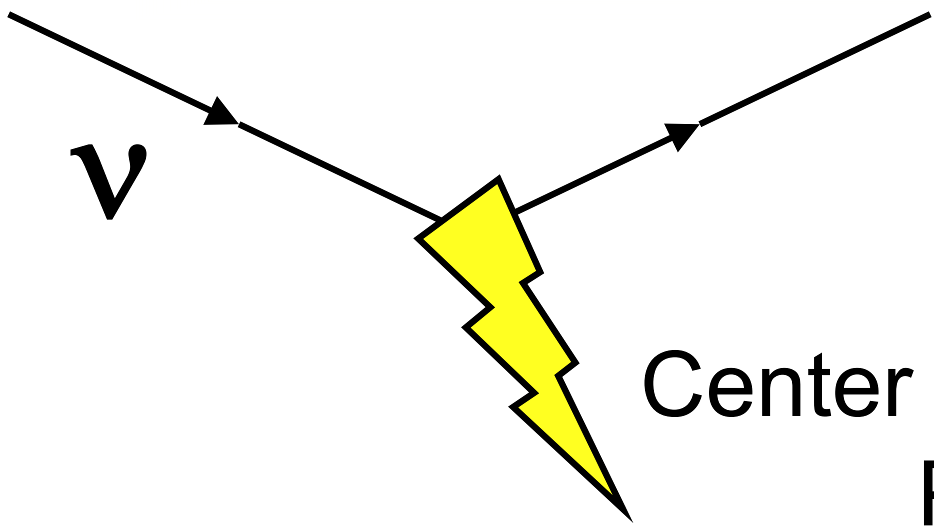
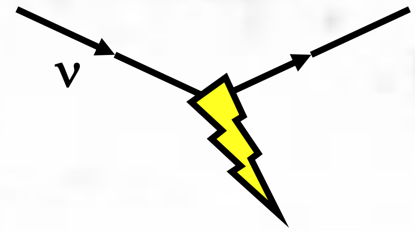
Interactions of Neutrinos



Kevin McFarland
University of Rochester
Center for Excellence in Particle Physics (CCEPP) School
25-26 August 2021

Outline

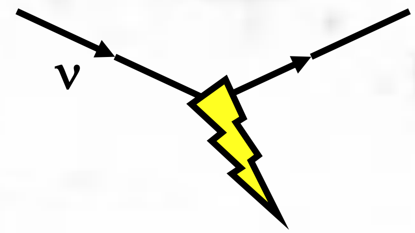


- Brief Motivation for Measuring Interactions
- Weak interactions and neutrinos
  - Elastic and pointlike processes, e.g., νe scattering
  - Deep inelastic (νq) scattering
  - Complication of Targets with Structure: Inverse Beta Decay
- Interactions with nucleons
  - Elastic and nearly elastic scattering
- Interactions with nuclei
  - Phenomena at very low to moderate momentum transfer
  - Recent experimental results
  - Theory and implementation in generators
- Conclusions

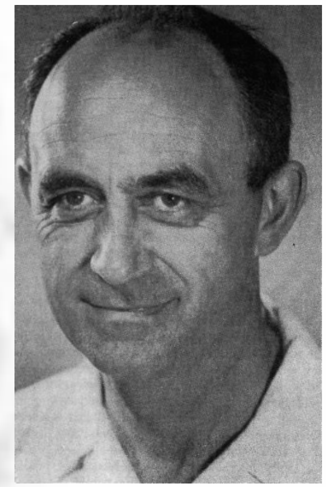
Focus of These Lectures

- This is not a comprehensive review of all the interesting physics associated with neutrino interactions
- Choice of topics will focus on:
  - Cross-sections useful for studying neutrino properties
  - Estimating cross-sections
  - Understanding the most important effects qualitatively or semi-quantitatively
  - Understanding how we use our knowledge of cross-sections in experiments

Weak Interactions



- Current-current interaction $H_W = \frac{G_F}{\sqrt{2}} J^{\mu} J_{\mu}$
  Fermi, Z. Physik, 88, 161 (1934)
  - Paper famously rejected by *Nature:*
    "it contains speculations too remote from reality to be of interest to the reader"

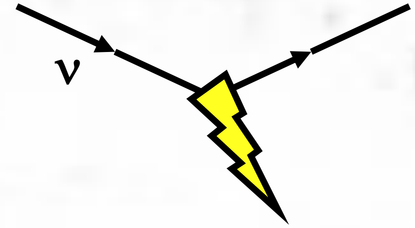


- Prediction for neutrino interactions
  - If $n \to p e^{-} \bar{\nu}$, then $\bar{\nu} p \to e^{+} n$
  - Better yet, it is robustly predicted by Fermi theory
    - o Bethe and Peirels, Nature 133, 532 (1934)
  - For neutrinos of a few MeV from a reactor, a typical cross-section was found to be

$$\sigma_{\bar{\nu} p} \sim 5 \times 10^{-44}\,\mathrm{cm}^2$$

This is wrong by a factor of two (parity violation)

How Weak is This?

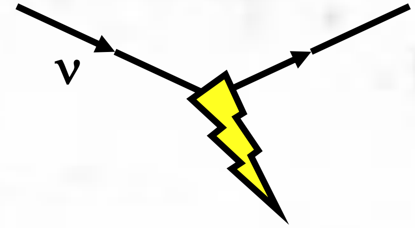


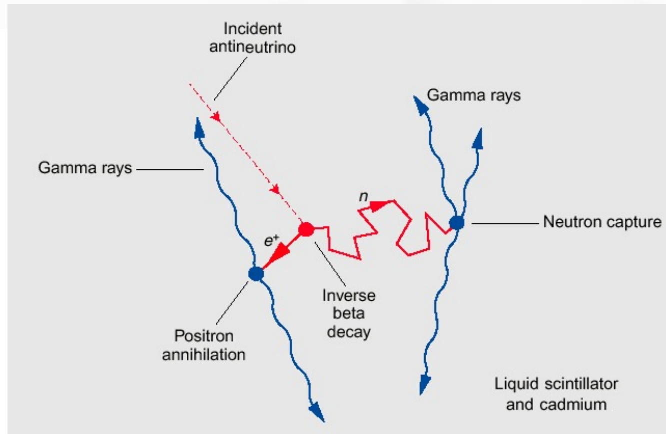
- $\sigma \sim 5 \times 10^{-44} cm^2$ compared with
  - $\sigma_{\gamma p} \sim 10^{-25}\ cm^2$ at similar energies, for example
- The cross-section of these few MeV neutrinos is such that the mean free path in steel would be 10 light-years

"I have done something very bad today by proposing a particle that cannot be detected; it is something no theorist should ever do."

