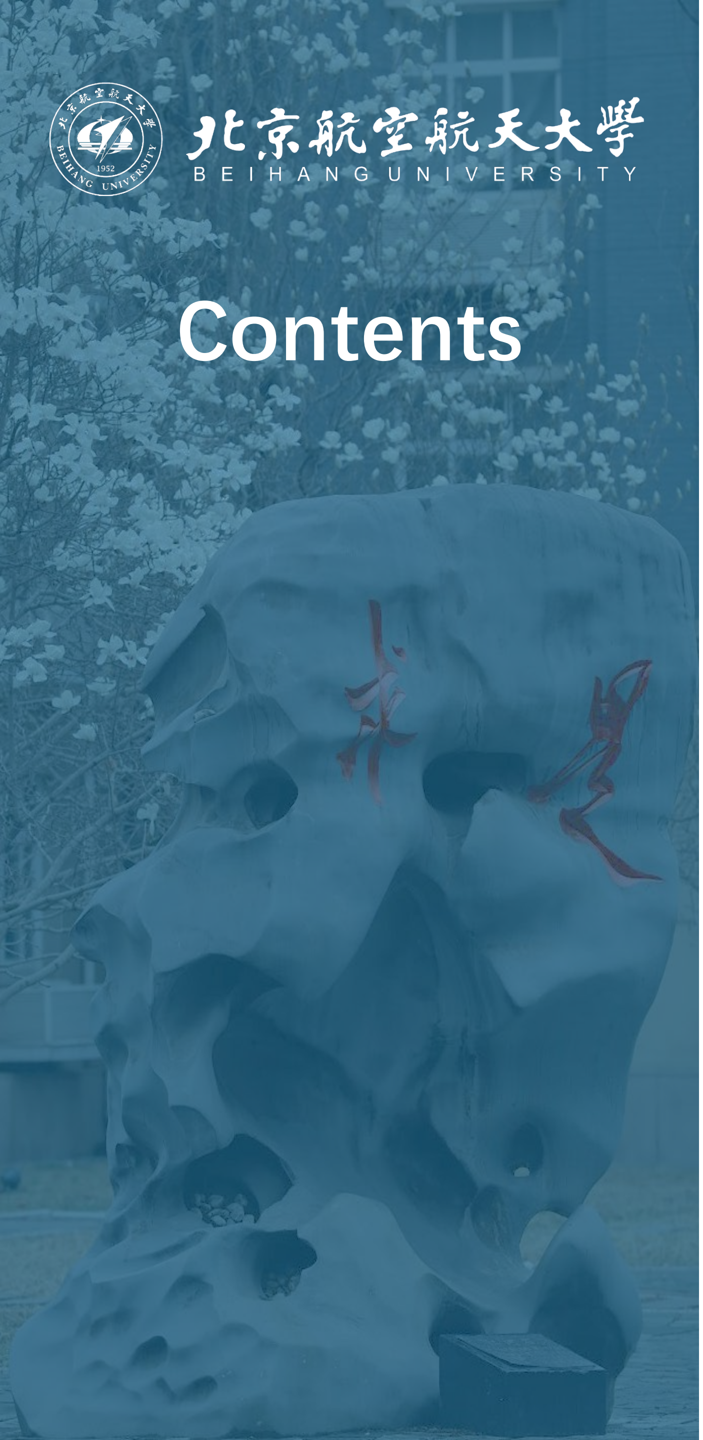
Strangeness S = -2 BB interactions and femtoscopic correlation functions in relativistic ChEFT

Zhi-Wei Liu（刘志伟）

School of Physics, Beihang University, China

Oct. 15th, 2022

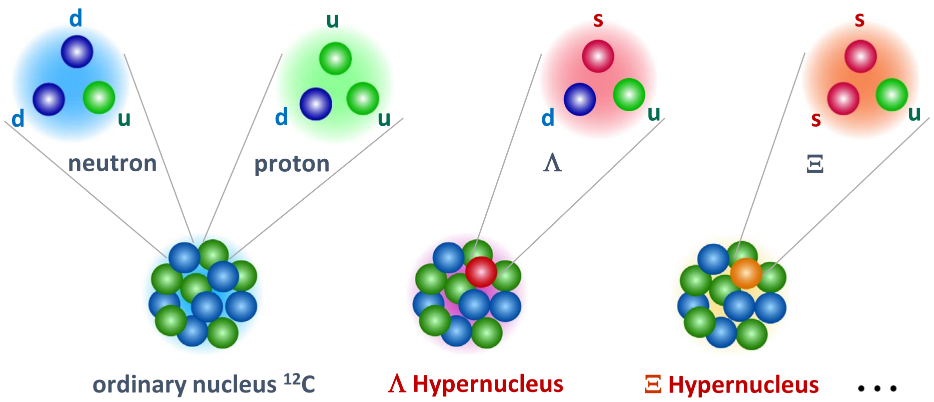
Reference:
Z. W. Liu, K. W. Li, L. S. Geng*, arXiv:2201.04997 *(accepted for publication in CPC)*



Contents

YN and YY interactions

- **Fundamental inputs to hypernuclear physics and nuclear astrophysics**



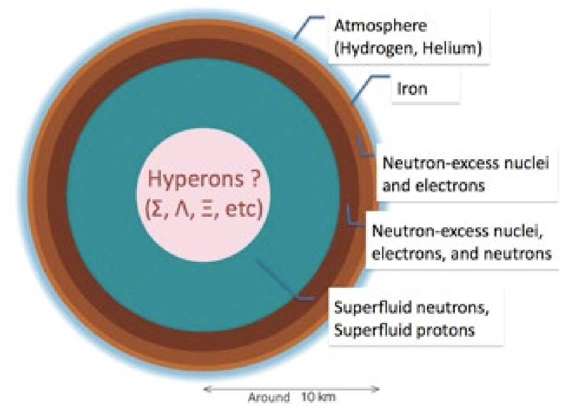


Figure from KEK

- Important open questions

A bound H-dibaryon (uuddss) ?	*R. L. Jaffe, Phys. Rev. Lett.* ***38*** *(1977) 195* *T. Inoue et al., Phys. Rev. Lett.* ***106*** *(2011) 162002*
Spin-Dependent ΛN CSB ?	*T. O. Yamamoto et al., Phys. Rev. Lett.* ***115*** *(2015) 222501*
Hyperon puzzle in neutron star ?	*D. Lonardoni et al., Phys. Rev. Lett.* ***114*** *(2015) 092301*
…	

Research status

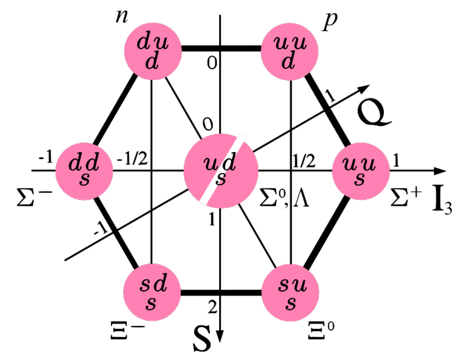
- **Scattering Experiment**
  - S= 0 (NN) : an amount of high-quality scattering data
  - S=-1 (ΛN, ΣN) : small quantity

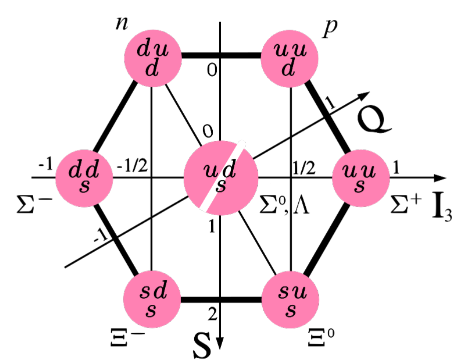
    *Engelmann, et al., Phys. Lett. **21** (1966) 587*
    *G. Alexander, et al., Phys. Rev. **173** (1968) 1452*
    *B. Sechi-Zorn, et al., Phys. Rev. **175** (1968) 1735*
    *F. Eisele, et al., Phys. Lett. **37B** (1971) 204*
    *V. Hepp and H. Schleich, Z. Phys. **214** (1968) 71*
    CLAS Collaboration, Phys. Rev. Lett. 127 (2021) 272303
    J-PARC E40 Collaboration, Phys. Rev. Lett. 128 (2022) 072501
  - S=-2 (ΛΛ, ΞN, ΛΣ, ΣΣ) : scarcity

    *J. K. Ahn et al., Phys. Lett. B **633** (2006) 214*
  - S=-3 (ΞΛ, ΞΣ) : complete lack of scattering data
  - S=-4 (ΞΞ) : complete lack of scattering data

lack of exp. constrains

Future: Sup. J/ψ factory





- **Theoretical Description**
  - One-boson-exchange model

    *V. G. J. Stoks and T. A. Rijken, Phys. Rev. C **59** (1999) 3009*
  - SU(6) quark cluster model

    *Y. Fujiwara, Y. Suzuki, and C. Nakamoto, Prog. Part. Nucl. Phys. **58** (2007) 439*
  - Non-relativistic Chiral effective field theory

