Neutrino cosmology

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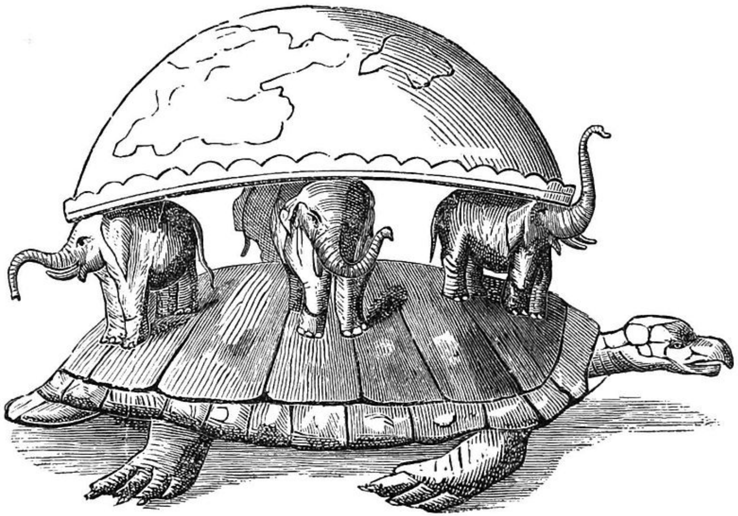
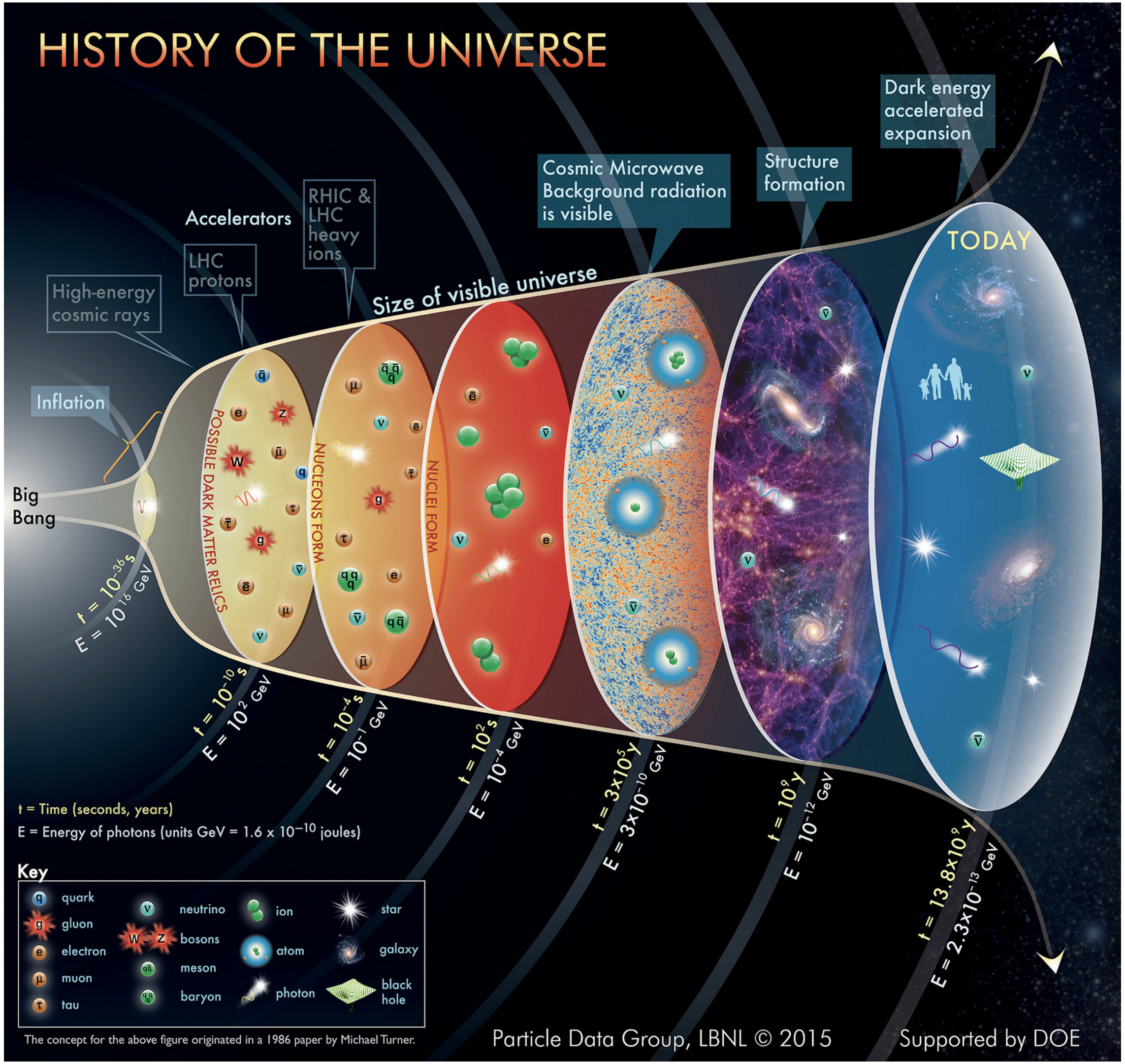


CCEPP Summer School on Neutrino Physics, August 20 – 28, 2021

Or, how neutrinos fit into this grand scheme?



The concept for the above figure originated in a 1986 paper by Michael Turner.

Particle Data Group, LBNL © 2015

Supported by DOE

The grand lecture plan...

Lecture 1: Neutrinos in homogeneous cosmology

1. The homogeneous and isotropic universe
2. The hot universe and the relic neutrino background
3. Measuring the relic neutrino background via N_{eff}

Lecture 2: Neutrinos in inhomogeneous cosmology

Lecture 1: Neutrinos in homogeneous cosmology

1. The homogeneous and isotropic universe

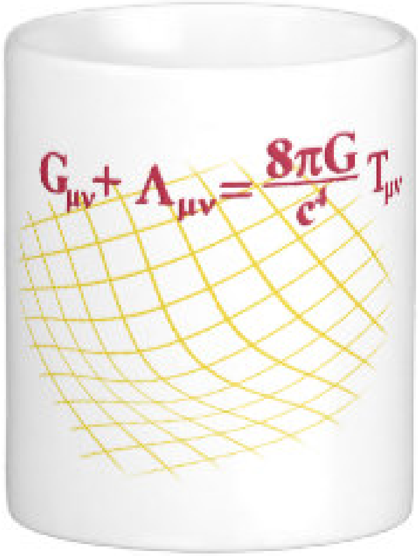
2. The hot universe and the relic neutrino background

3. Measuring the relic neutrino background via N_{eff}

Useful references...

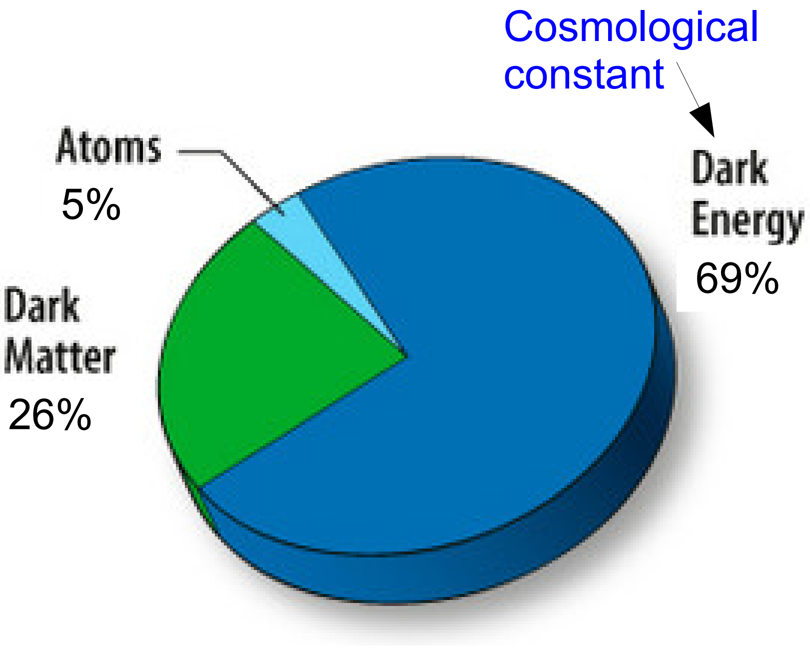
- **Lecture notes**
  - A. D. Dolgov, *Neutrinos in cosmology,* Phys. Rept. **370** (2002) 333 [hep-ph/0202122]
  - J. Lesgourgues & S. Pastor, *Massive neutrinos and cosmology*, Phys. Rep. **429** (2006) 307 [astro-ph/0603494].
  -
- **Textbooks**
  - J. Lesgourgues, G. Mangano, G. Miele & S. Pastor, *Neutrino cosmology*

1. The homogeneous and isotropic universe...

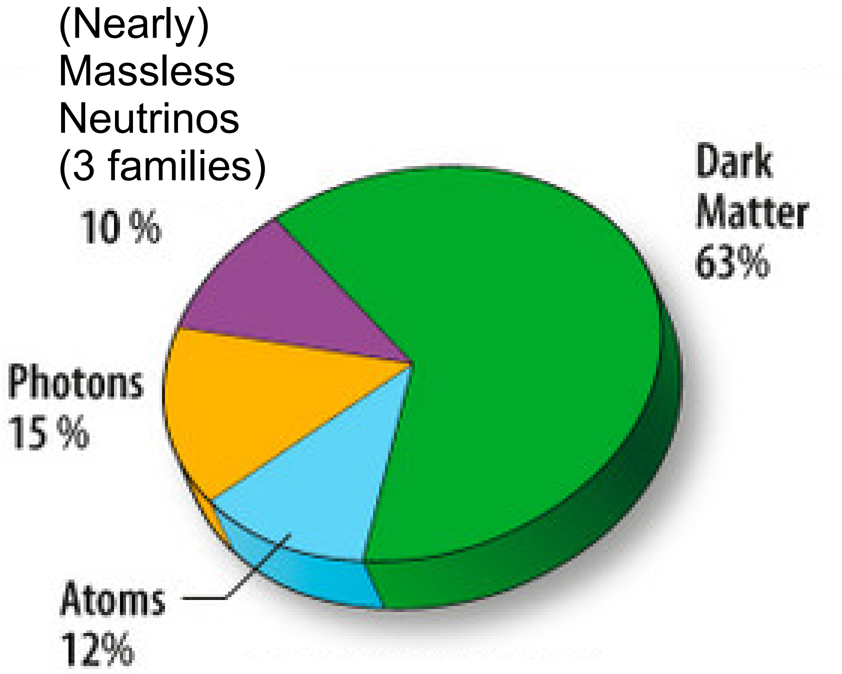


The concordance flat ΛCDM model...

- The simplest model consistent with **present observations**.



Composition today



13.4 billion years ago
(at photon decoupling)

Plus flat spatial geometry+initial conditions from single-field inflation

Friedmann-Lemaître-Robertson-Walker universe...

- **Cosmological principle**: our universe is spatially homogeneous and isotropic on sufficiently large length scales (i.e., we are not special).
  - Homogeneous → same everywhere
  - Isotropic → same in all directions
  - Sufficiently large scales → > O(100 Mpc)

Size of visible universe
~ O(10 Gpc)

Isotropic but
not homogeneous

Homogeneous but
not isotropic

Homogeneous
and isotropic

Friedmann-Lemaître-Robertson-Walker universe...

- Homogeneity and isotropy imply maximally symmetric 3-spaces (3 translational and 3 rotational symmetries).
  - A spacetime geometry that satisfies these requirements:

$$\boxed{ds^2 = -dt^2 + a^2(t)\left[\frac{dr^2}{1-Kr^2} + r^2\left(d\theta^2 + \sin^2\theta\, d\phi^2\right)\right]}$$

FLRW metric

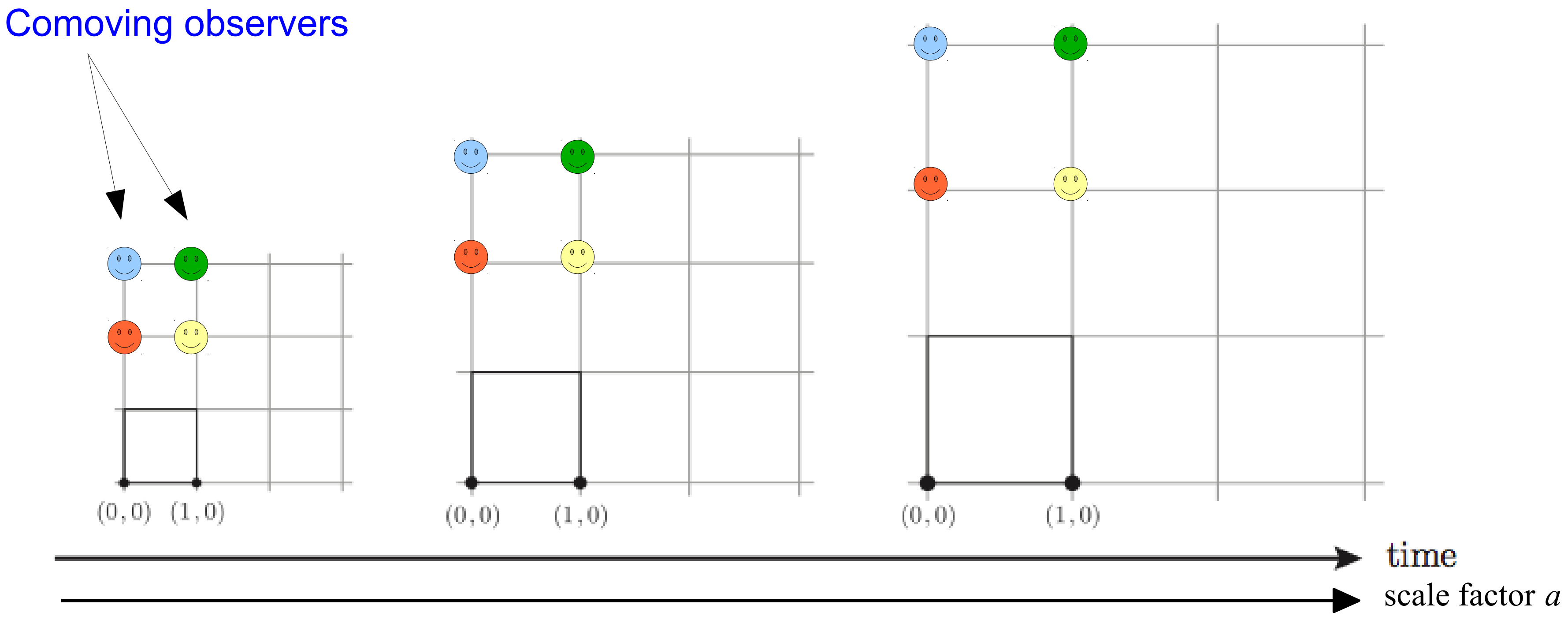
$a(t)$ = Scale factor

Spatial geometry
K = -1 (hyperbolic), 0 (flat), +1 (spherical)

$\frac{a(t_2)}{a(t_1)} =$ Factor by which a physical length scale increases between time t_1 and t_2.

- An observer at rest with the FLRW spatial coordinates is a **comoving observer**.

$$ds^2 = -dt^2 + a^2(t)\left[\frac{dr^2}{1-Kr^2} + r^2\left(d\theta^2 + \sin^2\theta\, d\phi^2\right)\right]$$



→ The **physical distance** between two comoving observers increases with time, but the coordinate distance between them remains unchanged.

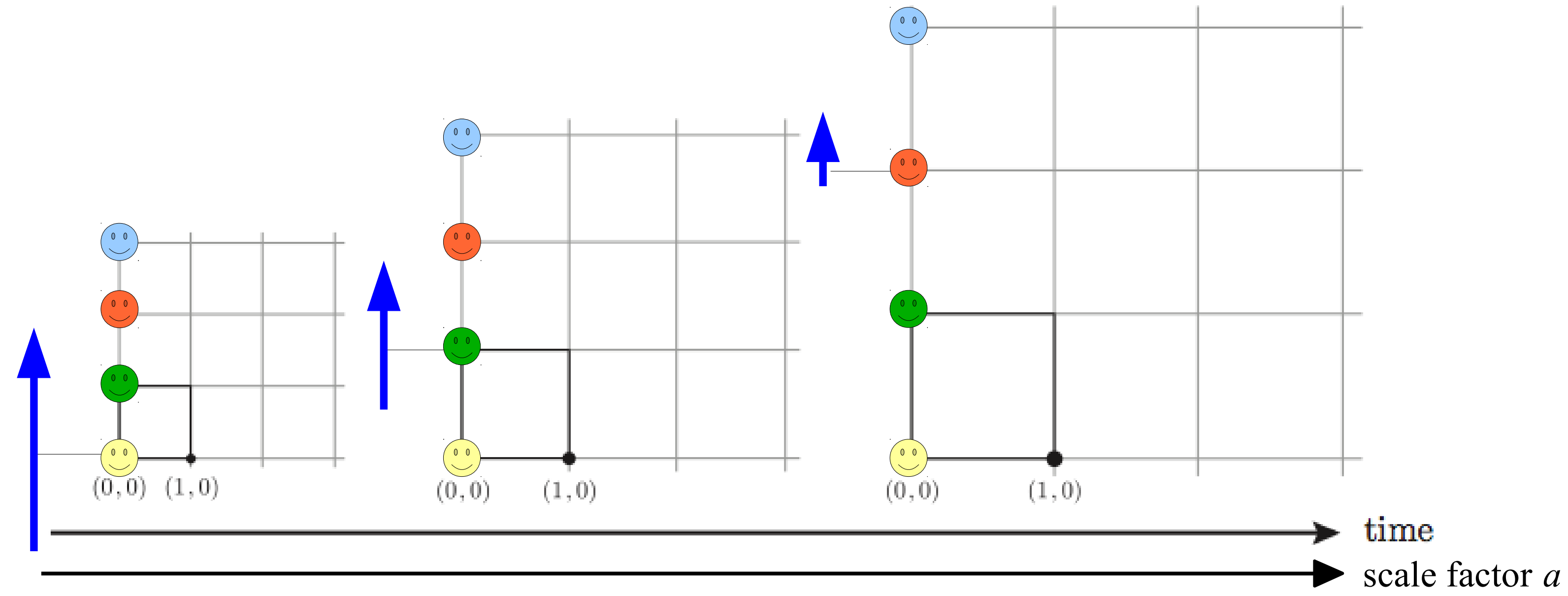
Cosmological redshift...

- All test particles (massive or massless) moving on geodesics of an FLRW universe suffer cosmological redshift:

Momentum of a point particle measured by comoving observers →

$$\left|\vec{p}\right| \propto a^{-1}$$

→ Physical momentum of a particle decreases with expanding space.



- Alternatively, in terms of wavelength: $\lambda \propto 1/|\vec{p}|$

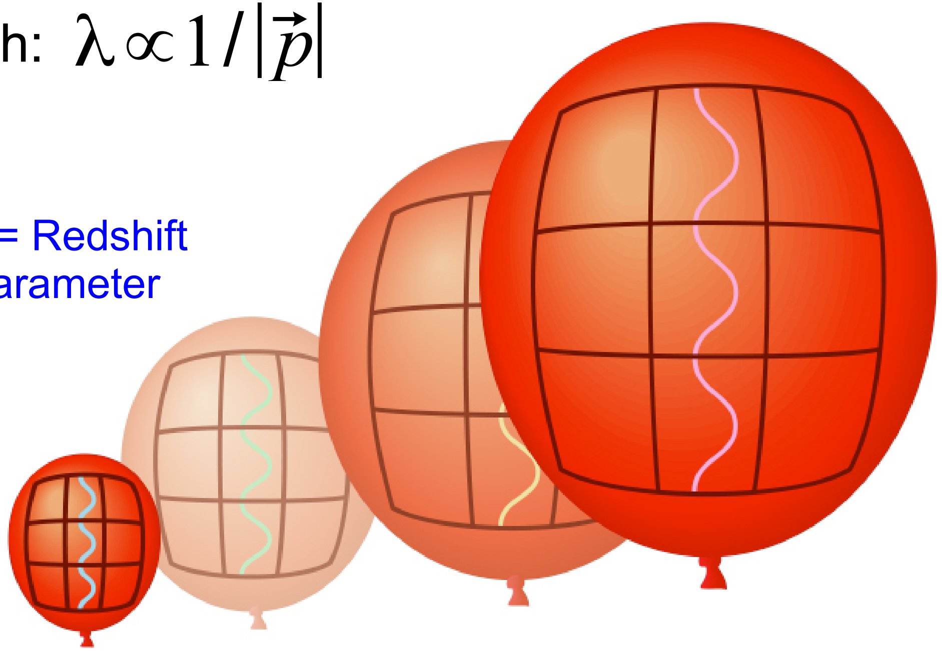
Wavelength measured by comoving observer

t_0 = today

$$\boxed{\frac{\lambda_0}{\lambda_e} = \frac{a(t_0)}{a(t_e)} \equiv 1 + z}$$

z = Redshift parameter

Wavelength of particle (usually photon) emitted by comoving emitter



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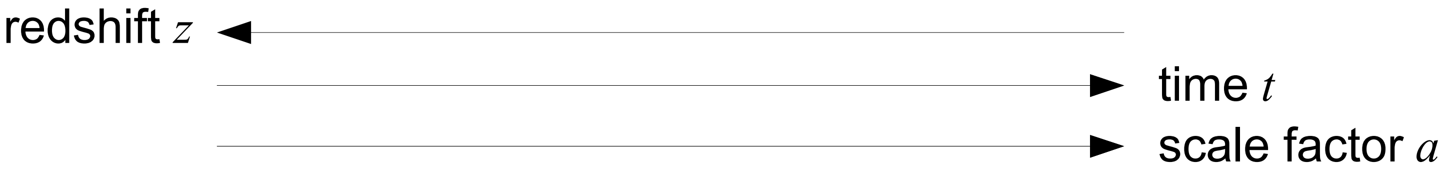
- In an FLRW universe, there is a one-to-one correspondence between t, a, and z:



→ We use them interchangeably as a measure of time.

Matter/energy content: conservation law...

$$\nabla_\mu T^{\mu\nu}_{(\alpha)}=0$$

- Local conservation of energy-momentum in an FLRW universe implies:

Energy density

Pressure

$$\boxed{\frac{d\rho_\alpha}{dt}+3\frac{\dot{a}}{a}\left(\rho_\alpha+P_\alpha\right)=0}$$

Continuity equation

  - There is **one such equation for each substance α**.

- We need in addition to specify a relation between $\rho(t)$ and $P(t)$ (a property of the substance).

$$w_\alpha(t)\equiv\frac{P_\alpha(t)}{\rho_\alpha(t)}$$

w = Equation of state parameter

- Assuming a constant w_α:

$$\boxed{\rho_\alpha(t)\propto a^{-3(1+w_\alpha)}}$$

How energy density evolves with the scale factor.

