One-loop Matching of the Seesaw Model onto the SM Effective Field Theory

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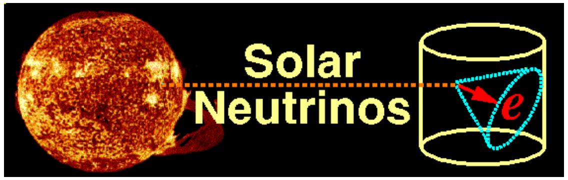
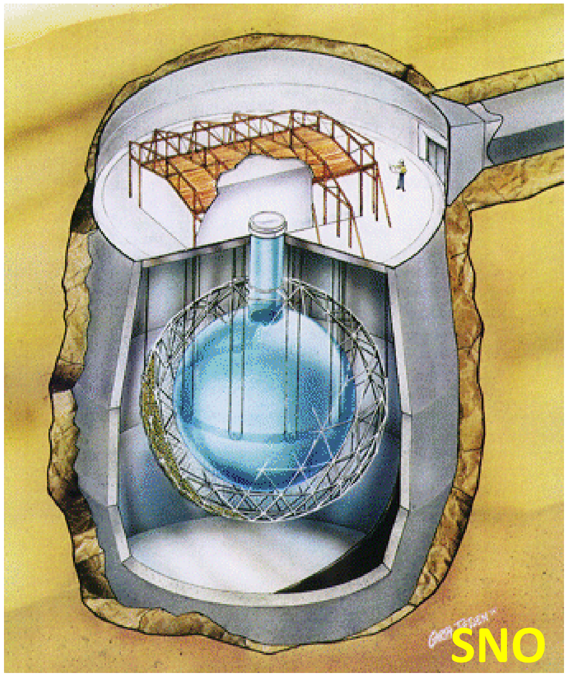
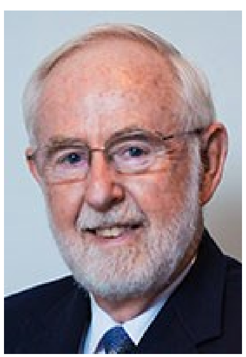
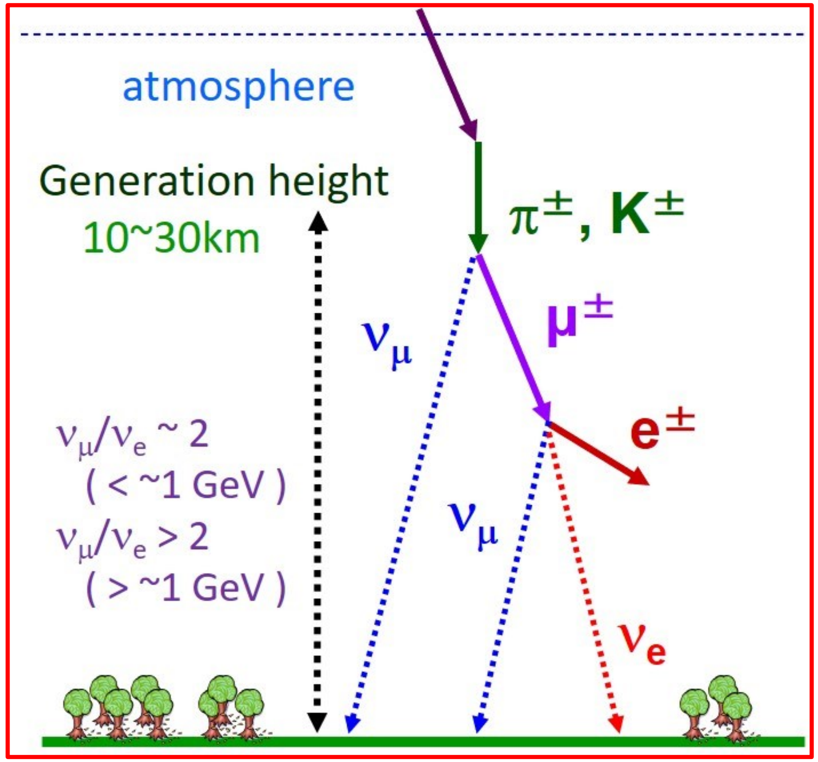
Massive Neutrinos: New Physics beyond the SM

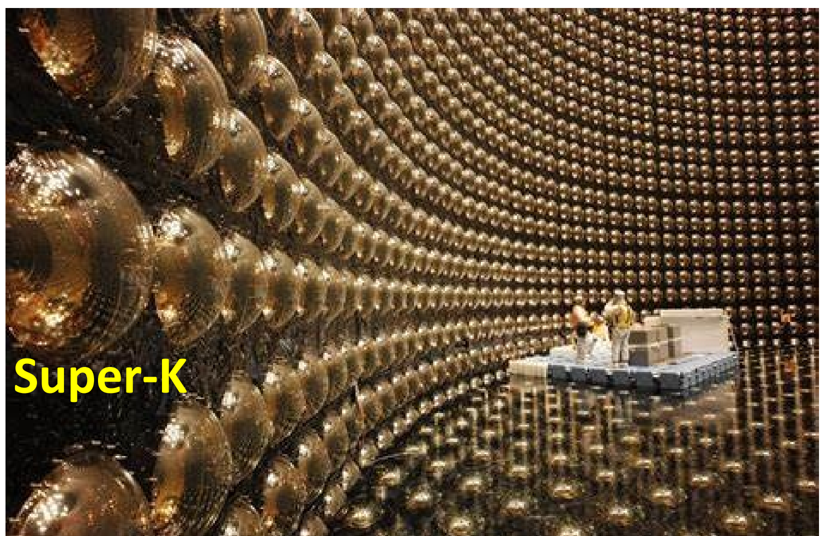


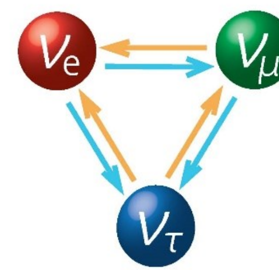




Nobel Prize in 2015







- **Neutrinos are massive !!!**
- **New physics beyond the SM**

Current Status of Neutrino Oscillations

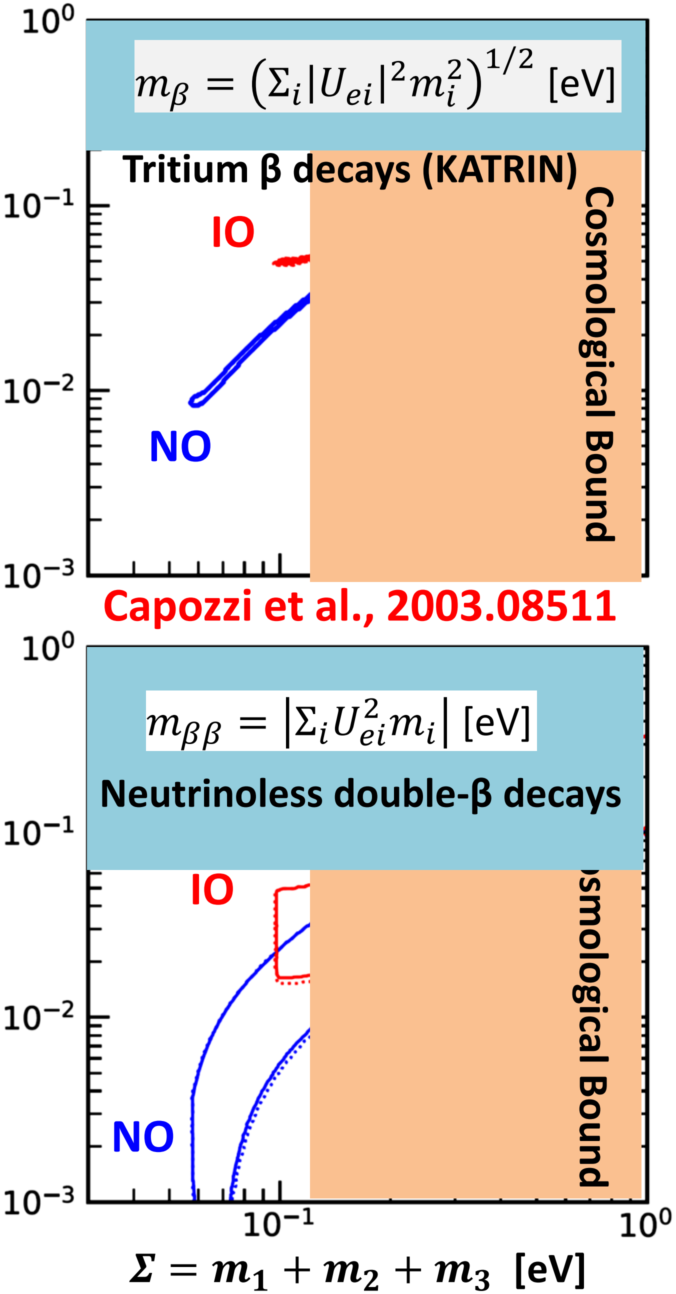
Basic neutrino parameters — Esteban *et al.*, 2007.14792, NuFIT 5.0 (2020)

without SK atmospheric data

	Normal Ordering (best fit)		Inverted Ordering ($\Delta\chi^2 = 2.7$)	
	bfp $\pm 1\sigma$	3σ range	bfp $\pm 1\sigma$	3σ range
$\sin^2\theta_{12}$	$0.304^{+0.013}_{-0.012}$	$0.269 \to 0.343$	$0.304^{+0.013}_{-0.012}$	$0.269 \to 0.343$
$\theta_{12}/^\circ$	$33.44^{+0.78}_{-0.75}$	$31.27 \to 35.86$	$33.45^{+0.78}_{-0.75}$	$31.27 \to 35.87$
$\sin^2\theta_{23}$	$0.570^{+0.018}_{-0.024}$	$0.407 \to 0.618$	$0.575^{+0.017}_{-0.021}$	$0.411 \to 0.621$
$\theta_{23}/^\circ$	$49.0^{+1.1}_{-1.4}$	$39.6 \to 51.8$	$49.3^{+1.0}_{-1.2}$	$39.9 \to 52.0$
$\sin^2\theta_{13}$	$0.02221^{+0.00068}_{-0.00062}$	$0.02034 \to 0.02430$	$0.02240^{+0.00062}_{-0.00062}$	$0.02053 \to 0.02436$
$\theta_{13}/^\circ$	$8.57^{+0.13}_{-0.12}$	$8.20 \to 8.97$	$8.61^{+0.12}_{-0.12}$	$8.24 \to 8.98$
$\delta_{\mathrm{CP}}/^\circ$	195^{+51}_{-25}	$107 \to 403$	286^{+27}_{-32}	$192 \to 360$
$\dfrac{\Delta m^2_{21}}{10^{-5}\ \mathrm{eV}^2}$	$7.42^{+0.21}_{-0.20}$	$6.82 \to 8.04$	$7.42^{+0.21}_{-0.20}$	$6.82 \to 8.04$
$\dfrac{\Delta m^2_{3\ell}}{10^{-3}\ \mathrm{eV}^2}$	$+2.514^{+0.028}_{-0.027}$	$+2.431 \to +2.598$	$-2.497^{+0.028}_{-0.028}$	$-2.583 \to -2.412$