    *H. Polinder, J. Haidenbauer and U.-G. Meißner, Nucl. Phys. A **779** (2006) 244*
    *J. Haidenbauer, U.-G. Meißner and S. Petschauer, Nucl. Phys. A **954** (2016) 273*
    *J. Haidenbauer and U.-G. Meißner, Phys. Lett. B **684** (2010) 275*

model dependent

New progress in experiment: CFs

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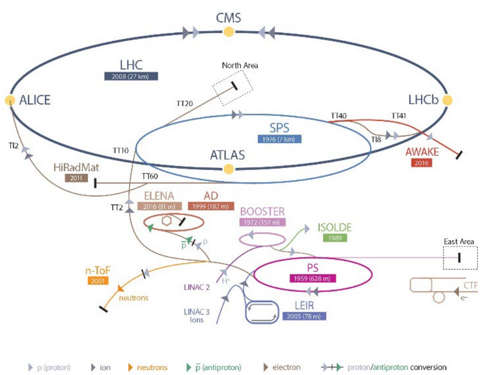
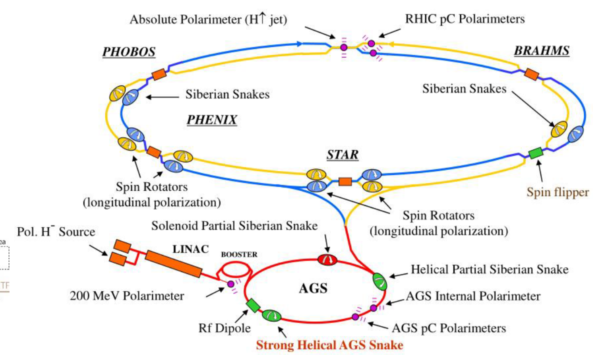
Large Hadron Collider



Brookhaven National Laboratory

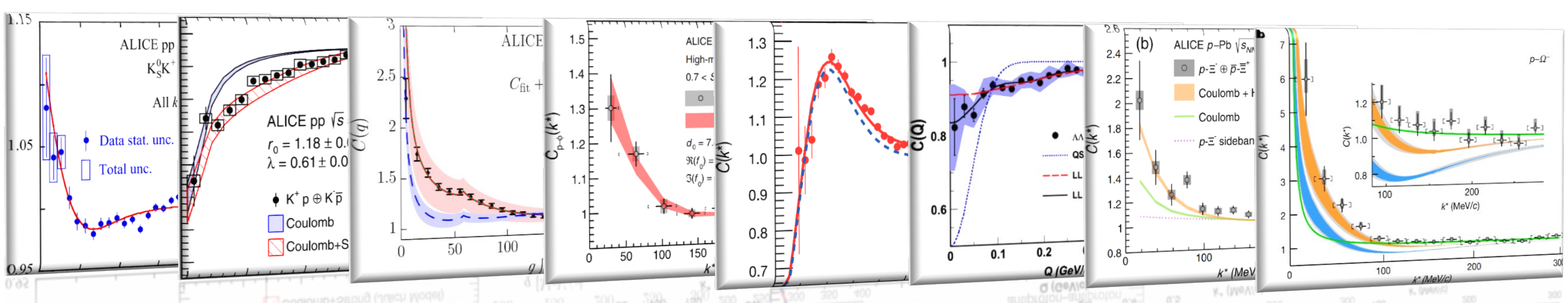
RHIC Relativistic Heavy Ion Collider



Experimental correlation function

- Relativistic heavy-ion collisions can produce hadrons with strange quarks in abundance.
- Correlation function can be used to probe the short-range nature of the strong interaction.
- The capabilities of new detector are excellent enough in identifying particle and measuring their momenta.

$K^0_S K^\pm$ $K^\pm p$ $K^- p$ ϕp $\bar{p}\,\bar{p}$ $\Lambda\Lambda$ $\Xi^- p$ $\Omega^- p$



$K^- d$
$\Sigma^0 p$
$\Lambda\Xi$
$\Xi\Xi$
$\Omega\Omega$
pD
ppp
$pp\Lambda$

ALICE Collaboration, Phys. Lett. B **790** (2019) 22
ALICE Collaboration, Phys. Rev. Lett. **124** (2020) 092301
Y. Kamiya and et al., Phys. Rev. Lett. **124** (2020) 132501
ALICE Collaboration, Phys. Rev. Lett. **127** (2021) 172301
STAR Collaboration, Nature **527** (2015) 345
STAR Collaboration, Phys. Rev. Lett. **114** (2015) 022301
ALICE Collaboration, Phys. Rev. Lett. **123** (2019) 112002
ALICE Collaboration, Nature **588** (2020) 232
L. Fabbietti, Ann. Rev. Nucl. Part. Sci. **71** (2021) 377

New progress for S=-2 sector

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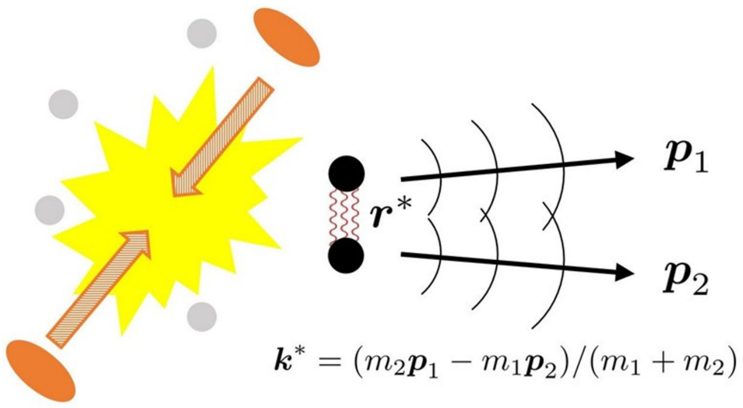
- **Experimental Correlation Functions**
  - $\Lambda\Lambda$ correlation functions

    STAR Collaboration, Phys. Rev. Lett. ***114*** *(2015) 022301*
    ALICE Collaboration, Phys. Rev. C ***99*** *(2019) 024001*
    ALICE Collaboration, Phys. Lett. B ***797*** *(2019) 134822*
  - ΞN correlation functions

    ALICE Collaboration, Phys. Rev. Lett. ***123*** *(2019) 112002*
    ALICE Collaboration, Nature ***588*** *(2020) 232*

- **Lattice QCD Simulation**
  - $\Lambda\Lambda$ interaction
  - ΞN interaction

  K. Sasaki et al. (HAL QCD Collaboration), Nucl. Phys. A ***998*** *(2020) 121737*



Emission source $S_{12}(r^*)$ Two-particle wavefunction $\psi(k^*, r^*)$

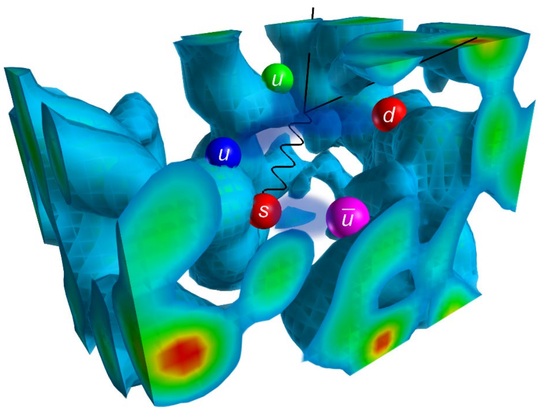


Figure from D. B. Leinweber.

opportunity

Constraints on S = −2 BB ($\Lambda\Lambda$, ΞN, $\Lambda\Sigma$, $\Sigma\Sigma$) interactions from experiment and first-principle

Relativistic ChEFT

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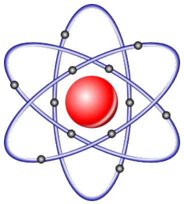
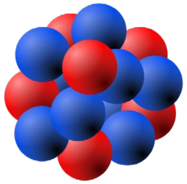
- **Advantages of ChEFT**
  - ✓ improve calculations systematically
  - ✓ estimate theoretical uncertainties
  - ✓ consistent treatment of three- and four-baryon interactions

S. Weinberg, Phys. Lett. B ***251*** *(1990) 288*
S. Weinberg, Nucl. Phys. B ***363*** *(1991) 3*

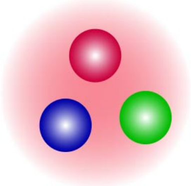
- **Why relativistic ? (kinematical effect + dynamical effect)**



atom
~ 10^{-8}cm



nucleus
~ 10^{-12}cm



baryon
~ 10^{-13}cm

- ✓ liquid mercury
- ✓ yellow gold
- …

- ✓ large spin-orbit splitting
- ✓ pseudo-spin symmetry
- ✓ consistent time-odd fields
- ✓ connection to QCD
- ✓ relativistic saturation mechanism
- ✓ covariance restricts parameters
- …

- ✓ octet baryon mass
- ✓ magnetic moments
- ✓ vector and axial couplings
- …

- ✓ Development of relativistic many-body methods

motivation

Using **lattice QCD simulation and experimental CFs**
to construct and test the **chiral S = −2 BB interactions in relativistic ChEFT**

SU(3) Rel. ChEFT

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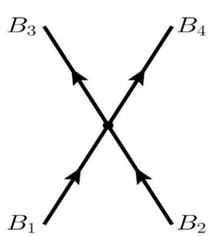
Contact-Terms potentials



$$\mathcal{L}_{\rm CT} = \sum_{i=1}^{5}\left[\frac{\tilde{C}_i^1}{2}\,\mathrm{tr}\left(\bar{B}_1\bar{B}_2(\Gamma_i B)_2(\Gamma_i B)_1\right) + \frac{\tilde{C}_i^2}{2}\,\mathrm{tr}\left(\bar{B}_1(\Gamma_i B)_1\bar{B}_2(\Gamma_i B)_2\right) + \frac{\tilde{C}_i^3}{2}\,\mathrm{tr}\left(\bar{B}_1(\Gamma_i B)_1\right)\mathrm{tr}\left(\bar{B}_2(\Gamma_i B)_2\right)\right]$$

$$\Gamma_1 = 1,\qquad \Gamma_2 = \gamma^\mu,\qquad \Gamma_3 = \sigma^{\mu\nu},\qquad \Gamma_4 = \gamma^\mu\gamma_5,\qquad \Gamma_5 = \gamma_5.$$

