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Asia/Shanghai 时区

Chiral condensate and spontaneous chiral symmetry breaking on the lattice

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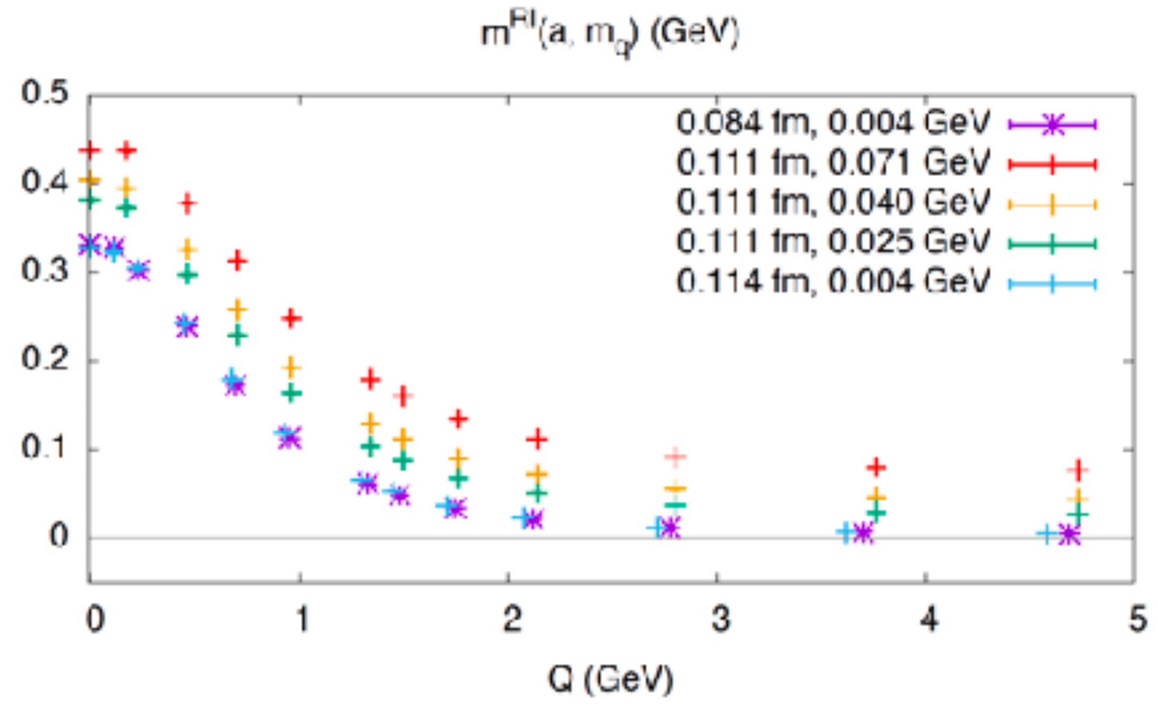
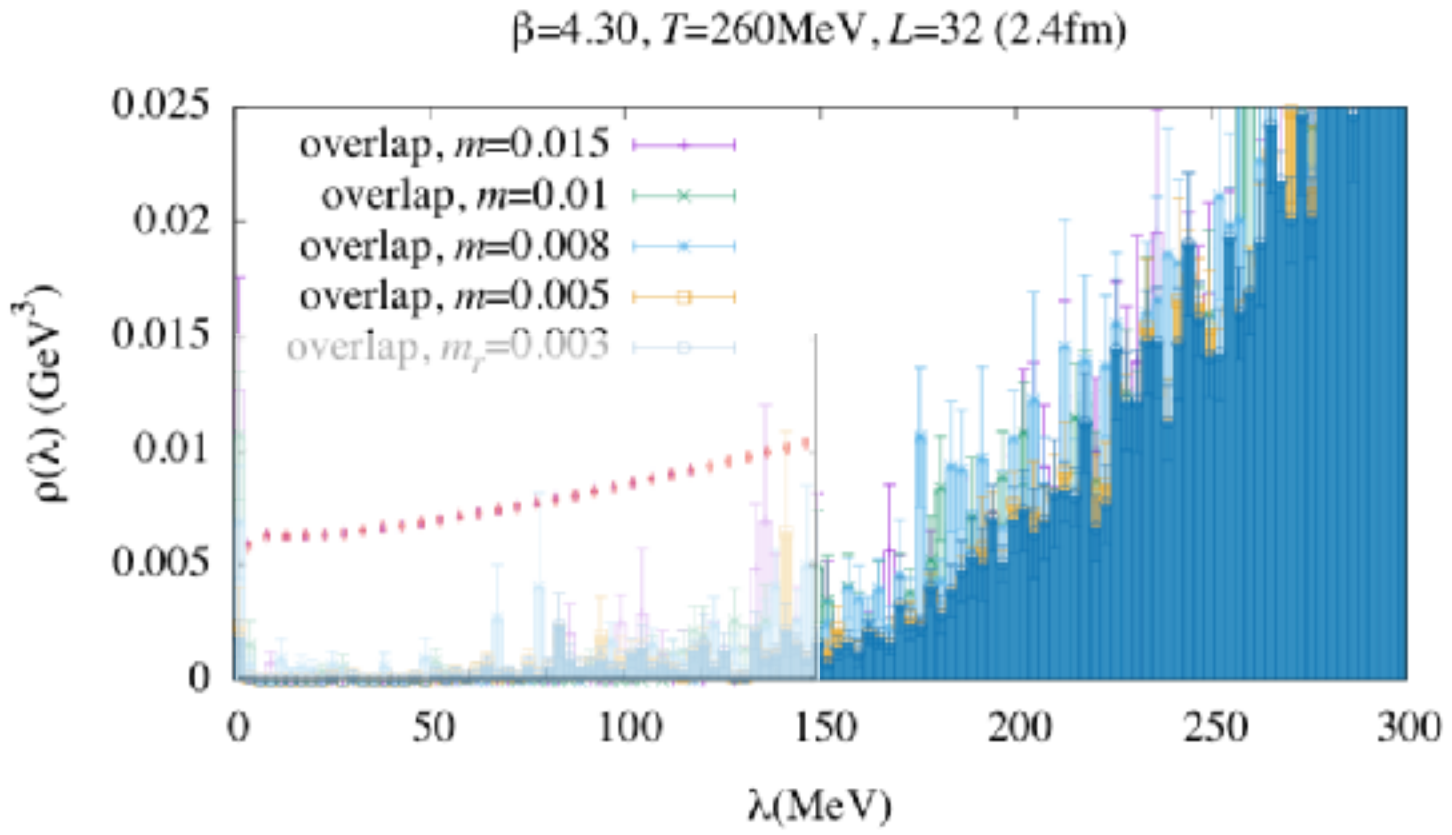
Outline

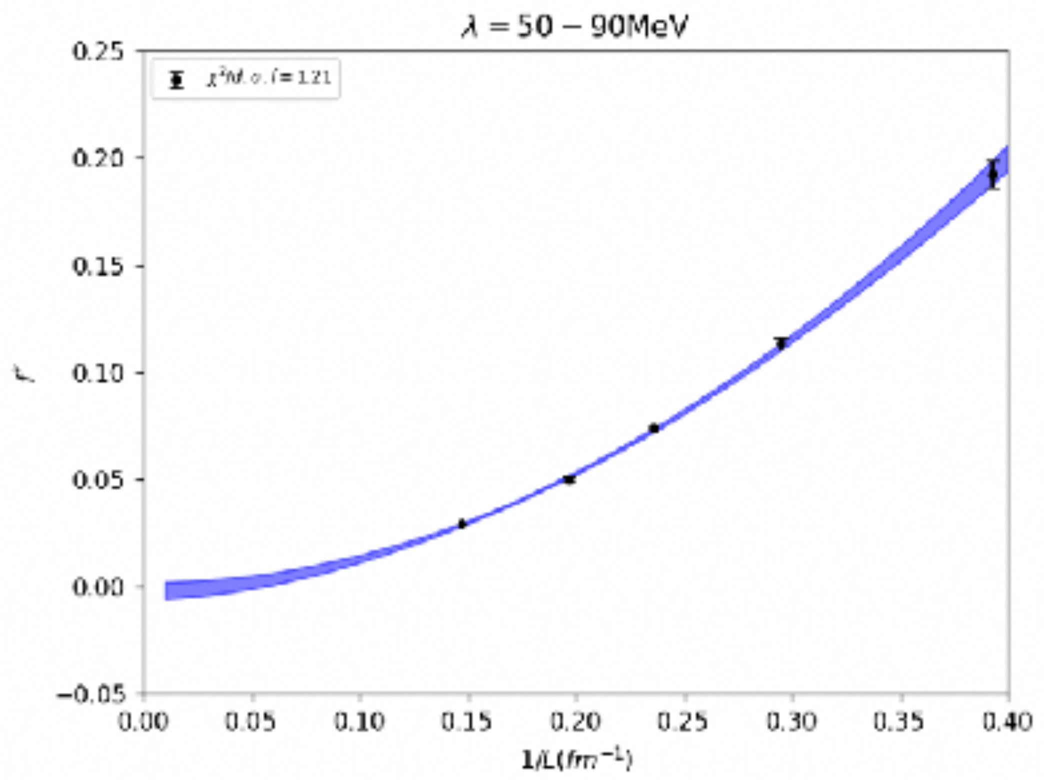
- **Spontaneous chiral symmetry breaking;**



- Dirac spectrum…

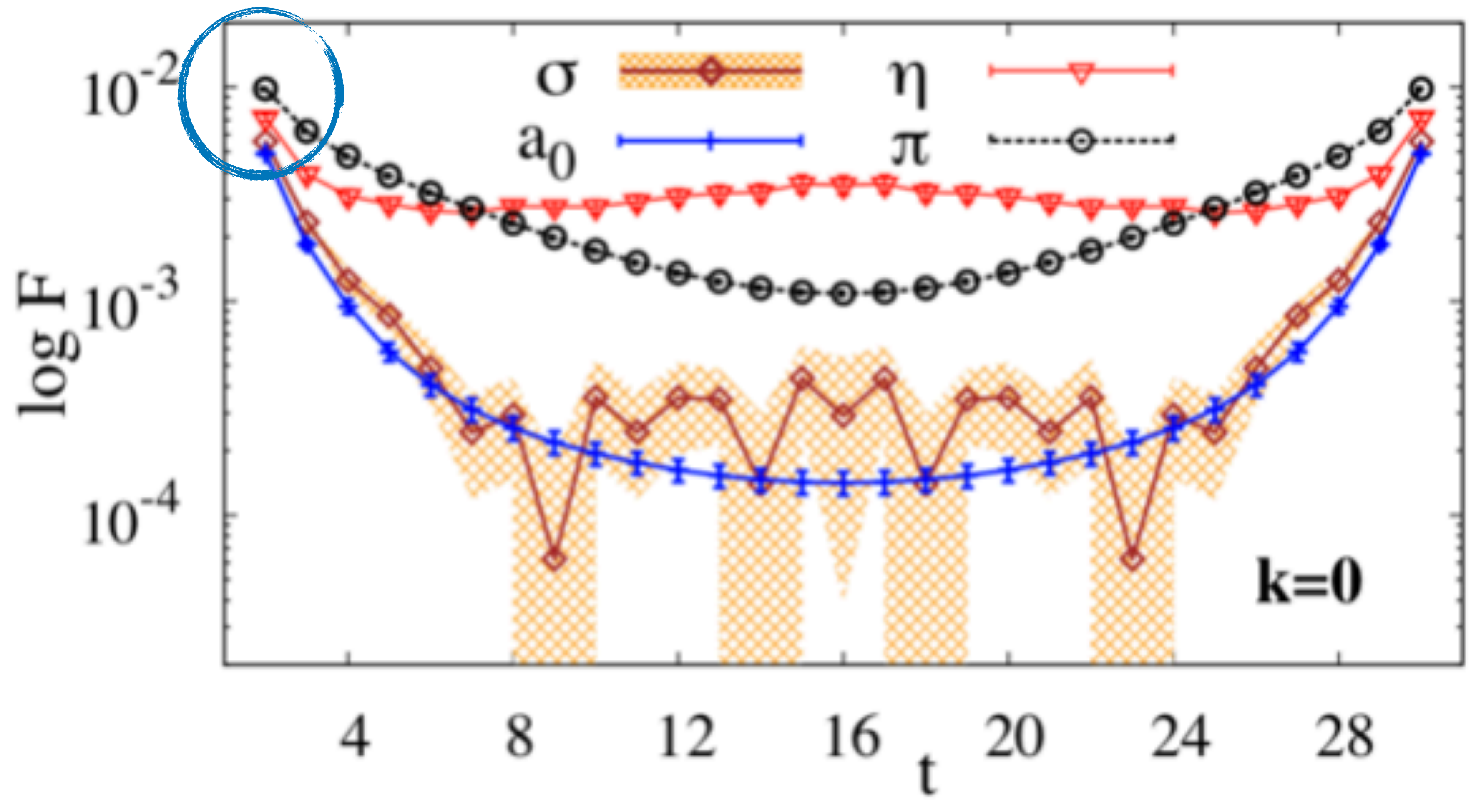


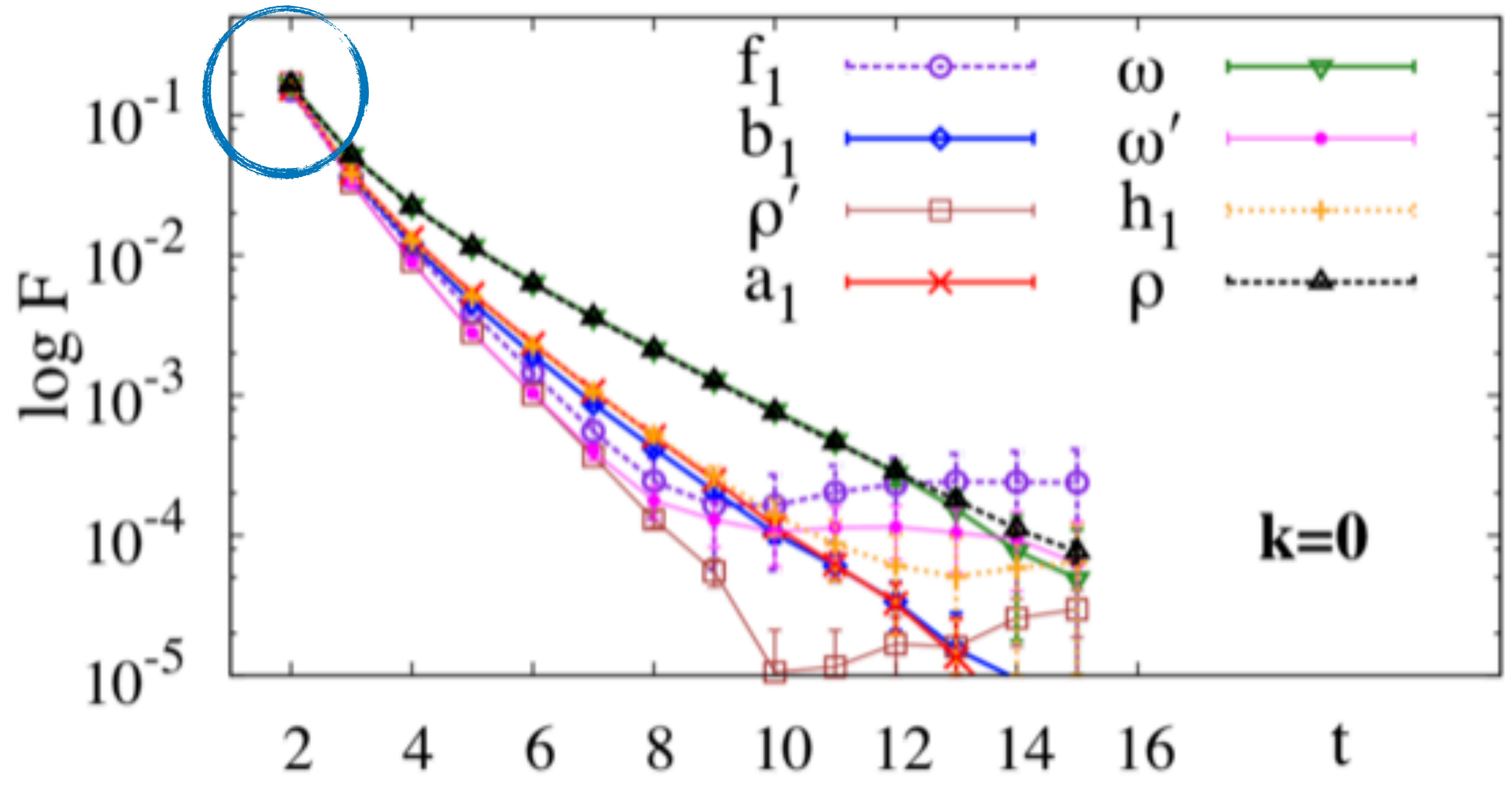
- …and its low dimension modes.



