



東南大學  
SOUTHEAST UNIVERSITY



# 第七届手征有效场论研讨会

The nature of  $\Lambda(1405)$

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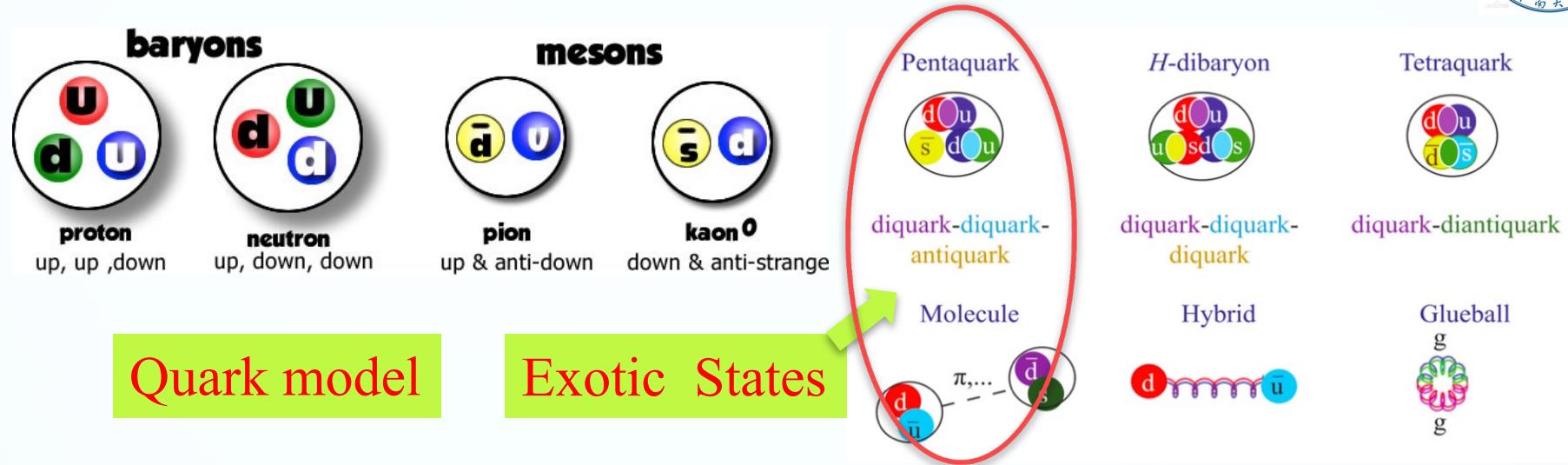
2022. 南京



# Outline

1. Introduction
2. Formalism
3. Results for the  $\Lambda(1405)$
4. Summary

# §1. Introduction



- H.-X. Chen, W. Chen, X. Liu and S.-L. Zhu, **Phys. Rept.** **639** (2016) 1.
- A. Esposito, A. Pilloni and A. D. Polosa, **Phys. Rept.** **668** (2016) 1.
- F.-K. Guo, C. Hanhart, U.-G. Meißner, Q. Wang, Q. Zhao and B.-S. Zou, **Rev. Mod. Phys.** **90** (2018) 0115004.
- S. L. Olsen, T. Skwarnicki and D. Zieminska, **Rev. Mod. Phys.** **90** (2018) 0115003.
- C. Z. Yuan, **Int. J. Mod. Phys. A** **33**, 1830018 (2018).
- Y. R. Liu, H. X. Chen, W. Chen, X. Liu and S. L. Zhu, **Prog. Part. Nucl. Phys.** **107**, 237 (2019).
- N. Brambilla, S. Eidelman, C. Hanhart, A. Nefediev, C. P. Shen, C. E. Thomas, A. Vairo and C. Z. Yuan, **Phys. Rept.** **873** (2020) 1.



## Background

$\Lambda(1405)$  was **predicted**:

*R. H. Dalitz and S. F. Tuan, Phys. Rev. Lett. 2, 425 (1959)*

$\Lambda(1405)$  was **discovered**:

*M. H. Alston, et.al., Phys. Rev. Lett. 6, 698 (1961)*

But, its properties are still **NOT** known well.....



## 1) Normal three-quark baryon: constituent quark models (failed)

*N. Isgur and G. Karl, Phys. Rev. D 18, 4187 (1978)*  
*S. Capstick and N. Isgur, Phys. Rev. D 34, 2809 (1986)*

## 2) Five-quark baryon: MIT bag model

*D. Strottman, Phys. Rev. D 20, 748 (1979)*

## 3) $J^P = \frac{1}{2}^-$ pentaquark state: Jaffe and Wilczek's diquark model

*A. Zhang, Y. R. Liu, P. Z. Huang, W. Z. Deng, X. L. Chen and S. L. Zhu, HEPNP 29, 250 (2005)*



## 4) Kaon bound state: Skyrme model

C. G. Callan, Jr. and I. R. Klebanov, *Nucl. Phys. B* 262, 365 (1985)

## 5) $\bar{K}N$ quasi-bound state: Coupled channel approach / molecular model

N. Kaiser, P. B. Siegel and W. Weise, *Nucl. Phys. A* 594, 325-345 (1995)

N. Kaiser, T. Waas and W. Weise, *Nucl. Phys. A* 612, 297-320 (1997)

E. Oset and A. Ramos, *Nucl. Phys. A* 635, 99 (1998)

## 6) Nnarrow $\bar{K}N$ Feshbach resonance: Skyrme model

T. Ezoe and A. Hosaka, *Phys. Rev. D* 102, 014046 (2020)

# More about **Coupled channel approach**: LO+NLO+Born Term



*B. Borasoy, R. Nissler and W. Weise, Eur. Phys. J. A 25, 79-96 (2005)*

*J. A. Oller, J. Prades and M. Verbeni, Phys. Rev. Lett. 95, 172502 (2005)*

*B. Borasoy, U. G. Meissner and R. Nissler, Phys. Rev. C 74, 055201 (2006)*

*Y. Ikeda, T. Hyodo and W. Weise, Phys. Lett. B 706, 63-67 (2011)*

*M. Mai and U.-G. Meißner, Nucl. Phys. A 900, 51 - 64 (2013)*

*Z. H. Guo and J. A. Oller, Phys. Rev. C 87, 035202 (2013)*

*M. Mai and U.-G. Meißner, Eur. Phys. J. A 51, 30 (2015)*

*A. Ramos, A. Feijoo and V. K. Magas, Nucl. Phys. A 954, 58-74 (2016)*



## About two-poles structure :

*P. J. Fink, Jr., G. He, R. H. Landau and J. W. Schnick, Phys. Rev. C 41, 2720 (1990) ----first prediction (also with coupled channel approach)*

*J. A. Oller and U.-G. Meißner, Phys. Lett. B 500, 263 (2001)*

*D. Jido, A. Hosaka, J. C. Nacher, E. Oset and A. Ramos, Phys. Rev. C 66, 025203 (2002)*

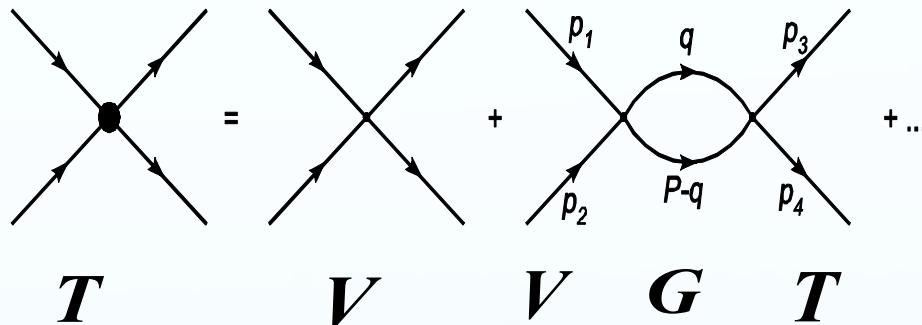
*C. Garcia-Recio, J. Nieves, E. Ruiz Arriola and M. J. Vicente Vacas, Phys. Rev. D 67, 076009 (2003)*

*D. Jido, J. A. Oller, E. Oset, A. Ramos and U.-G. Meißner, Nucl. Phys. A 725, 181 (2003)*

## §2. Formalism

- **Coupled Channel Unitary Approach:** solving Bethe-Salpeter equations, which take on-shell approximation for the loops.

$$T = V + V G T, \quad T = [1 - V G]^{-1} V$$



where **V** matrix (potentials) can be evaluated from the interaction Lagrangians.

