Theory and Phenomenology of Neutrinoless Double-Beta Decay

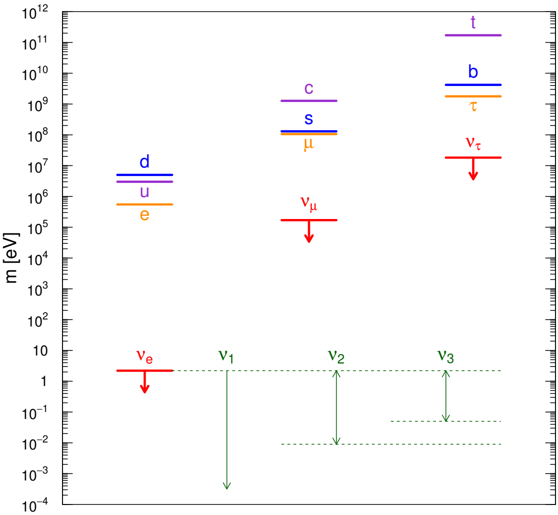
Carlo Giunti

INFN, Torino, Italy

CCEPP Summer School 2021 on Neutrino Physics

20–28 August 2021

Neutrino Masses



Origin of Neutrino Masses

	1st Generation	2nd Generation	3rd Generation
Quarks:	$\begin{pmatrix} u_L \\ d_L \end{pmatrix} \quad \begin{matrix} u_R \\ d_R \end{matrix}$	$\begin{pmatrix} c_L \\ s_L \end{pmatrix} \quad \begin{matrix} c_R \\ s_R \end{matrix}$	$\begin{pmatrix} t_L \\ b_L \end{pmatrix} \quad \begin{matrix} t_R \\ b_R \end{matrix}$
Leptons:	$\begin{pmatrix} \nu_{eL} \\ e_L \end{pmatrix} \quad \begin{matrix} \boxed{\nu_{eR}} \\ e_R \end{matrix}$	$\begin{pmatrix} \nu_{\mu L} \\ \mu_L \end{pmatrix} \quad \begin{matrix} \boxed{\nu_{\mu R}} \\ \mu_R \end{matrix}$	$\begin{pmatrix} \nu_{\tau L} \\ \tau_L \end{pmatrix} \quad \begin{matrix} \boxed{\nu_{\tau R}} \\ \tau_R \end{matrix}$

- Standard Model extension: $\nu_R \Rightarrow$ Dirac mass Lagrangian

$$\mathscr{L}_\mathrm{D} \sim m_\mathrm{D} \, \overline{\nu_L} \nu_R$$

- This is Standard Model physics, because m_D is generated by the standard Higgs mechanism through the Yukawa Lagrangian:

$$\mathscr{L}_\mathrm{Y} \sim y \, \overline{L_L} \widetilde{\Phi} \nu_R \quad \xrightarrow[\text{Breaking}]{\text{Symmetry}} \quad y \, v \, \overline{\nu_L} \nu_R \quad \Rightarrow \quad m_\mathrm{D} \sim y \, v = y \, 246 \, \mathrm{GeV}$$

- Extremely small Yukawa couplings are needed to get $m_\mathrm{D} \lesssim 1 \, \mathrm{eV}$:

$$y \lesssim 10^{-11}$$

It is considered unnatural, unless there is a protecting BSM symmetry.

Beyond the Standard Model

- The introduction of ν_R leads us beyond the Standard Model because they can have the Majorana mass Lagrangian

$$\mathscr{L}_R^{\mathrm{M}} \sim m_R \, \overline{\nu_R^c} \nu_R \qquad \text{singlet under SM symmetries!}$$

- This is beyond the Standard Model because m_R is not generated by the Higgs mechanism of the Standard Model $\Rightarrow$ new BSM physics is required.
- The Majorana mass Lagrangian can be avoided by imposing lepton number conservation which should anyway be explained by some physics beyond the Standard Model.
  - Dirac mass Lagrangian: $m_{\mathrm{D}} \overline{\nu_L} \nu_R$

$$L(\nu_L) = L(\nu_R) = +1 \Longrightarrow \text{Lepton number conservation}$$

  - Majorana mass Lagrangian: $m_R \, \overline{\nu_R^c} \nu_R$

$$\boxed{\nu_R^c = \mathcal{C} \overline{\nu_R}^T = \mathcal{C} \gamma_0^T (\nu_R^\dagger)^T}$$

$$L(\nu_R^c) = -L(\nu_R) \Longrightarrow \text{Lepton number violation } |\Delta L| = 2$$

- A Majorana mass Lagrangian for ν_L is forbidden by the symmetries of the Standard Model

No Majorana Neutrino Mass in the SM

- Majorana Mass Lagrangian for SM ν_L: $\mathscr{L}_L^{\mathrm{M}} \sim m_L \, \overline{\nu_L^c} \nu_L = -\nu_L^T \, \mathcal{C}^\dagger \, \nu_L$
- Eigenvalues of the weak isospin I, of its third component I_3, of the hypercharge Y and of the charge Q of the lepton and Higgs multiplets:

	I	I_3	Y	$Q = I_3 + \frac{Y}{2}$
lepton doublet $L_L = \begin{pmatrix} \nu_L \\ \ell_L \end{pmatrix}$	$1/2$	$1/2$ $-1/2$	-1	0 -1
lepton singlet ℓ_R	0	0	-2	-1
Higgs doublet $\Phi(x) = \begin{pmatrix} \phi_+(x) \\ \phi_0(x) \end{pmatrix}$	$1/2$	$1/2$ $-1/2$	$+1$	1 0

- $\nu_L^T \, \mathcal{C}^\dagger \, \nu_L$ has $I_3 = 1$ and $Y = -2$ $\Longrightarrow$ needed $Y = 2$ Higgs triplet ($I = 1$, $I_3 = -1$)
- Compare with Dirac Mass Lagrangian $\propto \overline{\nu_R} \nu_L$ with $I_3 = 1/2$ and $Y = -1$ balanced by $\phi_0 \to v$ with $I_3 = -1/2$ and $Y = +1$

Dirac-Majorana Mass Lagrangian

- One-Generation Dirac-Majorana mass Lagrangian:

$$\mathscr{L}^{\text{D+M}}_{\text{mass}} = \mathscr{L}^{\text{D}}_{\text{mass}} + \mathscr{L}^{R}_{\text{mass}}$$

$$= -\, m_{\text{D}} \left(\overline{\nu_L}\, \nu_R + \overline{\nu_R}\, \nu_L \right) - \frac{1}{2}\, m_R \left(\overline{\nu_R^c}\, \nu_R + \overline{\nu_R}\, \nu_R^c \right)$$

$$= -\frac{1}{2} \begin{pmatrix} \overline{\nu_L} & \overline{\nu_R^c} \end{pmatrix} \begin{pmatrix} 0 & m_{\text{D}} \\ m_{\text{D}} & m_R \end{pmatrix} \begin{pmatrix} \nu_L^c \\ \nu_R \end{pmatrix} - \frac{1}{2} \begin{pmatrix} \overline{\nu_L^c} & \overline{\nu_R} \end{pmatrix} \begin{pmatrix} 0 & m_{\text{D}} \\ m_{\text{D}} & m_R \end{pmatrix} \begin{pmatrix} \nu_L \\ \nu_R^c \end{pmatrix}$$

- Since the Dirac mass m_{D} couples ν_L and ν_R, these chiral fields are not mass eigenstates.