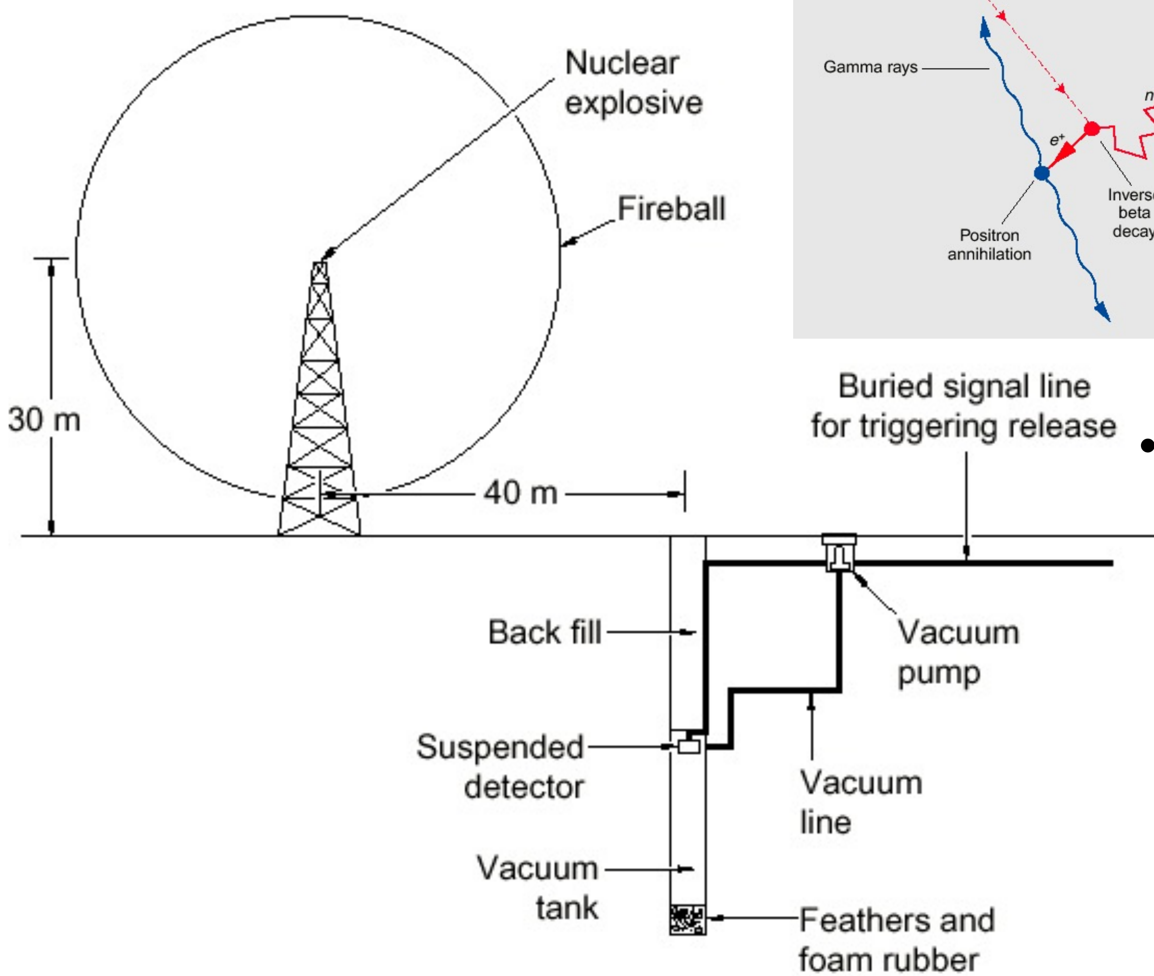
Wolfgang Pauli

Extreme Measures to Overcome Weakness *(Reines and Cowan, 1946)*



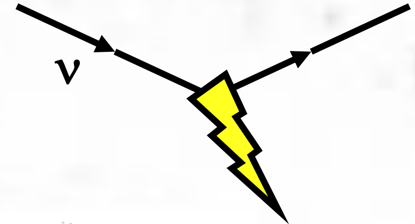


$$\bar{\nu}\, p \to e^{+} n$$

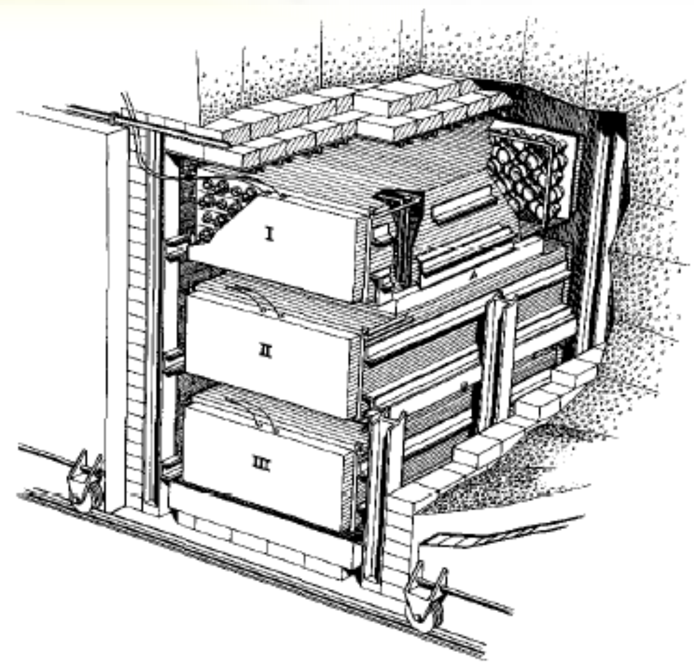


- Why inverse neutron beta decay?
  - clean prediction of Fermi weak theory
  - clean signature of prompt gammas from e^+ plus delayed neutron signal.
    - Latter not as useful with bomb source.