Matter/energy content: what is out there?

$$\rho_\alpha(t) \propto a^{-3(1+w_\alpha)}$$

- **Nonrelativistic** matter
  - Atoms (or constituents thereof); “baryons” in cosmology-speak.
  - Dark matter (does not emit light but feels gravity); GR people call it “dust”.

$$w_m \simeq 0$$

$$\Rightarrow \rho_m \propto a^{-3}$$

Volume expansion

- **Ultra-relativistic** radiation (at least for a significant part of their evolution history)
  - Photons (mainly the CMB)
  - Relic neutrinos (analogous to CMB)
  - Gravitational waves

$$w_r = 1/3$$

$$\Rightarrow \rho_r \propto a^{-4}$$

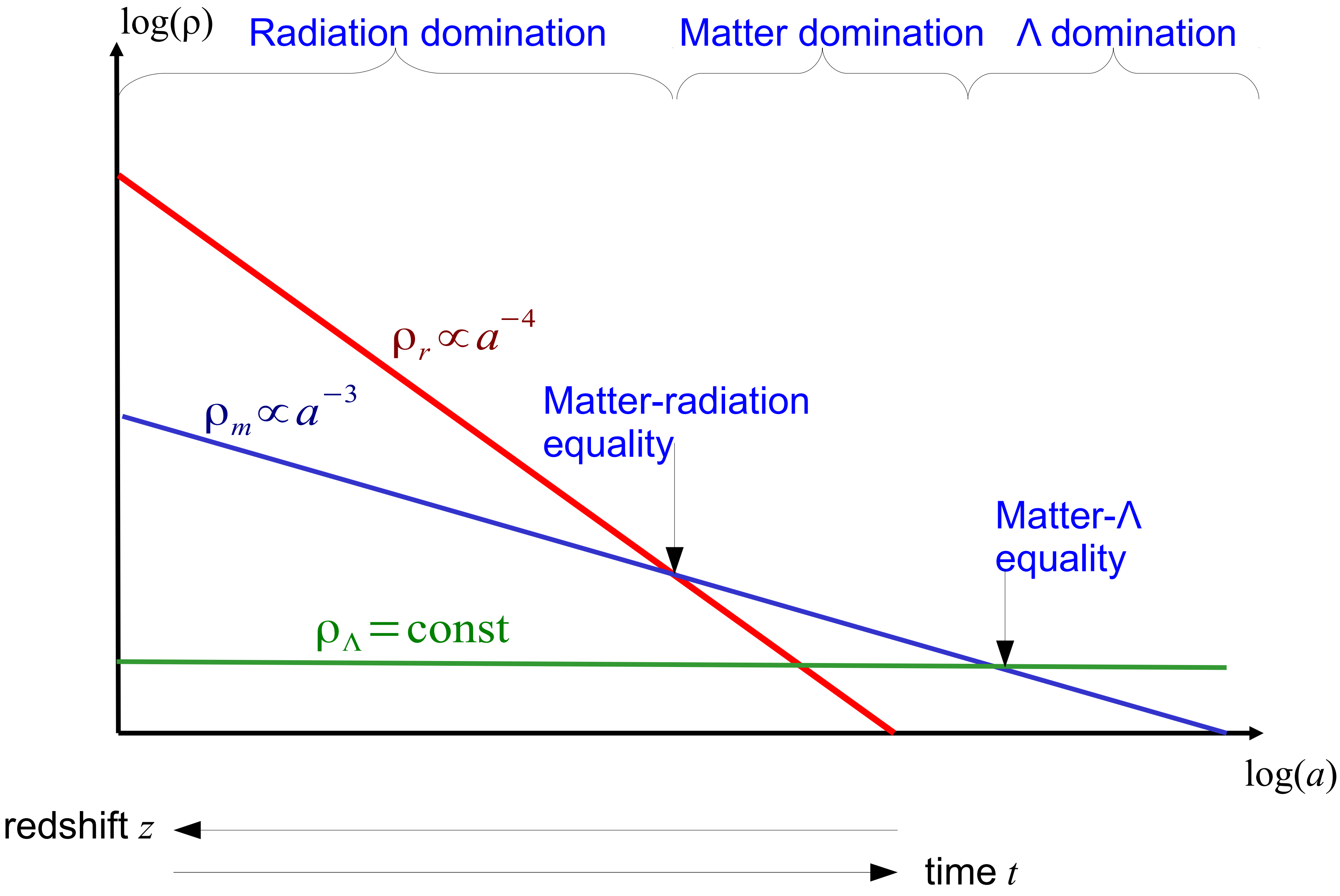
Volume expansion
+ momentum redshift

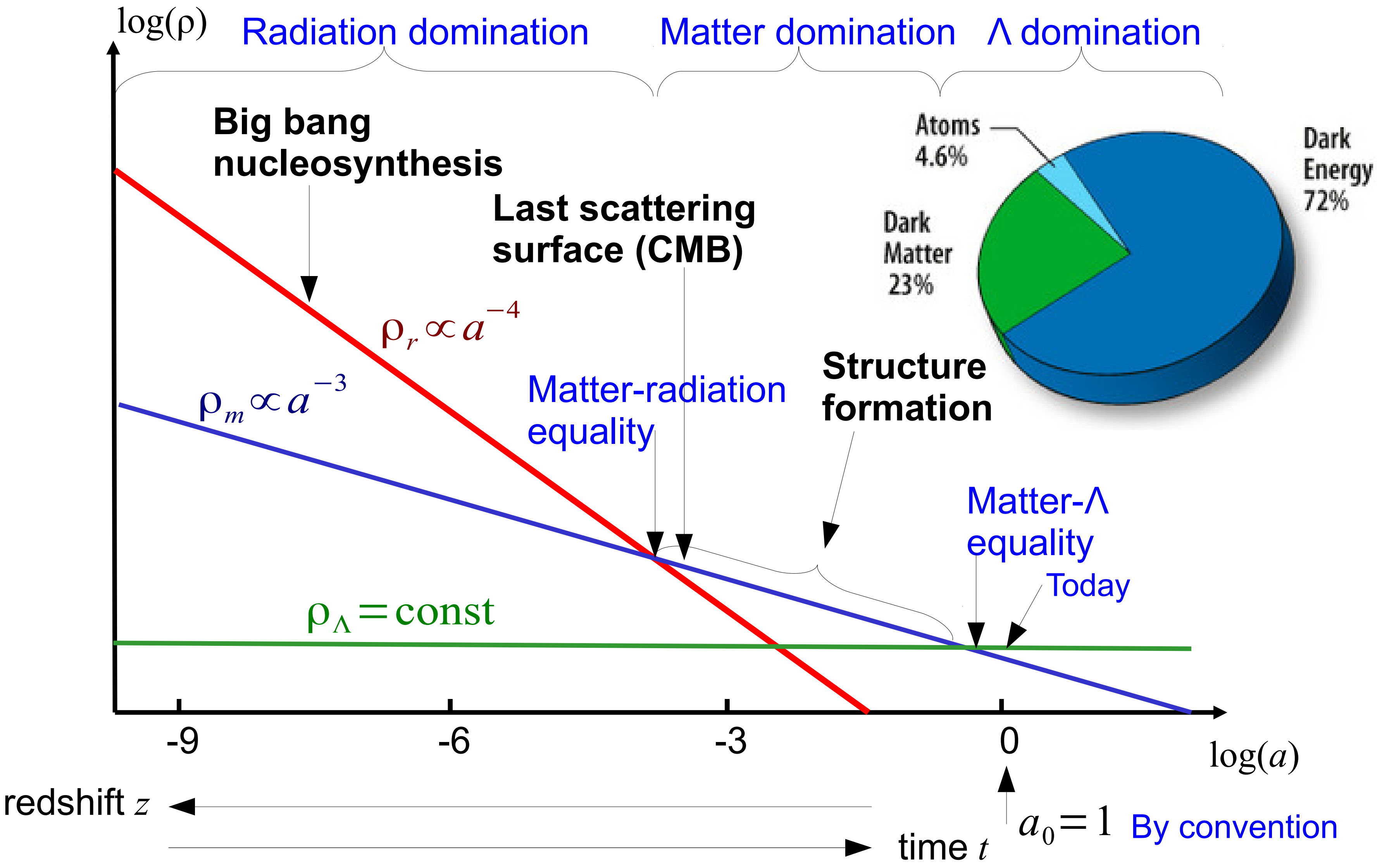
- **Other** funny things
  - Cosmological constant/vacuum energy
  - ??

$$w_\Lambda = -1$$

$$\Rightarrow \rho_\Lambda \propto \text{constant}$$

More space,
more energy


log(ρ)
Radiation domination
Matter domination
Λ domination
$\rho_r \propto a^{-4}$
$\rho_m \propto a^{-3}$
Matter-radiation equality
Matter-Λ equality
$\rho_\Lambda = \mathrm{const}$
log(a)
redshift z
time t


$\log(\rho)$
Radiation domination
Matter domination
Λ domination
Big bang nucleosynthesis
Last scattering surface (CMB)
Atoms 4.6%
Dark Energy 72%
Dark Matter 23%
$\rho_r \propto a^{-4}$
$\rho_m \propto a^{-3}$
Matter-radiation equality
Structure formation
Matter-Λ equality
Today
$\rho_\Lambda = \mathrm{const}$
-9
-6
-3
0
$\log(a)$
redshift z
time t
$a_0 = 1$
By convention

Friedmann equation...

- Derived from the Einstein equation:

Newton's constant

R = Ricci scalar and tensor (nonlinear functions of the 2nd derivative of the spacetime metric)

$$R_{\mu\nu} - \frac{1}{2} g_{\mu\nu} R = 8\pi G T_{\mu\nu}$$

Stress-energy tensor

- The Friedmann equation is an evolution equation for the scale factor $a(t)$:

$$\boxed{H^2(t) \equiv \left(\frac{\dot{a}}{a}\right)^2 = \frac{8\pi G}{3} \sum_\alpha \rho_\alpha - K}$$

Friedmann equation

$H(t)$ = Hubble parameter

- **Friedmann+continuity** equations → specify the whole system.

Friedmann equation...

- You may also have seen the Friedmann equation in this form:

$$\boxed{H^2(t) = H^2(t_0)\left[\Omega_m a^{-3} + \Omega_r a^{-4} + \Omega_\Lambda + \Omega_K a^{-2}\right]}$$

$$\Omega_\alpha = \frac{\bar{\rho}_\alpha(t_0)}{\rho_{\text{crit}}(t_0)}, \qquad \rho_{\text{crit}}(t) \equiv \frac{3H^2(t)}{8\pi G}, \qquad \Omega_K \equiv -\frac{K}{a^2 H^2(t_0)}$$

Critical density

- A **flat universe** means:

$$\Omega_m + \Omega_r + \Omega_\Lambda \simeq \Omega_m + \Omega_\Lambda = 1$$

From measuring
the CMB temperature
and energy spectrum.

Radiation energy density is negligibly small today: $\Omega_r \sim 10^{-5}$

Friedmann equation...

- You may also have seen the Friedmann equation in this form:

$$\boxed{H^2(t) = H^2(t_0)\left[\Omega_m a^{-3} + \Omega_r a^{-4} + \Omega_\Lambda + \Omega_K a^{-2}\right]}$$

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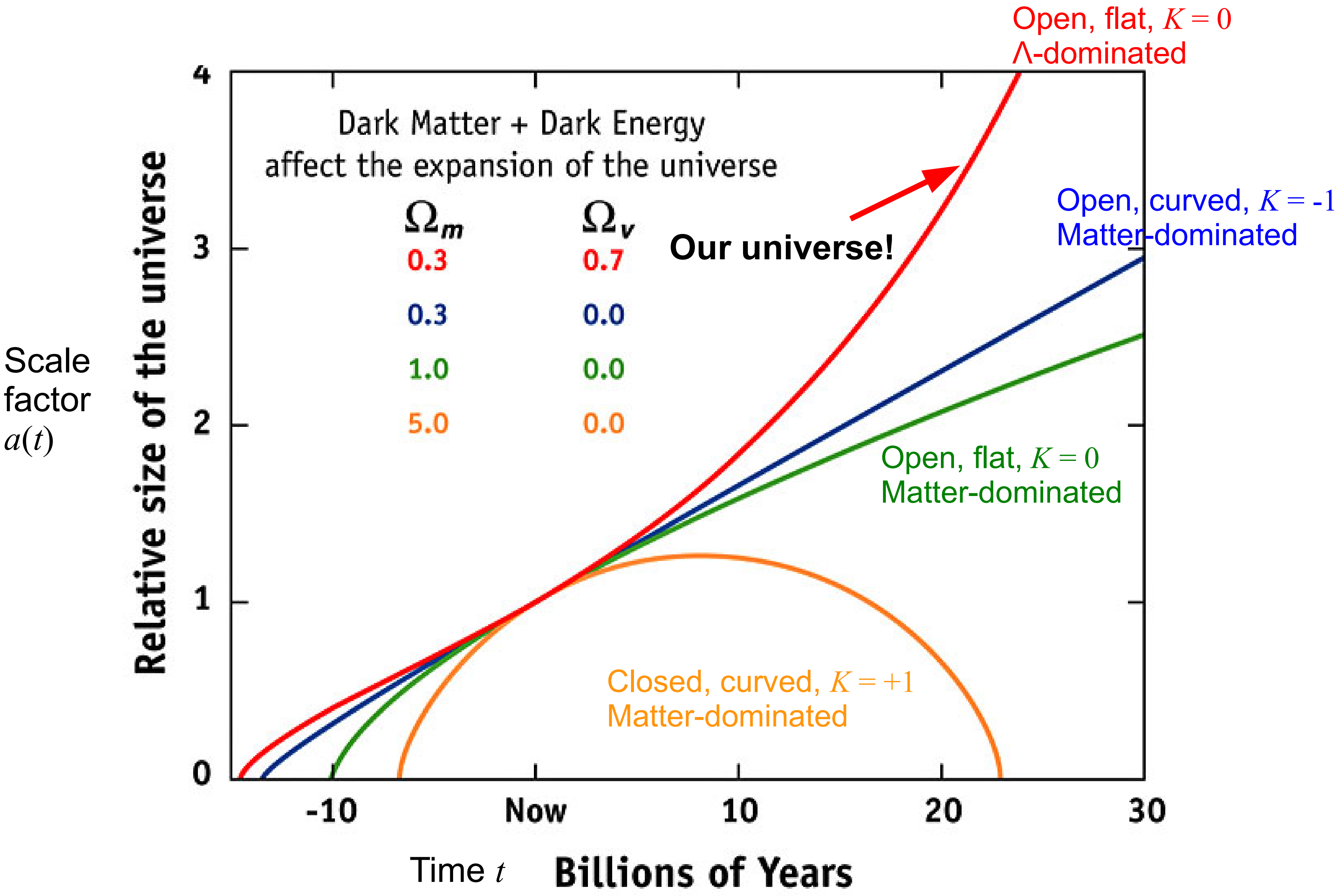
Critical density

- **Current observations**:

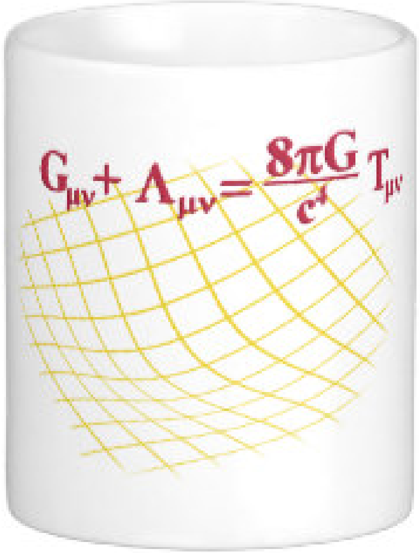
$$\Omega_m \sim 0.3, \quad \Omega_\Lambda \sim 0.7, \quad |\Omega_K| < 0.01$$

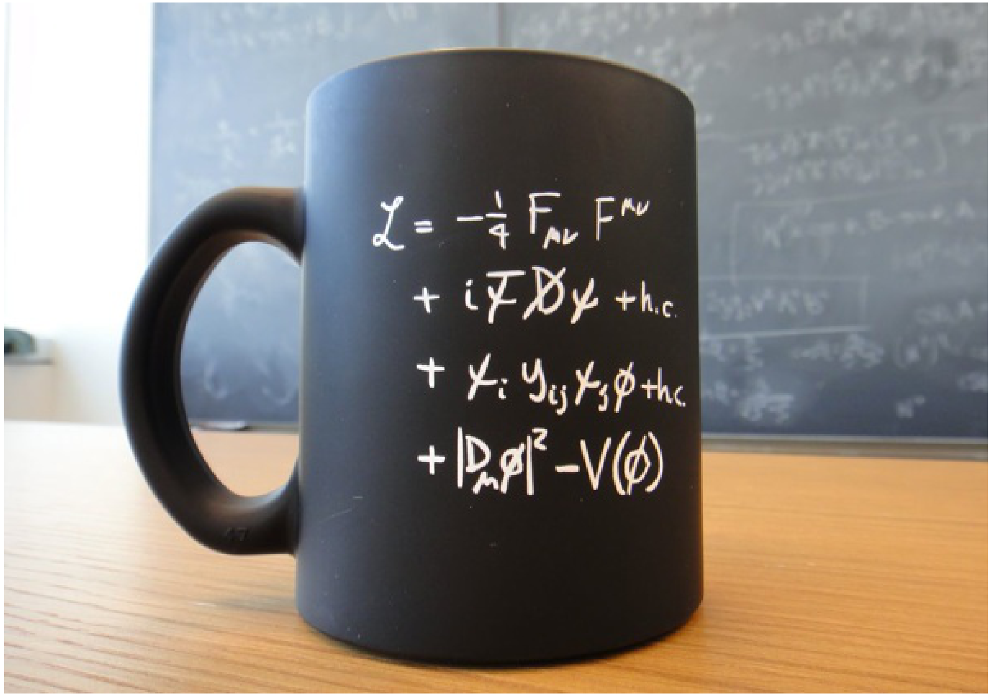
$$H_0 \equiv H(t_0) \sim 70 \ \ \text{km}\,\text{s}^{-1}\,\text{Mpc}^{-1}$$

e.g., Ade et al.
[Planck collaboration]
arXiv:1502.01589


Open, flat, $K = 0$
Λ-dominated
Dark Matter + Dark Energy
affect the expansion of the universe
Ω_m
Ω_v
0.3 0.7
0.3 0.0
1.0 0.0
5.0 0.0
Our universe!
Open, curved, $K = -1$
Matter-dominated
Open, flat, $K = 0$
Matter-dominated
Closed, curved, $K = +1$
Matter-dominated
Scale factor $a(t)$
Relative size of the universe
4
3
2
1
0
-10
Now
10
20
30
Time t
Billions of Years

2. The hot universe and the relic neutrino background...





The hot universe...

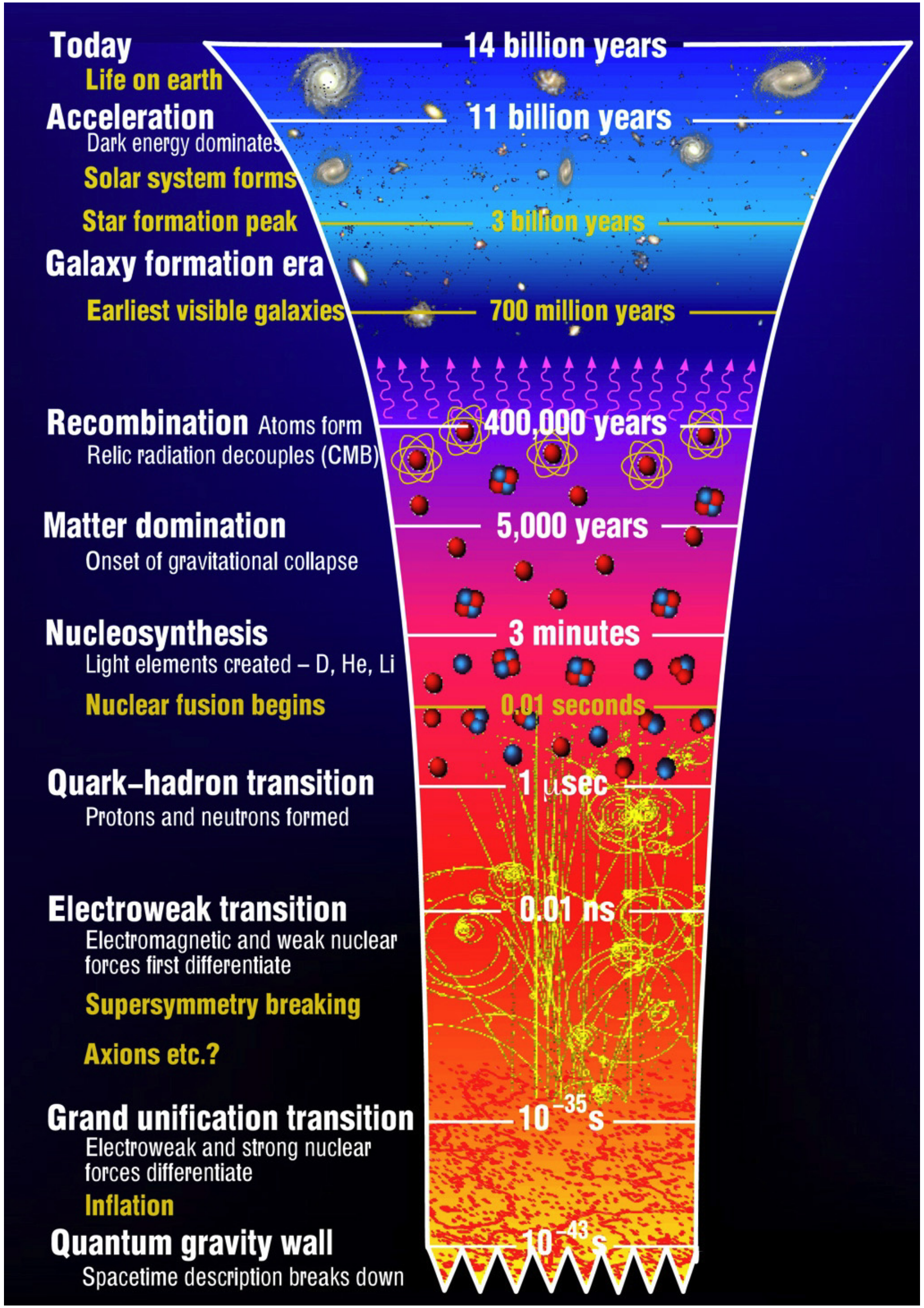
- The early universe was a very hot and dense place.

  → Frequent particle interactions.

- If the interaction rate (per particle) is so large that

  Interaction rate Γ >> Expansion rate, H

  → the interaction process is in a state of **equilibrium**.



Equilibrium...

$$G_F \sim 10^{-5}\ \mathrm{GeV}^{-2}$$
$$m_{\mathrm{pl}} \sim 10^{19}\ \mathrm{GeV}$$

- Consider the **weak interaction**. The interaction rate per particle is:

Fermi constant

Number density $n \sim T^3$

$$\Gamma = n \langle \sigma v \rangle \sim G_F^2 T^5$$

T = temperature

Cross section $\sigma \sim G_F^2 T^2$

Relative velocity $v \sim 1$

- The Hubble expansion goes like

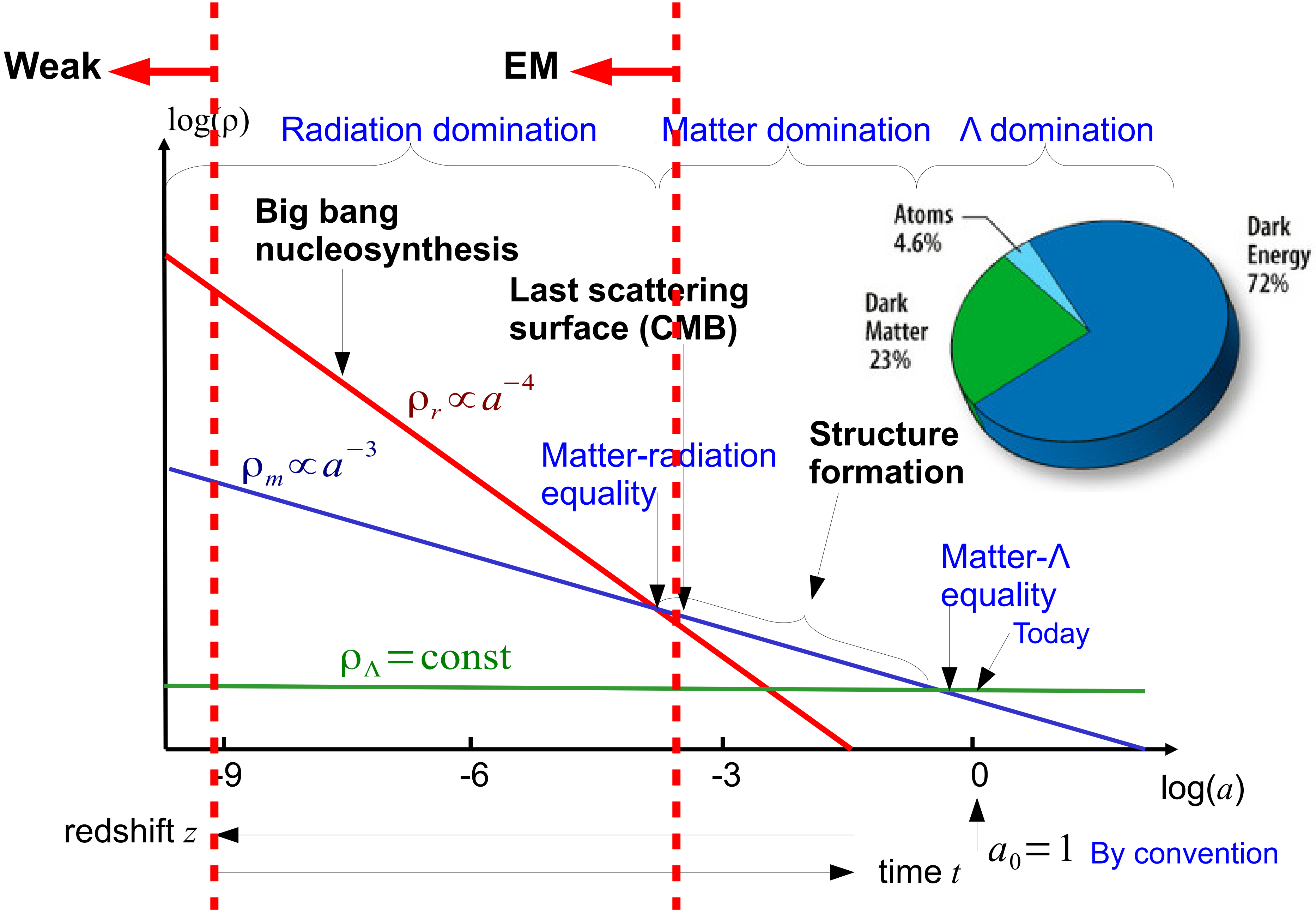
$$H = \sqrt{\frac{8\pi G}{3} \sum_\alpha \rho_\alpha} \sim \frac{T^2}{m_{\mathrm{pl}}}$$

Planck mass

$$\frac{\Gamma}{H} \sim m_{\mathrm{pl}} G_F^2 T^3 \sim \left(\frac{T}{1\ \mathrm{MeV}} \right)^3$$

→ When the temperature exceeds O(1) MeV, even weak interaction processes are in a state of equilibrium!