J. A. Oller and E. Oset, *Nucl. Phys. A* 620 (1997) 438

E. Oset and A. Ramos, *Nucl. Phys. A* 635 (1998) 99

J. A. Oller and U. G. Meißner, *Phys. Lett. B* 500 (2001) 263



$G$  is a diagonal matrix with the loop functions of each channels:

$$G_{ll}(s) = i \int \frac{d^4 q}{(2\pi)^4} \frac{2M_l}{(P-q)^2 - m_{l1}^2 + i\varepsilon} \frac{1}{q^2 - m_{l2}^2 + i\varepsilon}$$

The coupled channel scattering amplitudes **T matrix satisfy the unitary** :

$$\text{Im } T_{ij} = T_{in} \sigma_{nn} T_{nj}^*$$
$$\sigma_{nn} \equiv \text{Im } G_{nn} = - \frac{q_{cm}}{8\pi\sqrt{s}} \theta(s - (m_1 + m_2)^2))$$

To search the poles of the resonances, we should extrapolate the scattering amplitudes to the second Riemann sheets:

$$G_{ll}^{II}(s) = G_{ll}^I(s) + \boxed{\frac{i}{2\pi} \frac{M_l q_{cml}(s)}{\sqrt{s}}}$$



## Three-momentum cutoff (CO) method

$$G_l(s) = \int_0^{q_{max}} \frac{q^2 dq}{2\pi^2} \frac{1}{2\omega_l(q)} \frac{M_l}{E_l(q)} \frac{1}{p^0 + k^0 - \omega_l(q) - E_l(q) + i\epsilon}.$$

## Dimensional regularization (DR) method

$$\begin{aligned} G_l(s) = & \frac{2M_l}{16\pi^2} \left\{ a_\mu + \ln \frac{M_l^2}{\mu^2} + \frac{m_l^2 - M_l^2 + s}{2s} \ln \frac{m_l^2}{M_l^2} \right. \\ & + \frac{q_{cml}(s)}{\sqrt{s}} \left[ \ln \left( s - (M_l^2 - m_l^2) \right) + 2q_{cml}(s)\sqrt{s} \right] \\ & + \ln \left( s + (M_l^2 - m_l^2) \right) + 2q_{cml}(s)\sqrt{s} \\ & - \ln \left( -s - (M_l^2 - m_l^2) \right) + 2q_{cml}(s)\sqrt{s} \\ & \left. - \ln \left( -s + (M_l^2 - m_l^2) \right) + 2q_{cml}(s)\sqrt{s} \right\}, \end{aligned}$$

The couplings can be defined by

$$g_i^2 = \lim_{\sqrt{s} \rightarrow \sqrt{s_p}} (\sqrt{s} - \sqrt{s_p}) T_{ii}$$

The sum rule can be evaluated by

$$-\sum_i g_i^2 \left[ \frac{dG_l}{d\sqrt{s}} \right]_{\sqrt{s}=\sqrt{s_p}} = 1 - Z.$$

The wave function

$$\phi(\vec{r}) = \frac{1}{(2\pi)^{3/2}} \frac{4\pi}{r} \frac{1}{C} \int_{q_{\max}} pdp \sin(pr) \times \frac{\Theta(q_{\max} - |\vec{p}|)}{E - \omega_1(\vec{p}) - \omega_2(\vec{p})}.$$

The form factor

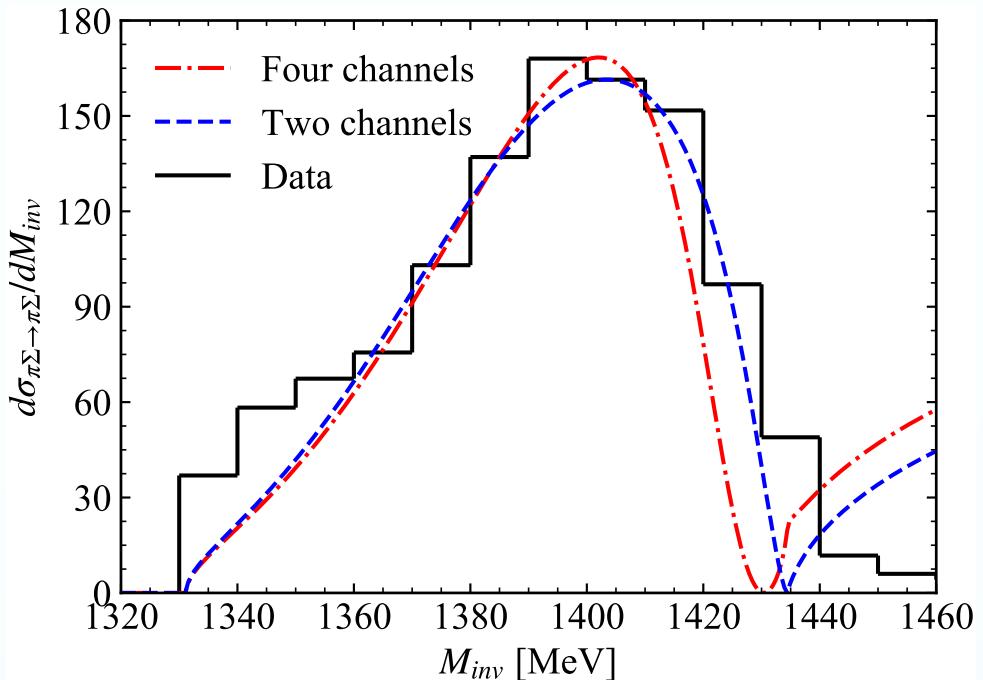
$$\begin{aligned} F(\vec{q}) &= \int d^3\vec{r} \phi(\vec{r}) \phi^*(\vec{r}) e^{-i\vec{q}' \cdot \vec{r}} \\ &= \int d^3\vec{p} \times \frac{\theta(\Lambda - p)\theta(\Lambda - |\vec{p} - \vec{q}|)}{[E - \omega_1(p) - \omega_2(p)] [E - \omega_1(\vec{p} - \vec{q}) - \omega_2(\vec{p} - \vec{q})]} \end{aligned}$$

The radius of the resonance

$$\langle r^2 \rangle = -6 \left[ \frac{dF(q)}{dq^2} \right]_{q^2=0}$$

## §3. Results for the $\Lambda(1405)$

### 1) Results of coupled channels



Four coupled channels

$\mu = q_{max} = 630 \text{ MeV}$  ← E. Oset and A. Ramos, Nucl. Phys. A 635, 99 (1998)

