

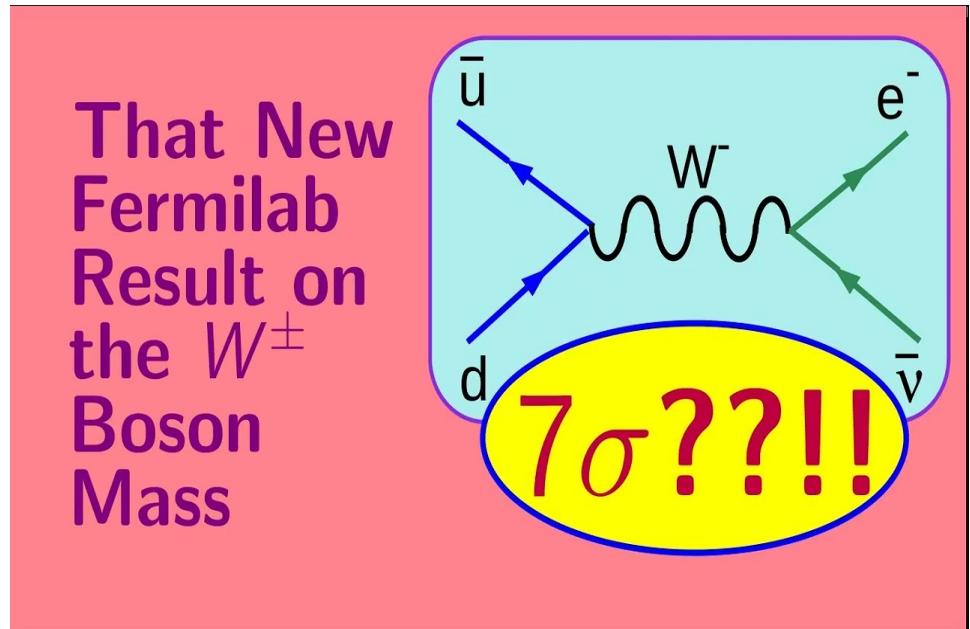


W boson mass: EW Fits and New Physics

武雷

1. W mass in a nutshell

April 8, 2022



$$m_W^{\text{CDF}} = 80.4335 \pm 0.0094 \text{ GeV}$$

Science, 2022, CDF Collaboration

$$m_W^{\text{SM}} = 80.357 \pm 0.006 \text{ GeV}$$

1. W mass in a nutshell



Weak theory

...

1961 : Glashow, IVB Z boson

1964 : Salam and Ward, EW and W boson mass

1967 : Weinberg, EW and W/Z boson masses

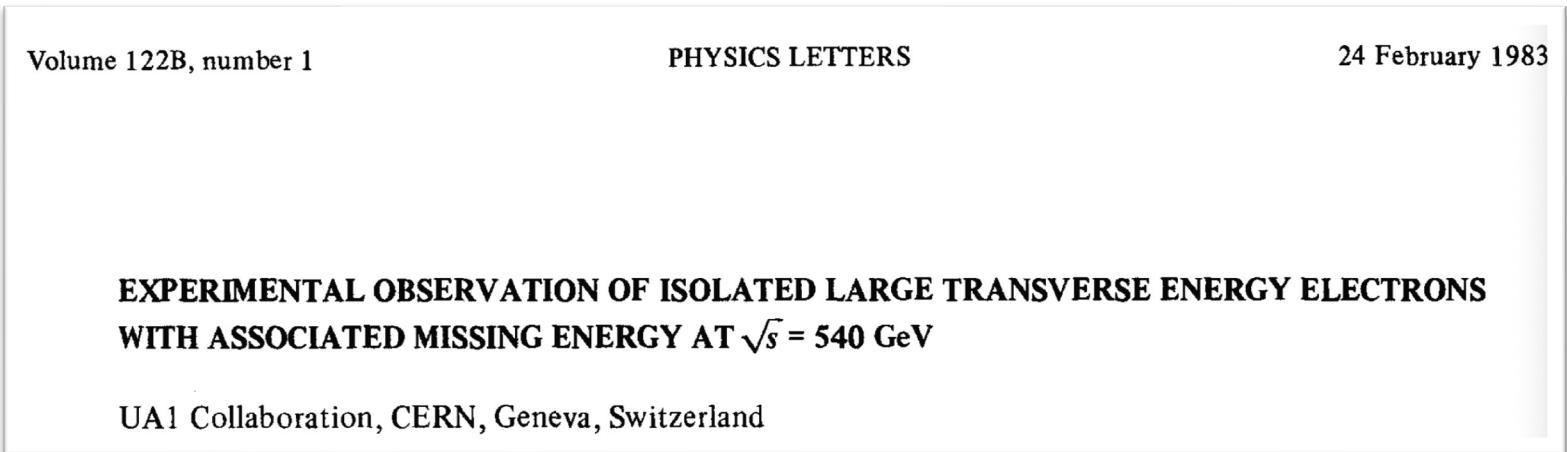
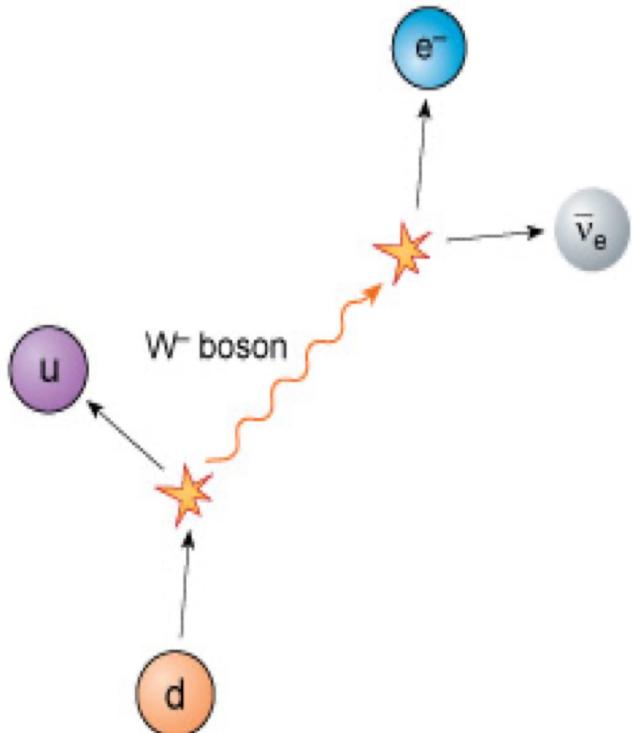
...

1979 Nobel Prize



1. W mass in a nutshell

1983, UA1+2 Collab :



Exp:

The result of a fit on electron angle and energy and neutrino transverse energy with allowance for systematic errors, is

$$m_W = (81^{+5}_{-5}) \text{ GeV}/c^2 ,$$

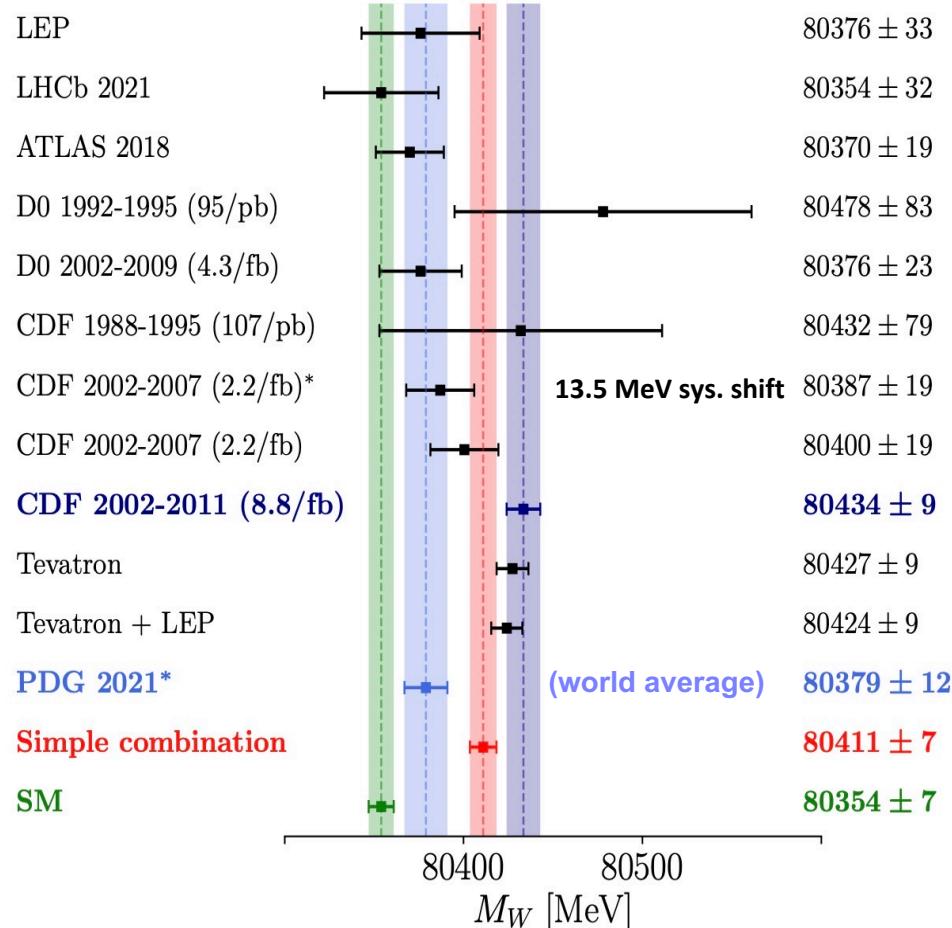
in excellent agreement with the expectation of the Weinberg–Salam model [2].

Th:

Properties of IVBs become better specified within the theoretical frame of the unified weak and electromagnetic theory and of the Weinberg–Salam model [2]. The mass of the IVB is precisely predicted [3]:

$$M_{W^\pm} = (82 \pm 2.4) \text{ GeV}/c^2$$

1. W mass in a nutshell



2204.03996, Atron, Fowlie, Lu, Wu, Wu, Zhu

$$M_W^2 = M_Z^2 \left\{ \frac{1}{2} + \sqrt{\frac{1}{4} - \frac{\pi \alpha}{\sqrt{2} G_F M_Z^2} (1 + \Delta r)} \right\}$$

* iterative calculation

α is extracted from **low-energy experiments**,
 G_F is extracted from the **muon lifetime**,
 m_Z is measured from **e^+e^- annihilation** near the Z mass,
 Δr is **radiative corrections**.

1-loop [1980],
2-loop level [1987-2002, QCD, EW],
leading 3- and 4-loop corrections [1994–2005],
unknown higher-order corrections [2003, 2021 updated],

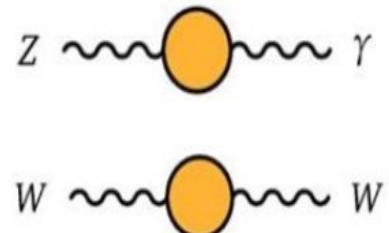
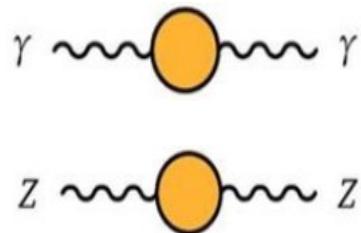
$$\delta M_W^{\text{theo}} \approx 4 \text{ MeV}$$

0311148, Awramik, Czakon, Freitas, Weiglein

1. W mass in a nutshell

New Physics:

$$(M_W^2)_{\text{phys}} = (M_W^{SM})^2 \left[1 - \frac{\alpha S}{2(c_w^2 - s_w^2)} + \frac{c_w^2 \alpha T}{c_w^2 - s_w^2} + \frac{\alpha U}{4s_w^2} \right]$$



$$\alpha S = 4s_W^2 c_W^2 [\Pi_{ZZ}(0) - (c_W^2 - s_W^2)/(s_W c_W) \cdot \Pi'_{Z\gamma}(0) - \Pi'_{\gamma\gamma}(0)]$$

$$\alpha T = \Pi_{WW}(0)/M_W^2 - \Pi_{ZZ}(0)/M_Z^2$$

$$\alpha U = 4s_W^2 [\Pi'_{WW}(0) - c_W^2 \Pi'_{ZZ}(0) - 2s_W c_W \Pi'_{Z\gamma}(0) - s_W^2 \Pi'_{\gamma\gamma}(0)]$$

PRD 46 (1992) 381-409, Peskin and Takeuchi

- (1): The electroweak gauge group must be $SU_L(2) \times U_Y(1)$, with no new electroweak gauge bosons apart from the photon, the W^\pm and the Z .
- (2): The couplings of the new physics to light fermions are suppressed compared to its couplings to the gauge bosons.
- (3): The intrinsic scale, M , of the new physics is large in comparison with M_w and M_z .

- **Exception-1: NP is comparatively light.**
- **Exception-2: NP is comparatively large, but the low-energy measurements become sufficiently accurate.**

9306267, Maksymyk, Burgess, London



2. EW Precision Fits and NP Hints

Three important questions:

- 1. Internal Consistency of the SM ?**
- 2. New Physics Scale ?**
- 3. Possible New Physics Models ?**

2. EW Precision Fits and NP Hints

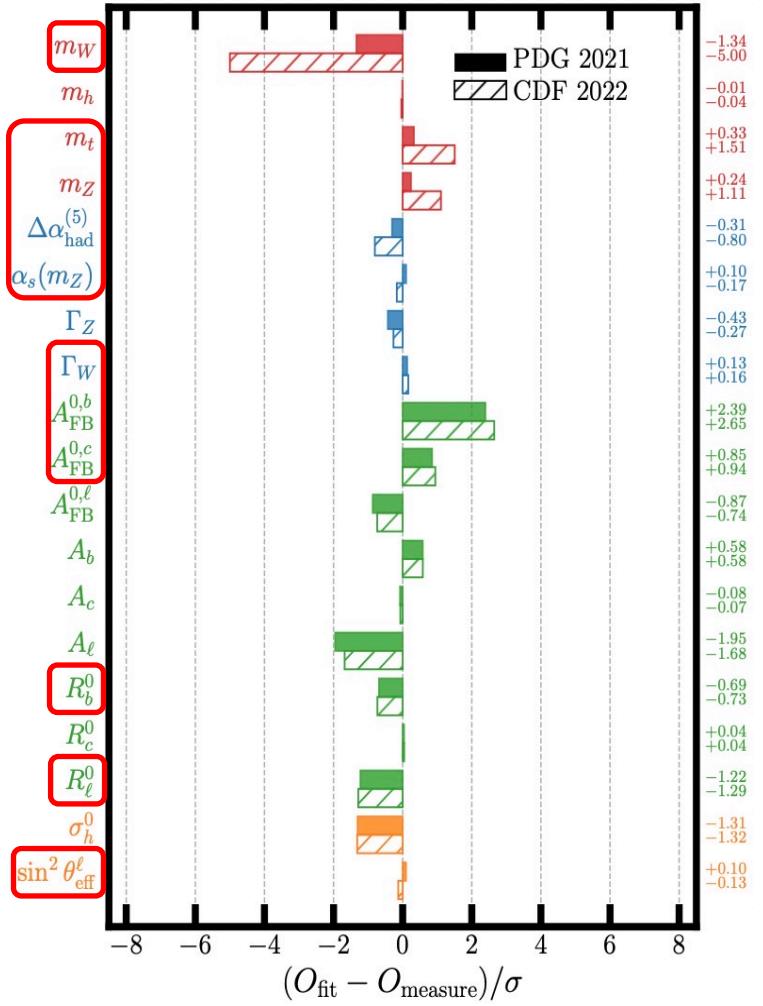
Internal Consistency of the SM :

2022.03796, Lu, Wu, Wu, Zhu

Parameter	Input Value	PDG 2021				CDF 2022			
		$\chi^2_{\text{min}}(\text{dof}) = 18.74(16)$	Fit Result	Pull	Fit w/o Input	Pull	$\chi^2_{\text{min}}(\text{dof}) = 62.58(16)$	Fit Result	Pull
m_W [GeV]	80.379(12)	80.361(6)	-1.34	80.357(6)	-1.65	-	-	-	-
	80.4335(94)	-	-	-	-	80.380(5)	-5.00	80.357(6)	-6.83
$\Delta\alpha_{\text{had}}^{(5)}$ ^a	0.02761(11)	0.02756(11)	-0.31	0.02717(38)	-2.85	0.02747(10)	-0.95	0.02609(36)	-3.99
m_h [GeV]	125.25(17)	125.25(17)	-0.01	92 ⁽²¹⁾ ₍₁₈₎	-1.56	125.24(17)	-0.04	44 ⁽¹⁰⁾ ₍₈₎	-7.96
m_t [GeV] ^b	172.76(58)	173.02(56)	0.33	176.2(2.0)	1.65	173.97(55)	1.51	184.1(16)	6.47
$\alpha_s(m_Z)$	0.1179(9)	0.1180(9)	0.10	0.1193(9)	0.46	0.1177(9)	-0.17	0.1155(29)	-0.80
Γ_W [GeV]	2.085(42)	2.0905(5)	0.13	2.0905(5)	0.13	2.0918(5)	0.16	2.0918(5)	0.16
Γ_Z [GeV]	2.4952(23)	2.4942(6)	-0.43	2.4940(7)	-0.49	2.4946(6)	-0.27	2.4945(7)	-0.31
m_Z [GeV]	91.1875(21)	91.1882(21)	0.24	91.2037(90)	1.75	91.1907(20)	1.11	91.2386(79)	6.28
$A_{FB}^{0,b}$	0.0992(16)	0.1031(3)	2.39	0.1033(3)	2.48	0.1035(3)	2.65	0.1037(3)	2.75
$A_{FB}^{0,c}$	0.0707(35)	0.0737(3)	0.85	0.0737(3)	0.85	0.07401(25)	0.94	0.07402(25)	0.94
$A_{FB}^{0,\ell}$	0.0171(10)	0.01623(10)	-0.87	0.01622(10)	-0.88	0.01636(10)	-0.74	0.01635(10)	-0.75
A_b	0.923(20)	0.93462(4)	0.58	0.93462(4)	0.58	0.93464(4)	0.58	0.93464(4)	0.58
A_c	0.670(27)	0.6679(2)	-0.08	0.6679(2)	-0.08	0.6682(2)	-0.07	0.6682(2)	-0.07
A_ℓ (SLD)	0.1513(21)	0.1471(5)	-1.95	0.1469(5)	-2.05	0.1477(5)	-1.68	0.1475(5)	-1.76
A_ℓ (LEP)	0.1465(33)	0.1471(5)	0.18	0.1469(5)	0.12	0.1477(5)	0.36	0.1475(5)	0.30
R_b^0	0.21629(66)	0.21583(10)	-0.69	0.21582(10)	-0.70	0.21580(10)	-0.73	0.21579(10)	-0.75
R_c^0	0.1721(30)	0.17222(6)	0.04	0.17222(6)	0.04	0.17223(6)	0.04	0.17223(6)	0.04
R_ℓ^0	20.767(25)	20.735(8)	-1.22	20.732(8)	-1.33	20.733(8)	-1.29	20.730(8)	-1.41
σ_h^0 [nb]	41.540(37)	41.491(8)	-1.30	41.489(8)	-1.36	41.490(8)	-1.32	41.488(8)	-1.36
$\sin^2 \theta_{\text{eff}}^{\ell}(Q_{FB})$	0.2324(12)	0.23151(6)	-0.74	0.23151(6)	-0.74	0.23144(6)	-0.80	0.23143(6)	-0.80
$\sin^2 \theta_{\text{eff}}^{\ell}(\text{Teva})$	0.23148(33)	0.23151(6)	0.10	0.23151(6)	0.10	0.23144(6)	-0.13	0.23144(6)	-0.13
\bar{m}_c [GeV]	1.27(2)	1.27(2)	0.00	-	-	1.27(2)	0.00	-	-
\bar{m}_b [GeV]	4.18 ⁽³⁾ ₍₂₎	4.18 ⁽³⁾ ₍₂₎	0.00	-	-	4.18 ⁽³⁾ ₍₂₎	0.00	-	-

^a Scaled with $\alpha_s(m_Z)$.