Natural units $k_B = 1$


Weak
EM
log(ρ)
Radiation domination
Matter domination
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Big bang nucleosynthesis
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Atoms 4.6%
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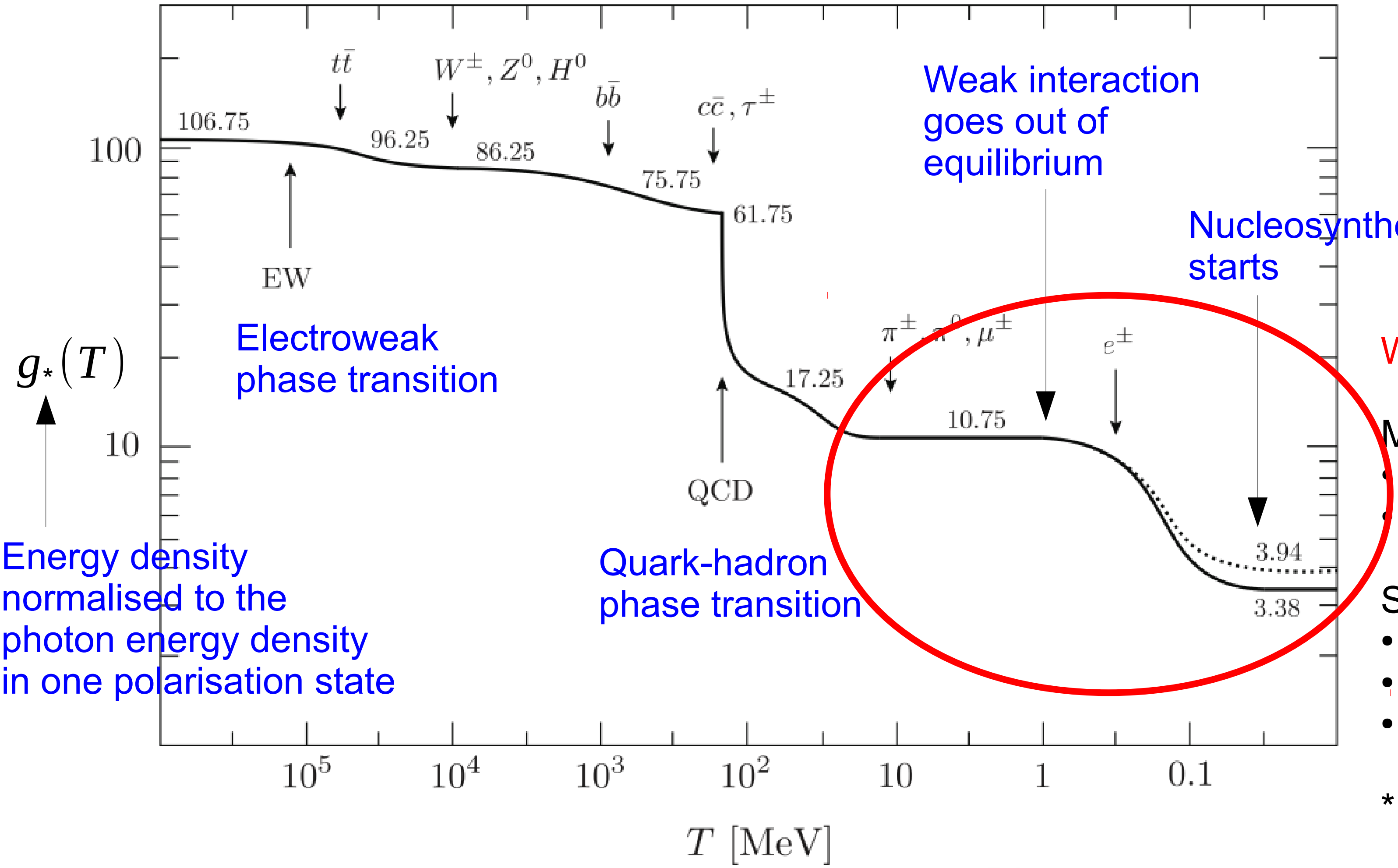
Today
$\rho_\Lambda = \mathrm{const}$
-9
-6
-3
0
log(a)
redshift z
time t
$a_0 = 1$
By convention

$g*$ for the **standard model of particle physics:**



Weak interaction goes out of equilibrium

Nucleosynthesis starts

Electroweak phase transition

Energy density normalised to the photon energy density in one polarisation state

Quark-hadron phase transition

What's left?

Mainly
- Photons
- Neutrinos

Small numbers* of
- Electrons
- Nucleons
- Nuclei

* Small means $<10^{-9} n_{\gamma}$

Photon temperature

Time

Particle content & interactions at 0.1 < T < 100 MeV...

- **QED plasma**

$$e^+, e^-, \gamma$$

EM interactions:

$$e^+ e^- \leftrightarrow e^+ e^-$$
$$e\, e \leftrightarrow e\, e$$
$$\gamma\, e \leftrightarrow \gamma\, e$$
$$\gamma\, \gamma \leftrightarrow e^+ e^-$$
$$\text{etc.}$$

$$\Gamma \sim G_F^2 T^5$$
$$H \sim T^2 / m_{\text{pl}}$$

- **3 families of neutrinos + antineutrinos**

$$\nu_e, \bar{\nu}_e, \nu_\mu, \bar{\nu}_\mu, \nu_\tau, \bar{\nu}_\tau$$

Weak interactions @ T > O(1) MeV:

$$\nu_i \nu_j \leftrightarrow \nu_i \nu_j$$
$$\nu_i \bar{\nu}_j \leftrightarrow \nu_i \bar{\nu}_j$$
$$\nu_i \nu_i \leftrightarrow \nu_i \nu_i$$
$$\nu_i \bar{\nu}_i \leftrightarrow \nu_j \bar{\nu}_j$$
$$\text{etc.}$$

Coupled

$$\nu\, e \leftrightarrow \nu\, e$$
$$\nu\, \bar{\nu} \leftrightarrow e^+ e^-$$

Weak interactions @ T > O(1) MeV:

Particle content & interactions at 0.1 < T < 100 MeV...

- **QED plasma**

$$e^{+}, e^{-}, \gamma$$

EM interactions:

$$e^{+}e^{-} \leftrightarrow e^{+}e^{-}$$
$$e\,e \leftrightarrow e\,e$$
$$\gamma\, e \leftrightarrow \gamma\, e$$
$$\gamma\,\gamma \leftrightarrow e^{+}e^{-}$$
$$\text{etc.}$$

$$\Gamma \sim G_F^2 T^5$$
$$H \sim T^2 / m_{\text{pl}}$$

Decoupled

- **3 families of neutrinos + antineutrinos**

$$\nu_e, \bar{\nu}_e, \nu_\mu, \bar{\nu}_\mu, \nu_\tau, \bar{\nu}_\tau$$

Weak interactions @ T > O(1) MeV:

$$\cancel{\nu_i\,\nu_j \leftrightarrow \nu_i\,\nu_j}$$
$$\cancel{\nu_i\,\bar{\nu}_j \leftrightarrow \nu_i\,\bar{\nu}_j}$$
$$\cancel{\nu_i\,\nu_i \leftrightarrow \nu_i\,\nu_i}$$
$$\nu_i\,\bar{\nu}_i \leftrightarrow \nu_j\,\bar{\nu}_j$$
$$\text{etc.}$$

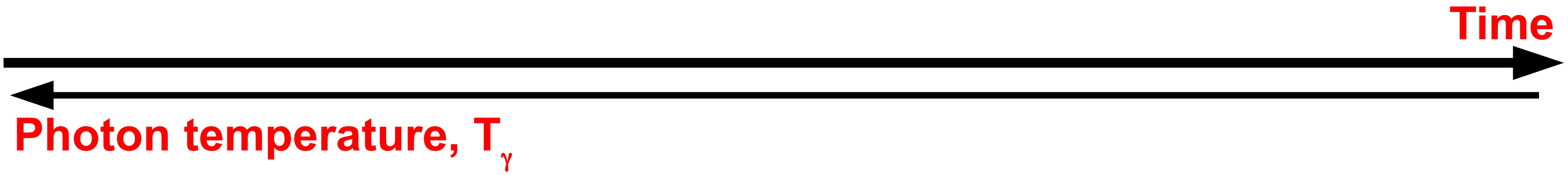
$$\cancel{\nu\, e \leftrightarrow \nu\, e}$$
$$\cancel{\nu\,\bar{\nu} \leftrightarrow e^{+} e^{-}}$$

Weak interactions @ T > O(1) MeV:

Not efficient
T< O(1) MeV

Thermal history of neutrinos...

Events


Time

Photon temperature, T_γ

Neutrino
temperature

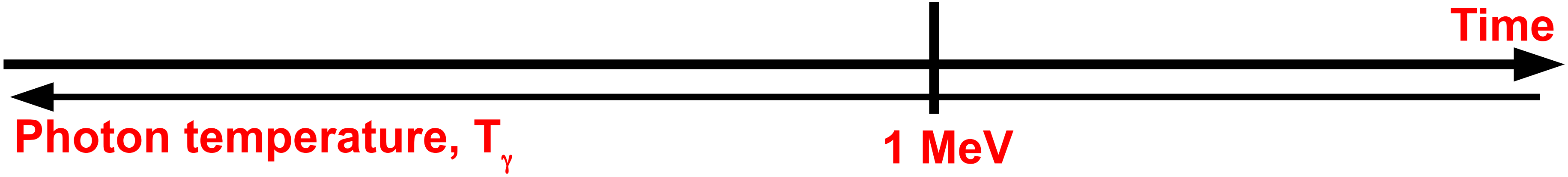
Phase space
density

Thermal history of neutrinos...

Events

Neutrinos in thermal contact with QED plasma

$$\left.\begin{array}{ll}\text{Weak interaction:} & \Gamma \sim G_F^2\, T^5 \\[1em] \text{Expansion rate:} & H \sim \dfrac{T^2}{m_{\text{planck}}}\end{array}\right\} \quad \Gamma > H$$



Time

Photon temperature, T_γ

1 MeV

Neutrino temperature

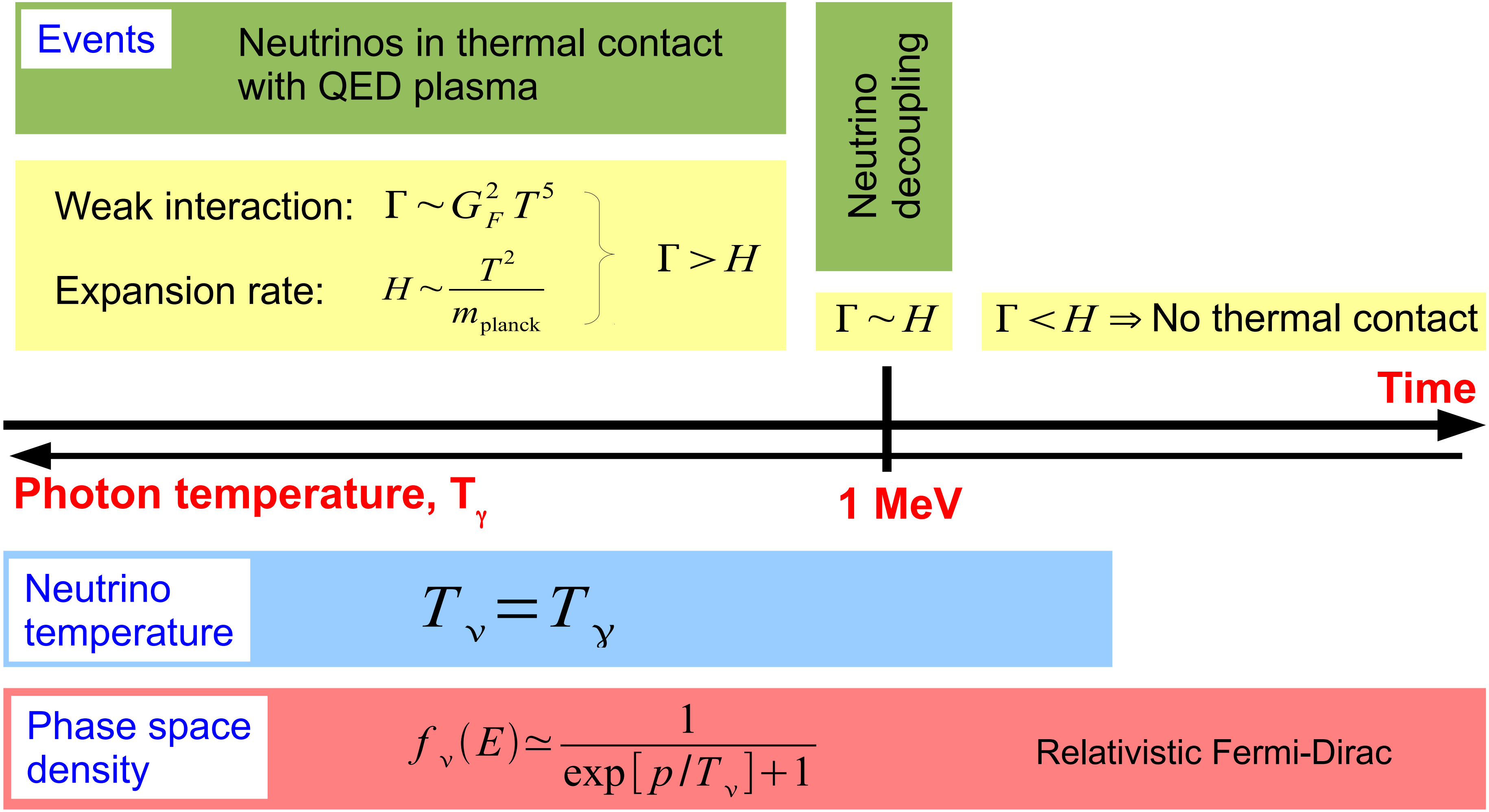
$$T_\nu = T_\gamma$$

Phase space density

$$f_\nu(E) \simeq \frac{1}{\exp[p/T_\nu]+1}$$

Relativistic Fermi-Dirac

Thermal history of neutrinos...



Events

Neutrinos in thermal contact with QED plasma

Weak interaction: $\Gamma \sim G_F^2 T^5$

Expansion rate: $H \sim \frac{T^2}{m_{\text{planck}}}$

$\Big\} \; \Gamma > H$

Neutrino decoupling

$\Gamma \sim H$

$\Gamma < H \Rightarrow$ No thermal contact

Time

Photon temperature, T$_\gamma$

1 MeV

Neutrino temperature

$$T_\nu = T_\gamma$$

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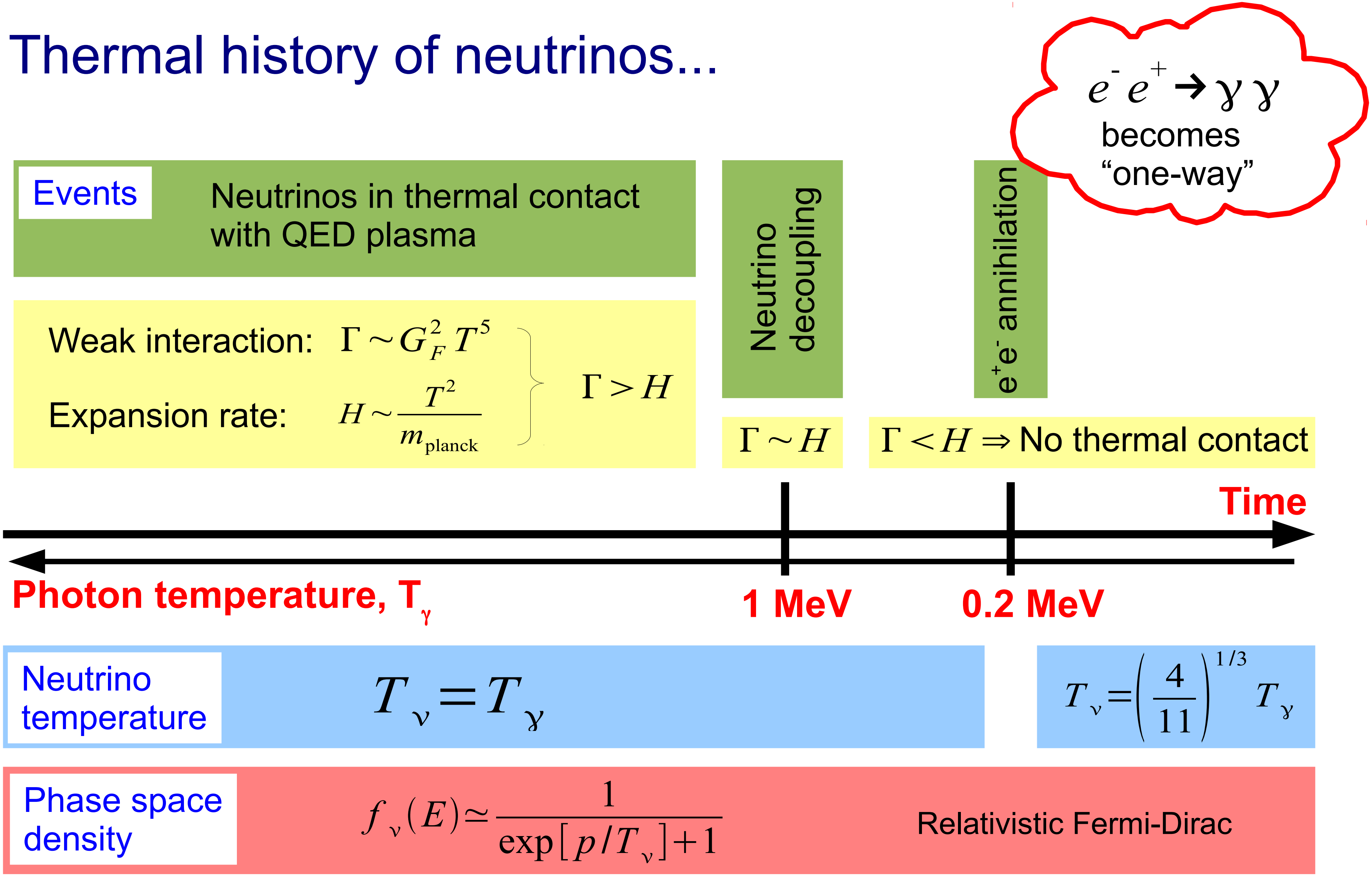


$e^- e^+ \rightarrow \gamma \gamma$

becomes
“one-way”

Events

Neutrinos in thermal contact with QED plasma

Weak interaction: $\Gamma \sim G_F^2 T^5$

Expansion rate: $H \sim \frac{T^2}{m_{\text{planck}}}$

$\Gamma > H$

Neutrino decoupling

$\Gamma \sim H$

e^+e^- annihilation

$\Gamma < H \Rightarrow$ No thermal contact

Time

Photon temperature, T_γ

1 MeV

0.2 MeV

Neutrino temperature

$$T_\nu = T_\gamma$$

$$T_\nu = \left(\frac{4}{11}\right)^{1/3} T_\gamma$$

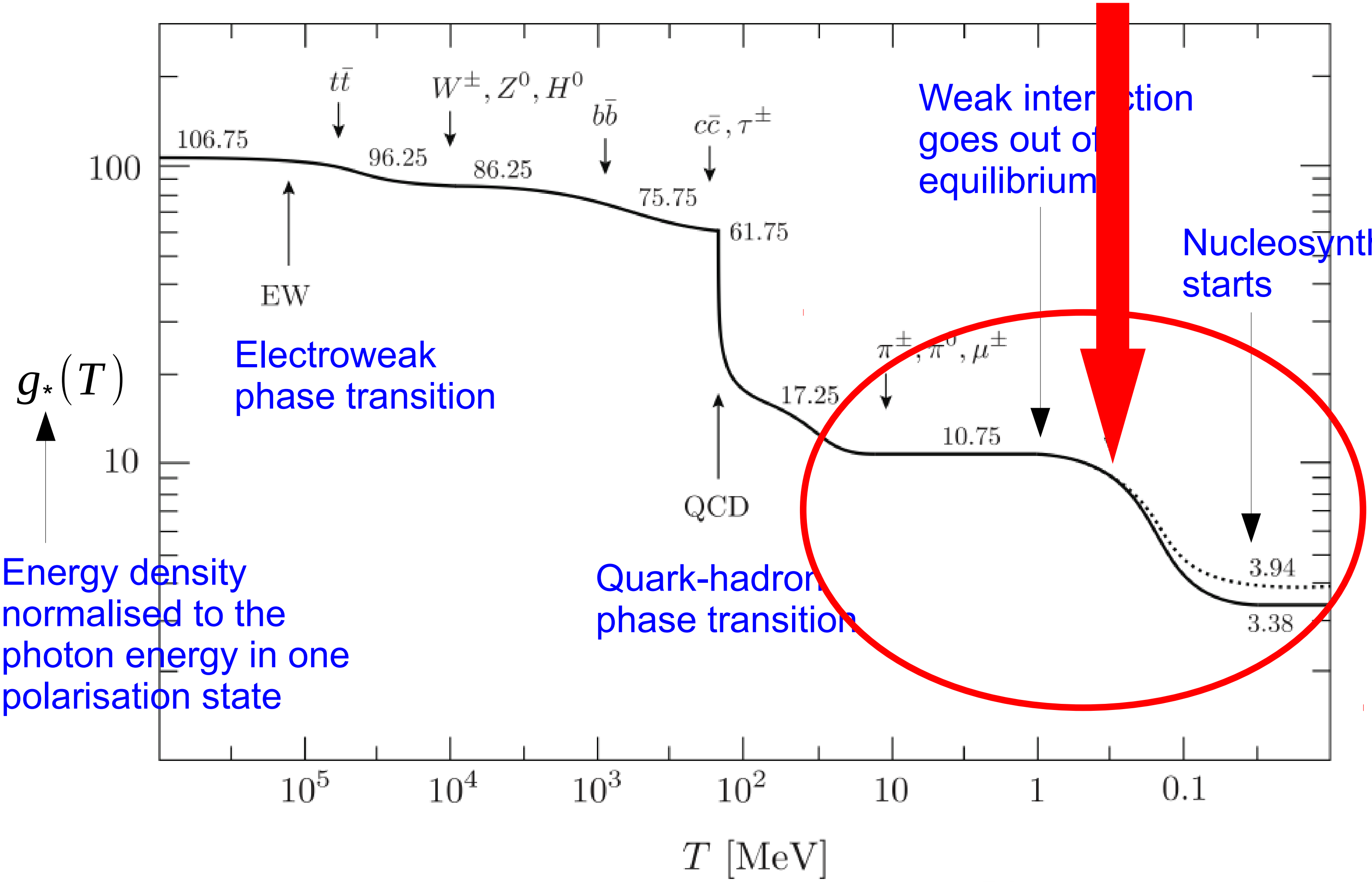
Phase space density

$$f_\nu(E) \simeq \frac{1}{\exp[p/T_\nu]+1}$$

Relativistic Fermi-Dirac

$g*$ for the **standard model of particle physics:**

This drop here.