- To get the mass eigenstate fields that describe the physical massive neutrinos we need to diagonalize the Dirac-Majorana mass Lagrangian.

- Dirac-Majorana mass Lagrangian:

$$\mathscr{L}_{\text{mass}}^{\text{D+M}} = -\frac{1}{2} \begin{pmatrix} \overline{\nu_L^c} & \overline{\nu_R} \end{pmatrix} \begin{pmatrix} 0 & m_{\text{D}} \\ m_{\text{D}} & m_R \end{pmatrix} \begin{pmatrix} \nu_L \\ \nu_R^c \end{pmatrix} + \text{H.c.}$$

- In matrix form: $\mathscr{L}_{\text{mass}}^{\text{D+M}} = -\frac{1}{2} \overline{N_L^c} \, M \, N_L + \text{H.c.}$

with $N_L = \begin{pmatrix} \nu_L \\ \nu_R^c \end{pmatrix}$ and $M = \begin{pmatrix} 0 & m_{\text{D}} \\ m_{\text{D}} & m_R \end{pmatrix}$

- Diagonalization: $N_L = U \, n_L$ with $n_L = \begin{pmatrix} \nu_{1L} \\ \nu_{2L} \end{pmatrix}$

$$\mathscr{L}_{\text{mass}}^{\text{D+M}} = -\frac{1}{2} \overline{n_L^c} \, U^T \, M \, U \, n_L + \text{H.c.}$$

$$U^T \, M \, U = \begin{pmatrix} m_1 & 0 \\ 0 & m_2 \end{pmatrix} \quad \text{with real} \quad m_k \geq 0$$

$$\mathscr{L}_{\text{mass}}^{\text{D+M}} = -\frac{1}{2} \sum_{k=1,2} m_k \left(\overline{\nu_{kL}^c} \, \nu_{kL} + \overline{\nu_{kL}} \, \nu_{kL}^c \right) = -\frac{1}{2} \sum_{k=1,2} m_k \, \overline{\nu_k} \, \nu_k$$

$$\nu_k = \nu_{kL} + \nu_{kL}^c \quad \Longrightarrow \quad \boxed{\nu_k = \nu_k^c}$$

Massive neutrinos are Majorana!

- The treatment can be generalized to three generations with the same conlusion:

The Dirac-Majorana mass Lagrangian implies Majorana massive neutrinos!

- A definition of a total lepton number for the Majorana massive neutrino field ν_k is forbidden by the Majorana constraint:

$$\cancel{L = +1} \quad \longleftarrow \quad \boxed{\nu_k = \nu_k^c} \quad \longrightarrow \quad \cancel{L = -1}$$

- We can still assign a lepton number to ν_{kL} in the SM limit:

$$L(\nu_{kL}) = +1 \quad \Longrightarrow \quad L(\nu_{kL}^c) = -1$$

- The conservation of this lepton number is violated by the Majorana mass Lagrangian:

$$\mathscr{L}_{\text{mass}}^{\text{D+M}} = -\frac{1}{2} \sum_k m_k \left(\overline{\nu_{kL}^c} \, \nu_{kL} + \overline{\nu_{kL}} \, \nu_{kL}^c \right) \quad \Longrightarrow \quad \boxed{|\Delta L| = 2}$$

- Best process to find $|\Delta L| = 2$: Neutrinoless Double-β Decay

Seesaw Mechanism

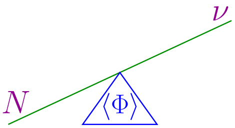
$$\mathscr{L}_{\text{mass}}^{\text{D+M}} = -\frac{1}{2}\begin{pmatrix}\overline{\nu_L^c} & \overline{\nu_R}\end{pmatrix}\begin{pmatrix}0 & m_{\text{D}} \\ m_{\text{D}} & m_R\end{pmatrix}\begin{pmatrix}\nu_L \\ \nu_R^c\end{pmatrix} + \text{H.c.}$$

m_R can be arbitrarily large (not protected by SM symmetries)

$m_R \sim$ scale of new physics beyond Standard Model $\Rightarrow m_R \gg m_{\text{D}}$

diagonalization of $\begin{pmatrix}0 & m_{\text{D}} \\ m_{\text{D}} & m_R\end{pmatrix} \Longrightarrow \quad m_\nu \simeq \dfrac{m_{\text{D}}^2}{m_R} \qquad m_N \simeq m_R$

natural explanation of smallness
of light neutrino masses



seesaw mechanism

massive neutrinos are Majorana $\Rightarrow$ $\boxed{\beta\beta_{0\nu}}$

$$\nu \simeq -i\left(\nu_L - \nu_L^c\right) \qquad N \simeq \nu_R + \nu_R^c$$

3-GEN $\Rightarrow$ effective low-energy 3-ν mixing

Majorana Neutrinos

There are compelling arguments in favor of Majorana Neutrinos:

- A Majorana field is simpler than a Dirac field:
  - A Majorana field corresponds to the fundamental spinor representation of the Lorentz group.
  - A Dirac field is made of two Majorana fields degenerate in mass.

  Therefore, if there is no additional constraint (as L conservation), a neutral elementary particle as the neutrino is naturally Majorana.

- The seesaw mechanism if ν_R is introduced to generate neutrino masses.

- A general Effective Field Theory argument from high-energy new physics:

$$\mathscr{L} = \mathscr{L}_{\text{SM}} + \frac{g_5}{\mathcal{M}}\,\mathscr{O}_5 + \frac{g_6}{\mathcal{M}^2}\,\mathscr{O}_6 + \ldots$$

  - $\mathscr{O}_5$: Majorana neutrino masses (Lepton number violation and $\beta\beta_{0\nu}$ decay).

$$\mathscr{O}_5 = (\overline{L}\,\widetilde{\Phi})\,(\widetilde{\Phi}^T L^c) \qquad L = \begin{pmatrix} \nu_L \\ \ell_L \end{pmatrix} \qquad \widetilde{\Phi} = \begin{pmatrix} \phi_0 \\ -\phi_+ \end{pmatrix}$$

  - $\mathscr{O}_6$: Baryon number violation (proton decay), neutrino Non-Standard Interactions (NSI), neutrino magnetic moments.

- The only $\mathrm{SU}(2)_L \times \mathrm{U}(1)_Y$ invariant dim-5 Lagrangian term that can be constructed with SM fields:

$$\mathscr{L}_5 = -\frac{g_5}{\mathcal{M}} \left[\left(\overline{L_L}\, \widetilde{\Phi} \right) \left(\widetilde{\Phi}^T L_L^c \right) + \left(\overline{L_L^c}\, \widetilde{\Phi}^* \right) \left(\widetilde{\Phi}^\dagger L_L \right) \right]$$

- Electroweak Symmetry Breaking:

$$\widetilde{\Phi} = i\sigma_2 \Phi^* \quad \xrightarrow[\text{Breaking}]{\text{EW Symmetry}} \quad \frac{1}{\sqrt{2}} \begin{pmatrix} v + H(x) \\ 0 \end{pmatrix}$$

- $\mathscr{L}_5 \xrightarrow[\text{Breaking}]{\text{EW Symmetry}} \mathscr{L}^{\mathrm{M}}_{\mathrm{mass}} = -\frac{1}{2} \frac{g_5\, v^2}{\mathcal{M}} \left(\overline{\nu_L}\, \nu_L^c + \overline{\nu_L^c}\, \nu_L \right)$