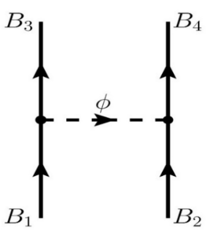
Dirac spinor

$$u_B(p,s) = N_p \begin{pmatrix} 1 \\ \frac{\sigma\cdot p}{E_p+M_B} \end{pmatrix} \chi_s$$

$$N_p = \sqrt{\frac{E_p+M_B}{2M_B}}$$

Chin. Phys. C **42** (2018) 014105

One-Pseudoscalar-Meson-Exchange potentials



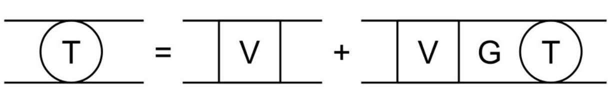
$$\mathcal{L}_{MB}^{(1)} = \mathrm{tr}\left(\bar{B}\left(i\gamma_\mu D^\mu - M_B\right)B - \frac{D}{2}\bar{B}\gamma^\mu\gamma_5\{u_\mu, B\} - \frac{F}{2}\bar{B}\gamma^\mu\gamma_5[u_\mu, B]\right)$$

$$B = \begin{pmatrix} \frac{\Sigma^0}{\sqrt{2}} + \frac{\Lambda}{\sqrt{6}} & \Sigma^+ & p \\ \Sigma^- & -\frac{\Sigma^0}{\sqrt{2}} + \frac{\Lambda}{\sqrt{6}} & n \\ \Xi^- & \Xi^0 & -\frac{2\Lambda}{\sqrt{6}} \end{pmatrix} \qquad \phi = \begin{pmatrix} \frac{\pi^0}{\sqrt{2}} + \frac{\eta}{\sqrt{6}} & \pi^+ & K^+ \\ \pi^- & -\frac{\pi^0}{\sqrt{2}} + \frac{\eta}{\sqrt{6}} & K^0 \\ K^- & \bar{K}^0 & -\frac{2\eta}{\sqrt{6}} \end{pmatrix}$$

Partial wave analysis

momentum space —> helicity basis —> $|JM\rangle$ basis —> $|LSJ\rangle$ basis

Coupled-channel Kadyshevsky Equation



$$T_{\rho\rho'}^{\nu\nu',J}(\boldsymbol{p}',\boldsymbol{p};\sqrt{s}) = V_{\rho\rho'}^{\nu\nu',J}(\boldsymbol{p}',\boldsymbol{p}) + \sum_{\rho'',\nu''}\int_0^\infty \frac{\mathrm{d}p''p''^2}{(2\pi)^3}\frac{M_{B_{1,\nu''}}M_{B_{2,\nu''}}V_{\rho\rho''}^{\nu\nu'',J}(\boldsymbol{p}',\boldsymbol{p}'')T_{\rho''\rho'}^{\nu''\nu',J}(\boldsymbol{p}'',\boldsymbol{p};\sqrt{s})}{E_{1,\nu''}E_{2,\nu''}(\sqrt{s}-E_{1,\nu''}-E_{2,\nu''}+i\epsilon)}$$

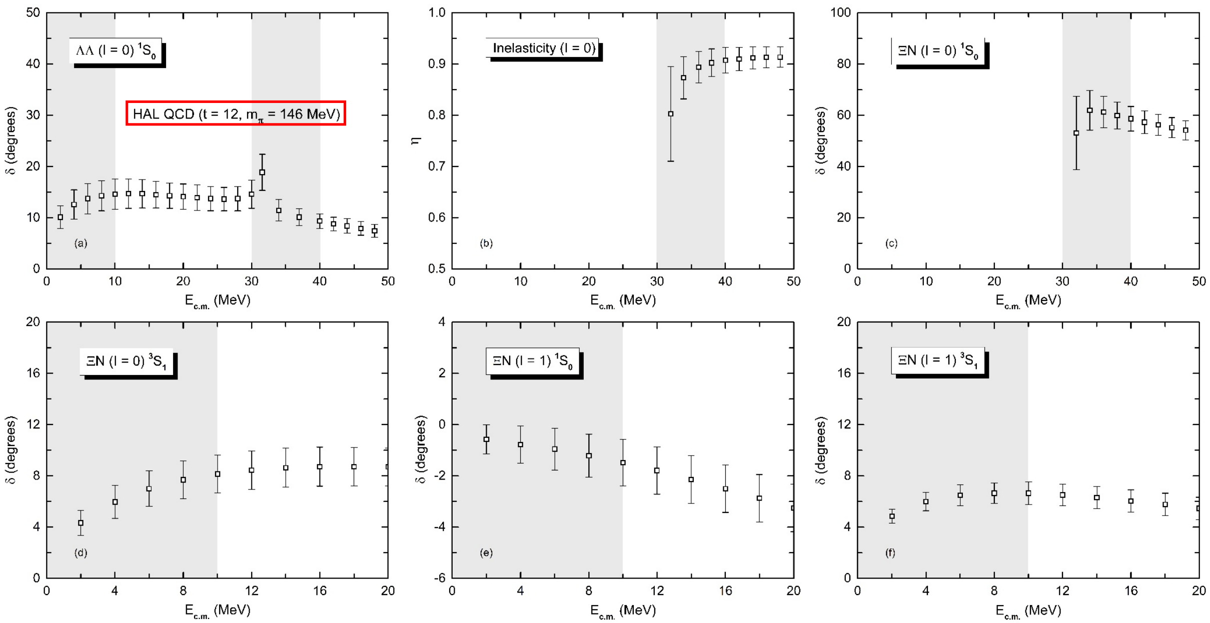
$$f_{\Lambda_F}(\boldsymbol{p},\boldsymbol{p}') = \exp\left[-\left(\frac{\boldsymbol{p}}{\Lambda_F}\right)^4 - \left(\frac{\boldsymbol{p}'}{\Lambda_F}\right)^4\right] \qquad \Lambda_F = 550\text{–}700\ \mathrm{MeV}$$

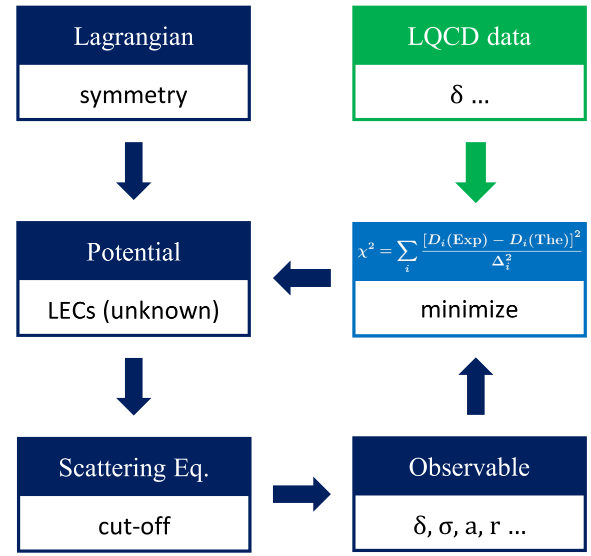
JXL, QQB @ 17th pm

Fits to the S = -2 YN/YY LQCD data

YN and YY lattice QCD phase shifts (data in the gray region is used)

K. Sasaki et al. (HAL QCD Collaboration), Nucl. Phys. A ***998*** *(2020) 121737*





Low-energy constants (LECs) for S=-2 system

Λ_F	$C^{\Lambda\Lambda}_{1S0}$	$C^{\Sigma\Sigma}_{1S0}$	$C^{\Lambda\Lambda}_{3S1}$	$C^{\Sigma\Sigma}_{3S1}$	$C^{\Lambda\Sigma}_{3S1}$	$C^{4\Lambda}_{1S0}$	$\hat{C}^{\Lambda\Lambda}_{1S0}$	$\hat{C}^{\Sigma\Sigma}_{1S0}$	$\hat{C}^{\Lambda\Lambda}_{3S1}$	$\hat{C}^{\Sigma\Sigma}_{3S1}$	$\hat{C}^{\Lambda\Sigma}_{3S1}$	$\hat{C}^{4\Lambda}_{1S0}$	χ^2/d.o.f.
550	−0.0274	−0.0412	−0.0078	0.0255	0.0024	−0.0242	2.3493	2.5353	1.3695	1.0552	−0.0423	1.9485	0.366
600	−0.0175	−0.0300	−0.0076	0.0472	0.0026	−0.0176	2.0832	2.2246	1.0521	1.1759	0.0793	1.8207	0.333
650	−0.0049	−0.0169	−0.0070	0.0720	0.0026	−0.0075	1.9847	2.0755	0.8493	1.1768	0.0793	1.8207	0.324
700	0.0089	−0.0053	−0.0064	0.1049	0.0026	0.0066	1.8566	1.8869	0.7072	1.1768	0.0793	1.8206	0.333