- **Future neutrino oscillation experiments will measure the octant of θ_{23}, the CP-violating phase δ, and the neutrino mass ordering**
- **The most restrictive bound on absolute neutrino masses is coming from cosmological observations: $m_1 + m_2 + m_3 < 0.12$ eV (Planck)**

Current Status of Absolute Neutrino Masses



$m_1 < m_2 < m_3$ (NO) or $m_3 < m_1 < m_2$ (IO)

Constraints on absolute neutrino masses

- **Tritium β decays (95% C.L.)**

  $m_\beta < 0.8$ eV (KATRIN 2021)
- **Neutrinoless double-β decays (90% C.L.)**

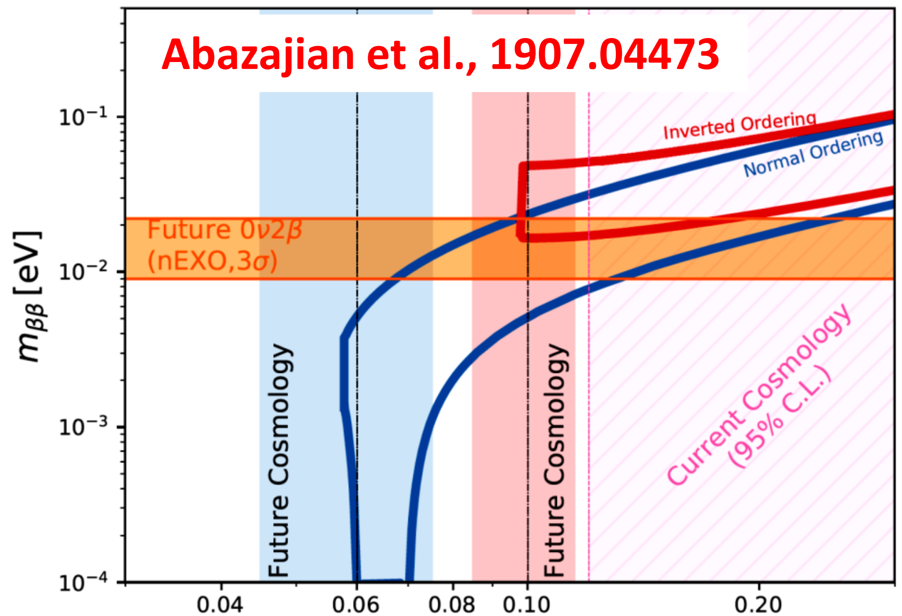
  $m_{\beta\beta} < (0.036\sim0.156)$ eV (KamLAND-Zen)

  $(0.15\sim0.40)$ eV (EXO-200)

  $(0.08\sim0.18)$ eV (GERDA-II)

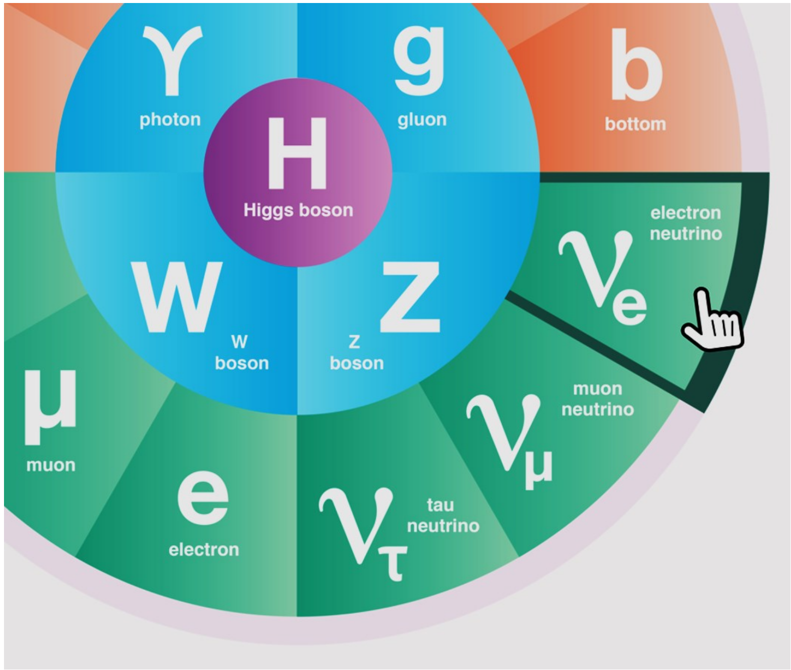
  $(0.08\sim0.35)$ eV (CUORE)
- **Cosmological observations (95% probability)**

  $\Sigma < 0.12$ eV (Planck)



Open Questions in Neutrino Physics

- Normal or Inverted (sign of Δm^2_{31}?)
- Leptonic CP Violation (δ = ?)
- Octant of θ_{23} (> or < 45º?)
- Absolute Neutrino Masses ($m_{\text{lightest}} = 0$?)
- Majorana or Dirac Nature ($\nu=\nu^C$?)
- Majorana CP-Violating Phases (how?)



- Extra Neutrino Species
- Exotic Neutrino Interactions
- Various LNV & LFV Processes
- Leptonic Unitarity Violation

- Origin of Neutrino Masses
- Flavor Structure (Symmetry?)
- Quark-Lepton Connection
- Relations to DM and/or BAU

Zero Neutrino Masses in the SM

Unified Electroweak Theory with the $SU(2)_L xU(1)_Y$ gauge symmetry

- ☐ **Particle content**
  minimality
- ☐ **Symmetries**
  SU(2)xU(1)
- ☐ **Renormalizability**
  predictive power

Particle content	Weak isospin I^3	Hypercharge Y	Electric charge Q
$Q_L \equiv \begin{pmatrix} u_L \\ d_L \end{pmatrix}, \begin{pmatrix} c_L \\ s_L \end{pmatrix}, \begin{pmatrix} t_L \\ b_L \end{pmatrix}$	$\begin{pmatrix} +1/2 \\ -1/2 \end{pmatrix}$	$+1/6$	$\begin{pmatrix} +2/3 \\ -1/3 \end{pmatrix}$
$\ell_L \equiv \begin{pmatrix} \nu_{eL} \\ e_L \end{pmatrix}, \begin{pmatrix} \nu_{\mu L} \\ \mu_L \end{pmatrix}, \begin{pmatrix} \nu_{\tau L} \\ \tau_L \end{pmatrix}$	$\begin{pmatrix} +1/2 \\ -1/2 \end{pmatrix}$	$-1/2$	$\begin{pmatrix} 0 \\ -1 \end{pmatrix}$
$U_R \equiv u_R, c_R, t_R$	0	$+2/3$	$+2/3$
$D_R \equiv d_R, s_R, b_R$	0	$-1/3$	$-1/3$
$E_R \equiv e_R, \mu_R, \tau_R$	0	-1	-1

Glashow, 61; Weinberg, 67; Salam, 68

The reason is rather SIMPLE **NO right-handed neutrinos**

- **Neutrinos experience only the weak force**
- **Weak interactions violate parity**
- **Only LH neutrinos/RH antineutrinos in weak interactions**

But neutrino oscillations show that neutrinos are massive particles

The simplest way to accommodate tiny neutrino masses

- **Dirac Neutrinos**
- **Majorana Neutrinos**

$$\mathcal{L} = \mathcal{L}_{\rm SM} + \overline{\nu_{\rm R}} {\rm i} \partial\!\!\!/ \nu_{\rm R} - \left[\overline{\ell_{\rm L}} Y_\nu \tilde{H} \nu_{\rm R} + {\rm h.c.}\right] \qquad - \left[\frac{1}{2}\overline{\nu^{\rm C}_{\rm R}} M_{\rm R} \nu_{\rm R} + {\rm h.c.}\right]$$

Generate Dirac ν masses in a similar way to that for quarks and charged leptons, after the spontaneous gauge symmetry breaking

$$M_\nu = Y_\nu v$$

$v \approx \mathbf{174\ GeV}$

M_ν: $\boldsymbol{O(0.1\ \mathrm{eV})}$

Y_ν: $\boldsymbol{O(10^{-12})}$

Generate tiny Majorana ν masses via the so-called seesaw mechanism

$$M_\nu = v^2 Y_\nu M_{\rm R}^{-1} Y_\nu^{\rm T}$$

M_ν: $\boldsymbol{O(0.1\ \mathrm{eV})}$

$M_{\rm R}$: $\boldsymbol{O(10^{14}\ \mathrm{GeV})}$

Difficulties with Dirac neutrinos

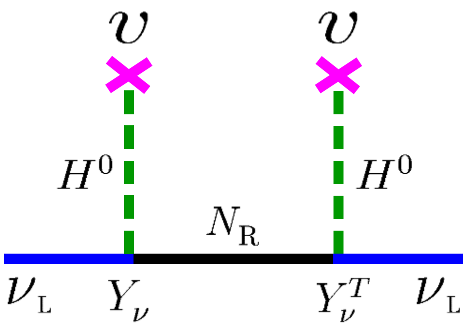
- **Tiny Dirac masses worsen fermion mass hierarchy problem (i.e., $m_i/m_t < 10^{-12}$)**
- **Mandatory lepton number conservation, which is actually accidental in the SM**

- **Retain the SM symmetries**
- **Well motivated by GUTs**

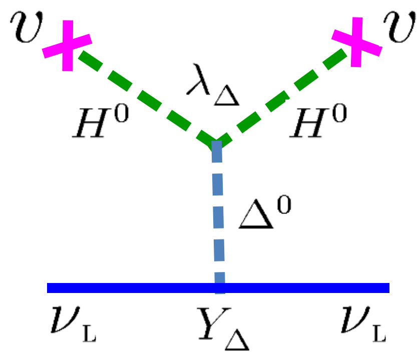
All the terms allowed by the SM gauge symmetries

Origin of Neutrino Masses

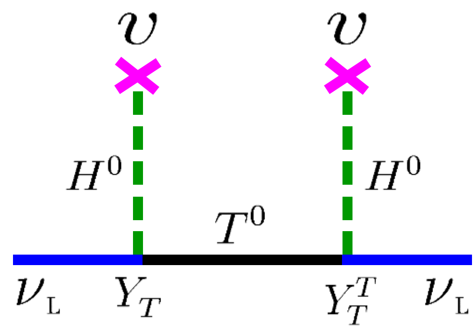
Majorana neutrinos: a natural way to understand neutrino masses



$$M_\nu \approx -v^2 Y_\nu \frac{1}{M_R} Y_\nu^T$$



$$M_\nu \approx \lambda_\Delta Y_\Delta \frac{v^2}{M_\Delta}$$



$$M_\nu \approx -v^2 Y_T \frac{1}{M_T} Y_T^T$$

Type-I: SM + 3 right-handed Majorana ν's (Minkowski 77; Yanagida 79; Glashow 79; Gell-Mann, Ramond, Slanski 79; Mohapatra, Senjanovic 79)

Type-II: SM + 1 Higgs triplet (Magg, Wetterich 80; Schechter, Valle 80; Lazarides et al 80; Mohapatra, Senjanovic 80; Gelmini, Roncadelli 80)

Type-III: SM + 3 triplet fermions (Foot, Lew, He, Joshi 89)

- **Can naturally be embedded into the Grand Unified Theories, e.g., SO(10) GUT**
- **Responsible for both tiny neutrino masses and matter-antimatter asymmetry**

Baryon- and Lepton-Nonconserving Processes

Weinberg, 79

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(Received 13 August 1979)

A number of properties of possible baryon- and lepton-nonconserving processes are shown to follow under very general assumptions. Attention is drawn to the importance of measuring μ^+ polarizations and $\bar{\nu}_e/e^+$ ratios in nucleon decay as a means of discriminating among specific models.