- Majorana neutrino mass: $\boxed{m = \frac{g_5\, v^2}{\mathcal{M}}}$

- General Seesaw Mechanism: $$m \propto \frac{v^2}{\mathcal{M}} = v \, \frac{v}{\mathcal{M}}$$

  natural explanation of the strong suppression of neutrino masses with respect to the electroweak scale

- Example: $\mathcal{M} \sim 10^{15}\,\text{GeV}$ (GUT scale)

$$v \sim 10^2\,\text{GeV} \implies \frac{v}{\mathcal{M}} \sim 10^{-13} \implies m \sim 10^{-2}\,\text{eV}$$

Effective Low-Energy 3ν Majorana Mixing

$$\nu_\alpha = \sum_{k=1}^{3} U_{\alpha k} \nu_k \quad \text{for} \quad \alpha = e, \mu, \tau \quad \text{with} \quad \boxed{\nu_k = \nu_k^c}$$

Standard Parameterization of Mixing Matrix (as CKM)

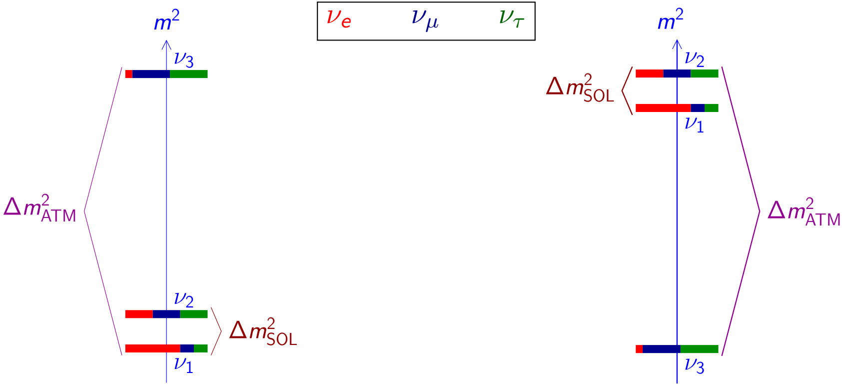
$$U = \begin{pmatrix} c_{12}c_{13} & s_{12}c_{13} & s_{13}e^{-i\delta_{13}} \\ -s_{12}c_{23}-c_{12}s_{23}s_{13}e^{i\delta_{13}} & c_{12}c_{23}-s_{12}s_{23}s_{13}e^{i\delta_{13}} & s_{23}c_{13} \\ s_{12}s_{23}-c_{12}c_{23}s_{13}e^{i\delta_{13}} & -c_{12}s_{23}-s_{12}c_{23}s_{13}e^{i\delta_{13}} & c_{23}c_{13} \end{pmatrix} \begin{pmatrix} 1 & 0 & 0 \\ 0 & e^{i\lambda_{21}} & 0 \\ 0 & 0 & e^{i\lambda_{31}} \end{pmatrix}$$

$$c_{ab} \equiv \cos\vartheta_{ab} \qquad s_{ab} \equiv \sin\vartheta_{ab} \qquad 0 \leq \vartheta_{ab} \leq \frac{\pi}{2} \qquad 0 \leq \delta_{13}, \lambda_{21}, \lambda_{31} < 2\pi$$

OSCILLATION PARAMETERS
- 3 Mixing Angles: ϑ_{12}, ϑ_{23}, ϑ_{13}
- 1 CPV Dirac Phase: δ_{13}
- 2 independent $\Delta m^2_{kj} \equiv m^2_k - m^2_j$: Δm^2_{21}, Δm^2_{31}

2 CPV Majorana Phases: λ_{21}, λ_{31} $\Longleftrightarrow$ $\boxed{|\Delta L| = 2}$ processes

Neutrino Mass Ordering



Normal Ordering

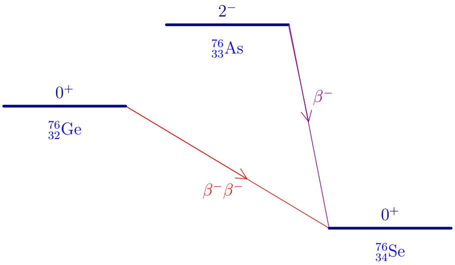
Inverted Ordering

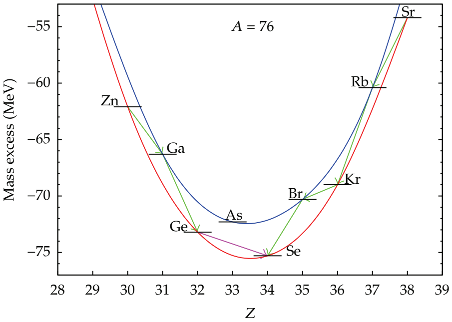
$$\Delta m^2_{31} > \Delta m^2_{32} > 0$$

$$\Delta m^2_{32} < \Delta m^2_{31} < 0$$

absolute scale is not determined by neutrino oscillation data

Neutrinoless Double-Beta Decay





- Semi-empirical Bethe-Weizsäcker mass formula (liquid drop model):

$$M = Z m_p + N m_n - E_{\mathrm{B}}(Z,N) \quad \leftarrow \quad \text{Binding Energy}$$

$$E_{\mathrm{B}}(Z,N) = a_{\mathrm{V}} A - a_{\mathrm{S}} A^{2/3} - a_{\mathrm{C}} \frac{Z(Z-1)}{A^{1/3}} - a_{\mathrm{A}} \frac{(N-Z)^2}{A} + \delta_{\mathrm{P}}(Z,N)$$

- Pairing term due to spin-coupling:

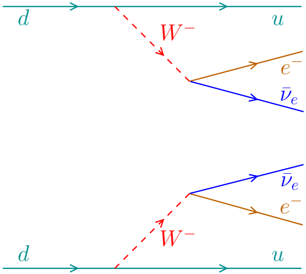
$$\delta_{\mathrm{P}}(Z,N) = \begin{cases} a_{\mathrm{P}} A^{k_{\mathrm{P}}} & \text{if both } Z \text{ and } N \text{ are even } (A \text{ is even}) \\ -a_{\mathrm{P}} A^{k_{\mathrm{P}}} & \text{if both } Z \text{ and } N \text{ are odd } (A \text{ is even}) \\ 0 & \text{if } A \text{ is odd} \end{cases}$$

Two-Neutrino Double-β Decay: $\Delta L = 0$

$$\mathcal{N}(A,Z) \to \mathcal{N}(A,Z+2) + e^- + e^- + \bar{\nu}_e + \bar{\nu}_e$$

$$(T^{2\nu}_{1/2})^{-1} = G_{2\nu}\,|\mathcal{M}_{2\nu}|^2$$

second order weak interaction process in the Standard Model

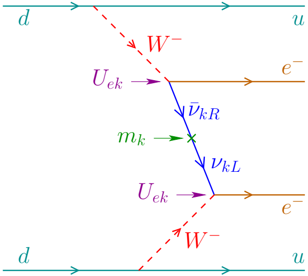


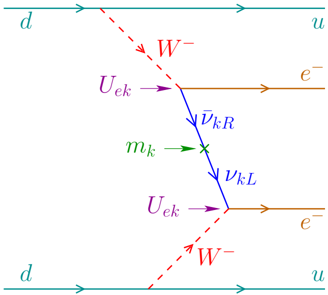
Neutrinoless Double-β Decay: $\Delta L = 2$

$$\mathcal{N}(A,Z) \to \mathcal{N}(A,Z+2) + e^- + e^-$$

$$(T^{0\nu}_{1/2})^{-1} = G_{0\nu}\,|\mathcal{M}_{0\nu}|^2\,|m_{\beta\beta}|^2$$

effective Majorana mass $\quad |m_{\beta\beta}| = \left|\sum_k U_{ek}^2\, m_k\right|$