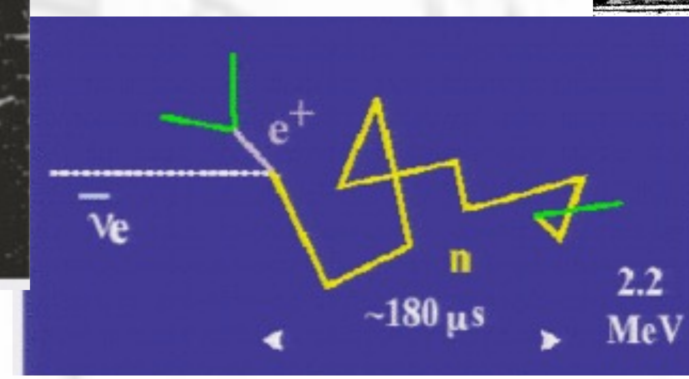
Discovery of the Neutrino

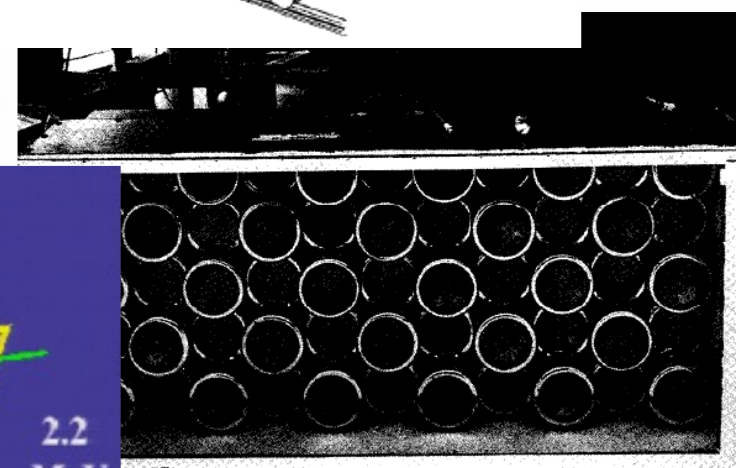


- Reines and Cowan (1955)
  - Chose a constant source, nuclear reactor (Savannah River)
  - 1956 message to Pauli: ”We are happy to inform you [Pauli] that we have definitely detected neutrinos…”
  - 1995 Nobel Prize for Reines

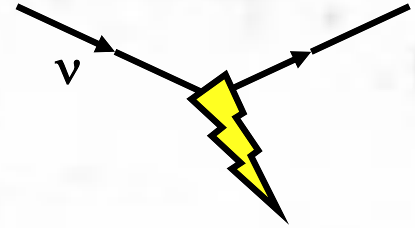


$$\bar{\nu}\, p \rightarrow e^{+} n$$





Better than the Nobel Prize?



Frederick REINES and Clyde COWAN
Box 1663, LOS ALAMOS, New Mexico

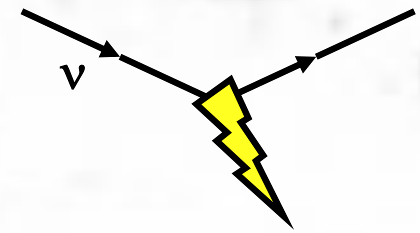
Thanks for message. Everything comes to him who knows how to wait.

Pauli

Thanks for the message. Everything comes to him who knows how to wait.

erh. 15.6.56 / 15.35 h
als Night Letter

Lecture Questions



- Lectures are a hard format for active learning.
- I like to ask my students questions in lectures.
- Here's a "Question #0" to try it out.

PHYSICAL REVIEW VOLUME 97, NUMBER 3 FEBRUARY 1, 1955

Attempt to Detect the Antineutrinos from a Nuclear Reactor by the $Cl^{37}(\bar{\nu},e^-)A^{37}$ Reaction*

RAYMOND DAVIS, JR.
Department of Chemistry, Brookhaven National Laboratory, Upton, Long Island, New York

(Received September 21, 1954)

Tanks containing 200 and 3900 liters of carbon tetrachloride were irradiated outside of the shield of the Brookhaven reactor in an attempt to induce the reaction $Cl^{37}(\bar{\nu},e^-)A^{37}$ with fission product antineutrinos. The experiments serve to place an upper limit on the antineutrino capture cross section for the reaction of 2×10^{-42} cm² per atom. Cosmic-ray-induced A^{37} was observed and the production rate measured at 14 100 feet altitude and sea level. Measurements with the 3900-liter container shielded from cosmic rays with 19 feet of earth permit placing an upper limit on the neutrino flux from the sun.

I. INTRODUCTION

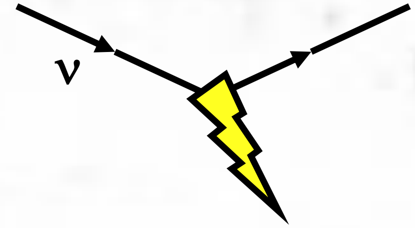
THERE have been a number of experiments performed in the past to detect the neutrino by scattering processes and nuclear interactions.[1] The most sensitive of these experiments serve to place a limit on the scattering cross section for neutrinos on electrons of less than 14×10^{-40} cm²/electron and for nuclear interaction of less than 10^{-30} cm²/atom. Recently Reines and Cowan of the Los Alamos Laboratory performed an experiment with a large hydrocarbon liquid scintillator having a high sensitivity for detecting the interaction $p(\bar{\nu},e^+)n$ within the liquid.[2] Measurements were made with this scintillator located adjacent to the Hanford reactor within a shield designed to absorb other radiations from the reactor to which the scintillator was sensitive. Under these conditions decay a neutrino (ν) is emitted which may be formally distinguished from an antineutrino ($\bar{\nu}$) which accompanies negative beta emission. A nuclear reactor emits antineutrinos which arise from the negative beta decays of fission products. In our experiment an attempt is made to observe an inverse electron capture process which requires neutrinos, using a source emitting antineutrinos. If neutrinos and antineutrinos are identical in their interactions with nucleons one should be able to observe the process upon carrying the experiment to the required sensitivity. However, if neutrinos and antineutrinos differ in their interactions with nucleons one would not expect to induce the reaction $Cl^{37}(\bar{\nu},e^-)A^{37}$. A positive experiment of this type would show that these particles are not to be distinguished in their nuclear reactions. A negative experiment

Question #0: Raymond Davis first tried out his chlorine experiment that discovered neutrinos from the sun at a reactor, to look for

$$\bar{\nu} + {}^{37}Cl \rightarrow {}^{37}Ar + e^-.$$

Davis didn't find it. Why not?

Lecture Answer #0



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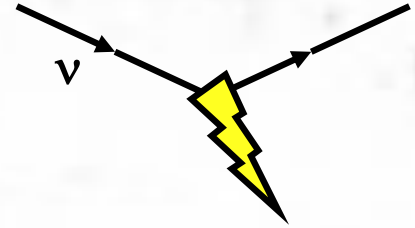
Question #0: Raymond Davis first tried out his chlorine experiment that discovered neutrinos from the sun at a reactor, to look for
$\bar{\nu} + {}^{37}Cl \rightarrow 37Ar + e^-$.
Davis didn't find it. Why not?