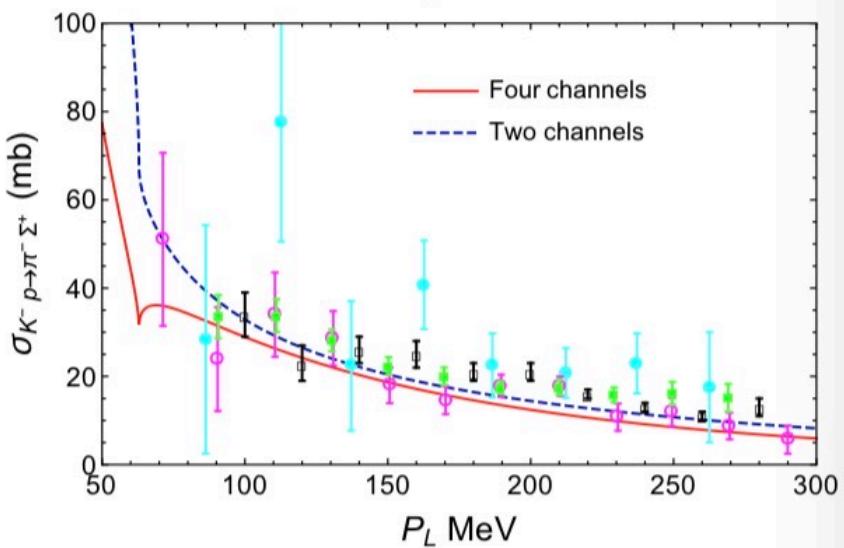
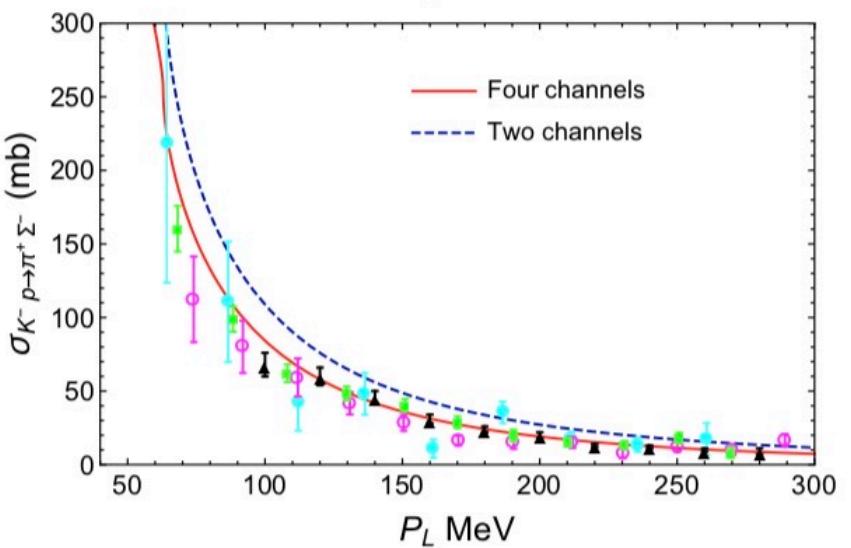
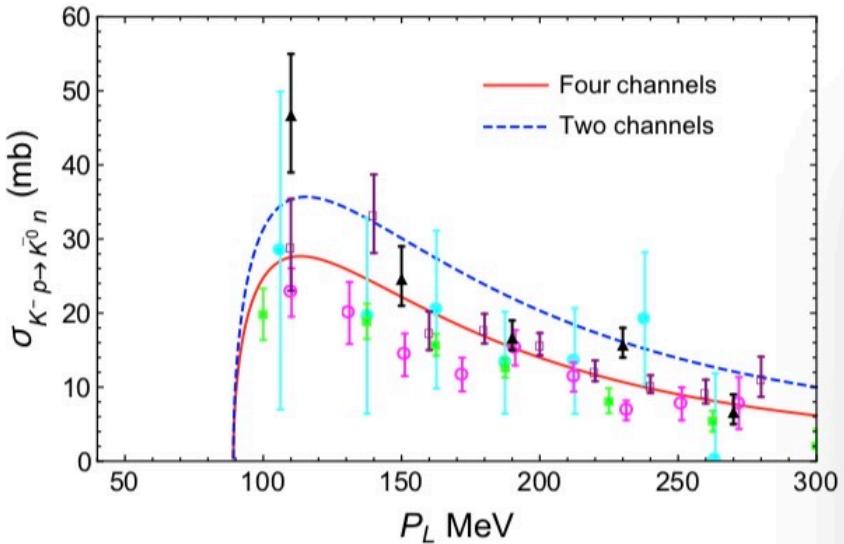
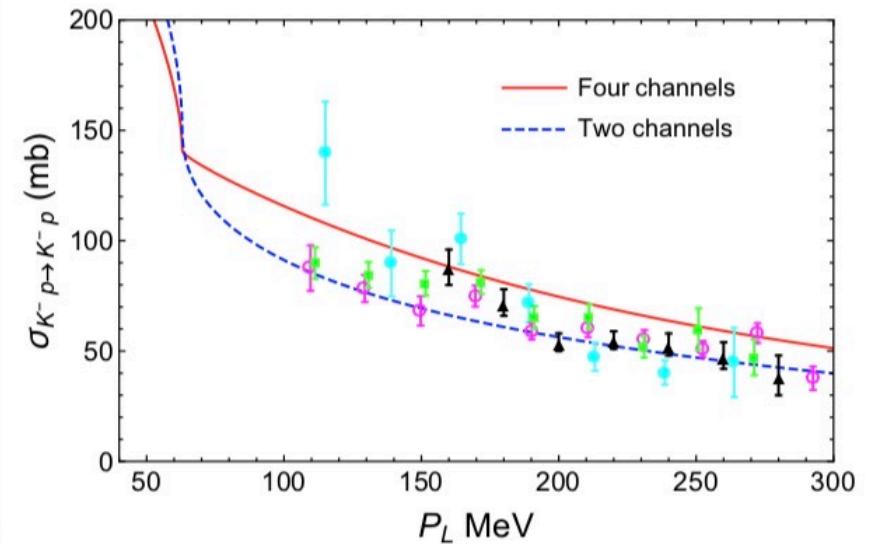
$a_{\bar{K}N} = -1.84, a_{\pi\Sigma} = -2.00, a_{\eta\Lambda} = -2.25, a_{K\Xi} = -2.67$