- First vertex (below): leptonic CC current in β and $\beta\beta$ decay:

$$j_\mu^{(\beta)}(x) = \overline{e}(x)\gamma_\mu\,(1-\gamma_5)\,\nu_e(x) = \sum_k \overline{e}(x)\gamma_\mu\,(1-\gamma_5)\,U_{ek}\nu_k(x)$$

  - $\overline{e}(x)$ creates e^- as needed
  - $(1-\gamma_5)\,\nu_k(x) = 2\nu_{kL}(x)$ destroys ν_{kL} as needed (and creates $\bar{\nu}_{kR}$)

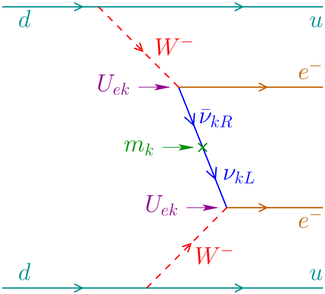
- Second vertex (above): $j_\mu^{(\beta)\dagger}(x) = \sum_k \overline{\nu_k}(x)U_{ek}^*\gamma_\mu\,(1-\gamma_5)\,e(x)$?
  - Does not work, because $e(x)$ destroys e^- and creates e^+
  - We need to rearrange $j_\mu^{(\beta)}(x)$ that contains the needed e^- creation operator

- Charge conjugation: $\psi^c = \mathcal{C}\,\overline{\psi}^T \qquad \overline{\psi^c} = -\psi^T \mathcal{C}^\dagger$

- Charge-conjugation matrix: $\mathcal{C}\,\gamma_\mu^T\,\mathcal{C}^{-1} = -\gamma_\mu$ and $\begin{cases} \mathcal{C}^\dagger = \mathcal{C}^{-1} \\ \mathcal{C}^T = -\mathcal{C} \\ \mathcal{C}\,\gamma_5^T\,\mathcal{C}^{-1} = \gamma_5 \end{cases}$

- Second vertex: the same $j_\mu^{(\beta)}(x)$ as in the first vertex:

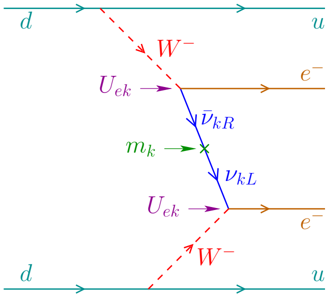
$$
\begin{aligned}
j_\mu^{(\beta)}(x) &= \overline{e}(x)\gamma_\mu\,(1-\gamma_5)\,\nu_e(x) = [\overline{e}(x)\gamma_\mu\,(1-\gamma_5)\,\nu_e(x)]^T \\
&= -\,\nu_e^T(x)\left(1-\gamma_5^T\right)\gamma_\mu^T\overline{e}^T(x) \\
&= -\,\nu_e^T(x)\mathcal{C}^\dagger\mathcal{C}\left(1-\gamma_5^T\right)\mathcal{C}^\dagger\mathcal{C}\gamma_\mu^T\mathcal{C}^\dagger\mathcal{C}\overline{e}^T(x) \\
&= -\,\overline{\nu_e^c}(x)\,(1-\gamma_5)\,\gamma_\mu e^c(x) = -\sum_k \overline{\nu_k^c}(x)U_{ek}\,(1-\gamma_5)\,\gamma_\mu e^c(x)
\end{aligned}
$$



- First vertex (below): $$j_\mu^{(\beta)}(x) = \sum_k \overline{e}(x)\gamma_\mu \left(1-\gamma_5\right) U_{ek}\nu_k(x)$$
  - $\overline{e}(x)$ creates e^- as needed
  - $(1-\gamma_5)\,\nu_k(x)$ destroys ν_{kL} as needed (and creates $\bar{\nu}_{kR}$)

- Second vertex (above): $$j_\mu^{(\beta)}(x) = -\sum_k \overline{\nu_k^c}(x) U_{ek}\left(1-\gamma_5\right)\gamma_\mu e^c(x)$$

  Obviously the properties of this version of $j_\mu^{(\beta)}(x)$ are the same as those of $j_\mu^{(\beta)}(x)$ in the first vertex:
  - $e^c(x)$ creates e^- as needed
  - $\overline{\nu_k^c}(x)\,(1-\gamma_5) = \overline{\nu_{kR}^c}(x)$ creates $\bar{\nu}_{kR}$ as needed (and destroys ν_{kL})



- Leptonic tensor in the $\beta\beta_{0\nu}$ amplitude:

$$A_{\mu\nu} = -\sum_{k,j} \overline{e}(x)\gamma_\mu (1-\gamma_5) U_{ek}\nu_k(x)\overline{\nu_j^c}(y) U_{ej} (1-\gamma_5) \gamma_\nu e^c(y)$$

- $\nu_k(x)\overline{\nu_j^c}(y)$ gives a propagator only if $\nu_k(x)$ and $\nu_j^c(x)$ are the same field: $k = j$ and $\nu_j^c = \nu_j$ $\quad\leftarrow\quad$ massive Majorana neutrinos!
- Propagator for Majorana massive neutrinos:

$$\langle 0| T\left[\nu_k(x)\overline{\nu_j^c}(y)\right]|0\rangle = \langle 0| T[\nu_k(x)\overline{\nu_k}(y)]|0\rangle \delta_{kj}$$

- The propagator $\langle 0|\, T[\nu_k(x)\overline{\nu_k}(y)]\,|0\rangle$ is the same as for a Dirac field:

$$\langle 0|\, T[\nu_k(x)\overline{\nu_k}(y)]\,|0\rangle = \lim_{\epsilon\to 0} i \int \frac{d^4p}{(2\pi)^4} \frac{\not{p} + m_k}{p^2 - m_k^2 + i\,\epsilon}\, e^{-ip\cdot(x-y)}$$

- Leptonic tensor:

$$A_{\mu\nu} \propto \sum_k U_{ek}^2 \int \frac{d^4p}{(2\pi)^4} \overline{e}(x)\gamma_\mu\,(1-\gamma_5)\,\frac{\not{p} + m_k}{p^2 - m_k^2}\,(1-\gamma_5)\,\gamma_\nu e^c(y)\, e^{-ip\cdot(x-y)}$$

  - Numerator: $(1-\gamma_5)\not{p}(1-\gamma_5) = \not{p}(1+\gamma_5)(1-\gamma_5) = 0 \Rightarrow \boxed{m_k > 0}$ needed!
  - Denominator: for light massive neutrinos $m_k^2 \ll p^2$

$$p \gtrsim 1/R \quad \text{with} \quad R \approx 1.2\, A^{1/3}\,\text{fm}$$

$$A \lesssim 200 \quad \Longrightarrow \quad R \lesssim 7\,\text{fm} \quad \Longrightarrow \quad p \gtrsim 30\,\text{MeV} \gg m_k$$

- Therefore, m_k in the denominator can be neglected and we obtain:

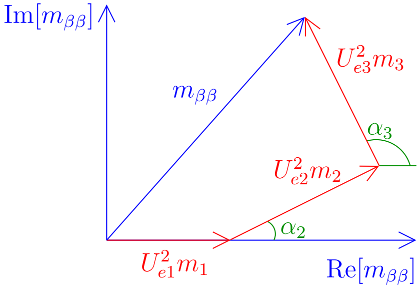
$$A_{\mu\nu} \propto \underbrace{\sum_k U_{ek}^2 m_k}_{m_{\beta\beta}} \int \frac{d^4p}{(2\pi)^4} \overline{e}(x)\gamma_\mu\,(1-\gamma_5)\,\gamma_\nu e^c(y)\,\frac{e^{-ip\cdot(x-y)}}{p^2}$$

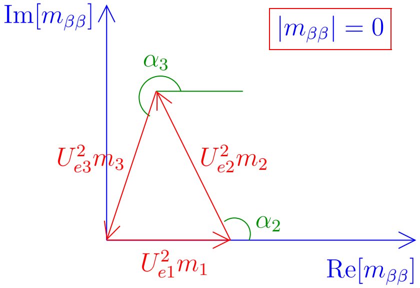
Effective Majorana Neutrino Mass

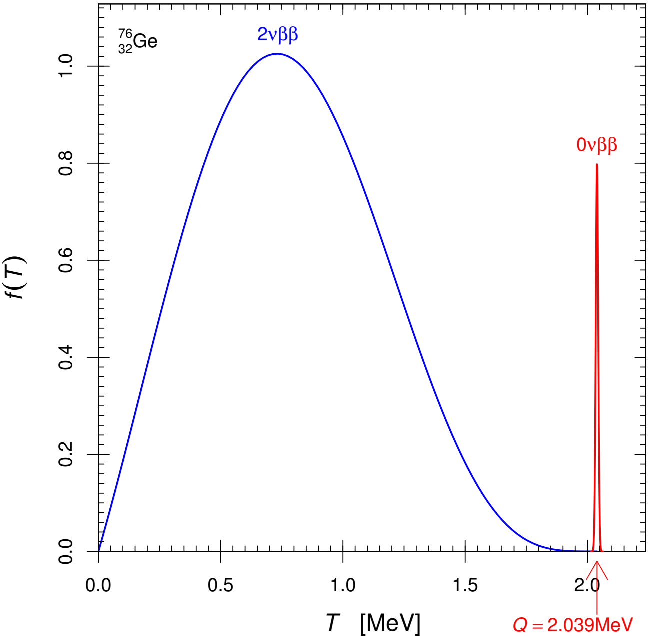
$$m_{\beta\beta} = \sum_k U_{ek}^2 \, m_k \qquad \text{complex } U_{ek} \Rightarrow \text{possible cancellations}$$

$$m_{\beta\beta} = |U_{e1}|^2 \, m_1 + |U_{e2}|^2 \, e^{i\alpha_2} \, m_2 + |U_{e3}|^2 \, e^{i\alpha_3} \, m_3$$

$$\alpha_2 = 2\lambda_2 \qquad \alpha_3 = 2\left(\lambda_3 - \delta_{13}\right)$$

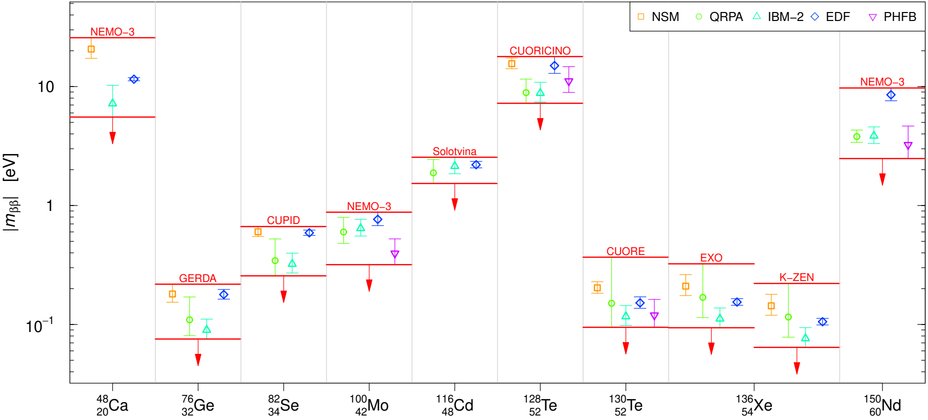





$^{76}_{32}$Ge
2νββ
0νββ
f(T)
T [MeV]
Q = 2.039MeV
0.0
0.5
1.0
1.5
2.0
0.2
0.4
0.6
0.8

90% C.L. Experimental Bounds

$\beta\beta^-$ decay	experiment	$T^{0\nu}_{1/2}$ [y]	$m_{\beta\beta}$ [eV]
${}^{48}_{20}\text{Ca} \to {}^{48}_{22}\text{Ti}$	ELEGANT-VI	$> 1.4 \times 10^{22}$	$< 6.6 - 31$
${}^{76}_{32}\text{Ge} \to {}^{76}_{34}\text{Se}$	Heidelberg-Moscow	$> 1.9 \times 10^{25}$	$< 0.23 - 0.67$
	IGEX	$> 1.6 \times 10^{25}$	$< 0.25 - 0.73$
	Majorana	$> 4.8 \times 10^{25}$	$< 0.20 - 0.43$
	GERDA	$> 8.0 \times 10^{25}$	$< 0.12 - 0.26$
${}^{82}_{34}\text{Se} \to {}^{82}_{36}\text{Kr}$	NEMO-3	$> 1.0 \times 10^{23}$	$< 1.8 - 4.7$
${}^{100}_{42}\text{Mo} \to {}^{100}_{44}\text{Ru}$	NEMO-3	$> 2.1 \times 10^{25}$	$< 0.32 - 0.88$
${}^{116}_{48}\text{Cd} \to {}^{116}_{50}\text{Sn}$	Solotvina	$> 1.7 \times 10^{23}$	$< 1.5 - 2.5$
${}^{128}_{52}\text{Te} \to {}^{128}_{54}\text{Xe}$	CUORICINO	$> 1.1 \times 10^{23}$	$< 7.2 - 18$
${}^{130}_{52}\text{Te} \to {}^{130}_{54}\text{Xe}$	CUORE	$> 1.5 \times 10^{25}$	$< 0.11 - 0.52$
${}^{136}_{54}\text{Xe} \to {}^{136}_{56}\text{Ba}$	EXO	$> 1.1 \times 10^{25}$	$< 0.17 - 0.49$
	KamLAND-Zen	$> 1.1 \times 10^{26}$	$< 0.06 - 0.16$
${}^{150}_{60}\text{Nd} \to {}^{150}_{62}\text{Sm}$	NEMO-3	$> 2.1 \times 10^{25}$	$< 2.6 - 10$



$$\boxed{|m_{\beta\beta}| = \left(G_{0\nu} \, |\mathcal{M}_{0\nu}|^2 \, T^{0\nu}_{1/2} \right)^{-1/2}}$$

Light-ν Exchange in a Nucleus

$$[T_{1/2}^{0\nu}]^{-1} = G(Z,N)\,|M_{0\nu}|^2\, m_{\beta\beta}^2$$

Phase-space factor

Nuclear matrix element

"Traditional" part of matrix element:

$$\boxed{M_{0\nu} = M_{0\nu}^{GT} - \frac{g_V^2}{g_A^2} M_{0\nu}^{F} + \ldots} \quad \times\, g_A^{\,2}$$

with

$$M_{0\nu}^{GT} = \langle F | \sum_{i,j} H(r_{ij})\, \boldsymbol{\sigma}_i \cdot \boldsymbol{\sigma}_j\, \tau_i^+ \tau_j^+ \; | I \rangle + \ldots$$

$$M_{0\nu}^{F} = \langle F | \sum_{i,j} H(r_{ij})\, \tau_i^+ \tau_j^+ \; | I \rangle + \ldots$$

$$H(r) \approx \frac{2R}{\pi r} \int_0^\infty dq \frac{\sin qr}{q + \overline{E} - (E_i + E_f)/2} \quad \text{roughly} \propto 1/r$$