Chiral symmetry breaking

in the hadron masses





M. Denissenya, et al., PRD91(2015)034505, 1410.8751

- Based on the 2pt correlator $C_2(t,\Gamma) = \sum_{\vec{x}} \mathrm{Tr}[\langle \Gamma S(\vec{0},0;\vec{x},t)\Gamma S(\vec{x},t;\vec{0},0)\rangle]$ in the Euclidean space, the ground state mass with given interpolation field $\bar{q}_1\Gamma q_2$ can be defined by:

$$m_\Gamma \equiv \frac{1}{a}\lim_{t\to\infty} \log\frac{C_2(t,\Gamma)}{C_2(t+a,\Gamma)}.$$

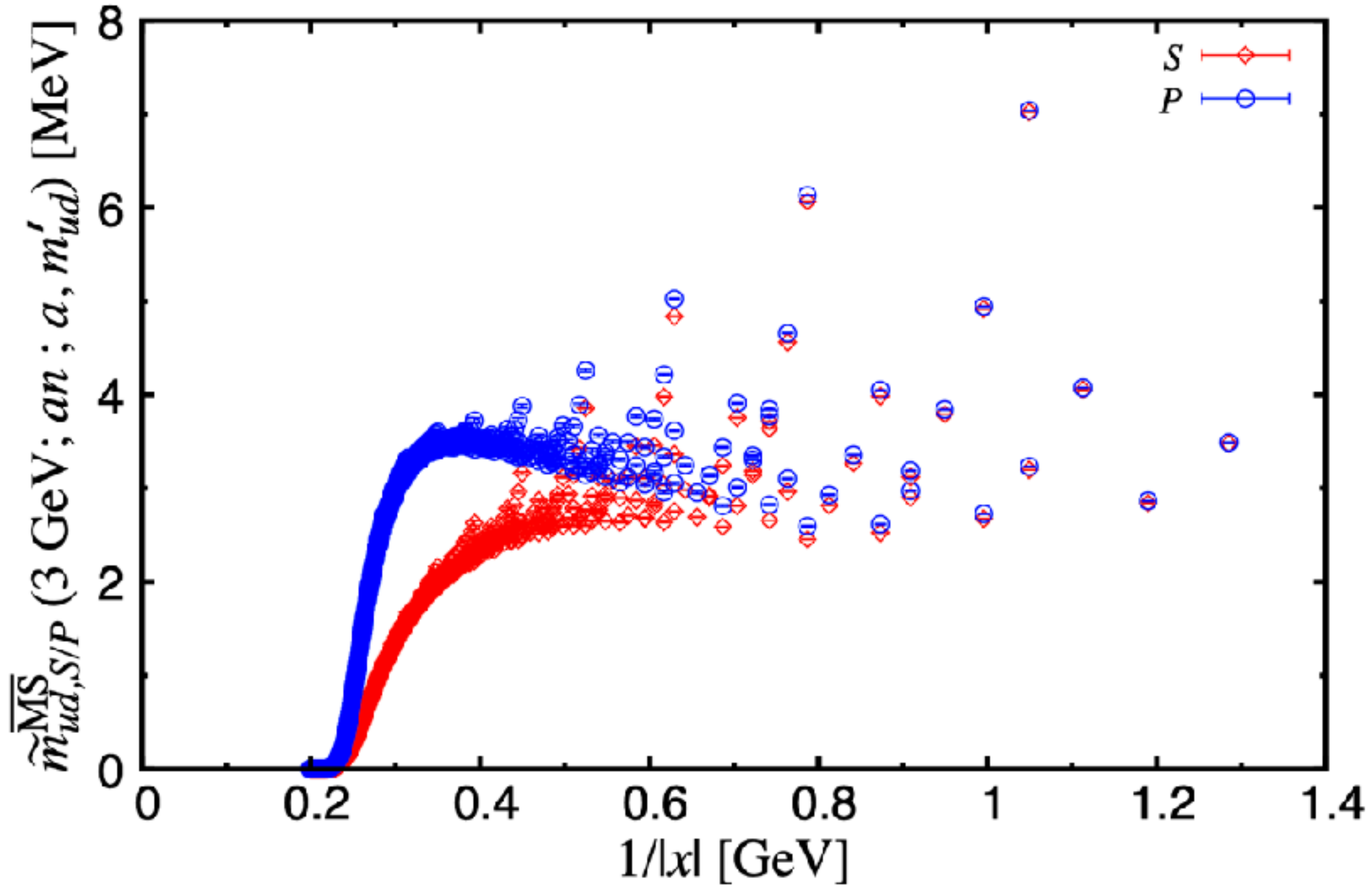
- The spontaneous chiral symmetry breaking makes

$$m_{a_0} \equiv m_I \neq m_{\gamma_5} \equiv m_\pi,$$
$$m_{a_1} \equiv m_{\gamma_5\gamma_i} \neq m_{\gamma_i} \equiv m_\rho.$$

- But the correlators seem to be closer at smaller t…

Chiral symmetry breaking

in the correlators



M. Tomii, et al., PRD99(2019)014515, 1811.11238

- Based on the 2pt correlator $C_2(|x|,\Gamma) = \mathrm{Tr}[\langle \Gamma S(0;x)\Gamma S(x;0)\rangle]$ in the Euclidean space, the renormalized quark mass can be defined by:

$$m^{\overline{\rm MS}}_{ud,S/P}(1/|x|) = \sqrt{\frac{C_2(|x|,I/\gamma_5)}{C_2^{\overline{\rm MS}}(|x|,I/\gamma_5)}}m^{\rm bare}_{ud};$$

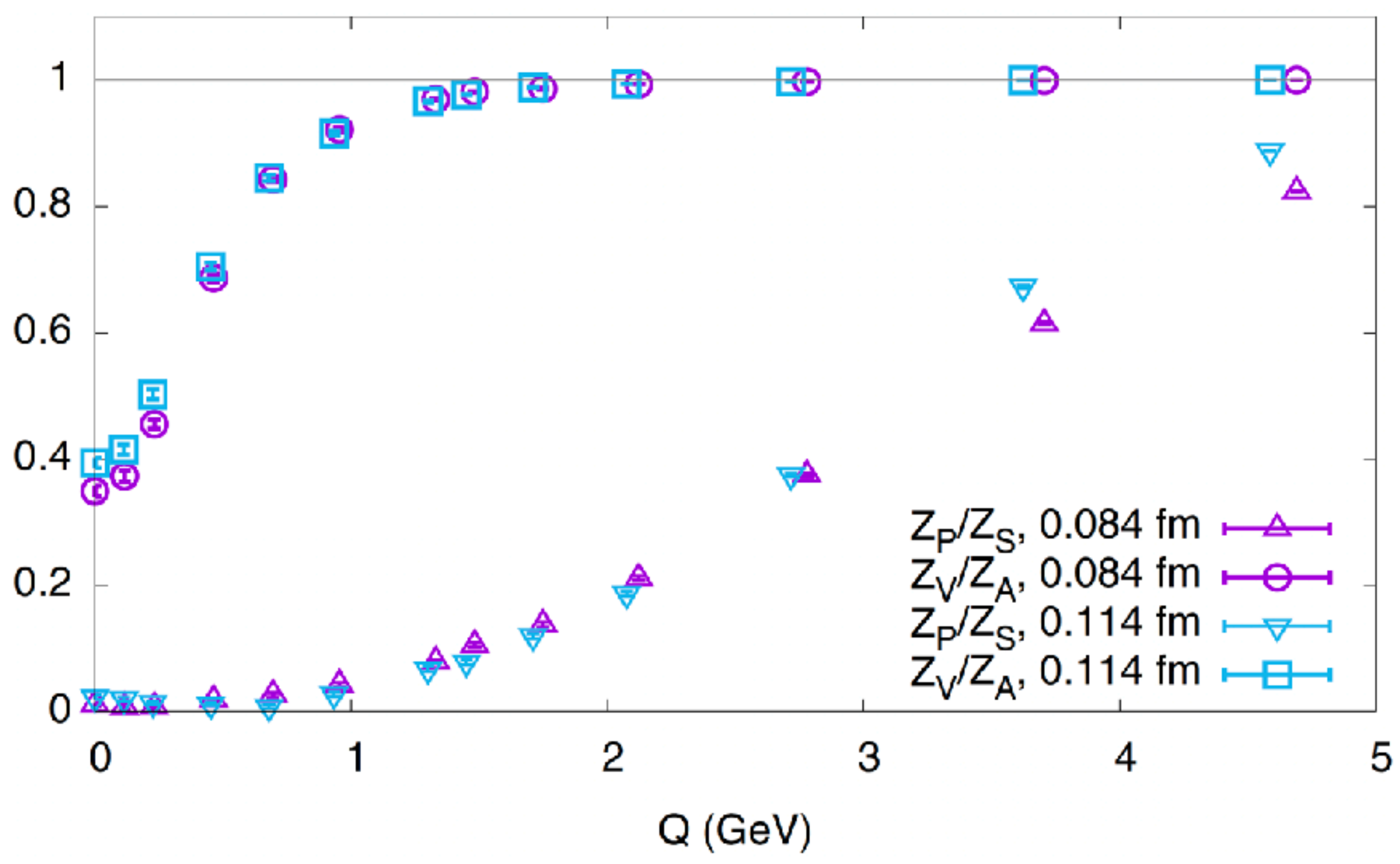
- Then we have

1. $m^{\overline{\rm MS}}_{ud,S}(1/|x|) \simeq m^{\overline{\rm MS}}_{ud,P}(1/|x|), C_2(|x|,I) \simeq C_2(|x|,\gamma_5)$, if $1/|x| \gg 1$ GeV;

2. $m^{\overline{\rm MS}}_{ud,S}(1/|x|) \neq m^{\overline{\rm MS}}_{ud,P}(1/|x|), C_2(|x|,I) \simeq C_2(|x|,\gamma_5)$, if $1/|x| \ll 1$ GeV.

Chiral symmetry breaking

in the vertex correction under the Landau gauge



- Under the Landau gauge, we can define the vertex correction at given off-shell scale as:

$$Z_\Gamma(Q) \equiv \frac{Z_q(Q)}{\mathrm{Tr}[\langle S(p)\rangle^{-1}.\langle S(p).\Gamma.S(p)\rangle.\langle S(p)\rangle^{-1}]}|_{Q=\sqrt{-p^2}}.$$

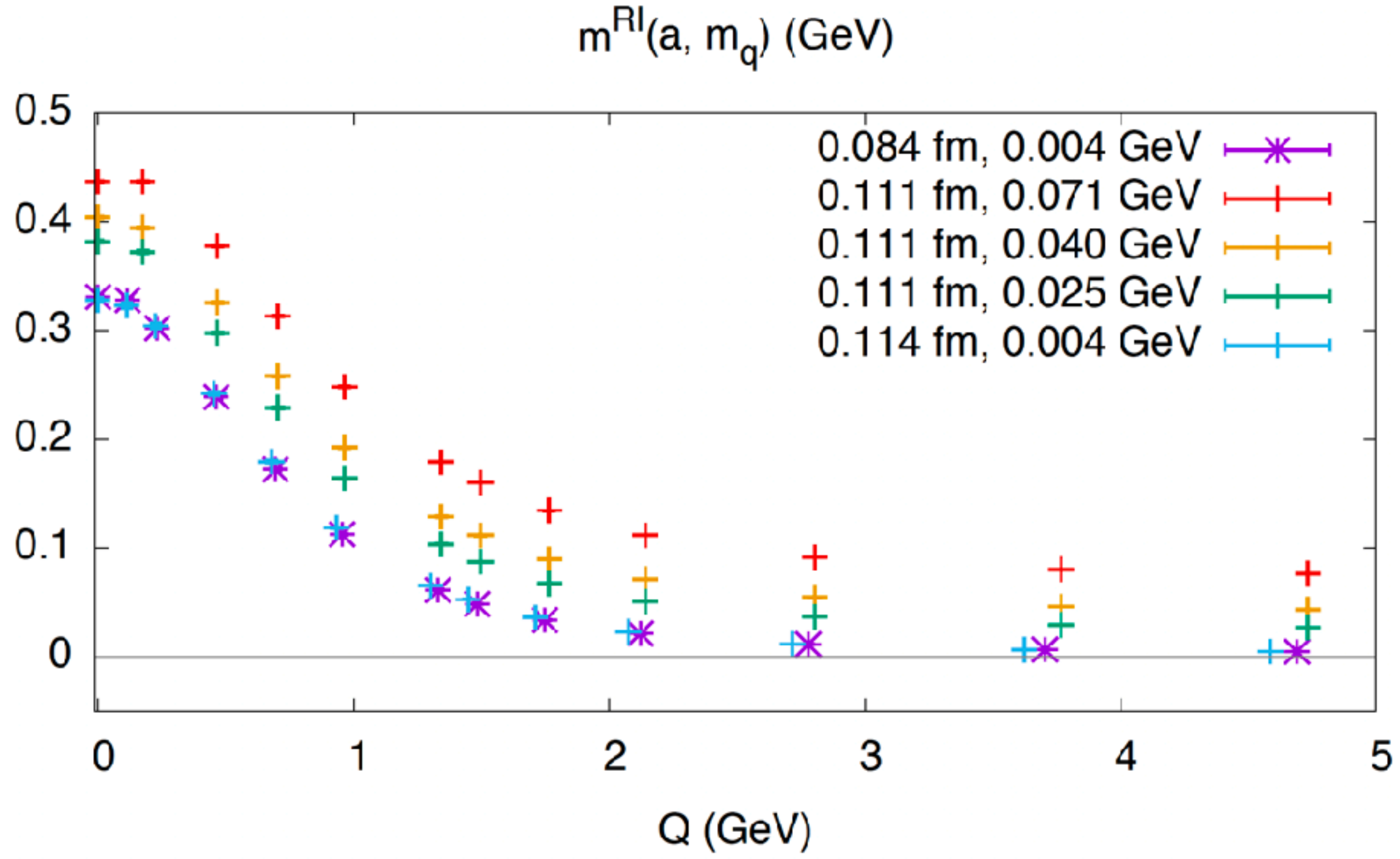
- Then we have

$Z_S \neq Z_P,\, Z_V \neq Z_A$, if Q is small.

- And we can see $Z_P = Z_m^{-1}$ approaches zero when $Q \to 0$.

Chiral symmetry breaking

dynamical quark mass under the Landau gauge



YBY, χQCD, Lattice2019 001, 2003.12914

L. Chang et.al., PRD104(2021)094509, 2105.06596

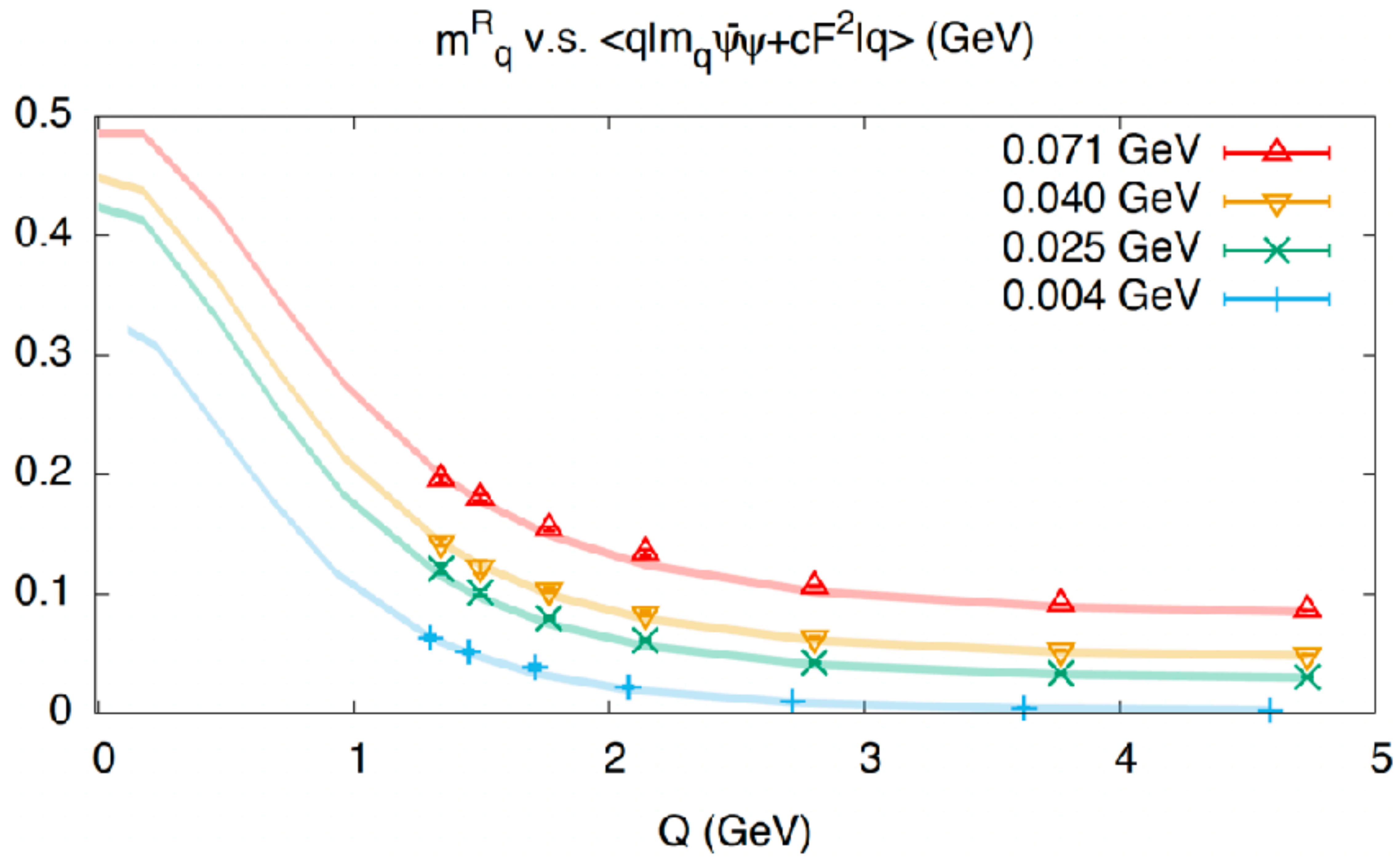
- If we define the mass renormalization constant through $m^{RI}(Q) = \frac{Tr[\langle S(p)\rangle^{-1}]}{Z_q(Q)}|_{Q=\sqrt{-p^2}}$, then we have $Z_p(Q)Z_m(Q) = 1$ for arbitrary quark mass and scale, and then a non-zero "dynamical mass" will appear in the renormalized quark mass

$$m_q^R(a, m_q; Q) = Z_m m_q^{\text{bare}}(a, m_q).$$

- It is an important feature in the DSE approach to understand the IR physics of QCD.
- But where does the feature come from?