^b 0.5 GeV theoretical uncertainty is included.





2. EW Precision Fits and NP Hints

0311148, Awramik, Czakon, Freitas, Weiglein

$$M_W = [M_W^0 + c_t \Delta_t + c'_t \Delta_t^2 + c_Z \Delta_Z + c_\alpha \Delta_\alpha + c_{\alpha_s} \Delta_{\alpha_s}] \text{ MeV},$$

with the definitions,

~ 0.5 MeV precision wrt exact formulae

$$\Delta_t \equiv \left(\frac{m_t}{173 \text{ GeV}} \right)^2 - 1, \quad \Delta_Z \equiv \frac{M_Z}{91.1876 \text{ GeV}} - 1, \quad \Delta_\alpha \equiv \frac{\Delta \alpha_{\text{had}}^{(5)}(M_Z^2)}{0.0276} - 1, \quad \Delta_{\alpha_s} \equiv \frac{\alpha_s(M_Z^2)}{0.119} - 1,$$

and the numerical values,

$$M_W^0 = 80359.5 \quad c_t = 520.5 \quad c'_t = -67.7 \quad c_Z = 115000. \quad c_\alpha = -503. \quad c_{\alpha_s} = -71.6$$

M_W correlates **positively** with m_t and M_Z
negatively with α(M_Z) and α_s(M_Z)

2. EW Precision Fits and NP Hints

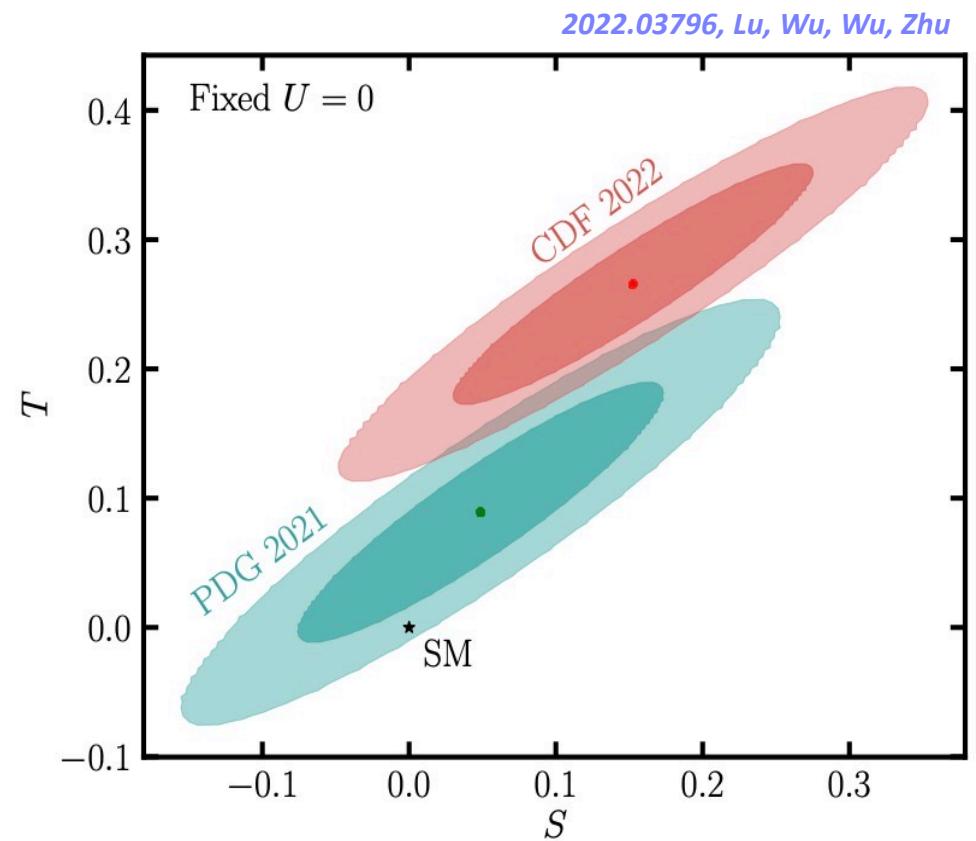
S, T and U:

13 dof	PDG 2021			CDF 2022				
	Result	Correlation		Result	Correlation			
		S	T	U		S	T	U
	$\chi^2_{\min} = 15.42$			$\chi^2_{\min} = 15.44$				
S	0.06 ± 0.10	1.00	0.90	-0.57	0.06 ± 0.10	1.00	0.90	-0.59
T	0.11 ± 0.12		1.00	-0.82	0.11 ± 0.12		1.00	-0.85
U	-0.02 ± 0.09			1.00	0.14 ± 0.09			1.00

due to W width in fits

$U = 0$	PDG 2021			CDF 2022		
	Result	Correlation		Result	Correlation	
	S	T		S	T	
	$\chi^2_{\min} = 15.48$			$\chi^2_{\min} = 17.82$		
S	0.05 ± 0.08	1.00	0.92	0.15 ± 0.08	1.00	0.93
T	0.09 ± 0.07		1.00	0.27 ± 0.06		1.00

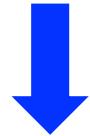
favor positive



2. EW Precision Fits and NP Hints

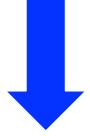
New Physics scale:

$$|H^\dagger D_\mu H|^2 / \Lambda^2$$



T parameter

$$H^\dagger W^{\mu\nu} B_{\mu\nu} H / \Lambda^2$$



S parameter

$$(H^\dagger W^{\mu\nu} H) (H^\dagger W_{\mu\nu} H) / \Lambda^4$$



U parameter

$$\mathcal{O}(0.1) \xrightarrow{\text{pert. couplings}} \Lambda \sim 10 \text{ TeV}$$

$$\mathcal{O}(0.1) \xrightarrow{\text{pert. couplings}} \Lambda \sim 0.1 \text{ TeV}$$

2. EW Precision Fits and NP Hints

Possible NP models:

How to make T larger: $\alpha T = \rho$

$$\begin{aligned}\rho &= \frac{M_W^2}{c_W^2 M_Z^2} \\ &= 1 \\ (\text{Custodial symmetry})\end{aligned}$$

NP @ tree-level

$$\rho = \frac{\sum_i [I_i(I_i + 1) - (I_i^3)^2] v_i^2}{2 \sum_i (I_i^3)^2 v_i^2} \quad (\text{higher Higgs representations: Triplet etc})$$

NP @ loop-level

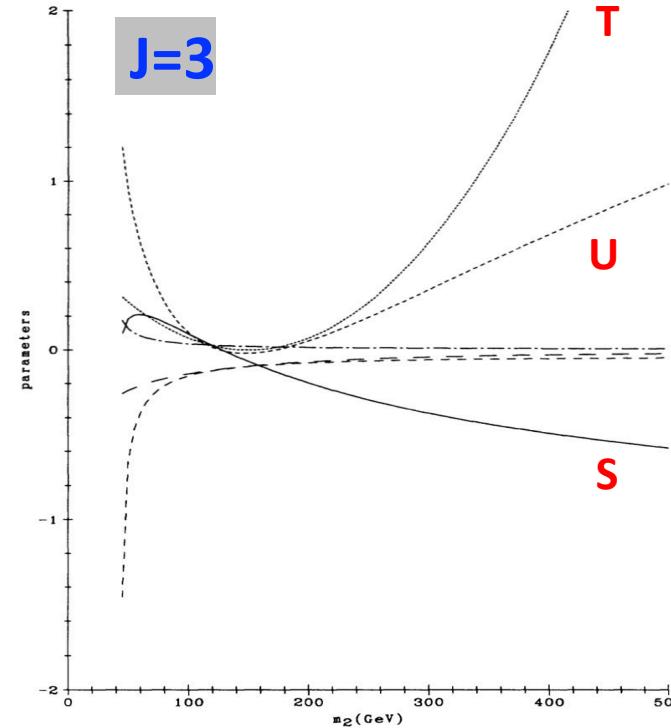
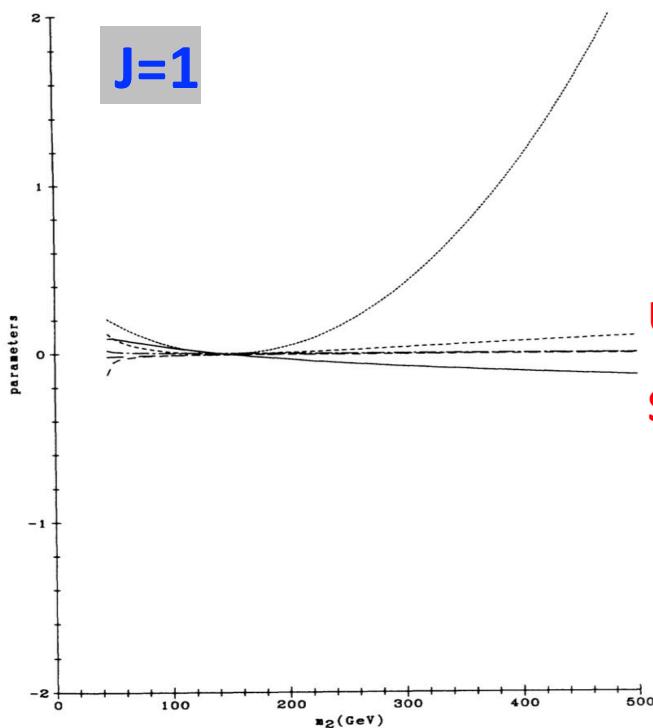
$$\rho = 1 + \frac{3g^2}{64\pi^2} \frac{m_t^2 - m_b^2}{M_W^2} - \frac{11}{192\pi^2} g'^2 \log \frac{M_h^2}{M_Z^2} + \dots \quad (\text{new particles: 2HDM etc})$$

2. EW Precision Fits and NP Hints

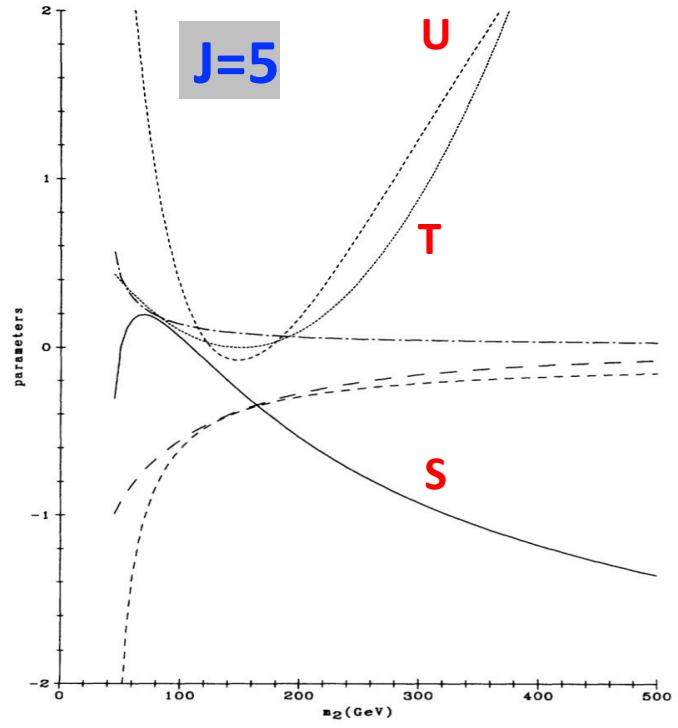
How to make S and U larger:

PRD 49 (1994) 1409-1416, Lavoura and Li

e.g. multiplet scalars



high isospin, light states





3. Inert Higgs DM and W mass

2204.03693, Fan, Tang, Sming, Wu

i2HDM:

Discrete Z_2 symmetry ($H_1 \rightarrow H_1$ and $H_2 \rightarrow -H_2$)

$$H_1 = \begin{pmatrix} G^+ \\ \frac{1}{\sqrt{2}}(v + h + iG^0) \end{pmatrix}, \quad H_2 = \begin{pmatrix} H^+ \\ \frac{1}{\sqrt{2}}(S + iA) \end{pmatrix}$$

$$V = \mu_1^2 |H_1|^2 + \lambda_1 |H_1|^4 + \mu_2^2 |H_2|^2 + \lambda_2 |H_2|^4 + \lambda_3 |H_1|^2 |H_2|^2 + \lambda_4 |H_1^\dagger H_2|^2 + \frac{\lambda_5}{2} \left\{ (H_1^\dagger H_2)^2 + h.c. \right\}$$

PRD 18 (1978) 2574, Deshpande, Ma

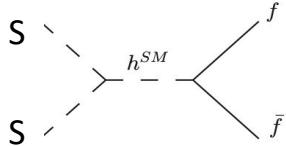


3. Inert Higgs DM and W mass

Oblique Parameter $\left\{ \begin{array}{l} S = \frac{1}{2\pi} \left[\frac{1}{6} \log\left(\frac{m_S^2}{m_{H^\pm}^2}\right) - \frac{5}{36} + \frac{m_S^2 m_A^2}{3(m_A^2 - m_S^2)^2} + \frac{m_A^4(m_A^2 - 3m_S^2)}{6(m_A^2 - m_S^2)^3} \log\left(\frac{m_A^2}{m_S^2}\right) \right] \\ T = \frac{1}{32\pi^2 \alpha v^2} \left[F(m_{H^\pm}^2, m_A^2) + F(m_{H^\pm}^2, m_S^2) - F(m_A^2, m_S^2) \right] \\ \hline = 32i\pi^2 \int \frac{d^4 k}{(2\pi)^4} k^2 \frac{(m_+^2 - m_1^2)(m_+^2 - m_2^2)}{(k^2 - m_+^2)^2(k^2 - m_1^2)(k^2 - m_2^2)} \end{array} \right.$

3. Inert Higgs DM and W mass

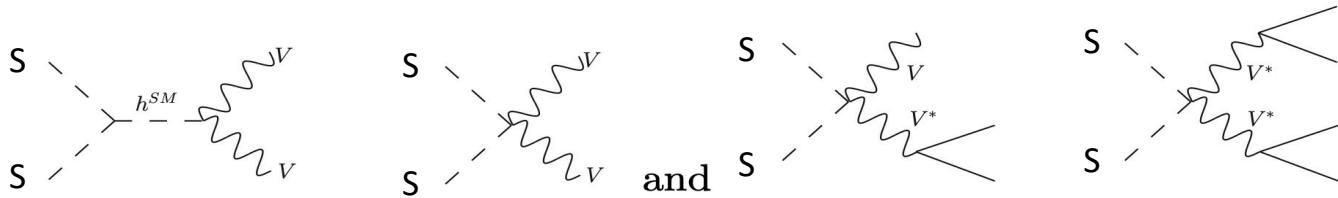
- annihilation through Higgs into fermions



depends on g_{DMh} coupling; dominant channel for $M_{DM} < M_h/2$

- annihilation into gauge bosons (also into virtual states)

Relic Density of DM



crucial for heavy masses; non-negligible for $M_h/2 < M_{DM} < M_W$

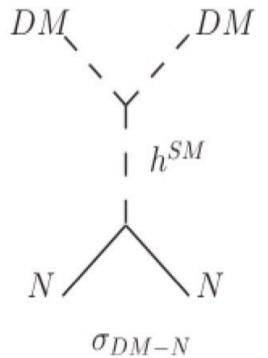
- coannihilation



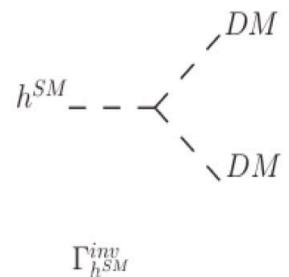
very important when particles have similar masses

3. Inert Higgs DM and W mass

Direct Detection



Higgs invisible decay



DM-scattering on nucleus through the Higgs exchange

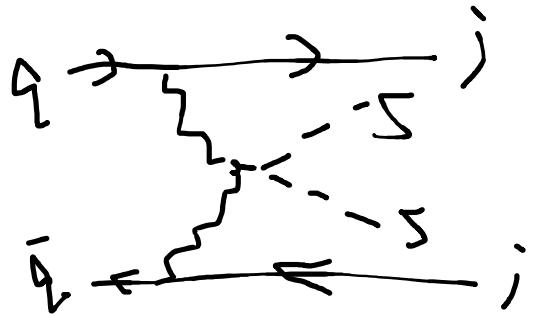
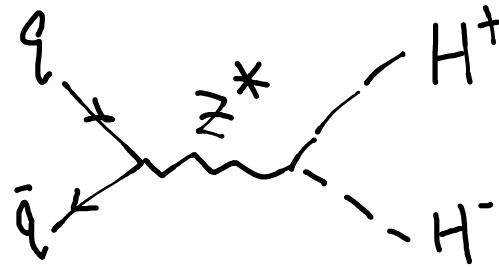
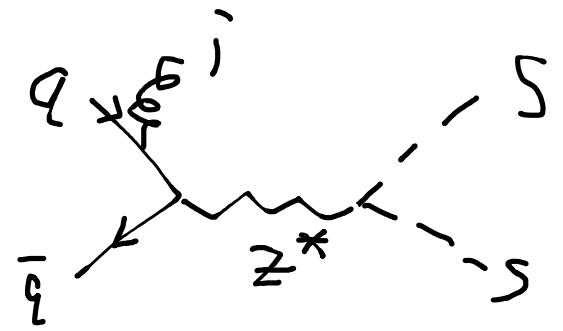
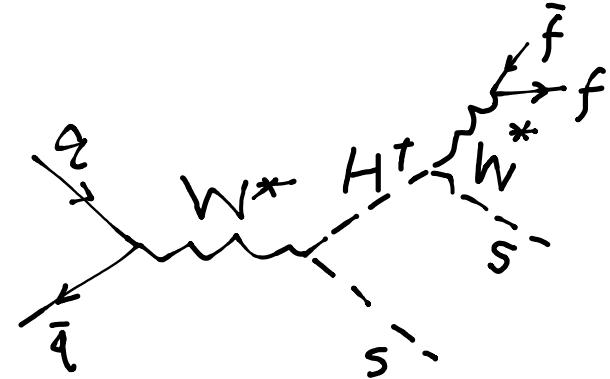
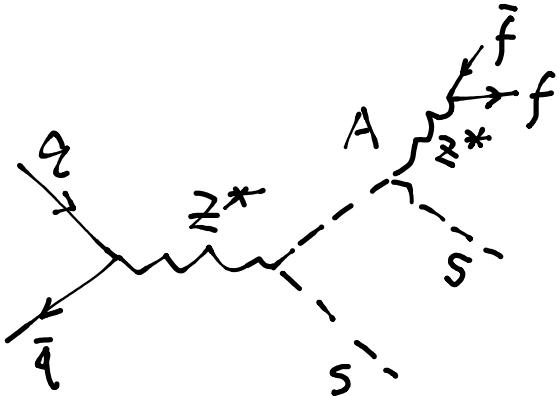
$$\sigma_{DM-N} \propto g_{DMh}^2 / (M_{H_1} + M_N)^2$$