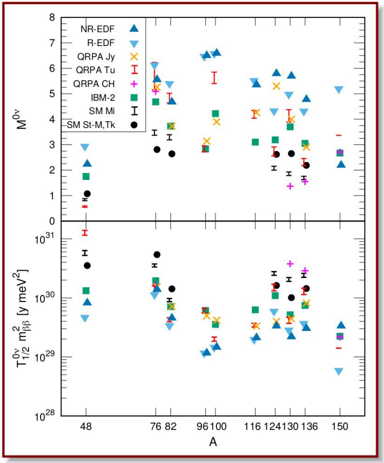
Corrections are from "forbidden" terms, weak nucleon form factors, many-body currents, other effects of high-energy physics that depend on framework.

[Jonathan Engel @ Mini Workshop: Nuclear Theory of Neutrinoless Double-Beta Decay, 22 July 2020]

Light-ν-Exchange Matrix Elements

Significant spread. And all the models may miss important physics.

Uncertainty hard to quantify.

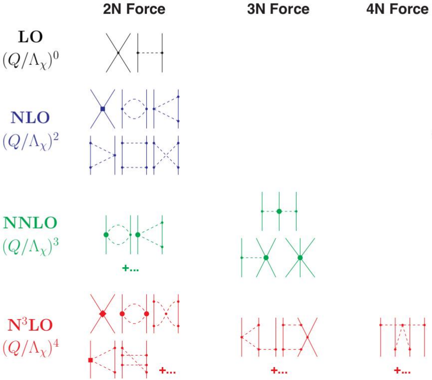


[Jonathan Engel @ Mini Workshop: Nuclear Theory of Neutrinoless Double-Beta Decay, 22 July 2020]

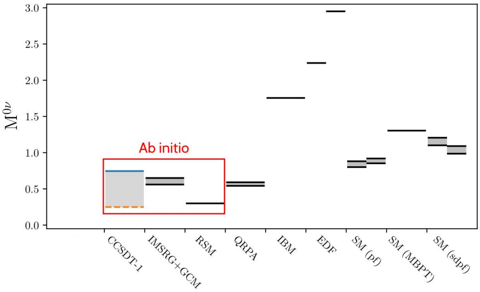
The Way Forward: Ab Initio Nuclear Theory

Starts with chiral effective field theory.

Nucleons, pions sufficient below chiral-symmetry breaking scale.

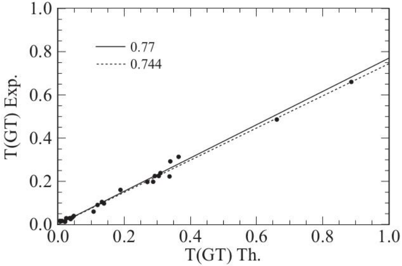


^{48}Ca: Ab-Inito $0\nu\beta\beta$ Matrix Elements vs. Older Ones

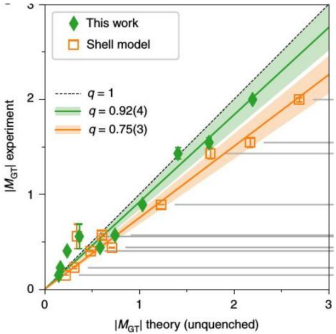


[Jonathan Engel @ Mini Workshop: Nuclear Theory of Neutrinoless Double-Beta Decay, 22 July 2020]

β decays (e^- capture) challenge for nuclear theory



Martinez-Pinedo et al. PRC53 2602(1996)



Gysbers et al. Nature Phys. 15 428 (2019)

$$\langle F| \sum_i [g_A \, \boldsymbol{\sigma}_i \tau_i^-]^{\text{eff}} \, |I\rangle \, , \quad [\boldsymbol{\sigma}_i \tau]^{\text{eff}} \approx 0.7 \boldsymbol{\sigma}_i \tau$$

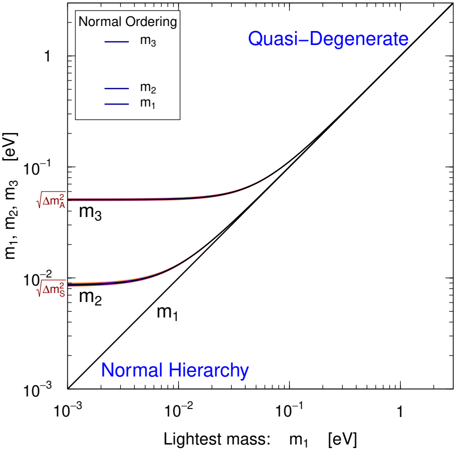
Phenomenological models need $\boldsymbol{\sigma}_i \tau$ "quenching"

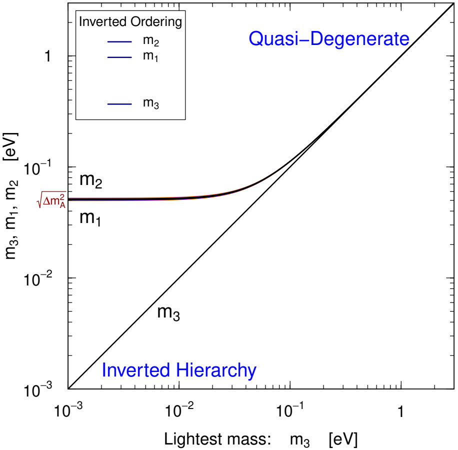
Ab initio calculations including meson-exchange currents do not need any "quenching"

[Javier Menendez @ Mini Workshop: Nuclear Theory of Neutrinoless Double-Beta Decay, 22 July 2020]

Predictions from Neutrino Oscillations

$$m_{\beta\beta} = |U_{e1}|^2\, m_1 + |U_{e2}|^2\, e^{i\alpha_2}\, m_2 + |U_{e3}|^2\, e^{i\alpha_3}\, m_3$$