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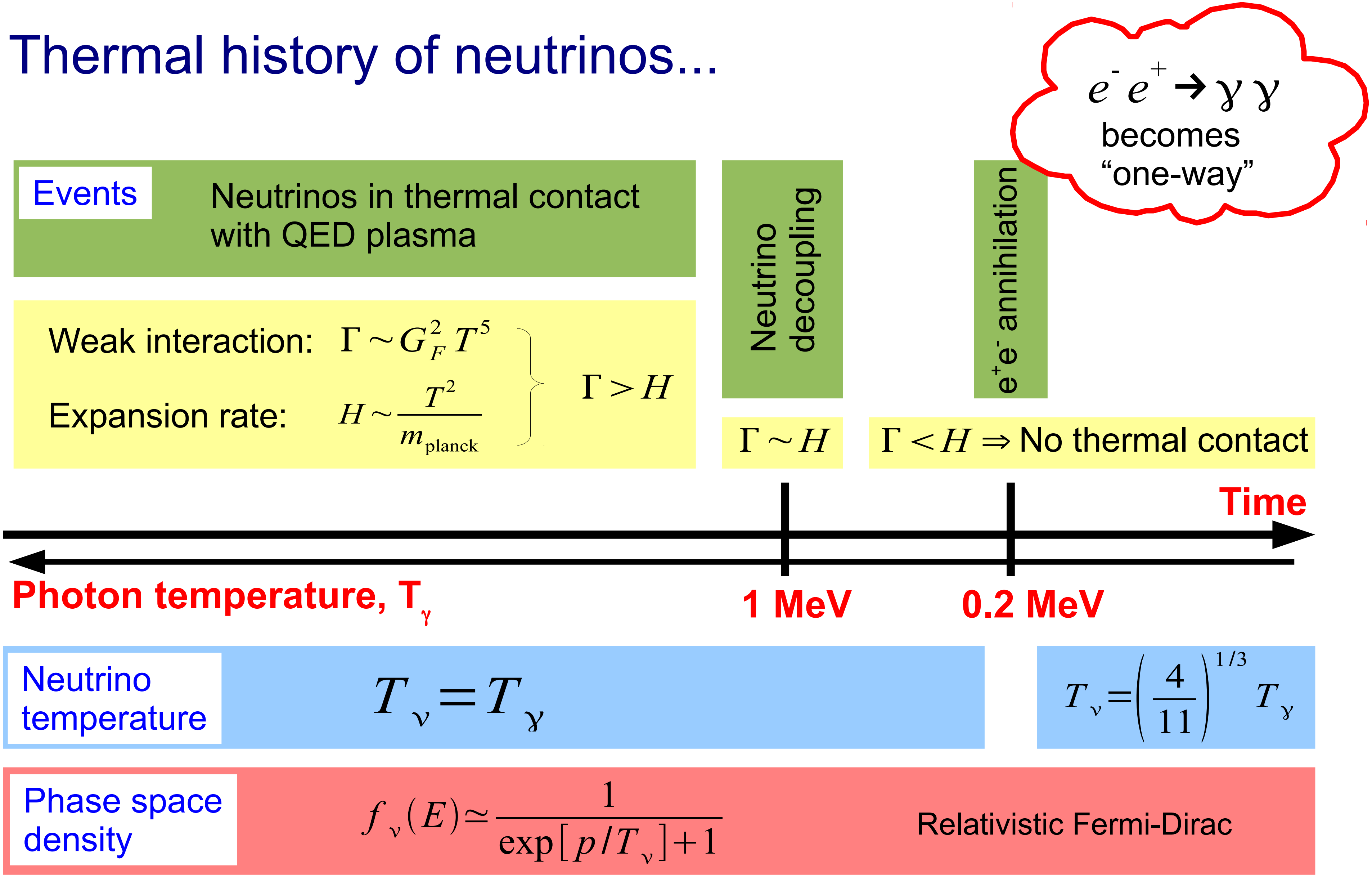
* Small means

$$<10^{-9} n_\gamma$$

Photon temperature

Time

Thermal history of neutrinos...



$e^- e^+ \rightarrow \gamma\,\gamma$
becomes
"one-way"

Events

Neutrinos in thermal contact with QED plasma

Neutrino decoupling

e^+e^- annihilation

Weak interaction: $\Gamma \sim G_F^2\, T^5$

Expansion rate: $H \sim \dfrac{T^2}{m_{\text{planck}}}$

$\Gamma > H$

$\Gamma \sim H$

$\Gamma < H \Rightarrow$ No thermal contact

Time

Photon temperature, T_γ

1 MeV

0.2 MeV

Neutrino temperature

$$T_\nu = T_\gamma$$

$$T_\nu = \left(\frac{4}{11}\right)^{1/3} T_\gamma$$

Phase space density

$$f_\nu(E) \simeq \frac{1}{\exp[p/T_\nu]+1}$$

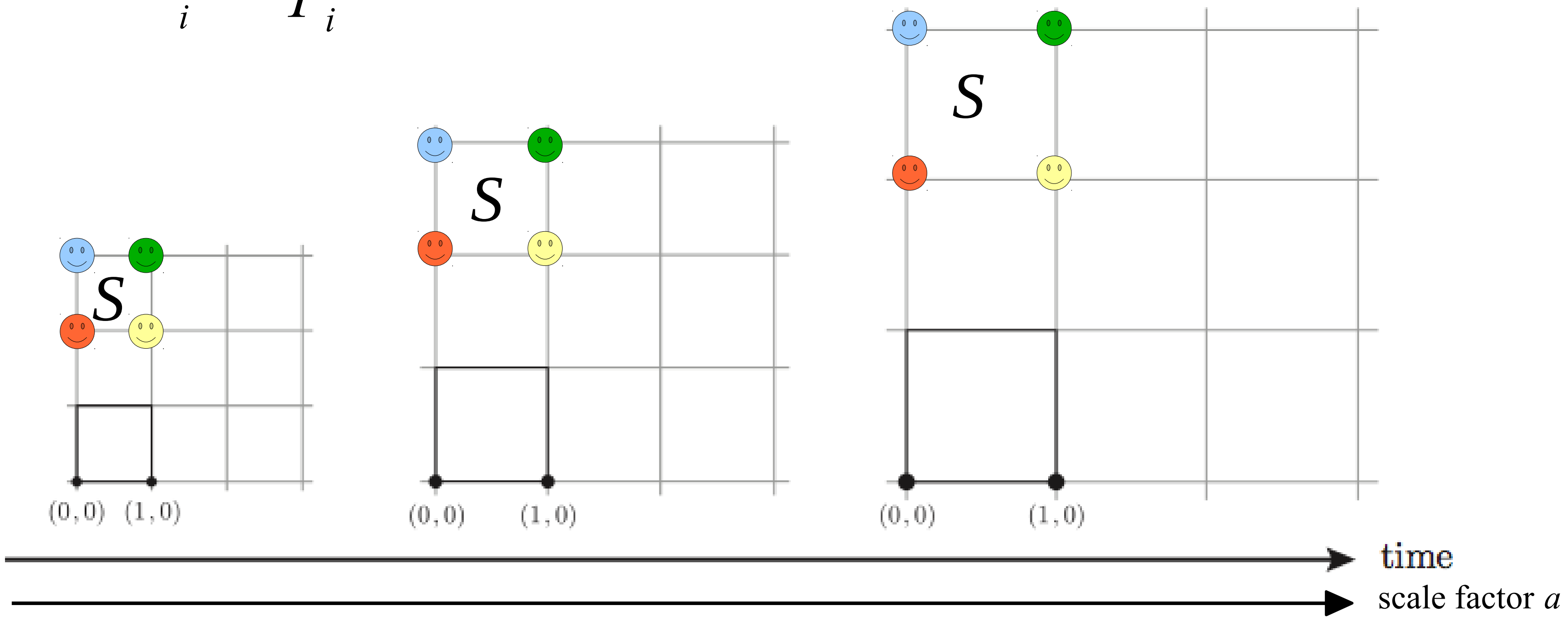
Relativistic Fermi-Dirac

Comoving entropy density & its conservation...

- In a universe that expands **quasi-statically** (so that equilibrium is always maintained), the comoving entropy denisty S is approximately conserved.

$$S \equiv a^3 \sum_i \frac{\rho_i + P_i}{T_i}$$

Comoving entropy density



Entropy conservation after neutrino decoupling...

QED plasma

$$e^{+}, e^{-}, \gamma$$

$$S_{e\gamma} = \frac{11}{45} \pi^2 T_\gamma^3 a^3$$

Same temperature pre-e⁺e⁻ anniliation

3 families of neutrinos + antineutrinos

$$\nu_e, \bar{\nu}_e, \nu_\mu, \bar{\nu}_\mu, \nu_\tau, \bar{\nu}_\tau$$

$$S_\nu = \frac{7}{30} \pi^2 T_\nu^3 a^3$$

Pre-e⁺e⁻ annihilation

$T_\gamma \sim m_e$

Post-e⁺e⁻ annihilation

$$\gamma$$

$$S_{e\gamma} = \frac{4}{45} \pi^2 T_\gamma^3 a^3$$

Time

Conservation of $S_{e\gamma}$ across the annihilation era gives:

$$T_\nu = \left(\frac{4}{11}\right)^{1/3} T_\gamma$$

$$\rho_\nu = \frac{7}{8} \left(\frac{4}{11}\right)^{4/3} \rho_\gamma$$

Energy density per neutrino family

Effective number of neutrinos N_{eff}...

- It is convenient to express the neutrino energy density relative to the photon energy density in terms of the **N_{eff} parameter**:

Total energy density in neutrinos

$$\sum_i \rho_{\nu_i} \equiv N_{\text{eff}} \times \left[\frac{7}{8} \left(\frac{4}{11} \right)^{4/3} \rho_\gamma \right]$$

"Standard" energy density per flavour assuming the "standard" neutrino temperature

- In the idealised scenario just discussed:

$$N_{\text{eff}} = 3$$

For 3 families

Three small corrections...

… make the neutrinos a little more energetic than is implied by $N_{\mathrm{eff}} = 3$.

- **Non-relativistic (m_e/T) correction**
- **Finite-temperature QED**
- **Non-instantaneous neutrino decoupling**

Non-relativistic (m_e/T) correction...

QED plasma

Expression assumes T >> m_e, not quite valid at T < 1 MeV.

$$e^+, e^-, \gamma$$

Same temperature pre-e^+e^- anniliation

$$S_{e\gamma} = \frac{11}{45}\pi^2 T_\gamma^3 a^3$$

Pre-e^+e^- annihilation

$$T_\gamma \sim m_e$$

Post-e^+e^- annihilation

$$\gamma$$

$$S_{e\gamma} = \frac{4}{45}\pi^2 T_\gamma^3 a^3$$

Time

3 families of neutrinos + antineutrinos

$$\nu_e, \bar{\nu}_e, \nu_\mu, \bar{\nu}_\mu, \nu_\tau, \bar{\nu}_\tau$$

$$S_\nu = \frac{7}{30}\pi^2 T_\nu^3 a^3$$

Conservation of $S_{e\gamma}$ across the annihilation era gives:

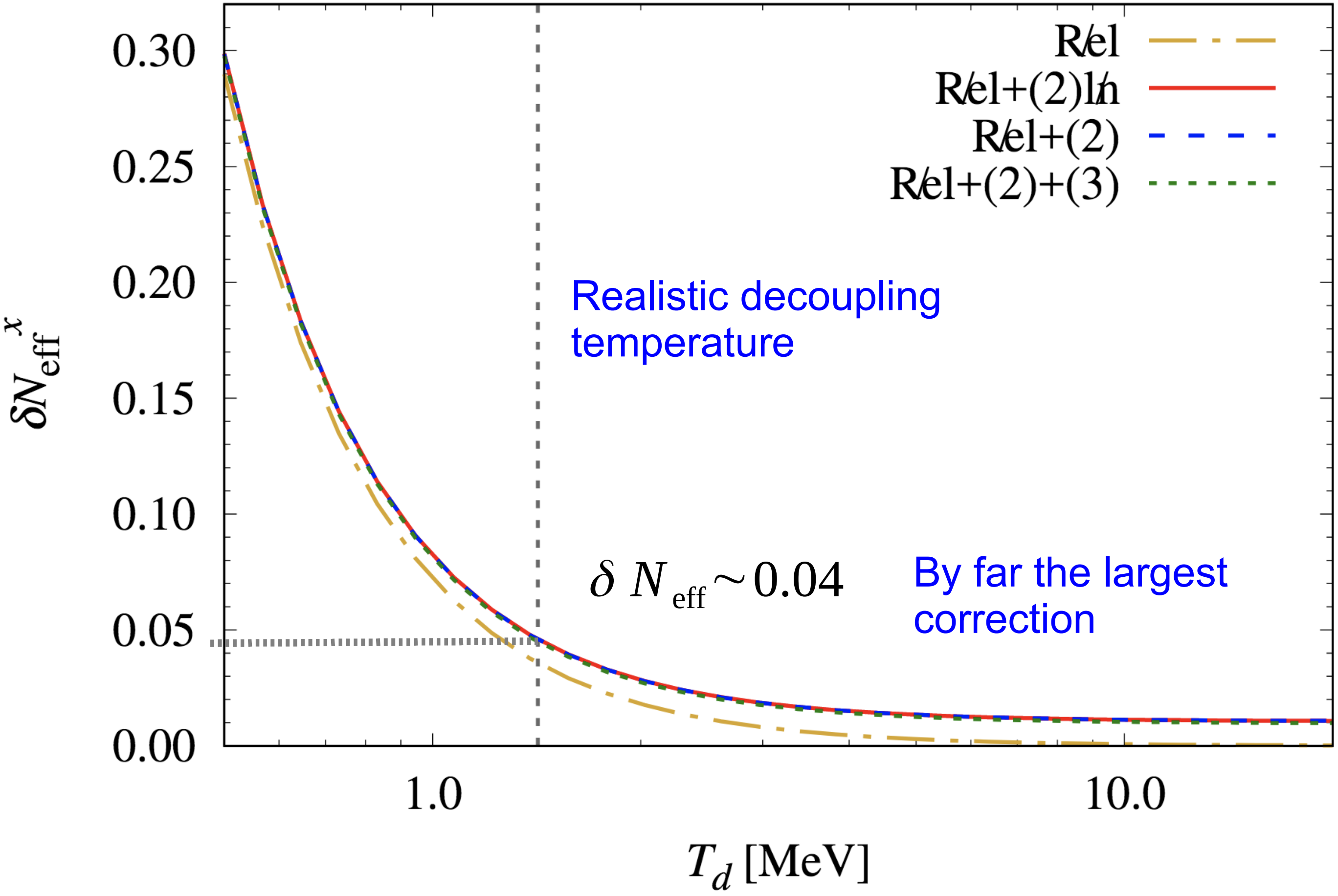
Not strictly correct

$$T_\nu = \left(\frac{4}{11}\right)^{1/3} T_\gamma$$

$$\rho_\nu = \frac{7}{8}\left(\frac{4}{11}\right)^{4/3} \rho_\gamma$$

Energy density per neutrino family

Non-relativistic (m_e/T) correction...



Realistic decoupling temperature

$\delta N_{\rm eff} \sim 0.04$

By far the largest correction

Change in $N_{\rm eff}$

Neutrino decoupling temperature

Bennett, Buldgen, Drewes & Y^3W 2019

Three small corrections...

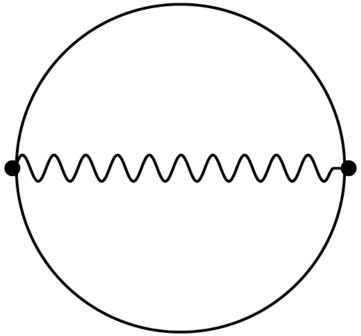
… make the neutrinos a little more energetic than is implied by $N_{\rm eff} = 3$.

- **Non-relativistic (m_e/T) correction**
- **Finite-temperature QED**
- **Non-instantaneous neutrino decoupling**

Finite-temperature QED...

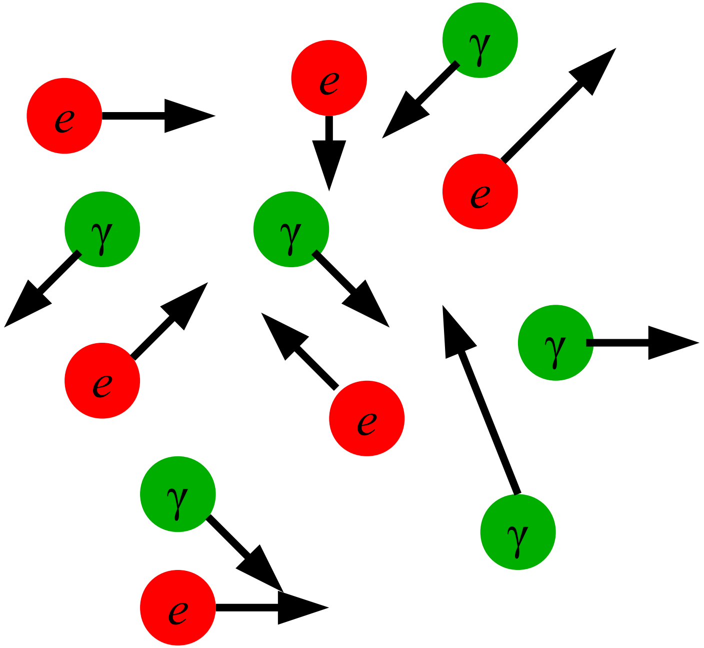
- **Interactions** in the **QED plasma** cause its thermodynamical properties to deviate from an ideal gas description.
- Lowest-order O(e^2) correction to the QED partition function:

$$\ln Z^{(2)} \;=\; -\frac{1}{2}$$



Finite-temperature QED...

Ideal gas

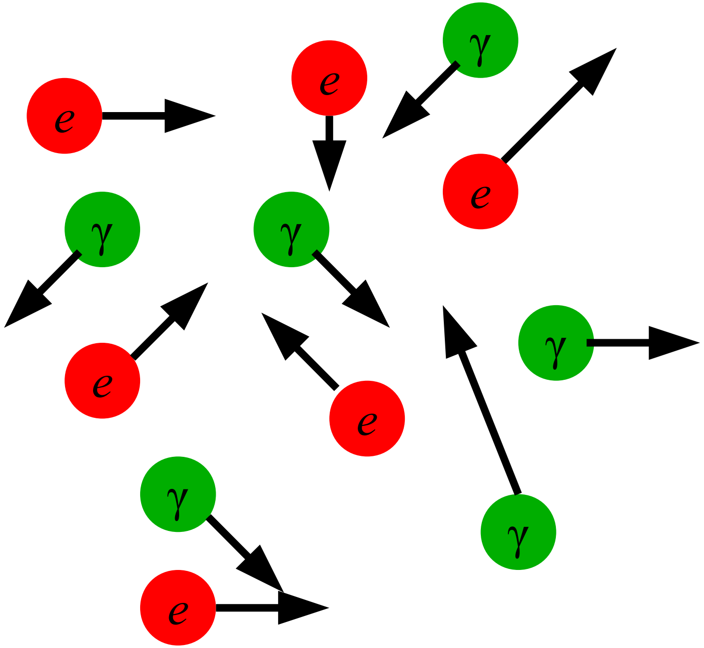


Energy = kinetic energy + rest mass

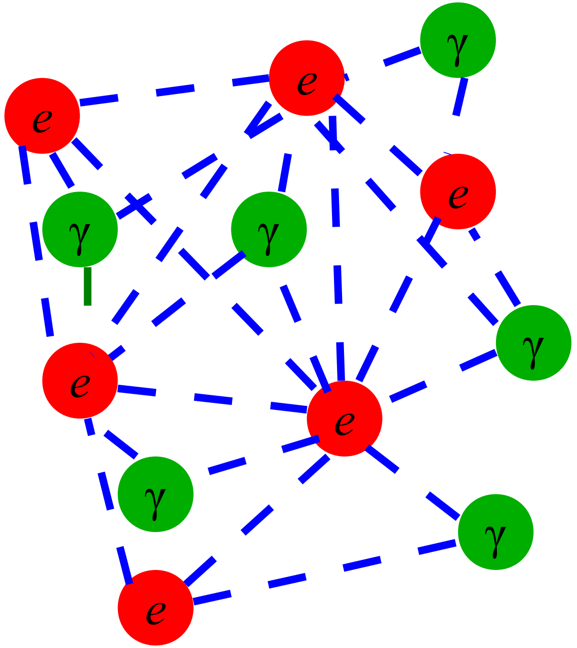
Pressure = from kinetic energy

Finite-temperature QED...

Ideal gas



+ EM interactions



Temperature
-dependent
dispersion relation

Energy = kinetic energy + rest mass

Pressure = from kinetic energy

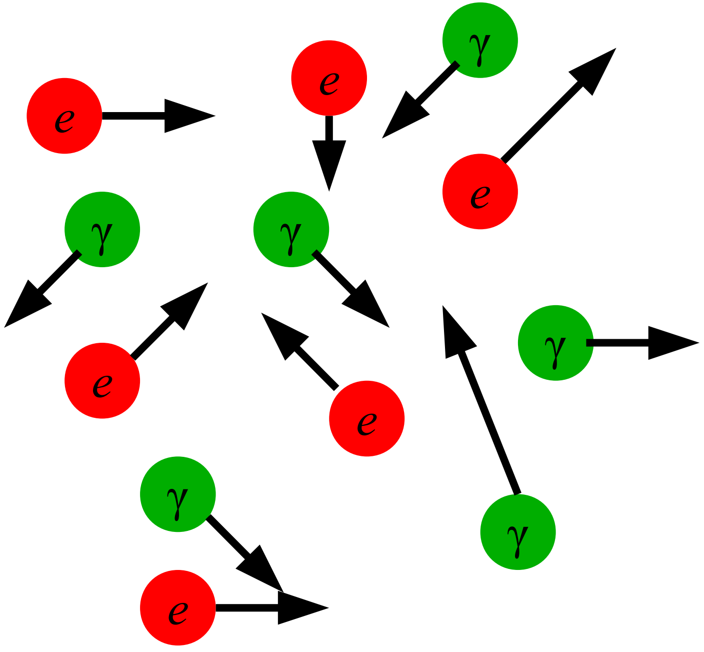
Energy = **modified** kinetic energy + **T-dependent masses**

Pressure = from **modified** kinetic energy

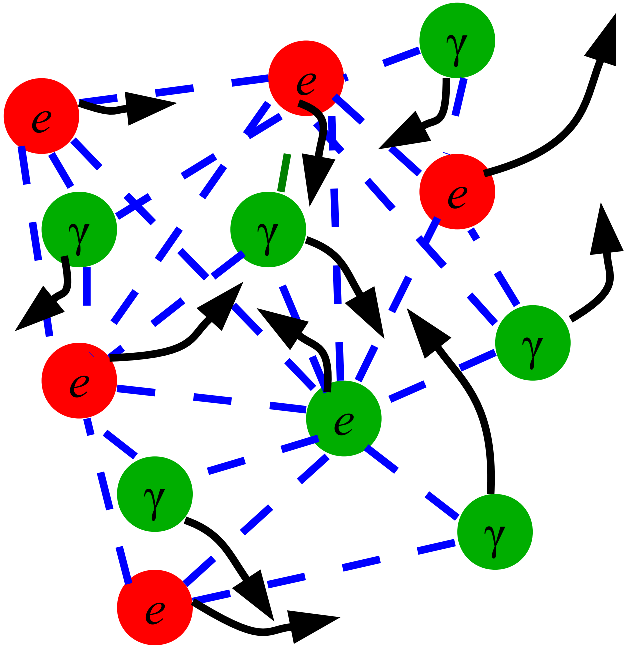
⇒ Modified QED equation of state

Finite-temperature QED...