Chiral symmetry breaking

dynamical quark mass and trace anomaly

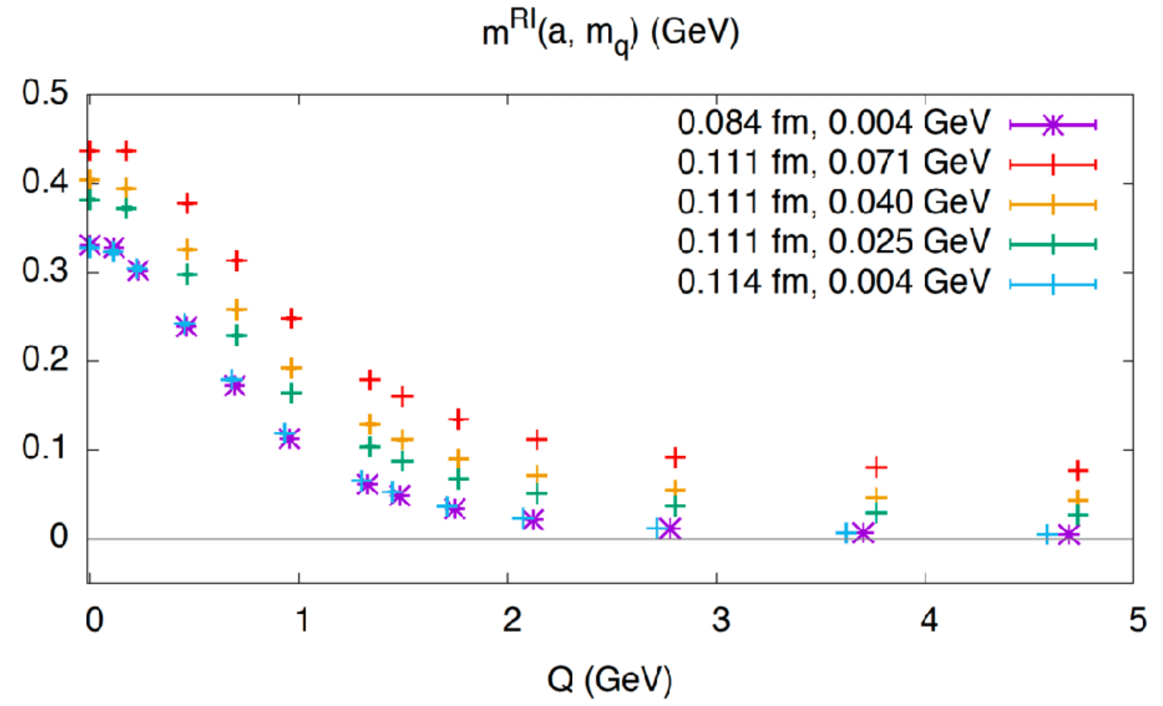


YBY, χQCD, Lattice2019 001, 2003.12914

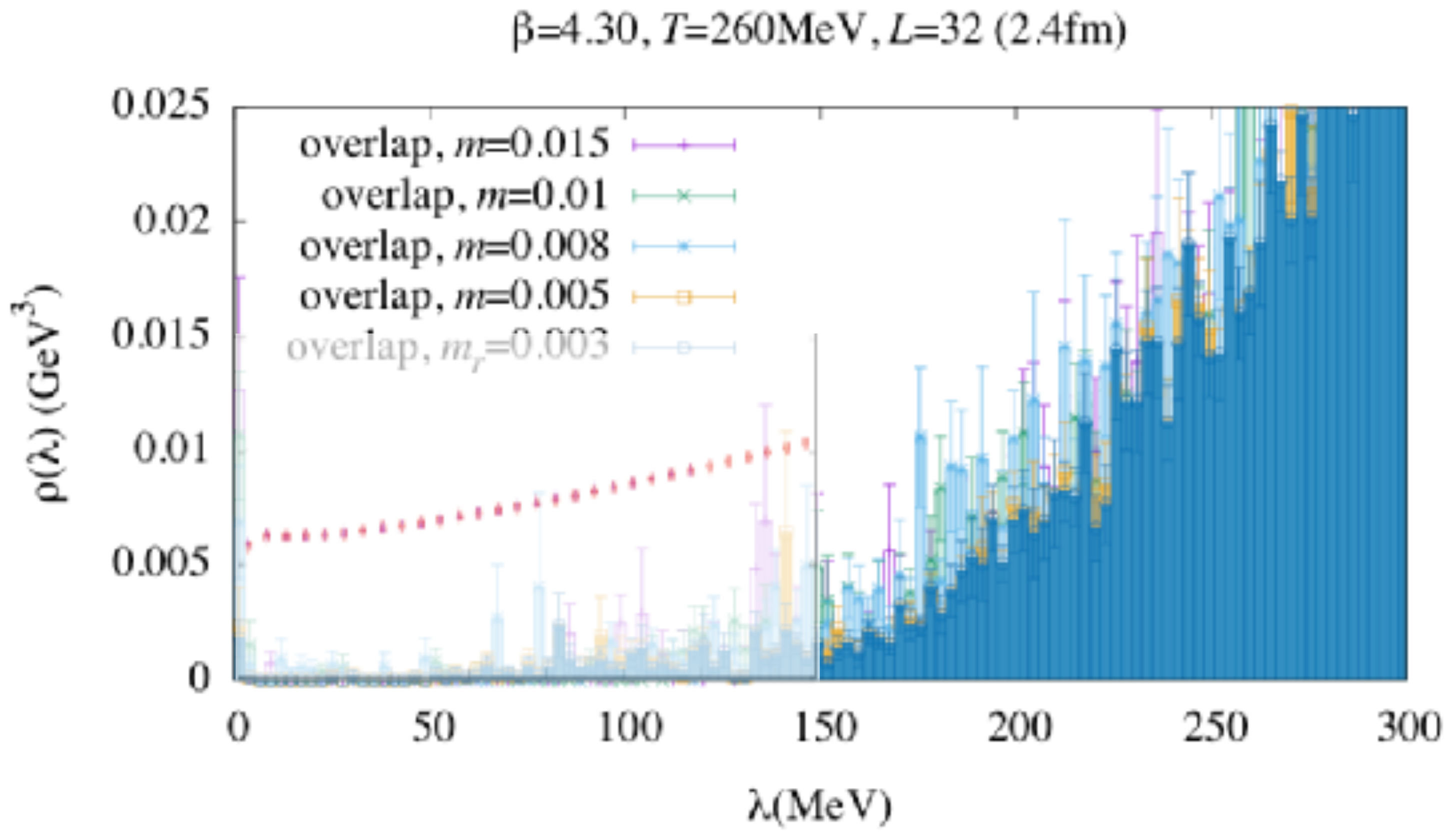
- If we compare $m_q^R(a, m_q; Q) = Z_m m_q^{\text{bare}}(a, m_q)$ with $\langle q\,|\,m_q\bar\psi\psi - \frac{\beta}{2g}F^2\,|\,q\rangle$, they are somehow close to each other at large Q.
- But it would not be a well-defined comparison and requires further investigation.
- Let us consider the problem in another way, through the Dirac spectrum…

Outline

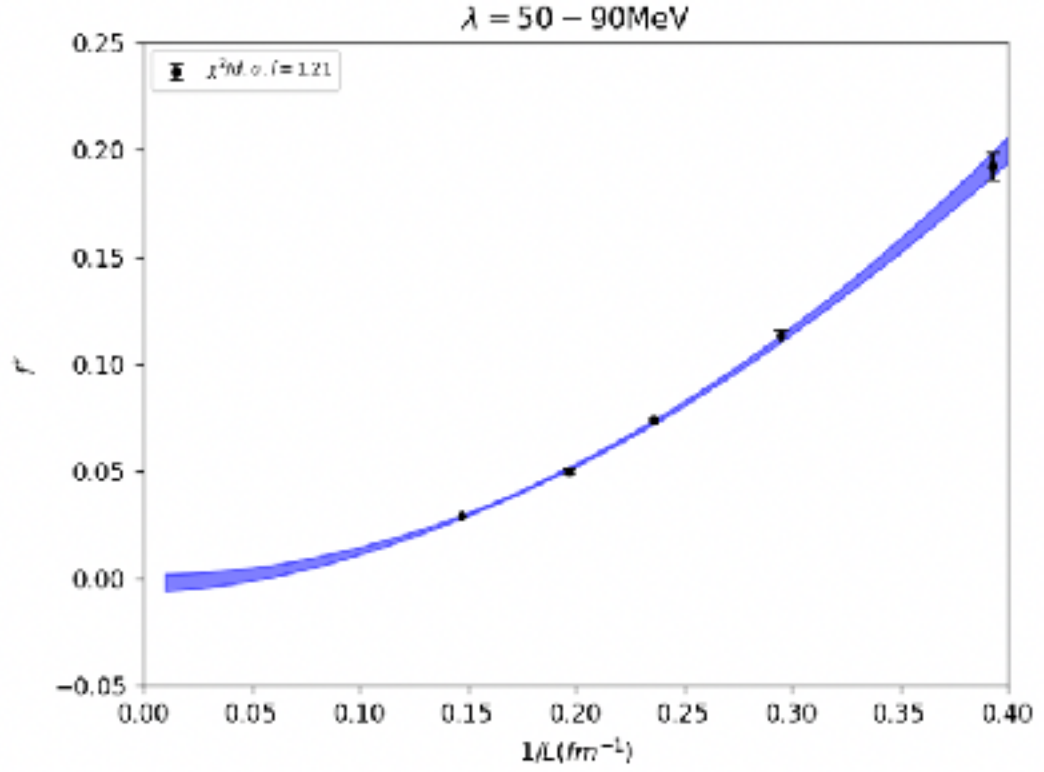
- Spontaneous chiral symmetry breaking;



- **Dirac spectrum…**



- …and its low dimension modes.



Dirac spectrum

and Ginsburg-Wilson fermion

- The overlap fermion operator satisfies the Ginsburg-Wilson,

$$\gamma_5 D_{ov} + D_{ov}\gamma_5 = \frac{a}{\rho} D_{ov}\gamma_5 D_{ov}$$

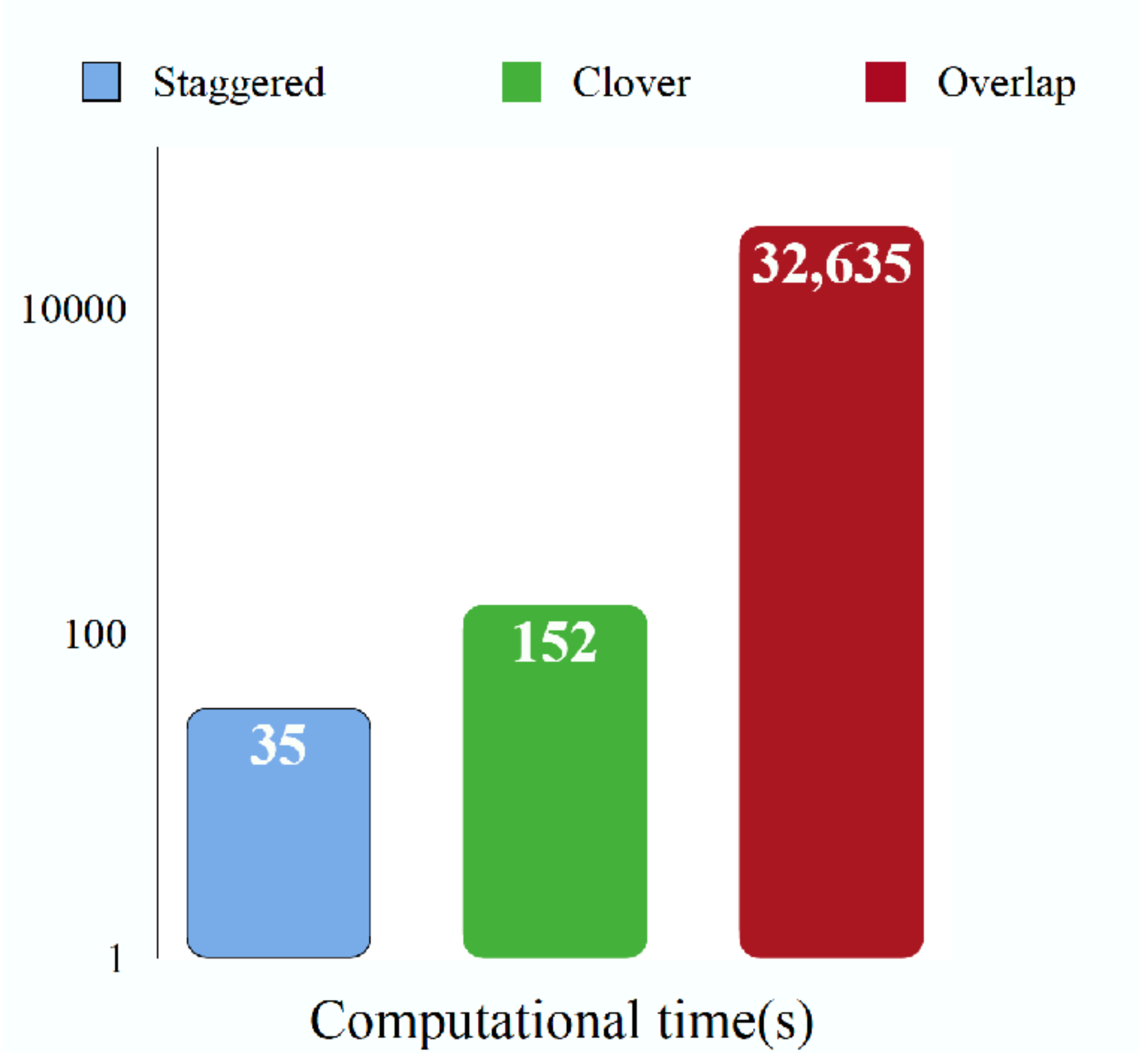
- It can be rewritten into

$$D_{ov}^{-1}\gamma_5 + \gamma_5 D_{ov}^{-1} = \frac{a}{\rho}\gamma_5, \quad (D_{ov}^{-1} - \frac{1}{2\rho})\gamma_5 + \gamma_5(D_{ov}^{-1} - \frac{1}{2\rho}) = 0$$

- Thus the chiral fermion operator satisfying $\gamma_5 D_c = -D_c\gamma_5$ can be defined through the overlap fermion operator:

$$D_c + m_q = \frac{D_{ov}}{1 - \frac{1}{2\rho}D_{ov}} + m_q, \; D_{ov} = \rho(1 + \gamma_5\epsilon_{ov}(\rho)) \,.$$

- D_c has infinite eigenvalue (for the case with non-zero topological charge) and then is not well-defined, but $\dfrac{1}{D_c + m_q}$ is finite with non-zero m_q.



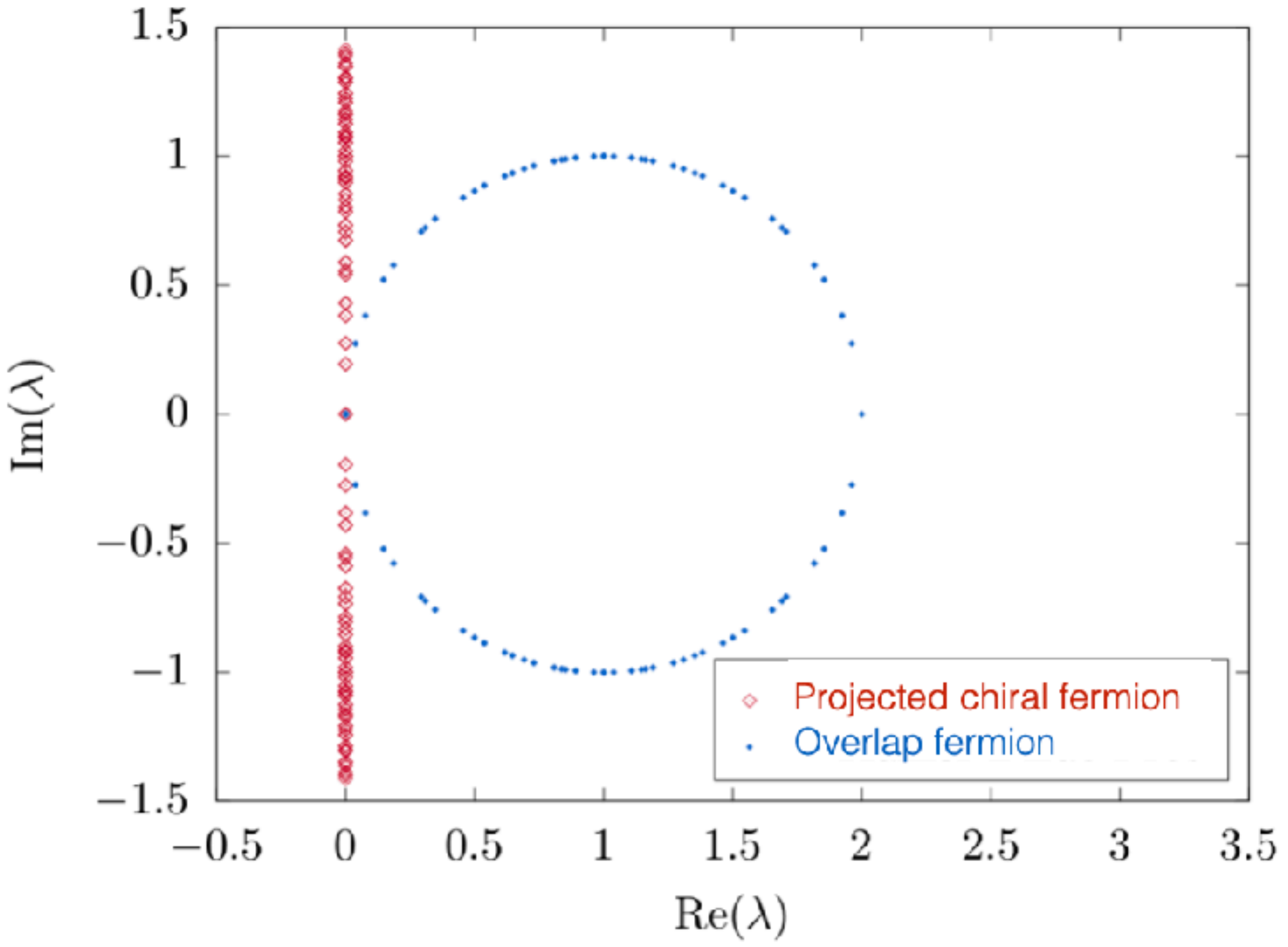
Dirac spectrum

and chiral symmetry breaking

- The non-zero and finite eigenvalues of the overlap fermion are all paired, $D_c v = \mathrm{i}\lambda_c v,\ D_c\gamma_5 v = -\mathrm{i}\lambda_c\gamma_5 v$

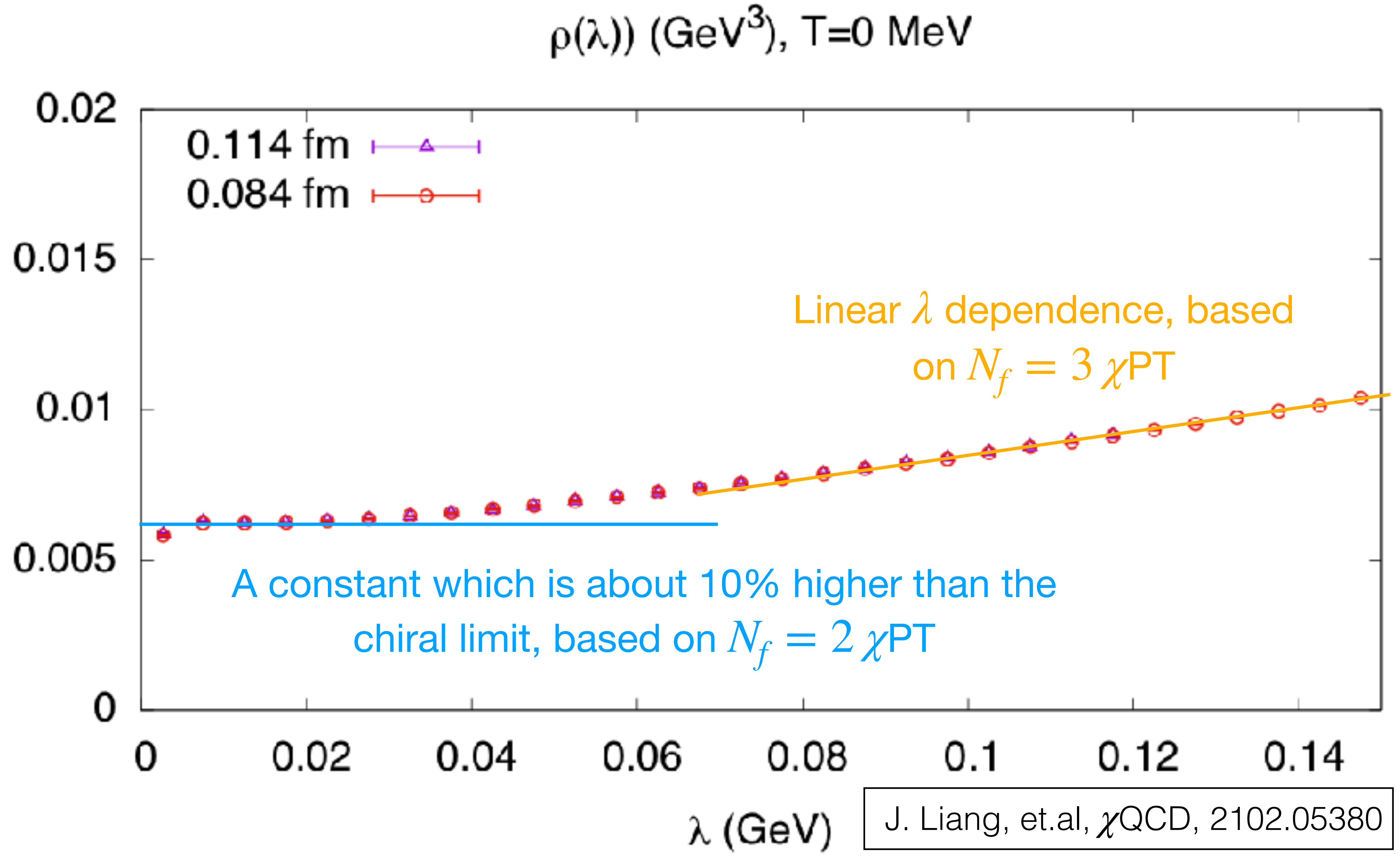
- Thus if $|\lambda_c|$ has a lower band $\lambda_0 \gg m_q$, then $\gamma_5 \frac{1}{D_c + m_q} + \frac{1}{D_c + m_q}\gamma_5 \propto \frac{m_q}{\lambda_0^2} \propto \frac{m_q}{\lambda_0}\frac{\gamma_5}{D_c + m_q}$

- and then the chiral symmetry restores in such a case.