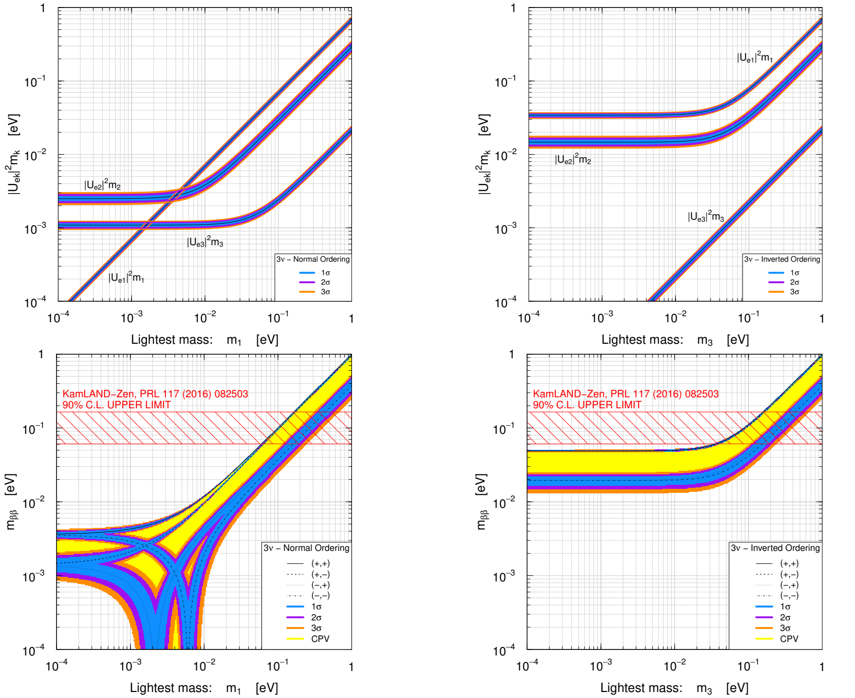


$$m_2^2 = m_1^2 + \Delta m_{21}^2 = m_1^2 + \Delta m_S^2$$

$$m_3^2 = m_1^2 + \Delta m_{31}^2 = m_1^2 + \Delta m_A^2$$

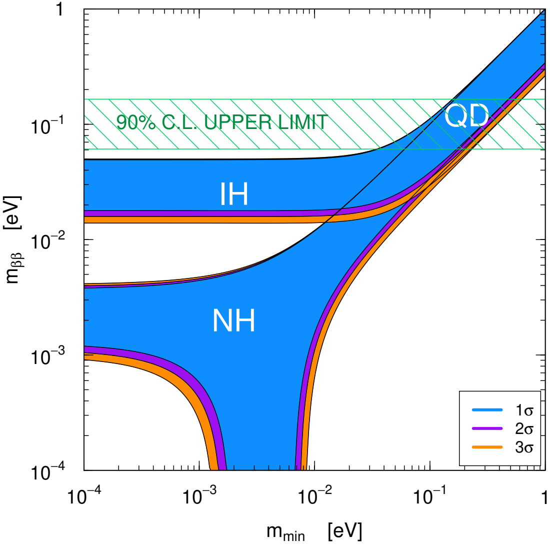
$$m_1^2 = m_3^2 - \Delta m_{31}^2 = m_3^2 + \Delta m_A^2$$

$$m_2^2 = m_1^2 + \Delta m_{21}^2 \simeq m_3^2 + \Delta m_A^2$$



$|U_{e2}|^2 m_2$
$|U_{e3}|^2 m_3$
$|U_{e1}|^2 m_1$
3ν – Normal Ordering
1σ
2σ
3σ
$|U_{ek}|^2 m_k$ [eV]
Lightest mass: m_1 [eV]
$|U_{e1}|^2 m_1$
$|U_{e2}|^2 m_2$
$|U_{e3}|^2 m_3$
3ν – Inverted Ordering
1σ
2σ
3σ
$|U_{ek}|^2 m_k$ [eV]
Lightest mass: m_3 [eV]
KamLAND–Zen, PRL 117 (2016) 082503
90% C.L. UPPER LIMIT
3ν – Normal Ordering
(+,+)
(+,−)
(−,+)
(−,−)
1σ
2σ
3σ
CPV
$m_{\beta\beta}$ [eV]
Lightest mass: m_1 [eV]
KamLAND–Zen, PRL 117 (2016) 082503
90% C.L. UPPER LIMIT
3ν – Inverted Ordering
(+,+)
(+,−)
(−,+)
(−,−)
1σ
2σ
3σ
CPV
$m_{\beta\beta}$ [eV]
Lightest mass: m_3 [eV]

$$m_{\beta\beta} = |U_{e1}|^2\, m_1 + |U_{e2}|^2\, e^{i\alpha_2}\, m_2 + |U_{e3}|^2\, e^{i\alpha_3}\, m_3$$



- Quasi-Degenerate:

$$|m_{\beta\beta}| \simeq m_\nu \sqrt{1 - s^2_{2\vartheta_{12}} s^2_{\alpha_2}}$$

- Inverted Hierarchy:

$$|m_{\beta\beta}| \simeq \sqrt{\Delta m^2_{\text{A}} (1 - s^2_{2\vartheta_{12}} s^2_{\alpha_2})}$$

- Normal Hierarchy:

$$|m_{\beta\beta}| \simeq |s^2_{12} \sqrt{\Delta m^2_{\text{S}}} + e^{i\alpha} s^2_{13} \sqrt{\Delta m^2_{\text{A}}}|$$

$$\simeq |2.7 + 1.2 e^{i\alpha}| \times 10^{-3}\,\text{eV}$$

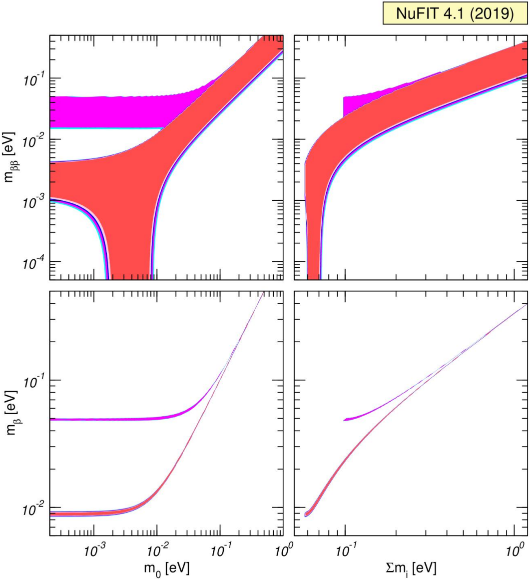
- If $|m_{\beta\beta}| \lesssim 10^{-2}\,\text{eV}$

$$\Downarrow$$

Normal Spectrum

Status of m_β and $m_{\beta\beta}$

- Results of the global fit of oscillation data can be projected onto m_β and $m_{\beta\beta}$ as a function of lightest ν mass m_0 (or $\sum m_i$);
- no neutrino ordering assumed: both cases considered on equal footing $\Rightarrow$ **IO** region disfavored at $\Delta\chi^2 = 6.2$ by oscillation data (growing to $\Delta\chi^2 = 10.4$ if Super-K atmospheric data also included);
- extension of $m_{\beta\beta}$ regions dominated by unknown $\eta_i \Rightarrow$ flat χ^2 valley closed by steep walls $\Rightarrow 1\sigma$, 2σ, 3σ, ... ranges very similar.

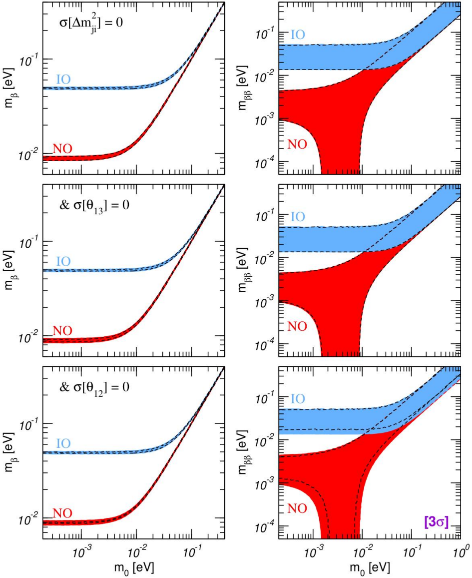


[Michele Maltoni @ Mini Workshop: Particle Theory of Neutrinoless Double-Beta Decay, 15 July 2020]

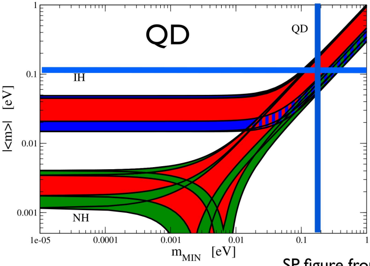
Impact of osc. parameters

- Uncertainty on Δm^2_{21} and Δm^2_{31} has negligible impact on the extension of the m_β and $m_{\beta\beta}$ regions;
- uncertainty on θ_{13} marginally affect m_β, and is irrelevant for $m_{\beta\beta}$;
- the only oscillation parameter whose precision has a visible (albeit small) impact on m_β and $m_{\beta\beta}$ ranges is θ_{12};

$\Rightarrow$ the present phenomenological picture will not be significantly affected by future improvements in the determination of the oscillation parameters, **except for what concerns the neutrino <u>mass ordering</u>**.