Two coupled channels

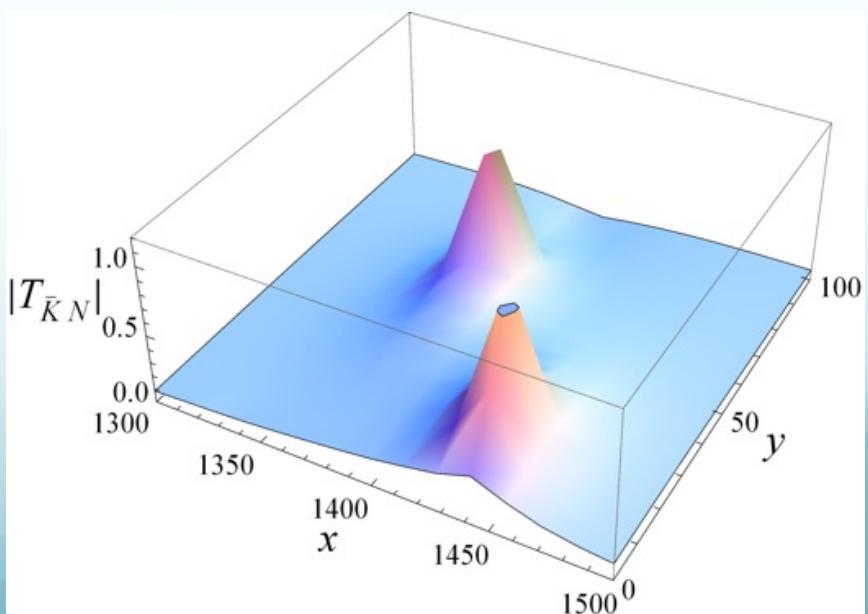
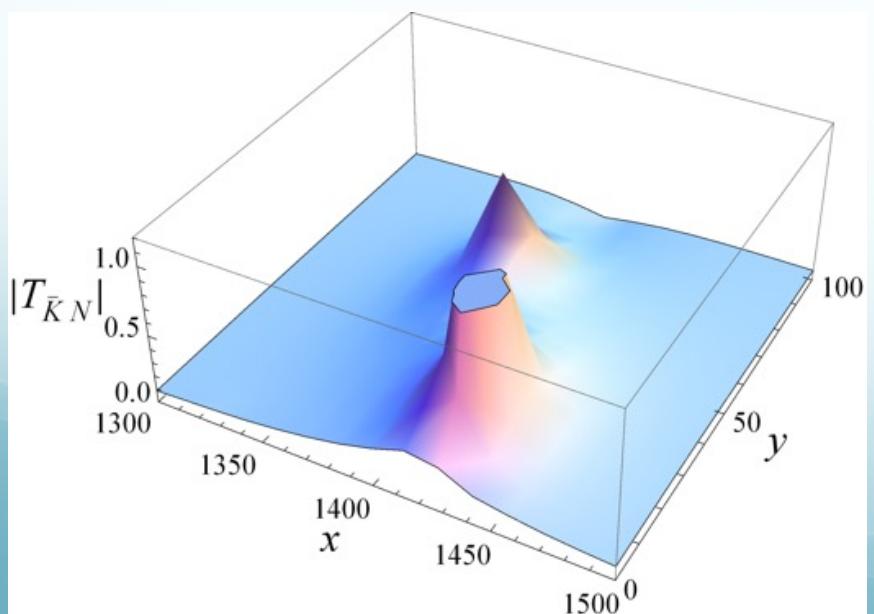
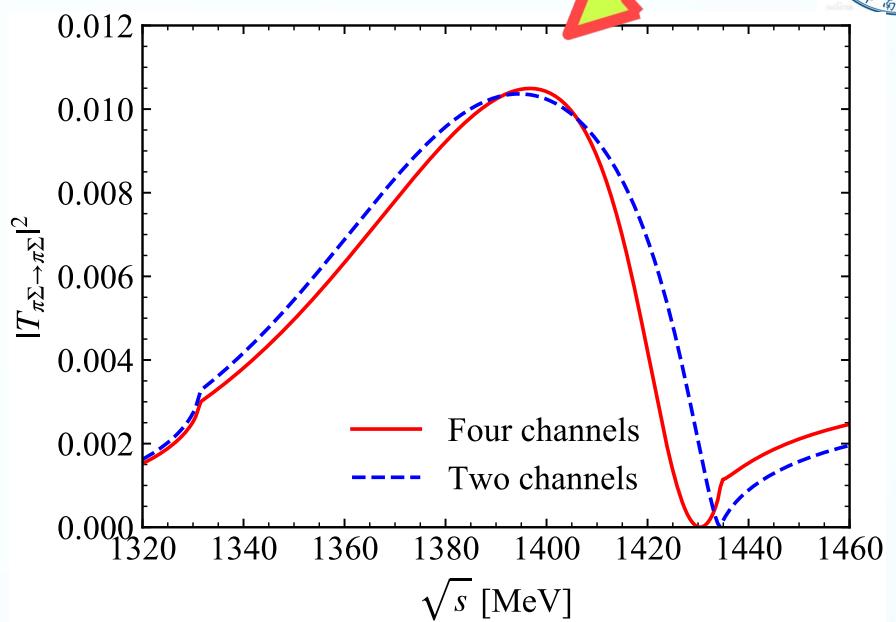
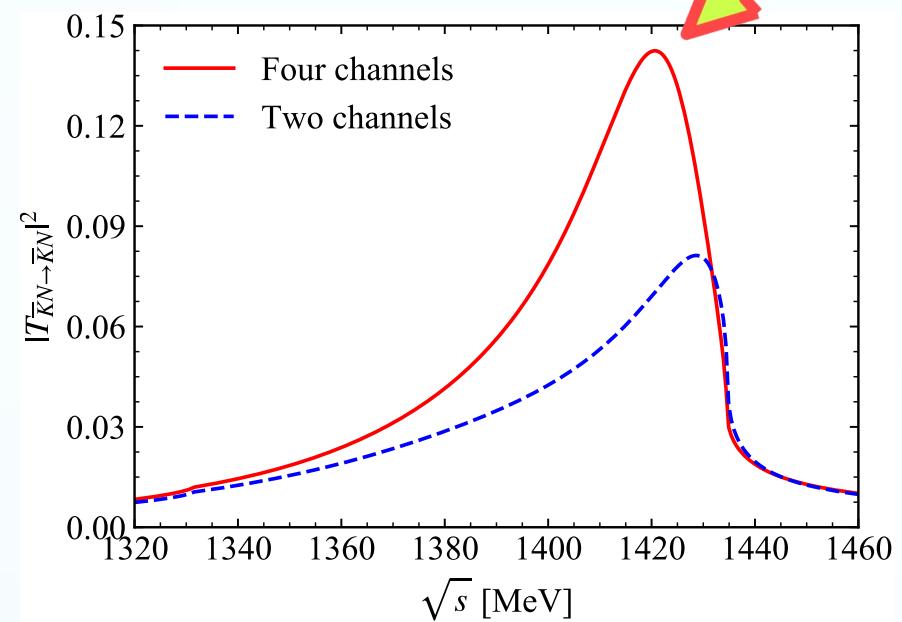
D. Jido, J. A. Oller, E. Oset, A. Ramos and U.-G. Meißner, Nucl. Phys. A 725, 181 (2003)

$\mu = q_{max} = 623 \text{ MeV}$   $a_{\bar{K}N} = -1.86, a_{\pi\Sigma} = -2.09$

# Cross sections for different channel



# Two-poles structure



# Poles positions

	$1^{st}$ pole	$2^{st}$ pole	$3^{st}$ pole
Four coupled channels (DR)	$1390.61 + 65.91i$	$1426.77 + 16.32i$	$1680.38 + 20.18i$
Four coupled channels (CO)	$1377.35 + 58.46i$	$1427.96 + 18.10i$	$1762.8 + 27.51i$
Two coupled channels (DR)	$1384.54 + 60.98i$	$1438.95 + 12.44i$	
Two coupled channels (CO)	$1375.56 + 62.63i$	$1438.04 + 11.44i$	
Ref. [48], NLO	$1381_{-6}^{+18} - 81_{-8}^{+19}i$	$1424_{-23}^{+7} - 26_{-14}^{+3}i$	
Ref. [49] solution II	$1388_{-9}^{+9} - 114_{-25}^{+24}i$	$1421_{-2}^{+3} - 19_{-5}^{+8}i$	In the same Remann sheet
Ref. [53] solution II	$1330_{-5}^{+4} - 56_{-11}^{+17}i$	$1434_{-2}^{+2} - 10_{-1}^{+2}i$	
Ref. [53] solution IV	$1325_{-15}^{+15} - 90_{-18}^{+12}i$	$1429_{-7}^{+8} - 12_{-3}^{+2}i$	
Ref. [94] one-pole solution		$(1421 \pm 3) - (23 \pm 3)i$	
Ref. [94] two-pole solution	$1380 - 90i$ (fixed)	$(1423 \pm 3) - (20 \pm 3)i$	

[48] Y. Ikeda, T. Hyodo and W. Weise, Nucl. Phys. A 881, 98-114 (2012).

[49] Z. H. Guo and J. A. Oller, Phys. Rev. C 87, 035202 (2013).

[53] M. Mai and U.-G. Meißner, Eur. Phys. J. A 51, 30 (2015).

[94] D. L. B. Sombillo, Y. Ikeda, T. Sato and A. Hosaka, Phys. Rev. D 104, 036001 (2021).

# Couplings for each coupled channel

$\sqrt{s_p}$	$g_{\bar{K}N}$	$ g_{\bar{K}N} $	$g_{\pi\Sigma}$	$ g_{\pi\Sigma} $	$g_{\eta\Lambda}$	$ g_{\eta\Lambda} $	$g_{K\Xi}$	$ g_{K\Xi} $
Four coupled channels								
$1390.61 + 65.91i$	$1.20 + 1.74i$	2.12	$2.46 + 1.51i$	2.88	$0.01 + 0.76i$	0.76	$0.45 + 0.41i$	0.61
$1426.77 + 16.32i$	$2.55 - 0.94i$	2.71	$0.42 - 1.45i$	1.51	$1.40 - 0.20i$	1.41	$0.11 - 0.33i$	0.35
$1680.38 + 20.18i$	$0.30 + 0.72i$	0.78	$0.01 + 0.27i$	0.27	$1.05 + 0.11i$	1.06	$3.45 + 0.12i$	3.46
Two coupled channels								
$1384.54 + 60.98i$	$1.46 + 1.45i$	2.06	$2.38 + 1.29i$	2.71	—	—	—	—
$1438.95 + 12.44i$	$2.00 - 1.34i$	2.41	$0.07 + 1.42i$	1.42	—	—	—	—
Ref. [36]								
$1390 + 66i$	$1.2 + 1.7i$	2.1	$-2.5 - 1.5i$	2.9	$0.010 + 0.77i$	0.77	$-0.45 - 0.41i$	0.61
$1426 + 16i$	$-2.5 + 0.94i$	2.7	$0.42 - 1.4i$	1.5	$-1.4 + 0.21i$	1.4	$0.11 - 0.33i$	0.35
$1680 + 20i$	$0.30 + 0.71i$	0.77	$-0.003 - 0.27i$	0.27	$-1.1 - 0.12i$	1.1	$3.4 + 0.14i$	3.5
Ref. [95]								
$1391 - 66i$	$-0.86 + 1.26i$	—	$-1.42 + 0.88i$	—	$-0.01 + 0.79i$	—	$-0.33 + 0.30i$	—
$1426 - 17i$	$1.84 + 0.67i$	—	$0.26 + 0.85i$	—	$1.44 + 0.21i$	—	$0.09 + 0.24i$	—

[36] D. Jido, J. A. Oller, E. Oset, A. Ramos and U.-G. Meißner, Nucl. Phys. A 725, 181 (2003).