The sort of analysis used here in treating baryon nonconservation can also be applied to lepton nonconservation. A great difference is that there is a possible lepton-nonconserving term in the effective Lagrangian with dimensionality $d=5$:

$$\boxed{f_{abmn}\bar{l}^C_{iaL}\, l_{jbL}\,\varphi_k^{(m)}\varphi_l^{(n)}\epsilon_{ik}\epsilon_{jl} + f_{abmn}'\,\bar{l}^C_{iaL}\, l_{jbL}\,\varphi_k^{(m)}\varphi_l^{(n)}\epsilon_{ij}\epsilon_{kl}\,,} \tag{20}$$

where $\varphi^{(m)}$ are one or more scalar doublets. We expect f and f' to be roughly of order $1/M$; one-loop graphs would give values of order α^2/M.[13] The interaction (20) would produce a neutrino mass $m_\nu \simeq G_F^{-1}f$, or roughly 10^{-5} to 10^{-1} eV. This is well below any existing laboratory or cosmological limits, but there is no reason why this neutrino-mass matrix should be diagonal, and masses of this order might perhaps be observable in neutrino oscillation experiments.

Unique dim-5 Weinberg operator for Majorana neutrino masses

SM Effective Field Theory (SMEFT)

$$\mathcal{L}_{\rm SMEFT} = \mathcal{L}_{\rm SM} + \sum_d \mathcal{L}^{(d)} = \mathcal{L}_{\rm SM} + \sum_{i=1}^{n_d} \frac{1}{\Lambda^{d-4}} C_i^{(d)} \mathcal{O}_i^{(d)}$$

SM Effective Field Theory

X^3		φ^6 and $\varphi^4 D^2$		$\psi^2\varphi^3$	
Q_G	$f^{ABC}G_\mu^{A\nu}G_\nu^{B\rho}G_\rho^{C\mu}$	Q_φ	$(\varphi^\dagger\varphi)^3$	$Q_{e\varphi}$	$(\varphi^\dagger\varphi)(\bar{l}_p e_r \varphi)$
$Q_{\widetilde{G}}$	$f^{ABC}\widetilde{G}_\mu^{A\nu}G_\nu^{B\rho}G_\rho^{C\mu}$	$Q_{\varphi\Box}$	$(\varphi^\dagger\varphi)\Box(\varphi^\dagger\varphi)$	$Q_{u\varphi}$	$(\varphi^\dagger\varphi)(\bar{q}_p u_r \widetilde{\varphi})$
Q_W	$\varepsilon^{IJK}W_\mu^{I\nu}W_\nu^{J\rho}W_\rho^{K\mu}$	$Q_{\varphi D}$	$\left(\varphi^\dagger D^\mu\varphi\right)^\star\left(\varphi^\dagger D_\mu\varphi\right)$	$Q_{d\varphi}$	$(\varphi^\dagger\varphi)(\bar{q}_p d_r \varphi)$
$Q_{\widetilde{W}}$	$\varepsilon^{IJK}\widetilde{W}_\mu^{I\nu}W_\nu^{J\rho}W_\rho^{K\mu}$				
$X^2\varphi^2$		$\psi^2 X\varphi$		$\psi^2\varphi^2 D$	
$Q_{\varphi G}$	$\varphi^\dagger\varphi\, G^A_{\mu\nu}G^{A\mu\nu}$	Q_{eW}	$(\bar{l}_p\sigma^{\mu\nu}e_r)\tau^I\varphi W^I_{\mu\nu}$	$Q_{\varphi l}^{(1)}$	$(\varphi^\dagger i\overleftrightarrow{D}_\mu\,\varphi)(\bar{l}_p\gamma^\mu l_r)$
$Q_{\varphi\widetilde{G}}$	$\varphi^\dagger\varphi\,\widetilde{G}^A_{\mu\nu}G^{A\mu\nu}$	Q_{eB}	$(\bar{l}_p\sigma^{\mu\nu}e_r)\varphi B_{\mu\nu}$	$Q_{\varphi l}^{(3)}$	$(\varphi^\dagger i\overleftrightarrow{D}^I_\mu\,\varphi)(\bar{l}_p\tau^I\gamma^\mu l_r)$
$Q_{\varphi W}$	$\varphi^\dagger\varphi\, W^I_{\mu\nu}W^{I\mu\nu}$	Q_{uG}	$(\bar{q}_p\sigma^{\mu\nu}T^A u_r)\widetilde{\varphi}\, G^A_{\mu\nu}$	$Q_{\varphi e}$	$(\varphi^\dagger i\overleftrightarrow{D}_\mu\,\varphi)(\bar{e}_p\gamma^\mu e_r)$
$Q_{\varphi\widetilde{W}}$	$\varphi^\dagger\varphi\,\widetilde{W}^I_{\mu\nu}W^{I\mu\nu}$	Q_{uW}	$(\bar{q}_p\sigma^{\mu\nu}u_r)\tau^I\widetilde{\varphi}\, W^I_{\mu\nu}$	$Q_{\varphi q}^{(1)}$	$(\varphi^\dagger i\overleftrightarrow{D}_\mu\,\varphi)(\bar{q}_p\gamma^\mu q_r)$
$Q_{\varphi B}$	$\varphi^\dagger\varphi\, B_{\mu\nu}B^{\mu\nu}$	Q_{uB}	$(\bar{q}_p\sigma^{\mu\nu}u_r)\widetilde{\varphi}\, B_{\mu\nu}$	$Q_{\varphi q}^{(3)}$	$(\varphi^\dagger i\overleftrightarrow{D}^I_\mu\,\varphi)(\bar{q}_p\tau^I\gamma^\mu q_r)$
$Q_{\varphi\widetilde{B}}$	$\varphi^\dagger\varphi\,\widetilde{B}_{\mu\nu}B^{\mu\nu}$	Q_{dG}	$(\bar{q}_p\sigma^{\mu\nu}T^A d_r)\varphi\, G^A_{\mu\nu}$	$Q_{\varphi u}$	$(\varphi^\dagger i\overleftrightarrow{D}_\mu\,\varphi)(\bar{u}_p\gamma^\mu u_r)$
$Q_{\varphi WB}$	$\varphi^\dagger\tau^I\varphi\, W^I_{\mu\nu}B^{\mu\nu}$	Q_{dW}	$(\bar{q}_p\sigma^{\mu\nu}d_r)\tau^I\varphi\, W^I_{\mu\nu}$	$Q_{\varphi d}$	$(\varphi^\dagger i\overleftrightarrow{D}_\mu\,\varphi)(\bar{d}_p\gamma^\mu d_r)$
$Q_{\varphi\widetilde{W}B}$	$\varphi^\dagger\tau^I\varphi\,\widetilde{W}^I_{\mu\nu}B^{\mu\nu}$	Q_{dB}	$(\bar{q}_p\sigma^{\mu\nu}d_r)\varphi\, B_{\mu\nu}$	$Q_{\varphi ud}$	$i(\widetilde{\varphi}^\dagger D_\mu\varphi)(\bar{u}_p\gamma^\mu d_r)$

Table 2: Dimension-six operators other than the four-fermion ones.

$(\bar{L}L)(\bar{L}L)$		$(\bar{R}R)(\bar{R}R)$		$(\bar{L}L)(\bar{R}R)$	
Q_{ll}	$(\bar{l}_p\gamma_\mu l_r)(\bar{l}_s\gamma^\mu l_t)$	Q_{ee}	$(\bar{e}_p\gamma_\mu e_r)(\bar{e}_s\gamma^\mu e_t)$	Q_{le}	$(\bar{l}_p\gamma_\mu l_r)(\bar{e}_s\gamma^\mu e_t)$
$Q_{qq}^{(1)}$	$(\bar{q}_p\gamma_\mu q_r)(\bar{q}_s\gamma^\mu q_t)$	Q_{uu}	$(\bar{u}_p\gamma_\mu u_r)(\bar{u}_s\gamma^\mu u_t)$	Q_{lu}	$(\bar{l}_p\gamma_\mu l_r)(\bar{u}_s\gamma^\mu u_t)$
$Q_{qq}^{(3)}$	$(\bar{q}_p\gamma_\mu\tau^I q_r)(\bar{q}_s\gamma^\mu\tau^I q_t)$	Q_{dd}	$(\bar{d}_p\gamma_\mu d_r)(\bar{d}_s\gamma^\mu d_t)$	Q_{ld}	$(\bar{l}_p\gamma_\mu l_r)(\bar{d}_s\gamma^\mu d_t)$
$Q_{lq}^{(1)}$	$(\bar{l}_p\gamma_\mu l_r)(\bar{q}_s\gamma^\mu q_t)$	Q_{eu}	$(\bar{e}_p\gamma_\mu e_r)(\bar{u}_s\gamma^\mu u_t)$	Q_{qe}	$(\bar{q}_p\gamma_\mu q_r)(\bar{e}_s\gamma^\mu e_t)$
$Q_{lq}^{(3)}$	$(\bar{l}_p\gamma_\mu\tau^I l_r)(\bar{q}_s\gamma^\mu\tau^I q_t)$	Q_{ed}	$(\bar{e}_p\gamma_\mu e_r)(\bar{d}_s\gamma^\mu d_t)$	$Q_{qu}^{(1)}$	$(\bar{q}_p\gamma_\mu q_r)(\bar{u}_s\gamma^\mu u_t)$
		$Q_{ud}^{(1)}$	$(\bar{u}_p\gamma_\mu u_r)(\bar{d}_s\gamma^\mu d_t)$	$Q_{qu}^{(8)}$	$(\bar{q}_p\gamma_\mu T^A q_r)(\bar{u}_s\gamma^\mu T^A u_t)$
		$Q_{ud}^{(8)}$	$(\bar{u}_p\gamma_\mu T^A u_r)(\bar{d}_s\gamma^\mu T^A d_t)$	$Q_{qd}^{(1)}$	$(\bar{q}_p\gamma_\mu q_r)(\bar{d}_s\gamma^\mu d_t)$
				$Q_{qd}^{(8)}$	$(\bar{q}_p\gamma_\mu T^A q_r)(\bar{d}_s\gamma^\mu T^A d_t)$