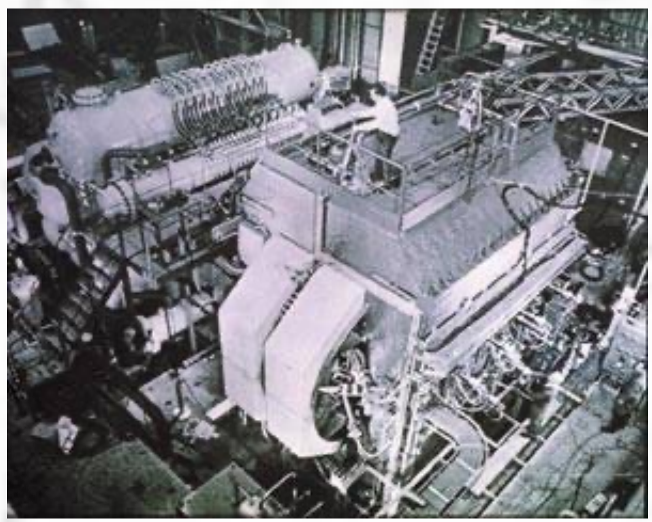
- Subsequent questions will mostly be multiple choice and require some short calculations.
- Paper may be helpful.
- Please participate!

Another Neutrino Interaction Discovery

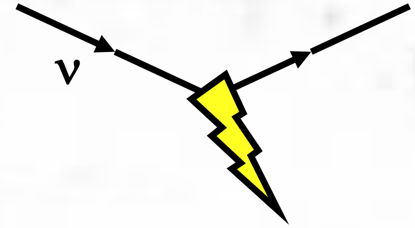


- Neutrinos only feel the weak force
  - a great way to study the weak force!
- Search for neutral current
  - arguably the most famous neutrino interaction ever observed is shown at right

$$\bar{\nu}_\mu e^- \rightarrow \bar{\nu}_\mu e^-$$



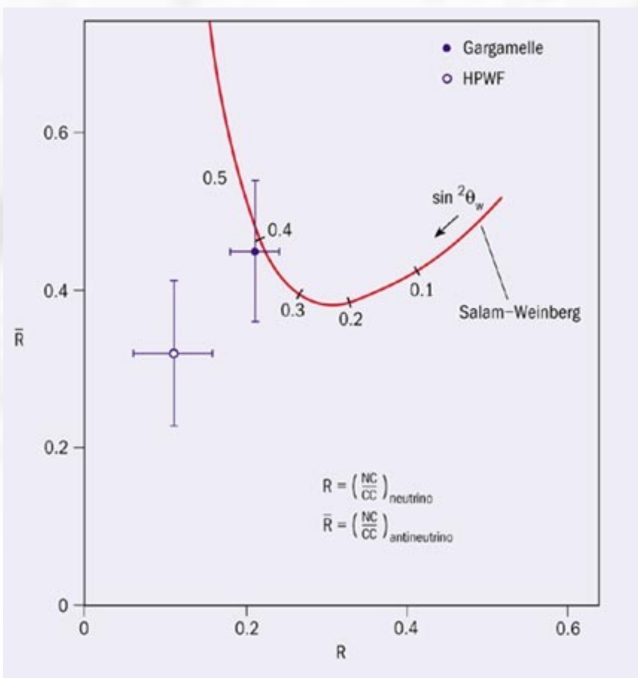
Gargamelle, event from neutral weak force



An Illuminating Aside

- The "discovery signal" for the neutral current was really neutrino scattering from nuclei
  - usually quoted as a ratio of muon-less interactions to events containing muons

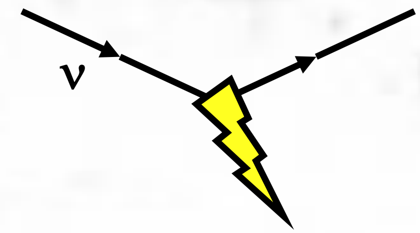
$$R^{\nu} = \frac{\sigma(\nu_{\mu} N \to \nu_{\mu} X)}{\sigma(\nu_{\mu} N \to \mu^{-} X)}$$



- But this discovery was complicated for 12-18 months by a lack of understanding of neutrino interactions
  - backgrounds from neutrons induced by neutrino interactions outside the detector
  - not understanding fragmentation to high energy hadrons which then "punched through" to fake muons

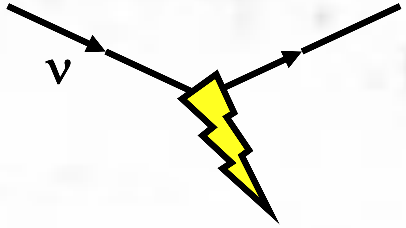
See P. Gallison, Rev Mod Phys 55, 477 (1983)

The Future: Interactions and Oscillation Experiments



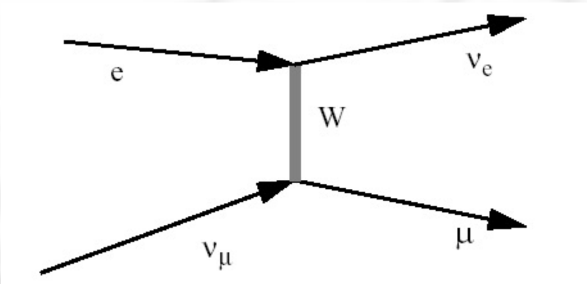
- Oscillation experiments point us to a rich physics potential at L/E~400 km/GeV *(and L/E~N·(400 km/GeV) as well)*
  - ordering of neutrino masses, CP violation
- But transition probabilities as a function of energy must be precisely measured for mass ordering and CP violation
- In other words, there are no neutrino flavor measurements in which distinguishing 1 from 0 or 1/3 gives you a trip to Stockholm to receive a Nobel Prize.
  - Difficulties are more like the neutral current discovery experiments.
- What are these difficulties?

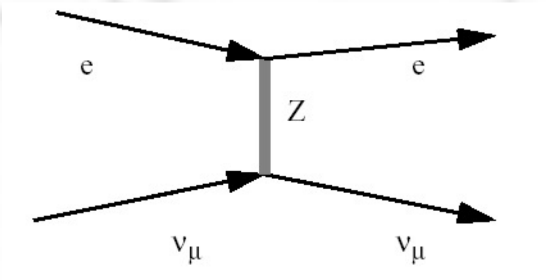
Needs of Oscillation Experiments, Question #1

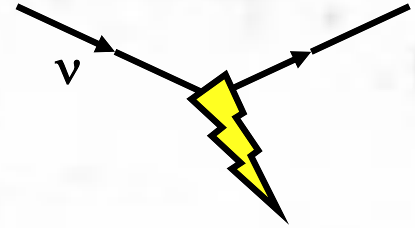


- Oscillation experiments need neutrino flavor and L/E_ν to relate oscillation probabilities to mixing and mass parameters.
  - If a neutrinos arrive with unknown flavor and energy, we must infer these from the final state.
  - Measurements are made with charged-current reactions.

