



Fully heavy tetraquarks and dibaryons

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Outline

I. Introduction

II. Fully heavy tetraquarks

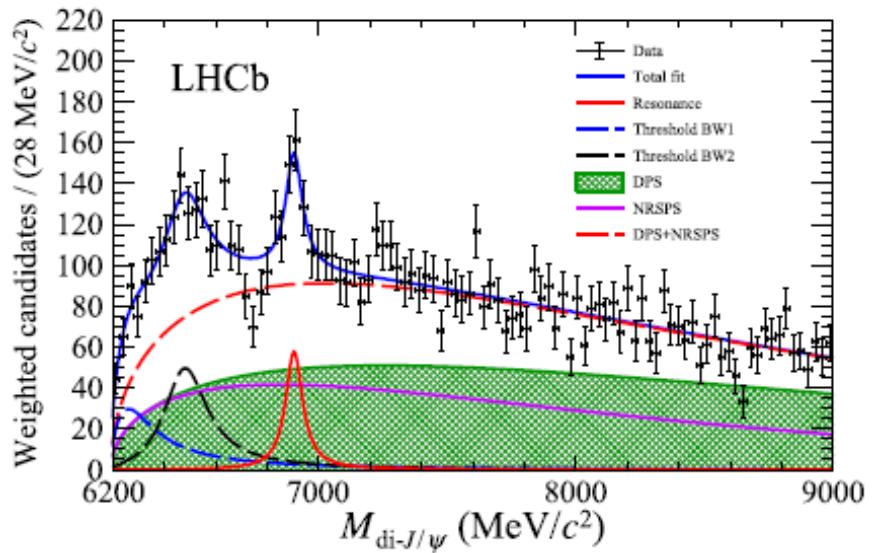
III. Fully heavy dibaryons

VI. Summary

I. Introduction

➤ Experimental results

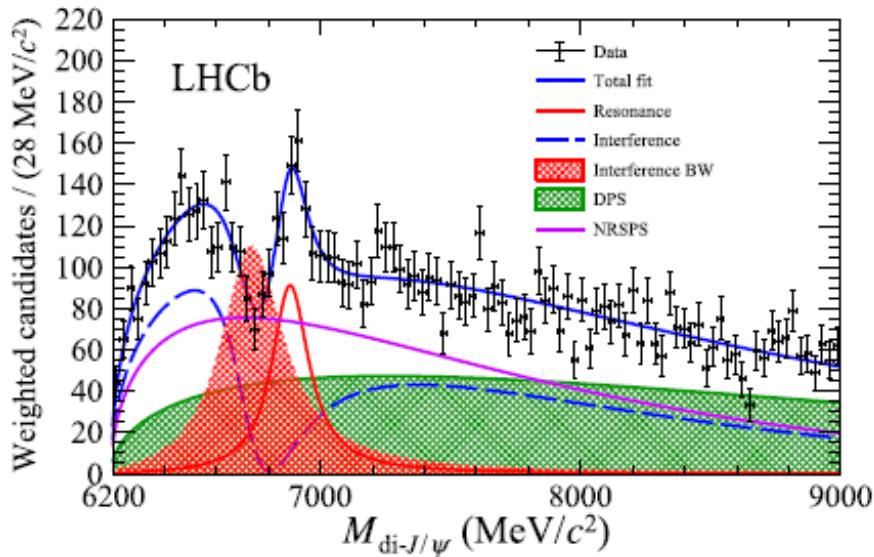
LHCb Collaboration, Science Bulletin 65 (2020) 1983–1993



$$m[X(6900)] = 6905 \pm 11 \pm 7 \text{ MeV}/c^2,$$

and

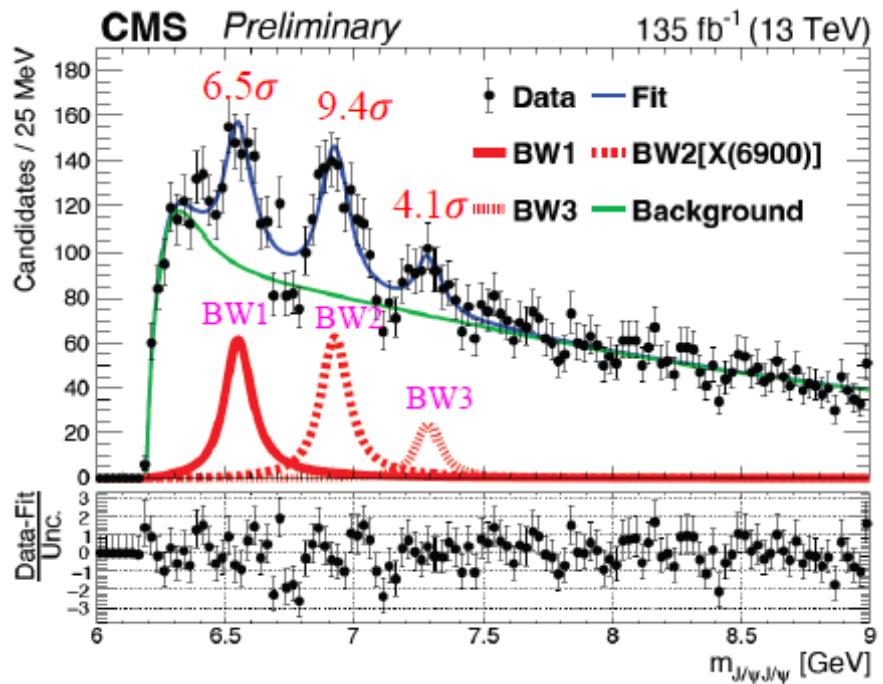
$$\Gamma[X(6900)] = 80 \pm 19 \pm 33 \text{ MeV},$$



$$m[X(6900)] = 6886 \pm 11 \pm 11 \text{ MeV}/c^2$$

and

$$\Gamma[X(6900)] = 168 \pm 33 \pm 69 \text{ MeV}.$$



CMS Collaboration,
 CMS-PAS-BPH-21-003

CMS found 3 significant structures using 135 fb^{-1} 13 TeV data

$$M[\text{BW1}] = 6552 \pm 10 \pm 12 \text{ MeV} \quad \Gamma[\text{BW1}] = 124 \pm 29 \pm 34 \text{ MeV} \quad >5.7\sigma$$

$$M[\text{BW2}] = 6927 \pm 9 \pm 5 \text{ MeV} \quad \Gamma[\text{BW2}] = 122 \pm 22 \pm 19 \text{ MeV} \quad >9.4\sigma$$

$$M[\text{BW3}] = 7287 \pm 19 \pm 5 \text{ MeV} \quad \Gamma[\text{BW3}] = 95 \pm 46 \pm 20 \text{ MeV} \quad >4.1\sigma$$

- BW2 consistent with X(6900) reported by LHCb
- CMS found two new structures, provisionally named as X(6600), X(7200)
- A family of structures which are candidates for all-charm tetra-quarks!