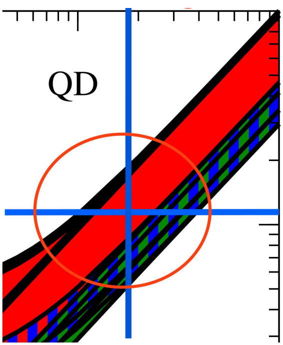


[Michele Maltoni @ Mini Workshop: Particle Theory of Neutrinoless Double-Beta Decay, 15 July 2020]



SP, figure from PDG

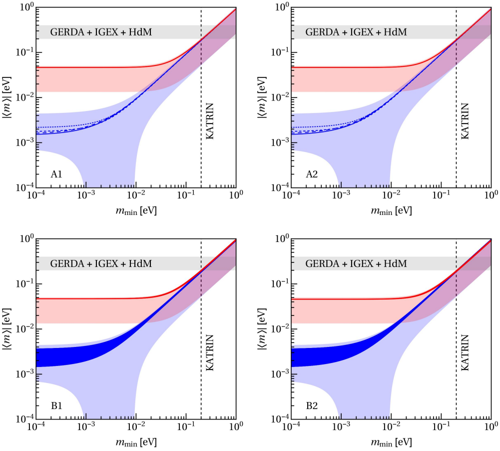
If m_{ee} and neutrino masses are measured with sufficient precision, then it may be possible to establish CPV due to Majorana phases.



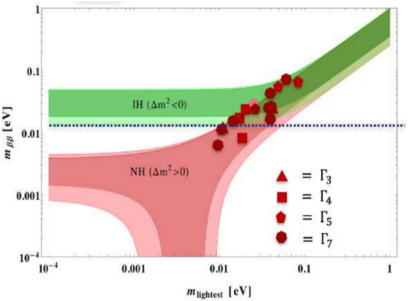
However, this requires also a very precise determination of NME.

[Silvia Pascoli @ Mini Workshop: Particle Theory of Neutrinoless Double-Beta Decay, 15 July 2020]

Examples of model predictions



I. Girardi, S. T. Petcov, A. Titov, 1605.04172

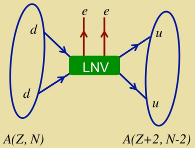


F. Feruglio, Bethe Colloquium, 18 June 2020,

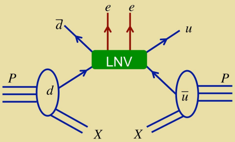
[Silvia Pascoli @ Mini Workshop: Particle Theory of Neutrinoless Double-Beta Decay, 15 July 2020]

TeV Scale LNV: $0\nu\beta\beta$-Decay & Colliders

$0\nu\beta\beta$-Decay



pp Collisions



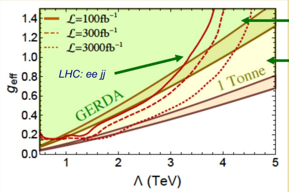
Simplified Model: Illustrative Case

$$\boxed{\mathcal{L}_{\rm INT} = g_1 \bar{Q}_i^a d^a S_i + g_2 \epsilon^{ij} \bar{L}_i F S_j^* + \text{H.c.}}$$

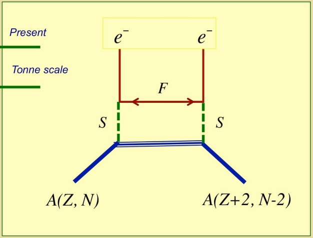
S: (1, 2, ½)

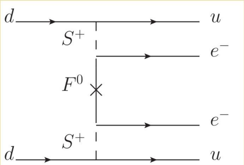
F: (1, 0, 0) Majorana

Benchmark Sensitivity: TeV LNV



T. Peng, MRM, P. Winslow 1508.04444



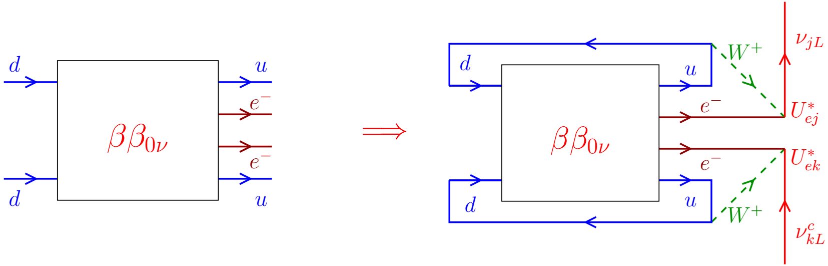


LHC: $pp \to jj\, e^- e^-$

[Michael Ramsey-Musolf @ Mini Workshop: Particle Theory of Neutrinoless Double-Beta Decay, 15 July 2020]

$\beta\beta_{0\nu}$ Decay $\Leftrightarrow$ Majorana Neutrino Mass

- $|m_{\beta\beta}|$ can vanish because of unfortunate cancellations among the ν_1, ν_2, ν_3 contributions or because neutrinos are Dirac particles.
- However, $\beta\beta_{0\nu}$ decay can be generated by another BSM mechanism.
- In this case, Majorana masses are generated by radiative corrections:



[Schechter, Valle, PRD 25 (1982) 2951; Takasugi, PLB 149 (1984) 372]

- Majorana Mass Lagrangian: $\boxed{\mathscr{L}^{\text{M}}_{\text{mass}} = -\frac{1}{2}\, m_{\text{box}} \sum_{j,k} U^*_{ej} U^*_{ek} \overline{\nu_{jL}} \nu^c_{kL} + \text{H.c.}}$

- Very small four-loop diagram contribution: $m_{\text{box}} \sim 10^{-24}\,\text{eV}$

[Duerr, Lindner, Merle, arXiv:1105.0901]

Conclusions

- It is likely that light neutrinos are Majorana particles and generate $\beta\beta_{0\nu}$ decay through $m_{\beta\beta}$.
- Unfortunate cancellations among the light massive neutrino contributions can suppress $|m_{\beta\beta}|$.
- $\beta\beta_{0\nu}$ decay can also be generated by heavy Majorana neutrinos or other BSM physics.
- In any case finding $\beta\beta_{0\nu}$ decay is important for:
  - Finding total Lepton number violation $(|\Delta L| = 2)$ (BSM physics).
  - Establishing the Majorana nature of neutrinos (BSM physics).
- On the other hand, even if $\beta\beta_{0\nu}$ decay is not found, it is very difficult to prove experimentally that neutrinos are Dirac particles, because
  - A Dirac neutrino is equivalent to 2 Majorana neutrinos with the same mass.
  - It is impossible to prove experimentally that the mass splitting is exactly zero.