Higgs invisible decays for $M_{DM} < M_h/2$

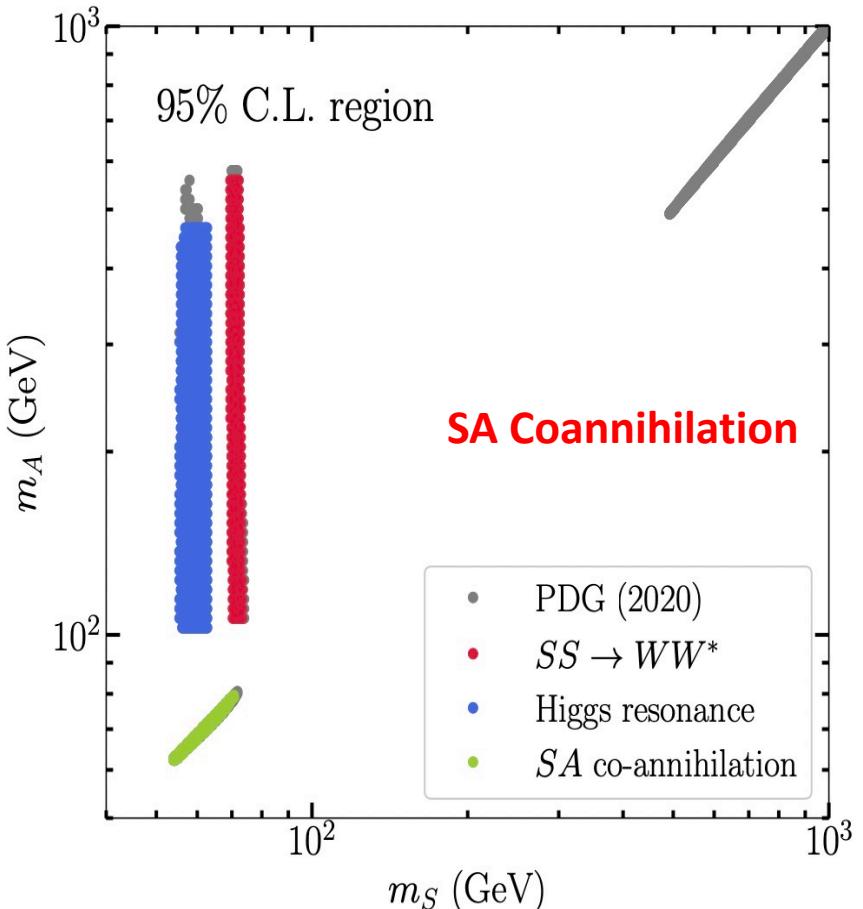
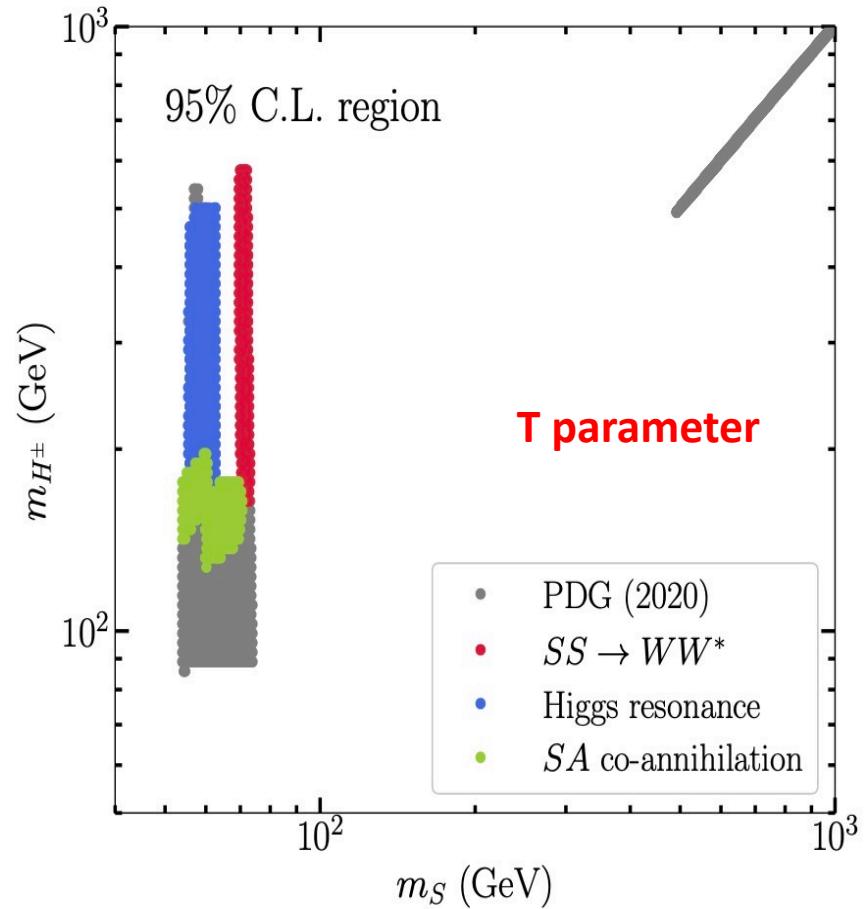
$$\Gamma(h \rightarrow H_1 H_1) = \frac{g_{DMh}^2 v^2}{32\pi M_h} \sqrt{1 - \frac{4M_{H_1}^2}{M_h^2}}$$

3. Inert Higgs DM and W mass

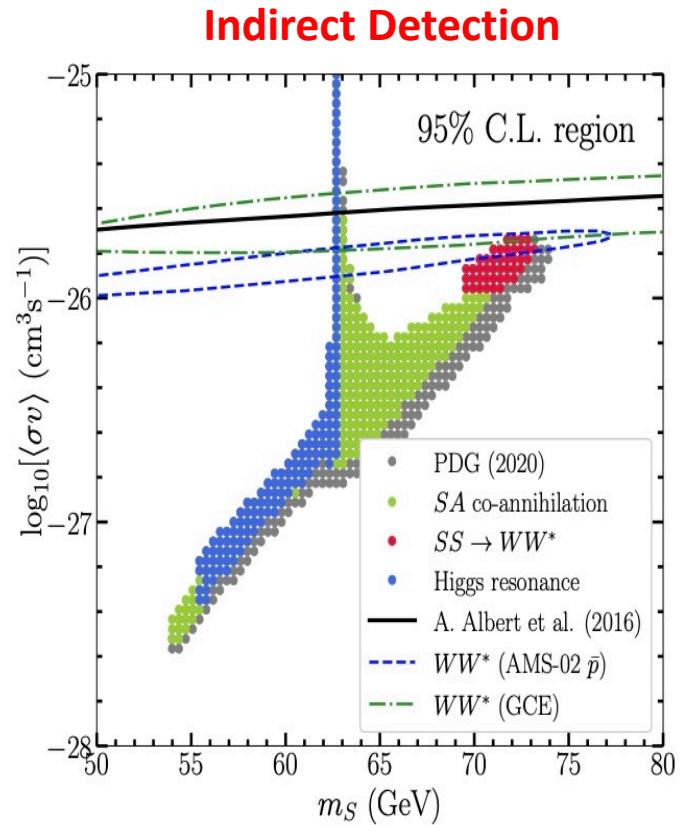
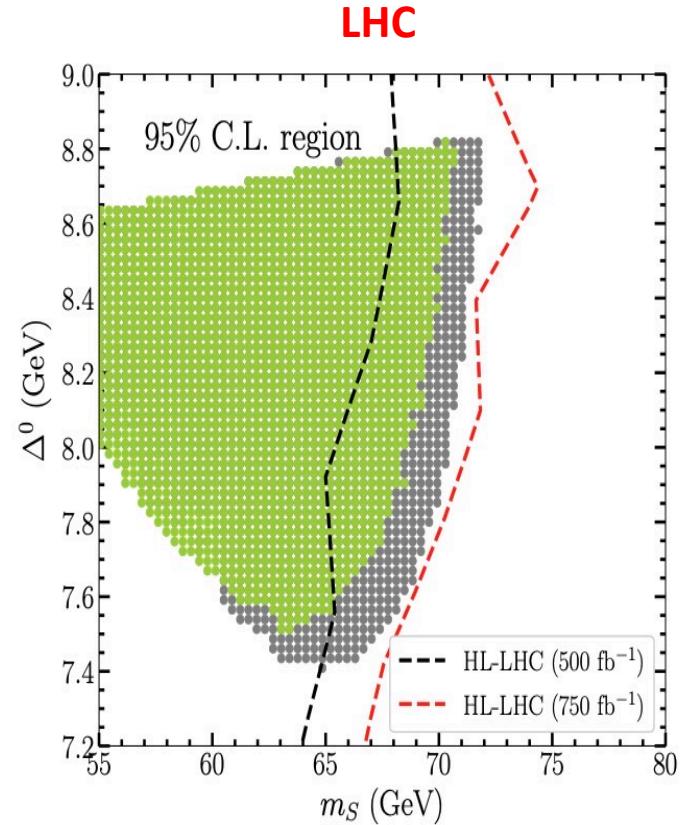
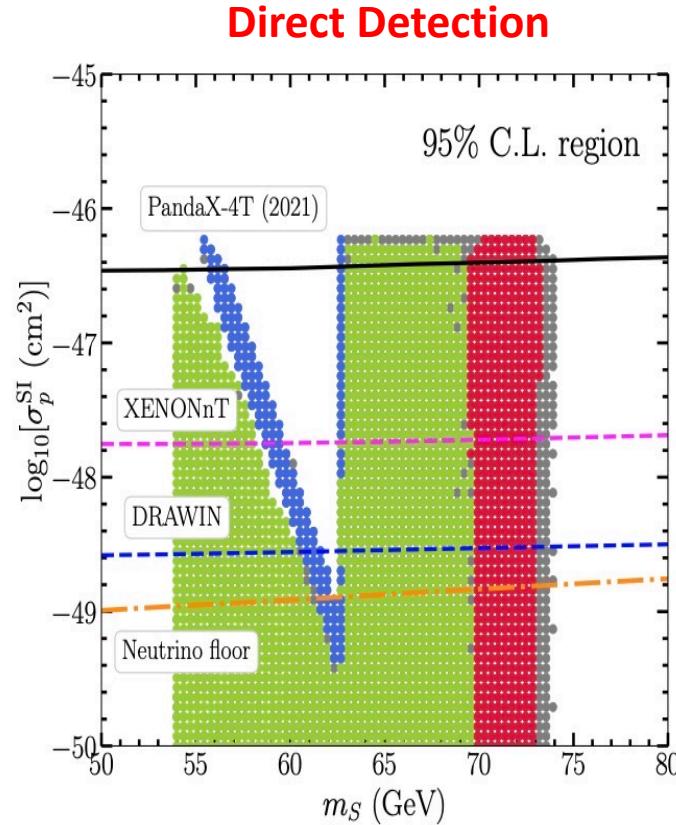
LHC signatures



3. Inert Higgs DM and W mass



3. Inert Higgs DM and W mass



It is very predictive, and can be tested soon!

4. Personal View

I believe that the extraordinary
should be pursued. But
extraordinary claims require
extraordinary evidence.

Carl Sagan

Thank you!



Backup

Questions:

Higher order ✓

PDF ✓

Width ✓

Generator

Detector

New Physics

...

May 6, 2022

[27] [arXiv:2205.02788 \[pdf, other\]](#)

ResBos2 and the CDF W Mass Measurement

[Joshua Isaacson, Yao Fu, C.-P. Yuan](#)

Comments: 11 pages, 13 figures

Subjects: High Energy Physics – Phenomenology (hep-ph); High Energy Physics – Experiment (hep-ex)

The recent CDF W mass measurement of $80,433 \pm 9$ MeV is the most precise direct measurement. However, this result deviates from the Standard Model predicted mass of $80,359.1 \pm 5.2$ MeV by 7σ . The CDF experiment used an older version of the ResBos code that was only accurate at NNLL+NLO, while the state-of-the-art ResBos2 code is able to make predictions at $N^3LL+NNLO$ accuracy. We determine that the data-driven techniques used by CDF capture most of the higher order corrections, and using higher order corrections would result in a decrease in the value reported by CDF by at most 10 MeV.

$\sim 6\sigma$

1. W mass in a nutshell

9306267, Maksymyk, Burgess, London

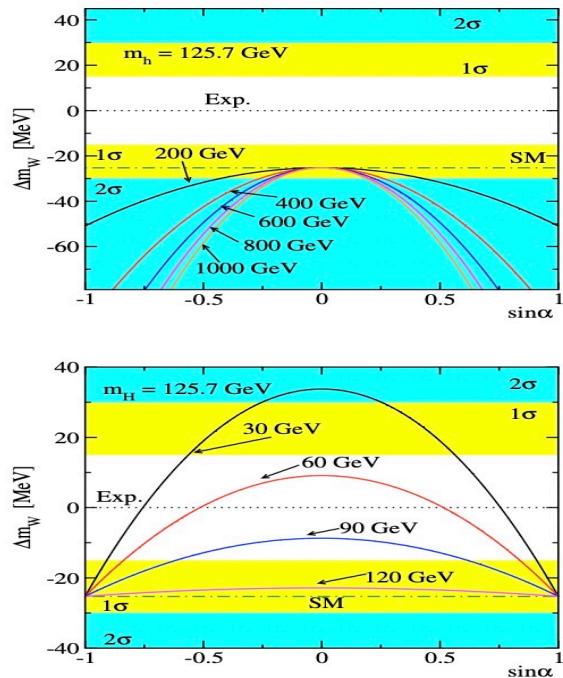
$$\begin{aligned}\Gamma_Z &= 2.4950 - 0.0092S + 0.026T + 0.019V - 0.020X \\ \sigma_{had}^0 &= 41.484 + 0.014S - 0.0098T + 0.031X \\ R_\ell &= 20.743 - 0.062S + 0.042T - 0.14X \\ A_{FB}^\ell &= 0.01626 - 0.0061S + 0.0042T - 0.013X \\ A_\ell &= 0.1472 - 0.028S + 0.019T - 0.061X \\ A_c &= 0.6680 - 0.012S + 0.0084T - 0.027X \\ A_b &= 0.93463 - 0.0023S + 0.0016T - 0.0050X \\ A_{FB}^c &= 0.0738 - 0.015S + 0.010T - 0.033X \\ A_{FB}^b &= 0.1032 - 0.020S + 0.014T - 0.043X \\ R_c &= 0.17226 - 0.00021S + 0.00015T - 0.00046X \\ R_b &= 0.21578 + 0.00013S - 0.000091T + 0.00030X \\ s_{\theta_{\text{eff}}}^2 &= 0.23150 + 0.0035S - 0.0024T + 0.0078X \\ m_W &= 80.364 - 0.28S + 0.43T + 0.35U \\ \Gamma_W &= 2.091 - 0.015S + 0.023T + 0.018U + 0.016W\end{aligned}$$



U only contributes to W mass and width

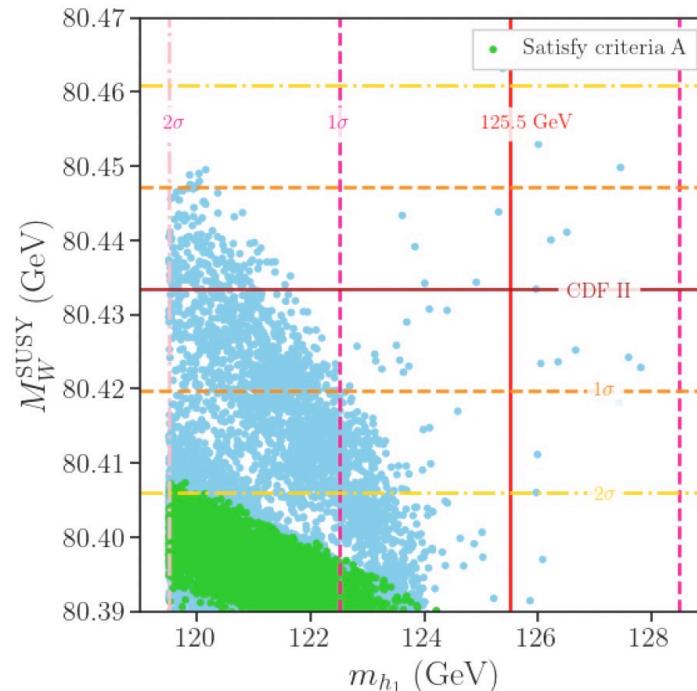
2. EW Precision Fits and NP Hints

Real Singlet:

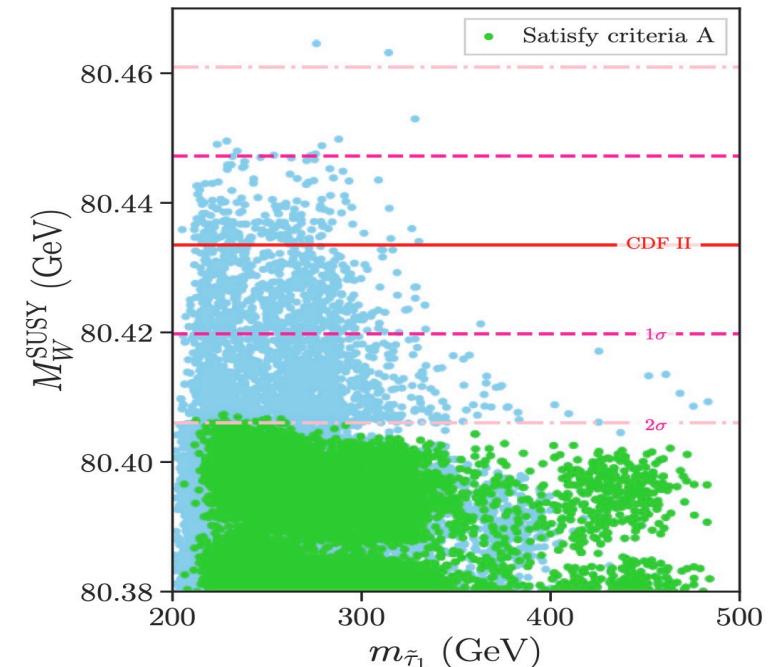


1406.1043, López-Val, Robens

MSSM:



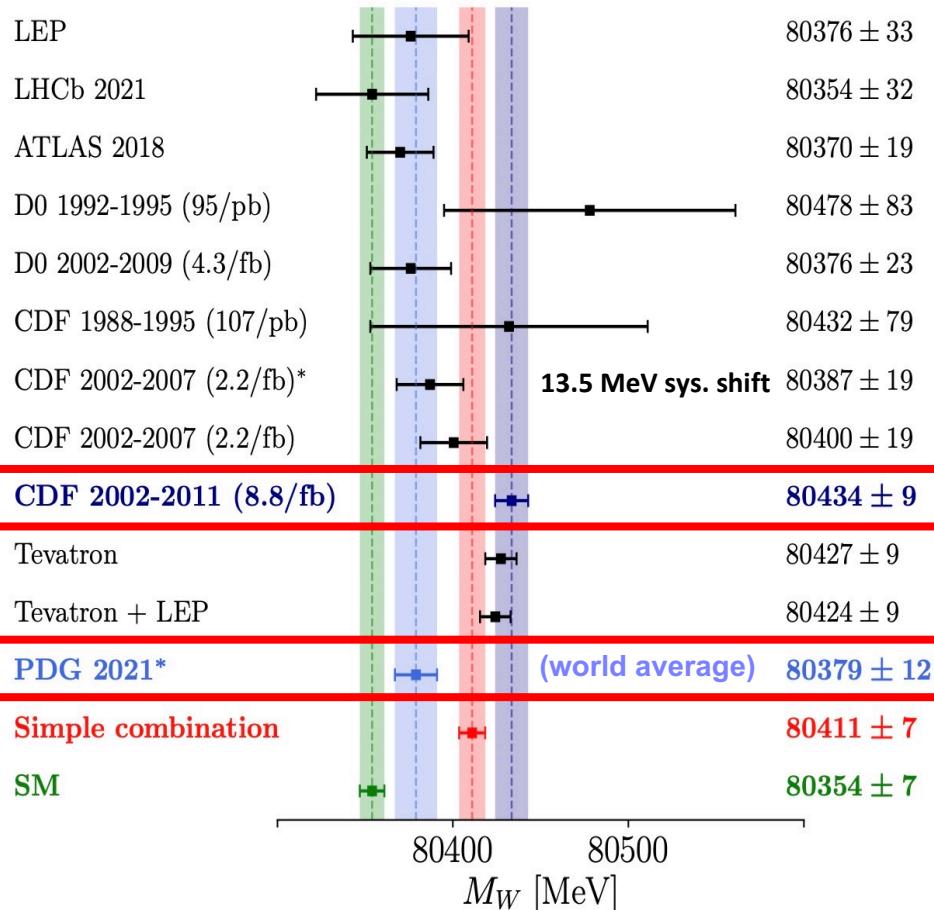
2204.04202, Yang, Zhang



Other solutions: Z', Leptoquark, Vector-like lepton/quark, Triplet scalar etc

1. W mass in a nutshell

2204.03996, Athron, Fowlie, Lu, Wu, Wu, Zhu



What is experimentally measured?