➤ Theoretical studies

- Before LHCb's results

1) Possible to exist

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- **Tetraquark states**

meson-meson ($c\bar{c}$)—($c\bar{c}$)

diquark-antidiquark (cc)—($\bar{c}\bar{c}$)

- **Hybrid ($c\bar{c}\bar{c}\bar{c}g$)**

- **Coupled-channel interaction**

- **Triangle-singularities**

- **CUSP**

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➤ Our work

Eur. Phys. J. C. 80, 1083 (2020)

- Bound state calculation

meson-meson

diquark-antidiquark

- Effective potentials
- A stabilization method (real scaling method)



II. Fully heavy tetraquarks

Eur. Phys. J. C. 80, 1083 (2020)

1. Quark Models

(1) Quark delocalization color screening model (QDCSM)

- QDCSM was developed by Nanjing-Los Alamos collaboration in 1990s aimed to multi-quark study. (PRL 69, 2901, 1992)
- Two new ingredients (based on quark cluster model configuration)
 - quark delocalization (orbital excitation)
 - color screening (color structure)
- Apply to the study of baryon-baryon interaction and dibaryons
deuteron, d^ , NN , $NN\Lambda$, $NN\Omega$, ...*
- Apply to the study of baryon-meson interaction and pentaquarks
 NK , Npi , Pc , ...



$$H = \sum_{i=1}^4 \left(m_i + \frac{p_i^2}{2m_i} \right) - T_{cm} + \sum_{i=1 < j}^4 (V_{ij}^{CON} + V_{ij}^{OGE})$$

$$V_{ij}^{CON} = \begin{cases} -\lambda_i^c \cdot \lambda_j^c (a_c r_{ij}^2 + V_0), & i,j \text{ in the same cluster} \\ -\lambda_i^c \cdot \lambda_j^c a_c \frac{1-e^{-\mu_{ij} r_{ij}^2}}{\mu_{ij}}, & \text{otherwise} \end{cases}$$

$$V_{ij}^{OGE} = \frac{\alpha_s}{4} \lambda_i^c \cdot \lambda_j^c \left[\frac{1}{r_{ij}} - \frac{\pi}{2} \delta(\mathbf{r}_{ij}) \left(\frac{1}{m_i^2} + \frac{1}{m_j^2} + \frac{4\sigma_i \cdot \sigma_j}{3m_i m_j} \right) \right]$$

$$\phi_\alpha(\mathbf{S}_i) = \left(\frac{1}{\pi b^2} \right)^{\frac{3}{4}} e^{-\frac{(\mathbf{r}-\mathbf{S}_i/2)^2}{2b^2}}, \quad \psi_\alpha(\mathbf{S}_i, \epsilon) = (\phi_\alpha(\mathbf{S}_i) + \epsilon \phi_\alpha(-\mathbf{S}_i)) / N(\epsilon),$$

$$\phi_\beta(-\mathbf{S}_i) = \left(\frac{1}{\pi b^2} \right)^{\frac{3}{4}} e^{-\frac{(\mathbf{r}+\mathbf{S}_i/2)^2}{2b^2}}. \quad \psi_\beta(-\mathbf{S}_i, \epsilon) = (\phi_\beta(-\mathbf{S}_i) + \epsilon \phi_\beta(\mathbf{S}_i)) / N(\epsilon),$$

→

$$N(\epsilon) = \sqrt{1 + \epsilon^2 + 2\epsilon e^{-S_i^2/4b^2}}.$$



(2) Chrial Quark Model (ChQM)

Provide the intermediate-range attraction by σ meson-exchange.

Rep. Prog. Phys. 68, 965 (2005)

SU(2) ChQM: only σ meson-exchange;

$$V_{ij}^\sigma = -\frac{g_{ch}^2}{4\pi} \frac{\Lambda_\sigma^2}{\Lambda_\sigma^2 - m_\sigma^2} m_\sigma \left[Y(m_\sigma r_{ij}) - \frac{\Lambda_\sigma}{m_\sigma} Y(\Lambda_\sigma r_{ij}) \right]$$

SU(3) ChQM: full scalar octet meson-exchange.

PRC 75, 034002 (2007)

$$V_{ij}^{\sigma_a} = V_{a_0}(r_{ij}) \sum_{a=1}^3 \lambda_i^a \cdot \lambda_j^a + V_\kappa(r_{ij}) \sum_{a=4}^7 \lambda_i^a \cdot \lambda_j^a + V_{f_0}(r_{ij}) \lambda_i^8 \cdot \lambda_j^8 + V_\sigma(r_{ij}) \lambda_i^0 \cdot \lambda_j^0$$

$$V_{ij}^{CON} = -\lambda_i^c \cdot \lambda_j^c (a_c r_{ij}^2 + V_0)$$



2. Wave functions

The wave function of the four-quark system

$$\Psi = \mathcal{A}[[\psi^L \chi^\sigma]_{JM} \chi^f \chi^c].$$

The meson-meson structure $\mathcal{A} = 1 - P_{13} - P_{24} + P_{13}P_{24}$

The diquark-antidiquark structure $\mathcal{A} = 1 - P_{12} - P_{34} + P_{12}P_{34}$

$$\psi^L = \psi_1(\mathbf{R}_1)\psi_2(\mathbf{R}_2)\chi_L(\mathbf{R})$$

$$\begin{aligned}\chi_L(\mathbf{R}) &= \frac{1}{\sqrt{4\pi}} \left(\frac{3}{2\pi b^2} \right) \sum_{i=1}^n C_i \\ &\times \int \exp \left[-\frac{3}{4b^2} (\mathbf{R} - \mathbf{S}_i)^2 \right] Y_{LM}(\hat{S}_i) d\hat{S}_i\end{aligned}$$



- Flavor wave functions

$$\chi_{00}^{fm1} = b\bar{b}b\bar{b}$$

$$\chi_{00}^{fd1} = b\bar{b}\bar{b}\bar{b}$$

$$\chi_{00}^{fm2} = c\bar{c}c\bar{c}$$

$$\chi_{00}^{fd2} = c\bar{c}\bar{c}\bar{c}$$

- Spin wave functions

$$\chi_{00}^{\sigma 1} = \frac{1}{2}(\alpha\beta\alpha\beta - \beta\alpha\alpha\beta - \beta\beta\alpha + \beta\alpha\beta\alpha)$$

$$\begin{aligned}\chi_{00}^{\sigma 2} = \sqrt{\frac{1}{12}} & (2\alpha\alpha\beta\beta + 2\beta\beta\alpha\alpha \\ & - \alpha\beta\alpha\beta - \beta\alpha\alpha\beta - \beta\beta\alpha - \beta\alpha\beta\alpha)\end{aligned}$$

$$\chi_{11}^{\sigma 3} = \sqrt{\frac{1}{2}}(\alpha\beta\alpha\alpha - \beta\alpha\alpha\alpha)$$

$$\chi_{11}^{\sigma 4} = \sqrt{\frac{1}{2}}(\alpha\alpha\alpha\beta - \alpha\alpha\beta\alpha)$$

$$\chi_{11}^{\sigma 5} = \frac{1}{2}(\alpha\alpha\alpha\beta + \alpha\alpha\beta\alpha - \alpha\beta\alpha\alpha - \beta\alpha\alpha\alpha)$$

$$\chi_{22}^{\sigma 6} = \alpha\alpha\alpha\alpha$$



- Color wave functions

Meson-meson structure

$$\chi_{c[111]}^1 = \sqrt{\frac{1}{3}}(r\bar{r} + g\bar{g} + b\bar{b})$$

$$\chi_{c[21]}^2 = r\bar{b} \quad \chi_{c[21]}^3 = -r\bar{g}$$

$$\chi_{c[21]}^4 = g\bar{b} \quad \chi_{c[21]}^5 = -b\bar{g}$$

$$\chi_{c[21]}^6 = g\bar{r} \quad \chi_{c[21]}^7 = b\bar{r}$$

$$\chi_{c[21]}^8 = \sqrt{\frac{1}{2}}(r\bar{r} - g\bar{g})$$

$$\chi_{c[21]}^9 = \sqrt{\frac{1}{6}}(-r\bar{r} - g\bar{g} + 2b\bar{b})$$

$$\chi^{cm1} = \chi_{c[111]}^1 \chi_{c[111]}^1$$

$$\begin{aligned} \chi^{cm2} = & \sqrt{\frac{1}{8}}(\chi_{c[21]}^2 \chi_{c[21]}^7 - \chi_{c[21]}^4 \chi_{c[21]}^5 - \chi_{c[21]}^3 \chi_{c[21]}^6 \\ & + \chi_{c[21]}^8 \chi_{c[21]}^8 - \chi_{c[21]}^6 \chi_{c[21]}^3 + \chi_{c[21]}^9 \chi_{c[21]}^9 \\ & - \chi_{c[21]}^5 \chi_{c[21]}^4 + \chi_{c[21]}^7 \chi_{c[21]}^2) \end{aligned}$$