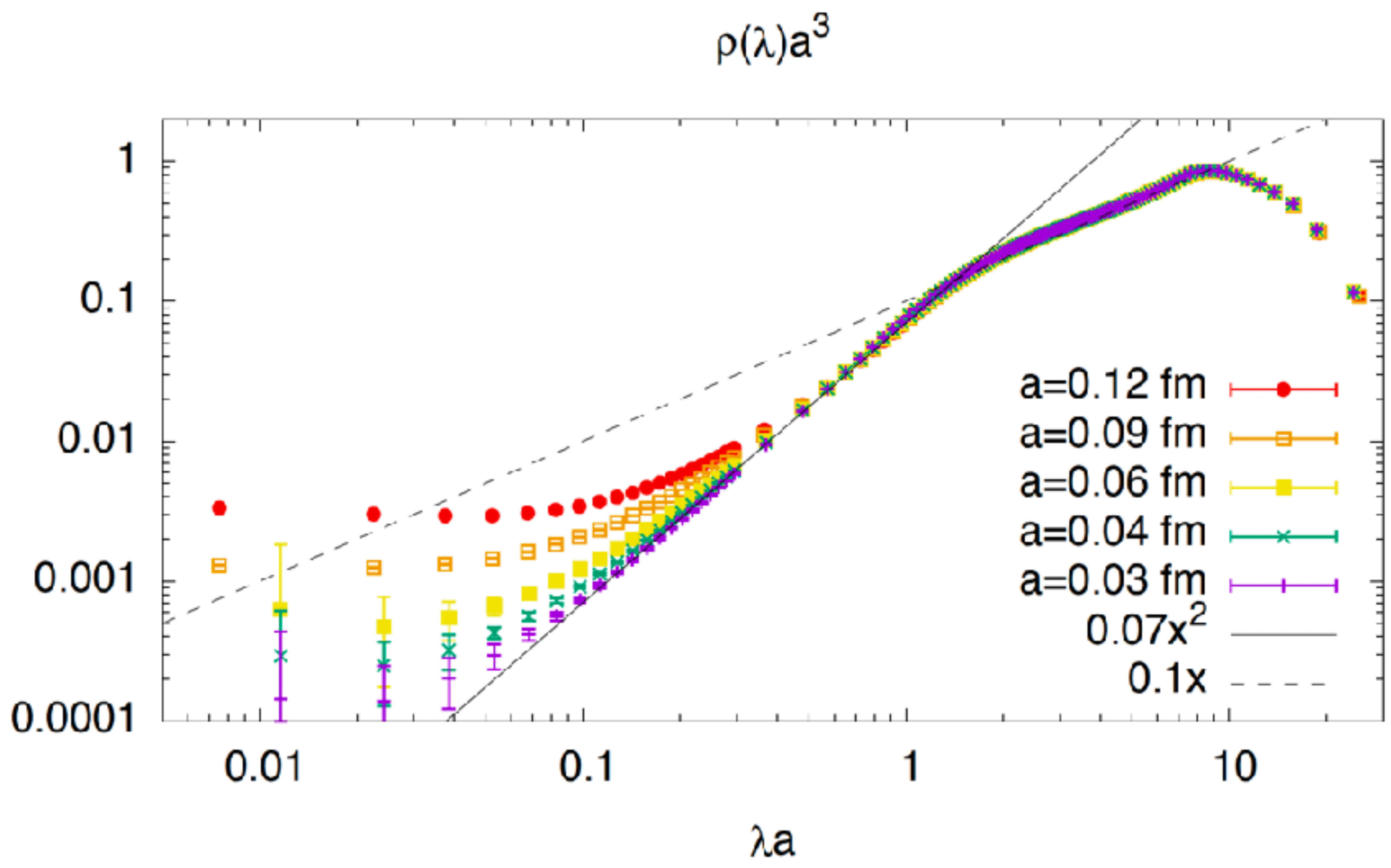


Dirac spectrum

Small and large λ region



J. Liang, et.al, χQCD, 2102.05380



$$\rho(\lambda, V) = \frac{\Sigma}{\pi}\left(1 + \frac{N_f^2 - 4}{N_f}\frac{\lambda\Sigma}{32\pi F^4}\right) + \mathcal{O}\left(\frac{1}{\sqrt{V}}, m_q^{\rm sea}, \lambda^2\right)$$

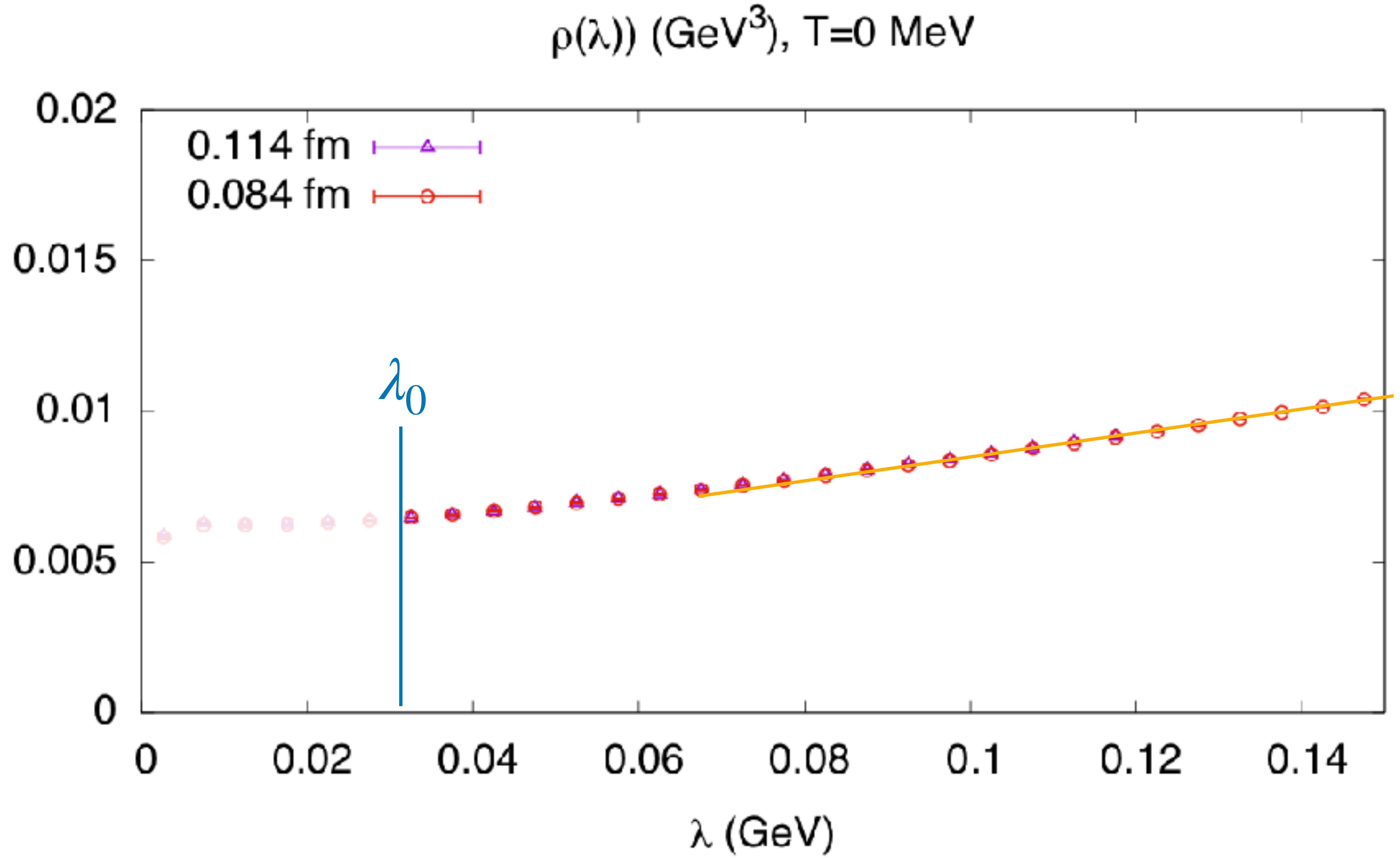
P. H. Damgaard and H Fukaya, JHEP01(2009),052, 0812.2797

- $\rho(\lambda)$ has $\dfrac{1}{a^2}$ and $\dfrac{1}{a}$ UV divergences at large λ
- Only the small λ region is physical.

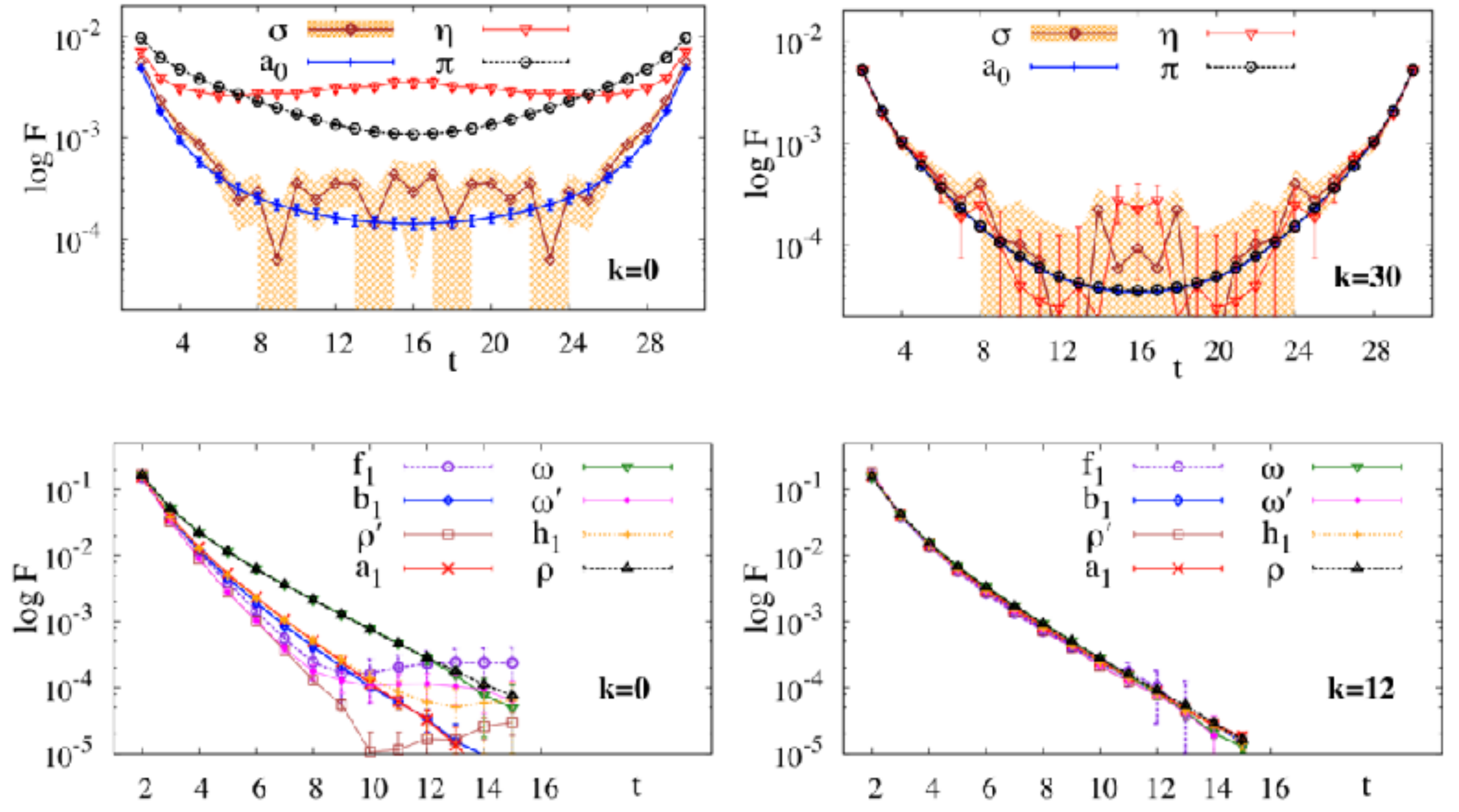
Dirac spectrum

with hard cutoff at small eigenvalues

$$S_h(y,x;\lambda_c) = \langle \bar{\psi}(x)\psi(y)\rangle - \sum_{-\lambda_0<\lambda<\lambda_0} v_\lambda(x)\frac{1}{i\lambda+m}v_\lambda^\dagger(y)$$



YBY, χQCD, Lattice2019 001, 2003.12914

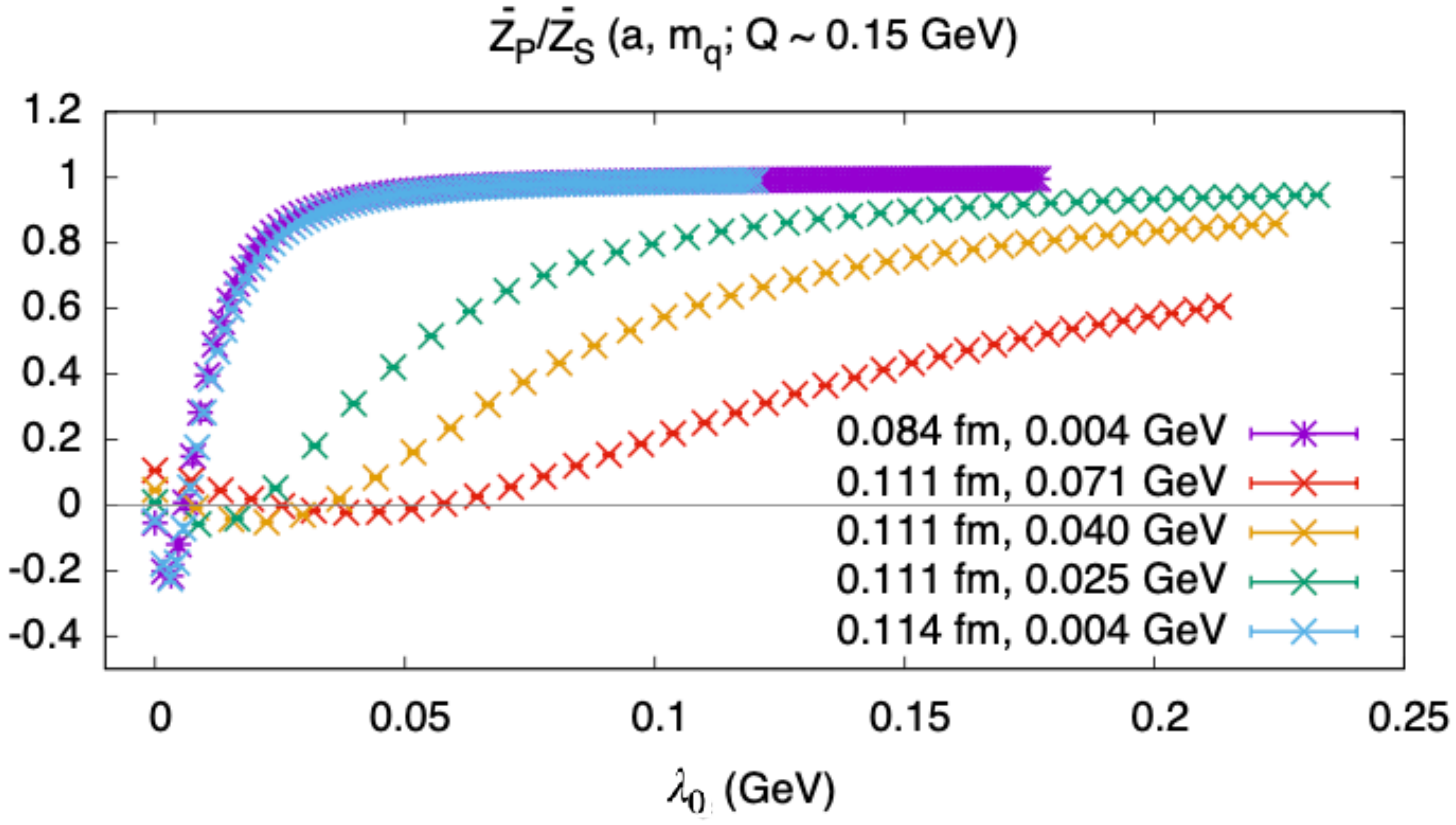


M. Denissenya, et al., PRD91(2015)034505, 1410.8751

If we put a hard cutoff λ_0 at small λ, then one would expect that the chiral condensate "vanishes", and chiral symmetry "restores" with a "spin symmetry".