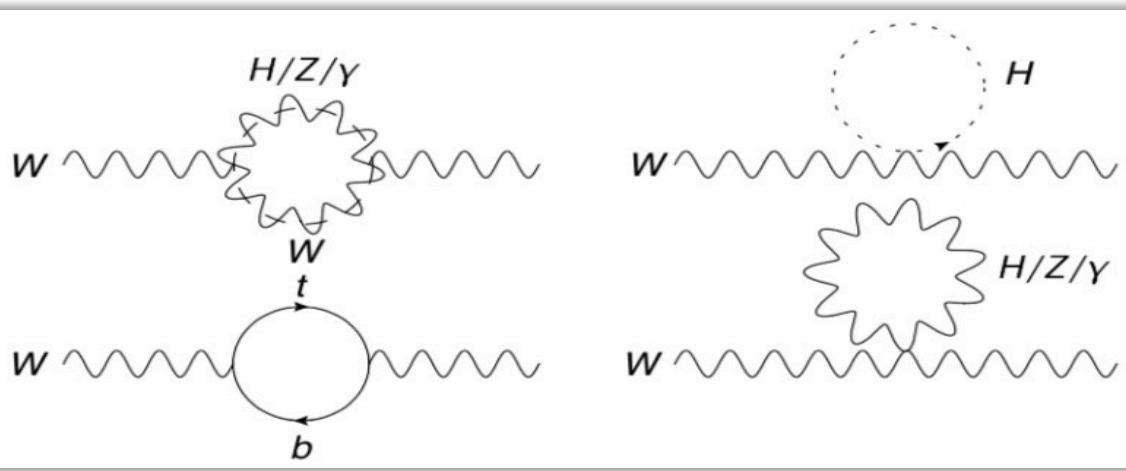
- LEP: $e^+e^- \rightarrow W^+ W^-$ in the continuum and at threshold (small amount of data);
- Tevatron/LHC: transverse mass distribution

Backup

$$\bar{s}_W^2 = 1 - \frac{M_W^2}{M_Z^2} + c_W^2 \left(\frac{\Pi_{WW}(M_W^2)}{M_W^2} - \frac{\Pi_{ZZ}(M_Z^2)}{M_Z^2} \right) \sim 1 - \frac{M_W^2}{M_Z^2} + c_W^2 \Delta\rho$$

$\rho_0 = 1.00038 \pm 0.00020$ (1.9 σ , global fit, PDG 2021)

$$\Delta\rho = \frac{\Pi_{WW}(0)}{M_W^2} - \frac{\Pi_{ZZ}(0)}{M_Z^2}$$



Before electroweak symmetry breaking there was a **global SU(2)_LSU(2)_R symmetry** in the **Higgs potential**, which is broken to just **SU(2)_V** after electroweak symmetry breaking. This remnant symmetry is called custodial symmetry. The **total standard model lagrangian** would be custodial symmetric if the **yukawa couplings** are the same, i.e. $Y_u=Y_d$ and **hypercharge coupling is zero**. It is very important to see beyond the standard model effect by including new terms which violate custodial symmetry.

1. W boson mass in the SM and BSM

$$\Delta r = \Delta\alpha - \cot^2 \theta_W \Delta\rho + \Delta_r^{\text{rem}}$$

scale dependence of α

$$\alpha(M_Z^2) = \frac{\alpha(0)}{1 - \Delta\alpha}$$

$$\Delta\alpha(M_Z^2) = \Pi_{\gamma\gamma}(0) - \Pi_{\gamma\gamma}(M_Z^2)$$

$$\Delta\alpha^{\text{lept}}(M_Z^2) = \sum_{\ell=e,\mu,\tau} \frac{\alpha}{3\pi} \left[\log \frac{M_Z^2}{m_\ell^2} - \frac{5}{3} \right] + \mathcal{O}\left(\frac{m_\ell^2}{M_Z^2}\right) + \mathcal{O}(\alpha^2) + \mathcal{O}(\alpha^3) \simeq 0.0315$$

$$\Delta\alpha^{\text{had}}(M_Z^2) = -\frac{\alpha M_Z^2}{3\pi} \text{Re} \left(\int_{4m_\pi^2}^\infty ds' \frac{R_{\gamma\gamma}(s')}{s'(s' - M_Z^2)} \right)$$

$$R_{\gamma\gamma}(s) = \frac{\sigma(e^+e^- \rightarrow \gamma^* \rightarrow \text{had.})}{\sigma(e^+e^- \rightarrow \gamma^* \rightarrow \mu^+\mu^-)}$$

$$\Delta\alpha^{\text{had}}(M_Z^2) = 0.02761 \pm 0.00036$$

New Physics

- (1) parity violation effects in electron scattering;
- (2) atomic parity violation (weak charge), Lamb shift
- (3) electron and muon magnetic moments

1. W boson mass in the SM and BSM

$$\Delta r = \Delta\alpha - \cot^2 \theta_W \Delta\rho + \Delta_r^{\text{rem}}$$

Strength of the ratio of NC to CC at zero-momentum transfer in DIS

$$\rho = \frac{1}{1 - \Delta\rho}$$

$$\rho = \frac{M_W^2}{c_W^2 M_Z^2} \quad (=1, \text{Custodial symmetry})$$

$$\Delta\rho = \frac{\Pi_{WW}(0)}{M_W^2} - \frac{\Pi_{ZZ}(0)}{M_Z^2}$$

$$\rho = 1 + \frac{3g^2}{64\pi^2} \frac{m_t^2 - m_b^2}{M_W^2} - \frac{11}{192\pi^2} g'^2 \log \frac{M_h^2}{M_Z^2} + \dots$$

$$\alpha T = \rho$$

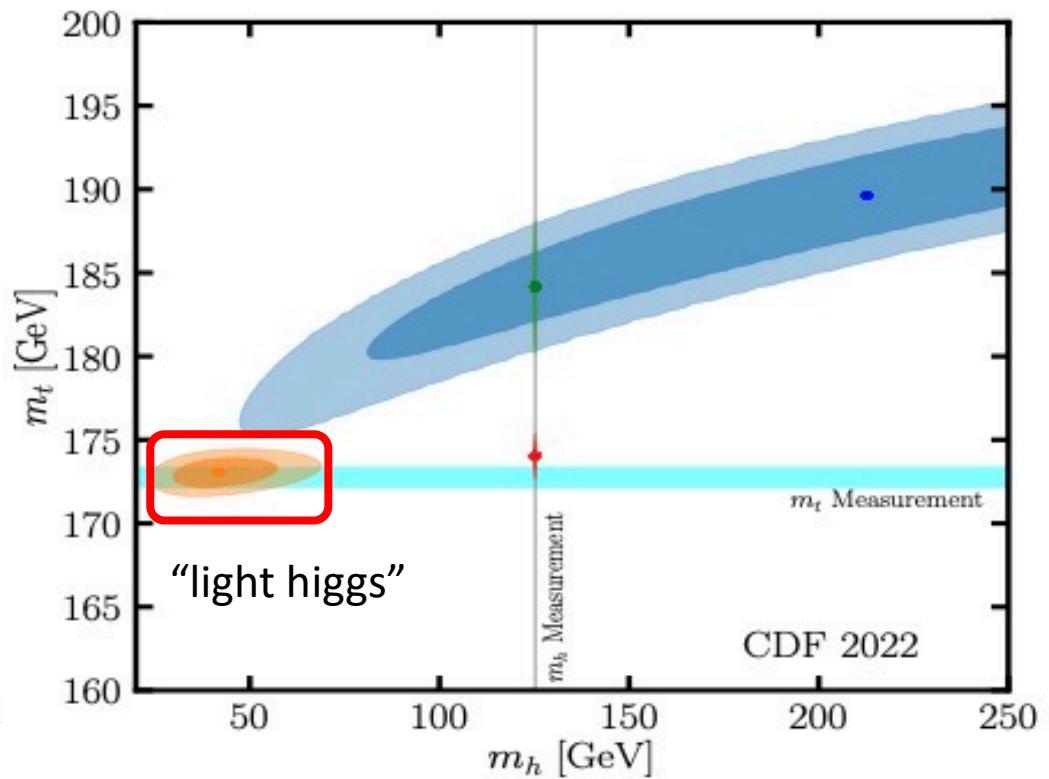
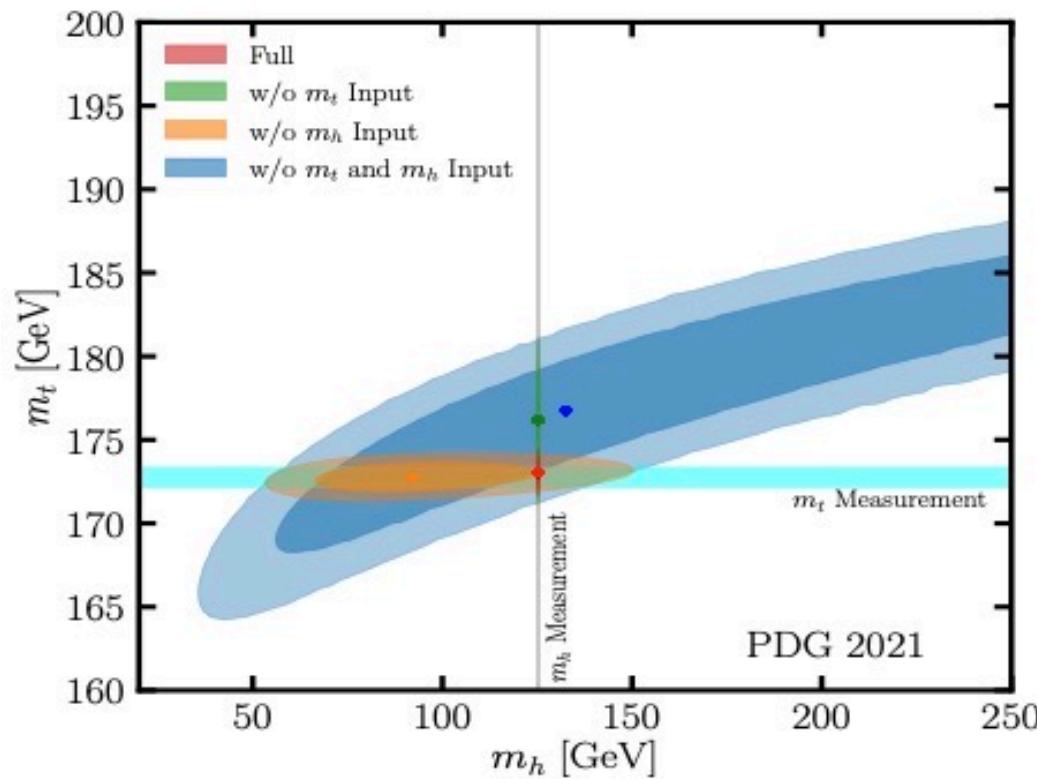
$$\rho = \frac{\sum_i [I_i(I_i + 1) - (I_i^3)^2] v_i^2}{2 \sum_i (I_i^3)^2 v_i^2}$$

(other Higgs representations)

New Physics

- (1) non-degenerate SU(2) doublet :
2HDM, SUSY, vector-like fermion...
- (2) heavy Z' bosons
- (3) Other higher representations

2. EW Precision Fits and Possible NP

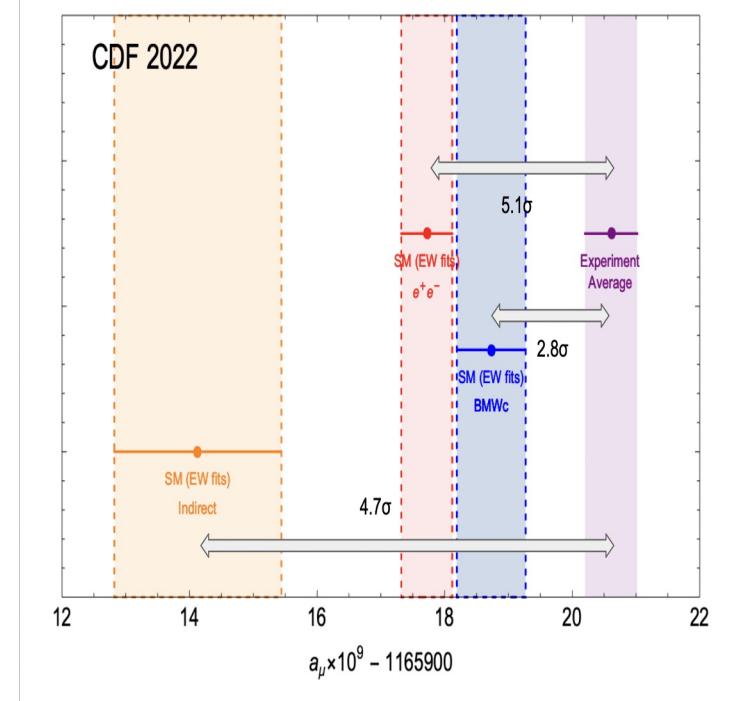
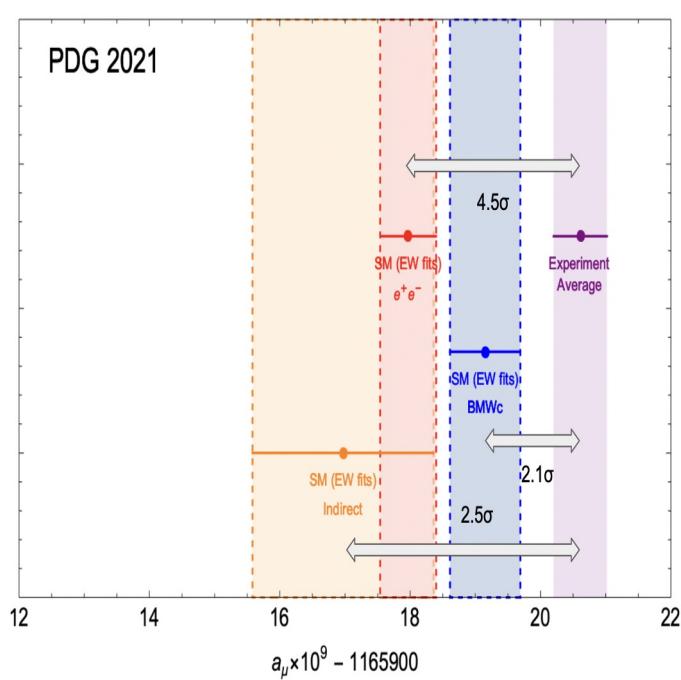
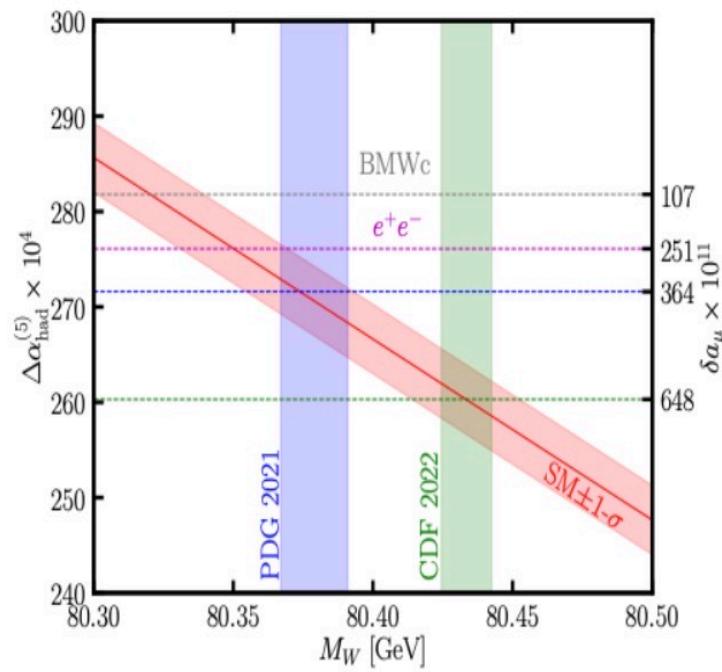


2. EW Precision Fits and NP

Muon g-2:

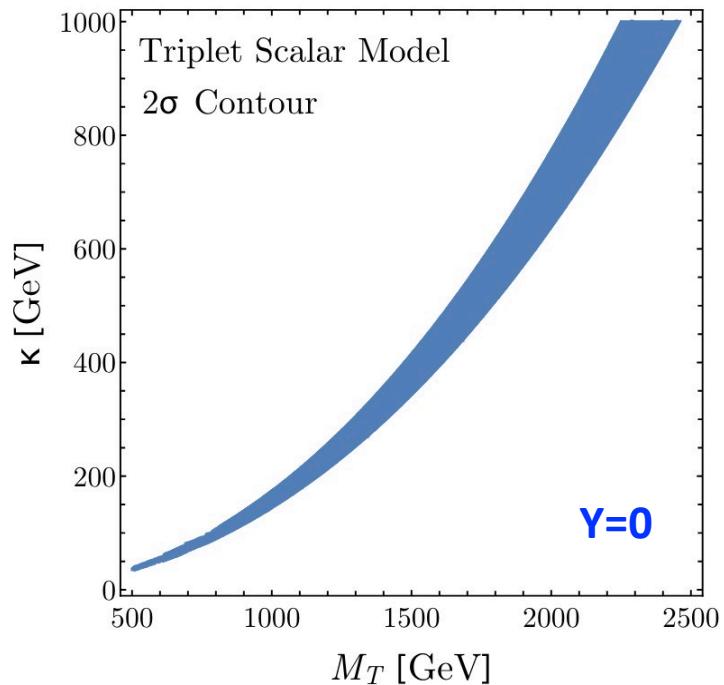
$$\Delta\alpha_{had}^{(5)}(ew) = \Delta\alpha_{had}^{(5)}(e^+e^-) * \frac{\delta a_\mu^{HVP}(ew)}{\delta a_\mu^{HVP}(e^+e^-)}$$