Fits to the S = -2 YN/YY LQCD data

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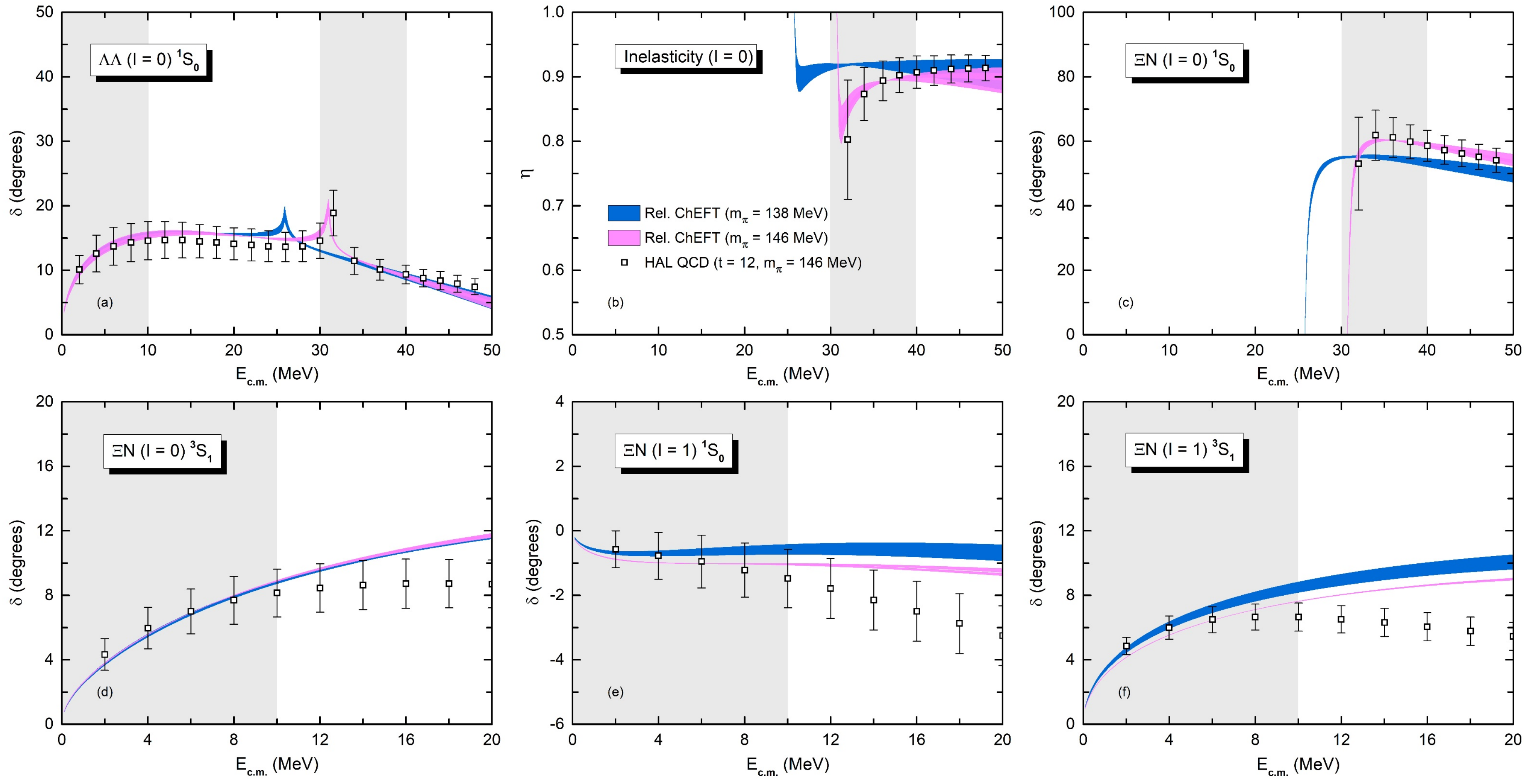
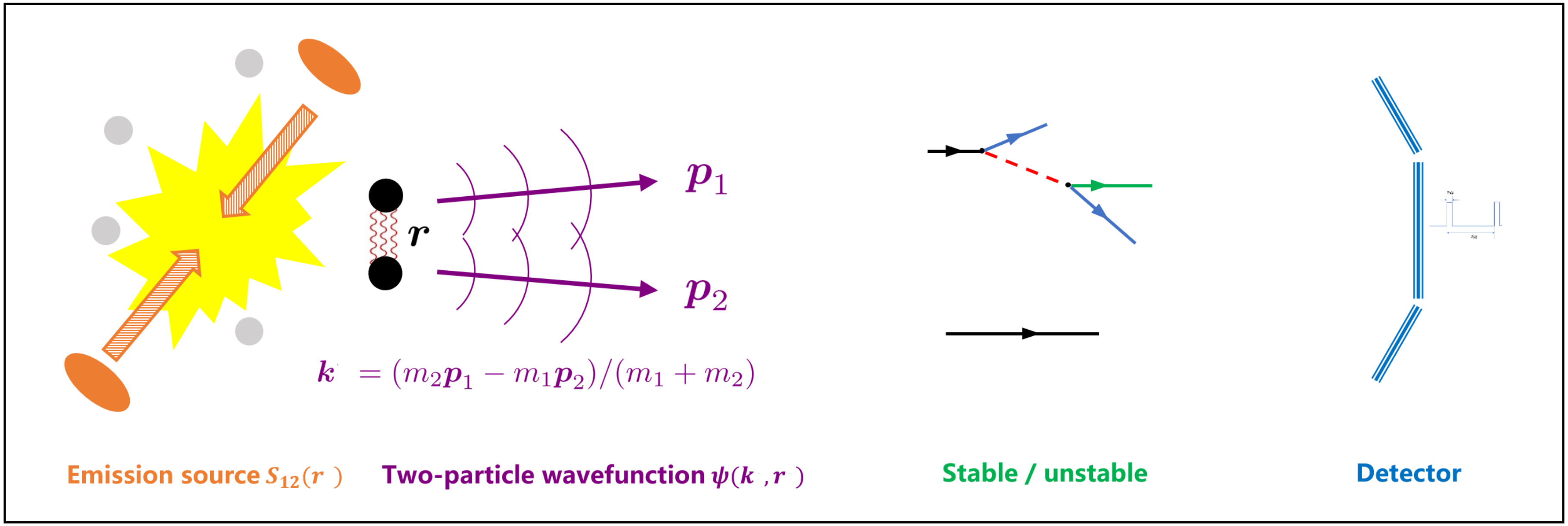


✓ Based on the full coupled-channel framework, relativistic ChEFT can describe LQCD S-wave phase shifts rather well.

Correlation functions



Theo. description
Koonin–Pratt formula

Exp. measurement
mixed-event technique

$$C(k) = \int S_{12}(\boldsymbol{r})\ |\psi(\boldsymbol{k},\boldsymbol{r})|^2\ \mathrm{d}\boldsymbol{r} = \xi(k)\frac{N_{\mathrm{same}}(k)}{N_{\mathrm{mixed}}(k)}$$

- **>1 if the interaction is attractive**
- **=1 if there is no interaction**
- **<1 if the interaction is repulsive**

$S_{12}(\boldsymbol{r})$: **spacial structure**

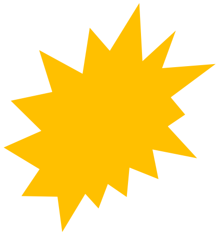
$|\psi(\boldsymbol{k},\boldsymbol{r})|^2$:
- **final-state interactions**
- **quantum statistics effects**
- **coupled-channel effects**

$N_{\mathrm{same}}(k)/N_{\mathrm{mixed}}(k)$: **the same and mixed event distributions**

$\xi(k)$: **the corrections for experimental effects**

Correlation functions

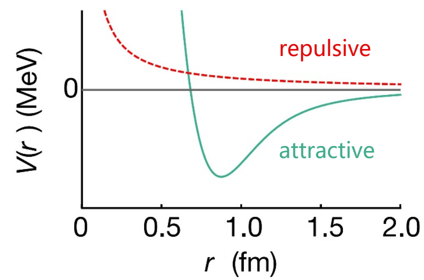
Source parametrisation



Gaussian source

$$S_{12}(r) = (4\pi R^2)^{-3/2} \cdot \exp\left(\frac{-r^2}{4R^2}\right)$$

Interacting potential

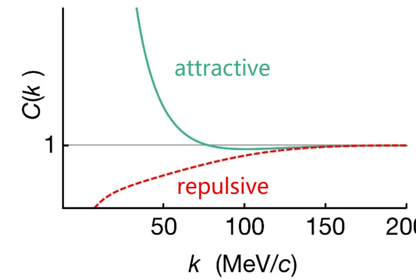


Schrödinger equation

↓

Two-particle wavefunction $\psi(k, r)$

Correlation function



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$S_{12}(\boldsymbol{r})$: **spacial structure**

$|\psi(\boldsymbol{k},\boldsymbol{r})|^2$:
- **final-state interactions**
- **quantum statistics effects**
- **coupled-channel effects**

$\frac{N_{\text{same}}(k)}{N_{\text{mixed}}(k)}$: **the same and mixed event distributions**

$\xi(k)$: **the corrections for experimental effects**

Theoretical details

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Koonin–Pratt (KP) formula

S. E. Koonin, Phys. Lett. B **70** (1) (1977) 43
A. Ohnishi, Nucl. Phys. A **954** (2016) 294

$$C(k) = \int S_{12}(r)\ |\Psi(\boldsymbol{r},\boldsymbol{k})|^2\ \mathrm{d}\boldsymbol{r}$$

- Relative wave function in the two-body outgoing state (consider only correlations in S-waves)

$$\Psi(\boldsymbol{r},\boldsymbol{k}) = e^{i\boldsymbol{k}\cdot\boldsymbol{r}} - j_0(kr) + \psi_0(r,k), \qquad \psi_0(r,k) \overset{r\to\infty}{\longmapsto} \frac{1}{2ikr}\left[e^{ikr} - e^{-2i\delta}e^{-ikr}\right]$$

- Correlation function for *non-identical particles without Coulomb interaction*

$$C(k) \simeq 1 + \int_0^\infty 4\pi r^2 \mathrm{d}r\ S_{12}(r)\ \left[|\psi_0(r,k)|^2 - |j_0(kr)|^2\right]$$

Scattering wave function

J. Haidenbauer, Nucl. Phys. A **981** (2019) 1

- Exploiting the relation $|\psi\rangle = |\phi\rangle + G_0 V|\psi\rangle, \quad V|\psi\rangle = T|\phi\rangle$, T-matrix from the Kadyshevsky Eq.