$(\bar{L}R)(\bar{R}L)$ and $(\bar{L}R)(\bar{L}R)$		B-violating	
Q_{ledq}	$(\bar{l}_p^j e_r)(\bar{d}_s q_t^j)$	Q_{duq}	$\varepsilon^{\alpha\beta\gamma}\varepsilon_{jk}\left[(d_p^\alpha)^T C u_r^\beta\right]\left[(q_s^{\gamma j})^T C l_t^k\right]$
$Q_{quqd}^{(1)}$	$(\bar{q}_p^j u_r)\varepsilon_{jk}(\bar{q}_s^k d_t)$	Q_{qqu}	$\varepsilon^{\alpha\beta\gamma}\varepsilon_{jk}\left[(q_p^{\alpha j})^T C q_r^{\beta k}\right]\left[(u_s^\gamma)^T C e_t\right]$
$Q_{quqd}^{(8)}$	$(\bar{q}_p^j T^A u_r)\varepsilon_{jk}(\bar{q}_s^k T^A d_t)$	Q_{qqq}	$\varepsilon^{\alpha\beta\gamma}\varepsilon_{jn}\varepsilon_{km}\left[(q_p^{\alpha j})^T C q_r^{\beta k}\right]\left[(q_s^{\gamma m})^T C l_t^n\right]$
$Q_{lequ}^{(1)}$	$(\bar{l}_p^j e_r)\varepsilon_{jk}(\bar{q}_s^k u_t)$	Q_{duu}	$\varepsilon^{\alpha\beta\gamma}\left[(d_p^\alpha)^T C u_r^\beta\right]\left[(u_s^\gamma)^T C e_t\right]$
$Q_{lequ}^{(3)}$	$(\bar{l}_p^j\sigma_{\mu\nu}e_r)\varepsilon_{jk}(\bar{q}_s^k\sigma^{\mu\nu}u_t)$		

Table 3: Four-fermion operators.

Buchmüller & Wyler, 86

Brivio & Trott, Phys. Rept. 793 (2019) 1

Dim-6 operators in the Warsaw basis

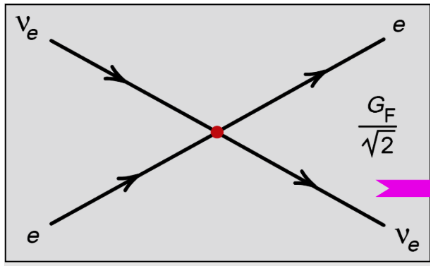
Dimension-Six Terms in the Standard Model Lagrangian

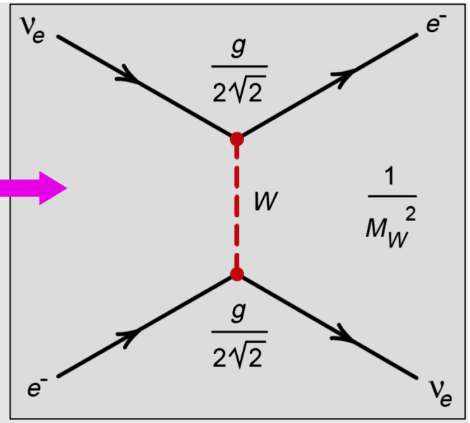
Here we perform their classification once again from the outset. Assuming baryon number conservation, we find $15 + 19 + 25 = 59$ independent operators (barring flavour structure and Hermitian conjugations), as compared to $16 + 35 + 29 = 80$ in Ref. [3]. The three summed numbers refer to operators containing 0, 2 and 4 fermion fields. If the assumption of baryon number conservation is relaxed, 4 new operators arise in the four-fermion sector.

Grzadkowski, Iskrzynski, Misiak, Rosiek, 1008.4884

- **Get rid of the redundancy**

Why EFTs?





$$\frac{G_F}{\sqrt{2}} = \frac{g^2}{8M_W^2}$$

$$M_W = 80.4\mathrm{GeV}$$

$$G_F = 1.166 \times 10^{-5}\ \mathrm{GeV}^{-2}$$

- **Bottom-up approach to new physics**

(a) Determine the renormalized SU(3) × SU(2) × U(1) couplings $g_i(\mu)$ in $\tilde{I}$ at renormalization scales μ of order M, by a *one-loop* functional integral over superheavy fields, as in (3). (Here and below, all renormalized couplings are to be defined by minimum subtraction [5], or by one of its simple modifications [10].)

(b) Calculate the renormalized SU(3) × SU(2) × U(1) couplings $g_i(\mu)$ at ordinary mass scales $\mu \approx m$, by integrating the *two-loop* renormalization group equations of the SU(3) × SU(2) × U(1) gauge theory (in which no superheavy fields appear) from $\mu \approx M$ to $\mu \approx m$, using the results of (a) as an initial condition at $\mu \approx M$.

EFFECTIVE GAUGE THEORIES ☆

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Received 7 January 1980

- **Top-down approach to precision tests of UV-complete models**

$$\ln \frac{M}{m} = \tfrac{4}{11}\pi^2 \left[\frac{1}{e^2(m)} - \frac{8}{3g_S^2(m)} \right] + O(1) \simeq 35\ , \qquad (1)$$

and the $Z^0-\gamma$ mixing parameter is ‡2

$$\sin^2\theta = \tfrac{1}{6} + \tfrac{5}{9} e^2(m)/g_S^2(m) + O(\alpha) \simeq 0.2\ , \qquad (2)$$

where $g_S(m)$ and $e(m)$ are the strong and electromagnetic couplings measured at some "ordinary" mass scale m, say $m \approx 100$ GeV.

$$\exp(i\tilde{I}[\phi]) = \int [d\Phi]\ \exp(iI[\phi, \Phi])\ . \qquad (3)$$

(c) Compare the results of (b) with the SU(3) × SU(2) × U(1) couplings determined from experiment at ordinary energy, including the effects of radiative corrections to *one-loop* order.

Low-energy Tests of Type-I Seesaw Model

- **The type-I seesaw model as a UV-complete theory**

$$\mathcal{L}_{\rm UV} = \mathcal{L}_{\rm SM} + \overline{N_{\rm R}} {\rm i}\partial\!\!\!/ N_{\rm R} - \left(\frac{1}{2}\overline{N^c_{\rm R}} M N_{\rm R} + \overline{\ell_{\rm L}} Y_\nu \widetilde{H} N_{\rm R} + {\rm h.c.} \right)$$

- **Tree-level matching: simply applying the EOM**

$$\left({\rm i}\partial\!\!\!/ - M\right) N - \left(Y^\dagger_\nu \widetilde{H}^\dagger \ell_{\rm L} + Y^{\rm T}_\nu \widetilde{H}^{\rm T} \ell^c_{\rm L} \right) = 0$$

- **Expansion up to *M*$^{-2}$ (dim-6 operators)**

$$N \simeq -\left(M^{-1} + M^{-2}{\rm i}\partial\!\!\!/\right) \left(Y^\dagger_\nu \widetilde{H}^\dagger \ell_{\rm L} + Y^{\rm T}_\nu \widetilde{H}^{\rm T} \ell^c_{\rm L} \right)$$

- **Seesaw Effective Field Theory (SEFT) @ tree level**

$$\mathcal{L}^{\rm tree}_{\rm SEFT} = \mathcal{L}_{\rm SM} + \left[\frac{1}{2} C^{(5)}_{\alpha\beta} \mathcal{O}^{(5)}_{\alpha\beta} + {\rm h.c.} \right] + C^{(6)}_{\alpha\beta} \mathcal{O}^{(6)}_{\alpha\beta}$$

$$\mathcal{O}^{(5)}_{\alpha\beta} = \overline{\ell_{\alpha{\rm L}}} \widetilde{H} \widetilde{H}^{\rm T} \ell^c_{\beta{\rm L}}$$

$$C^{(5)}_{\alpha\beta} = \left(Y_\nu M^{-1} Y^{\rm T}_\nu \right)_{\alpha\beta}$$

Neutrino masses & flavor mixing

$$\mathcal{O}^{(6)}_{\alpha\beta} = \left(\overline{\ell_{\alpha{\rm L}}} \widetilde{H} \right) {\rm i}\partial\!\!\!/ \left(\widetilde{H}^\dagger \ell_{\beta{\rm L}} \right)$$

$$C^{(6)}_{\alpha\beta} = \left(Y_\nu M^{-2} Y^\dagger_\nu \right)_{\alpha\beta}$$

Unitarity violation of flavor mixing matrix

After the spontaneous gauge symmetry breaking

$$\mathcal{L}_{\rm SEFT} = \overline{\nu_{\alpha \rm L}} \left(\mathbf{1} + M_{\rm D} M^{-2} M_{\rm D}^\dagger\right)_{\alpha\beta} {\rm i} \partial\!\!\!/ \nu_{\beta \rm L} - \left[\overline{l_{\alpha \rm L}} \left(M_l\right)_{\alpha\beta} l_{\beta \rm R} + \frac{1}{2} \overline{\nu_{\alpha \rm L}} \left(M_\nu\right)_{\alpha\beta} \nu^{\rm c}_{\beta \rm L} + {\rm h.c.}\right]$$
$$+ \left(\frac{g_2}{\sqrt{2}} \overline{l_{\alpha \rm L}} \gamma^\mu \nu_{\alpha \rm L} W^-_\mu + {\rm h.c.}\right) + \frac{g_2}{2\cos\theta_{\rm w}} \overline{\nu_{\alpha \rm L}} \gamma^\mu \nu_{\alpha \rm L} Z_\mu$$

Normalization: $\nu_{\rm L} \to V \nu_{\rm L}$ with $V = \mathbf{1} - RR^\dagger/2$ and $R \equiv M_{\rm D} M^{-1}$

Diagonalization: $U_0^\dagger V M_\nu V^T U_0^* = \widehat{M}_\nu = {\rm Diag}\{m_1, m_2, m_3\}$

The SEFT Lagrangian in the mass basis: $\boxed{U = V U_0}$

$$\mathcal{L}_{\rm SEFT} = \overline{\nu_{\rm L}} {\rm i} \partial\!\!\!/ \nu_{\rm L} - \left(\overline{l_{\rm L}} M_l l_{\rm R} + \frac{1}{2} \overline{\nu_{\rm L}} \widehat{M}_\nu \nu^{\rm c}_{\rm L} + {\rm h.c.}\right) + \left(\frac{g_2}{\sqrt{2}} \overline{l_{\rm L}} \gamma^\mu \boxed{U} \nu_{\rm L} W^-_\mu + {\rm h.c.}\right)$$
$$+ \frac{g_2}{2\cos\theta_{\rm w}} \overline{\nu_{\rm L}} \gamma^\mu \boxed{U^\dagger U} \nu_{\rm L} Z_\mu$$