Dirac spectrum

with hard cutoff at small eigenvalues



YBY, χQCD, Lattice2019 001, 2003.12914

- We define the subtracted propagator as

$$S_h(p,x;\lambda_c) = \sum_y e^{ipy}(\langle\bar{\psi}(x)\psi(y)\rangle - \sum_{-\lambda_0<\lambda<\lambda_0} \mathrm{v}_\lambda(x)\frac{1}{i\lambda+m}\mathrm{v}_\lambda^\dagger(y));$$

- And then the subtracted vertex correction is defined by

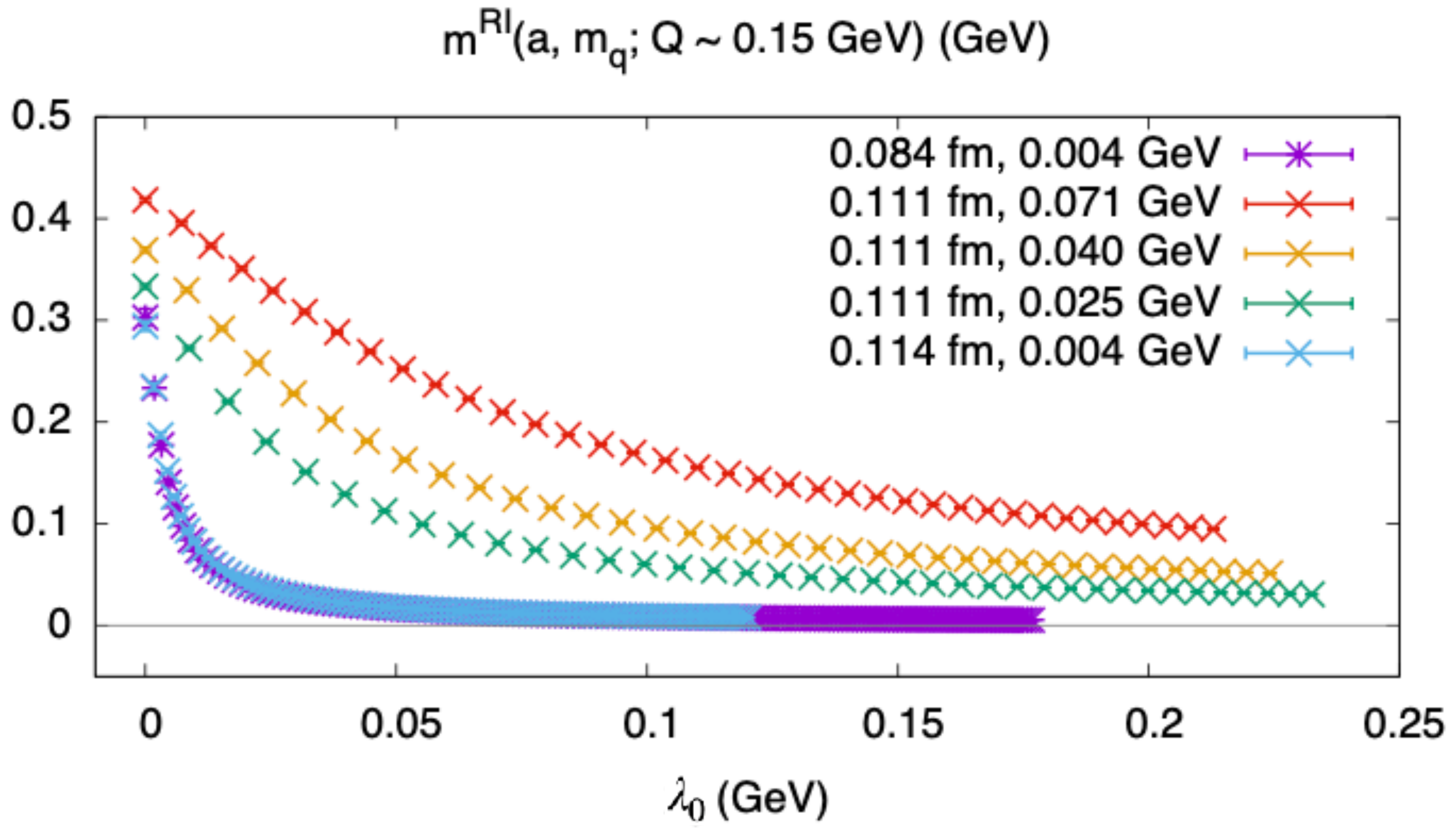
$$\frac{Z_p}{Z_s}(\lambda_0) = \frac{Tr[\langle S_h\rangle^{-1}.\langle S_h.\gamma_5.S_h\rangle.\langle S_h\rangle^{-1}]}{Tr[\langle S_h\rangle^{-1}.\langle S_h.S_h\rangle.\langle S_h\rangle^{-1}]}(\lambda_0).$$

- One can see that the chiral symmetry restores after the low mode with $\lambda < 10m_q$ are subtracted.

$$\gamma_5\frac{1}{D_c+m_q} + \frac{1}{D_c+m_q}\gamma_5 \propto \frac{m_q}{\lambda_0^2} \propto \frac{m_q}{\lambda_0}\frac{\gamma_5}{D_c+m_q}$$

Dirac spectrum

with hard cutoff at small eigenvalues



YBY, χQCD, Lattice2019 001, 2003.12914

- Similar feature appears in the "dynamical quark mass", when we define the subtracted quark mass as

$$m^{RI}(Q;\lambda_0) = \frac{Tr[\langle S_h(p;\lambda_0)\rangle^{-1}]}{Z_q(Q;\lambda_0)}|_{Q=\sqrt{-p^2}}.$$

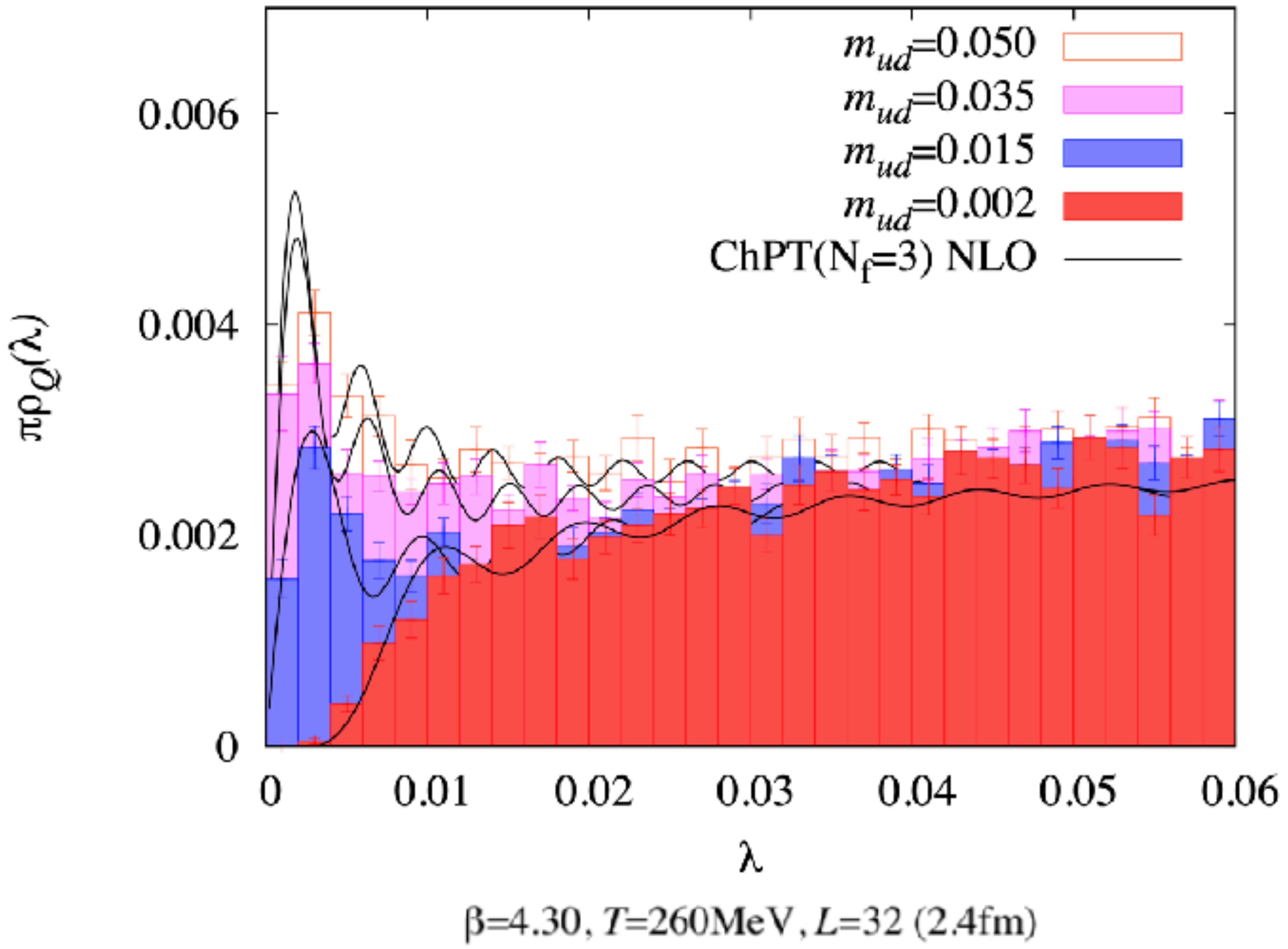
- Subtracted quark mass $m^{RI}(m_q;Q;\lambda_0) \propto m_q$ when the low mode with $\lambda < 10m_q$ are subtracted.

- It is consistent with the chiral symmetry "restoring" picture in the other examples.

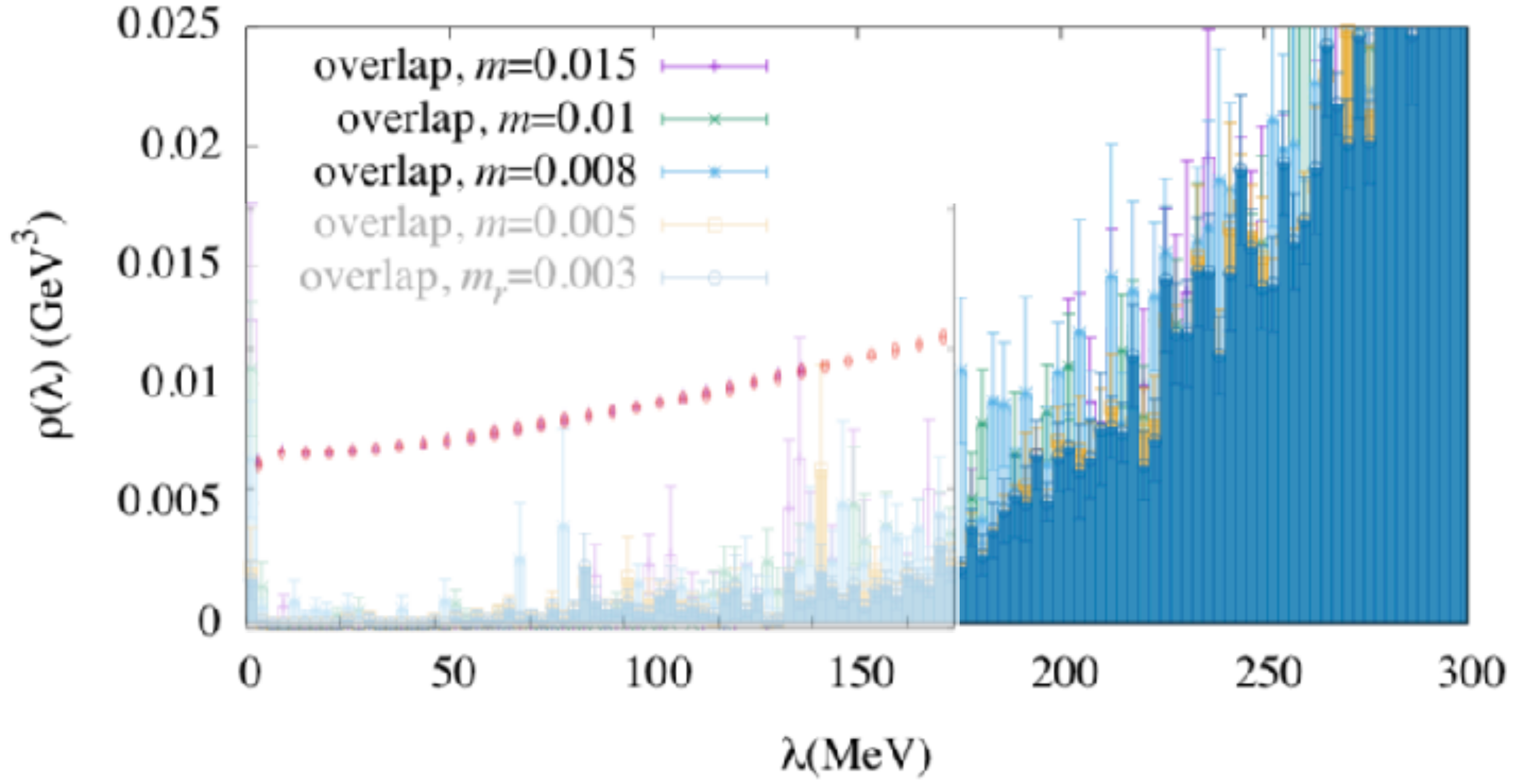
$$\gamma_5\frac{1}{D_c+m_q}+\frac{1}{D_c+m_q}\gamma_5 \propto \frac{m_q}{\lambda_0^2} \propto \frac{m_q}{\lambda_0}\frac{\gamma_5}{D_c+m_q}$$

Dirac spectrum

on a lattice with small volume $L_S^3 \times L_T$



JLQCD and TWQCD, PRD83(2010), 074501, 1012.4052



S. Aoki, et.al, JLQCD, PRD103(2021), 074506, 2011.01499

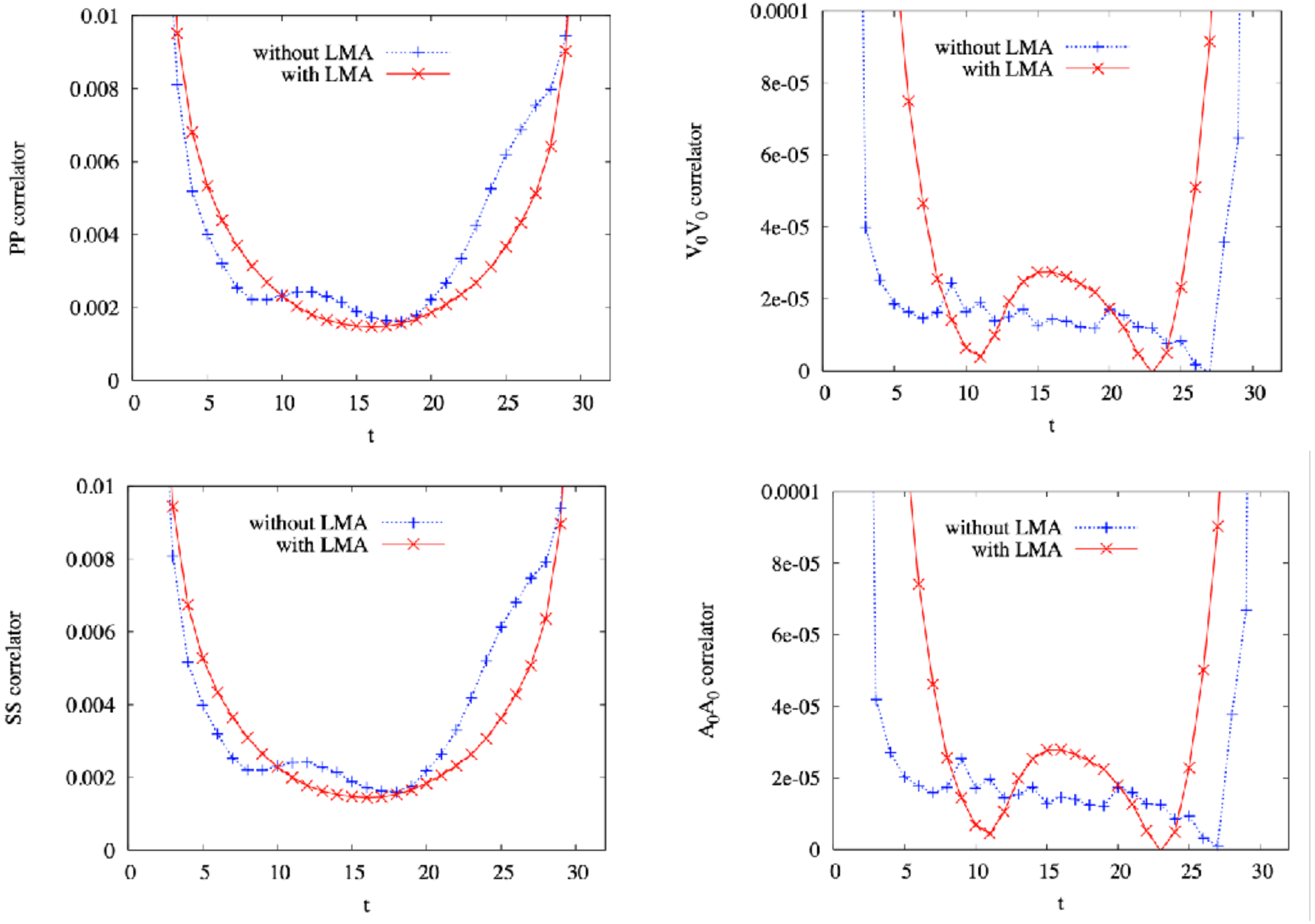
- The $\rho(\lambda)$ will be suppressed at small λ when L_S and/or $L_T = 1/T$ is small enough (*T* here corresponds the temperature):

1. $L_S < \dfrac{1}{m_\pi} \ll L_T$, ϵ-regime;
2. $L_T < \dfrac{1}{m_\pi} \ll L_S$, finite temperature regime.

- The chiral symmetry breaking are also suppressed in these two cases.