Ideal gas



+ EM interactions



Temperature
-dependent
dispersion relation
+
Forces

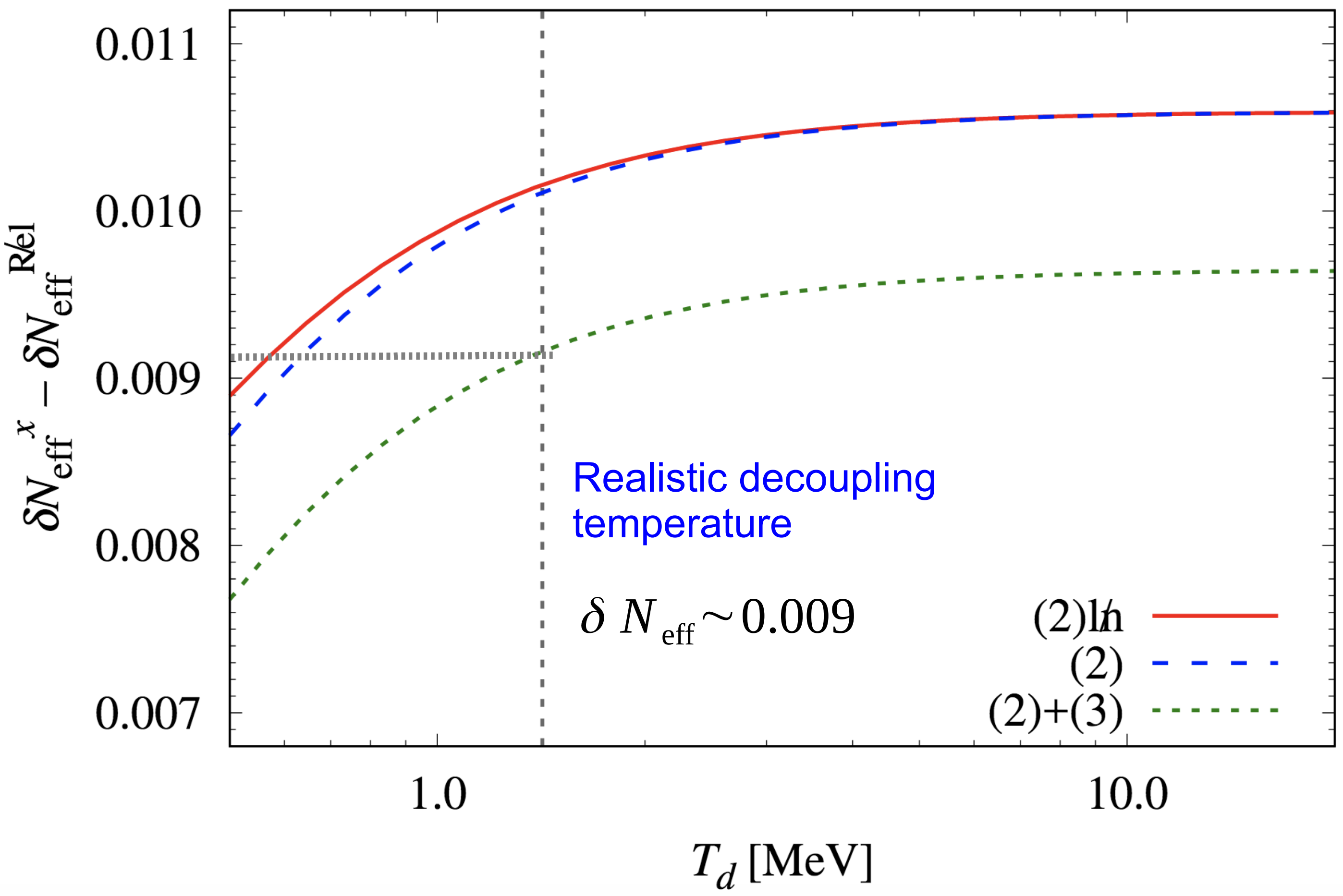
Energy = kinetic energy + rest mass

Pressure = from kinetic energy

Energy = **modified** kinetic energy + **T-dependent masses** + **interaction potential** energy
Pressure = from **modified** kinetic energy + **EM forces**

➡ Modified QED equation of state

Change in N_{eff}



Leading-order O(e^2) correction

+ O(e^3) correction (Debye-screened EM forces)

Realistic decoupling temperature

$\delta N_{\rm eff} \sim 0.009$

Neutrino decoupling temperature

Bennett, Buldgen, Drewes & Y^3W 2019

Three small corrections...

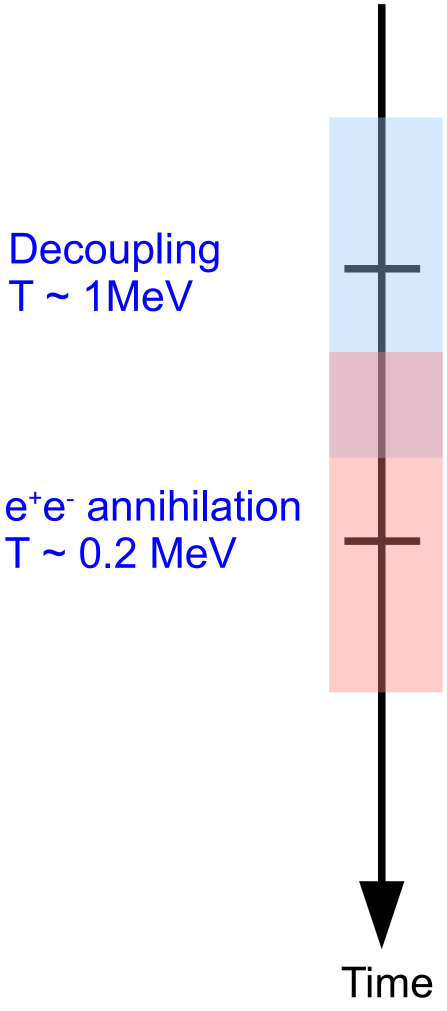
… make the neutrinos a little more energetic than is implied by $N_{\mathrm{eff}} = 3$.

- **Non-relativistic (m_e/T) correction**
- **Finite-temperature QED**
- **Non-instantaneous neutrino decoupling**

Non-instantaneous decoupling...

- **Neutrino decoupling** and **electron/positron annihilation** occur at similar times ($T \sim 1$ MeV vs $T \sim 0.2$ MeV).
  - Neither event is exactly localised in time.
  - Some neutrinos **are still decoupled** to the QED plasma when the annihilation happens.

    → Neutrinos at the high energy tail (where the interaction cross-section is larger) are affected by the entropy released in the annihilation.



- To track the neutrino decoupling process properly through the annihilation era, we need to use the **Boltzmann equation:**

Phase space density of particle species 1

$$\frac{\partial f_1}{\partial t} = -\{f_1, H\} + C[f_1]$$

Collision term

Hamiltonian for particle propagation

where the collision term for, e.g., 1+2 → 3+4 is

9D phase space integral

Energy-momentum conservation

Matrix element

$$C[f_1] = \frac{1}{2E_1} \int \prod_{i=2}^{4} \frac{d^3 p_i}{(2\pi)^3 2E_i} (2\pi)^4 \delta^4(P_1 + P_2 - P_3 - P_4) |M|^2$$
$$\times f_3 f_4 (1 \pm f_1)(1 \pm f_2) - f_1 f_2 (1 \pm f_3)(1 \pm f_4)$$

Quantum statistical factors

- To track **neutrino oscillations too**, we need to promote the classical Boltzmann equation for the phase space density to a quantum kinetic equation for the density matrix of the neutrino ensemble:

Boltzmann $$\frac{\partial f_1}{\partial t} = -\{f_1 , H\} + C[f_1]$$

The same formalism is also used to compute sterile neutrino thermalisation.

QKE $$\frac{\partial \hat{\rho}}{\partial t} = -\frac{1}{i\hbar}[\hat{\rho} , \hat{H}] + \hat{C}[\rho]$$

Collision term

Density matrix

Hamiltonian

$$\hat{\rho} = \begin{pmatrix} |\nu_e\rangle\langle\nu_e| & |\nu_e\rangle\langle\nu_\mu| & |\nu_e\rangle\langle\nu_\tau| \\ |\nu_\mu\rangle\langle\nu_e| & |\nu_\mu\rangle\langle\nu_\mu| & |\nu_\mu\rangle\langle\nu_\tau| \\ |\nu_\tau\rangle\langle\nu_e| & |\nu_\tau\rangle\langle\nu_\mu| & |\nu_\tau\rangle\langle\nu_\tau| \end{pmatrix}$$

$$\hat{H} = \frac{1}{2p} U \begin{pmatrix} m_1 & 0 & 0 \\ 0 & m_2 & 0 \\ 0 & 0 & m_3 \end{pmatrix} U^\dagger + \hat{V}_m$$

Diagonal ~ number densities
Off-diagonal ~ coherence

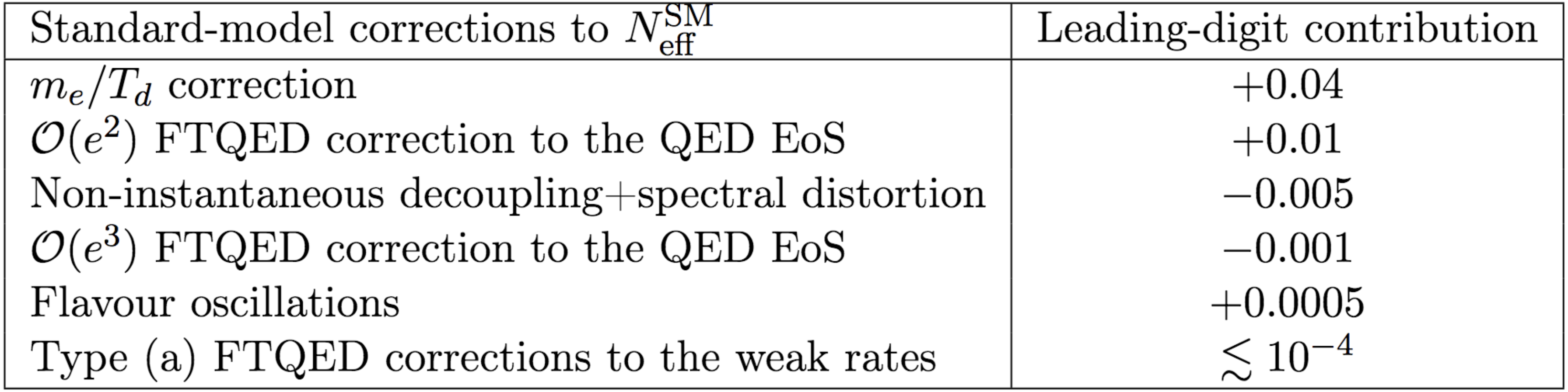
Vacuum + matter effects

Precision computation of N_{eff}…

Bennett, Buldgen, de Salas, Drewes,
Gariazzo, Pastor & Y^{3}W 2020
Froustey, Pitrou & Volpe 2020

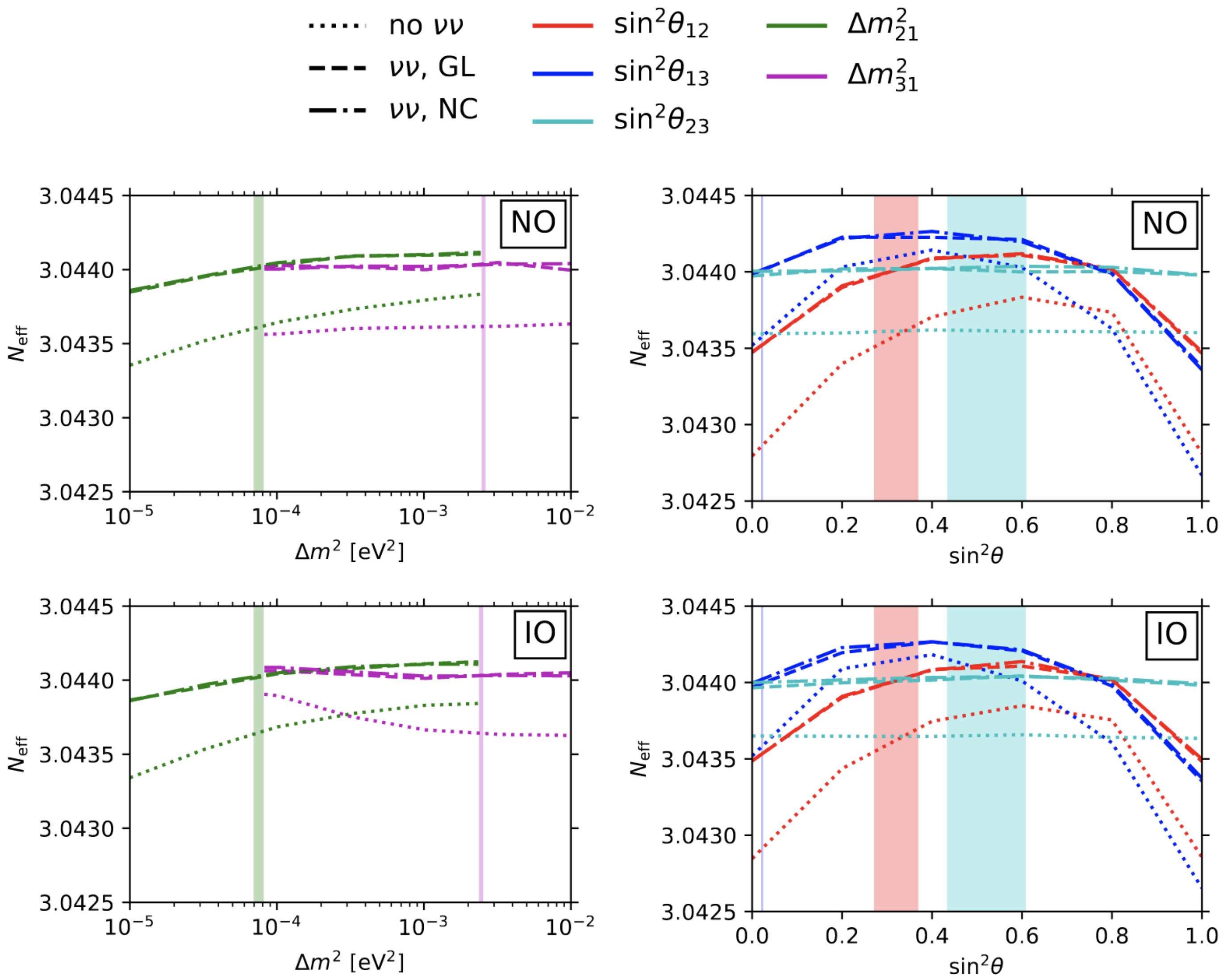
- Taking into account all three of the aforementioned corrections:

$$\boxed{N_{\text{eff}} = 3.0440 \pm 0.0002}$$



Standard-model corrections to $N_{\text{eff}}^{\text{SM}}$	Leading-digit contribution
m_e/T_d correction	$+0.04$
$\mathcal{O}(e^2)$ FTQED correction to the QED EoS	$+0.01$
Non-instantaneous decoupling+spectral distortion	-0.005
$\mathcal{O}(e^3)$ FTQED correction to the QED EoS	-0.001
Flavour oscillations	$+0.0005$
Type (a) FTQED corrections to the weak rates	$\lesssim 10^{-4}$

- Computed by two independent groups; agreement to **5 significant digits**
- Error estimate = numerical resolution + uncertainty in solar mixing angle



Variation of $N_{\rm eff}$ with respect to neutrino mixing parameters.

- Shaded regions = 5σ regions allowed by oscillation experiments

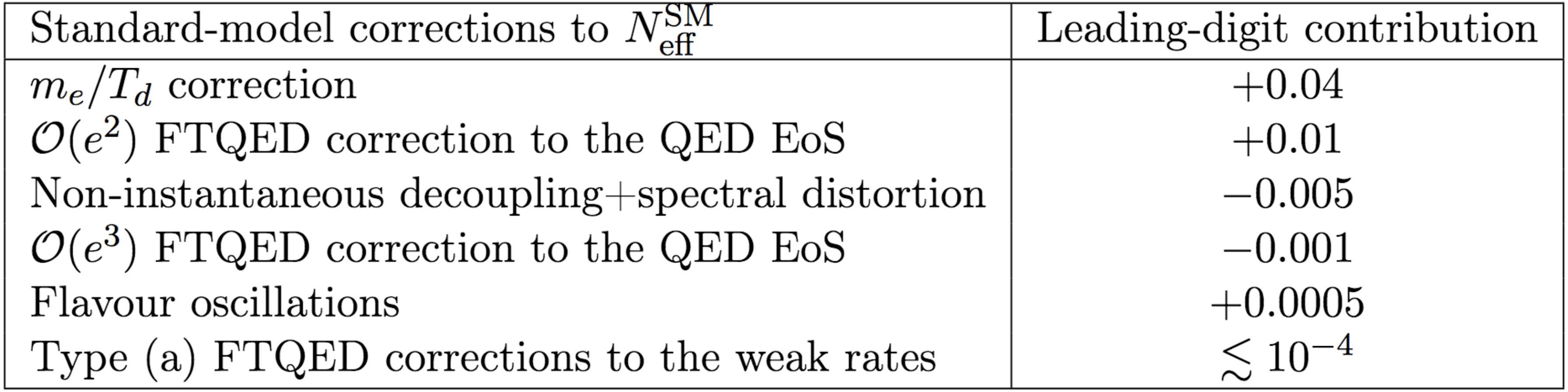
Bennett, Buldgen, de Salas, Drewes, Gariazzo, Pastor & Y^3W 2020

Take-home message...

Bennett, Buldgen, de Salas, Drewes,
Gariazzo, Pastor & Y^3W 2020
Froustey, Pitrou & Volpe 2020

- Precision calculations of the **standard-model effective number of neutrinos**, taking into finite-temperature effects, neutrino oscillations, etc., yield:

$$\boxed{N_{\text{eff}} = 3.0440 \pm 0.0002}$$



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Type (a) FTQED corrections to the weak rates	$\lesssim 10^{-4}$

3. Measuring the relic neutrino background via N_{eff}…

Direct detection of relic neutrinos...

… is a difficult business.

- Small interaction cross-section:
- Neutrino energy too small to cross most detection thresholds.
    - Conventional WIMP detection techniques via nuclear recoil don't work here.

cf WIMP detection, ~ 10^{-46} cm^2

$$\sigma_{\nu N} \sim \frac{G_F^2 m_\nu^2}{\pi} \simeq 10^{-56} \left(\frac{m_\nu}{\mathrm{eV}} \right)^2 \mathrm{cm}^2$$

$$\Delta p \sim m_\nu v_{\mathrm{earth}} \simeq 10^{-3} m_\nu$$

Speed of Earth with respect to the CMB, ~ 370 km s^{-1}

A zero threshold process?

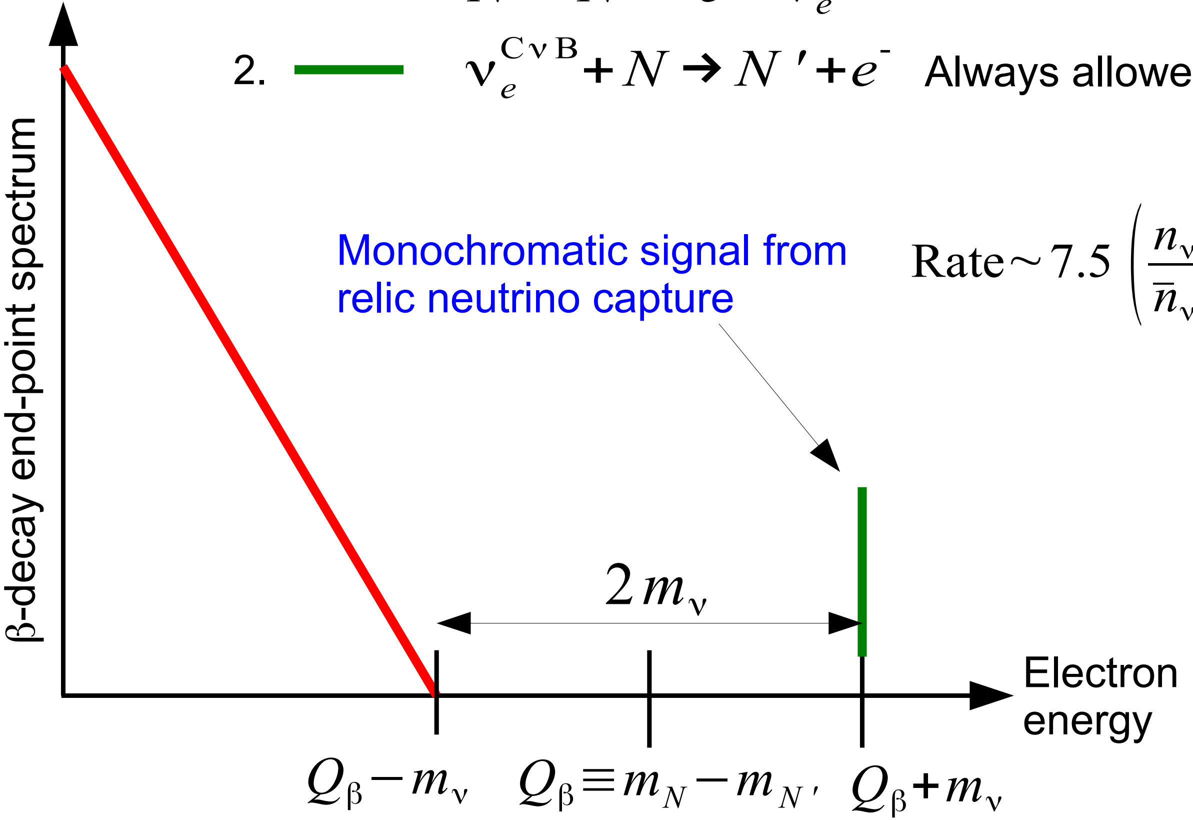
- One unique candidate here...

Direct detection by neutrino capture...