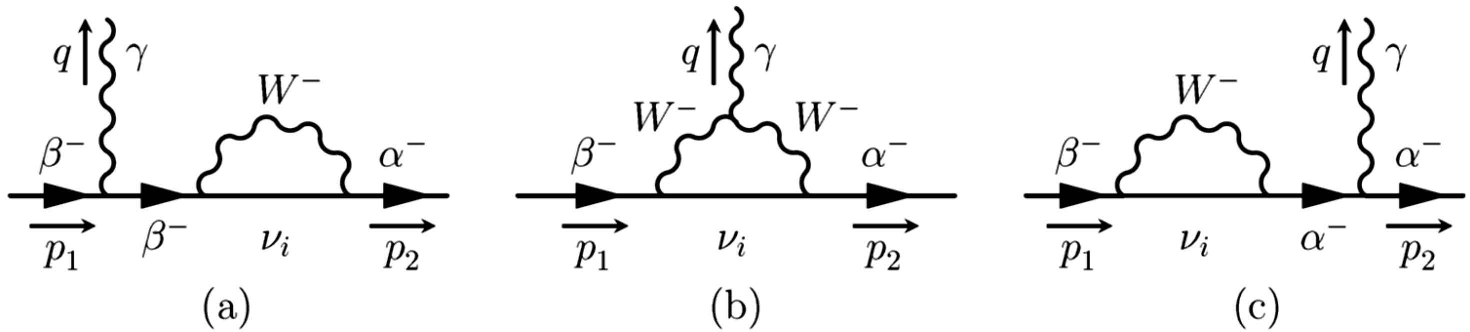
Non-unitary flavor mixing

Non-unitarity

Minimal unitarity violation (MUV) "equivalent" to SEFT @ tree level

Antusch et al., hep-ph/0607020; Antusch & Fischer, 1407.6607

LFV decays of charged leptons in the MUV scheme



The decay width

Xing & Zhang, 2009.09717

$$\Gamma\left(\beta^- \to \alpha^- + \gamma\right) \simeq \frac{\alpha_{\rm em} G_{\rm F}^2 m_\beta^5}{128\pi^4} \left| \sum_{i=1}^{3} U_{\alpha i} U_{\beta i}^* \left(-\frac{5}{6} + \frac{m_i^2}{4M_W^2}\right) \right|^2$$

However, the calculation in the full theory for $M_i >> M_W$ gives

$$\Gamma\left(\beta^- \to \alpha^- + \gamma\right) \simeq \frac{\alpha_{\rm em} G_{\rm F}^2 m_\beta^5}{128\pi^4} \left| \sum_{i=1}^{3} U_{\alpha i} U_{\beta i}^* \left(-\frac{5}{6} + \frac{m_i^2}{4M_W^2}\right) \boxed{- \frac{1}{3}\left(RR^\dagger\right)_{\alpha\beta}} \right|^2$$

$$\mathcal{B}\left(\mu^- \to e^- + \gamma\right) < 4.2 \times 10^{-13}$$
$$\mathcal{B}\left(\tau^- \to e^- + \gamma\right) < 3.3 \times 10^{-8}$$
$$\mathcal{B}\left(\tau^- \to \mu^- + \gamma\right) < 4.4 \times 10^{-8}$$

Question: What goes wrong with tree-level SEFT? EFT must give the same result as UV theory for low-energy observables

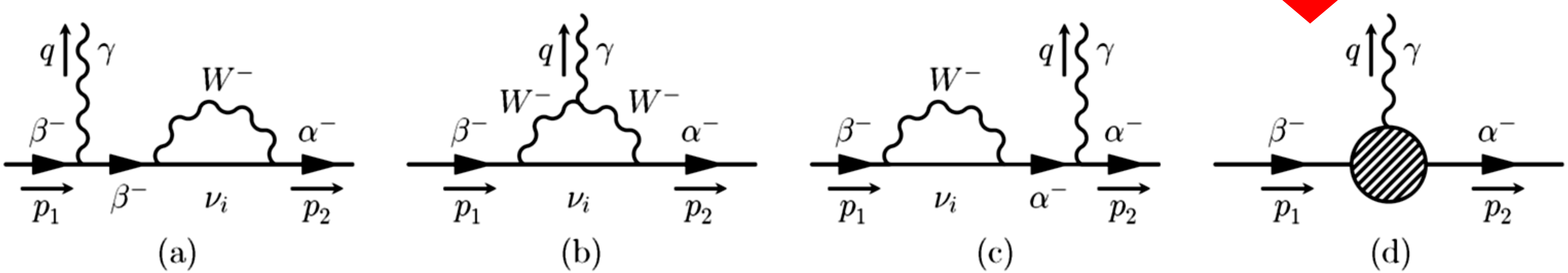
Answer: Radiative decays at one-loop require one-loop matching!

Another two relevant dim-6 operators @ one loop Zhang & S.Z., 2102.04954

$$\mathcal{L}^{(6)}_{\rm loop} = \frac{\left(Y_\nu M^{-2} Y^\dagger_\nu Y_l\right)_{\alpha\beta}}{24\,(4\pi)^2}\left[g_1\left(\overline{\ell_{\alpha \rm L}}\sigma_{\mu\nu}E_{\beta\rm R}\right)HB^{\mu\nu} + 5g_2\left(\overline{\ell_{\alpha\rm L}}\sigma_{\mu\nu}E_{\beta\rm R}\right)\tau^I H W^{I\mu\nu}\right] + {\rm h.c.}$$

leading to the direct EM-dipole vertex

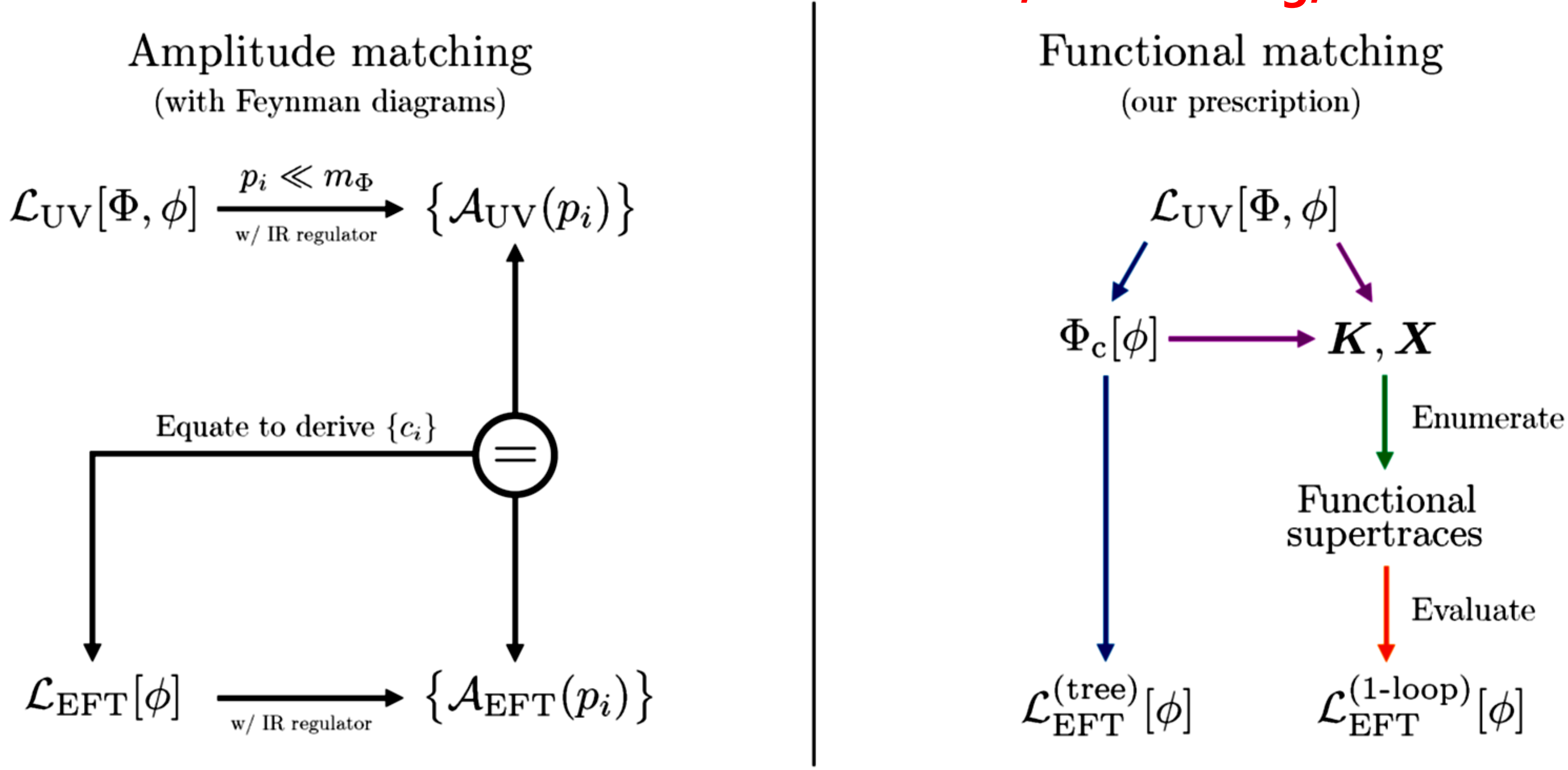
$$\mathcal{L}_{\rm SEFT} = \overline{\nu_{\rm L}}{\rm i}\partial\!\!\!/\nu_{\rm L} - \left(\overline{l_{\rm L}}M_l l_{\rm R} + \frac{1}{2}\overline{\nu_{\rm L}}\widehat{M}_\nu \nu^{\rm c}_{\rm L} + {\rm h.c.}\right) + \left(\frac{g_2}{\sqrt{2}}\overline{l_{\rm L}}\gamma^\mu U\nu_{\rm L}W^-_\mu + {\rm h.c.}\right)$$
$$+\frac{g_2}{2\cos\theta_{\rm w}}\overline{\nu_{\rm L}}\gamma^\mu U^\dagger U\nu_{\rm L}Z_\mu \boxed{- \frac{e g_2^2}{12\,(4\pi)^2 M_W^2}\overline{l_{\rm L}}\sigma_{\mu\nu}RR^\dagger M_l l_{\rm R}F^{\mu\nu}} + {\rm h.c.}\,,$$



exactly reproducing the result in the full theory (with $M_i >> M_W$)

Matching between UV theory and EFT

Cohen, Lu & Zhang, 2011.02484



- **Functional method for one-loop matching**
- **Covariant Derivative Expansion (CDE)** Gaillard, 86; Chen, 86; Cheyette, 88
- **Expansion by Regions (hard and soft loop momentum)** Beneke & Smirnov, 98; Smirnov, 02

Equating UV theory with EFT at the matching scale

$$\underline{\Gamma_{\rm L,UV}\left[\phi_{\rm B}\right]} = \Gamma_{\rm EFT}\left[\phi_{\rm B}\right]$$

One-Light-Particle-Irreducible (1LPI)

