"Minimal" $\mathcal{N}=1$ Theories

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Work in progress with Dan Xie

Minimal model in 2d

Consider 2d conformal field theory (CFT):

- IR fixed points with rational central charge c

  Ex: (p,q)-model $\quad p,q \in \mathbb{Z}_{>0}$. $\quad c_{p,q} = 1 - \frac{6(p-q)^2}{pq}$

- Spectrum: finite conformal primaries

  Ex. Ising model $(4,3)$ $\quad c = \frac{1}{2}$

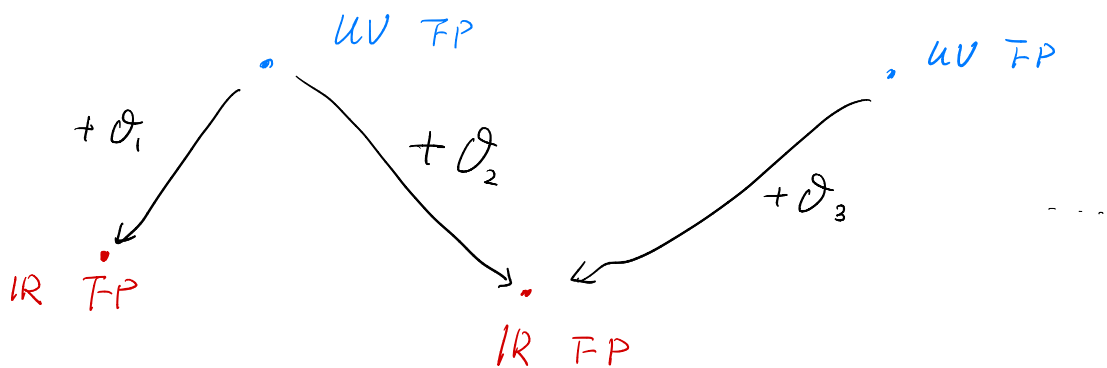
	$\mathbb{1}$	σ	ε
Δ	0	$\frac{1}{16}$	$\frac{1}{2}$

- Rationality of Virasoro alg.

Why CFT in any d

- Wilsonian picture of RG

UV fixed points + relevant def. $\longrightarrow$ IR fixed points
(CFT) (CFT)



4d analogue of minimal model?

- Looking at IR fixed points, may not have perturbative or Lagrangian description
- Global symmetry G_F. Spectrum $\{\mathcal{O}^i\}$
- For general QFT, such fixed points are still difficult to describe.

  Conformal symm. $d > 2$ less constraining than $d = 2$

Supersymmetry

- At fixed point, we have superconformal symm.
  more constraining
- Supersymmetric operators $\{\mathcal{O}_s^i\} \subset \{\mathcal{O}^i\}$

  $\mathcal{O}_s^i$: organized in short / BPS representations of the superconformal algebra, can work out even without Lagrangian, or strongly coupled system.
- Some theories have very simple $\{\mathcal{O}_s^i\}$ just like 2d minimal model $\longrightarrow$ 4d "minimal" model

4d $\mathcal{N}=2$ "minimal" model : Argyres - Douglas theory

- First example : Argyres - Douglas (A_1, A_2)

  Special fixed points of $\mathcal{N}=2$ pure $SU(3)$ SYM

- strongly coupled , Coulomb branch $\{u\}$ $\Delta = \frac{6}{5}$

Generalized AD theories

- Coulomb branch $\{u_i\}$ $i=1 \dots r$ $\Delta(u_i) \in \mathbb{Q}_{>1}$

- "minimal" model of 4d $\mathcal{N}=2$: superconformal index

  $\updownarrow$ 4d SCFT / 2d VOA correspondence

  2d minimal model

Tools

- no perturbation theory
- superconformal index

1. Susy partition function on $S^3 \times S^1 \overset{\text{conformal}}{\sim} \mathbb{R}^{3,1}$
2. Can be computed even without $\mathcal{L}$.
3. Counts susy operators we want and their relation

$\mathcal{N}=1$ deformation from AD theories

4d $\mathcal{N}=2$ AD $+$ $\mathcal{N}=1$ preserving def. $\int d\theta\, \mathcal{O}$

$$\Big\downarrow \text{RG}$$

$\mathcal{N}=1$ fixed points

- $\mathcal{N}=2$ $\mathcal{E}$-multiplets $\longrightarrow$ $\mathcal{N}=1$ chiral multiplets

  u_i $\qquad\qquad\qquad\qquad$ u_i, λ_i, S_i

  $\mathcal{O}$ has the form $u_i^a S_j^b \cdots$ and $\Delta(\mathcal{O}) <$ bound

- AD $+ \int d\theta\, \mathcal{O} \longrightarrow$ $\mathcal{N}=1$ "minimal" theories

- $\mathcal{N}=2$ Superconformal index $\longrightarrow$ $\mathcal{N}=1$ Superconformal index

Chiral operators and chiral ring relation

- AD theories, there is no relation between u_i's
- Not in $\mathcal{N}=1$

  Chiral operators $\{u_i, \lambda_i, S_i\}$ $i=1,\cdots r$

  also relations from $\mathcal{N}=2$ theory and deformation

- Relations (derived from superconformal index)

$$\begin{cases} \mathcal{O}=0 \qquad Q\mathcal{O}=0 \\ \lambda_{i\alpha}\lambda_j^{\alpha} + u_i S_j + u_j S_i + \cdots = 0 \qquad S_i\lambda_{j\alpha} + S_j\lambda_{i\alpha} + \cdots = 0 \\ S_i S_j + \cdots = 0 \end{cases}$$

Exact marginal deformation

- when there are multiple def. $\mathcal{O}_1$, $\mathcal{O}_2$ ⋯ with the same conformal dim $\Delta(\mathcal{O}_1) = \Delta(\mathcal{O}_2) = \cdots$
- can deform $\int d\theta \, (\alpha_1 \mathcal{O}_1 + \alpha_2 \mathcal{O}_2 + \cdots)$
- relative strength of α_i's $\Rightarrow$ exact marginal def.

  $\Rightarrow$ IR fixed point has a conformal manifold
- need to check there is no accidental symmetry in the IR

Examples.

1. (A_1, A_2) $\quad u_1$ $\quad \Delta(u_1) = \frac{6}{5}$

$\mathcal{O} = u_1^2$ $\quad a \sim 0.342$ $\quad c \sim 0.353$

chiral ring: u, λ_α, S

relation $\quad u^2 = 0 \quad u\lambda_\alpha = 0$

$\lambda_\alpha \lambda^\alpha + uS = 0, \quad \lambda_\alpha S = 0, \quad S^2 = 0$

2. (A_1, A_4) $\quad u_1 \quad u_2$

$\mathcal{O} = u_2$ $\quad a \sim 0.316$ $\quad c \sim 0.343$

this one is the know $\mathcal{N}=1$ theories with smallest central charges.

Summary and Future

1. We give a systematic way of constructing "minimal" $\mathcal{N}=1$ theories

2. Study duality, dynamics in the future

3. Large N limit, densed spectrum