$$\psi_{\beta\alpha;l}(r) = \delta_{\beta\alpha} j_l(k_\alpha r) + \frac{1}{\pi}\int \mathrm{d}q q^2 \frac{1}{\sqrt{s} - E_{\beta,1}(q) - E_{\beta,2}(q) + i\varepsilon} \cdot T_{\beta\alpha;l}(q,k_\alpha;\sqrt{s}) \cdot j_l(qr)$$

- Coupled-channel effect (sum over the outgoing channels)

$$|\psi_0(r,k)|^2 \to \sum_\beta \omega_\beta |\psi_{\beta\alpha;0}(r)|^2$$

ΛΛ correlation function

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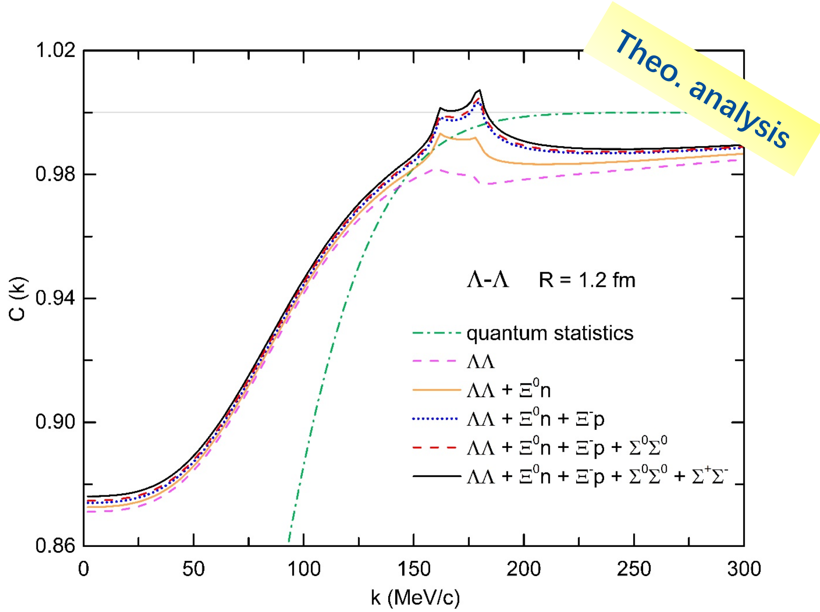
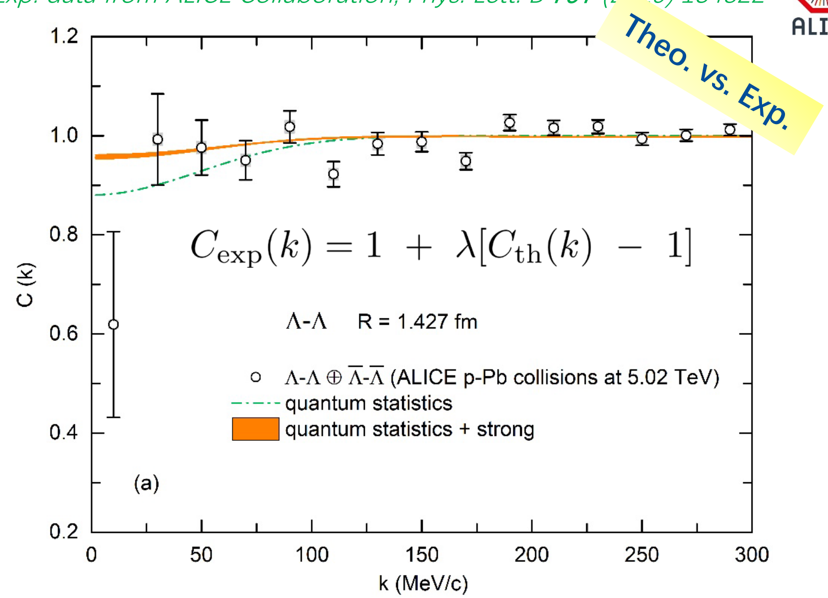
- **Correlation function for *identical neutral particles***

$$C_{\Lambda\Lambda}(k) \simeq 1 - \frac{1}{2}e^{-4k^2R^2} + \frac{1}{2}\int_0^\infty 4\pi r^2 \mathrm{d}r\ S_{12}(r)\ \left[|\psi_0(r,k)|^2 - |j_0(kr)|^2\right]$$

*Exp. data from ALICE Collaboration, Phys. Lett. B **797** (2019) 134822*

ALICE





- ✓ **There is an enhancement of the $C_{\Lambda\Lambda}$ due to the attractive strong interaction in the low-momentum region.**
- ✓ **The openings of the inelastic $\Xi^0 n$ and $\Xi^- p$ channels are remarkable as two cusp-like structures occurring at corresponding thresholds.**
- ✓ **The agreement of the orange (shaded) band with experimental data indicates a weak attraction in the ΛΛ channel, which rules out a deep bound state.**

$\Xi^- p$ correlation function

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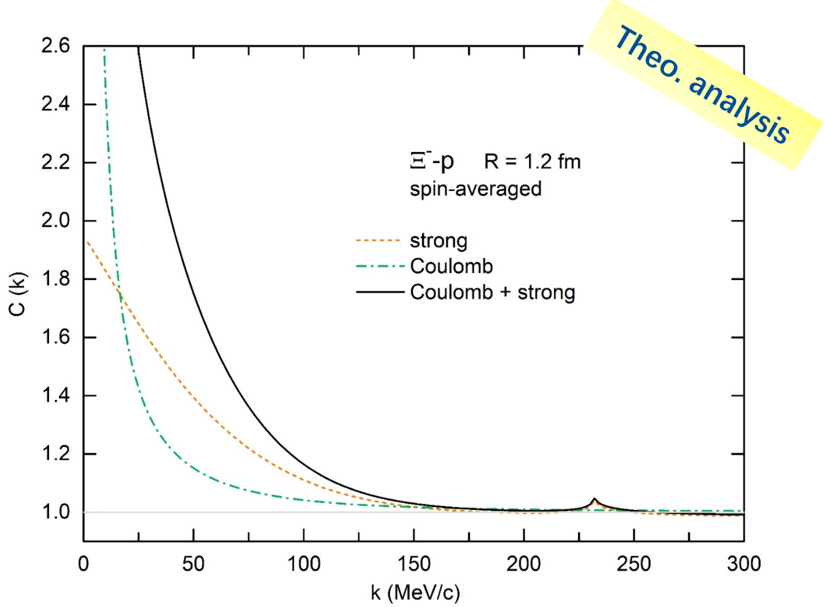
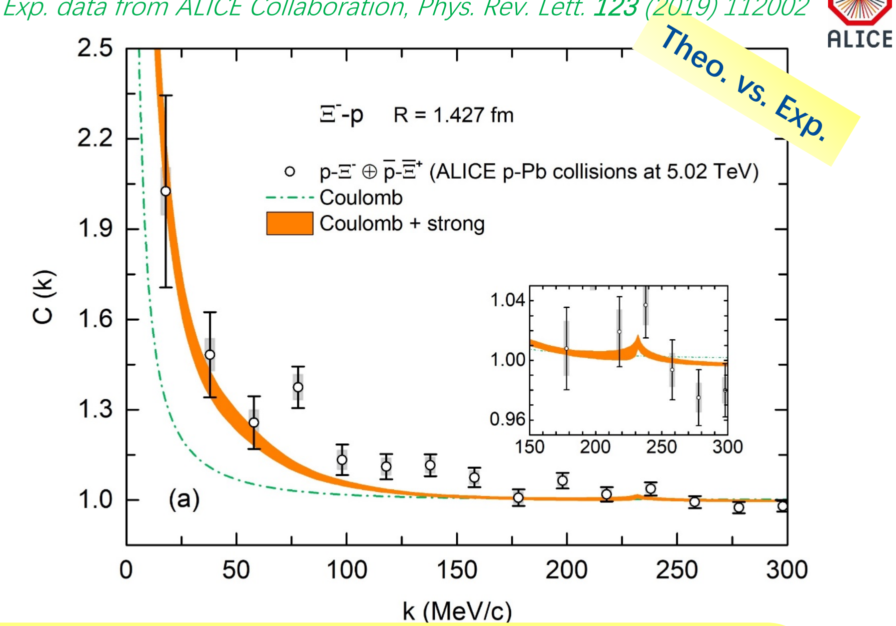
C. M. Vincent, S. C. Phatak, Phys. Rev. C ***10*** *(1974) 391*

- CF for *non-identical particles with Coulomb interaction* **Vincent-Phatak method**

$$C_{\Xi^- p}(k) \simeq \int \mathrm{d}\boldsymbol{r}\ S_{12}(r)\ |\phi^C(\boldsymbol{r},\boldsymbol{k})|^2 + \int_0^\infty 4\pi r^2 \mathrm{d}r\ S_{12}(r)\ \left[|\psi_0^{SC}(r,k)|^2 - |\phi_0^C(kr)|^2\right]$$