  - *Question #1: Why are neutral current reactions (mostly) not useful for oscillation experiments?*

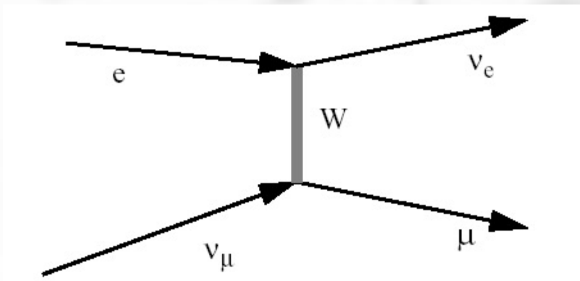


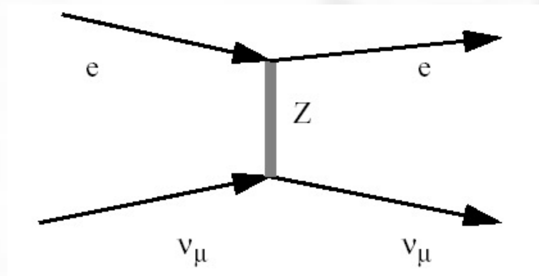




Answer #1

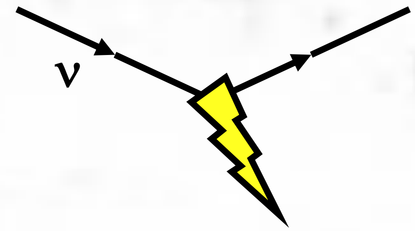
- *Question #1: Why are neutral current reactions (mostly) not useful for oscillation experiments?*





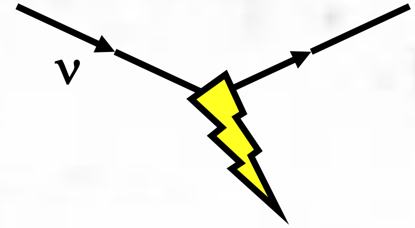
  - If a neutrinos arrive with unknown flavor and energy, we must infer these from the final state.
  - Neutral current reactions are independent of neutrino flavor, since the Z boson couples equally to all neutrinos, and we don't detect further interactions of the final neutrino.

What are the potential problems from interactions?

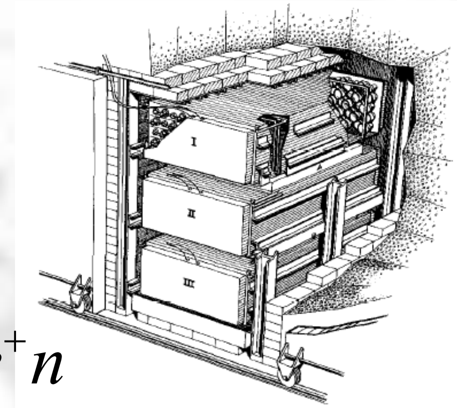


- For a fixed baseline oscillation experiment, the relationship between oscillation parameters and event rate depends on flavor and E_ν, both of which we measure from the final state.
- *Energy reconstruction*
  - Final state particles determine ability to reconstruct E_ν
- *Signal rate differ for different flavors*
- *Backgrounds to rare flavors*
  - Energetic pions have an annoying habit of faking leptons ($\pi^0 \rightarrow e$ or $\pi^\pm \rightarrow \mu$) in realistic detectors
  - Important to understand rate and spectrum of pions

Neutrino Reactions: *when nature is kind*



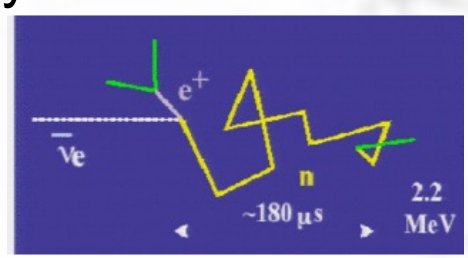
- Experiments at reactors, e.g., Reines and Cowan (1955) or Daya Bay or JUNO
  - Constant source of neutrinos, so backgrounds are high.
  - Coincidence technique… positron & neutron detected.
- This reaction has a calculable interaction cross-section because
  - It is on free nucleons.
  - Momentum transferred to nucleon is low and therefore “static” properties suffice.
- You can find the energy of the neutrino from the positron energy!

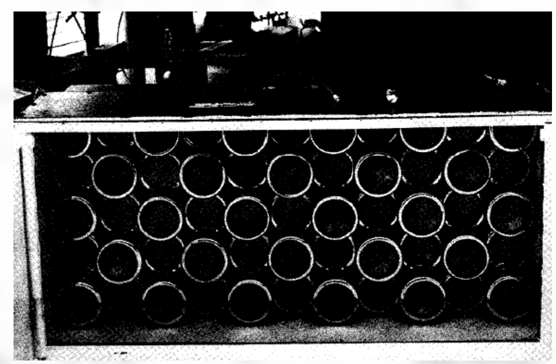


$$\bar{\nu} p \rightarrow e^{+} n$$

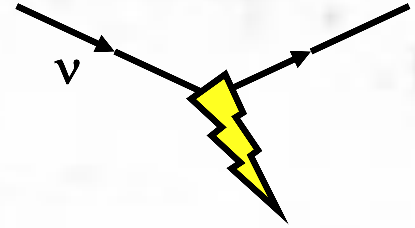
$$\langle E_e \rangle = \frac{2E_\nu M_p - M_n^2 + M_p^2 + M_e^2}{2(E_\nu + M_p)} \approx E_\nu - 1.3 \text{ MeV}$$

Variation of energy is small, ~1/2% at 4 MeV, since recoiling neutron is heavy compared to momentum transfer.





Neutrino Reactions: *when nature is cruel*



- Not everyone is so lucky. Consider the analogous problem of 10s to 100s MeV neutrinos striking argon.
- In the low energy region, 1-100 MeV the nuclear structure is critical for energy reconstruction

Reconstructing true neutrino energy:

Q is determined by measuring de-excitation gammas and nucleons

Outgoing e⁻ Energy

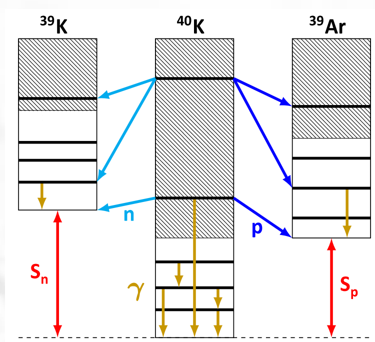
Energy donated to transition

Recoil Energy of Nucleus (negligible)

$$E_\nu = E_e + Q + K_{\text{recoil}}$$

… but detector may not see all energy, e.g., neutrons

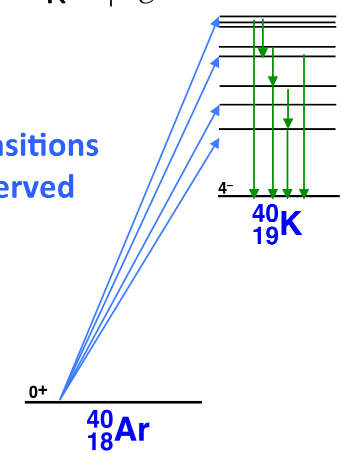
Protons and neutrons and photons, oh my!