Diquark-antidiquark structure

$$\chi_{c[2]}^1 = rr \quad \chi_{c[2]}^2 = \sqrt{\frac{1}{2}}(rg + gr) \quad \chi_{c[2]}^3 = gg$$

$$\chi_{c[2]}^4 = \sqrt{\frac{1}{2}}(rb + br) \quad \chi_{c[2]}^5 = \sqrt{\frac{1}{2}}(gb + bg)$$

$$\chi_{c[2]}^6 = bb \quad \chi_{c[11]}^7 = \sqrt{\frac{1}{2}}(rg - gr)$$

$$\chi_{c[11]}^8 = \sqrt{\frac{1}{2}}(rb - br) \quad \chi_{c[11]}^9 = \sqrt{\frac{1}{2}}(gb - bg)$$

$$\chi_{c[22]}^1 = \bar{r}\bar{r} \quad \chi_{c[22]}^2 = \sqrt{\frac{1}{2}}(\bar{r}\bar{g} + \bar{g}\bar{r}) \quad \chi_{c[22]}^3 = \bar{g}\bar{g}$$

$$\chi_{c[22]}^4 = \sqrt{\frac{1}{2}}(\bar{r}\bar{b} + \bar{b}\bar{r}) \quad \chi_{c[22]}^5 = \sqrt{\frac{1}{2}}(\bar{g}\bar{b} + \bar{b}\bar{g})$$

$$\chi_{c[22]}^6 = \bar{b}\bar{b} \quad \chi_{c[211]}^7 = \sqrt{\frac{1}{2}}(\bar{r}\bar{g} - \bar{g}\bar{r})$$

$$\chi_{c[211]}^8 = \sqrt{\frac{1}{2}}(\bar{r}\bar{b} - \bar{b}\bar{r}) \quad \chi_{c[211]}^9 = \sqrt{\frac{1}{2}}(\bar{g}\bar{b} - \bar{b}\bar{g})$$

$$\begin{aligned} \chi^{cd1} = & \sqrt{\frac{1}{6}}[\chi_{c[2]}^1 \chi_{c[22]}^1 - \chi_{c[2]}^2 \chi_{c[22]}^2 + \chi_{c[2]}^3 \chi_{c[22]}^3 \\ & + \chi_{c[2]}^4 \chi_{c[22]}^4 - \chi_{c[2]}^5 \chi_{c[22]}^5 + \chi_{c[2]}^6 \chi_{c[22]}^6] \end{aligned}$$

$$\chi^{cd2} = \sqrt{\frac{1}{3}}[\chi_{c[11]}^7 \chi_{c[211]}^7 - \chi_{c[11]}^8 \chi_{c[211]}^8 + \chi_{c[11]}^9 \chi_{c[211]}^9]$$

3. Fully charm tetraquarks

- Bound state calculation

Meson-meson structure

Table 4 The energies of $cc\bar{c}\bar{c}$ systems with meson–meson structure in ChQM and QDCSM. $[ijk]$ stand for the indices of spin, flavor and color wave functions χ^{σ_i} , χ^{fmj} , χ^{cmk} (unit: MeV). The thresholds of $\eta_c\eta_c$,

$J/\psi J/\psi$, $\eta_c J/\psi$, $J/\psi \eta_c$ are 5958 MeV, 6195 MeV, 6076 MeV and 6076 MeV, respectively

$[ijk]$	Channel	ChQM I		ChQM II		ChQM III		QDCSM I		QDCSM II		QDCSM III	
		E_{sc}	E_{cc2}	E_{sc}	E_{cc2}	E_{sc}	E_{cc2}	E_{sc}	E_{cc1}	E_{sc}	E_{cc1}	E_{sc}	E_{cc1}
$IJ^P = 00^+$													
111	$\eta_c\eta_c$	5969	5969	5962	5962	5961	5961	5969	5969	5962	5962	5961	5961
211	$J/\psi J/\psi$	6206		6198		6197		6206		6195		6197	
112	$\eta_c8\eta_c8$	6619		6632		6626							
212	$J/\psi_8 J/\psi_8$	6701		6666		6570							
$IJ^P = 01^+$													
311	$\eta_c J/\psi$	6088	6088	6080	6080	6079	6079	6088	6088	6080	6080	6079	6079
411	$J/\psi \eta_c$	6088		6080		6079		6088		6080		6079	
312	$\eta_c8 J/\psi_8$	6544		6570		6575							
412	$J/\psi_8 \eta_c8$	6544		6570		6575							
$IJ^P = 02^+$													
611	$J/\psi J/\psi$	6207	6206	6198	6198	6197	6197	6207		6198		6197	
612	$J/\psi_8 J/\psi_8$	6538		6577		6602							

- ✓ The energies are above the corresponding theoretical threshold in both two models with different sets of parameters.
- ✓ The channel-coupling effect is very small and cannot help much.
- ✓ There is no any bound states with the meson-meson structure in both two models.



Diquark-antidiquark structure

Table 5 The energies of $ccc\bar{c}$ systems with diquark-antidiquark structure in ChQM and QDCSM. $[ijk]$ stand for the indices of spin, flavor and color wave functions χ^{σ_i} , χ^{fdj} , χ^{cdk} (unit: MeV)

$[ijk]$	ChQM I		ChQM II		ChQM III		QDCSM I		QDCSM II		QDCSM III	
	E_{sc}	E_{cc1}	E_{sc}	E_{cc1}								
$IJ^P = 00^+$												
121	6729	6492	6717	6479	6669	6451	6174	6095	6320	6231	6405	6314
222	6493		6482		6466		6128		6270		6358	
$IJ^P = 01^+$												
522	6495		6488		6479		6149		6285		6375	
$IJ^P = 02^+$												
622	6498		6499		6505		6197		6314		6407	

- ✓ The energies of this configuration are higher than that of meson-meson configuration.
- ✓ The effect of the channel-coupling is also very small in both two models.

Is there any resonance state because of the color structure?

- Effective potentials

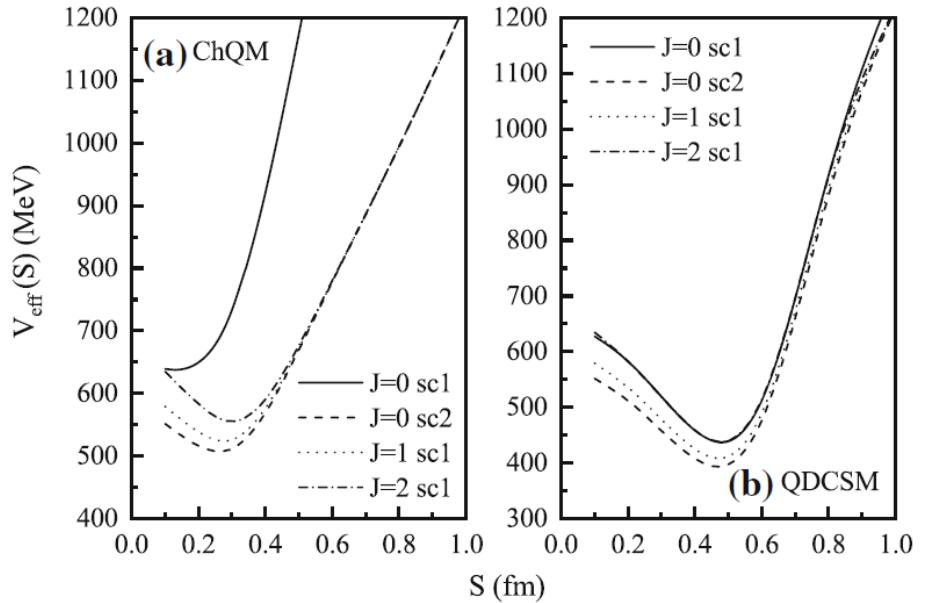


Fig. 7 The effective potentials for the diquark–antidiquark $cc\bar{c}\bar{c}$ systems in two quark models

- ✓ In ChQM, the minimum potential of each channel (except the first channel of $IJ=00$) appears at the separation of 0.3 fm, which indicates that two subclusters are not willing to huddle together or fall apart. So each state is possible to be a resonance state.
- ✓ In QDCSM, the results are similar. The minimum potential of each channel appears at the separation of 0.5 fm. So it is also possible for each channel to be a resonance state in QDCSM.