Weinberg 1962

β-decay end-point spectrum

1. 🟥 $N \rightarrow N' + e^- + \bar{\nu}_e$
2. 🟩 $\nu_e^{C\nu B} + N \rightarrow N' + e^-$ Always allowed if $m_\nu + m_N > m_{N'} + m_e$



Monochromatic signal from relic neutrino capture

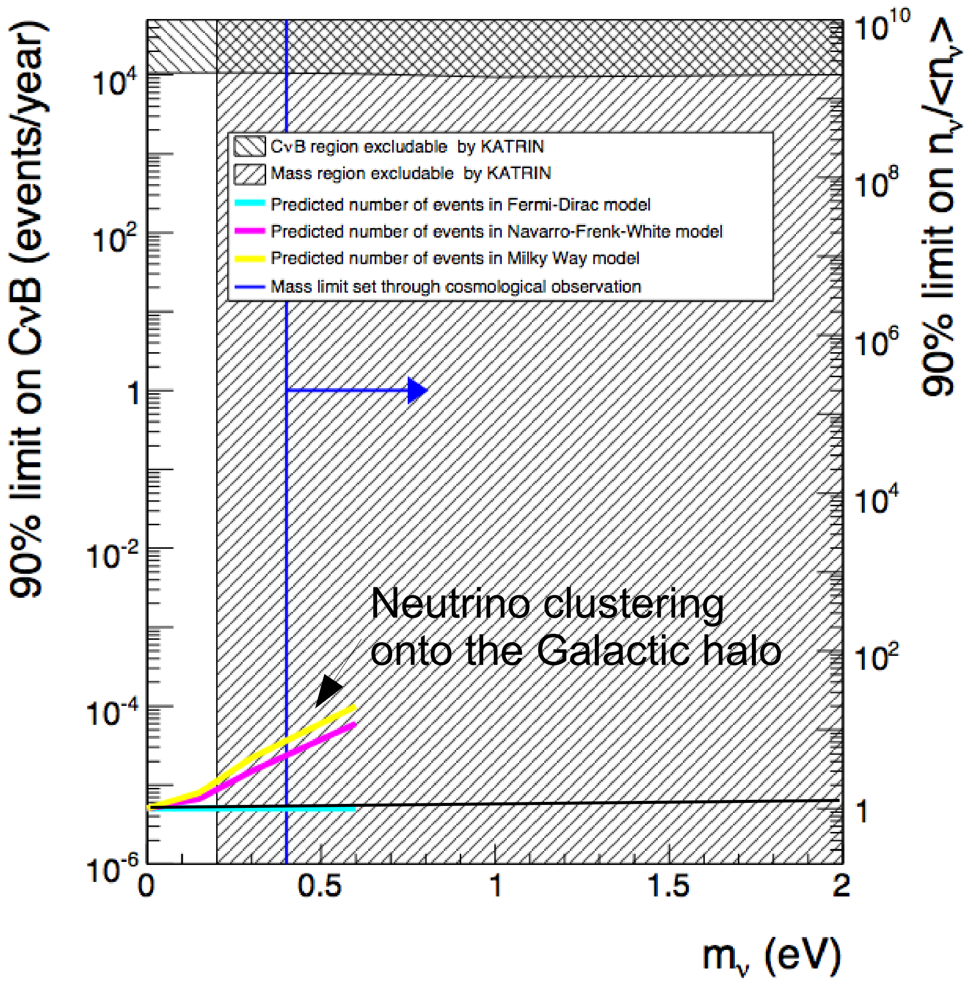
Local neutrino overdensity

$$\text{Rate} \sim 7.5 \left(\frac{n_\nu}{\bar{n}_\nu} \right) / \text{ year } / \left(100 \text{ g tritium} \right)$$

KATRIN ~ 0.1 mg tritium

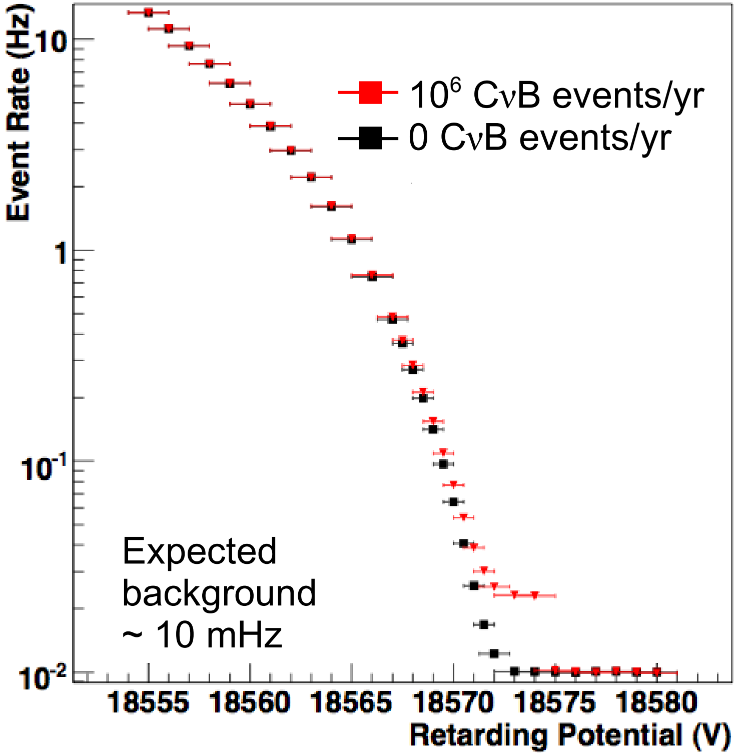
Neutrino capture with KATRIN...

Kaboth, Formaggio & Monreal 2010



Neutrino clustering onto the Galactic halo

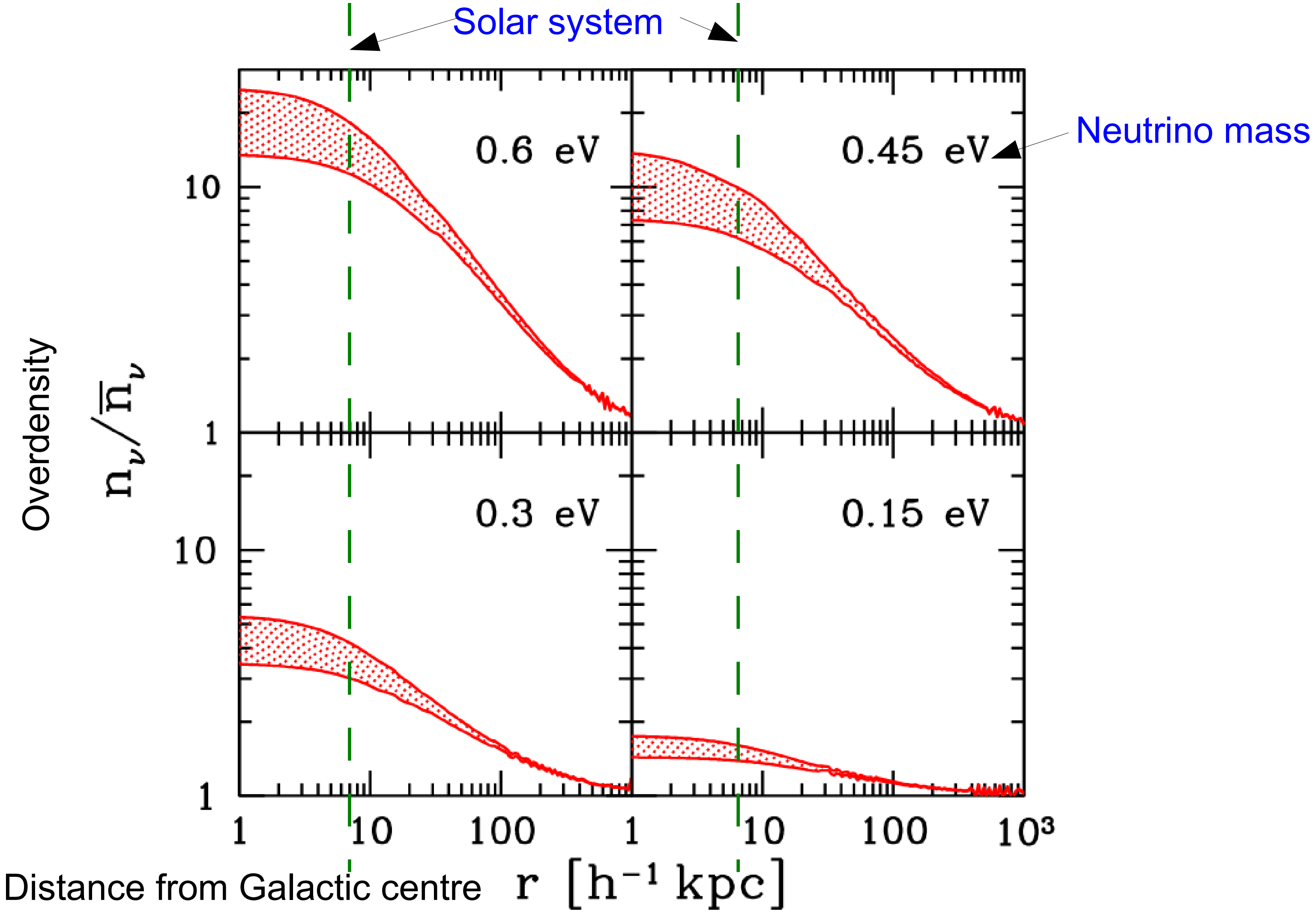
Requires a **10^{9} local overdensity of neutrinos** in a 3-year run for a 90% C.L. detection.



Expected background ~ 10 mHz

Realistic local neutrino overdensity...

Ringwald & Y^3W 2004



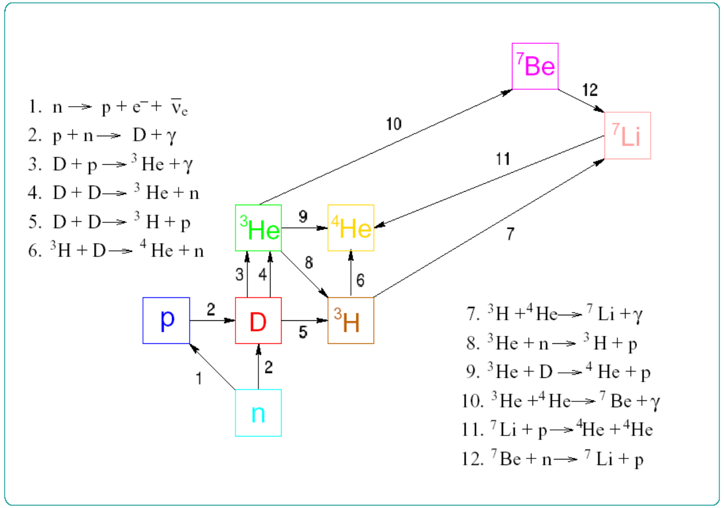
The bottom line:

Direct detection of the relic neutrinos is not going to happen anytime soon…

… but there are other ways to establish their presence...

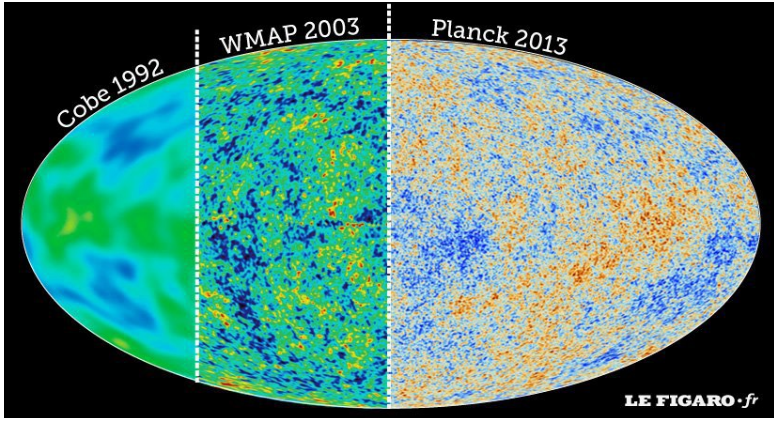
Indirect evidence for relic neutrinos...

- **Light element abundances**

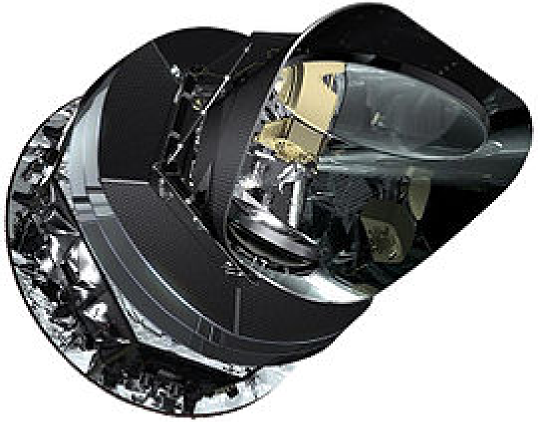


1. $n \to p + e^- + \bar{\nu}_e$
2. $p + n \to D + \gamma$
3. $D + p \to {}^3He + \gamma$
4. $D + D \to {}^3He + n$
5. $D + D \to {}^3H + p$
6. ${}^3H + D \to {}^4He + n$
7. ${}^3H + {}^4He \to {}^7Li + \gamma$
8. ${}^3He + n \to {}^3H + p$
9. ${}^3He + D \to {}^4He + p$
10. ${}^3He + {}^4He \to {}^7Be + \gamma$
11. ${}^7Li + p \to {}^4He + {}^4He$
12. ${}^7Be + n \to {}^7Li + p$

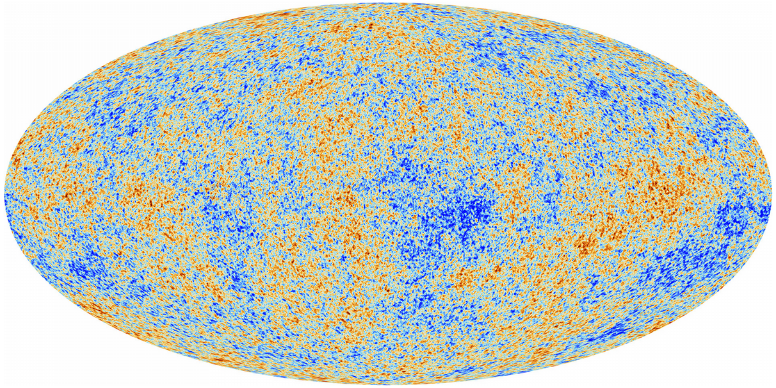
- **<u>CMB anisotropies</u>**

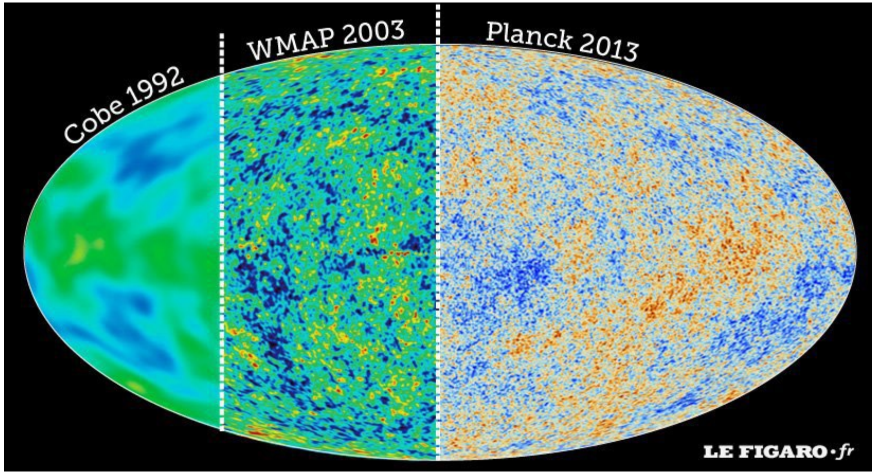


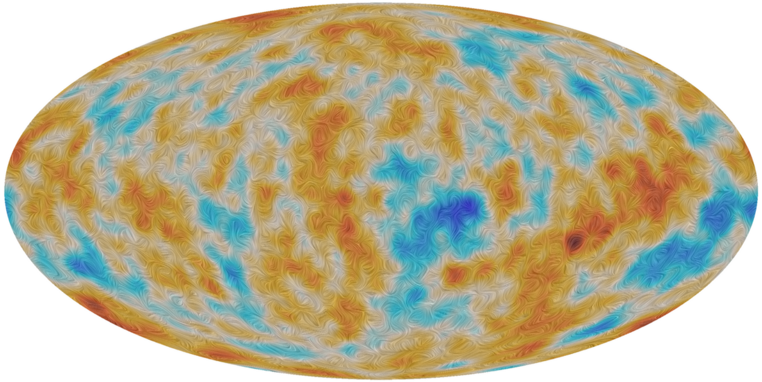
State-of-the-art: Temperature and polarisation fluctuations in the **cosmic microwave background** as seen by Planck. (Latest results 2018.)



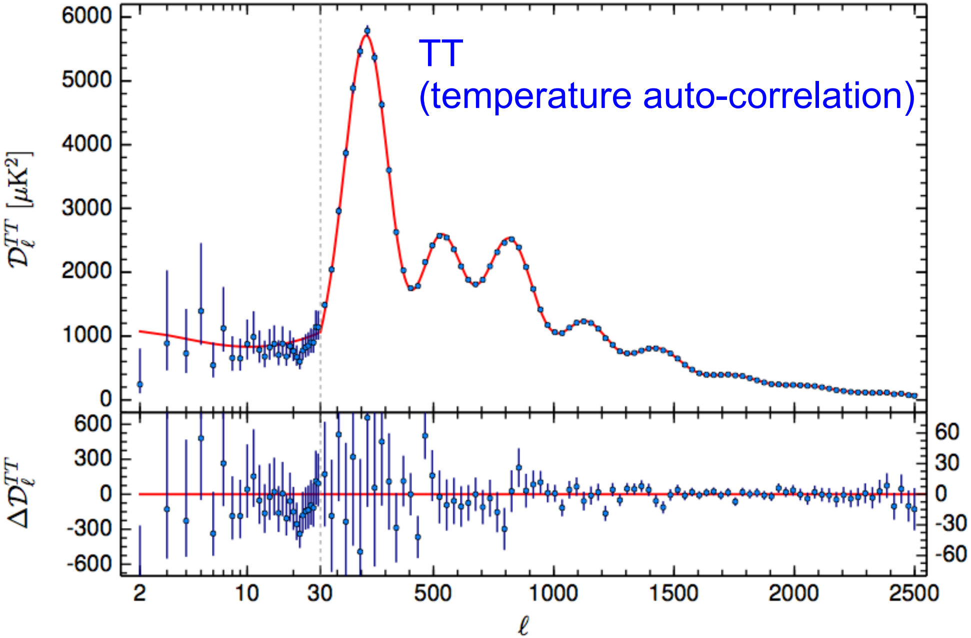
Temperature

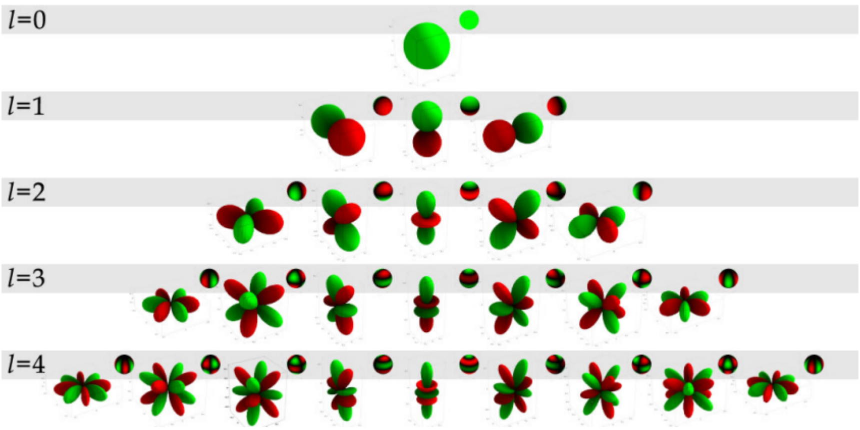


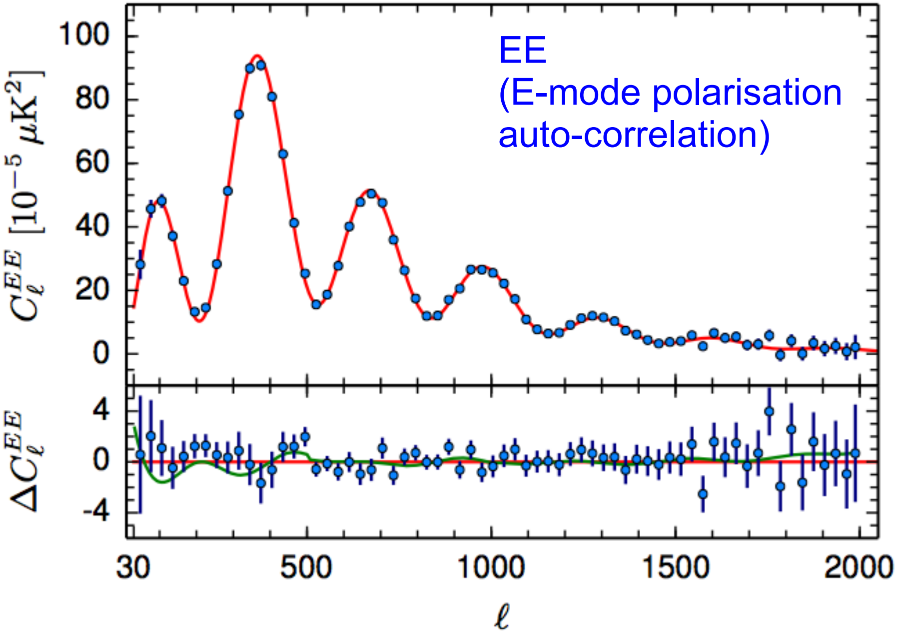


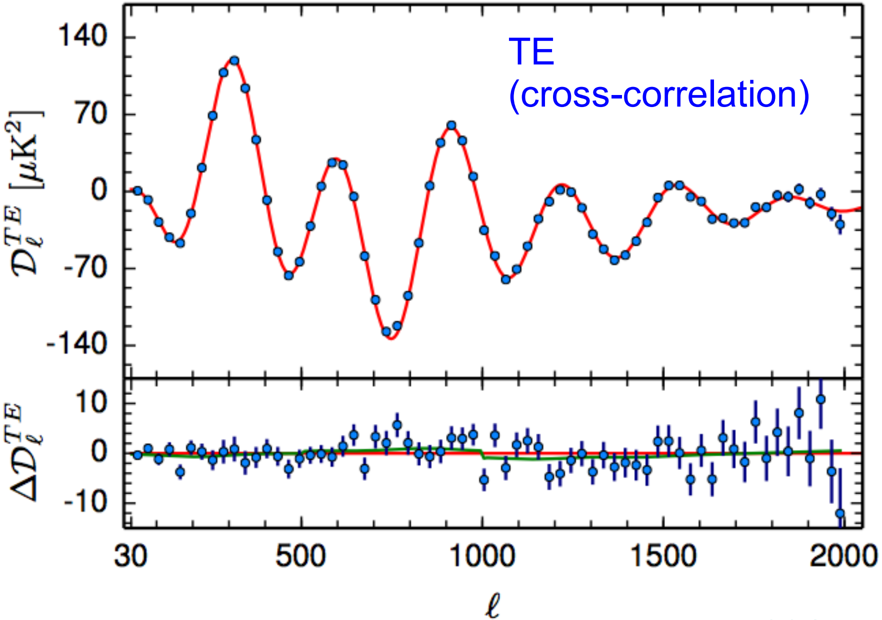


Polarisation





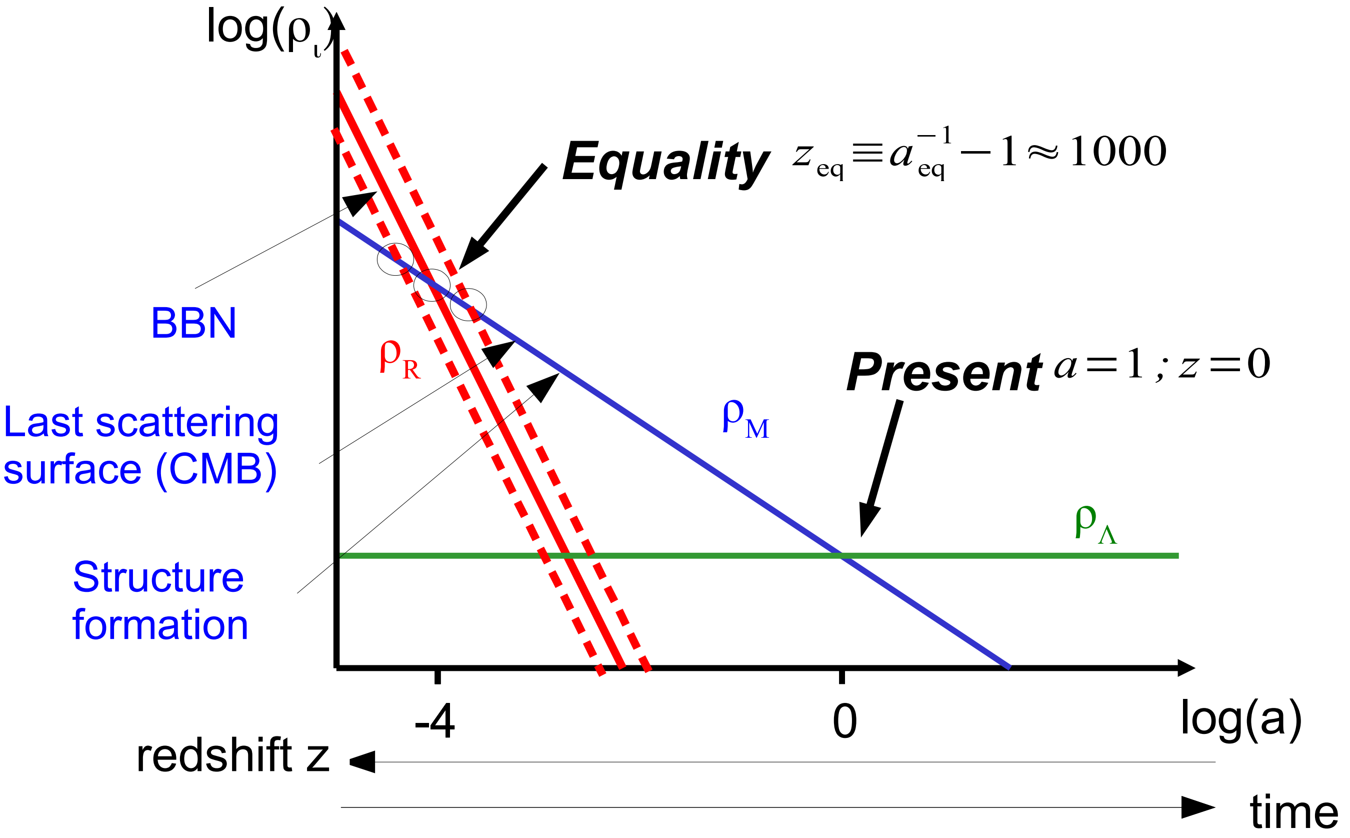




Ade et al. [Planck collaboration] 2015

CMB anisotropies are sensitive to N_{eff} too...