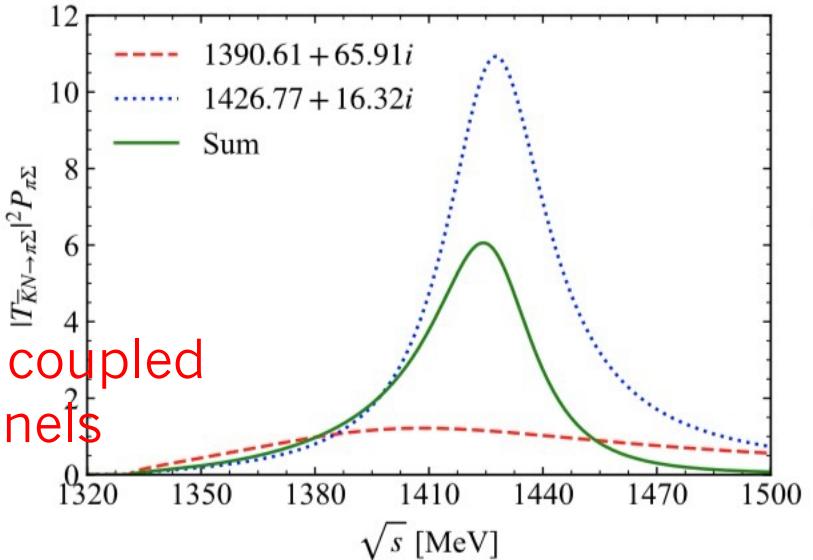
[95] A. V. Anisovich, et. al., Eur. Phys. J. A 56, no. 5, 139 (2020).

# Compositeness of the poles

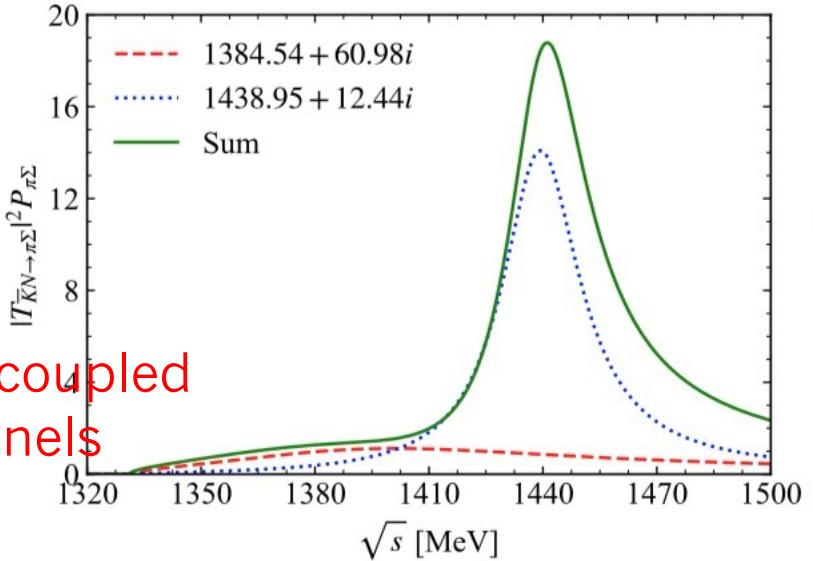
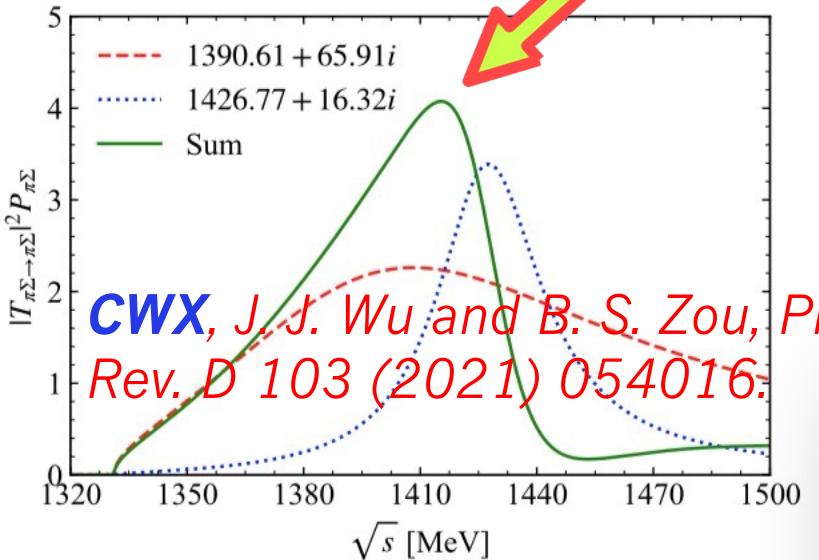
$\sqrt{s_p}$	$(1 - Z)_{\bar{K}N}$	$ (1 - Z)_{\bar{K}N} $	$(1 - Z)_{\pi\Sigma}$	$ (1 - Z)_{\pi\Sigma} $	$(1 - Z)_{\eta\Lambda}$	$ (1 - Z)_{\eta\Lambda} $	$(1 - Z)_{K\Xi}$	$ (1 - Z)_{K\Xi} $
Four coupled channels								
$1390.61 + 65.91i$	$-0.20 + 0.12i$	0.24	$0.36 - 0.52i$	0.63	$-0.01 - 0.002i$	0.01	$-0.0003 + 0.01i$	0.01
$1426.77 + 16.32i$	$0.98 - 0.04i$	0.98	$-0.04 + 0.15i$	0.15	$0.05 - 0.01i$	0.05	$-0.002 - 0.001i$	0.002
$1680.38 + 20.18i$	$0.02 + 0.002i$	0.02	$0.002 + 0.002i$	0.003	$-0.07 - 0.16i$	0.17	$0.52 + 0.09i$	0.52
Two coupled channels								
$1384.54 + 60.98i$	$-0.12 + 0.18i$	0.22	$0.27 - 0.51i$	0.58	—	—	—	—
$1438.95 + 12.44i$	$0.96 - 0.09i$	0.97	$0.05 + 0.12i$	0.13	—	—	—	—

$$T_{\bar{K}N \rightarrow \pi\Sigma} = g_{\bar{K}N}^{p_1} \frac{1}{\sqrt{s} - M_{p_1} + i\Gamma_{p_1}/2} g_{\pi\Sigma}^{p_1} + g_{\bar{K}N}^{p_2} \frac{1}{\sqrt{s} - M_{p_2} + i\Gamma_{p_2}/2} g_{\pi\Sigma}^{p_2},$$

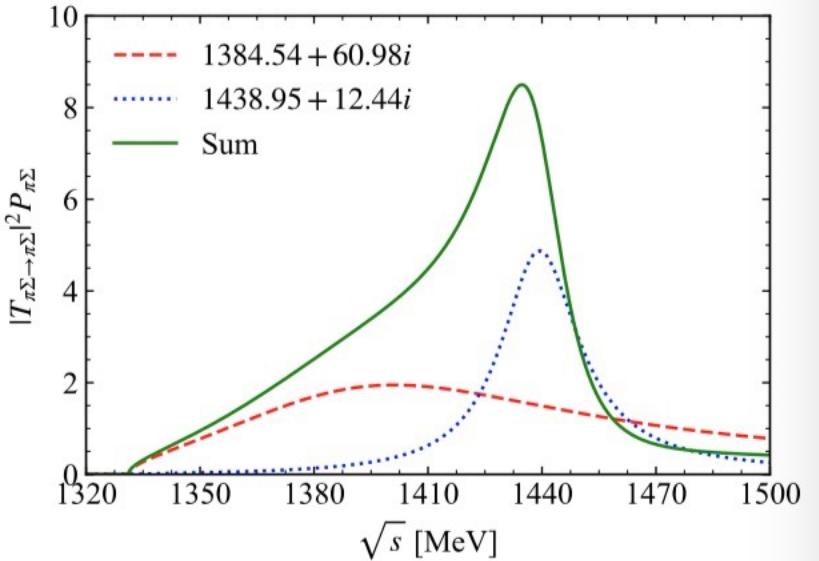
$$T_{\pi\Sigma \rightarrow \pi\Sigma} = g_{\pi\Sigma}^{p_1} \frac{1}{\sqrt{s} - M_{p_1} + i\Gamma_{p_1}/2} g_{\pi\Sigma}^{p_1} + g_{\pi\Sigma}^{p_2} \frac{1}{\sqrt{s} - M_{p_2} + i\Gamma_{p_2}/2} g_{\pi\Sigma}^{p_2},$$