Nature 593 (2021) 7857, 51-55



2204.03996, Athron, Fowlie, Lu, Wu, Wu, Zhu

Triplet Scalar:



2204.05283, Asadi, Cesarotti, Fraser, Homiller, Parikh

2HDM (alignment):

0207010, Gunion and Haber

$$\Phi_i = \begin{pmatrix} w_i^+ \\ \frac{v_i + h_i + i\eta_i}{\sqrt{2}} \end{pmatrix}, \quad i = 1, 2,$$

making a global SU(2) transformation in the scalar space spanned by the two doublets, it is always possible to work in the so-called **Higgs basis**,

$$H_1 = \begin{pmatrix} H_1^+ \\ H_1^0 \end{pmatrix} \equiv \frac{v_1 \Phi_1 + v_2 \Phi_2}{v}, \quad H_2 = \begin{pmatrix} H_2^+ \\ H_2^0 \end{pmatrix} \equiv \frac{-v_2 \Phi_1 + v_1 \Phi_2}{v},$$

$$\mathcal{V} \ni \frac{1}{2} Z_1 (H_1^\dagger H_1)^2 + \left\{ \frac{1}{2} Z_5 (H_1^\dagger H_2)^2 + Z_6 (H_1^\dagger H_1)(H_1^\dagger H_2) + \text{h.c.} \right\}$$

3. Inert Higgs Dark Matter and W mass

2HDM (alignment):

$$\mathcal{M}_H^2 = \begin{pmatrix} Z_1 v^2 & Z_6 v^2 \\ Z_6 v^2 & m_A^2 + Z_5 v^2 \end{pmatrix} \quad \begin{pmatrix} H \\ h \end{pmatrix} = \begin{pmatrix} c_{\beta-\alpha} & -s_{\beta-\alpha} \\ s_{\beta-\alpha} & c_{\beta-\alpha} \end{pmatrix} \begin{pmatrix} \sqrt{2} \operatorname{Re} H_1^0 - v \\ \sqrt{2} \operatorname{Re} H_2^0 \end{pmatrix}$$

$$Z_1 v^2 = m_h^2 s_{\beta-\alpha}^2 + m_H^2 c_{\beta-\alpha}^2 ,$$

h is SM-like if $|c_{\beta-\alpha}| \ll 1$

$$Z_6 v^2 = (m_h^2 - m_H^2) s_{\beta-\alpha} c_{\beta-\alpha} ,$$

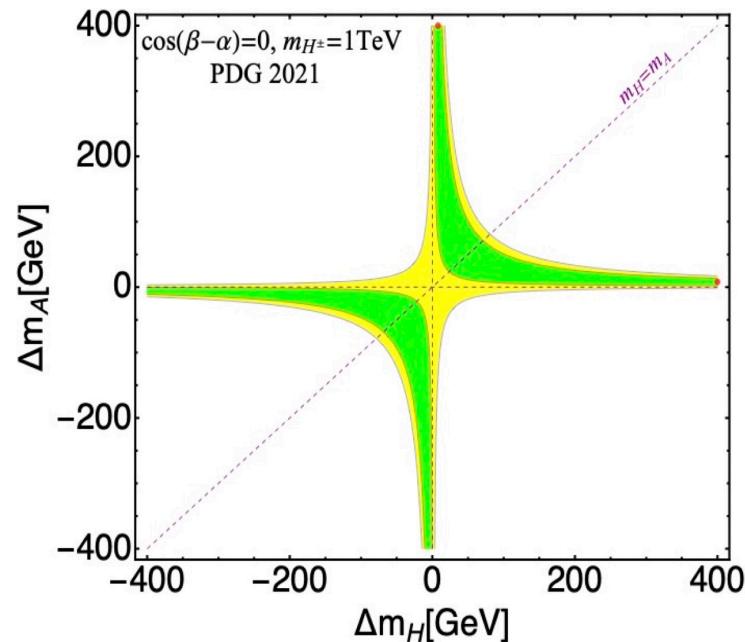
$$Z_5 v^2 = m_H^2 s_{\beta-\alpha}^2 + m_h^2 c_{\beta-\alpha}^2 - m_A^2 .$$

Alignment limit: Higgs base = Mass base

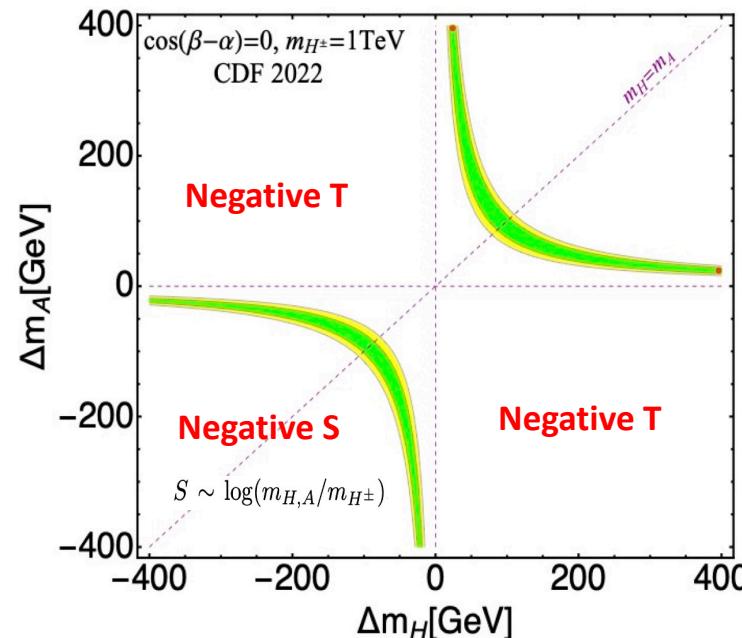
3. Inert Higgs Dark Matter and W mass

2HDM (alignment):

$$T = \frac{1}{32\pi \sin^2 \theta_W m_W^2} [\theta_+(m_+, m_1) + \theta_+(m_+, m_2) - \theta_+(m_1, m_2)]$$



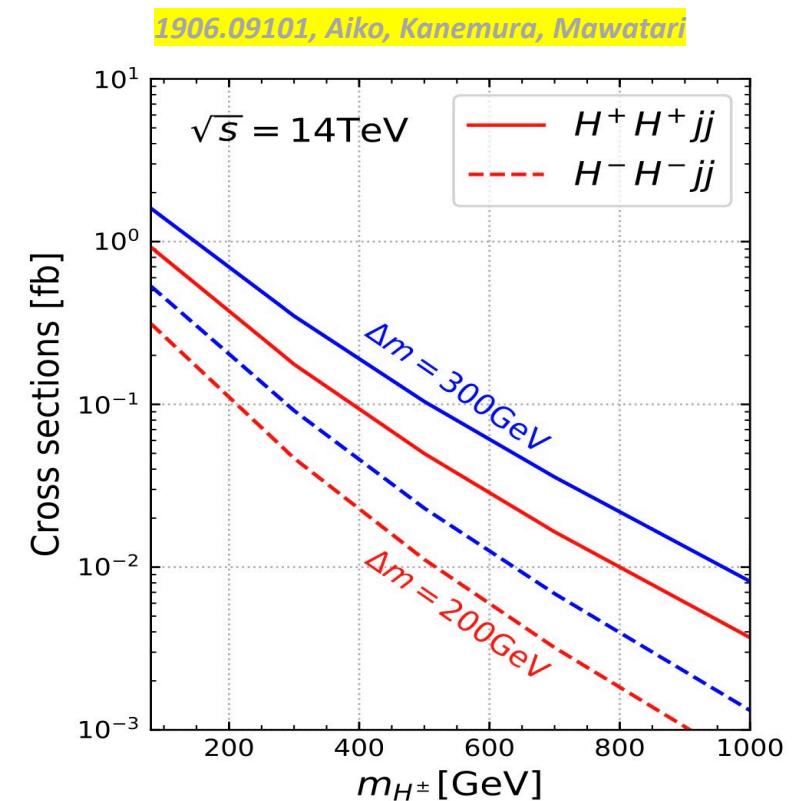
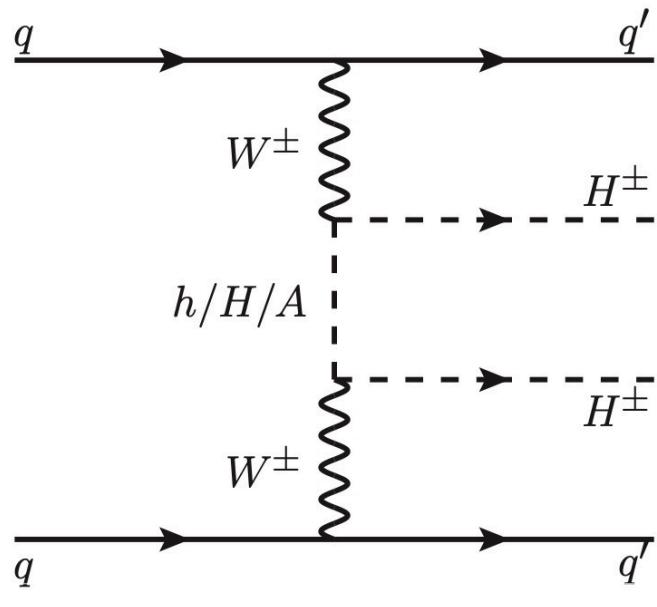
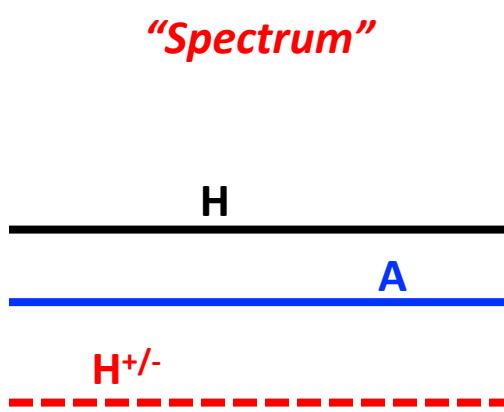
$$\begin{aligned} & \theta_+(m_+, m_1) + \theta_+(m_+, m_2) - \theta_+(m_1, m_2) \\ = & 32i\pi^2 \int \frac{d^4 k}{(2\pi)^4} k^2 \frac{(m_+^2 - m_1^2)(m_+^2 - m_2^2)}{(k^2 - m_+^2)^2(k^2 - m_1^2)(k^2 - m_2^2)}. \end{aligned}$$



2022.03796, Lu, Wu, Wu, Zhu

3. Inert Higgs Dark Matter and W mass

2HDM (alignment):



EW Global Fitting

- SM is well-known to explain our nature extremely well
- Global Fitting is a way to quantitatively show above statement
- In the fitting
 - Varying the independent input parameters of a model
 - Calculating observables
 - Comparing with measurements

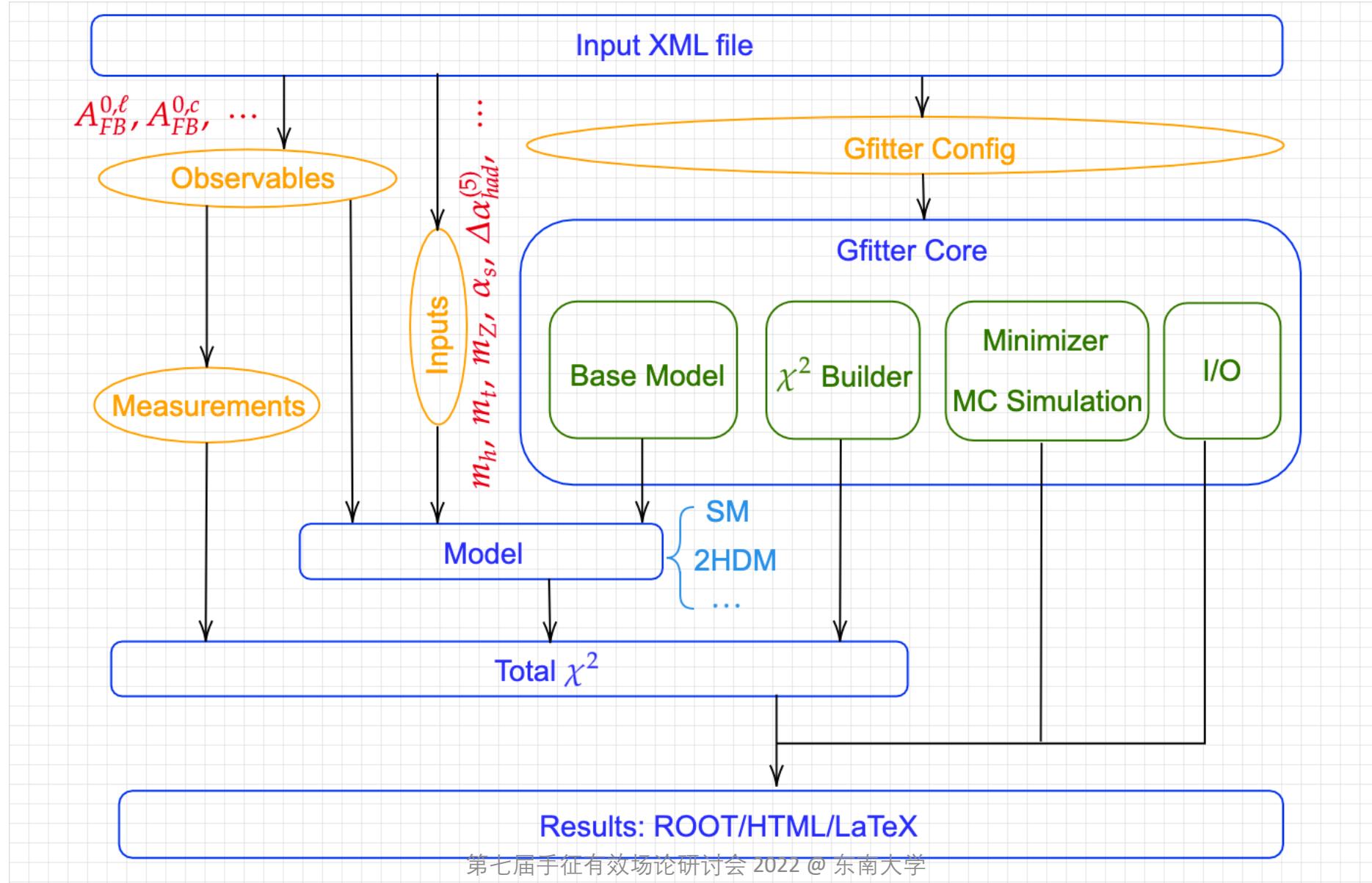
EW Global Fitting

- Input Parameters for SM
 - Fixed:
 - $G_F = 1.1663787 \times 10^{-5}$
 - m_u, m_d, m_s (MSbar)
 - Free in Fitting
 - $m_h, m_Z, m_t, \bar{m}_c, \bar{m}_b$
 - $\Delta\alpha_{had}^{(5)}, \alpha_s$
- All other parameters/observables can be expressed in terms of above parameters.
 - $M_W^2 = \frac{M_Z^2}{2} \left(1 + \sqrt{1 - \frac{\sqrt{8}\pi\alpha(1-\Delta r)}{G_F M_Z^2}} \right)$
 - Etc.

EW Observables

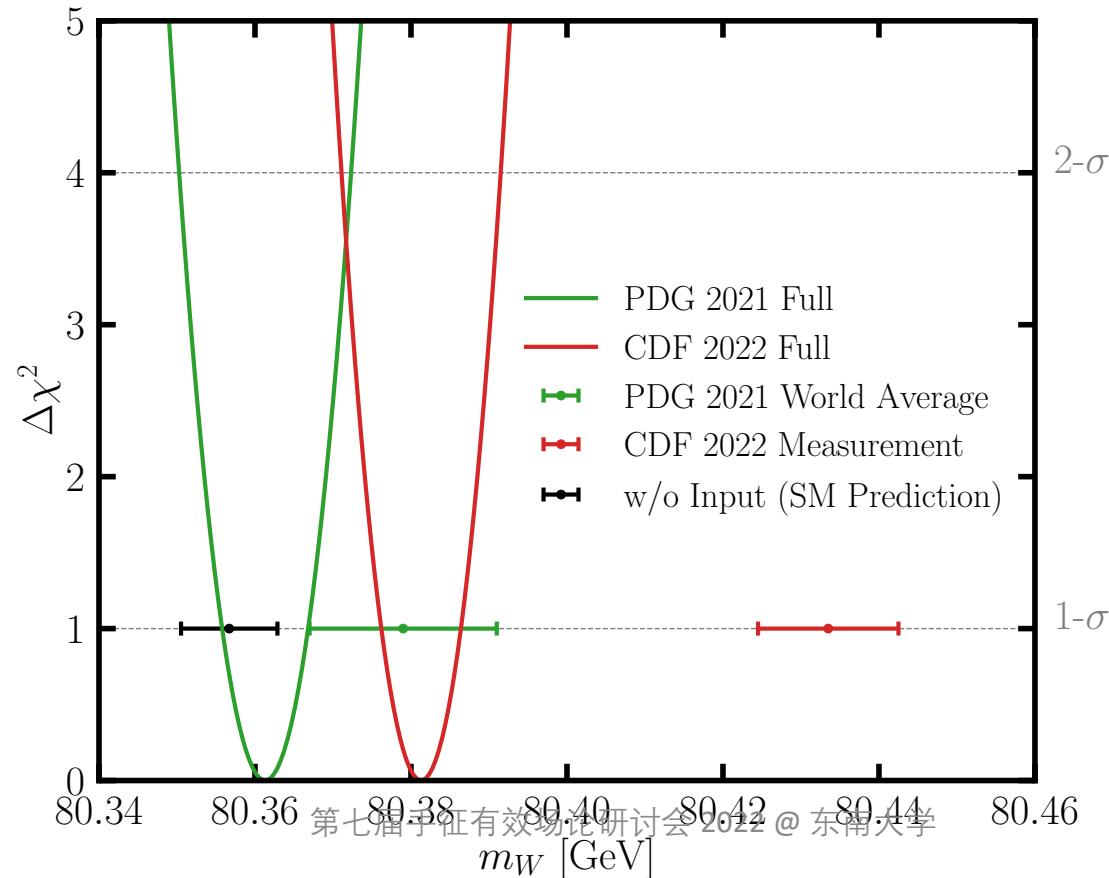
- Z resonance parameters:
 - M_Z, Γ_Z
 - $\sigma(e^+e^- \rightarrow Z \rightarrow hadron)$
- Partial Z cross section
 - R_ℓ^0, R_c^0, R_b^0
- Neutral current couplings
 - $\sin \theta_{eff}^\ell$
 - A_ℓ, A_c, A_b
 - $A_{FB}^{0,\ell}, A_{FB}^{0,c}, A_{FB}^{0,b}$
- W-boson parameters
 - M_W, Γ_W
- Higgs parameter
 - m_h
- Misc
 - m_t, m_b, m_c
 - $\alpha_s, \alpha \left(\Delta \alpha_{had}^{(5)} \right)$