Excitation of $^{40}K^*$

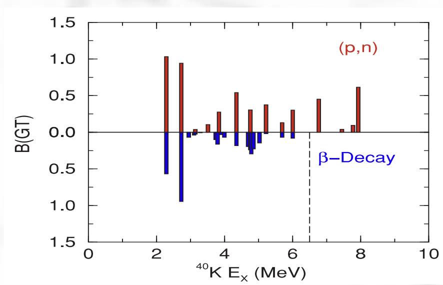
$$\nu_e + {}^{40}\text{Ar} \to {}^{40}\text{K}^* + e^-$$

At least 25 transitions have been observed indirectly



Figures from S. Gardiner, NuINT17

Decay of $^{40}K^*$

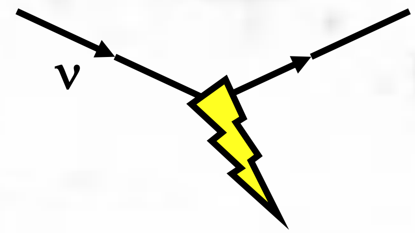


Q value in β-decay vs

$$p\ {}^{A}Z \to n\ {}^{A}(Z+1)$$

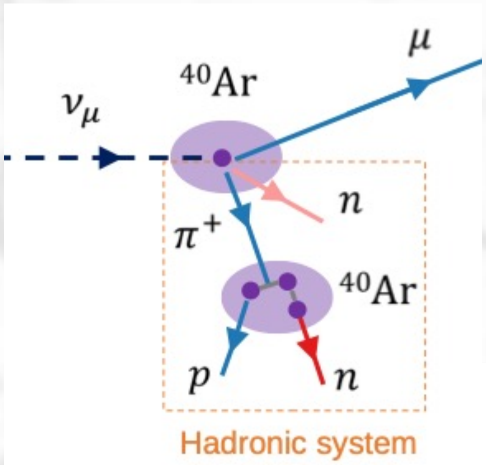
Significant difference means model for unseen energy is uncertain.

Neutrino Reactions: *when nature is cruel*



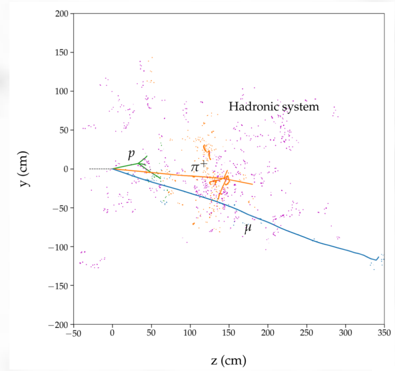
- Example from DUNE

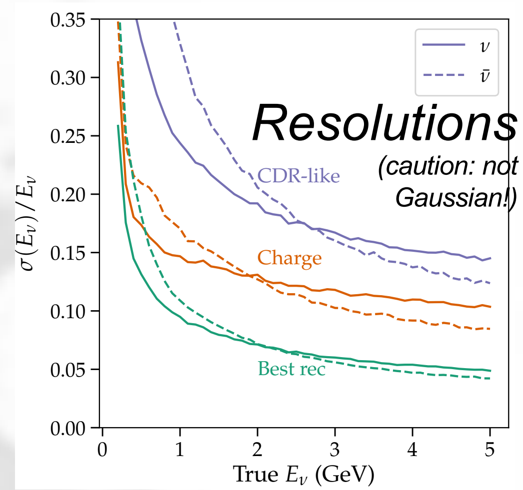
Need models of composition of final state to reconstruct energy.



Friedland and Li, Phys Rev D99, 036009 (2019)

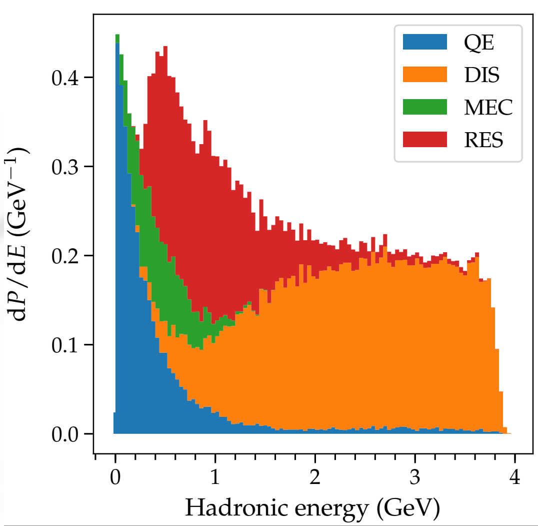
Hadronic energy for 4 GeV neutrinos by reaction type



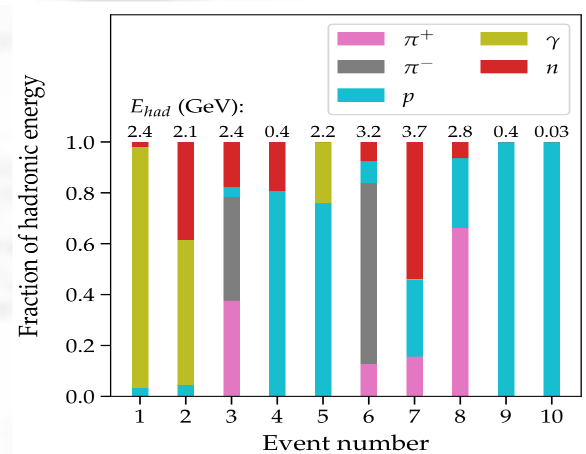


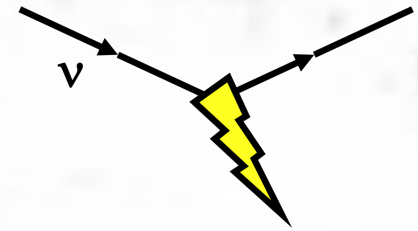
Resolutions

(caution: not Gaussian!)



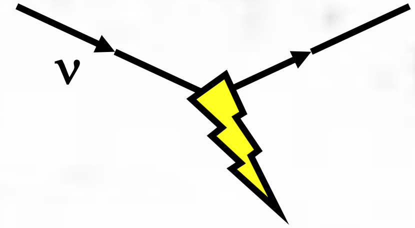
Stochastic fluctuations of hadronic energy.