- A stabilization method (real scaling method)

J. Simon, J. Chem. Phys. 75, 2465

Resonance state lifetimes from stabilization graphs

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(Received 20 January 1981; accepted 18 May 1981)

The stabilization method (SM) pioneered by Taylor and co-workers¹ has proven to be a valuable tool for estimating the energies of long-lived metastable states of electron–atom, electron–molecule, and atom–diatom complexes. In implementing the SM one searches for eigenvalues arising from a matrix representation of the relevant Hamiltonian H which are “stable” as the basis set used to construct H is varied.

To obtain lifetimes of metastable states, one can choose from among a variety of techniques^{2–7} (e.g., phase shift analysis, Feshbach projection “golden rule” formulas, Siegert methods, and complex coordinate scaling methods), many of which use the stabilized *eigenvector* as starting information. Here we demonstrate that one can obtain an *estimate* of the desired lifetime directly from the stabilization graph in a manner which makes a close connection with the complex coordinate rotation method (CRM) for which a satisfactory mathematical basis exists.

The starting point of our development is the observation that both the stable eigenvalue (E_r) and the eigenvalue(s) (E_c) which come from above and cross E_r (see Fig. 1 and Refs. 9–11 and 13) vary in a nearly linear manner (with α) near their avoided crossing points. This observation leads us to propose that the two eigenvalues arising in each such avoided crossing can be

thought of as arising from two “uncoupled” states having energies $\epsilon_r(\alpha) = \epsilon + S_r(\alpha - \alpha_c)$ and $\epsilon_c(\alpha) = \epsilon + S_c(\alpha - \alpha_c)$, where S_r and S_c are the *slopes* of the linear parts of the stable and “continuum” eigenvalues, respectively. α_c is the value of α at which these two straight lines would intersect, and ϵ is their common value at $\alpha = \alpha_c$. This modeling of ϵ_r and ϵ_c is simply based upon the *observa-*

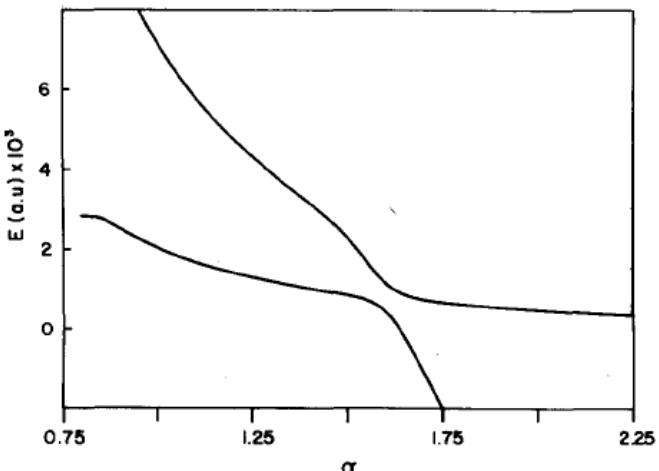
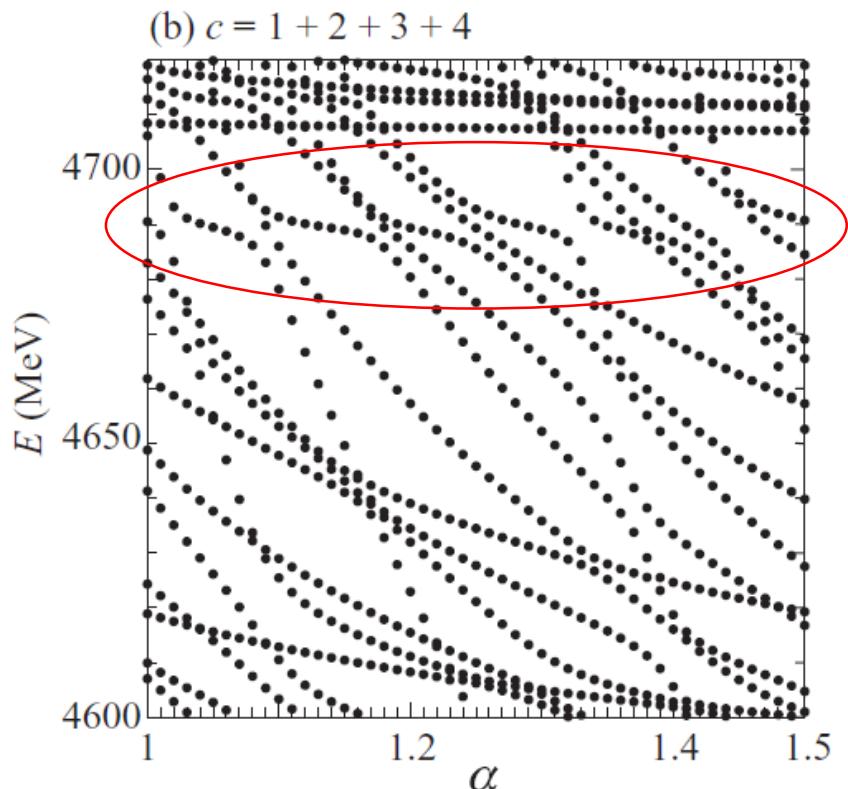
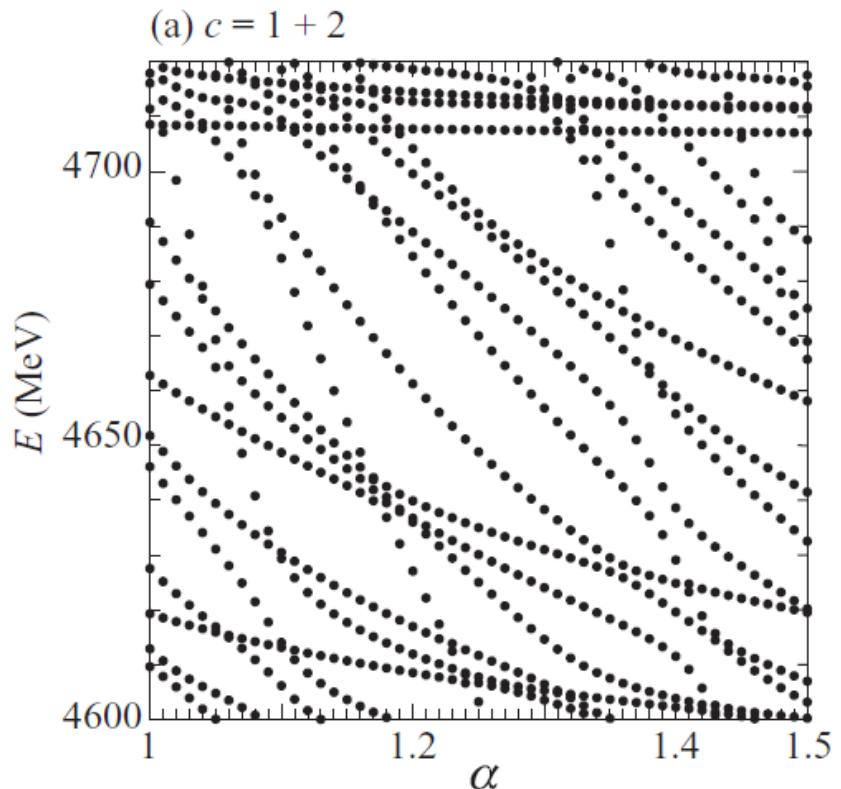


FIG. 1. Stabilization graph for the 2π shape resonance state of LiH^+ (Ref. 9).