Dirac spectrum

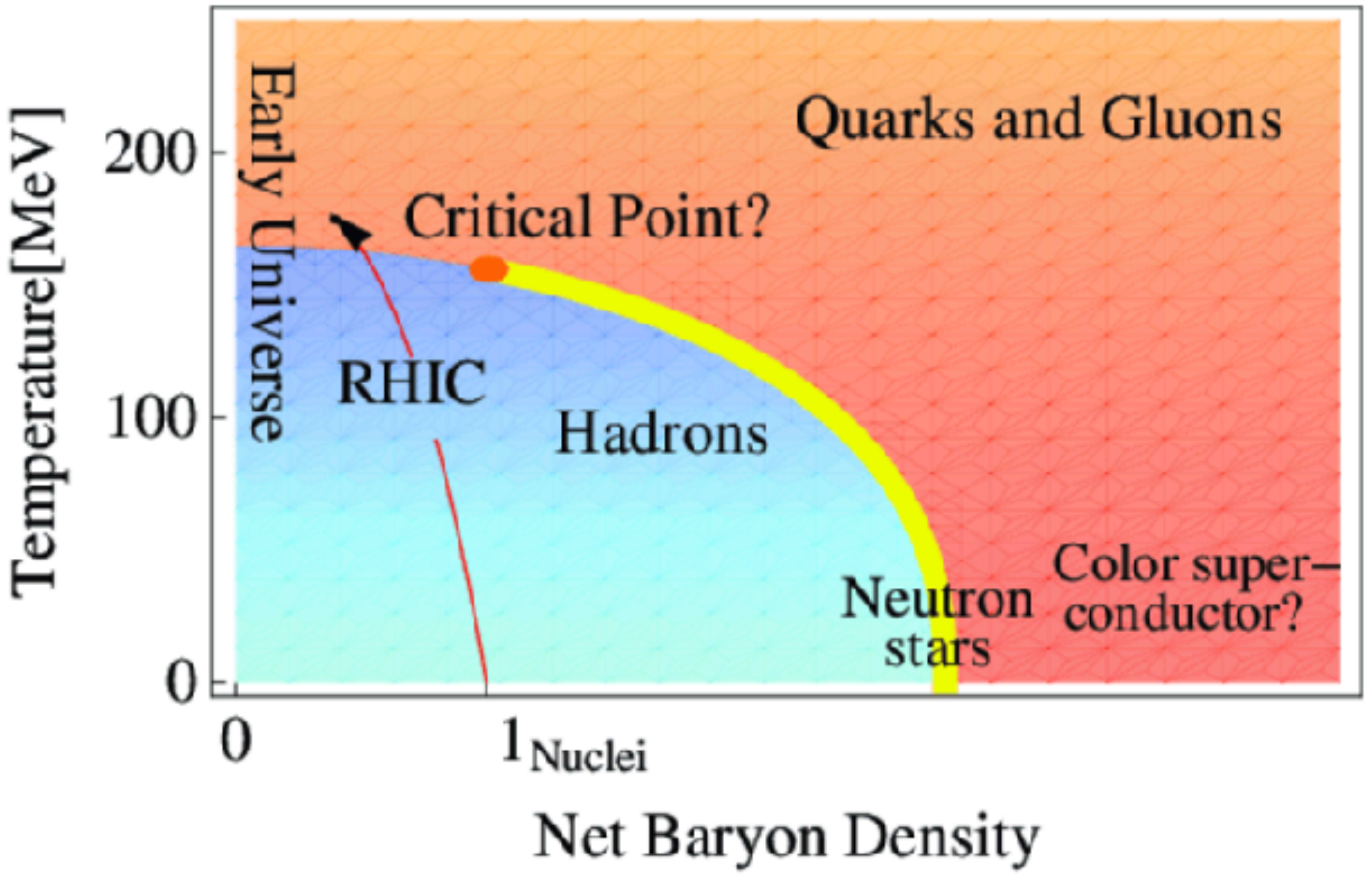
on a lattice with small spacial volume

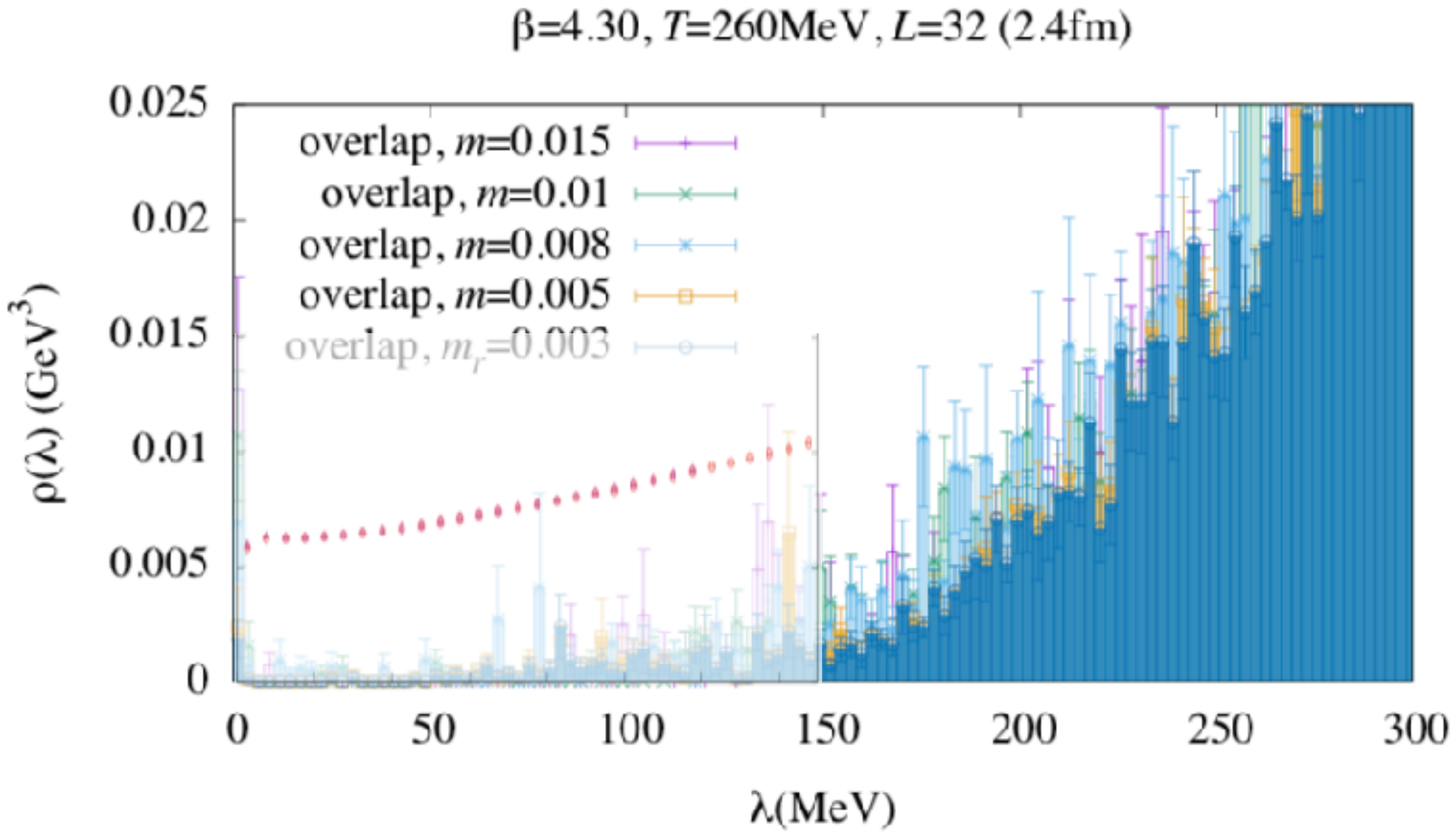


JLQCD, PRD77(2008), 074503, 0711.4965

- The result suggests that the chiral symmetry is restored in the ϵ-regime: $C_{2,P} = C_{2,S}$, $C_{2,V} = C_{2,A}$.
- The effective mass of “the vector meson” would be also small in such a case.

Dirac spectrum





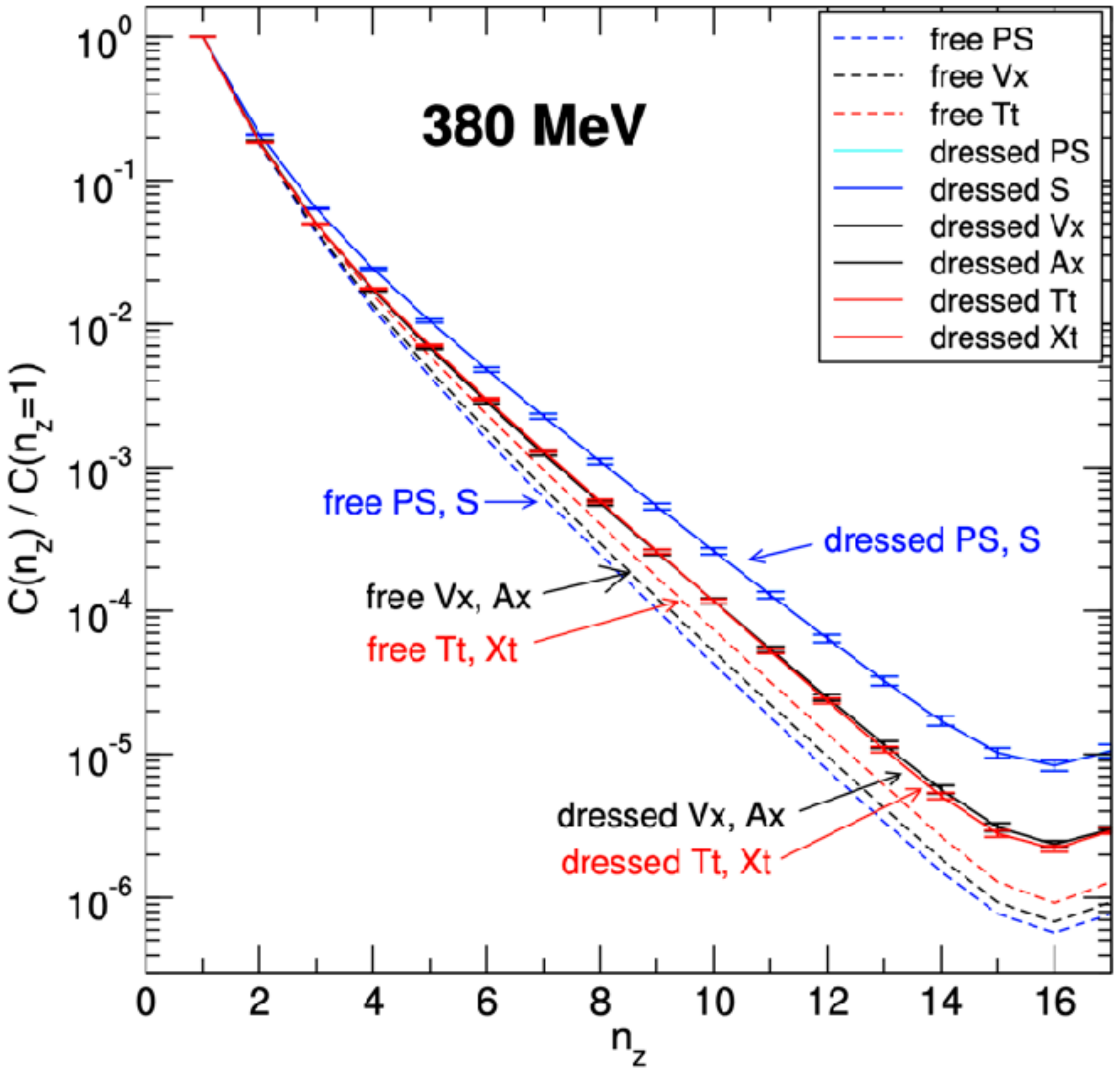
S. Aoki, et.al, JLQCD, PRD103(2021), 074506, 2011.01499

at high temperature

- Similarly, the $\rho(\lambda)$ is suppressed at small λ, when the temperature T is above the critical (cross-over) temperature ~ 150 MeV.
- The chiral condensates $\Sigma = \pi \lim_{\lambda\to 0} \lim_{V\to\infty} \rho(\lambda)$ vanishes;
- The chiral symmetry should also restores in such a case.

Dirac spectrum

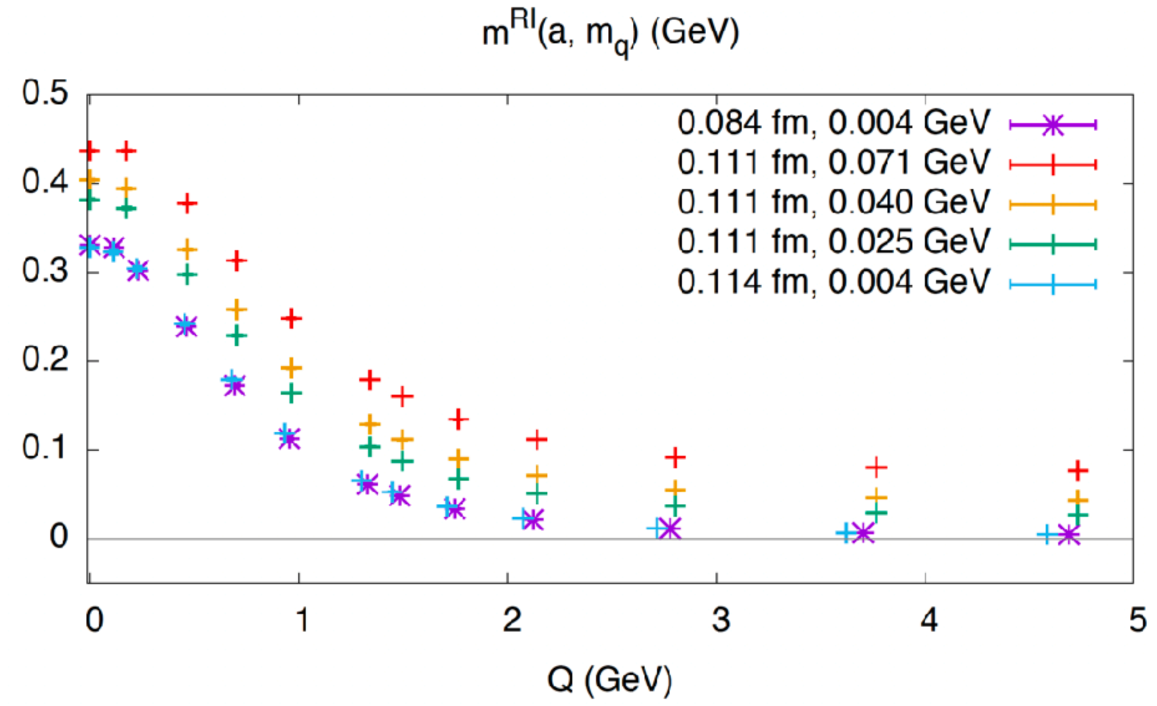
at high temperature



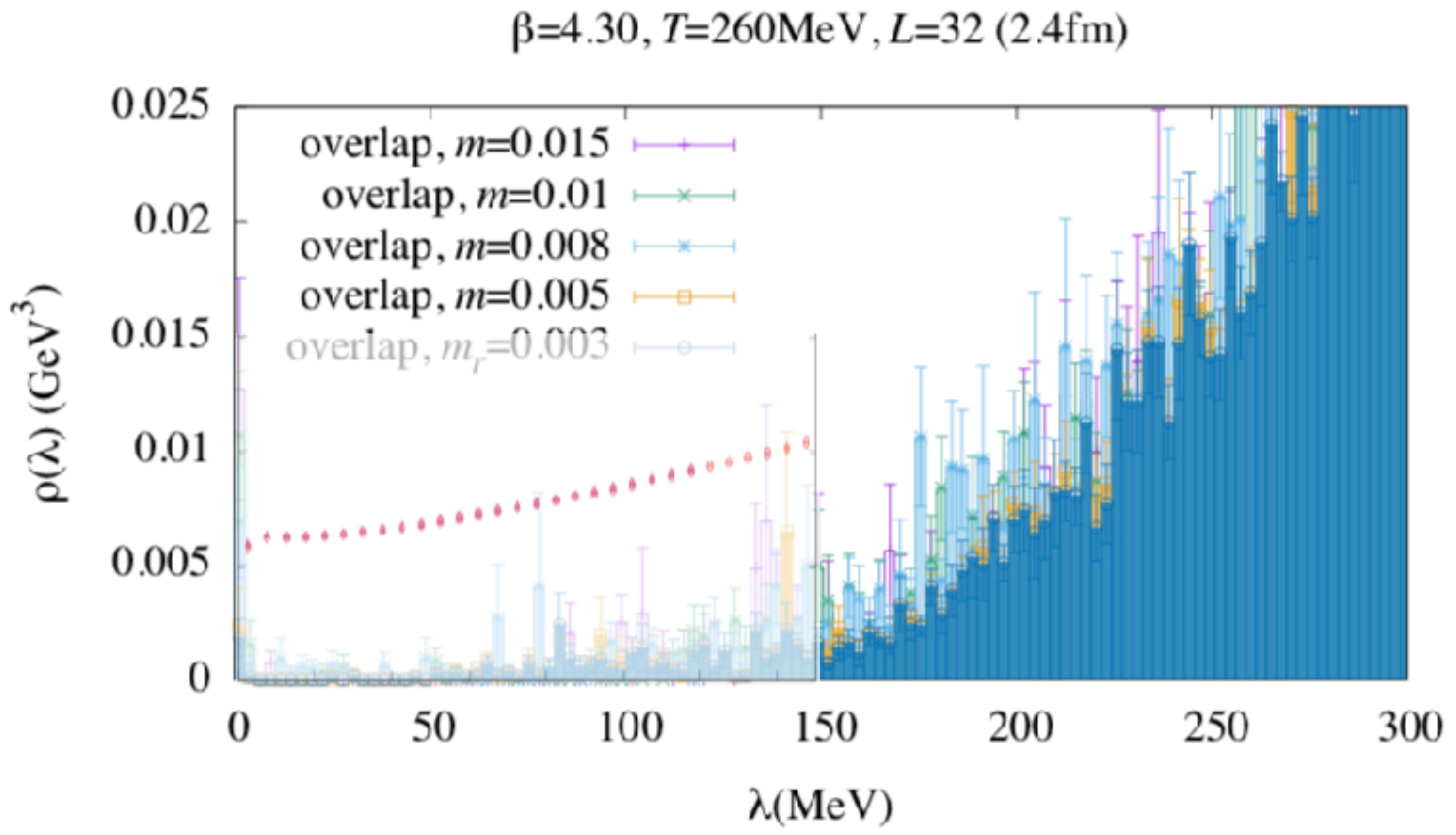
L. Ya Glozman, PRD101(2020), 074516, 1912.06505

- $32^3 \times 8,\ T = 380\ \mathrm{MeV} \sim 2.2T_c,\ N_f = 2$
- The chiral symmetry is restored, as $C_{2,P} = C_{2,S},\ C_{2,V} = C_{2,A}$.
- Even more, $C_{2,T} = C_{2,V} = C_{2,A}$.
- Chiral-spin symmetry within the $T_c - 3T_c$ intervals?

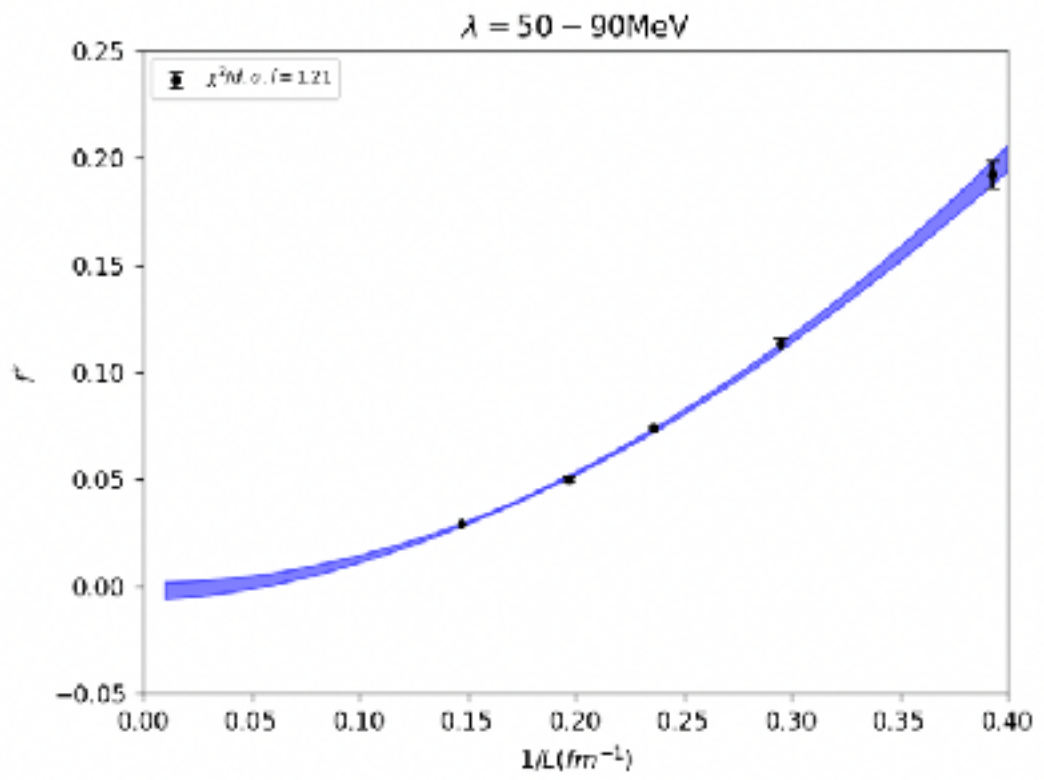
Outline



- Spontaneous chiral symmetry breaking;