Zhang & S.Z., 2107.12133

- **First, look at the UV theory**

$$Z_{\rm UV}\left[J_\Phi, J_\phi\right] = \int \mathcal{D}\Phi\mathcal{D}\phi \exp\left\{{\rm i}\int {\rm d}^d x \left(\mathcal{L}_{\rm UV}\left[\Phi,\phi\right] + J_\Phi \Phi + J_\phi \phi\right)\right\}$$

J_Φ and J_ϕ are external sources for Φ and ϕ $\boxed{d \equiv 4 - 2\varepsilon}$

Background & quantum fields $\Phi = \Phi_{\rm B} + \Phi'\ , \qquad \phi = \phi_{\rm B} + \phi'$

EOMs of background fields $\dfrac{\delta\mathcal{L}_{\rm UV}}{\delta\Phi}\left[\Phi_{\rm B},\phi_{\rm B}\right] + J_\Phi = 0 \quad \dfrac{\delta\mathcal{L}_{\rm UV}}{\delta\phi}\left[\Phi_{\rm B},\phi_{\rm B}\right] + J_\phi = 0$

Expansion up to 2$^{\rm nd}$-order of quantum fields

$$\mathcal{L}_{\rm UV}\left[\Phi,\phi\right] + J_\Phi\Phi + J_\phi\phi \simeq \mathcal{L}_{\rm UV}\left[\Phi_{\rm B},\phi_{\rm B}\right] + J_\Phi\Phi_{\rm B} + J_\phi\phi_{\rm B} - \frac{1}{2}\begin{pmatrix}\Phi'^{\rm T} & \phi'^{\rm T}\end{pmatrix}\mathcal{Q}_{\rm UV}\begin{pmatrix}\Phi' \\ \phi'\end{pmatrix}$$

Expansion up to 2nd-order of quantum fields

$$\mathcal{Q}_{\rm UV} \equiv \begin{pmatrix} -\dfrac{\delta^2 \mathcal{L}_{\rm UV}}{\delta \Phi^2}[\Phi_{\rm B}, \phi_{\rm B}] & -\dfrac{\delta^2 \mathcal{L}_{\rm UV}}{\delta \Phi \delta \phi}[\Phi_{\rm B}, \phi_{\rm B}] \\ -\dfrac{\delta^2 \mathcal{L}_{\rm UV}}{\delta \phi \delta \Phi}[\Phi_{\rm B}, \phi_{\rm B}] & -\dfrac{\delta^2 \mathcal{L}_{\rm UV}}{\delta \phi^2}[\Phi_{\rm B}, \phi_{\rm B}] \end{pmatrix} \equiv \begin{pmatrix} \Delta_\Phi & X_{\Phi\phi} \\ X_{\phi\Phi} & \Delta_\phi \end{pmatrix}$$

Evaluate the Gaussian integral

$$\begin{aligned} Z_{\rm UV}\left[J_\Phi, J_\phi\right] &\simeq \exp\left\{ {\rm i} \int {\rm d}^d x \left(\mathcal{L}_{\rm UV}[\Phi_{\rm B}, \phi_{\rm B}] + J_\Phi \Phi_{\rm B} + J_\phi \phi_{\rm B} \right) \right\} \\ &\quad \times \int \mathcal{D}\Phi' \mathcal{D}\phi' \exp\left\{ -\frac{\rm i}{2} \int {\rm d}^d x \left(\Phi'^{\rm T} \; \phi'^{\rm T} \right) \mathcal{Q}_{\rm UV} \begin{pmatrix} \Phi' \\ \phi' \end{pmatrix} \right\} \\ &\propto \exp\left\{ {\rm i} \int {\rm d}^d x \left(\mathcal{L}_{\rm UV}[\Phi_{\rm B}, \phi_{\rm B}] + J_\Phi \Phi_{\rm B} + J_\phi \phi_{\rm B} \right) \right\} \times \left(\det \mathcal{Q}_{\rm UV} \right)^{-c_s} \end{aligned}$$

$c_{\rm s} = 1/2$ for real bosonic fields $\qquad$ $c_{\rm s} = 1$ for complex bosonic fields

$c_{\rm s} = -1$ (or $-1/2$) for Dirac (or Majorana) fermionic fields

No external heavy fields

$$\left.\frac{\delta \mathcal{L}_{\rm UV}[\Phi, \phi]}{\delta \Phi}\right|_{\Phi = \Phi_{\rm c}[\phi_{\rm B}], \phi = \phi_{\rm B}} = 0$$

$$\boxed{\begin{aligned} \Gamma_{\rm L,UV}[\phi_{\rm B}] &\equiv -{\rm i} \ln Z_{\rm UV}\left[J_\Phi = 0, J_\phi\right] - \int {\rm d}^d x J_\phi \phi_{\rm B} \\ &\simeq \int {\rm d}^d x \mathcal{L}_{\rm UV}\left[\Phi_{\rm c}[\phi_{\rm B}], \phi_{\rm B}\right] + \frac{\rm i}{2} \ln \det \mathcal{Q}_{\rm UV}\left[\Phi_{\rm c}[\phi_{\rm B}], \phi_{\rm B}\right] \end{aligned}}$$

Seesaw Effective Field Theory @ one-loop

Classical heavy fields $\Phi_{\rm c}\left[\phi_{\rm B}\right] \equiv \Phi_{\rm B}\left[J_\Phi = 0, J_\phi\right]$ $\xrightarrow{\text{Expanded 1/M}}$ **localized** $\widehat{\Phi}_{\rm c}\left[\phi_{\rm B}\right]$

Tree- & 1-loop effective action

$$\Gamma^{\rm tree}_{\rm L,UV}\left[\phi_{\rm B}\right] = \int {\rm d}^d x \mathcal{L}_{\rm UV}\left[\widehat{\Phi}_{\rm c}\left[\phi_{\rm B}\right], \phi_{\rm B}\right] \ ,$$

$$\Gamma^{\rm 1-loop}_{\rm L,UV}\left[\phi_{\rm B}\right] = \frac{\rm i}{2} \ln\det \mathcal{Q}_{\rm UV}\left[\widehat{\Phi}_{\rm c}\left[\phi_{\rm B}\right], \phi_{\rm B}\right]$$

- **Then, look at the EFT**

$$Z_{\rm EFT}\left[J_\phi\right] = \int \mathcal{D}\phi \exp\left\{{\rm i}\int {\rm d}^d x \left(\mathcal{L}_{\rm EFT}\left[\phi\right] + J_\phi \phi\right)\right\}$$

$$\propto \exp\left\{{\rm i}\int {\rm d}^d x \left(\mathcal{L}^{\rm tree}_{\rm EFT}\left[\phi_{\rm B}\right] + \mathcal{L}^{\rm 1-loop}_{\rm EFT}\left[\phi_{\rm B}\right] + J_\phi \phi_{\rm B}\right)\right\} \times \left(\det \mathcal{Q}_{\rm EFT}\right)^{-1/2}$$

$$\boxed{\mathcal{Q}_{\rm EFT} \equiv -\frac{\delta^2 \mathcal{L}^{\rm tree}_{\rm EFT}}{\delta\phi^2}\left[\phi_{\rm B}\right]}$$

$$\Gamma_{\rm EFT}\left[\phi_{\rm B}\right] = -{\rm i}\ln Z_{\rm EFT}\left[J_\phi\right] - \int {\rm d}^d x J_\phi \phi_{\rm B}$$

$$\simeq \int {\rm d}^d x \left(\mathcal{L}^{\rm tree}_{\rm EFT}\left[\phi_{\rm B}\right] + \mathcal{L}^{\rm 1-loop}_{\rm EFT}\left[\phi_{\rm B}\right]\right) + \frac{\rm i}{2}\ln\det \mathcal{Q}_{\rm EFT}$$

$$\boxed{\begin{aligned} \Gamma^{\rm tree}_{\rm EFT}\left[\phi_{\rm B}\right] &= \int {\rm d}^d x \mathcal{L}^{\rm tree}_{\rm EFT}\left[\phi_{\rm B}\right] \ , \\ \Gamma^{\rm 1-loop}_{\rm EFT}\left[\phi_{\rm B}\right] &= \int {\rm d}^d x \mathcal{L}^{\rm 1-loop}_{\rm EFT}\left[\phi_{\rm B}\right] + \frac{\rm i}{2}\ln\det \mathcal{Q}_{\rm EFT} \end{aligned}}$$

- **Finally, matching UV theory with EFT**

$$\Gamma^{\rm tree}_{\rm L,UV}\left[\phi_{\rm B}\right] = \int {\rm d}^d x \mathcal{L}_{\rm UV}\left[\widehat{\Phi}_{\rm c}\left[\phi_{\rm B}\right], \phi_{\rm B}\right] \; ,$$
$$\Gamma^{\rm 1-loop}_{\rm L,UV}\left[\phi_{\rm B}\right] = \frac{\rm i}{2}\ln\det \mathcal{Q}_{\rm UV}\left[\widehat{\Phi}_{\rm c}\left[\phi_{\rm B}\right], \phi_{\rm B}\right]$$

$$\Gamma^{\rm tree}_{\rm EFT}\left[\phi_{\rm B}\right] = \int {\rm d}^d x \mathcal{L}^{\rm tree}_{\rm EFT}\left[\phi_{\rm B}\right] \; ,$$
$$\Gamma^{\rm 1-loop}_{\rm EFT}\left[\phi_{\rm B}\right] = \int {\rm d}^d x \mathcal{L}^{\rm 1-loop}_{\rm EFT}\left[\phi_{\rm B}\right] + \frac{\rm i}{2}\ln\det \mathcal{Q}_{\rm EFT}$$

$$\mathcal{L}^{\rm tree}_{\rm EFT}\left[\phi_{\rm B}\right] = \mathcal{L}_{\rm UV}\left[\widehat{\Phi}_{\rm c}\left[\phi_{\rm B}\right], \phi_{\rm B}\right]$$

$$\int {\rm d}^d x \mathcal{L}^{\rm 1-loop}_{\rm EFT}\left[\phi_{\rm B}\right] + \frac{\rm i}{2}\ln\det \mathcal{Q}_{\rm EFT}\left[\phi_{\rm B}\right] = \frac{\rm i}{2}\ln\det \mathcal{Q}_{\rm UV}\left[\widehat{\Phi}_{\rm c}\left[\phi_{\rm B}\right], \phi_{\rm B}\right]$$

Expansion by Regions

$$\int {\rm d}^d x \mathcal{L}^{\rm 1-loop}_{\rm EFT}\left[\phi_{\rm B}\right] = \frac{\rm i}{2}\ln\det\left(\Delta_\Phi - X_{\Phi\phi}\Delta_\phi^{-1}X_{\phi\Phi}\right)\left[\widehat{\Phi}_{\rm c}\left[\phi_{\rm B}\right], \phi_{\rm B}\right]\Big|_{\rm hard}$$
$$\int {\rm d}^d x \mathcal{L}^{\rm 1-loop}_{\rm EFT}\left[\phi_{\rm B}\right] = \Gamma^{\rm 1-loop}_{\rm L,UV}\left[\phi_{\rm B}\right]\Big|_{\rm hard} = \frac{\rm i}{2}\ln\det \mathcal{Q}_{\rm UV}\left[\widehat{\Phi}_{\rm c}\left[\phi_{\rm B}\right], \phi_{\rm B}\right]\Big|_{\rm hard}$$