E. Hiyama, A. Hosaka, M. Oka, and J-M. Richard, Phys. Rev. C 98, 045208 (2018)



One resonance at 4690 MeV

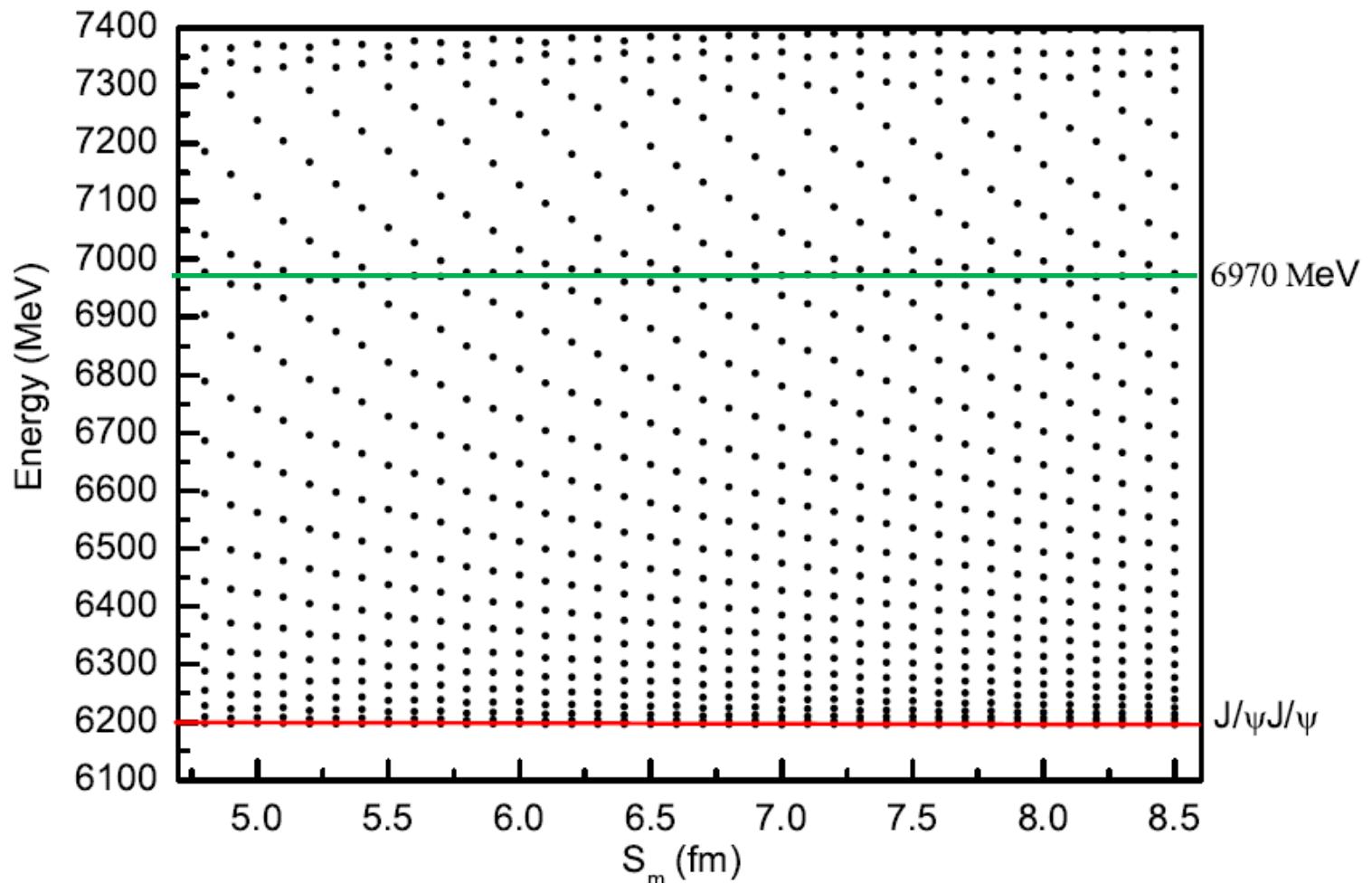


Fig. 10 The stabilization plots of the energies of the $cc\bar{c}\bar{c}$ system with $IJ^P = 02^+$ in ChQM III.

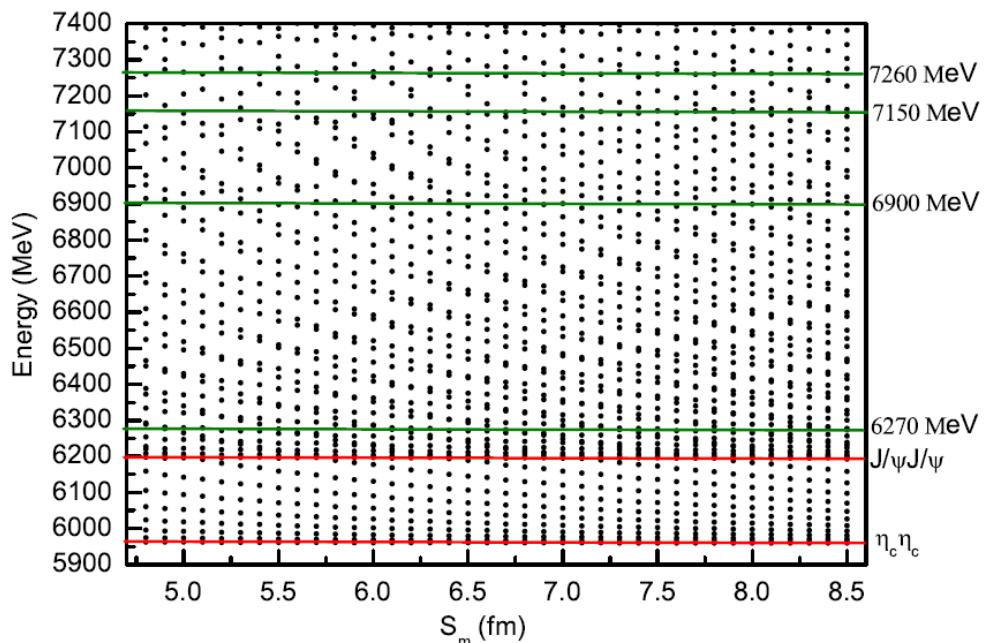


Fig. 8 The stabilization plots of the energies of the $ccc\bar{c}$ system with $IJ^P = 00^+$ in ChQM III.

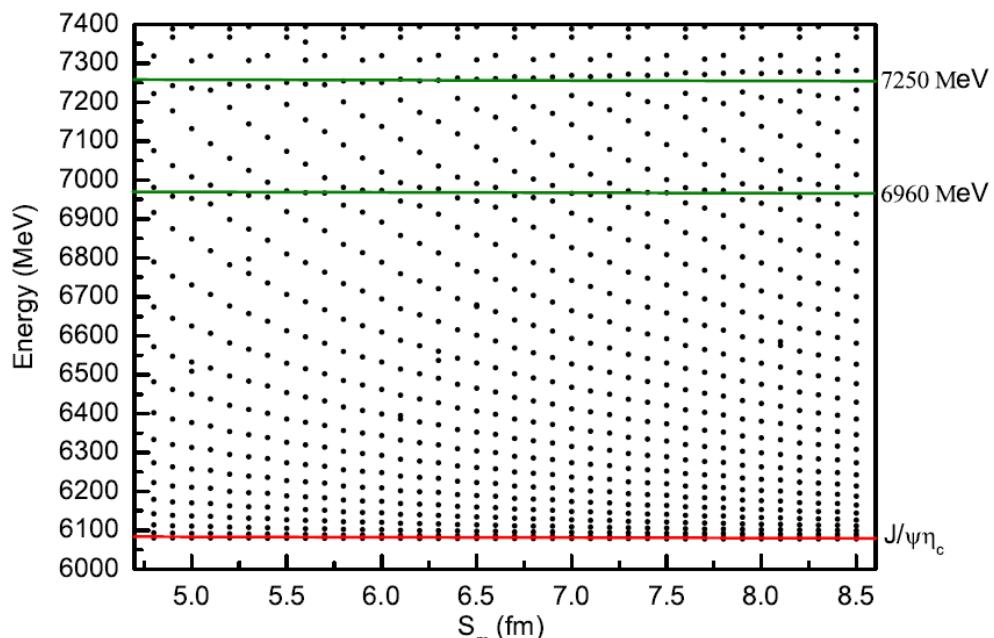


Fig. 9 The stabilization plots of the energies of the $ccc\bar{c}$ system with $IJ^P = 01^+$ in ChQM III.