EW Global Fitting in Gfitter



EW Global Fitting Individual Result

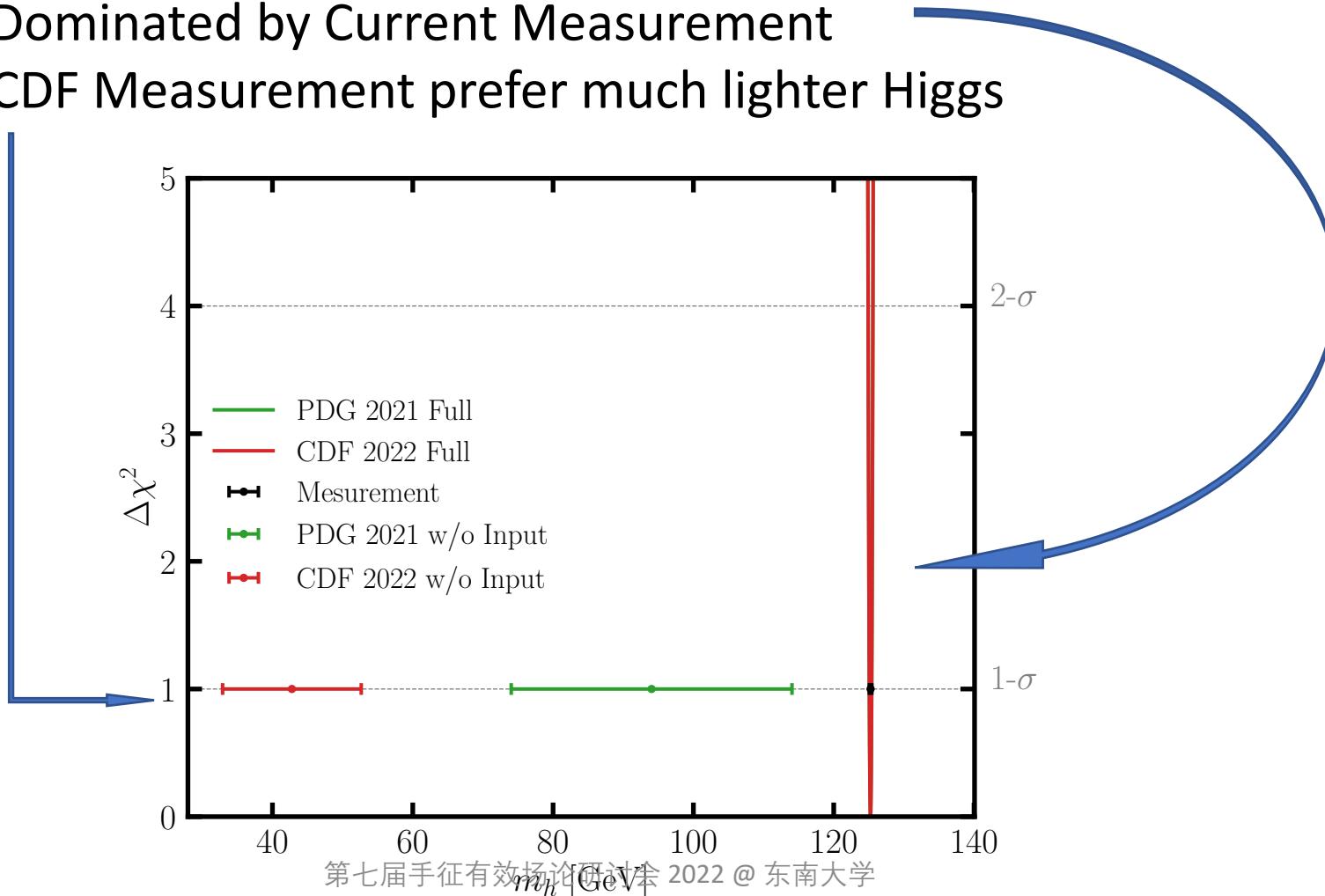
- m_W
 - There is huge deviation between
 - CDF measurement and SM prediction



EW Global Fitting Individual Result

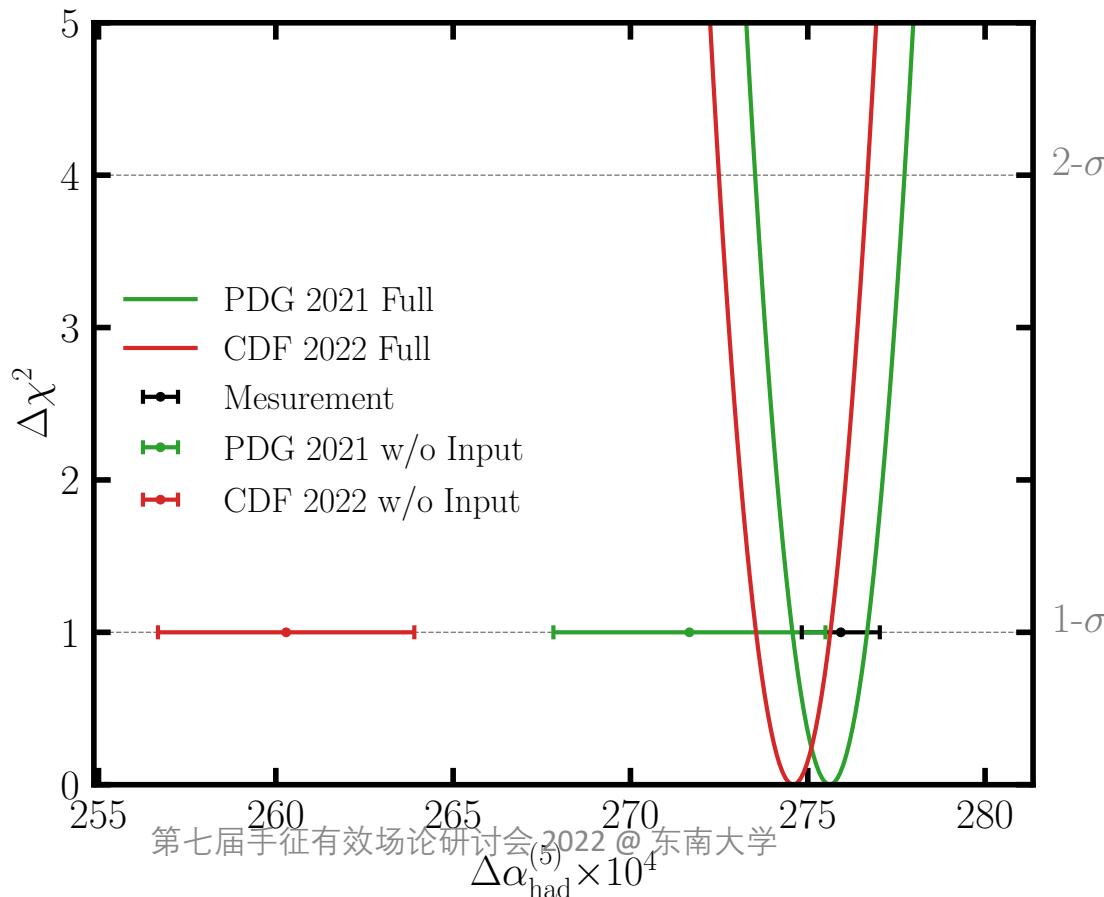
- **Higgs Mass**

- Dominated by Current Measurement
- CDF Measurement prefer much lighter Higgs



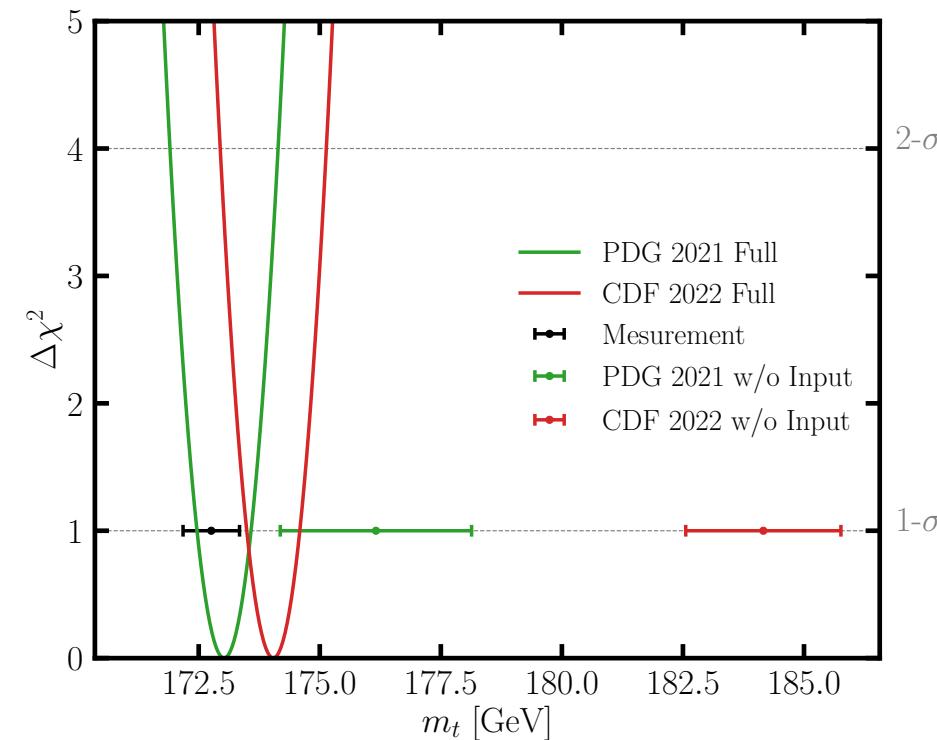
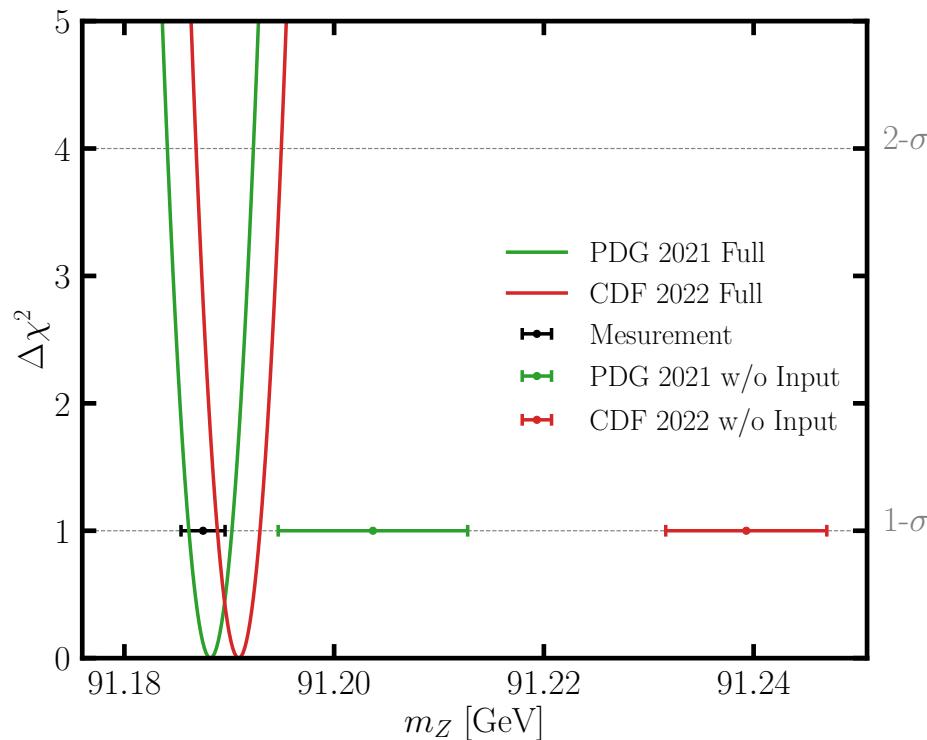
EW Global Fitting Individual Result

- $\Delta\alpha_{had}^{(5)}$
 - Is also important for g-2 interpretation
 - CDF prefer lower value which will enlarge the g-2 deviation



EW Global Fitting Individual Result

- m_Z, m_t
 - Fitting w/ CDF measurement prefer larger value



Oblique Parameters

- New CDF measurement worse the SM global fitting
- New Physics can be parameterized through
 - its correction to the self-energy of gauge bosons

$$\frac{\alpha S}{4s_w^2 c_w^2} = \left[\frac{\delta\Pi_{ZZ}(M_z^2) - \delta\Pi_{ZZ}(0)}{M_z^2} \right] - \frac{(c_w^2 - s_w^2)}{s_w c_w} \delta\Pi'_{Z\gamma}(0) - \delta\Pi'_{\gamma\gamma}(0),$$

$$\alpha T = \frac{\delta\Pi_{WW}(0)}{M_W^2} - \frac{\delta\Pi_{ZZ}(0)}{M_z^2},$$

$$\begin{aligned} \frac{\alpha U}{4s_w^2} &= \left[\frac{\delta\Pi_{WW}(M_w^2) - \delta\Pi_{WW}(0)}{M_w^2} \right] - c_w^2 \left[\frac{\delta\Pi_{ZZ}(M_z^2) - \delta\Pi_{ZZ}(0)}{M_z^2} \right] \\ &\quad - s_w^2 \delta\Pi'_{\gamma\gamma}(0) - 2s_w c_w \delta\Pi'_{Z\gamma}(0). \end{aligned}$$

$$m_W^2 = (m_W^{SM})^2 \left[1 - \frac{\alpha S}{2(c_w^2 - s_w^2)} + \frac{c_w^2 \alpha T}{c_w^2 - s_w^2} + \frac{\alpha U}{4s_w^2} \right]$$

Fitting with S/T/U

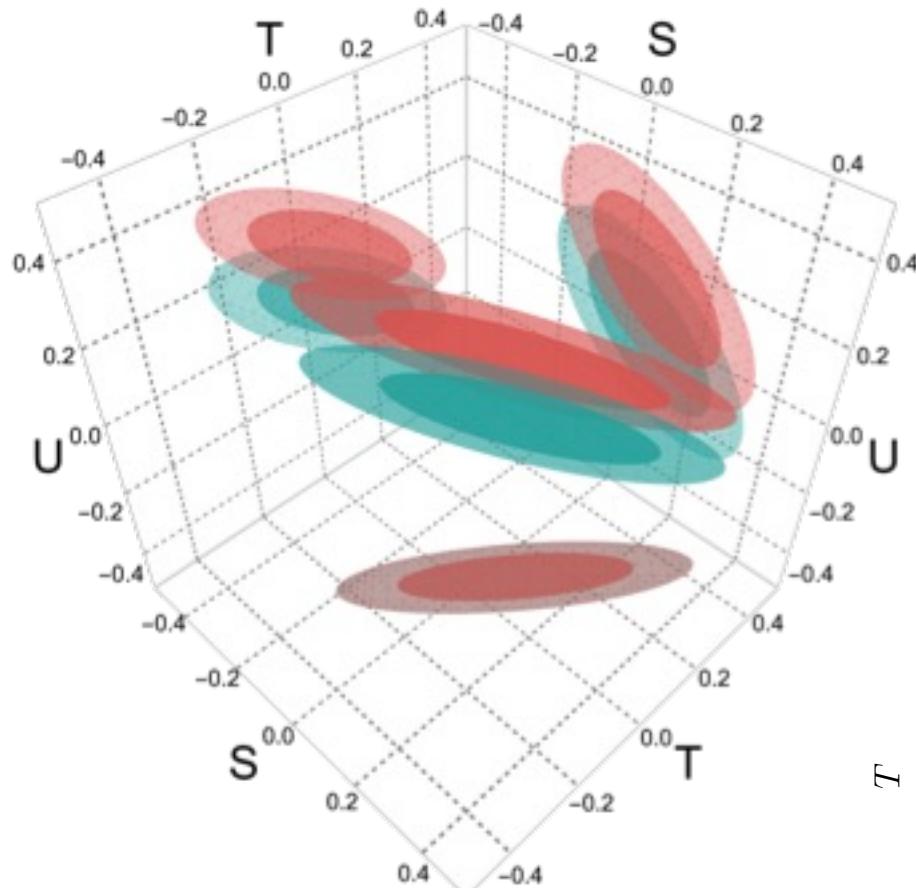
- Fitting with S/T/U
 - Need larger U

13 dof	PDG 2021			CDF 2022				
	Result	Correlation		Result	Correlation			
	$\chi^2_{\min} = 15.42$	S	T	U	$\chi^2_{\min} = 15.44$	S	T	U
S	0.06 ± 0.10	1.00	0.90	-0.57	0.06 ± 0.10	1.00	0.90	-0.59
T	0.11 ± 0.12		1.00	-0.82	0.11 ± 0.12		1.00	-0.85
U	-0.02 ± 0.09			1.00	0.14 ± 0.09			1.00

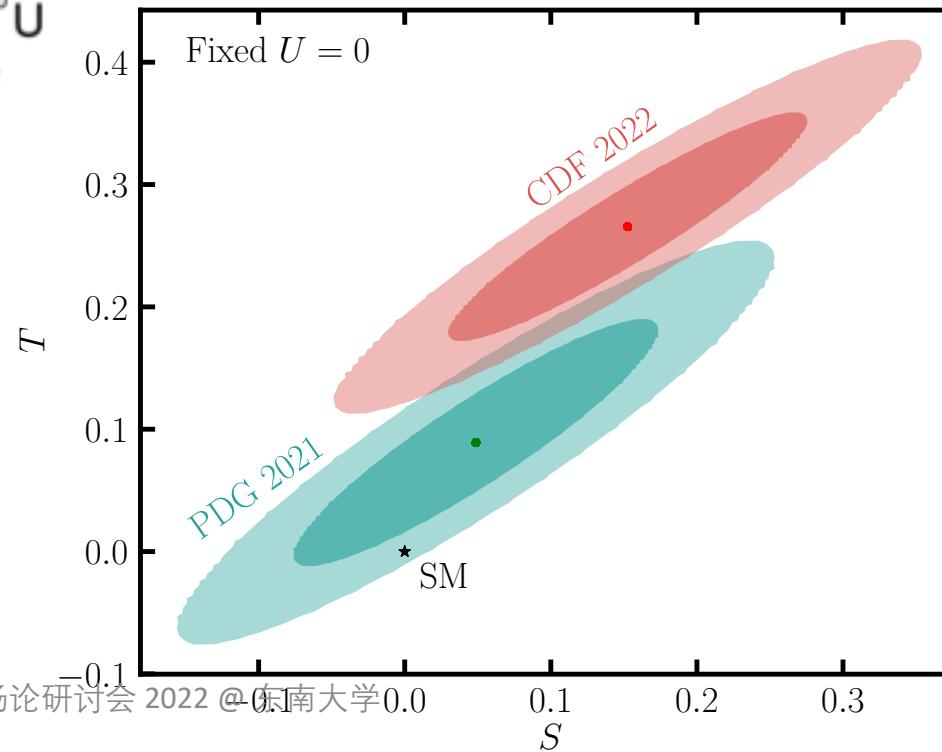
- Fix $U = 0$
 - S and T are significantly deviated from 0