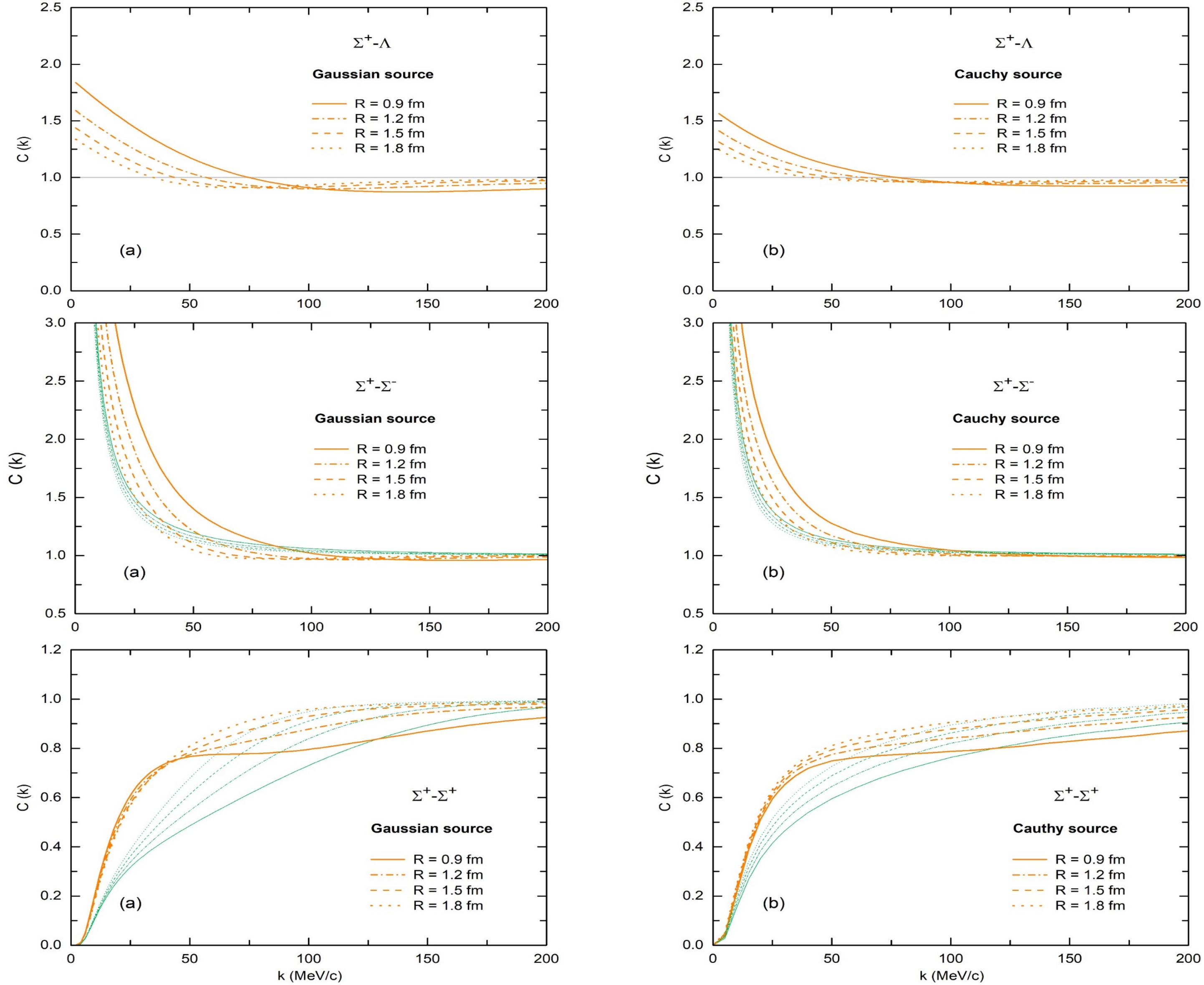
Exp. data from ALICE Collaboration, Phys. Rev. Lett. ***123*** *(2019) 112002*





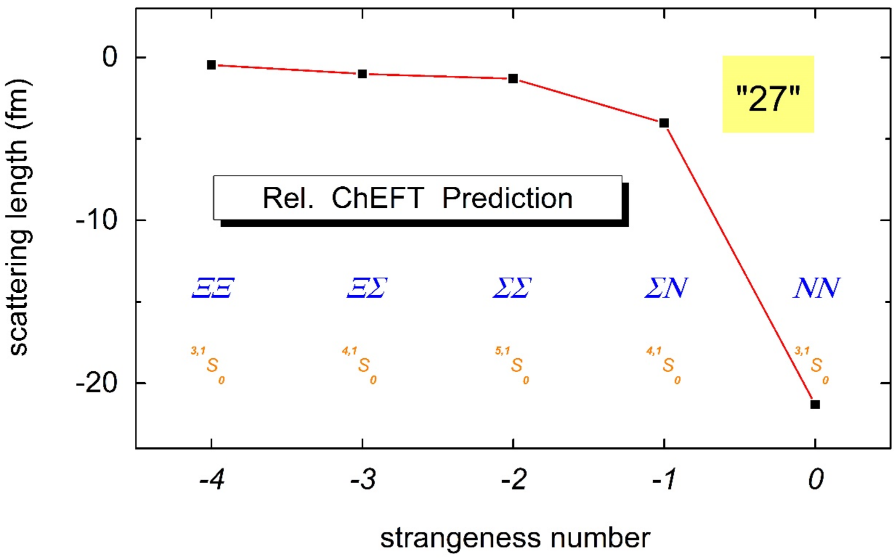
- ✓ The significant enhancement of the full $C_{\Xi^- p}$ below 150 MeV/c is consistent with the strong interaction contribution in the low-momentum region.
- ✓ There is an appreciable cusp-like structure around k ≈ 230 MeV/c.
- ✓ The reliability of $\Xi^- p$ interaction is demonstrated by the agreement between theoretical description and experimental measurement.

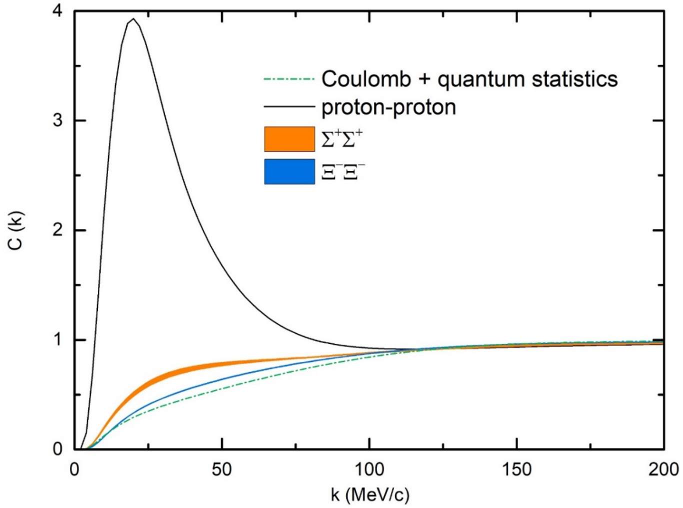
$\Sigma^+\Lambda$, $\Sigma^+\Sigma^-$, and $\Sigma^+\Sigma^+$ CFs

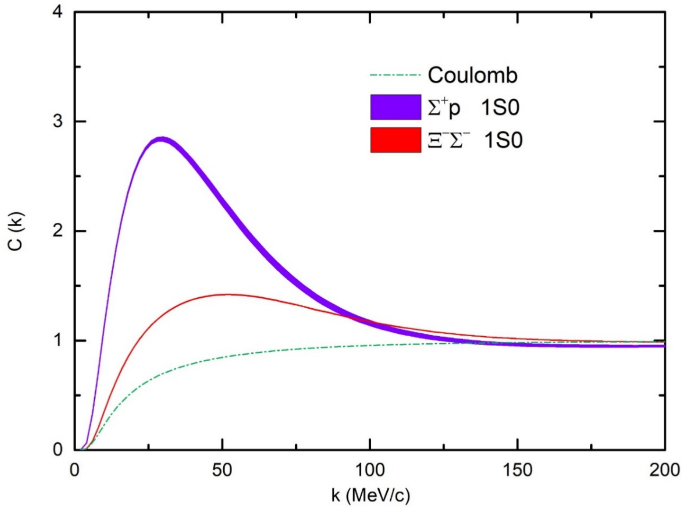


SU(3) symmetry breaking vs CFs

$$8 \otimes 8 = 1 \oplus 8_a \oplus 8_s \oplus 10^* \oplus 10 \oplus 27$$







- ✓ We predict the $\Sigma^+\Lambda$, $\Sigma^+\Sigma^-$, and $\Sigma^+\Sigma^+$ correlation functions for the first time.
- ✓ SU(3) flavor symmetry and its breaking can be tested quantitatively by measuring the correlation functions.

Summary

- **We studied the strangeness S = -2 BB interactions and the corresponding correlation functions in the relativistic ChEFT at leading order.**
  - ✓ The **full S =-2 BB S-wave interactions** are obtained by fitting the 12 LECs to the latest lattice QCD simulation data.
  - ✓ The reliability of obtained interactions is demonstrated by **the agreement between theoretical and experimental $\Lambda\Lambda$ and Ξ^-p correlation functions**.
  - ✓ We predict **the $\Sigma^+\Sigma^+$, $\Sigma^+\Lambda$, and $\Sigma^+\Sigma^-$ interactions and corresponding CFs** for the first time, and suggest **measuring CFs to test the SU(3) flavor symmetry and its breaking quantitatively**.

- **Collaborators**

Prof. Li-Sheng Geng

Dr. Kai-Wen Li

Thank you for your attention !