Table 6 The energies of $ccc\bar{c}$ excited states with the diquark-antidiquark structure in the ChQM and QDCSM (unit: MeV)

ChQM I	$IJ^P = 00^+$	<u>6210</u>	<u>6910</u>	<u>7160</u>	<u>7250</u>	$ J^P=00^+ : 6205 \sim 6270 \text{ MeV},$
	$IJ^P = 01^+$	6740	7275			$6825 \sim 6975 \text{ MeV},$
	$IJ^P = 02^+$	6725				$7140 \sim 7170 \text{ MeV},$
ChQM II						$7210 \sim 7260 \text{ MeV}$
	$IJ^P = 00^+$	<u>6210</u>	<u>6825</u>	<u>7150</u>	<u>7210</u>	$ J^P=01^+ : 6740 \sim 7150 \text{ MeV},$
	$IJ^P = 01^+$	6830	7280			$7250 \sim 7280 \text{ MeV}$
	$IJ^P = 02^+$	6825				$ J^P=02^+ : 6725 \sim 7050 \text{ MeV}$
ChQM III						
	$IJ^P = 00^+$	<u>6270</u>	<u>6900</u>	<u>7150</u>	<u>7260</u>	X(6900) $J^P=0^+$
	$IJ^P = 01^+$	6960	7250			X(7200) $J^P=0^+ \text{ or } 1^+$
	$IJ^P = 02^+$	6970				X(6600) ?
QDCSM I						X(6200), X(7100) $J^P=0^+$
	$IJ^P = 00^+$	<u>6205</u>	<u>6950</u>	<u>7170</u>	<u>7250</u>	
	$IJ^P = 01^+$	7040	7280			
	$IJ^P = 02^+$	7010				
QDCSM II						
	$IJ^P = 00^+$	<u>6205</u>	<u>6950</u>	<u>7150</u>	<u>7225</u>	
	$IJ^P = 01^+$	6925	7250			
	$IJ^P = 02^+$	6900				
QDCSM III						
	$IJ^P = 00^+$	<u>6205</u>	<u>6975</u>	<u>7140</u>	<u>7250</u>	
	$IJ^P = 01^+$	7150	7250			
	$IJ^P = 02^+$	7050				



4. Fully bottom tetraquarks

Table 2 The energies of $b\bar{b}\bar{b}\bar{b}$ systems with meson–meson structure in ChQM and QDCSM. $[ijk]$ stand for the indices of spin, flavor and color wave functions $\chi^{\sigma i}$, χ^{fmj} , χ^{cmk} (unit: MeV). The thresholds of $\eta_b\eta_b$, $\Upsilon\Upsilon$, $\eta_b\Upsilon$, $\Upsilon\eta_b$ are 18799 MeV, 18920 MeV, 18860 MeV and 18860 MeV, respectively

$[ijk]$	Channel	ChQM I		ChQM II		ChQM III		QDCSM I		QDCSM II		QDCSM III	
		E_{sc}	E_{cc2}	E_{sc}	E_{cc2}	E_{sc}	E_{cc2}	E_{sc}	E_{cc1}	E_{sc}	E_{cc1}	E_{sc}	E_{cc1}
$IJ^P = 00^+$													
111	$\eta_b\eta_b$	18803	18803	18800	18799	18800	18800	18803	18803	18800	18800	18800	18800
211	$\Upsilon\Upsilon$	18925		18922		18922		18925		18922		18922	
112	$\eta_{b8}\eta_{b8}$	19974		19975		19688							
212	$\Upsilon_8\Upsilon_8$	20041		20032		19696							
$IJ^P = 01^+$													
311	$\eta_b\Upsilon$	18864	18864	18861	18861	18860	18860	18864	18864	18861	18861	18860	18860
411	$\Upsilon\eta_b$	18864		18861		18860		18864		18861		18860	
312	$\eta_{b8}\Upsilon_8$	19925		19927		19641							
412	$\Upsilon_8\eta_{b8}$	19925		19927		19641							
$IJ^P = 02^+$													
611	$\Upsilon\Upsilon$	18925	18925	18922	18922	18921	18921	18925		18922		18921	
612	$\Upsilon_8\Upsilon_8$	19923		19933		19660							

Table 3 The energies of $b\bar{b}\bar{b}\bar{b}$ systems with diquark–antidiquark structure in ChQM and QDCSM. $[ijk]$ stand for the indices of spin, flavor and color wave functions $\chi^{\sigma i}$, χ^{fdj} , χ^{cdk} (unit: MeV)

$[ijk]$	ChQM I		ChQM II		ChQM III		QDCSM I		QDCSM II		QDCSM III	
	E_{sc}	E_{cc2}	E_{sc}	E_{cc2}								
$IJ^P = 00^+$												
121	20134	19466	20130	19456	19790	19310	19281	19237	19369	19317	19184	19122
222	19466		19456		19313		19256		19344		19165	
$IJ^P = 01^+$												
522	19467		19461		19323		19264		19354		19184	
$IJ^P = 02^+$												
622	19471		19471		19344		19279		19374		19236	

$|J^P=00^+|$:

19122 ~ 19344 MeV

$|J^P=01^+|$:

19184 ~ 19354 MeV

$|J^P=02^+|$:

19236 ~ 19374 MeV



III. Fully heavy dibaryons

1. Heavy-Antiquark-Diquark Symmetry (HADS)

Heavy diquark behaves as a heavy anti-quark from color freedom

Phys.Lett. B248 (1990) 177-180, Phys.Rev. D73 (2006) 054003

$$m_{\Xi_{cc\,3/2}} - m_{\Xi_{cc\,1/2}} = \frac{3}{4}(m_{D^*_s} - m_{D_s}) \approx 106.5 \text{ MeV}$$

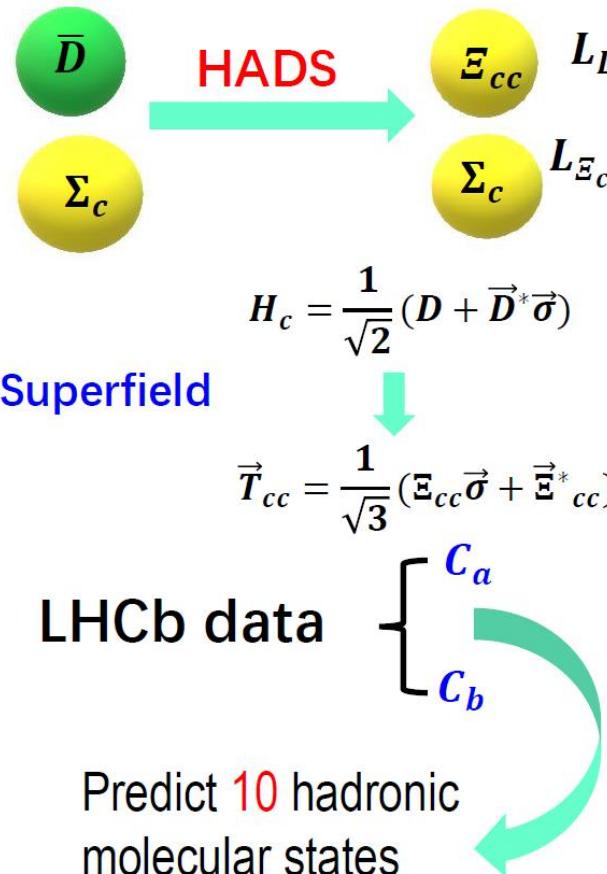
$$m_{\Omega_{cc\,3/2}} - m_{\Omega_{cc\,1/2}} = \frac{3}{4}(m_{D_{s*}} - m_{D_s}) \approx 107.9 \text{ MeV}$$

PHYSICAL REVIEW D 88, 054014 (2013)

Heavy-antiquark-diquark symmetry and heavy hadron molecules: Are there triply heavy pentaquarks?