Four coupled channels



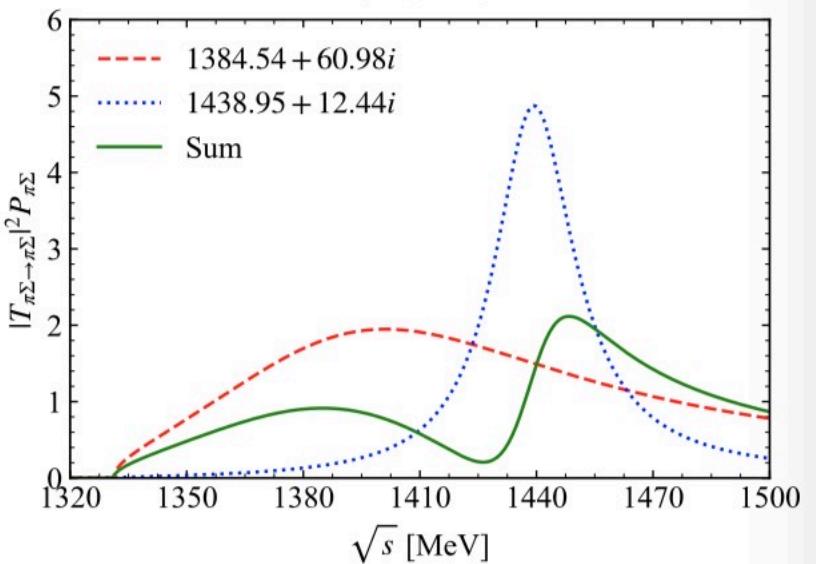
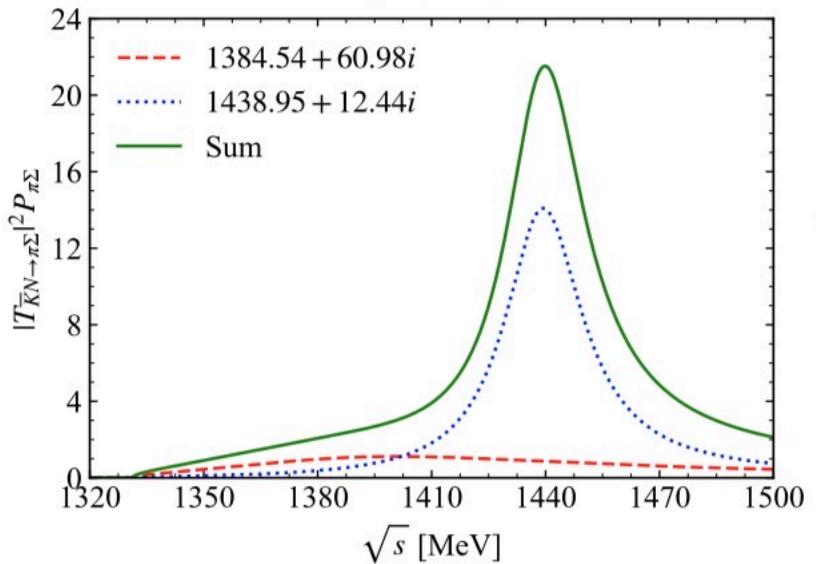
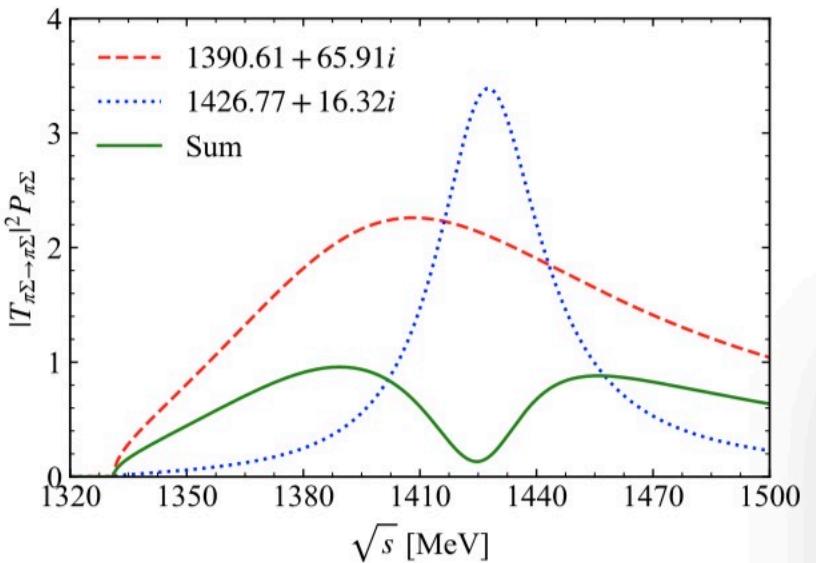
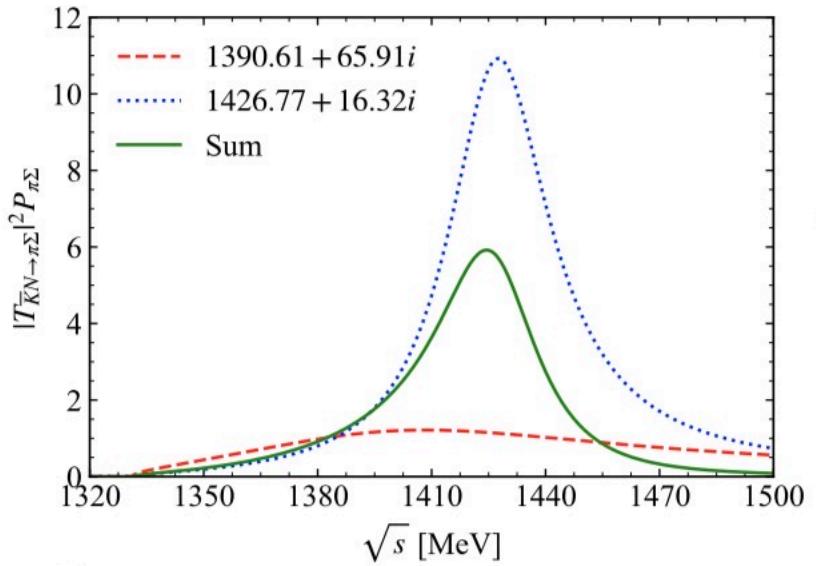
Two coupled channels