- At the most basic level, changing the **neutrino energy density** shifts the epoch of matter-radiation equality.



Radiation:
$\rho_R \propto a^{-4}$

Matter:
$\rho_M \propto a^{-3}$

Vacuum energy:
$\rho_\Lambda \propto \text{const}$

But it's not as simple as that...

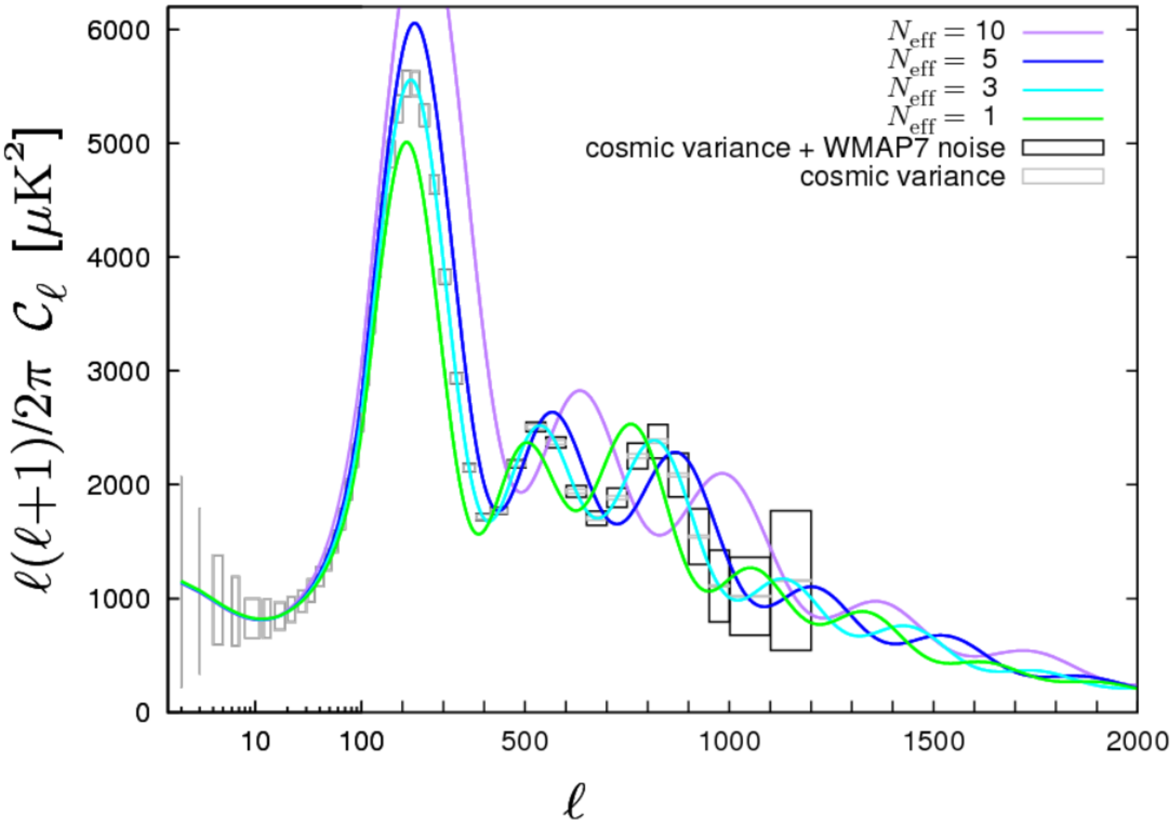
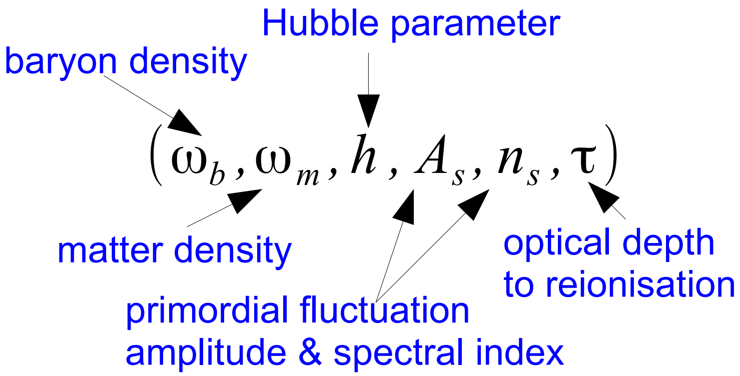


Figure courtesy of J. Hamann

- N_{eff} looks easy to detect in the CMB TT spectrum.
- But we also use the **same data** to measure at least 6 other parameters:



Hubble parameter

baryon density

$$(\omega_b, \omega_m, h, A_s, n_s, \tau)$$

matter density

optical depth to reionisation

primordial fluctuation amplitude & spectral index

- Plenty of parameter degeneracies!

What the CMB really probes: equality redshift...

Ratio of 3rd and 1st peaks sensitive to the redshift of matter-radiation equality via the early ISW an other time-dependent effects.

Exact degeneracy between the physical matter density ω_m and N_{eff}.

$$1+z_{eq}=\frac{\omega_m}{\omega_r}\simeq\frac{\omega_m}{\omega_\gamma}\frac{1}{1+0.2271\,N_{eff}}$$

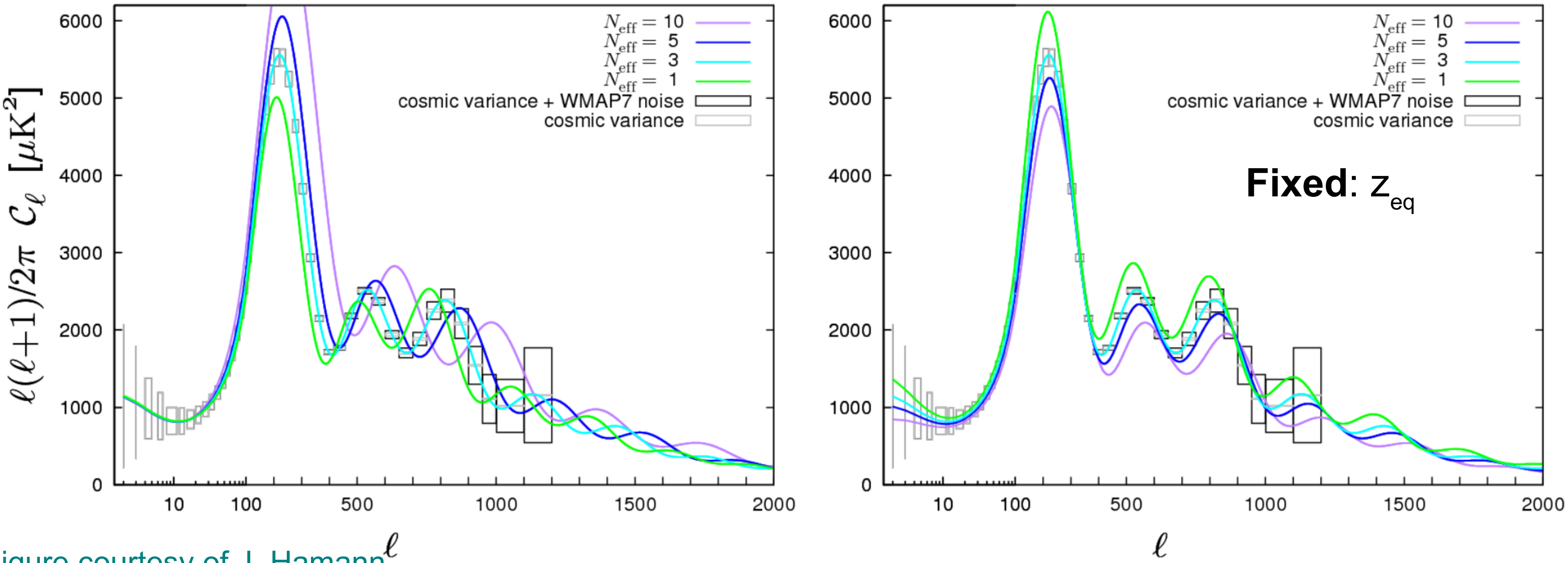


Figure courtesy of J. Hamann

What the CMB really probes: sound horizon...

Exact degeneracy between ω_m and the Hubble parameter h.

Peak positions depend on:

$$\theta_s = \frac{r_s}{D_A}$$

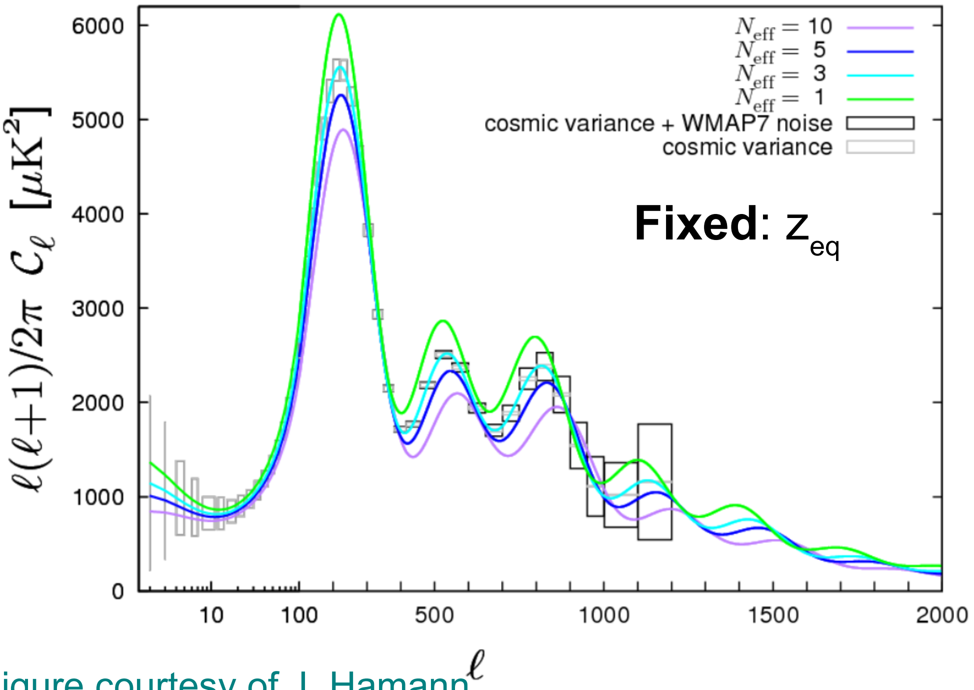
Sound horizon at decoupling

Angular distance to the last scattering surface

Flat ΛCDM

$$\theta_s \propto \frac{\left(\omega_m h^{-2}\right)^{-1/2}}{\displaystyle\int_{a_*}^{1} \frac{da}{\sqrt{\omega_m h^{-2} a^{-3} + \left(1 - \omega_m h^{-2}\right)}}}$$

Fixed z_{eq}, ω_b



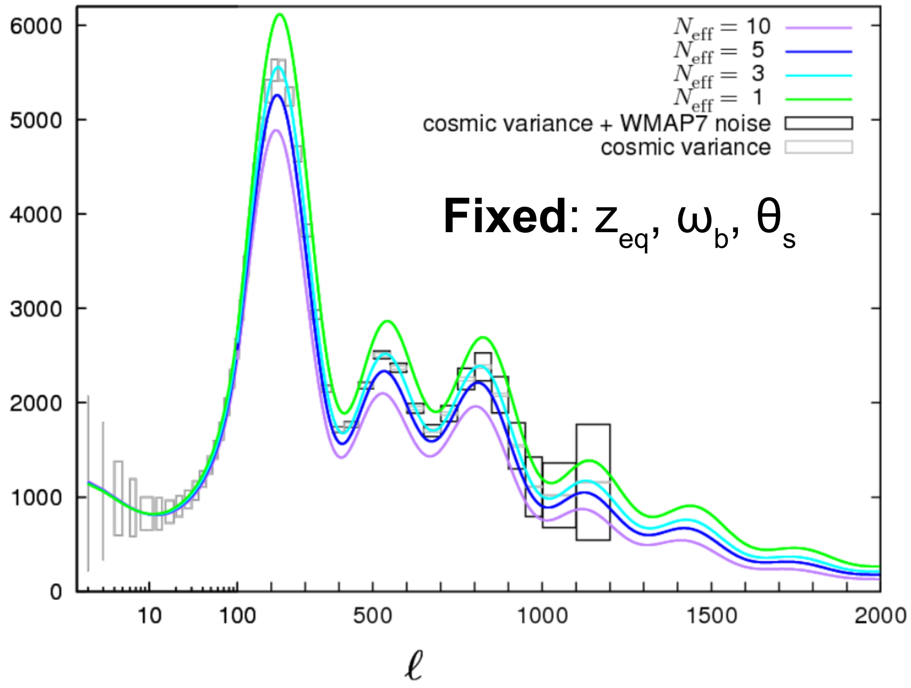
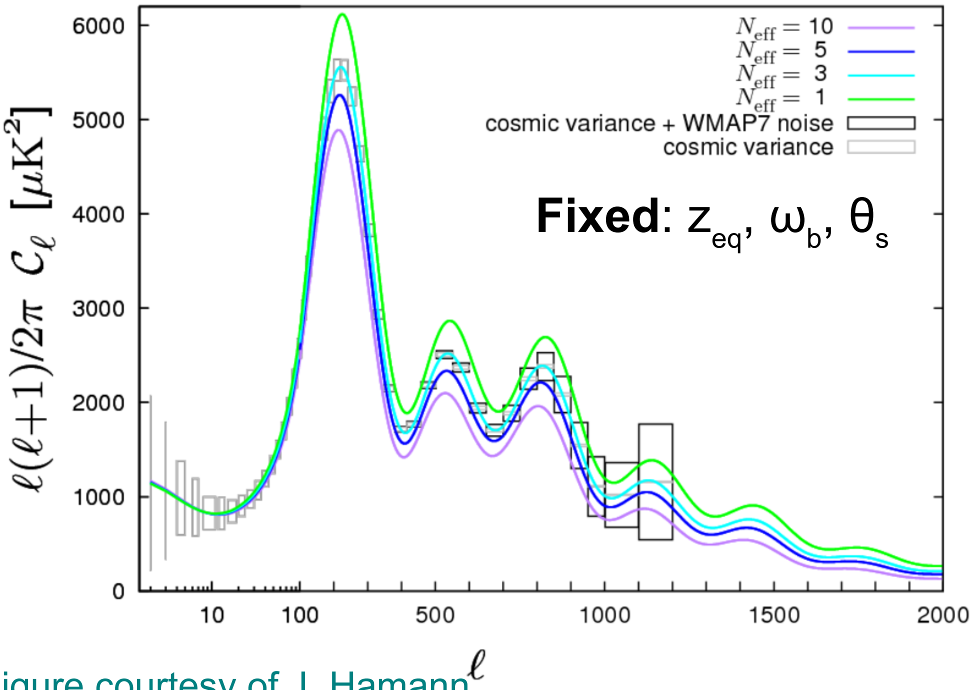


Figure courtesy of J. Hamann

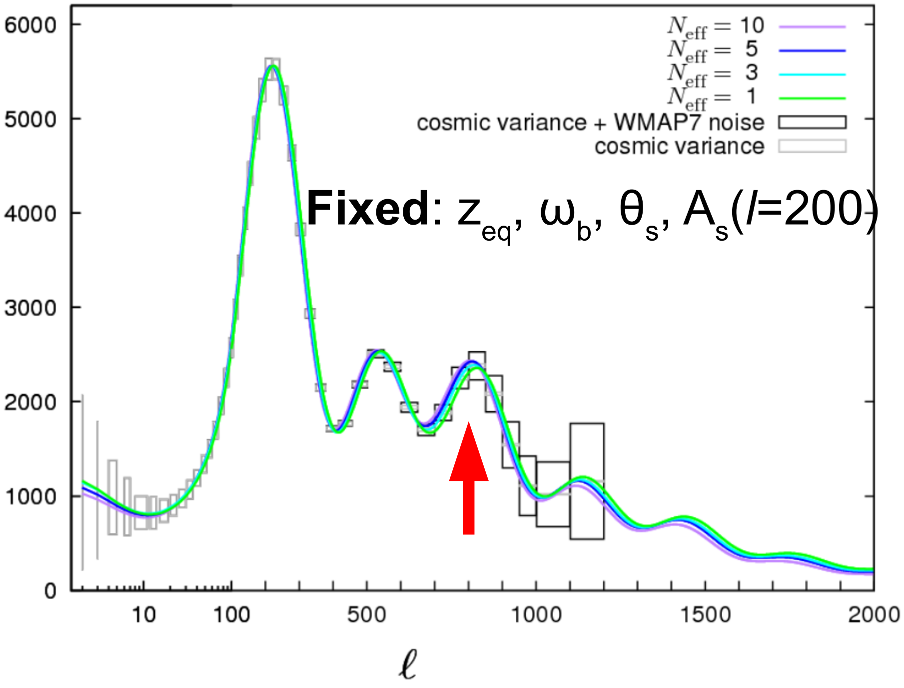
What the CMB really probes: anisotropic stress...

Apparent (i.e., not physical) partial degeneracies with inflationary parameters: primordial fluctuation amplitude A_s and spectral index n_s.



Fixed: z_{eq}, ω_b, θ_s

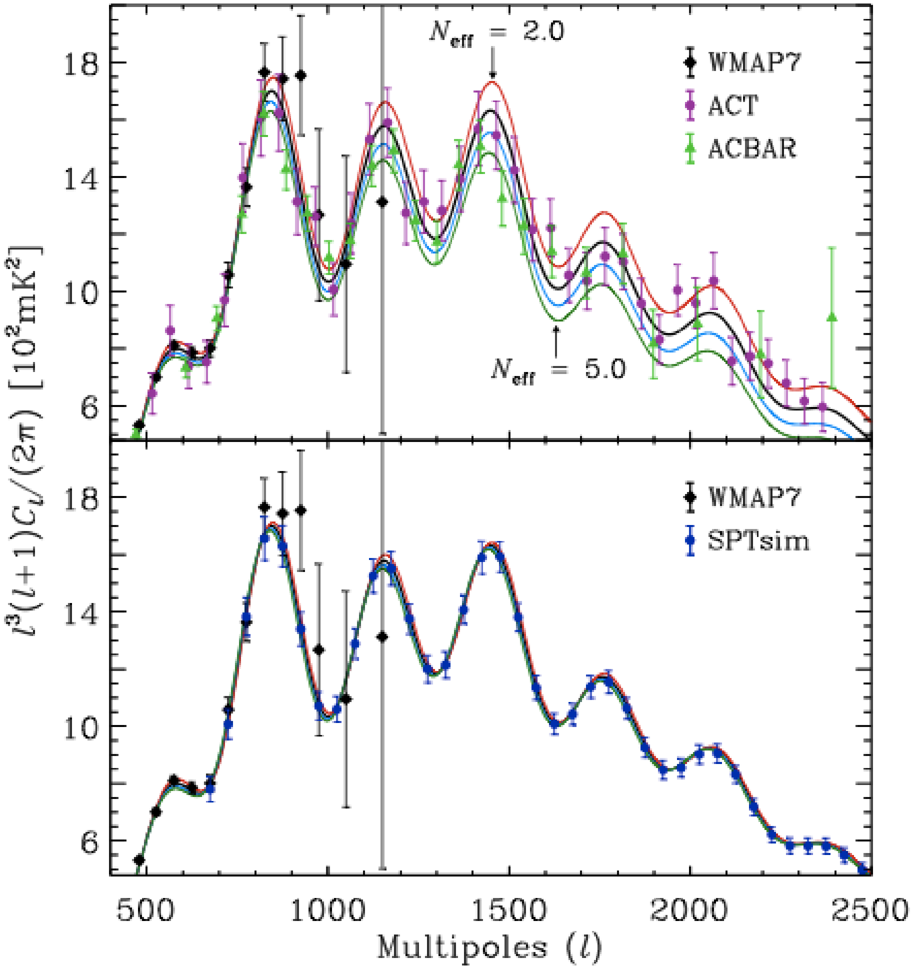
- However, free-streaming (non-interacting relativistic) particles have **anisotropic stress**.
- First real signature of N_{eff} in the 3rd peak!



Fixed: z_{eq}, ω_b, θ_s, $A_s(l=200)$

Figure courtesy of J. Hamann

N_{eff} signatures in the CMB damping tail...



Hou, Keisler, Knox et al. 2011

- Measured by **ACT** since 2010; **SPT** since 2011; **Planck** since 2013.
- Probe **photon diffusion scale**:

$$\theta_d = \frac{r_d}{D_A}$$

Diffusion scale at decoupling

- Primary signature of N_{eff} in the Planck era.

Current constraints on N_{eff} ...

Aghanim et al. [Planck] 2018
Ade et al. [Planck] 2015

Planck-inferred N_{eff} compatible with 3.046 at better than 2σ.

ΛCDM+Neff 7-parameter fit	Planck 2018 (95%)	Planck2015 (95%)
TT+lowE	$\mathbf{3.00^{+0.57}_{-0.53}}$	3.13±0.64
+lensing+BAO	$\mathbf{3.11^{+0.44}_{-0.43}}$	n/a
TT+lowE+TE+EE	$\mathbf{2.92^{+0.36}_{-0.37}}$	2.99±0.40
+lensing+BAO	$\mathbf{2.99^{+0.34}_{-0.33}}$	n/a

ΛCDM+Neff+neutrino mass
8-parameter fit

$$\left.\begin{array}{l} N_{\text{eff}} = 2.96^{+0.34}_{-0.33} \\ \sum m_\nu < 0.12 \ \text{eV} \end{array}\right\}$$

95% C. L.
Planck TT+TE+EE+lowE
+lensing+BAO

Flies in the ointment: the H_0 discrepancy...