$U = 0$	PDG 2021			CDF 2022		
	Result	Correlation		Result	Correlation	
14 dof	$\chi^2_{\min} = 15.48$	S	T	$\chi^2_{\min} = 17.82$	S	T
S	0.05 ± 0.08	1.00	0.92	0.15 ± 0.08	1.00	0.93
T	0.09 ± 0.07	1.00	0.27 ± 0.06			1.00

Fitting with S/T/U



- Most BSM has $U \approx 0$
 - Large correction to S/T is needed



The relationship between $\Delta\alpha_{\text{had}}$ and a_μ^{HVP}

The time-like and space-like master formulae for $\Delta\alpha_{\text{had}}^{\text{HVP}}$

1. **Time-like** : Using for $e^+ e^- \rightarrow \text{hadrons}$ data calculations

$$a_\mu^{\text{HVP, LO}} = \left(\frac{\alpha m_\mu}{3\pi} \right)^2 \int_{s_{\text{thr}}}^\infty ds \frac{\hat{K}(s)}{s^2} R_{\text{had}}(s) \quad R_{\text{had}}(s) = \frac{3s}{4\pi\alpha^2} \sigma(e^+ e^- \rightarrow \text{hadrons})$$

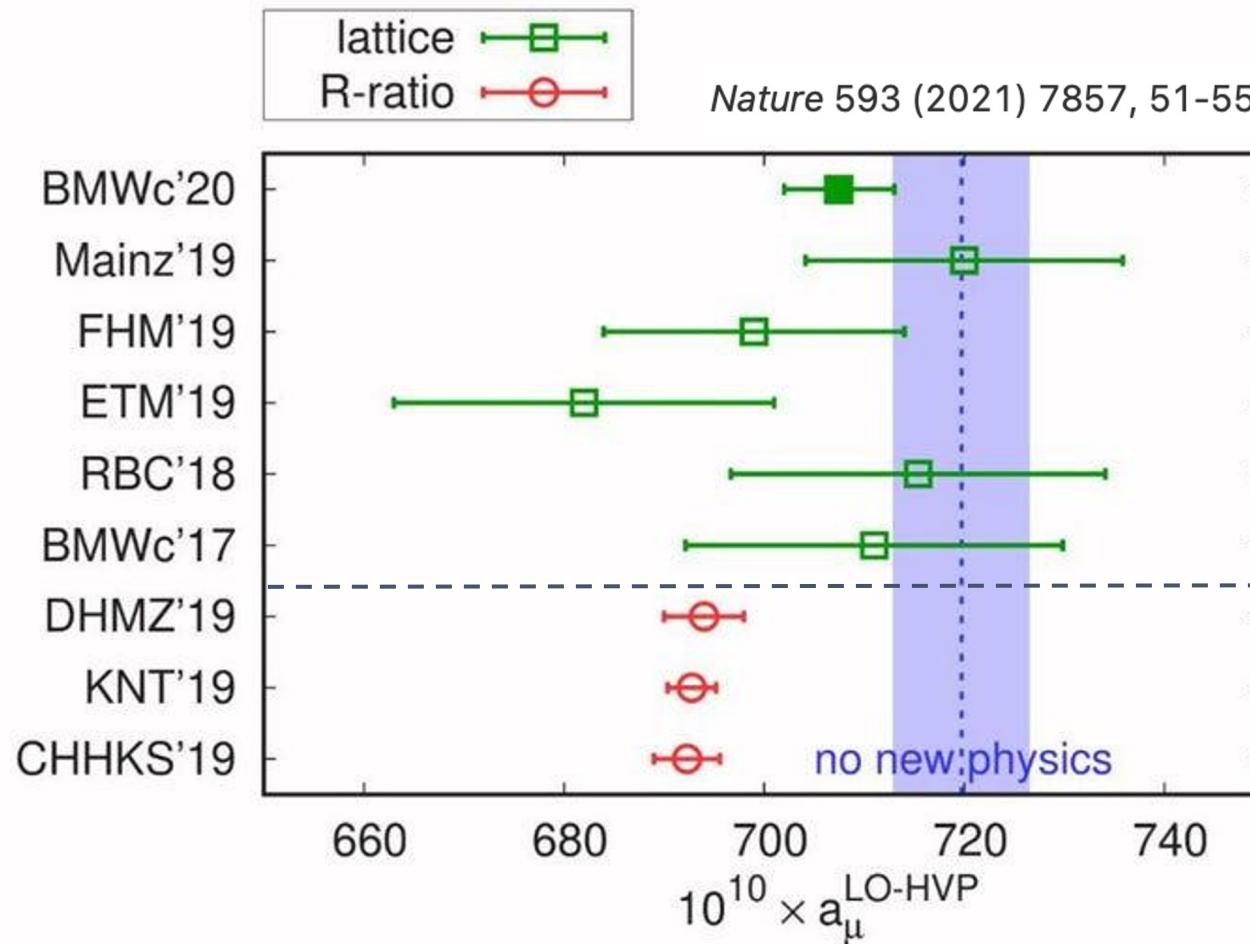
$$\Delta\alpha_{\text{had}}^{(5)}(M_Z^2) = \frac{\alpha M_Z^2}{3\pi} P \int_{s_{\text{thr}}}^\infty ds \frac{R_{\text{had}}(s)}{s(M_Z^2 - s)} \quad \text{where } s_{\text{thr}} = m_{\pi^0}^2$$

1. **Space-like** : Using for lattice QCD calculations

$$a_\mu^{\text{HVP, LO}} = \left(\frac{\alpha}{\pi} \right)^2 \int_0^\infty ds f(s) \hat{\Pi}(-s) \quad \hat{\Pi}(s) = 4\pi^2 [\Pi(s) - \Pi(0)]$$

$$\Delta\alpha_{\text{had}}^{(5)}(M_Z^2) = \frac{\alpha}{\pi} \hat{\Pi}(-M_Z^2) + \frac{\alpha}{\pi} (\hat{\Pi}(M_Z^2) - \hat{\Pi}(-M_Z^2))$$

A summary for a_μ^{HVP} from data-driven and lattice QCD calculations

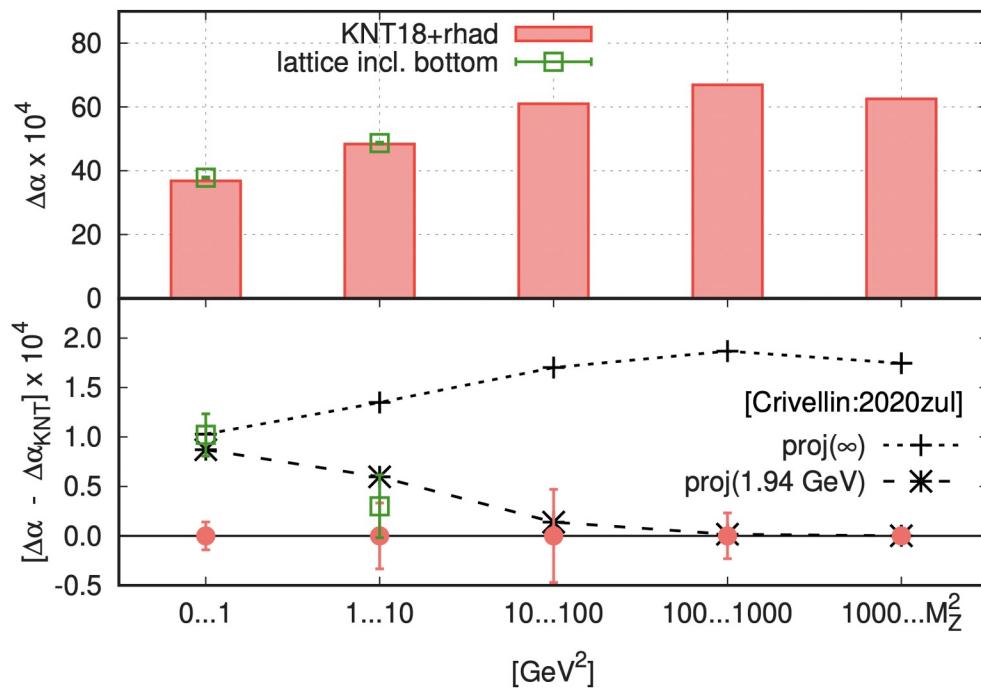


$$a_\mu^{\text{HVP, LO}} = \left(\frac{\alpha}{\pi}\right)^2 \int_0^\infty ds f(s) \hat{\Pi}(-s)$$

$$a_\mu^{\text{HVP, LO}} = \left(\frac{\alpha m_\mu}{3\pi}\right)^2 \int_{s_{\text{thr}}}^\infty ds \frac{\hat{K}(s)}{s^2} R_{\text{had}}(s)$$

The problem to compare $\Delta\alpha_{\text{had}}$ form data-driven and lattice QCD

1. The $\Delta\alpha_{\text{had}}$ is calculated at the scale M_Z for five quark flavors from data-driven method with $\Delta\alpha_{\text{had}}^{(5)}|_{e^+e^-} = 276.1(1.1) \times 10^{-4}$. KNT, DHMZ
2. However, we don't have enough information for $\Delta\alpha_{\text{had}}$ from lattice QCD side.



For example,
using the whole energy range project
[proj(∞)]:

$$a_\mu^{\text{HVP}}(\text{BMWc}) = 707.5(5.5) \times 10^{-10}$$

$$\Rightarrow \Delta\alpha_{\text{had}}(\text{BMWc}) = 276.1(1.1) \times 10^{-4} \times \frac{707.5}{693.1} = 281.8(1.5) \times 10^{-4}$$

Nature 593 (2021) 7857, 51-55

The 3rd way to extract

$\Delta\alpha_{\text{Had}}$ Global Electroweak Fits

P.Athron, A.Fowlie, C.T.Lu., L.Wu, Y.C.Wu, B.Zhu

GFitter

		Indirect			PDG 2021			CDF 2022			
	M_W $\Delta\alpha_{\text{had}}$	BMWc	e^+e^-	Indirect	BMWc	e^+e^-	Indirect	BMWc	e^+e^-	Indirect	
Input	M_W [GeV]	-	-	-	80.379(12)	80.379(12)	80.379(12)	80.4335(94)	80.4335(94)	80.4335(94)	
	$\Delta\alpha_{\text{had}} \times 10^4$	281.8(1.5)	276.1(1.1)	-	281.8(1.5)	276.1(1.1)	-	281.8(1.5)	276.1(1.1)	-	
Fitted	χ^2/dof	18.32/15	16.01/15	15.89/14	23.41/16	18.74/16	17.59/15	74.51/16	62.58/16	47.19/15	
	M_W [GeV]	80.348(6)	80.357(6)	80.359(9)	80.355(6)	80.361(6)	80.367(7)	80.375(5)	80.380(5)	80.396(7)	
	$\Delta\alpha_{\text{had}} \times 10^4$	280.9(1.4)	275.9(1.1)	274.4(4.4)	280.3(1.4)	275.6(1.1)	271.7(3.8)	278.6(1.4)	274.7(1.0)	260.9(3.6)	
	$\delta a_\mu \times 10^{11}$	-	-	294(166)	146(68)	264(59)	364(145)	188(68)	289(57)	648(137)	
	Tension	-	-	1.8σ	2.1σ	4.5σ	2.5σ	2.8σ	5.1σ	4.7σ	
		δM_W [MeV]	86(11)	77(11)	75(13)	79(11)	73(11)	67(12)	59(11)	54(11)	38(12)
		Tension	7.8σ	7.0σ	5.8σ	7.2σ	6.6σ	5.6σ	5.4σ	4.9σ	3.2σ

Then, how to transform the information between $\Delta\alpha_{\text{had}}$ and a_μ^{HVP} ?

Here we consider the whole energy range projection.

Using Global EW Fits to extract $\Delta\alpha_{\text{had}}$ Low energy projection

P.Athron, A.Fowlie, C.T.Lu., L.Wu, Y.C.Wu, B.Zhu

GFitter

M_W		Indirect			PDG 2021			CDF 2022		
	$\Delta\alpha_{\text{had}}$	BMWc	e^+e^-	Indirect	BMWc	e^+e^-	Indirect	BMWc	e^+e^-	Indirect
Input	M_W [GeV]	-	-	-	80.379(12)	80.379(12)	80.379(12)	80.4335(94)	80.4335(94)	80.4335(94)
	$\Delta\alpha_{\text{had}}^{(5)}(M_Z^2) \times 10^4$	277.4(1.2)	276.1(1.1)	-	277.4(1.2)	276.1(1.1)	-	277.4(1.2)	276.1(1.1)	-
Fitted	χ^2/dof	16.28/15	16.01/15	15.89/14	19.51/16	18.74/16	17.59/15	65.07/16	62.58/16	47.19/15
	M_W [GeV]	80.355(6)	80.357(6)	80.359(9)	80.360(6)	80.361(6)	80.367(7)	80.379(5)	80.380(5)	80.396(7)
	$\Delta\alpha_{\text{had}} \times 10^4$	277.1(1.2)	275.9(1.1)	274.4(4.4)	276.8(1.1)	275.6(1.1)	271.7(3.8)	275.6(1.1)	274.7(1.0)	260.9(3.6)
	$\delta a_\mu \times 10^{11}$	-	-	438(396)	173(54)	306(54)	748(339)	306(54)	416(54)	1997(320)
	Tension	-	-	1.1σ	3.2σ	5.7σ	2.2σ	5.7σ	7.7σ	6.2σ
	δM_W [MeV]	79(11)	77(11)	75(13)	74(11)	73(11)	67(12)	55(11)	54(11)	38(12)
	Tension	7.2σ	7.0σ	5.8σ	6.7σ	6.6σ	5.6σ	5.0σ	4.9σ	3.2σ

For the case of low energy projection, $\Delta\alpha_{\text{had}}$ is shrunk, but a_μ^{HVP} is enlarged after the transformation compared with the whole energy range projection.

Three various projections between $\Delta\alpha_{\text{had}}$ and a_μ^{HVP}

1. According to *Crivellin:2020zul*, there are three different hypotheses for the projection between

$\Delta\alpha_{\text{had}}$ and a_μ^{HVP} :

-
- (1) Low energy for the sum of exclusive channels : $m_{\pi_0} \leq \sqrt{s} \leq 1.937 \text{ GeV}$,
 - (2) Energy below the perturbative contributions : $m_{\pi_0} \leq \sqrt{s} \leq 11.199 \text{ GeV}$ or
 - (3) The whole energy range : $m_{\pi_0} \leq \sqrt{s} \leq \infty$,

(Hypothesis) : The part above the upper energy threshold is the same as data driven one and the uniform scaling is applied.)

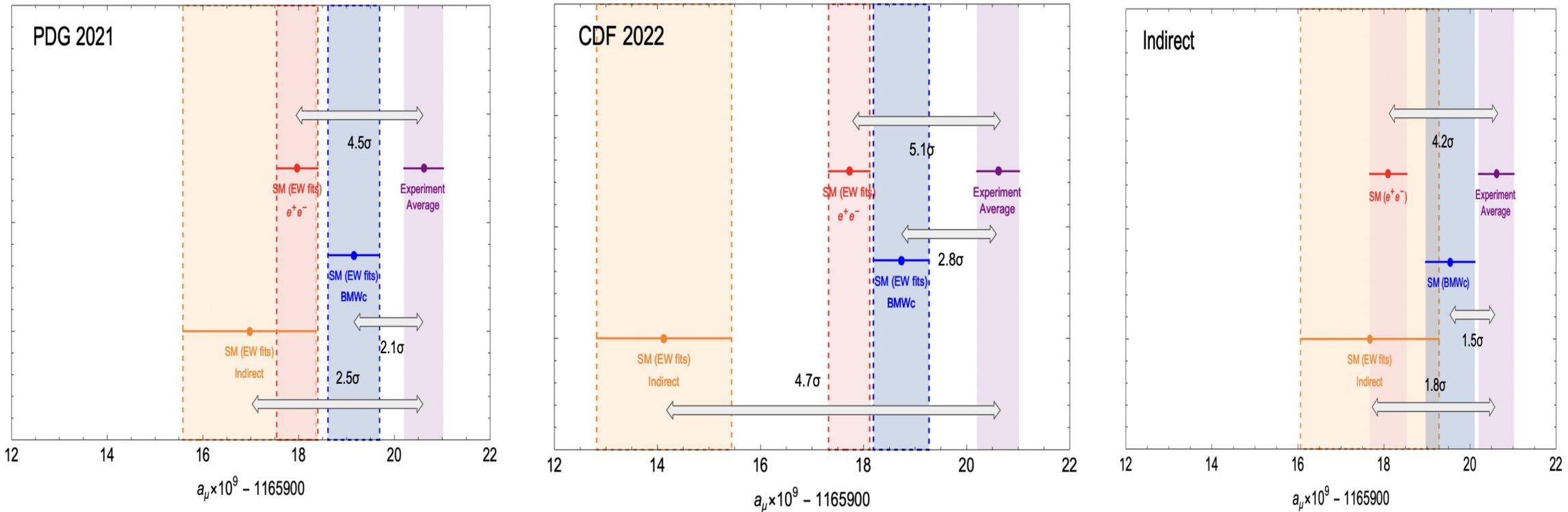
1. Open questions : (1) Which projection should be preferred ?

(The low energy projection agrees better with BMWc results.)