Calculate supertraces: CDE method

$$\begin{aligned}\int \mathrm{d}^d x \mathcal{L}_{\mathrm{EFT}}^{1-\mathrm{loop}}[\phi] &= \frac{\mathrm{i}}{2} \ln \mathrm{Sdet}\left(-\boldsymbol{K}+\boldsymbol{X}\right)\Big|_{\mathrm{hard}} = \frac{\mathrm{i}}{2}\mathrm{STr}\ln\left(-\boldsymbol{K}+\boldsymbol{X}\right)\Big|_{\mathrm{hard}} \\ &= \frac{\mathrm{i}}{2}\mathrm{STr}\ln\left(-\boldsymbol{K}\right)\Big|_{\mathrm{hard}} - \frac{\mathrm{i}}{2}\sum_{n=1}^{\infty}\frac{1}{n}\mathrm{STr}\left[\left(\boldsymbol{K}^{-1}\boldsymbol{X}\right)^n\right]\Big|_{\mathrm{hard}} ,\end{aligned}$$

$$\boldsymbol{K}_i = \begin{cases} P^2 - m_i^2 & \boxed{P_\mu \equiv \mathrm{i}D_\mu} \qquad (\text{scalar}) \\ \not{P} - m_i & \qquad (\text{fermion}) \\ -g^{\mu\nu}(P^2 - m_i^2) + (1-\xi^{-1})P^\mu P^\nu & \qquad (\text{vector}) \end{cases}$$

Application to type-I seesaw model

$$X_{HN} = \begin{pmatrix} \overline{\ell_{\mathrm{L}}} Y_\nu \epsilon \\ \overline{\ell^c_{\mathrm{L}}} Y^*_\nu \epsilon \end{pmatrix} , \qquad X_{NH} = \begin{pmatrix} \epsilon^{\mathrm{T}} Y^\dagger_\nu \ell_{\mathrm{L}} & \epsilon^{\mathrm{T}} Y^{\mathrm{T}}_\nu \ell^c_{\mathrm{L}} \end{pmatrix} ,$$

$$X_{\ell N} = \begin{pmatrix} Y_\nu \widetilde{H} P_{\mathrm{R}} \\ Y^*_\nu \widetilde{H}^* P_{\mathrm{L}} \end{pmatrix} , \qquad X_{N\ell} = \begin{pmatrix} \widetilde{H}^\dagger Y^\dagger_\nu P_{\mathrm{L}} & \widetilde{H}^{\mathrm{T}} Y^{\mathrm{T}}_\nu P_{\mathrm{R}} \end{pmatrix} ,$$

$$X_{\ell H} = \begin{pmatrix} Y_l E_{\mathrm{R}} & \epsilon Y_\nu P_{\mathrm{R}} N \\ \epsilon Y^*_\nu P_{\mathrm{L}} N & Y^*_l E^c_{\mathrm{R}} \end{pmatrix} , \qquad X_{H\ell} = \begin{pmatrix} \overline{E_{\mathrm{R}}} Y^\dagger_l & \epsilon^{\mathrm{T}} \overline{N} Y^{\mathrm{T}}_\nu P_{\mathrm{R}} \\ \epsilon^{\mathrm{T}} \overline{N} Y^\dagger_\nu P_{\mathrm{L}} & \overline{E^c_{\mathrm{R}}} Y^{\mathrm{T}}_l \end{pmatrix}$$

Calculate supertraces: CDE method

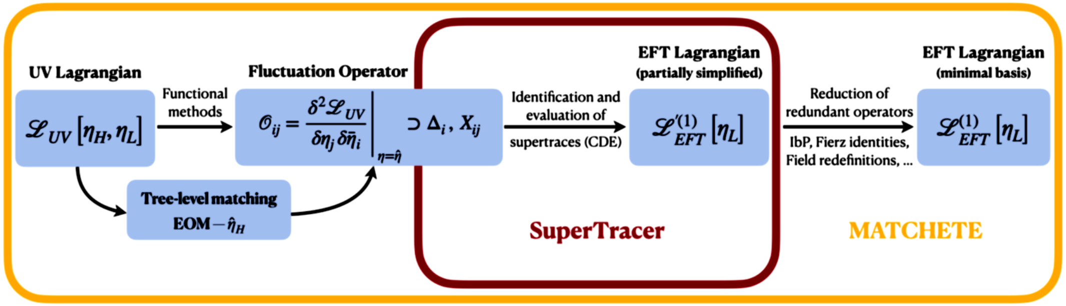
$$
\begin{aligned}
\int \mathrm{d}^d x \mathcal{L}_{\mathrm{EFT}}^{1-\mathrm{loop}} = -\frac{\mathrm{i}}{2} \Big\{ & \mathrm{STr}\left(K_N^{-1} X_{NH} K_H^{-1} X_{HN}\right) + \mathrm{STr}\left(K_N^{-1} X_{N\psi} K_\psi^{-1} X_{\psi N}\right) \\
& + \mathrm{STr}\left(K_N^{-1} X_{NH} K_H^{-1} X_{HH} K_H^{-1} X_{HN}\right) + \mathrm{STr}\left(K_N^{-1} X_{N\psi} K_\psi^{-1} X_{\psi\psi} K_\psi^{-1} X_{\psi N}\right) \\
& + \mathrm{STr}\left(K_N^{-1} X_{NH} K_H^{-1} X_{HV} K_V^{-1} X_{VH} K_H^{-1} X_{HN}\right) \\
& + \mathrm{STr}\left(K_N^{-1} X_{NH} K_H^{-1} X_{H\psi} K_\psi^{-1} X_{\psi H} K_H^{-1} X_{HN}\right) \\
& + \mathrm{STr}\left(K_N^{-1} X_{N\psi} K_\psi^{-1} X_{\psi V} K_V^{-1} X_{V\psi} K_\psi^{-1} X_{\psi N}\right) \\
& + \mathrm{STr}\left(K_N^{-1} X_{N\psi} K_\psi^{-1} X_{\psi H} K_H^{-1} X_{H\psi} K_\psi^{-1} X_{\psi N}\right) \\
& + \mathrm{STr}\left(K_N^{-1} X_{N\psi} K_\psi^{-1} X_{\psi\psi} K_\psi^{-1} X_{\psi\psi} K_\psi^{-1} X_{\psi N}\right) \\
& + \mathrm{STr}\left(K_N^{-1} X_{N\psi} K_\psi^{-1} X_{\psi\psi} K_\psi^{-1} X_{\psi\psi} K_\psi^{-1} X_{\psi\psi} K_\psi^{-1} X_{\psi N}\right) \\
& + \mathrm{STr}\left(K_N^{-1} X_{N\psi} K_\psi^{-1} X_{\psi\psi} K_\psi^{-1} X_{\psi\psi} K_\psi^{-1} X_{\psi\psi} K_\psi^{-1} X_{\psi\psi} K_\psi^{-1} X_{\psi N}\right) \\
& + \Big[\mathrm{STr}\left(K_N^{-1} X_{NH} K_H^{-1} X_{H\psi} K_\psi^{-1} X_{\psi N}\right) \\
& + \mathrm{STr}\left(K_N^{-1} X_{NH} K_H^{-1} X_{HV} K_V^{-1} X_{V\psi} K_\psi^{-1} X_{\psi N}\right) \\
& + \mathrm{STr}\left(K_N^{-1} X_{NH} K_H^{-1} X_{HH} K_H^{-1} X_{H\psi} K_\psi^{-1} X_{\psi N}\right) \\
& + \mathrm{STr}\left(K_N^{-1} X_{NH} K_H^{-1} X_{H\psi} K_\psi^{-1} X_{\psi\psi} K_\psi^{-1} X_{\psi N}\right) \\
& + \mathrm{STr}\left(K_N^{-1} X_{NH} K_H^{-1} X_{HV} K_V^{-1} X_{VH} K_H^{-1} X_{H\psi} K_\psi^{-1} X_{\psi N}\right) \\
& + \mathrm{STr}\left(K_N^{-1} X_{NH} K_H^{-1} X_{HV} K_V^{-1} X_{V\psi} K_\psi^{-1} X_{\psi\psi} K_\psi^{-1} X_{\psi N}\right) \\
& + \mathrm{STr}\left(K_N^{-1} X_{NH} K_H^{-1} X_{H\psi} K_\psi^{-1} X_{\psi\psi} K_\psi^{-1} X_{\psi\psi} K_\psi^{-1} X_{\psi N}\right) \\
& + \mathrm{STr}\left(K_N^{-1} X_{N\psi} K_\psi^{-1} X_{\psi V} K_V^{-1} X_{VH} K_H^{-1} X_{H\psi} K_\psi^{-1} X_{\psi N}\right) \\
& + \mathrm{STr}\left(K_N^{-1} X_{N\psi} K_\psi^{-1} X_{\psi V} K_V^{-1} X_{V\psi} K_\psi^{-1} X_{\psi\psi} K_\psi^{-1} X_{\psi N}\right) \\
& + \mathrm{STr}\left(K_N^{-1} X_{N\psi} K_\psi^{-1} X_{\psi H} K_H^{-1} X_{H\psi} K_\psi^{-1} X_{\psi\psi} K_\psi^{-1} X_{\psi N}\right) + \mathrm{h.c.}\Big] \Big\}\Big|_{\mathrm{hard}} \\
-\frac{\mathrm{i}}{2} \Big\{ & \frac{1}{2}\mathrm{STr}\left[\left(K_N^{-1} X_{NH} K_H^{-1} X_{HN}\right)^2\right] + \frac{1}{2}\mathrm{STr}\left[\left(K_N^{-1} X_{N\psi} K_\psi^{-1} X_{\psi N}\right)^2\right] \\
& + \frac{1}{2}\mathrm{STr}\left[\left(K_N^{-1} X_{N\psi} K_\psi^{-1} X_{\psi\psi} K_\psi^{-1} X_{\psi N}\right)^2\right] + \frac{1}{3}\mathrm{STr}\left[\left(K_N^{-1} X_{N\psi} K_\psi^{-1} X_{\psi N}\right)^3\right] \\
& + \mathrm{STr}\left(K_N^{-1} X_{N\psi} K_\psi^{-1} X_{\psi N} K_N^{-1} X_{NH} K_H^{-1} X_{HN}\right) \\
& + \mathrm{STr}\left(K_N^{-1} X_{N\psi} K_\psi^{-1} X_{\psi\psi} K_\psi^{-1} X_{\psi N} K_N^{-1} X_{NH} K_H^{-1} X_{HN}\right) \\
& + \mathrm{STr}\left(K_N^{-1} X_{N\psi} K_\psi^{-1} X_{\psi N} K_N^{-1} X_{N\psi} K_\psi^{-1} X_{\psi\psi} K_\psi^{-1} X_{\psi N}\right) \\
& + \mathrm{STr}\left(K_N^{-1} X_{N\psi} K_\psi^{-1} X_{\psi N} K_N^{-1} X_{N\psi} K_\psi^{-1} X_{\psi\psi} K_\psi^{-1} X_{\psi\psi} K_\psi^{-1} X_{\psi N}\right) \\
& + \Big[\mathrm{STr}\left(K_N^{-1} X_{N\psi} K_\psi^{-1} X_{\psi N} K_N^{-1} X_{NH} K_H^{-1} X_{H\psi} K_\psi^{-1} X_{\psi N}\right) + \mathrm{h.c.}\Big] \Big\}\Big|_{\mathrm{hard}} ,
\end{aligned}
$$