Feng-Kun Guo,^{1,*} Carlos Hidalgo-Duque,^{2,†} Juan Nieves,^{2,‡} and Manuel Pavón Valderrama^{3,§}

We explore the consequences of heavy flavor, heavy quark spin, and heavy antiquark-diquark symmetries for hadronic molecules within an effective field theory framework. Owing to heavy antiquark-diquark symmetry, the doubly heavy baryons have approximately the same light-quark structure as the heavy antimesons. As a consequence, the existence of a heavy meson-antimeson molecule implies the possibility of a partner composed of a heavy meson and a doubly heavy baryon. In this regard, the $D\bar{D}^*$ molecular nature of the $X(3872)$ will hint at the existence of several baryonic partners with isospin $I = 0$ and $J^P = \frac{5}{2}^-$ or $\frac{3}{2}^-$. Moreover, if the $Z_b(10650)$ turns out to be a $B^*\bar{B}^*$ bound state, we can be confident of the existence of $\Xi_{bb}^*\bar{B}^*$ hadronic molecules with quantum numbers $I(J^P) = 1(\frac{1}{2}^-)$ and $I(J^P) = 1(\frac{3}{2}^-)$. These states are of special interest since they can be considered to be triply heavy pentaquarks.



state	J^P	V	state	J^P	V
$\bar{D}\Sigma_c$	$1/2^-$	C_a	$\Xi_{cc}\Sigma_c$	0^+	$C_a + \frac{2}{3}C_b$
				1^+	$C_a - \frac{2}{9}C_b$
$\bar{D}\Sigma_c^*$	$3/2^-$	C_a	$\Xi_{cc}\Sigma_c^*$	1^+	$C_a + \frac{5}{9}C_b$
				2^+	$C_a - \frac{1}{3}C_b$
$\bar{D}^*\Sigma_c$	$1/2^-$	$C_a - \frac{4}{3}C_b$	$\Xi_{cc}^*\Sigma_c$	1^+	$C_a - \frac{10}{9}C_b$
	$3/2^-$	$C_a + \frac{2}{3}C_b$		2^+	$C_a + \frac{2}{3}C_b$
	$1/2^-$	$C_a - \frac{5}{3}C_b$		0^+	$C_a - \frac{5}{3}C_b$
$\bar{D}^*\Sigma_c^*$	$3/2^-$	$C_a - \frac{2}{3}C_b$	$\Xi_{cc}^*\Sigma_c^*$	1^+	$C_a - \frac{11}{9}C_b$
				2^+	$C_a - \frac{1}{3}C_b$
	$5/2^-$	$C_a + C_b$		3^+	$C_a + C_b$



From M. Liu's slide



2. Theoretical work of heavy dibaryons

- Deuteron like Heavy Dibaryons from Lattice Quantum Chromodynamics
Phys. Rev. Lett. 123 (2019) 162003
Bound states: $\Omega_c \Omega_{cc}(sscscc)$, $\Omega_b \Omega_{bb}(ssbsbb)$, and $\Omega_{ccb} \Omega_{cbb}(ccbcbb)$
The binding of these dibaryons becomes stronger as they become heavier in mass.
- Dibaryon with Highest Charm Number near Unitarity from Lattice QCD
Phys. Rev. Lett. 127 (2021) 072003
 $\Omega_{ccc} \Omega_{ccc}$ $B = 5.68(0.77)(^{+0.46})_{-1.02}$ MeV, $\sqrt{\langle r^2 \rangle} = 1.13(0.06)(^{+0.80})_{-0.03}$ fm.
- Prediction of an $\Omega_{bbb} \Omega_{bbb}$ Dibaryon in the Extended One-Boson Exchange Model
Chin. Phys. Lett. Vol. 38, No. 10 (2021) 101201
- Very Heavy Flavored Dibaryons
Phys. Rev. Lett. 124 (2020) 212001
using a constituent model
Very heavy dibaryons with three charm quarks and three beauty quarks
bbbccc, ccccc, bbbbb
no bound state is found

3. Quark model study of fully heavy dibaryons

- Bound state calculation

Table 4 The energies and binding energies (in MeV) of every single channel and the channel-coupling calculation of the $\Omega_{ccc}\Omega_{ccc}$ and $\Omega_{bbb}\Omega_{bbb}$ systems with $J^P = 0^+$

cccccc	bbbbbb		
E_{sc1}	B_{sc1}	E_{sc1}	B_{sc1}
9762.7 ± 2.2	-2.5 ± 0.1	$29,787.3 \pm 4.0$	-0.9 ± 0.2
E_{sc2}	B_{sc2}	E_{sc2}	B_{sc2}
9750.5 ± 2.5	-14.7 ± 0.3	$29,786.9 \pm 4.1$	-1.3 ± 0.1
E_{cc}	B_{cc}	E_{cc}	B_{cc}
9735.1 ± 2.7	-30.1 ± 0.5	$29,780.6 \pm 4.2$	-7.6 ± 0.1

Table 5 The energies and binding energies (in MeV) of every single channel and the channel-coupling calculation of the $\Omega_{ccb}\Omega_{bbc}$ and $\Omega_{ccc}\Omega_{bbb}$ systems with $J^P = 0^+$. E'_{cc} and B'_{cc} are the results by coupling all channels of both $\Omega_{ccb}\Omega_{bbc}$ and $\Omega_{ccc}\Omega_{bbb}$ systems

ccbbbc	cccbbb		
E_{sc1}	B_{sc1}	E_{sc1}	B_{sc1}
$19,776.7 \pm 3.1$	-1.1 ± 0.1	$19,777.7 \pm 3.1$	ub
E_{sc2}	B_{sc2}	E_{sc2}	B_{sc2}
$19,766.2 \pm 2.8$	-11.6 ± 0.3	$19,807.1 \pm 1.9$	ub
E_{cc}	B_{cc}	E_{cc}	B_{cc}
$19,742.2 \pm 1.2$	-35.6 ± 1.9	$19,777.7 \pm 3.1$	ub
E'_{cc}		B'_{cc}	
$19,696.6 \pm 3.2$		-80.1 ± 1.3	

Eur. Phys. J. C. 82, 805 (2022)

- ✓ The dibaryon composed of six c or b quarks with $J = 0$ is able to be bound.
- ✓ The channel coupling between the color-singlet and hidden-color channels causes the binding energy to increase.
- ✓ It is difficult for the dibaryon with the color-singlet type $\Omega_{ccc}\Omega_{bbb}$ to form any bound state.
- ✓ It is possible for the $\Omega_{ccb}\Omega_{bbc}$ state to be bound.
- ✓ The channel coupling of all channels of both $\Omega_{ccb}\Omega_{bbc}$ and $\Omega_{ccc}\Omega_{bbb}$ structures leads to the bound conclusion of this fully heavy system composed of three c quarks and three b.