4.2σ discrepancy between the Planck-inferred H_0 and local measurements:

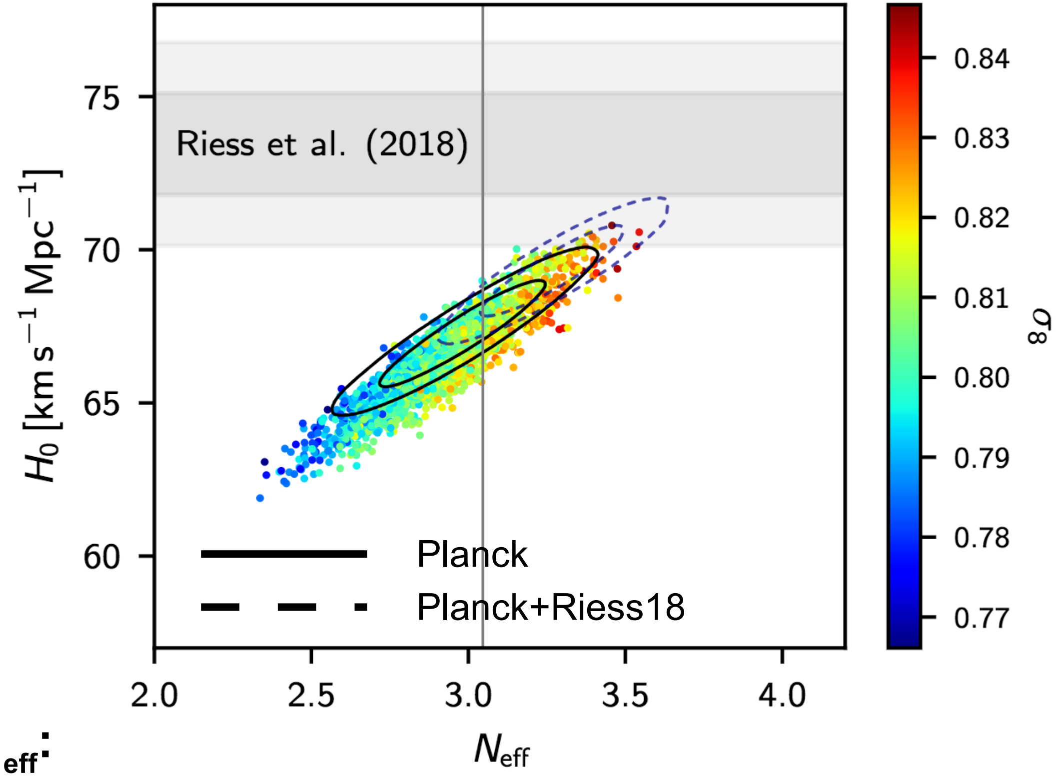
- **TT+TE+EE+lowE+lensing**

$$H_0 = 67.36 \pm 0.54\,\mathrm{km\,s^{-1}\,Mpc^{-1}}$$

- **Local measurement**:

$$H_0 = 73.2 \pm 1.3\,\mathrm{km\,s^{-1}\,Mpc^{-1}}$$

Riess et al. 2021



Joint Planck+Riess 2018 fit varying $N_{\rm eff}$:

$$\left.\begin{aligned} N_{\rm eff} &= 3.27 \pm 0.15 \\ H_0 &= 69.32 \pm 0.97\,\mathrm{km\,s^{-1}\,Mpc^{-1}} \end{aligned}\right\}$$

68% C. L.
Planck TT+TE+EE+lowE
+lensing+BAO+Riess

Take-home message...

- The prediction of a relic neutrino background is **as fundamental as** the prediction of the CMB.
- Direct detection based on scattering seems impossible at the moment...
- However, we can establish the CνB's presence through its effects on
  - The **light elemental abundances**
  - The **CMB temperature anisotropies**

→ The Planck measurements are consistent with $N_{\mathrm{eff}} = 3$.

→ However, a 4.2σ discrepancy between Planck and local measurements of H_0 remains in ΛCDM, which cannot be completely resolved with $N_{\mathrm{eff}}>3$. The discrepancy does however drive up slightly the preferred value of N_{eff} in a combined analysis.

Extra slides…

Primordial light elements...

Big bang nucleosynthesis...

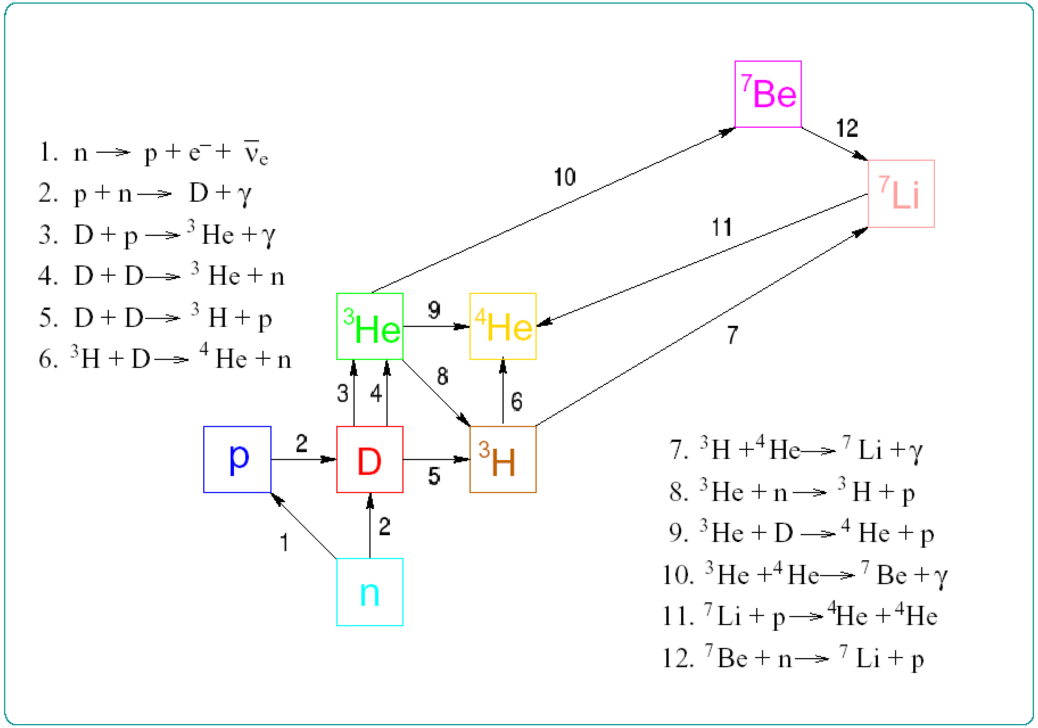
- The production of **light nuclei** at temperatures: $T \sim O(100) \rightarrow O(10)$ keV
  - Deuterium, Helium-3, Helium-4, Lithium-7, etc.

- **A 2-parameter problem**:

The initial neutron-to-proton ratio.

Relic neutrinos affect mainly this

The baryon-to-photon ratio; determines when the production of the first nucleus in the chain, Deuterium, should begin.



1. $n \rightarrow p + e^- + \bar{\nu}_e$
2. $p + n \rightarrow D + \gamma$
3. $D + p \rightarrow {}^3He + \gamma$
4. $D + D \rightarrow {}^3He + n$
5. $D + D \rightarrow {}^3H + p$
6. ${}^3H + D \rightarrow {}^4He + n$
7. ${}^3H + {}^4He \rightarrow {}^7Li + \gamma$
8. ${}^3He + n \rightarrow {}^3H + p$
9. ${}^3He + D \rightarrow {}^4He + p$
10. ${}^3He + {}^4He \rightarrow {}^7Be + \gamma$
11. ${}^7Li + p \rightarrow {}^4He + {}^4He$
12. ${}^7Be + n \rightarrow {}^7Li + p$

Setting the n/p ratio...

Relevant temperatures:
$T \sim 0.8 \rightarrow 0.1$ MeV

- **At T > 1 MeV** the neutron-to-proton ratio is set by the interactions:

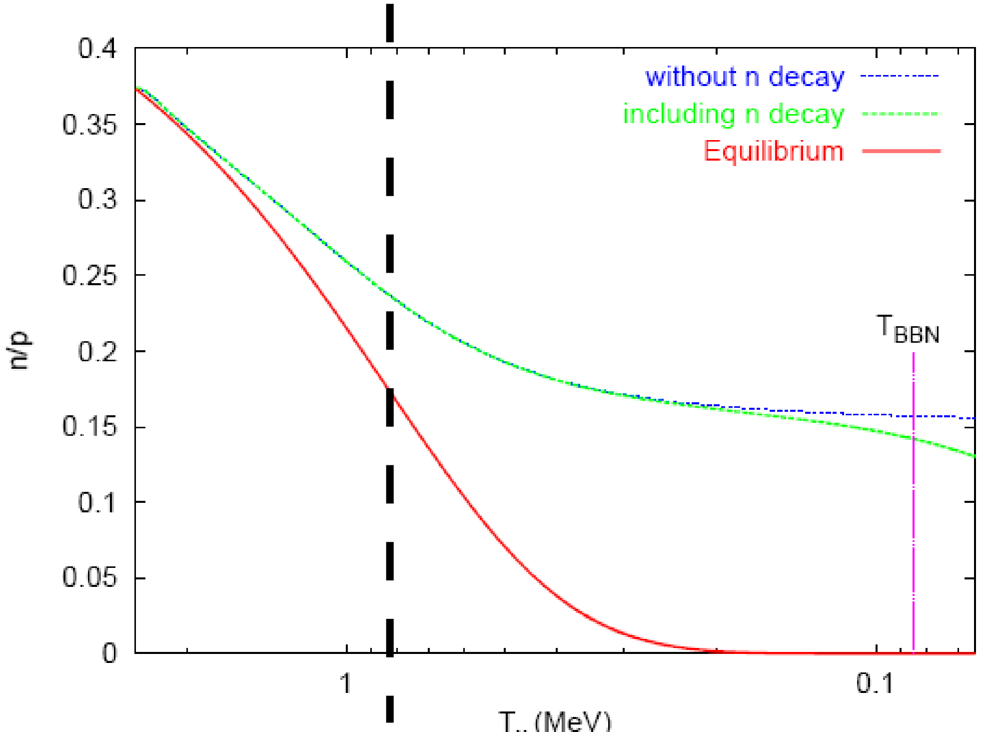
$$\bar{\nu}_e + p \rightarrow e^+ + n$$

$$\nu_e + n \rightarrow e^- + p$$

$$\left(\frac{n}{p}\right)_{\mathrm{eq}} \simeq \exp\left(-\frac{m_n - m_p}{T_\gamma}\right)$$

- **After freeze-out** neutron decay can still change the n/p ratio:

$$\tau_n = 885.6 \pm 0.8 \text{ s}$$



Freeze-out, i.e.,
Scattering rate per particle < Hubble expansion rate

Relic neutrinos and the n/p ratio...

- Hubble expansion rate:

Photons and neutrinos dominate the energy density at the time of nucleosynthesis

$$H^2=\left(\frac{1}{a}\frac{d\,a}{d\,t}\right)^2=\frac{8\pi G}{3}\rho_{\text{total}}\simeq\frac{8\pi G}{3}\left(\rho_\gamma+\sum_i\rho_{\nu,i}\right)$$

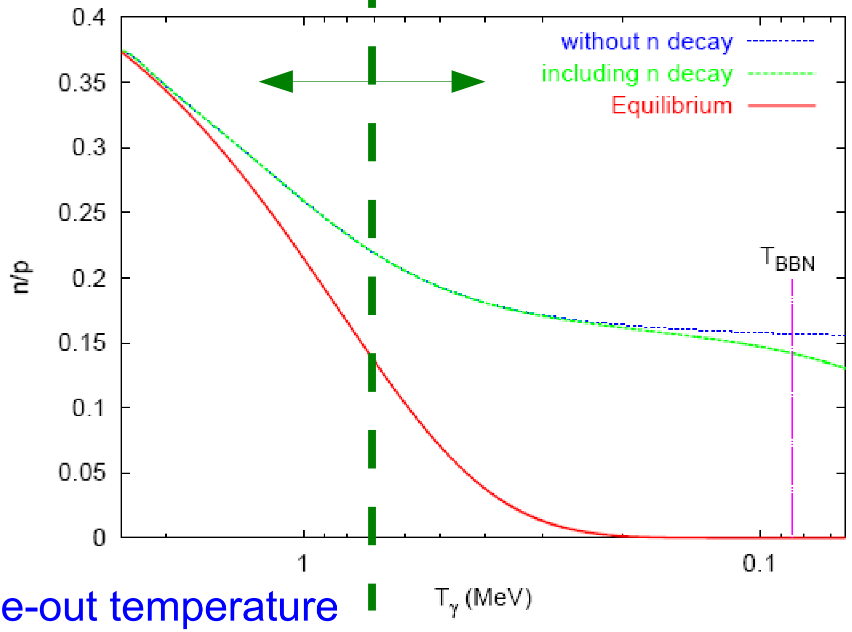
Total energy density: radiation, matter vacuum energy...

→ The **higher** the neutrino energy density the **earlier** the freeze-out (and vice versa)

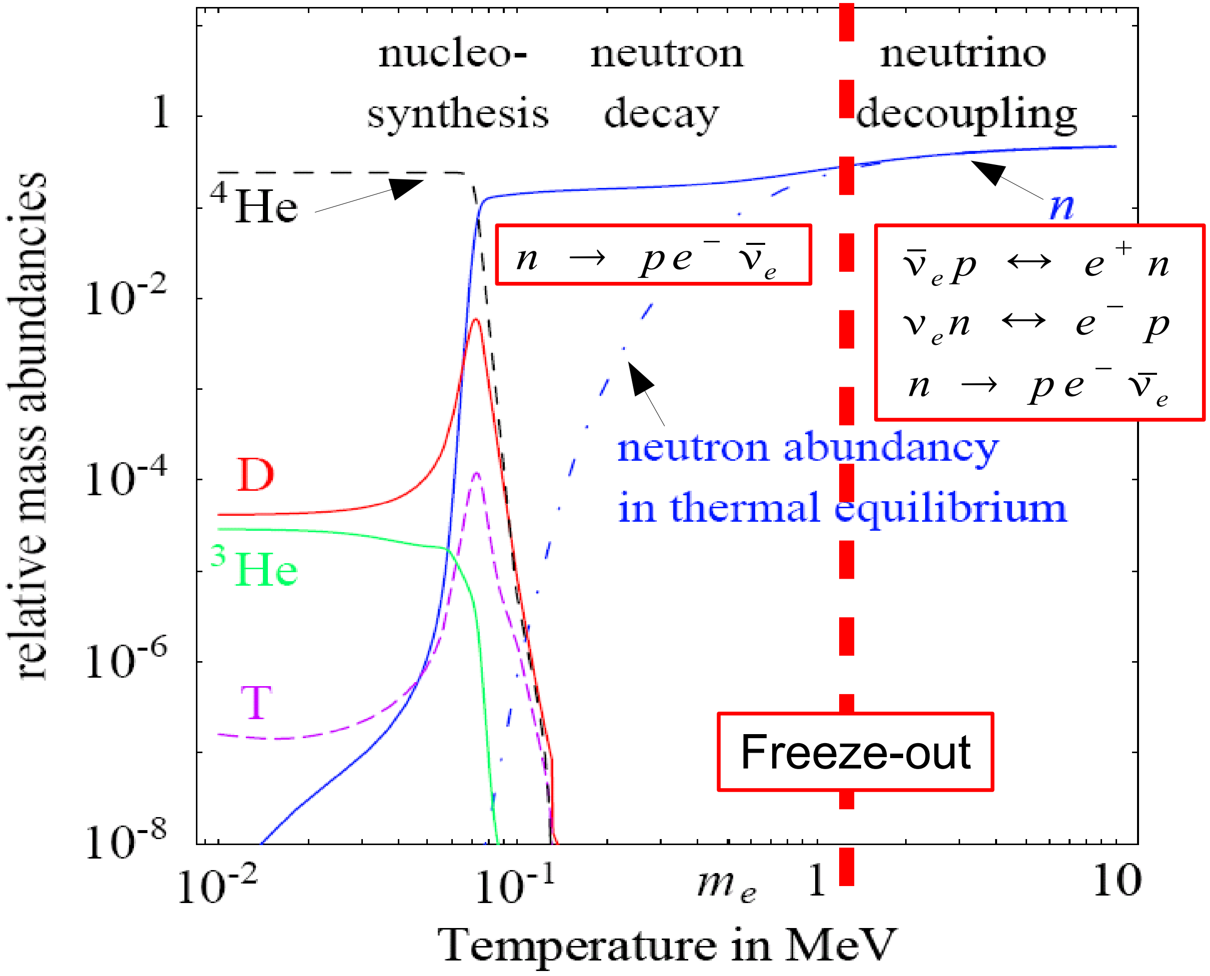
→ **Higher n/p ratio**:

$$\left(\frac{n}{p}\right)_{\text{freeze}}\simeq\exp\left(-\frac{m_n-m_p}{T_{\text{freeze}}}\right)$$

Freeze-out temperature



The n/p ratio affects **all** light elemental abundances.



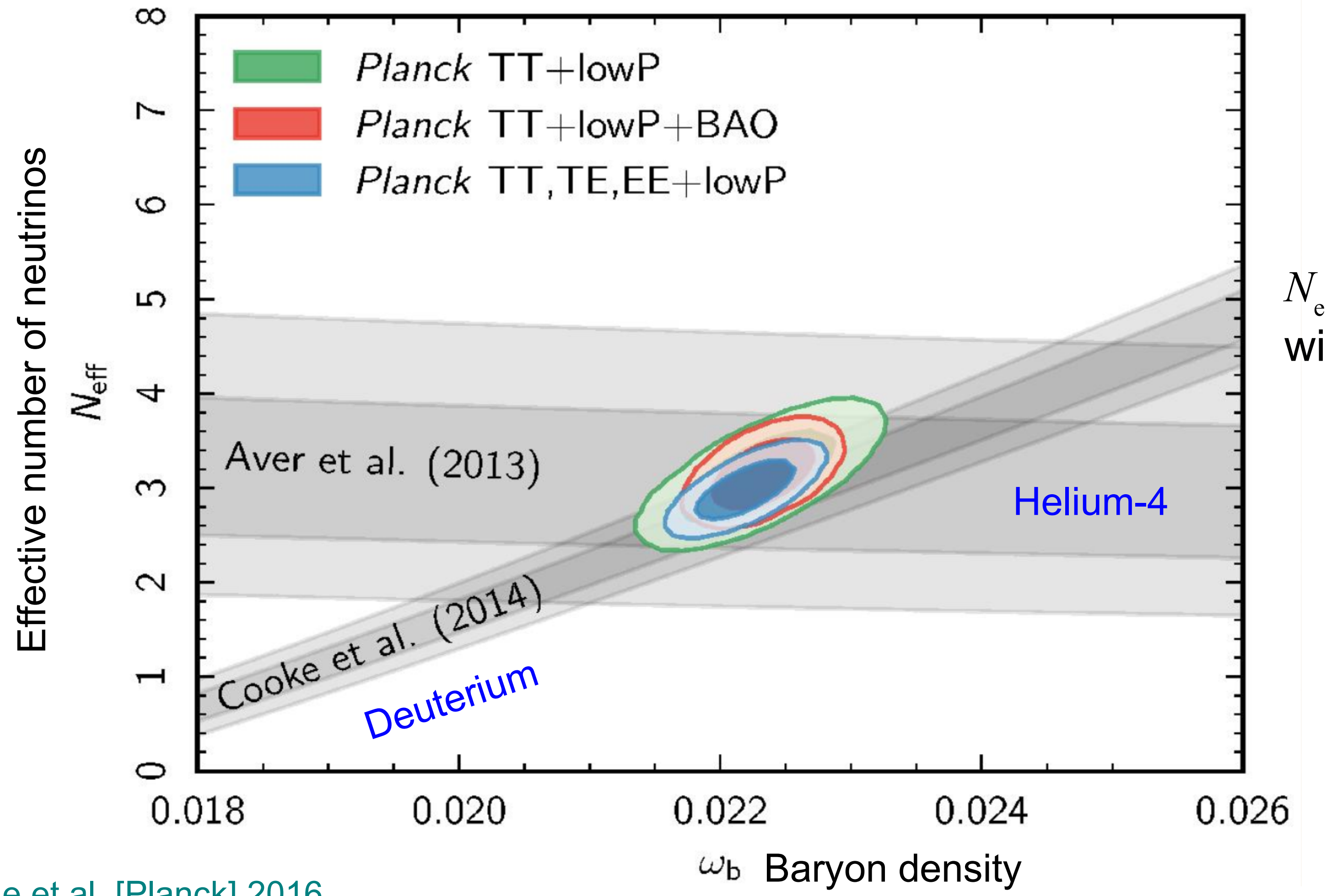
Measuring primordial abundances...

- Light element abundances we observe in astrophysical systems today are generally **not at their primordial values**.

Deuterium	Destroyed in stars.	Data from high-z, low metallicity QSO absorption line systems
Helium-3	Produced and destroyed in stars. Complicated evolution.	Data from solar system and galaxies, but not used in BBN analyses.
Helium-4	Produced in stars by H burning.	Data from low metallicity, extragalactic HII regions.
Lithium-7	Destroyed in stars, produced in cosmic ray interactions.	Data from the oldest, most metal poor stars in the Galaxy.

- For measurements, low metallicity systems with as little evolution as possible are the best bets!

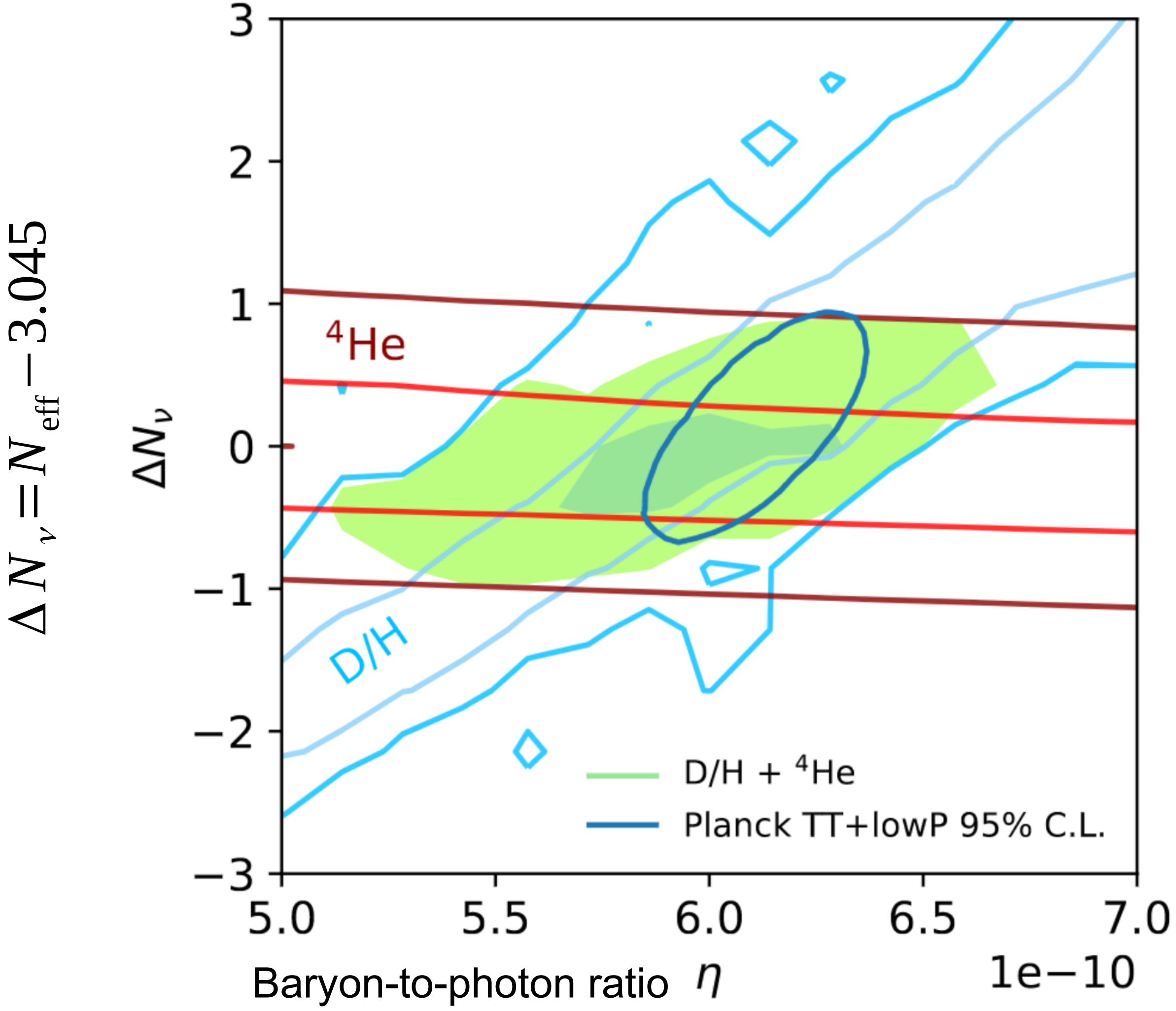
Light element constraints on N_{eff}...



$N_{\mathrm{eff}} = 3$ is consistent with measurements.

Ade et al. [Planck] 2016

Light element constraints on N_{eff}...



$N_{\mathrm{eff}} = 3$ is consistent with measurements.

Riemer-Sørensen
& Jenssen 2017

Planck CMB: flies in the ointment…

Small fly: the σ_8-Ω_m discrepancy...

Cosmic shear measurements tend to prefer lower values of σ_8 or Ω_m than Planck.

- Mostly mild to modest discrepancy
- (One claim of 2.6σ discrepancy from KiDS Joudaki et al. 2018)
- Appears amenable to improved treatment of lensing systematics.