2. Can we go beyond the uniform scaling (energy independent) hypothesis ?

The impact to muon g-2 from the global EW fits

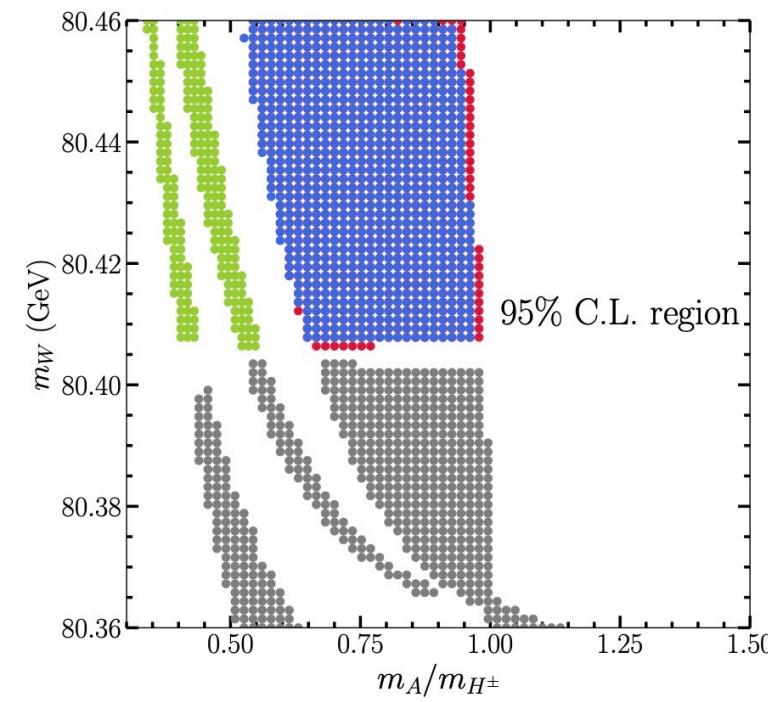
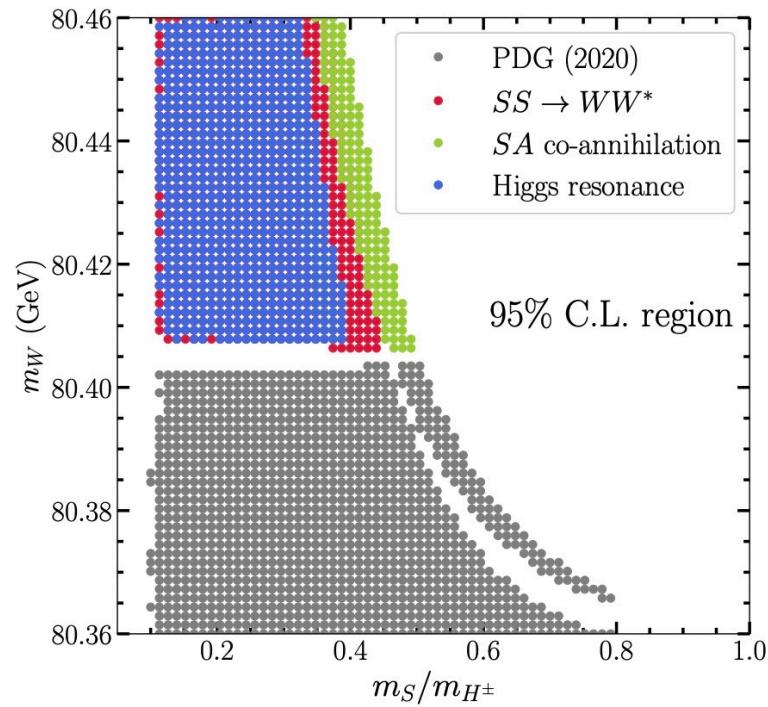
P.Athron, A.Fowlie, C.T.Lu., L.Wu, Y.C.Wu, B.Zhu

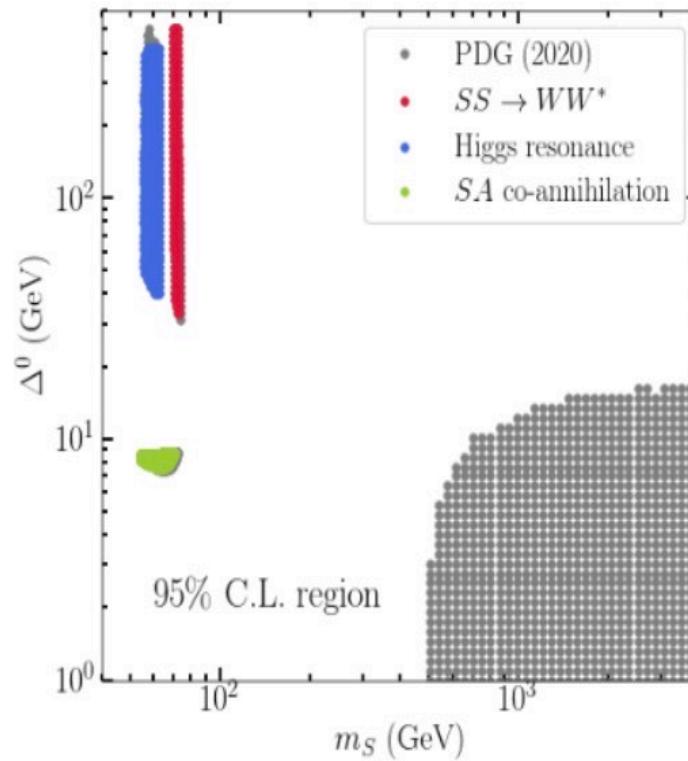
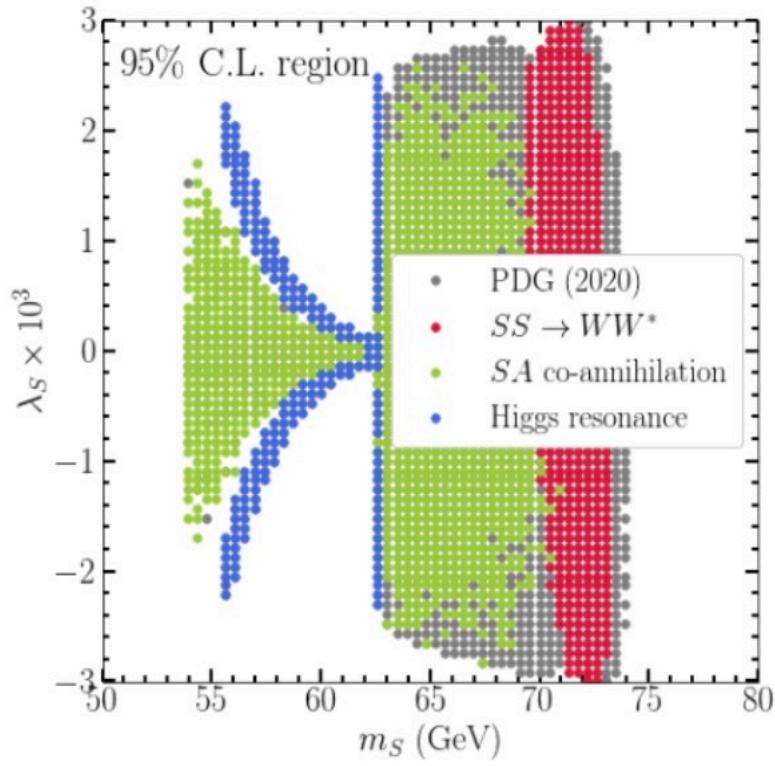


The EW fits from indirect and BMWc inputs go into opposite direction compared with the one fromm e^+e^- input.

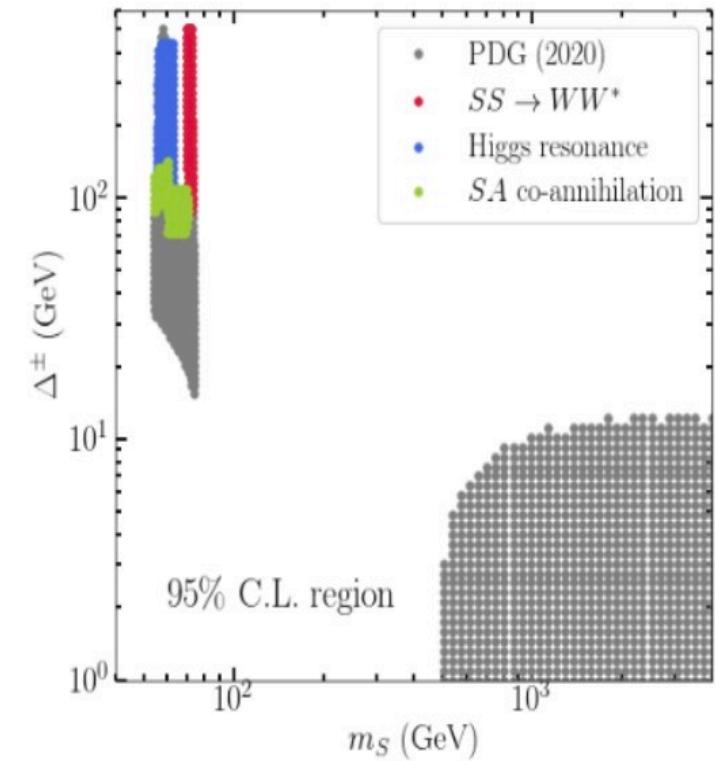
Channel	Energy range (GeV)	$a_\mu^{\text{had, LO VP}} \times 10^{10}$	$\Delta\alpha_{\text{had}}^{(5)}(M_Z^2) \times 10^4$	Our results
Chiral perturbation theory (ChPT) threshold contributions				
$\pi^0\gamma$	$m_\pi \leq \sqrt{s} \leq 0.600$	0.12 ± 0.01	0.00 ± 0.00	0.01 ± 0.00
$\pi^+\pi^-$	$2m_\pi \leq \sqrt{s} \leq 0.305$	0.87 ± 0.02	0.01 ± 0.00	0.01 ± 0.00
$\pi^+\pi^-\pi^0$	$3m_\pi \leq \sqrt{s} \leq 0.660$	0.01 ± 0.00	0.00 ± 0.00	0.01 ± 0.00
$\eta\gamma$	$m_\eta \leq \sqrt{s} \leq 0.660$	0.00 ± 0.00	0.00 ± 0.00	0.00 ± 0.00
Data based channels ($\sqrt{s} \leq 1.937$ GeV)				
$\pi^0\gamma$	$0.600 \leq \sqrt{s} \leq 1.350$	4.46 ± 0.10	0.36 ± 0.01	0.43 ± 0.01
$\pi^+\pi^-$	$0.305 \leq \sqrt{s} \leq 1.937$	502.97 ± 1.97	34.26 ± 0.12	26.60 ± 0.10
$\pi^+\pi^-\pi^0$	$0.660 \leq \sqrt{s} \leq 1.937$	47.79 ± 0.89	4.77 ± 0.08	6.51 ± 0.12
$\pi^+\pi^-\pi^+\pi^-$	$0.613 \leq \sqrt{s} \leq 1.937$	14.87 ± 0.20	4.02 ± 0.05	1.85 ± 0.02
$\pi^+\pi^-\pi^0\pi^0$	$0.850 \leq \sqrt{s} \leq 1.937$	19.39 ± 0.78	5.00 ± 0.20	3.57 ± 0.14
$(2\pi^+2\pi^-\pi^0)_{\text{no } \eta}$	$1.013 \leq \sqrt{s} \leq 1.937$	0.99 ± 0.09	0.33 ± 0.03	0.22 ± 0.02
$3\pi^+3\pi^-$	$1.313 \leq \sqrt{s} \leq 1.937$	0.23 ± 0.01	0.09 ± 0.01	0.07 ± 0.00
$(2\pi^+2\pi^-2\pi^0)_{\text{no } \eta\omega}$	$1.322 \leq \sqrt{s} \leq 1.937$	1.35 ± 0.17	0.51 ± 0.06	0.41 ± 0.05
K^+K^-	$0.988 \leq \sqrt{s} \leq 1.937$	23.03 ± 0.22	3.37 ± 0.03	5.05 ± 0.05
$K_S^0 K_L^0$	$1.004 \leq \sqrt{s} \leq 1.937$	13.04 ± 0.19	1.77 ± 0.03	2.91 ± 0.04
$KK\pi$	$1.260 \leq \sqrt{s} \leq 1.937$	2.71 ± 0.12	0.89 ± 0.04	0.78 ± 0.03
$KK2\pi$	$1.350 \leq \sqrt{s} \leq 1.937$	1.93 ± 0.08	0.75 ± 0.03	0.60 ± 0.02
$\eta\gamma$	$0.660 \leq \sqrt{s} \leq 1.760$	0.70 ± 0.02	0.09 ± 0.00	0.09 ± 0.02
$\eta\pi^+\pi^-$	$1.091 \leq \sqrt{s} \leq 1.937$	1.29 ± 0.06	0.39 ± 0.02	0.32 ± 0.01
$(\eta\pi^+\pi^-\pi^0)_{\text{no } \omega}$	$1.333 \leq \sqrt{s} \leq 1.937$	0.60 ± 0.15	0.21 ± 0.05	0.18 ± 0.05
$\eta 2\pi^+2\pi^-$	$1.338 \leq \sqrt{s} \leq 1.937$	0.08 ± 0.01	0.03 ± 0.00	0.02 ± 0.00
$\eta\omega$	$1.333 \leq \sqrt{s} \leq 1.937$	0.31 ± 0.03	0.10 ± 0.01	0.09 ± 0.01
$\omega(\rightarrow \pi^0\gamma)\pi^0$	$0.920 \leq \sqrt{s} \leq 1.937$	0.88 ± 0.02	0.19 ± 0.00	0.18 ± 0.00
$\eta\phi$	$1.569 \leq \sqrt{s} \leq 1.937$	0.42 ± 0.03	0.15 ± 0.01	0.15 ± 0.01
$\phi \rightarrow \text{unaccounted}$	$0.988 \leq \sqrt{s} \leq 1.029$	0.04 ± 0.04	0.01 ± 0.01	0.01 ± 0.01
$\eta\omega\pi^0$	$1.550 \leq \sqrt{s} \leq 1.937$	0.35 ± 0.09	0.14 ± 0.04	0.13 ± 0.03
$\eta(\rightarrow \text{npp})K\bar{K}_{\text{no } \phi \rightarrow K\bar{K}}$	$1.569 \leq \sqrt{s} \leq 1.937$	0.01 ± 0.02	0.00 ± 0.01	0.00 ± 0.01
$p\bar{p}$	$1.890 \leq \sqrt{s} \leq 1.937$	0.03 ± 0.00	0.01 ± 0.00	0.01 ± 0.00
$n\bar{n}$	$1.912 \leq \sqrt{s} \leq 1.937$	0.03 ± 0.01	0.01 ± 0.00	0.01 ± 0.00
Estimated contributions ($\sqrt{s} \leq 1.937$ GeV)				
$(\pi^+\pi^-3\pi^0)_{\text{no } \eta}$	$1.013 \leq \sqrt{s} \leq 1.937$	0.50 ± 0.04	0.16 ± 0.01	0.11 ± 0.01
$(\pi^+\pi^-4\pi^0)_{\text{no } \eta}$	$1.313 \leq \sqrt{s} \leq 1.937$	0.21 ± 0.21	0.08 ± 0.08	0.06 ± 0.06
$KK3\pi$	$1.569 \leq \sqrt{s} \leq 1.937$	0.03 ± 0.02	0.02 ± 0.01	0.01 ± 0.01
$\omega(\rightarrow \text{npp})2\pi$	$1.285 \leq \sqrt{s} \leq 1.937$	0.10 ± 0.02	0.03 ± 0.01	0.03 ± 0.01
$\omega(\rightarrow \text{npp})3\pi$	$1.322 \leq \sqrt{s} \leq 1.937$	0.17 ± 0.03	0.06 ± 0.01	0.05 ± 0.01
$\omega(\rightarrow \text{npp})KK$	$1.569 \leq \sqrt{s} \leq 1.937$	0.00 ± 0.00	0.00 ± 0.00	0.00 ± 0.00
$\eta\pi^+\pi^-2\pi^0$	$1.338 \leq \sqrt{s} \leq 1.937$	0.08 ± 0.04	0.03 ± 0.02	0.02 ± 0.01
Other contributions ($\sqrt{s} > 1.937$ GeV)				
Inclusive channel	$1.937 \leq \sqrt{s} \leq 11.199$	43.67 ± 0.67	82.82 ± 1.05	69.94 ± 1.07
J/ψ	-	6.26 ± 0.19	7.07 ± 0.22	10.03 ± 0.30
ψ'	-	1.58 ± 0.04	2.51 ± 0.06	2.53 ± 0.06
$\Upsilon(1S - 4S)$	-	0.09 ± 0.00	1.06 ± 0.02	0.14 ± 0.00
pQCD	$11.199 \leq \sqrt{s} \leq \infty$	2.07 ± 0.00	124.79 ± 0.10	127.25 ± 0.00
Total	$m_\pi \leq \sqrt{s} \leq \infty$	693.26 ± 2.46	276.11 ± 1.11	260.39 ± 1.14

2HDM (inert):

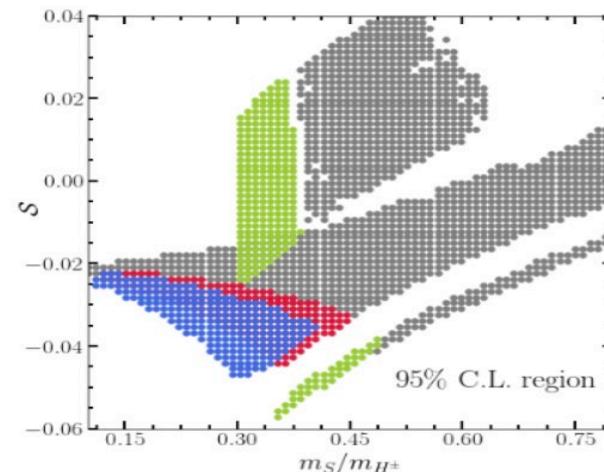




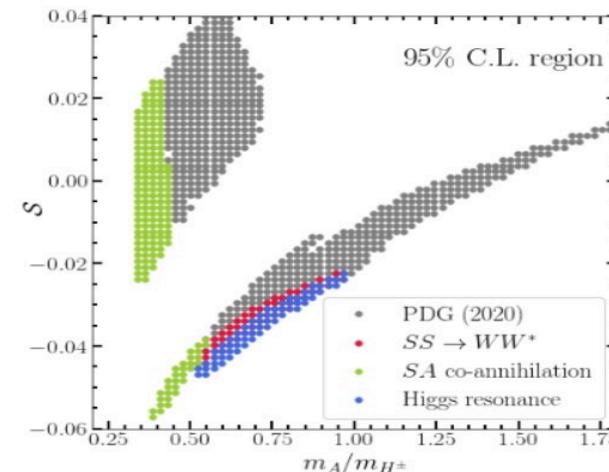
(a)



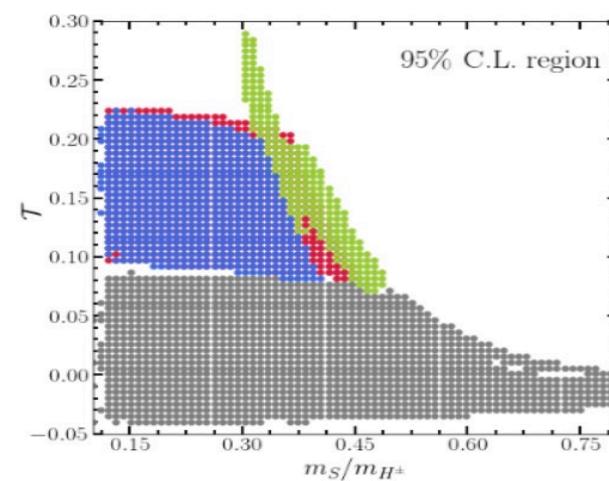
(b)



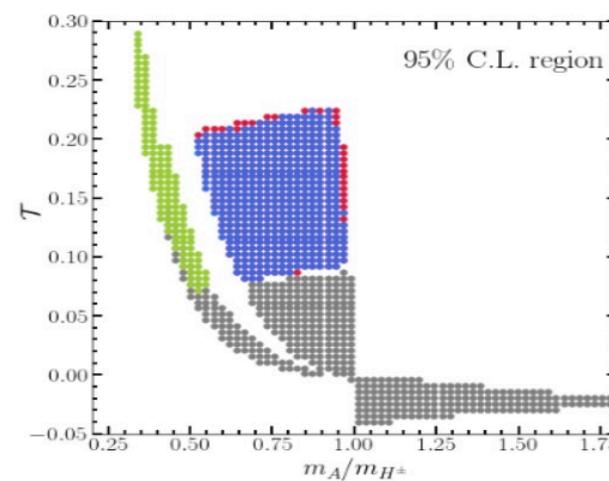
(a)



(b)



(c)



(d)