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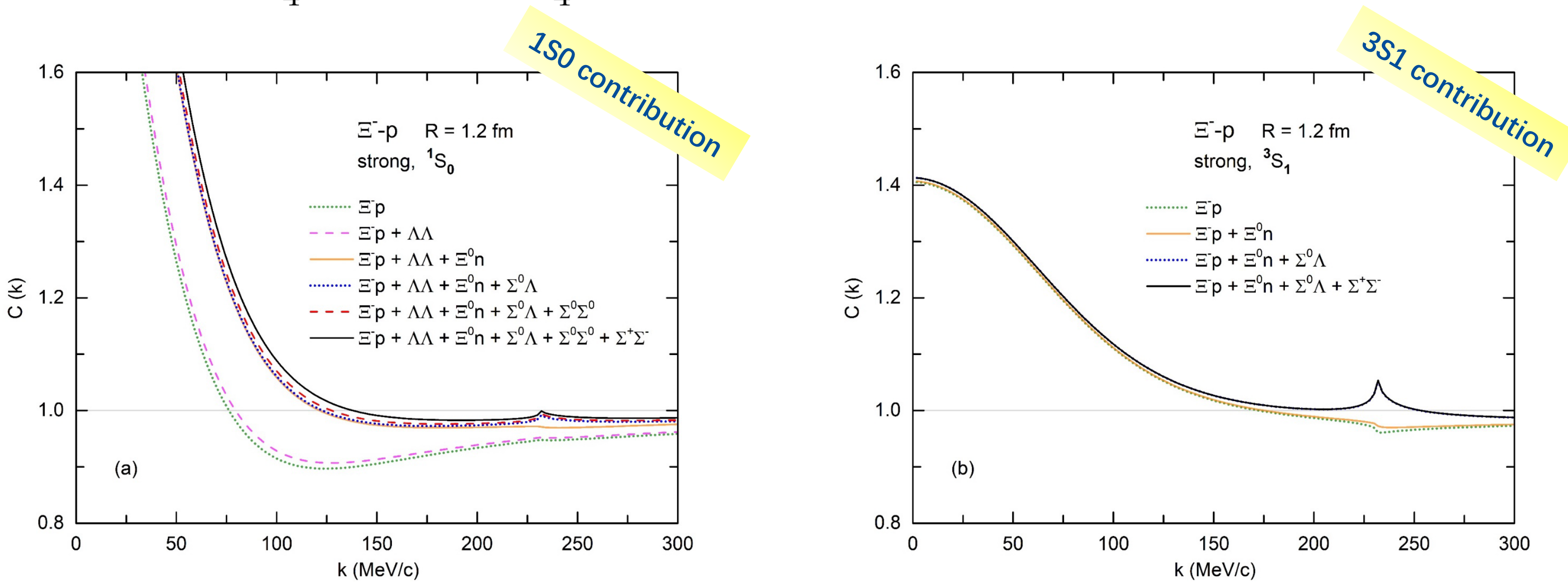
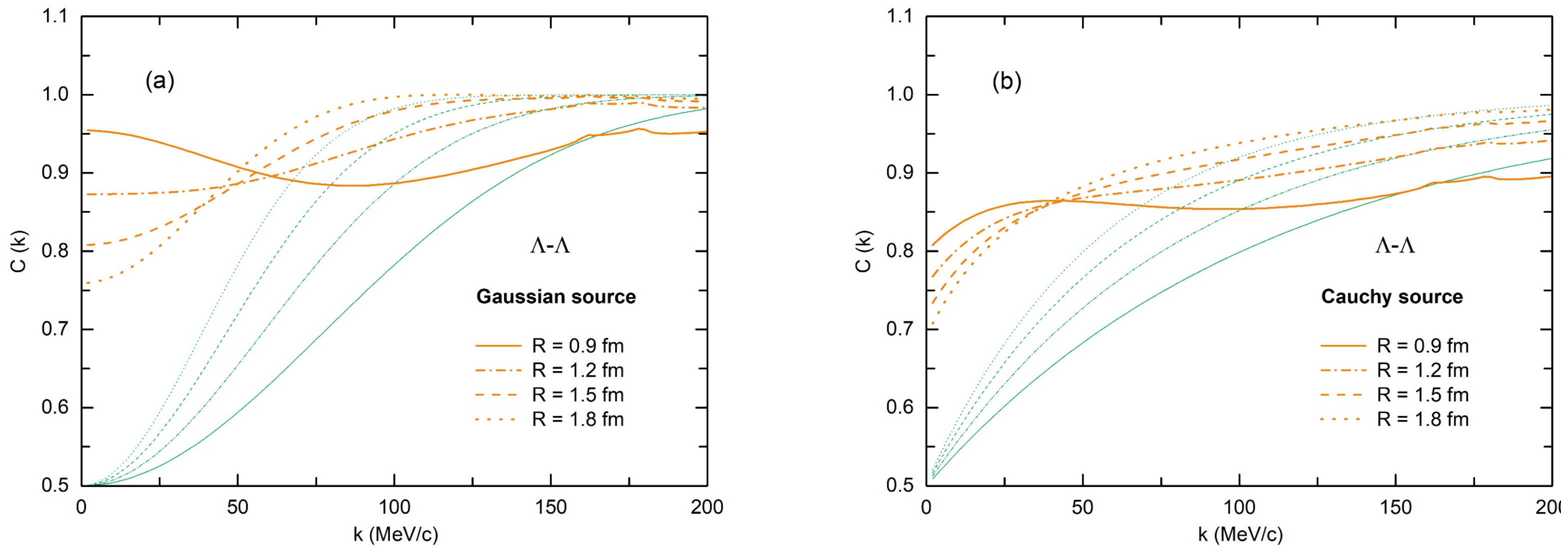
- **Breakdown of the strong interaction part of $\Xi^- \mathrm{p}$ correlation function**

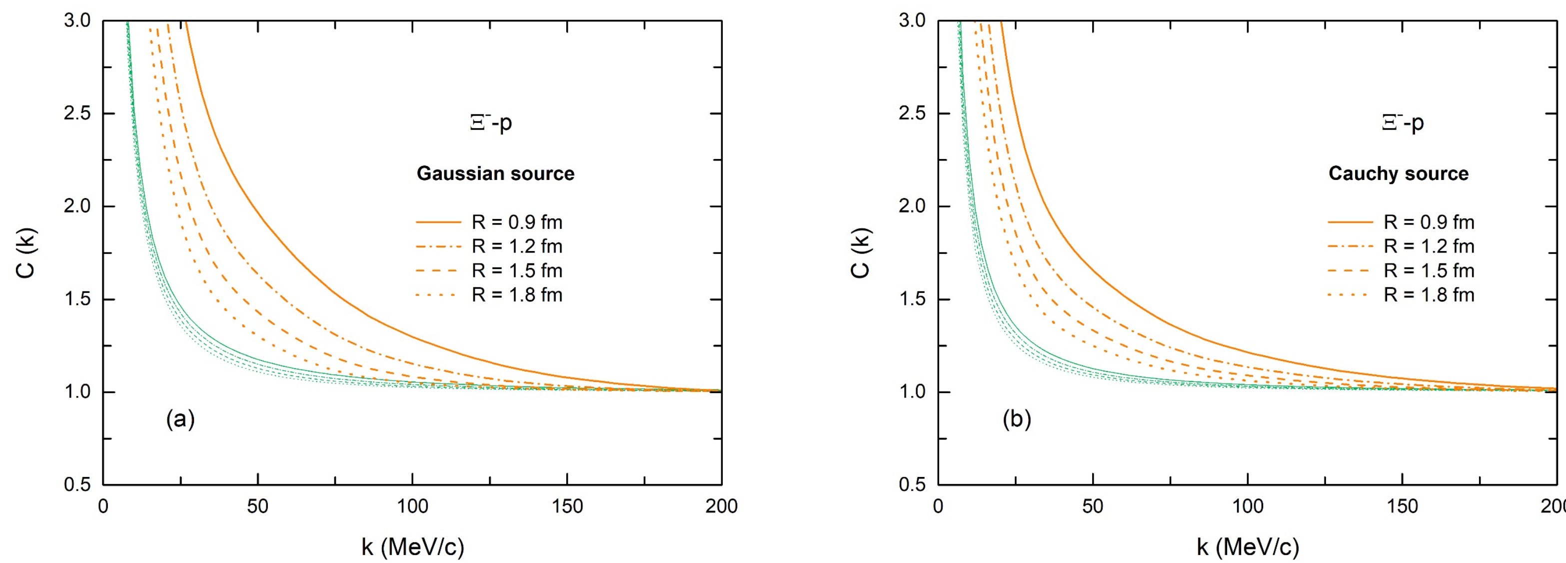
$$C_{\Xi^- p}(k) = \frac{1}{4} C^{1S0}_{\Xi^- p}(k) + \frac{3}{4} C^{3S1}_{\Xi^- p}(k)$$