- Study of interaction between two heavy baryons

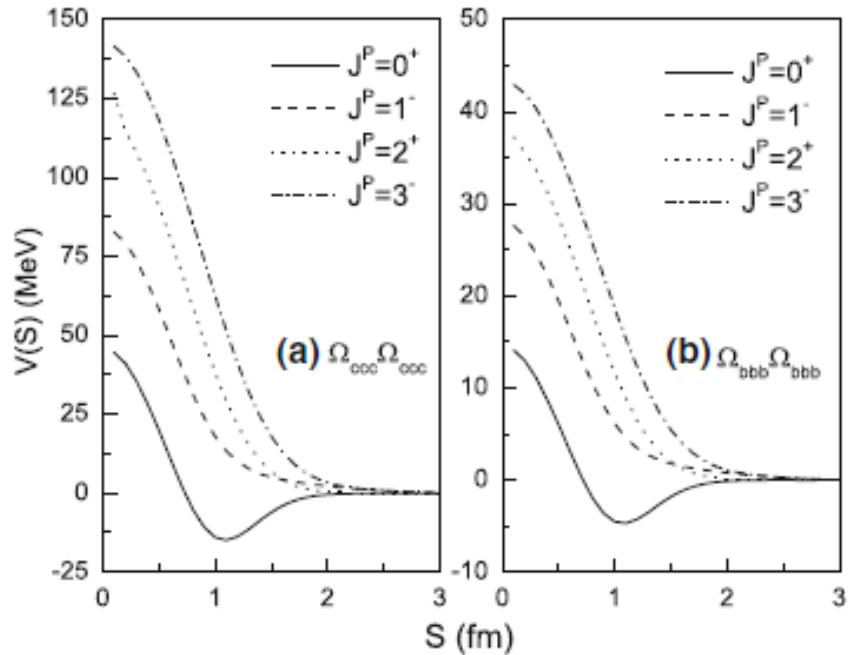


Fig. 1 The effective potential of the $\Omega_{ccc}\Omega_{ccc}$ and $\Omega_{bbb}\Omega_{bbb}$ systems

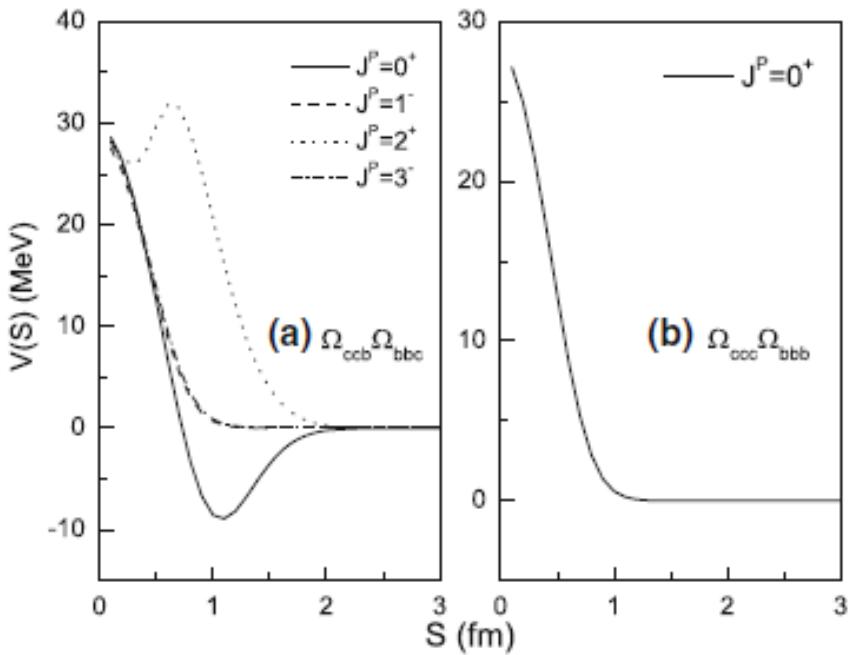


Fig. 2 The effective potential of the $\Omega_{ccb}\Omega_{bbc}$ and $\Omega_{ccc}\Omega_{bbb}$ systems

- ✓ The interactions between two Ω_{cccs} or $\Omega_{bbbb}s$ are attractive.
- ✓ There is no attractive interaction between Ω_{ccc} and Ω_{bbb} .
- ✓ The principal reason is that there is no symmetry requirement between the clusters Ω_{ccc} and Ω_{bbb} because the quark c and quark b is nonidentical quarks. While for the Ω_{ccc} or full Ω_{bbb} , the requirement of the antisymmetrization between the same baryon clusters introduce attractive interaction between two full heavy baryons, which leads to a super-heavy bound dibaryon.

- Low-energy scattering phase shifts

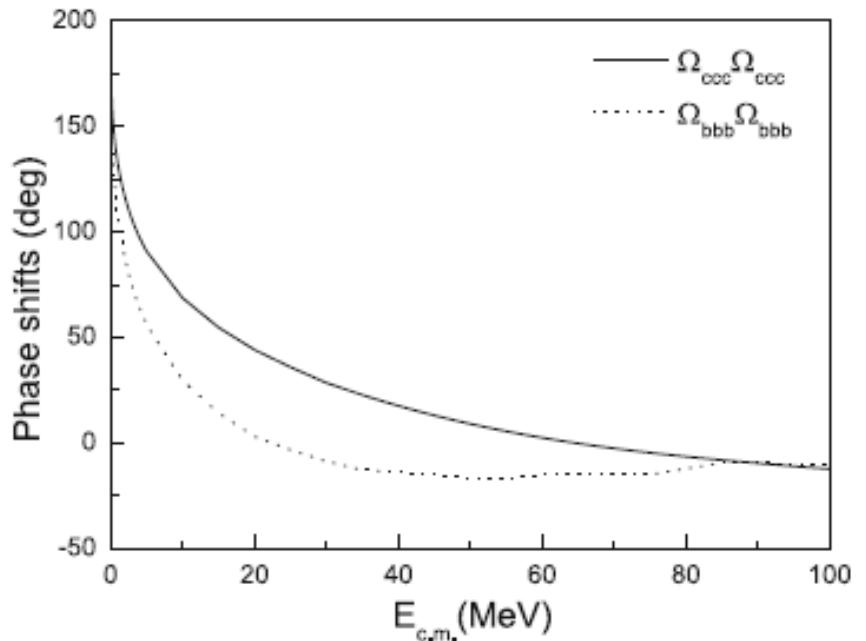


Fig. 4 The phase shifts of the S -wave $\Omega_{ccc}\Omega_{ccc}$ and $\Omega_{bbb}\Omega_{bbb}$ systems with $J^P = 0^+$

$$k \cot \delta = -\frac{1}{a_0} + \frac{1}{2} r_0 k^2 + \mathcal{O}(k^4)$$

$$r_0 = \frac{2}{\alpha} \left(1 - \frac{1}{\alpha a_0}\right)$$

$$B' = \frac{\hbar^2 \alpha^2}{2\mu}$$

Table 6 The scattering length a_0 , effective range r_0 , and binding energy B' of the $\Omega_{ccc}\Omega_{ccc}$ and $\Omega_{bbb}\Omega_{bbb}$ systems with $J^P = 0^+$

	a_0 (fm)	r_0 (fm)	B' (MeV)
$\Omega_{ccc}\Omega_{ccc}$	2.5576 ± 0.0233	1.1301 ± 0.0148	-2.72 ± 0.03
$\Omega_{bbb}\Omega_{bbb}$	2.4865 ± 0.0184	1.0760 ± 0.0091	-0.90 ± 0.01



IV. Summary

1. For the fully charm tetraquark systems, there may exist some resonance states, with masses range between 6.2 GeV to 7.4 GeV, and the quantum numbers $J^{\{P\}}=0^{\{+\}}$, $1^{\{+\}}$, and $2^{\{+\}}$.

X(6900), $J^{\{P\}}=0^{\{+\}}$; X(7200), $J^{\{P\}}=0^{\{+\}}$ or $1^{\{+\}}$; X(6600), ?;

X(6200), X(7100), $J^{\{P\}}=0^{\{+\}}$

The study of scattering process is needed.

2. The separation between the diquark and the antidiquark indicates that these states may be the compact resonance states.

3. For the fully beauty tetraquark systems, the wide resonances with masses around $19.1 \sim 19.4$ GeV are possible.

4. The dibaryons composed of six c/b quarks or 3c3b quarks is possible to be bound states.



Thanks for your attention!