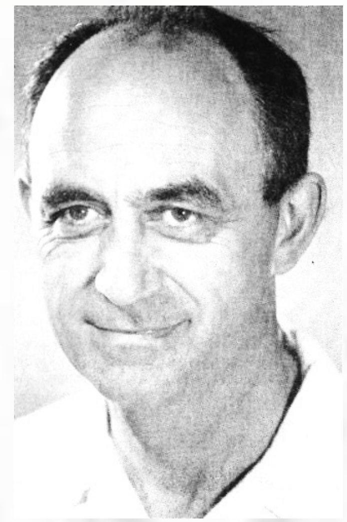


Calculating Neutrino Interactions from Electroweak Theory

Weak Interactions Revisited



- Current-current interaction (Fermi 1934) $\mathrm{H}_{w} = \frac{G_F}{\sqrt{2}} J^{\mu} J_{\mu}$

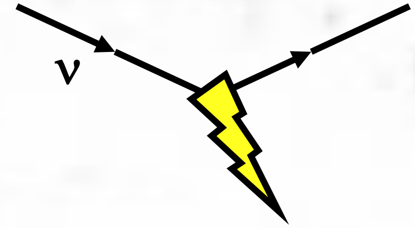


- Modern version:

$$\mathcal{H}_{weak} = \frac{G_F}{\sqrt{2}} \left[\bar{l} \gamma_{\mu} (1 - \gamma_5) \nu \right] \left[\bar{f} \gamma^{\mu} (V - A\gamma_5) f \right] + h.c.$$

- $P_L = 1/2(1-\gamma_5)$ is a projection operator onto left-handed components of fermions and right-handed components of anti-fermions

Helicity and Chirality

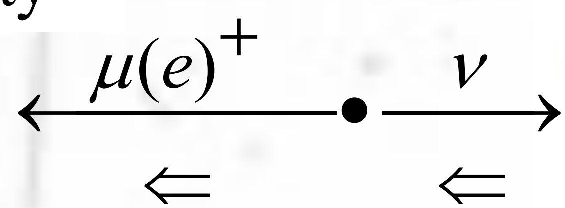


- ***Helicity*** is projection of spin along the particle's direction
  - Operator: **σ•p**
    - Frame dependent for massive particles

right-helicity left-helicity

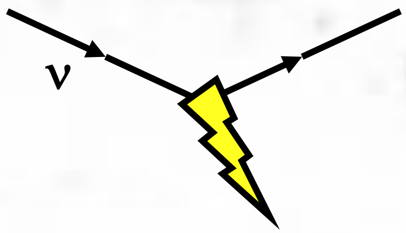
- However, ***chirality*** ("handedness") is Lorentz-invariant
- Operator: $P_{L(R)} = \frac{1}{2}(1 \mp \gamma_5)$
  – Couples to single helicity for massless particles
- Textbook example is pion decay to leptons

$$\pi^+(J=0) \rightarrow \mu(e)^+(J=\tfrac{1}{2})\nu_{\mu(e)}(J=\tfrac{1}{2})$$



$$R_{theory} = \frac{\Gamma(\pi^\pm \to e^\pm \nu_e)}{\Gamma(\pi^\pm \to \mu^\pm \nu_\mu)} = \left(\frac{m_e}{m_\mu}\right)^2 \left(\frac{m_\pi{}^2 - m_e{}^2}{m_\pi{}^2 - m_\mu{}^2}\right)^2 = 1.23 \times 10^{-4}$$

Helicity and Chirality



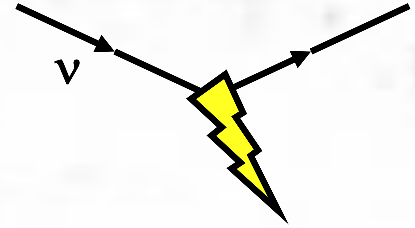
- ***Helicity*** is projection of spin along the particle's direction
  - Operator: **σ•p**
- However, ***chirality*** ("handedness") is Lorentz-invariant
  - Operator: $P_{L(R)} = \frac{1}{2}(1 \mp \gamma_5)$

- Neutrinos only interact weakly with a (V-A) interaction

$$\mathcal{H}_{weak} = \frac{G_F}{\sqrt{2}}\left[\bar{l}\gamma_\mu\left(1-\gamma_5\right)\nu\right]\left[\bar{f}\gamma^\mu\left(V - A\gamma_5\right)f\right] + h.c.$$

  - This interaction has only a **left-handed** coupling to **neutrinos** and only a **right-handed** coupling to **antineutrinos**
    - For a massless neutrino, this chirality implies a definite helicity neutrino

Helicity and Chirality

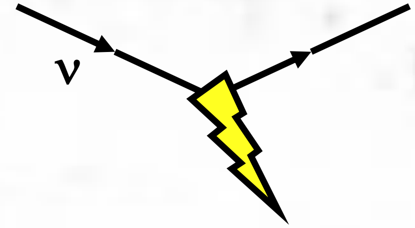


- ***Helicity*** is projection of spin along the particle's direction
- Operator: σ•**p**

- However, ***chirality*** ("handedness") is Lorentz-invariant
- Operator: $P_{L(R)} = \frac{1}{2}(1 \mp \gamma_5)$

- Since neutrinos have mass then the neutrino produced in a weak interaction is:
  - Overwhelmingly left-helicity
  - There is a small right-helicity component ∝ *m/E* but it can almost always be safely neglected for energies of interest in most applications

Two Weak Interactions



- W exchange gives Charged-Current (CC) events and Z exchange gives Neutral-Current (NC) events