MITP-20-076
TUM-HEP-1302/20
ZU-TH-54/20

SUPER TRACER

Fuentes-Martin et al., 2012.08506

SuperTracer: A Calculator of Functional Supertraces for One-Loop EFT Matching



Further reduction

- **Integration by parts**
- **Fierz identities**
- **Field redefinitions**
- **Equations of motion**

X^2H^2	
$\mathcal{O}_{HB}$	$B_{\mu\nu}B^{\mu\nu}H^\dagger H$
$\mathcal{O}_{HW}$	$W^I_{\mu\nu}W^{I\mu\nu}H^\dagger H$
$\mathcal{O}_{HWB}$	$W^I_{\mu\nu}B^{\mu\nu}\left(H^\dagger\tau^I H\right)$

H^4D^2	
$\mathcal{O}_{H\Box}$	$\left(H^\dagger H\right)\Box\left(H^\dagger H\right)$
$\mathcal{O}_{HD}$	$\left(H^\dagger D_\mu H\right)^*\left(H^\dagger D^\mu H\right)$

H^6	
$\mathcal{O}_{H}$	$\left(H^\dagger H\right)^3$

ψ^2XH	
$\mathcal{O}^{\alpha\beta}_{eB}$	$\left(\overline{\ell_{\alpha\mathrm{L}}}\sigma^{\mu\nu}E_{\beta\mathrm{R}}\right)HB_{\mu\nu}$
$\mathcal{O}^{\alpha\beta}_{eW}$	$\left(\overline{\ell_{\alpha\mathrm{L}}}\sigma^{\mu\nu}E_{\beta\mathrm{R}}\right)\tau^I HW^I_{\mu\nu}$

ψ^2DH^2	
$\mathcal{O}^{(1)\alpha\beta}_{HQ}$	$\left(\overline{Q_{\alpha\mathrm{L}}}\gamma^\mu Q_{\beta\mathrm{L}}\right)\left(H^\dagger\mathrm{i}\overleftrightarrow{D}_\mu H\right)$
$\mathcal{O}^{(3)\alpha\beta}_{HQ}$	$\left(\overline{Q_{\alpha\mathrm{L}}}\gamma^\mu\tau^I Q_{\beta\mathrm{L}}\right)\left(H^\dagger\mathrm{i}\overleftrightarrow{D}^I_\mu H\right)$
$\mathcal{O}^{\alpha\beta}_{HU}$	$\left(\overline{U_{\alpha\mathrm{R}}}\gamma^\mu U_{\beta\mathrm{R}}\right)\left(H^\dagger\mathrm{i}\overleftrightarrow{D}_\mu H\right)$
$\mathcal{O}^{\alpha\beta}_{Hd}$	$\left(\overline{D_{\alpha\mathrm{R}}}\gamma^\mu D_{\beta\mathrm{R}}\right)\left(H^\dagger\mathrm{i}\overleftrightarrow{D}_\mu H\right)$
$\mathcal{O}^{(1)\alpha\beta}_{H\ell}$	$\left(\overline{\ell_{\alpha\mathrm{L}}}\gamma^\mu \ell_{\beta\mathrm{L}}\right)\left(H^\dagger\mathrm{i}\overleftrightarrow{D}_\mu H\right)$
$\mathcal{O}^{(3)\alpha\beta}_{H\ell}$	$\left(\overline{\ell_{\alpha\mathrm{L}}}\gamma^\mu\tau^I \ell_{\beta\mathrm{L}}\right)\left(H^\dagger\mathrm{i}\overleftrightarrow{D}^I_\mu H\right)$
$\mathcal{O}^{\alpha\beta}_{He}$	$\left(\overline{E_{\alpha\mathrm{R}}}\gamma^\mu E_{\beta\mathrm{R}}\right)\left(H^\dagger\mathrm{i}\overleftrightarrow{D}_\mu H\right)$

ψ^2H^3	
$\mathcal{O}^{\alpha\beta}_{UH}$	$\left(\overline{Q_{\alpha\mathrm{L}}}\widetilde{H}U_{\beta\mathrm{R}}\right)\left(H^\dagger H\right)$
$\mathcal{O}^{\alpha\beta}_{dH}$	$\left(\overline{Q_{\alpha\mathrm{L}}}HD_{\beta\mathrm{R}}\right)\left(H^\dagger H\right)$
$\mathcal{O}^{\alpha\beta}_{eH}$	$\left(\overline{\ell_{\alpha\mathrm{L}}}HE_{\beta\mathrm{R}}\right)\left(H^\dagger H\right)$

Four-quark	
$\mathcal{O}^{(1)\alpha\beta\gamma\lambda}_{QU}$	$\left(\overline{Q_{\alpha\mathrm{L}}}\gamma^\mu Q_{\beta\mathrm{L}}\right)\left(\overline{U_{\gamma\mathrm{R}}}\gamma_\mu U_{\lambda\mathrm{R}}\right)$
$\mathcal{O}^{(8)\alpha\beta\gamma\lambda}_{QU}$	$\left(\overline{Q_{\alpha\mathrm{L}}}\gamma^\mu T^A Q_{\beta\mathrm{L}}\right)\left(\overline{U_{\gamma\mathrm{R}}}\gamma_\mu T^A U_{\lambda\mathrm{R}}\right)$
$\mathcal{O}^{(1)\alpha\beta\gamma\lambda}_{Qd}$	$\left(\overline{Q_{\alpha\mathrm{L}}}\gamma^\mu Q_{\beta\mathrm{L}}\right)\left(\overline{D_{\gamma\mathrm{R}}}\gamma_\mu D_{\lambda\mathrm{R}}\right)$
$\mathcal{O}^{(8)\alpha\beta\gamma\lambda}_{Qd}$	$\left(\overline{Q_{\alpha\mathrm{L}}}\gamma^\mu T^A Q_{\beta\mathrm{L}}\right)\left(\overline{D_{\gamma\mathrm{R}}}\gamma_\mu T^A D_{\lambda\mathrm{R}}\right)$
$\mathcal{O}^{(1)\alpha\beta\gamma\lambda}_{QUQd}$	$\left(\overline{Q^a_{\alpha\mathrm{L}}}U_{\beta\mathrm{R}}\right)\epsilon^{ab}\left(\overline{Q^b_{\gamma\mathrm{L}}}D_{\lambda\mathrm{R}}\right)$

Four-lepton	
$\mathcal{O}^{\alpha\beta\gamma\beta}_{\ell\ell}$	$\left(\overline{\ell_{\alpha\mathrm{L}}}\gamma^\mu \ell_{\beta\mathrm{L}}\right)\left(\overline{\ell_{\gamma\mathrm{L}}}\gamma_\mu \ell_{\lambda\mathrm{L}}\right)$
$\mathcal{O}^{\alpha\beta\gamma\lambda}_{\ell e}$	$\left(\overline{\ell_{\alpha\mathrm{L}}}\gamma^\mu \ell_{\beta\mathrm{L}}\right)\left(\overline{E_{\gamma\mathrm{R}}}\gamma_\mu E_{\lambda\mathrm{R}}\right)$

Semi-leptonic					
$\mathcal{O}^{(1)\alpha\beta\gamma\lambda}_{\ell Q}$	$\left(\overline{\ell_{\alpha\mathrm{L}}}\gamma^\mu \ell_{\beta\mathrm{L}}\right)\left(\overline{Q_{\gamma\mathrm{L}}}\gamma_\mu Q_{\lambda\mathrm{L}}\right)$	$\mathcal{O}^{\alpha\beta\gamma\lambda}_{\ell U}$	$\left(\overline{\ell_{\alpha\mathrm{L}}}\gamma^\mu \ell_{\beta\mathrm{L}}\right)\left(\overline{U_{\gamma\mathrm{R}}}\gamma_\mu U_{\lambda\mathrm{R}}\right)$	$\mathcal{O}^{\alpha\beta\gamma\lambda}_{\ell edQ}$	$\left(\overline{\ell_{\alpha\mathrm{L}}}E_{\beta\mathrm{R}}\right)\left(\overline{D_{\gamma\mathrm{R}}}Q_{\lambda\mathrm{L}}\right)$
$\mathcal{O}^{(3)\alpha\beta\gamma\lambda}_{\ell Q}$	$\left(\overline{\ell_{\alpha\mathrm{L}}}\gamma^\mu\tau^I \ell_{\beta\mathrm{L}}\right)\left(\overline{Q_{\gamma\mathrm{L}}}\gamma_\mu\tau^I Q_{\lambda\mathrm{L}}\right)$	$\mathcal{O}^{\alpha\beta\gamma\lambda}_{\ell d}$	$\left(\overline{\ell_{\alpha\mathrm{L}}}\gamma^\mu \ell_{\beta\mathrm{L}}\right)\left(D_{\gamma\mathrm{R}}\gamma_\mu D_{\lambda\mathrm{R}}\right)$	$\mathcal{O}^{(1)\alpha\beta\gamma\lambda}_{\ell eQU}$	$\left(\overline{\ell^a_{\alpha\mathrm{L}}}E_{\beta\mathrm{R}}\right)\epsilon^{ab}\left(\overline{Q^b_{\gamma\mathrm{L}}}U_{\lambda\mathrm{R}}\right)$

Out of 59 operators in the Warsaw basis, 31 dim-6 operators in SEFT

Summary

How to use the EFT

- **EFTs as a useful & powerful tool to probe new physics beyond the SM (e.g., SMEFT)**
- **A general idea to deal with a multi-scale system, e.g., mass/energy/time scales (e.g. the type-I seesaw model)**
- **SEFT@1-loop necessary for precision tests of neutrino mass models**

Zhang & S.Z., 2107.12133 | Du, Li & Yu, 2201.04646 | Liao & Ma, 2210.04270

Coy & Frigerio, 2110.09126 | Li, Zhang & S.Z., 2201.05082

Ohlsson & Penrow, 2201.00840 | **Seesaw models, Zee model, Scotogenic model**

Future directions:

1. **One-loop matching performed at the seesaw scale, but two-loop RGEs for the Wilson coefficients are lacking; Multi-step matching/running desired**

2. **Matching onto low-energy EFT and precision calculations of observables; a systematic comparison with experimental data**

Thanks a lot for your attention!