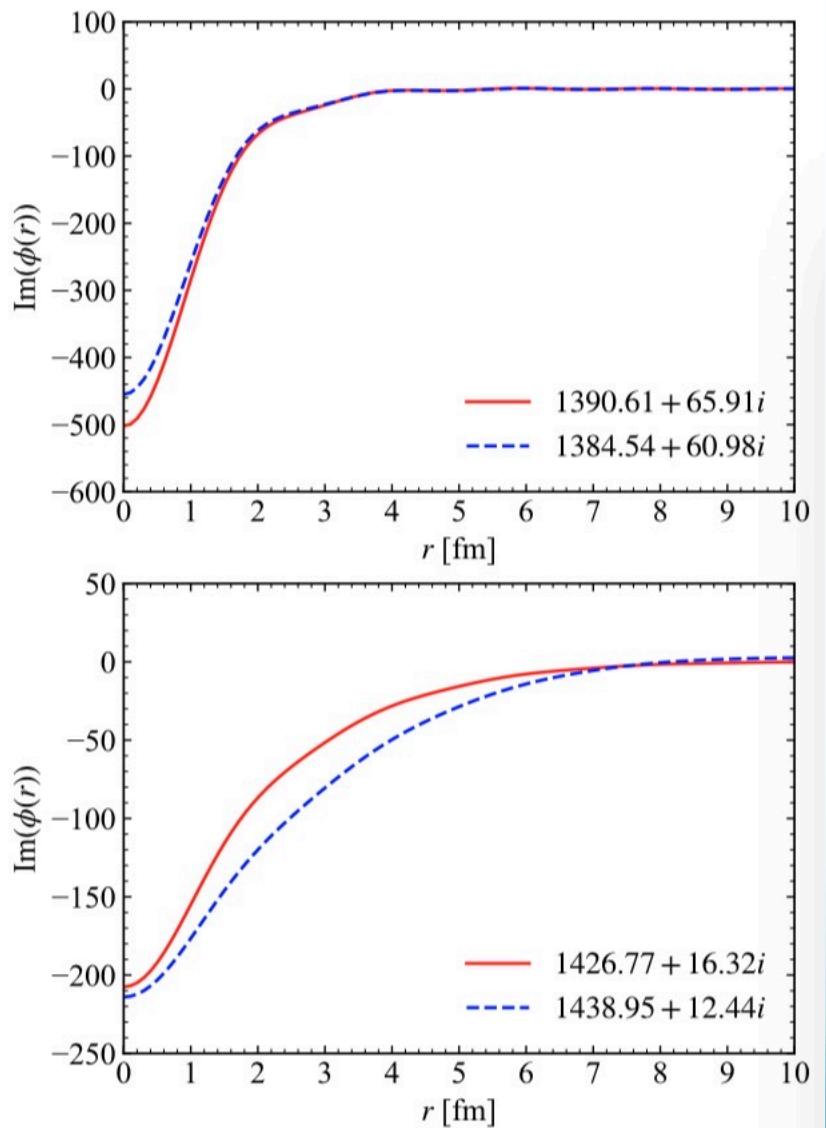
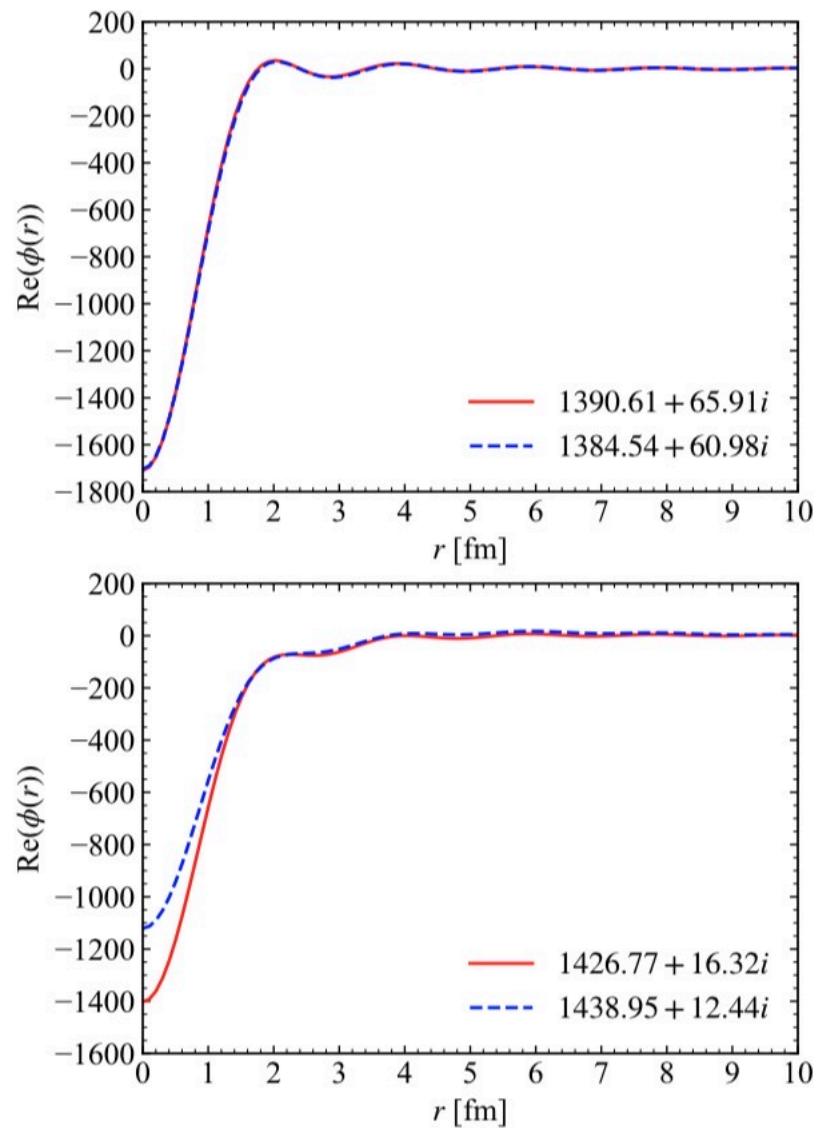
$$T_{\bar{K}N \rightarrow \pi\Sigma} = g_{\bar{K}N}^{p_1} \frac{1}{\sqrt{s} - M_{p_1} - i\Gamma_{p_1}/2} g_{\pi\Sigma}^{p_1} + g_{\bar{K}N}^{p_2} \frac{1}{\sqrt{s} - M_{p_2} - i\Gamma_{p_2}/2} g_{\pi\Sigma}^{p_2},$$

$$T_{\pi\Sigma \rightarrow \pi\Sigma} = g_{\pi\Sigma}^{p_1} \frac{1}{\sqrt{s} - M_{p_1} - i\Gamma_{p_1}/2} g_{\pi\Sigma}^{p_1} + g_{\pi\Sigma}^{p_2} \frac{1}{\sqrt{s} - M_{p_2} - i\Gamma_{p_2}/2} g_{\pi\Sigma}^{p_2},$$

Conjugated pole



# Wave functions of two poles for $\bar{K} N$ channel



## Radii of the poles

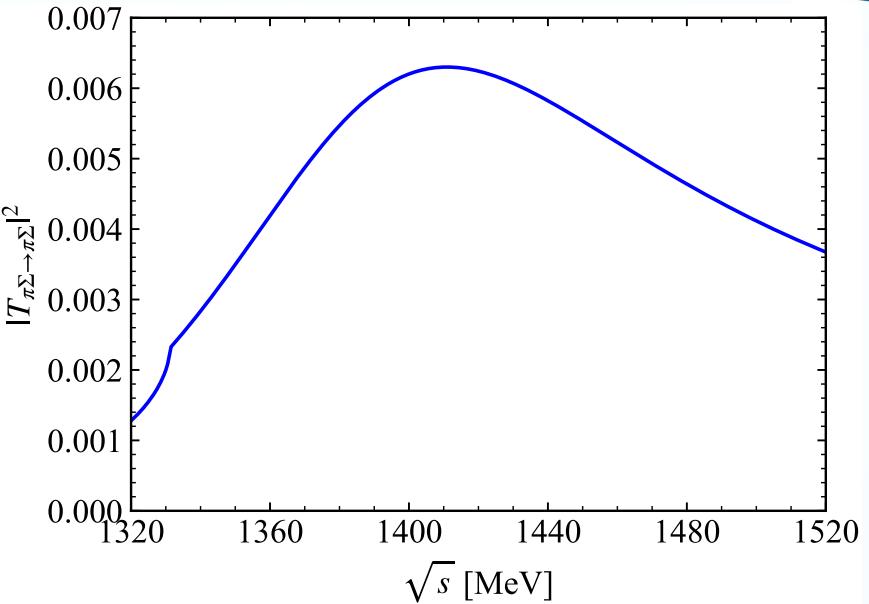
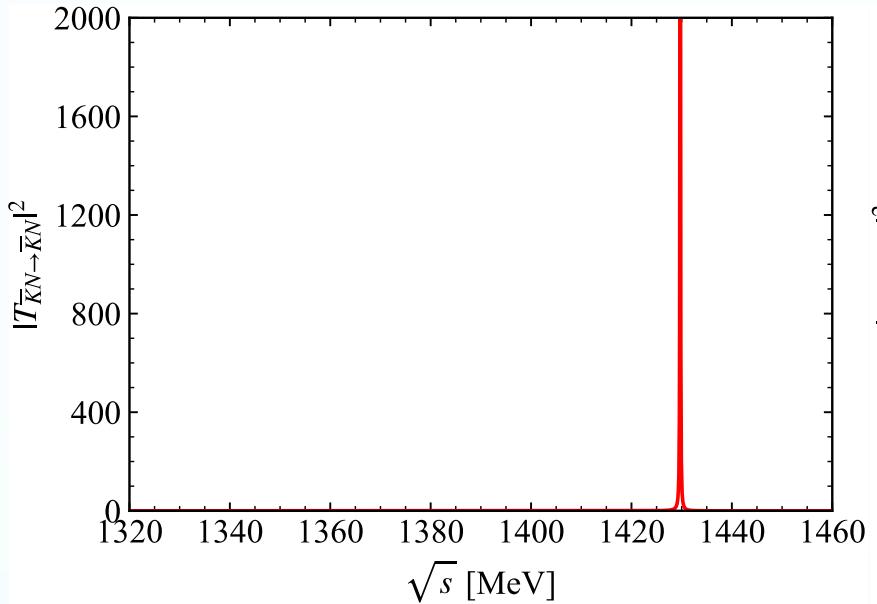
$\sqrt{s_p}$	$\sqrt{\langle r^2 \rangle_{\bar{K}N}}$	$ \sqrt{\langle r^2 \rangle_{\bar{K}N}} $	$\sqrt{\langle r^2 \rangle_{\pi\Sigma}}$	$ \sqrt{\langle r^2 \rangle_{\pi\Sigma}} $
Four coupled channels				
$1390.61 + 65.91i$	—	—	$(0.67 + 0.06i)$ fm	$0.67$ fm
$1426.77 + 16.32i$	$(1.46 + 0.57i)$ fm	$1.57$ fm	—	—
Two coupled channels				
$1384.54 + 60.98i$	—	—	$(0.65 + 0.10i)$ fm	$0.66$ fm
$1438.95 + 12.44i$	$(1.16 + 1.07i)$ fm	$1.58$ fm	—	—