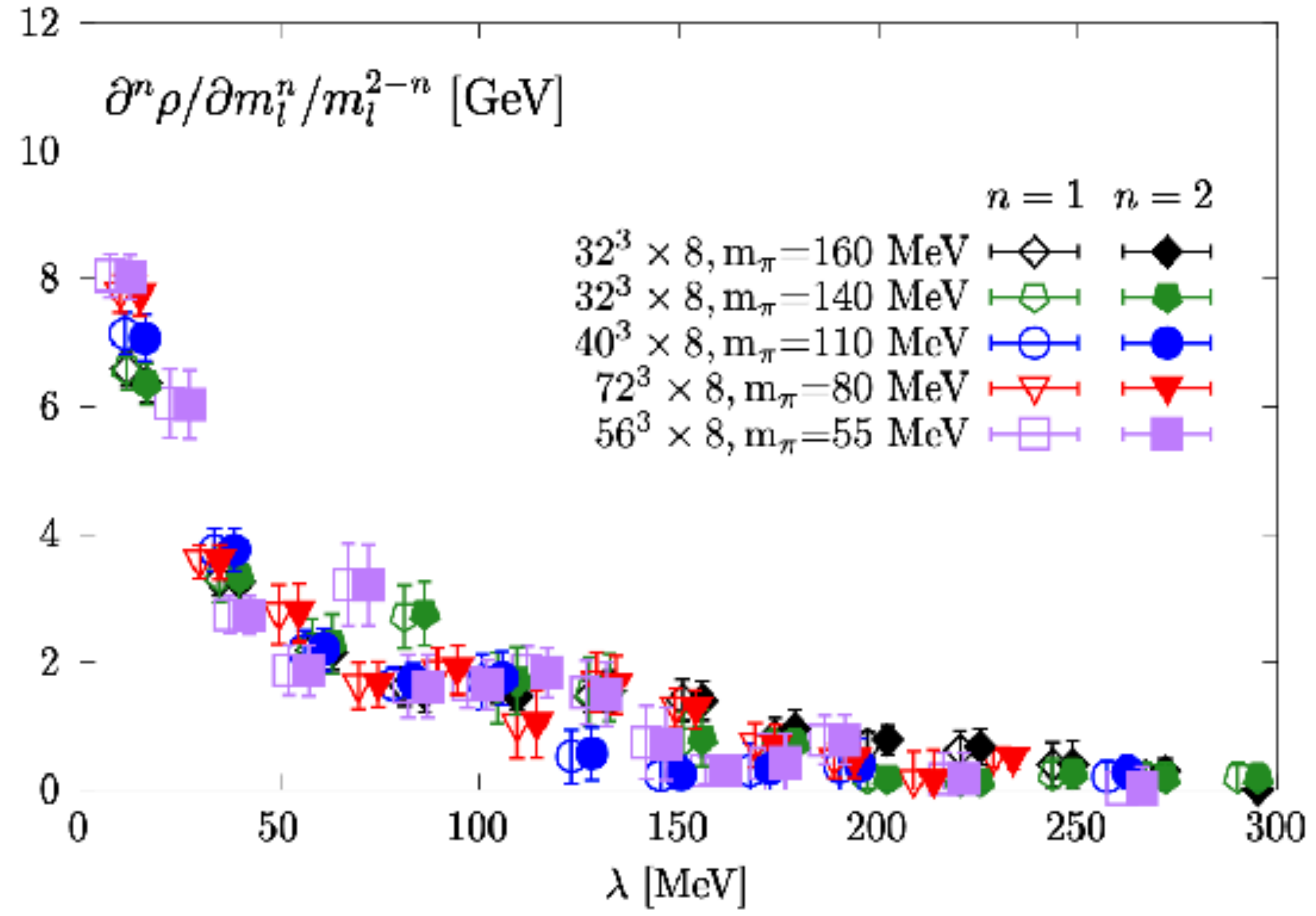
- Dirac spectrum…

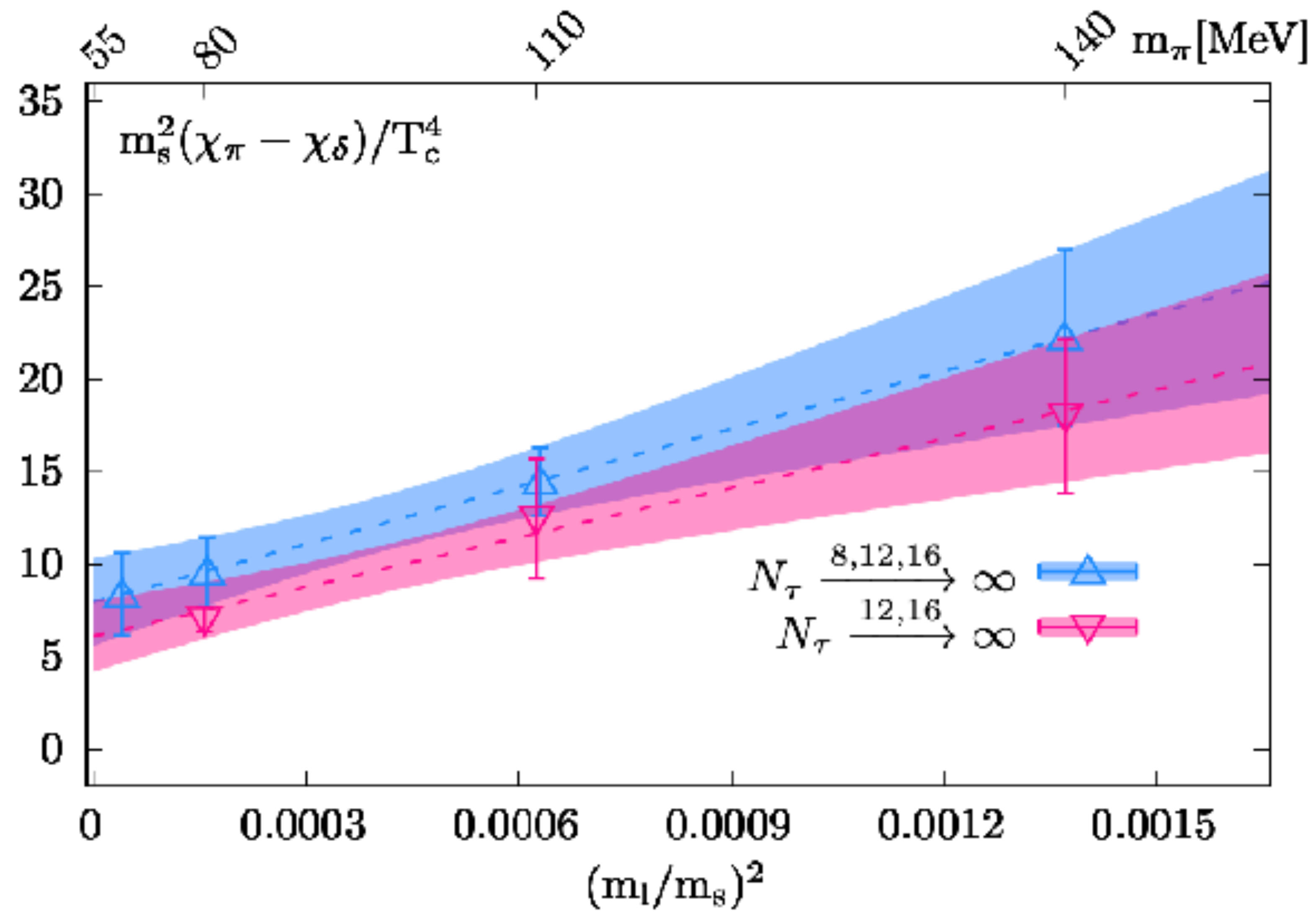


- **…and its low dimension modes.**

Dirac spectrum

above the cross-over temperature



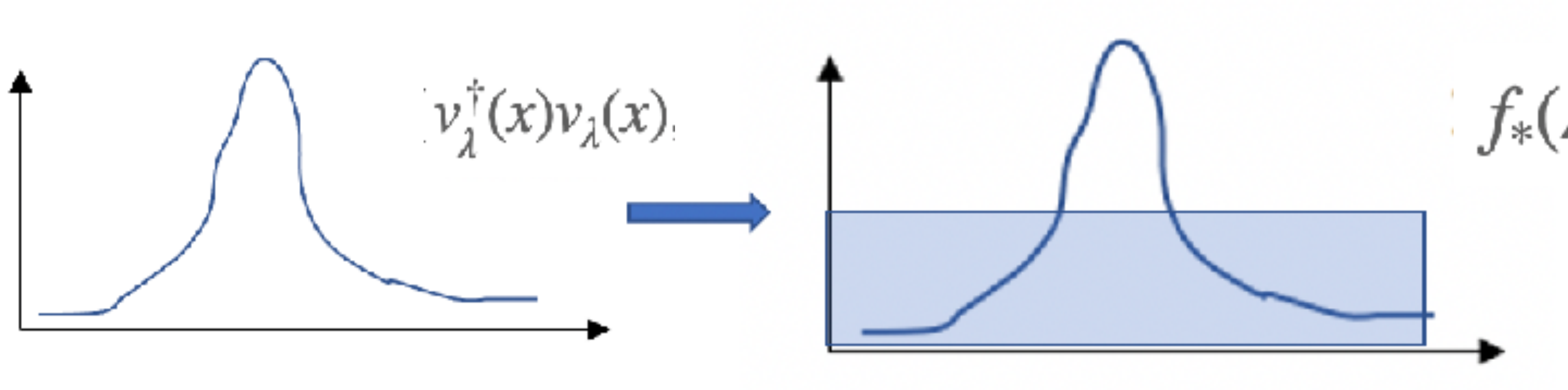


- The $\rho(\lambda)$ at $T \sim 205$ MeV has residual values at small λ, as $\rho(\lambda) \propto -m_q^2 \log \frac{\lambda^2 + m_q^2}{\Lambda_{QCD}^2}$.
- It corresponds to the axial anomaly in two-point correlation functions of light scalar and pseudoscalar mesons, $\int d^4x (C_{2,P}(x) - C_{2,S}(x)) \neq 0$, a confidence level above 95%.

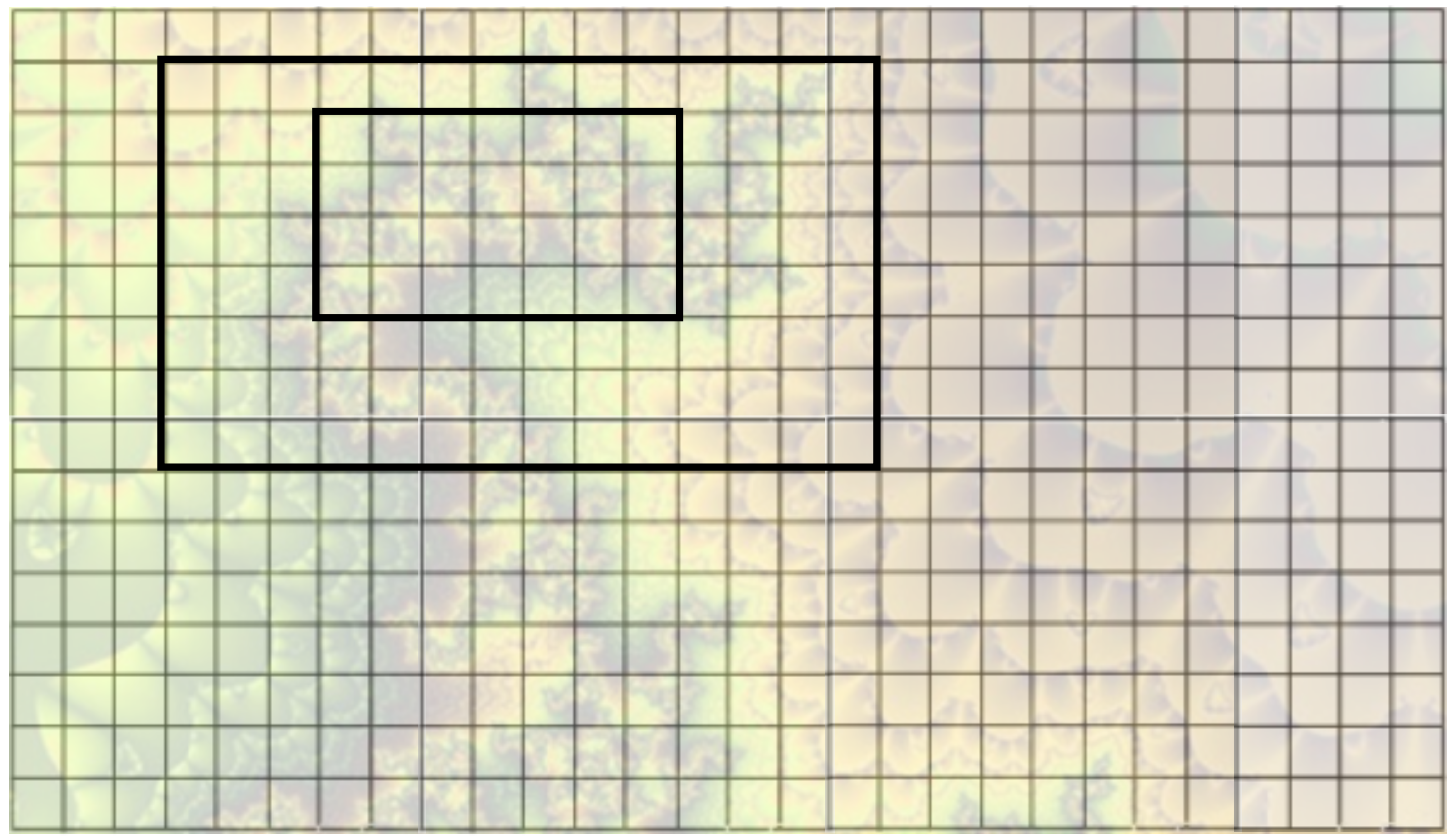
H.-T. Ding, et.al, PRL126(2021),082001, 2010.14836

Dirac spectrum

effective dimension of eigenvectors



$$f_*(\lambda) = \sum_{x\in V} \min\{v_\lambda^\dagger(x)v_\lambda(x), \frac{1}{V}\}, \qquad \sum_{x\in V} v_\lambda^\dagger(x)v_\lambda(x) = 1$$



- Comparing to the eigenvalue, the eigenvector includes more informations.

- One can define the function f_* to understand the effective dimension of the eigenvectors, as f_* of a low dimension mode decreases when the dimensionless volume is larger:

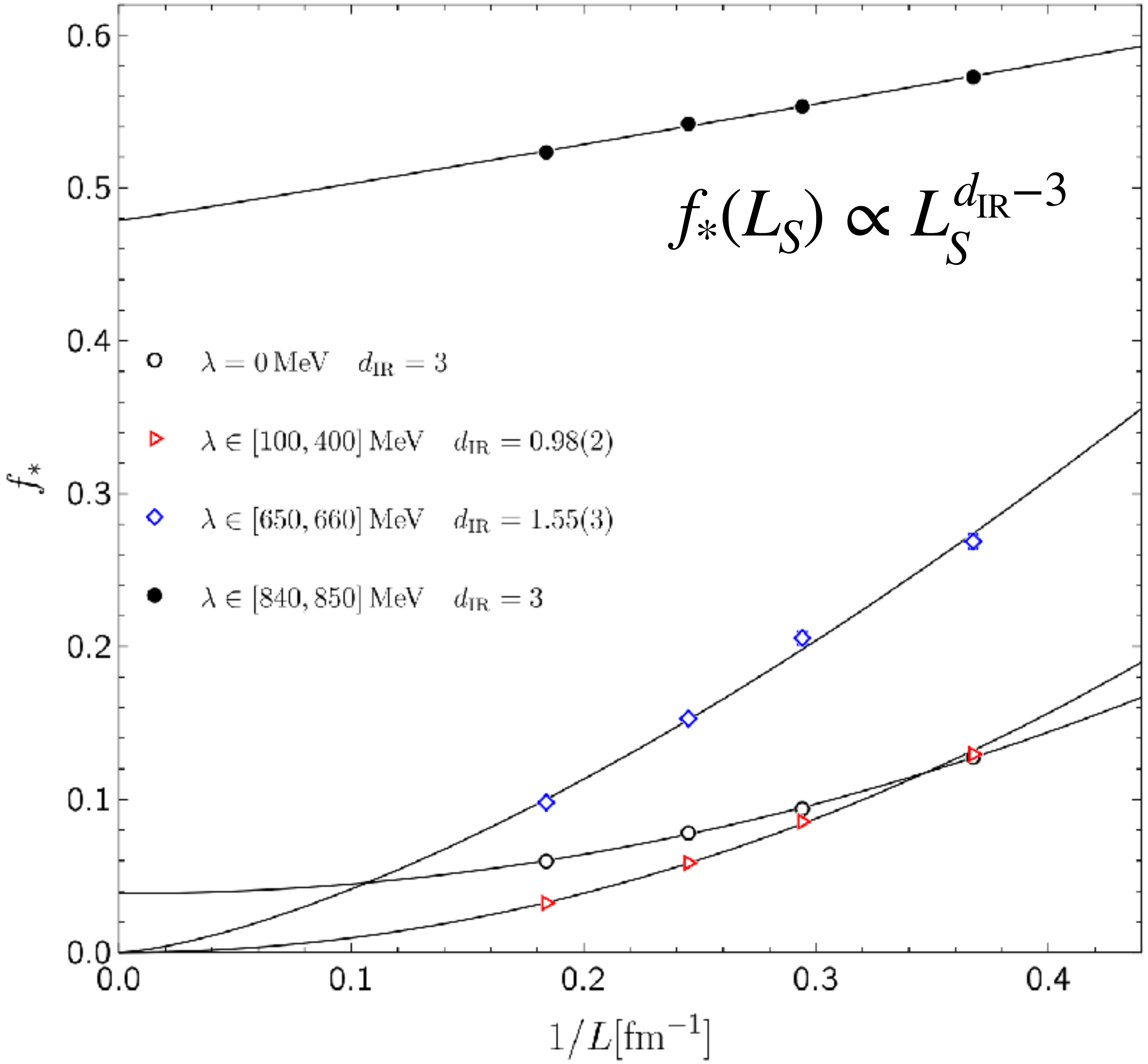
$$f_*(a) \propto a^{3-d_{\mathrm{UV}}}, \; f_*(L_S) \propto L_S^{d_{\mathrm{IR}}-3}$$

- $d_{\mathrm{UV}} = d_{\mathrm{IR}} = 3$, when $T = 0$.