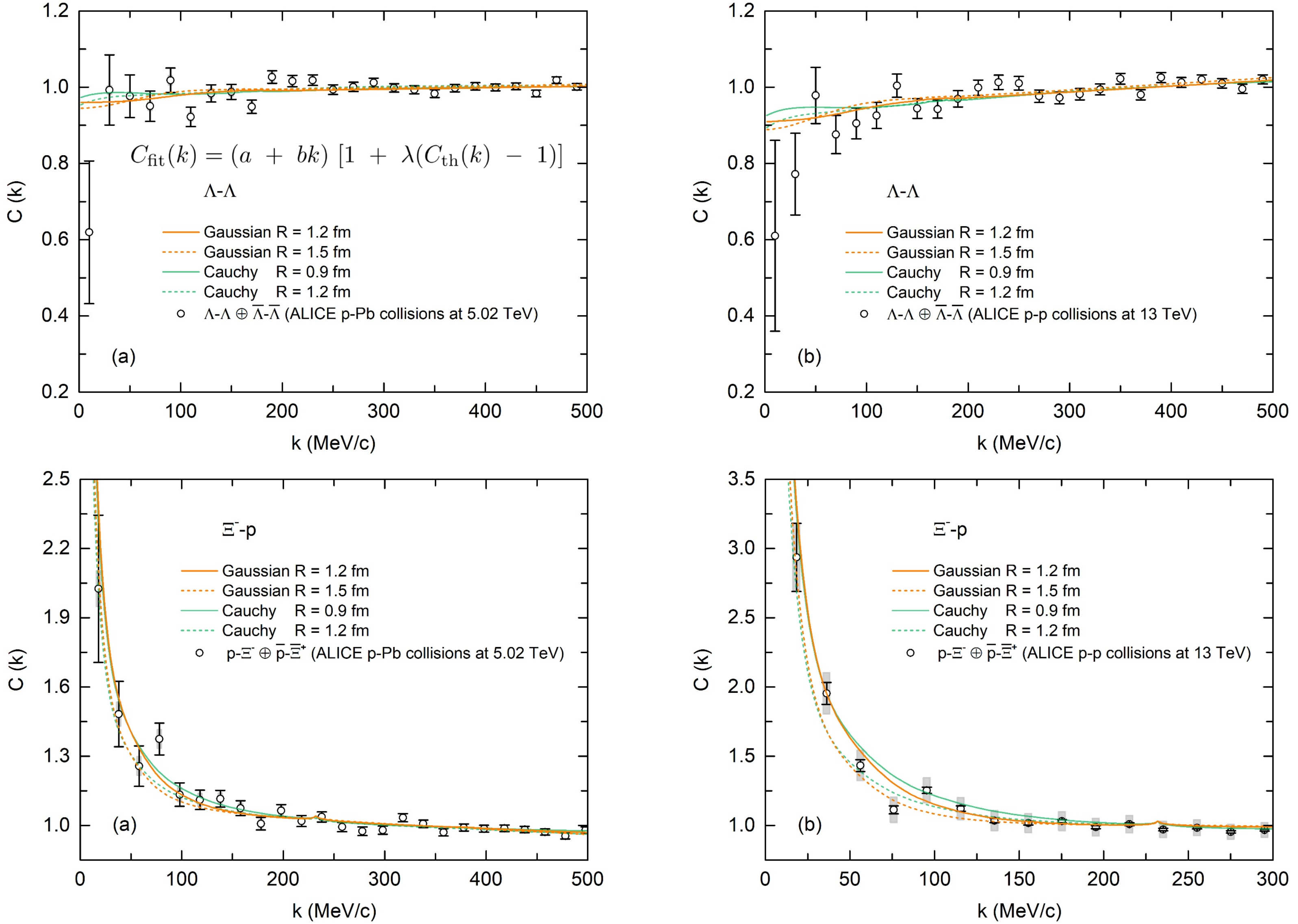
- ✓ Corresponding to the larger negative scattering length in the $\Xi^- p$ 1S0 channel, the correlation from the spin-singlet state is also stronger.
- ✓ It is clearly confirmed that the cusp-like structure comes from the contribution of $\Xi^- p - \Sigma^0 \Lambda$ coupled-channel, especially in the spin-triplet state.



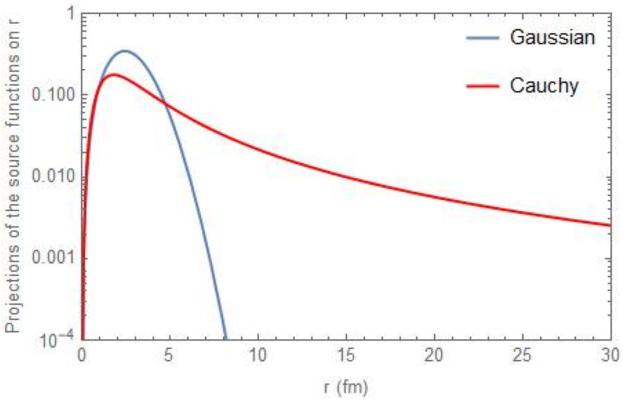
(a)
Λ-Λ
Gaussian source
R = 0.9 fm
R = 1.2 fm
R = 1.5 fm
R = 1.8 fm
C (k)
k (MeV/c)
(b)
Λ-Λ
Cauchy source
R = 0.9 fm
R = 1.2 fm
R = 1.5 fm
R = 1.8 fm
C (k)
k (MeV/c)

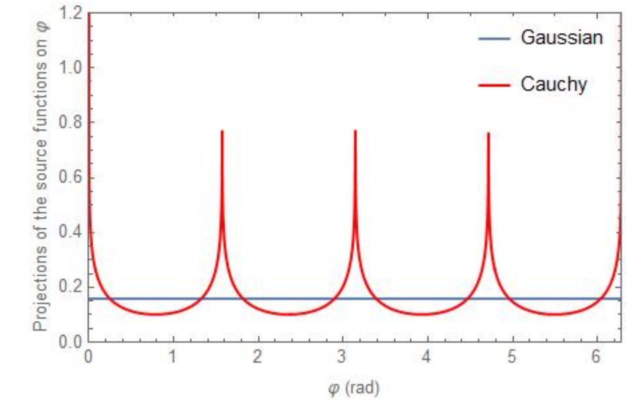


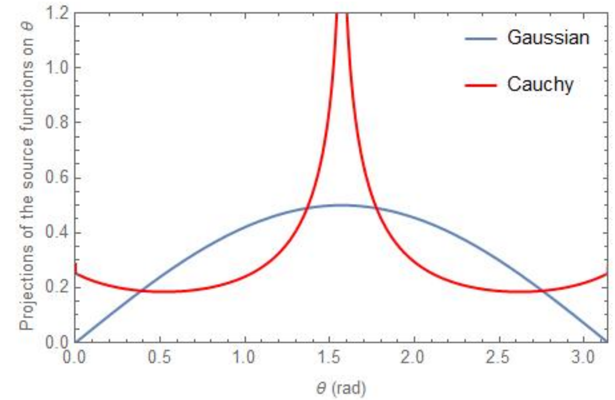
Ξ⁻-p
Gaussian source
R = 0.9 fm
R = 1.2 fm
R = 1.5 fm
R = 1.8 fm
(a)
C (k)
k (MeV/c)
Ξ⁻-p
Cauchy source
R = 0.9 fm
R = 1.2 fm
R = 1.5 fm
R = 1.8 fm
(b)
C (k)
k (MeV/c)

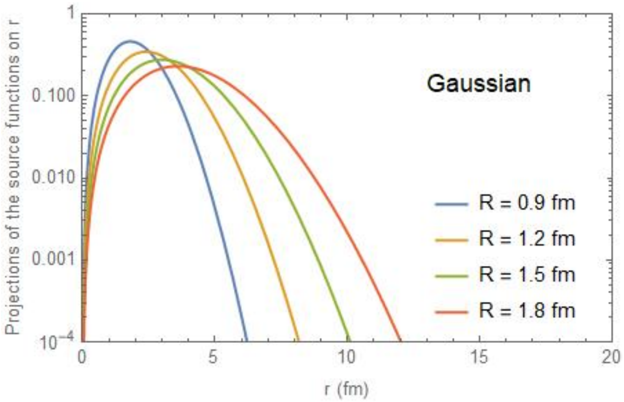

$C_{\rm fit}(k) = (a + bk)\,[1 + \lambda(C_{\rm th}(k) - 1)]$
Λ-Λ
Gaussian R = 1.2 fm
Gaussian R = 1.5 fm
Cauchy R = 0.9 fm
Cauchy R = 1.2 fm
Λ-Λ ⊕ $\overline{\Lambda}$-$\overline{\Lambda}$ (ALICE p-Pb collisions at 5.02 TeV)
(a)
Λ-Λ ⊕ $\overline{\Lambda}$-$\overline{\Lambda}$ (ALICE p-p collisions at 13 TeV)
(b)
C (k)
k (MeV/c)
Ξ^--p
p-Ξ^- ⊕ $\overline{\rm p}$-$\overline{\Xi}^+$ (ALICE p-Pb collisions at 5.02 TeV)
p-Ξ^- ⊕ $\overline{\rm p}$-$\overline{\Xi}^+$ (ALICE p-p collisions at 13 TeV)

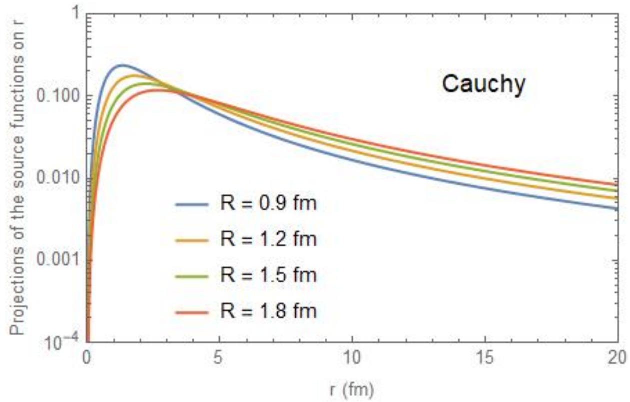
Appendix

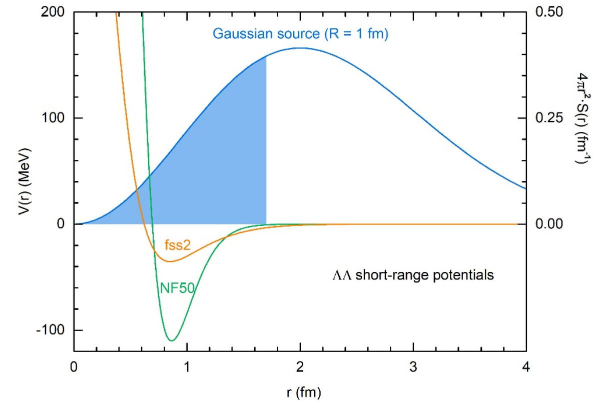












$$S_{12}^{\text{Gaussian}}(r,\theta) = \left(4\pi R^2\right)^{-3/2} \cdot \exp\left(\frac{-r^2}{4R^2}\right) \cdot r^2 \sin\theta$$

$$S_{12}^{\text{Cauchy}}(r,\theta,\varphi) = \left(\frac{R}{\pi}\right)^3 \frac{r^2\sin\theta}{(r\sin\theta\cos\varphi)^2 + R^2} \frac{1}{(r\sin\theta\sin\varphi)^2 + R^2} \frac{1}{(r\cos\theta)^2 + R^2}$$