# Scattering length of $K^- p$ channel

Approaches	$a_{K^- p}$ (fm)
Four coupled channels	$-0.73 + 1.31i$
Two coupled channels	$-0.15 + 1.48i$
Ref. [48], NLO	$-0.70 + 0.89i$
Ref. [49] (Fit I)	$-0.67^{+0.13}_{-0.12} + 0.92^{+0.07}_{-0.08}i$
Ref. [49] (Fit II)	$-0.61^{+0.13}_{-0.15} + 0.89^{+0.07}_{-0.06}i$
Exp. [98]	$(-0.78 \pm 0.15 \pm 0.03) + (0.49 \pm 0.25 \pm 0.12)i$
Exp. [92]	$-0.67 + 0.64i$
Exp [99]	$(-0.65 \pm 0.10) + (0.81 \pm 0.15)i$

- [48] Y. Ikeda, T. Hyodo and W. Weise, *Nucl. Phys. A* 881, 98-114 (2012).
- [49] Z. H. Guo and J. A. Oller, *Phys. Rev. C* 87, 035202 (2013).
- [92] J. K. Kim, *Columbia University Report No. NEVIS-149*, 1966.
- [98] Y. Bai and D. Y. Chen, *Phys. Rev. D* 99, no.7, 072007 (2019).
- [99] M. Iwasaki, et. al., *Phys. Rev. Lett.* 78, 3067-3069 (1997).

## 2) Results of single channel

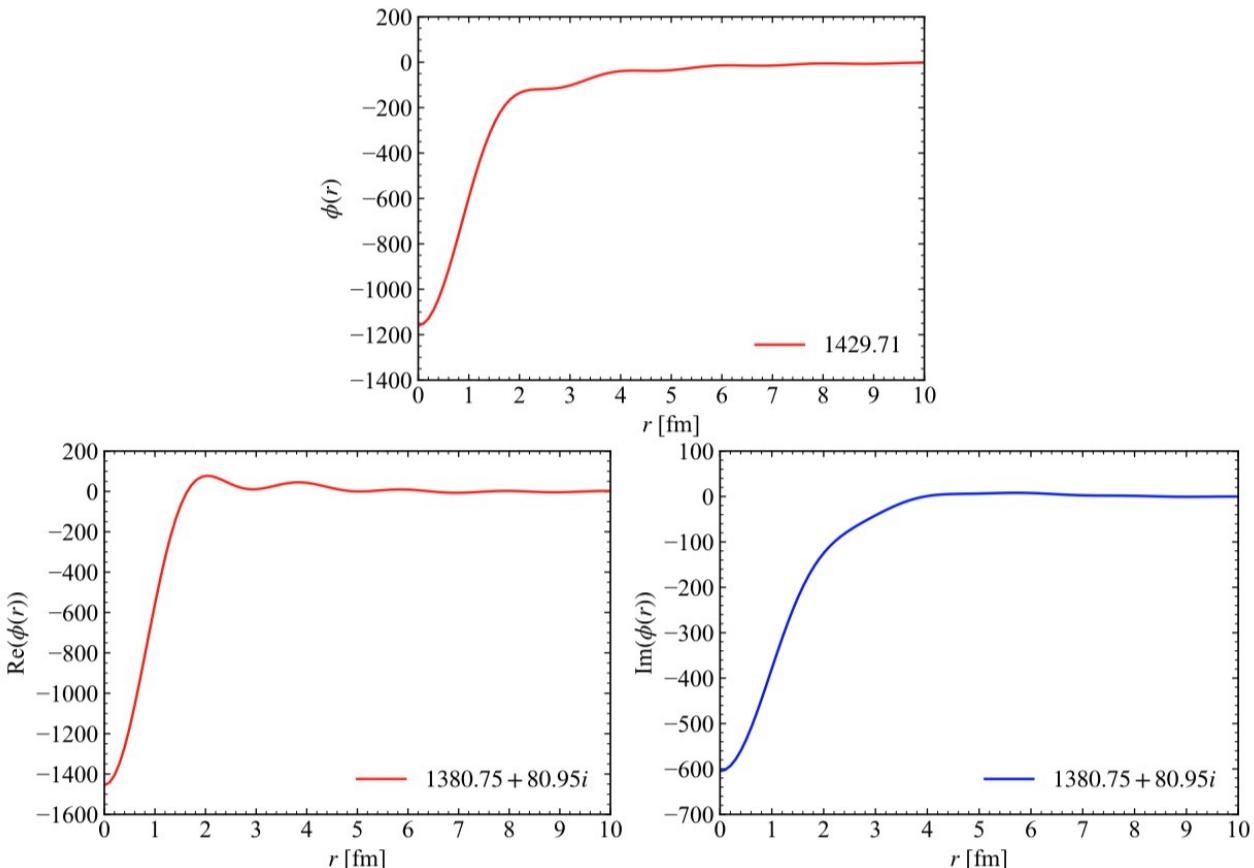


Pole positions  
and couplings

$\sqrt{s_p}$	$g_{\bar{K}N}$	$ g_{\bar{K}N} $	$g_{\pi\Sigma}$	$ g_{\pi\Sigma} $
$1380.75 + 80.95i$	—	—	$1.97 + 1.45i$	2.44
1429.71	1.81	1.81	—	—

Compositeness  
of the poles

$\sqrt{s_p}$	$(1 - Z)_{\bar{K}N}$	$ (1 - Z)_{\bar{K}N} $	$(1 - Z)_{\pi\Sigma}$	$ (1 - Z)_{\pi\Sigma} $
$1380.75 + 80.95i$	—	—	$0.30 - 0.35i$	0.45
1429.71	0.89	0.89	—	—



## Radii of the poles

$\sqrt{s_p}$	$\sqrt{\langle r^2 \rangle_{\bar{K}N}} \mid \sqrt{\langle r^2 \rangle_{\bar{K}N}} \mid$	$\sqrt{\langle r^2 \rangle_{\pi\Sigma}} \mid \sqrt{\langle r^2 \rangle_{\pi\Sigma}} \mid$
$1380.75 + 80.95i$	—	$(0.78 + 0.08i)$ fm
1429.71	2.79 fm	2.79 fm



# §4. Summary

➤ Two-poles structure of  $\Lambda(1405)$  (*molecules*)

➤ Couplings and Componess:

Lower one:  $\pi\Sigma$  ; Higher one:  $\bar{K}N$

➤ Wave function and radius:

Lower one: **0.67 fm** ; Higher one: **1.57 fm**

➤ Two poles can be reproduced in **single channel interaction** of  $\bar{K}N$  and  $\pi\Sigma$ , respectively.

**One can conclude that  $\Lambda(1405)$  is overlapped by  
TWO different states!**





谢谢大家！

*Thanks for your attention!*