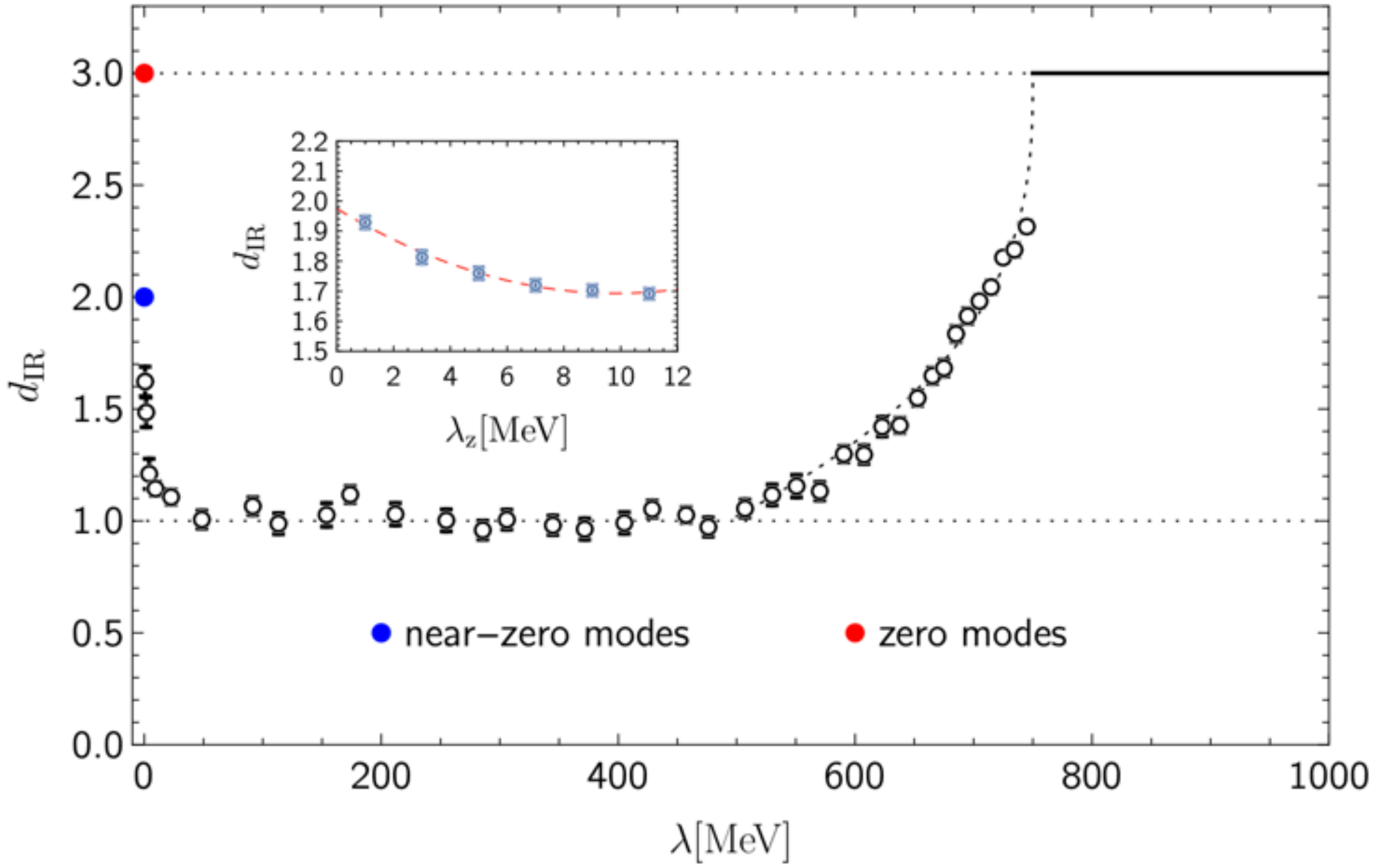
I Horvath, et al., Entropy 22(2020) 1273, 1807.03995
I Horvath, et al., 2205.11520

Dirac spectrum

effective dimension of eigenvectors



- $N_f = 0,\ T = 331$ MeV.
- $d_{\mathrm{IR}} = 3$ for the eigenvector with $\lambda = 0$ and $\lambda \geq 840$ MeV.
- $d_{\mathrm{IR}} = 1$ for the case with $\lambda \in [100,400]$ MeV.

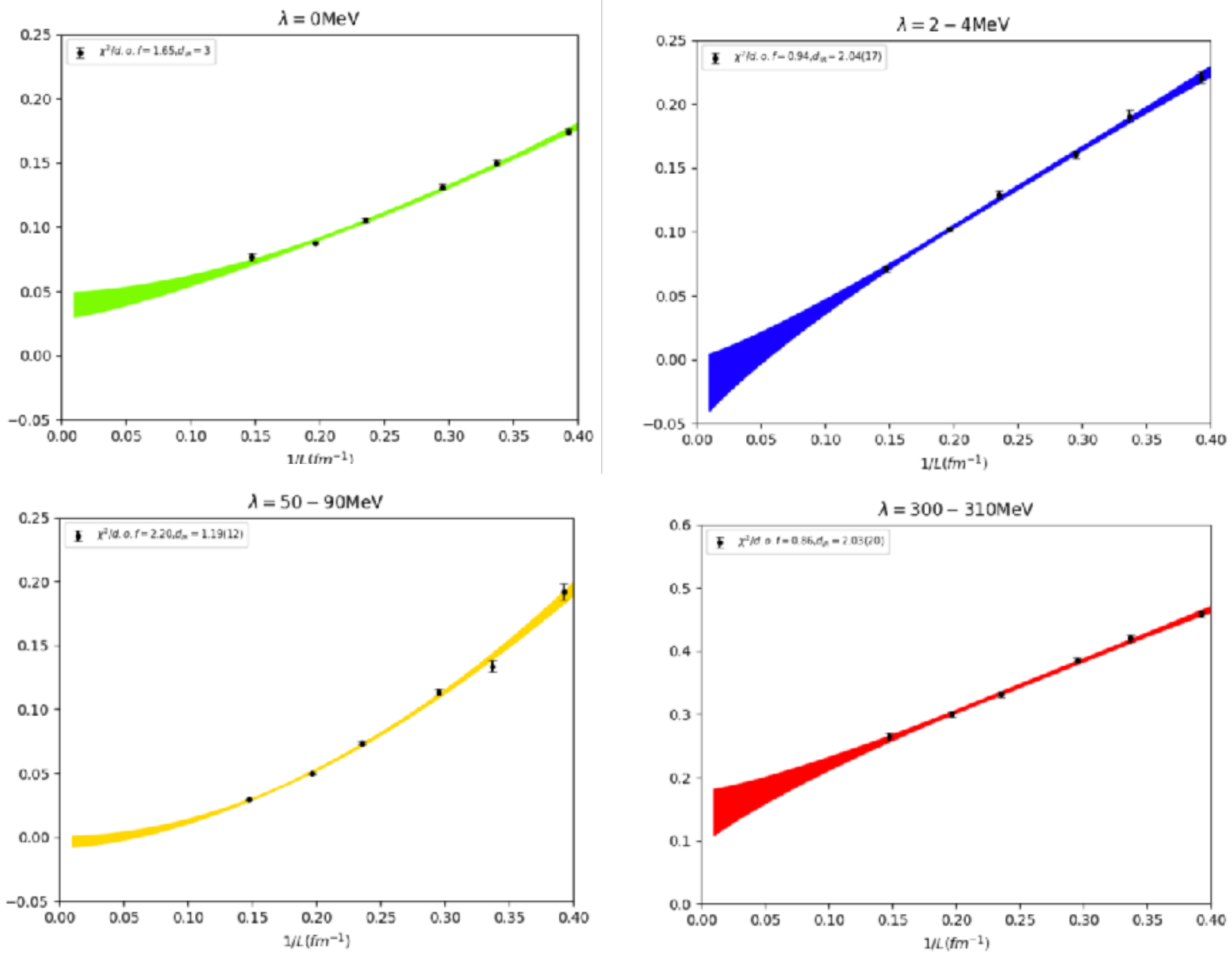


A. Alexandru, et.al, PRL127(2021),052303, 2103.05607

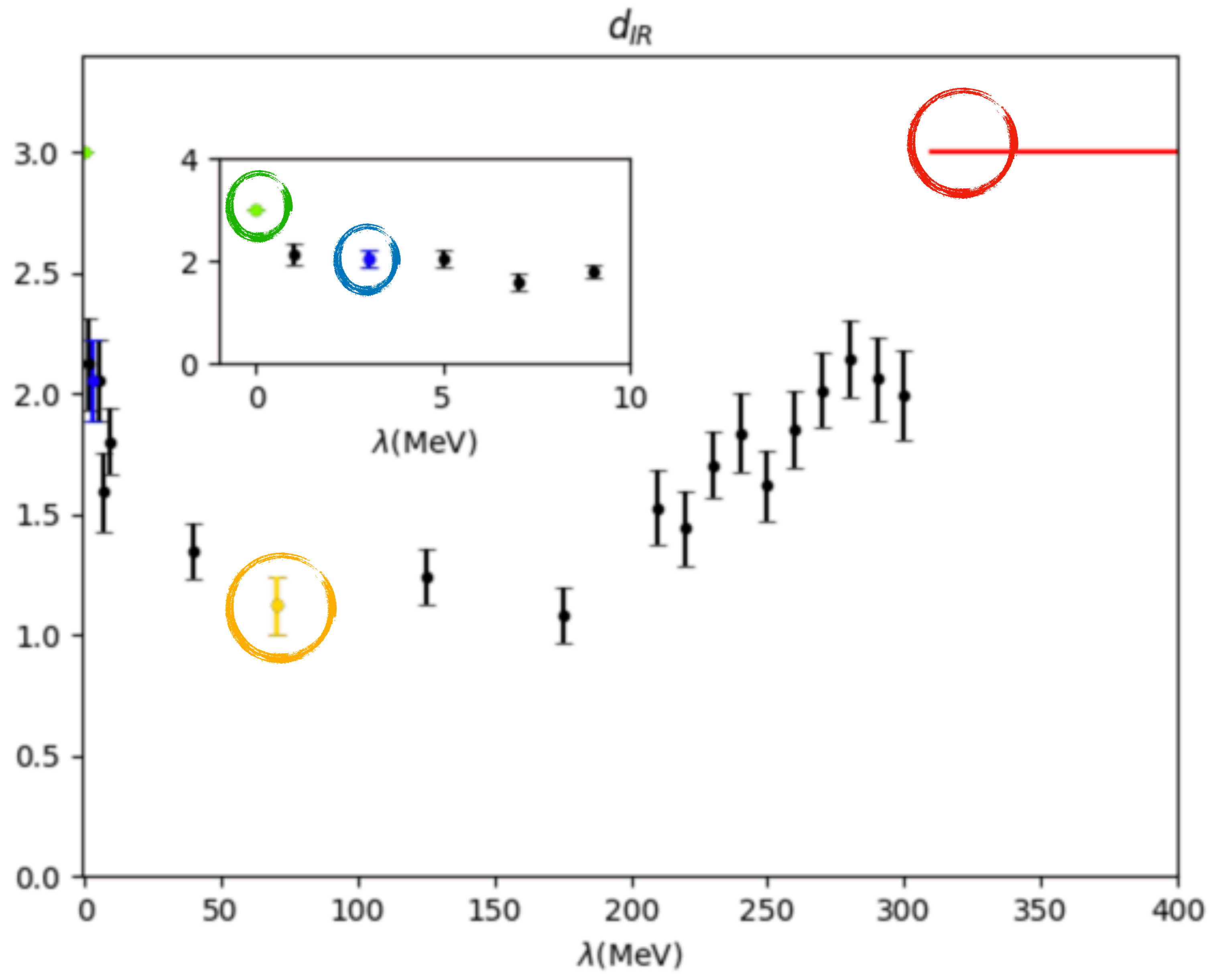
Dirac spectrum

effective dimension of eigenvectors

$$f_*(L_S) \propto L_S^{d_{\mathrm{IR}}-3}$$



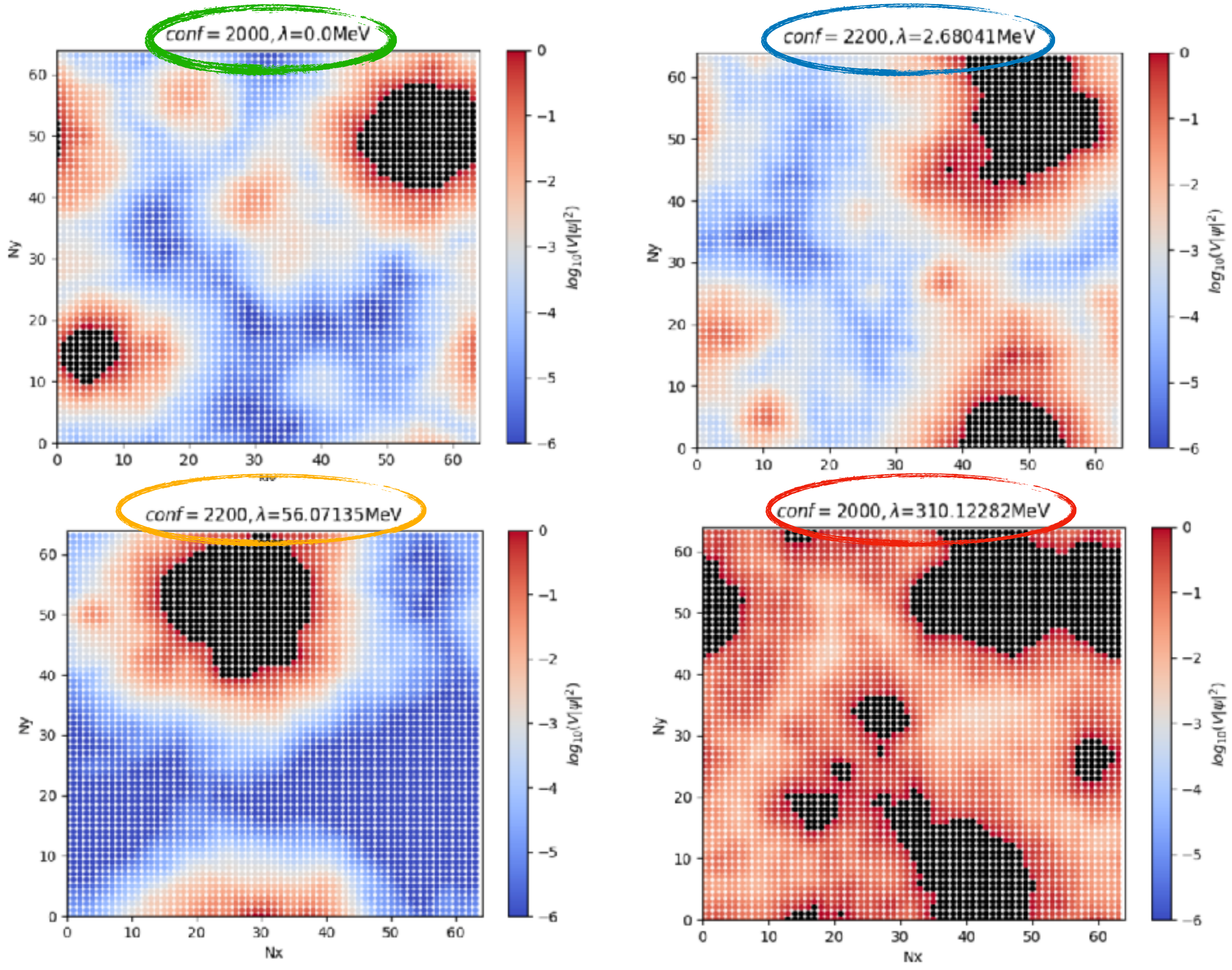
- $N_f = 2+1,\ T = 235\ \mathrm{MeV}, m_\pi = 135\ \mathrm{MeV}.$
- $d_{\mathrm{IR}} = 3$ for the eigenvector with $\lambda = 0$ and $\lambda \geq 300\ \mathrm{MeV}.$
- $d_{\mathrm{IR}} = 1$ for the case with $\lambda \in [30{,}190]\ \mathrm{MeV}.$



A. Alexandru, et.al, PRL127(2021),052303, 2103.05607

Dirac spectrum

Spacial distribution of given eigenvector



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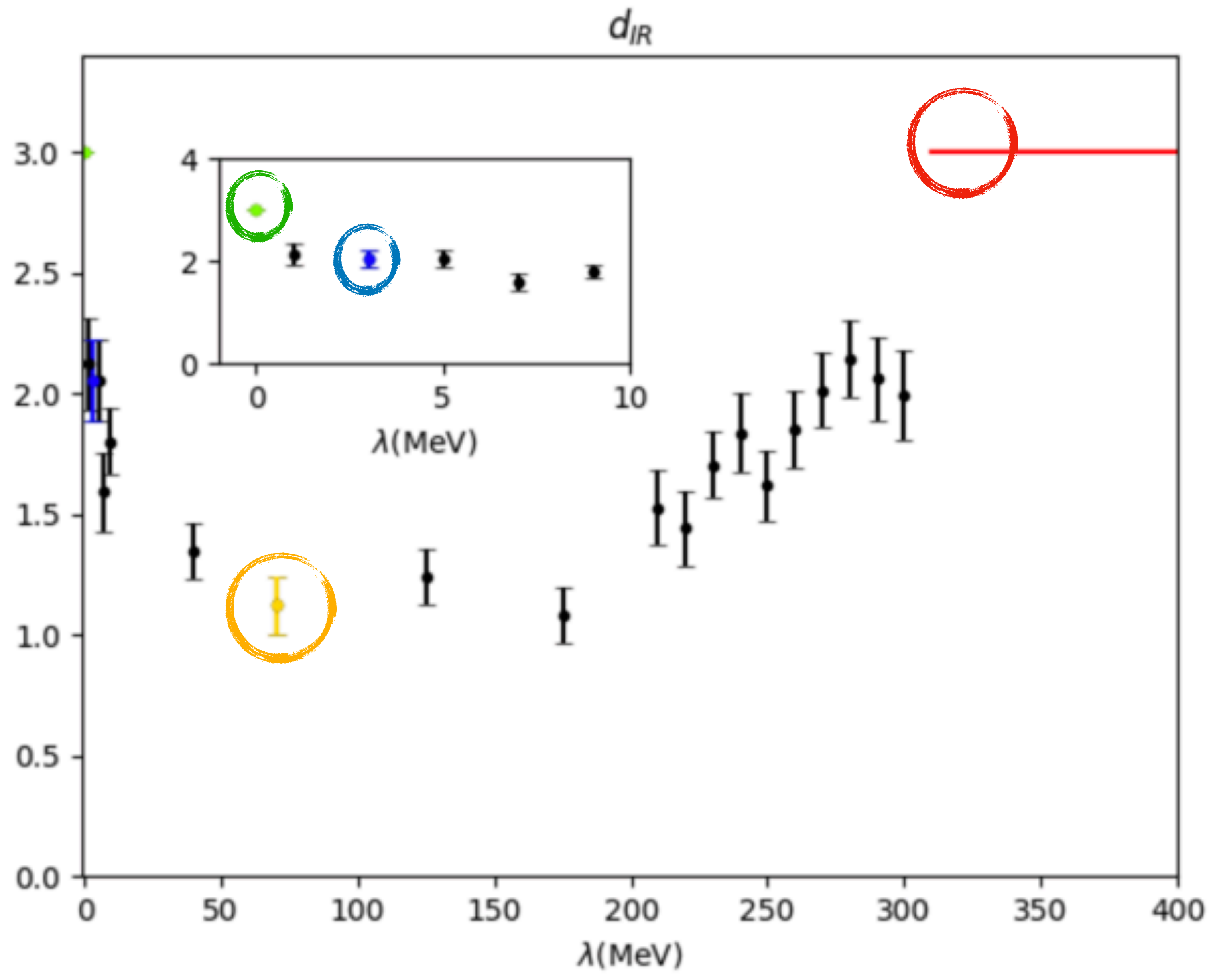
- $d_{\mathrm{IR}} = 3$ for the eigenvector with $\lambda = 0$ and $\lambda \geq 300\ \mathrm{MeV}.$
- $d_{\mathrm{IR}} = 1$ for the case with $\lambda \in [30{,}190]\ \mathrm{MeV}.$



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Summary

- The spontaneous chiral symmetry breaking has been observed in kinds of the lattice QCD calculations.
- It directly relates to the low lying eigen modes of the Dirac operator spectrum in the Euclidean space, and chiral symmetry shall restore when those eigen modes are removed or suppressed.
- Above the cross over temperature, the effective dimension of the spacial distribution of the eigenvector with $\lambda \in [30,190]\ \mathrm{MeV}$ are smaller than 3; and the zero mode would be the combination of the extended mode and poles.