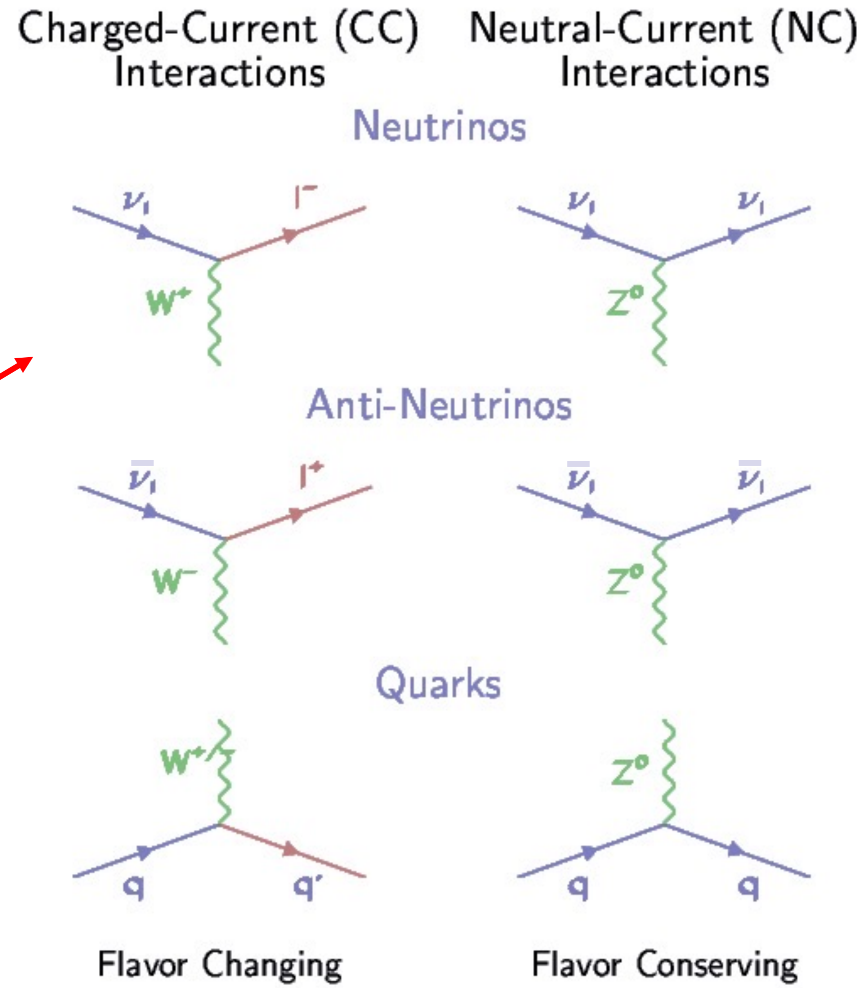
In charged-current events,

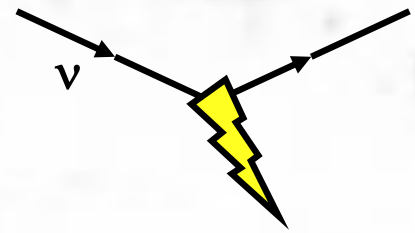
Flavor of outgoing lepton tags flavor of neutrino

Charge of outgoing lepton determines if neutrino or antineutrino

$$l^- \Rightarrow \nu_l$$

$$l^+ \Rightarrow \bar{\nu}_l$$



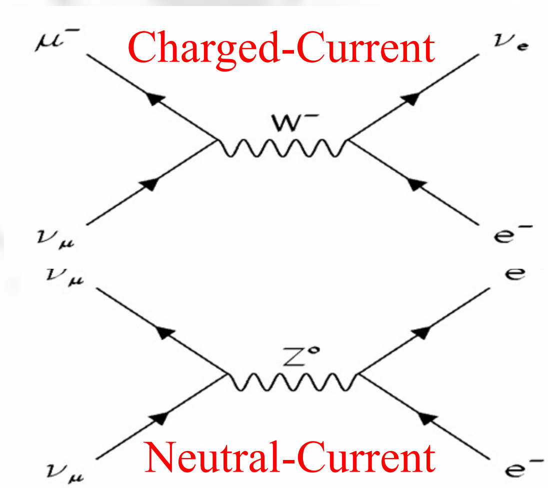


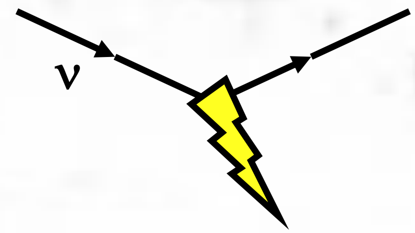
Electroweak Theory

- Standard Model
  - SU(2) ⊗ U(1) gauge theory unifying weak/EM
    $\Rightarrow$ weak NC follows from EM, Weak CC
  - Physical couplings related to mixing parameter for the interactions in the high energy theory

$$\mathcal{L}_{EW}^{\text{int}} = -Q_e A_\mu \bar{e}\gamma^\mu e + \frac{g}{\sqrt{2}} W_\mu^+ \bar{\nu}_L \gamma^\mu e_L + \frac{g}{\sqrt{2}} W_\mu^- \bar{e}_L \gamma^\mu \nu_L$$

$$+\frac{g}{\cos\theta_W} Z_\mu^0 \left\{ \begin{array}{l} \frac{1}{2}\bar{\nu}_L\gamma^\mu\nu_L \\ +\left(\sin^2\theta_W - \frac{1}{2}\right)\bar{e}_L\gamma^\mu e_L \\ +\sin^2\theta_W \bar{e}_R \gamma^\mu e_R \end{array} \right\}$$



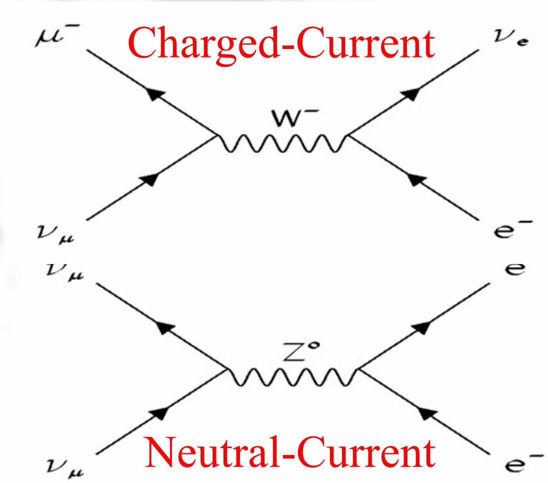


Electroweak Theory

- Standard Model
  - SU(2) ⊗ U(1) gauge theory unifying weak/EM
    - ⇒ weak NC follows from EM, Weak CC
  - Measured physical parameters related to mixing parameter for the couplings.

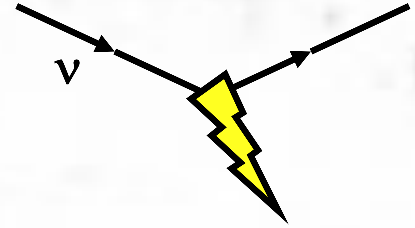
Z Couplings	g_L	g_R
ν_e, ν_μ, ν_τ	1/2	0
e, μ, τ	$-1/2 + \sin^2\theta_W$	$\sin^2\theta_W$
u, c, t	$1/2 - 2/3 \sin^2\theta_W$	$-2/3 \sin^2\theta_W$
d, s, b	$-1/2 + 1/3 \sin^2\theta_W$	$1/3 \sin^2\theta_W$

$$e = g\sin\theta_W,\ G_F = \frac{g^2\sqrt{2}}{8M_W^2},\ \frac{M_W}{M_Z} = \cos\theta_W$$



- Neutrinos are special in SM
  - ***NO*** right-handed interactions of neutrinos!

Why “Weak”?

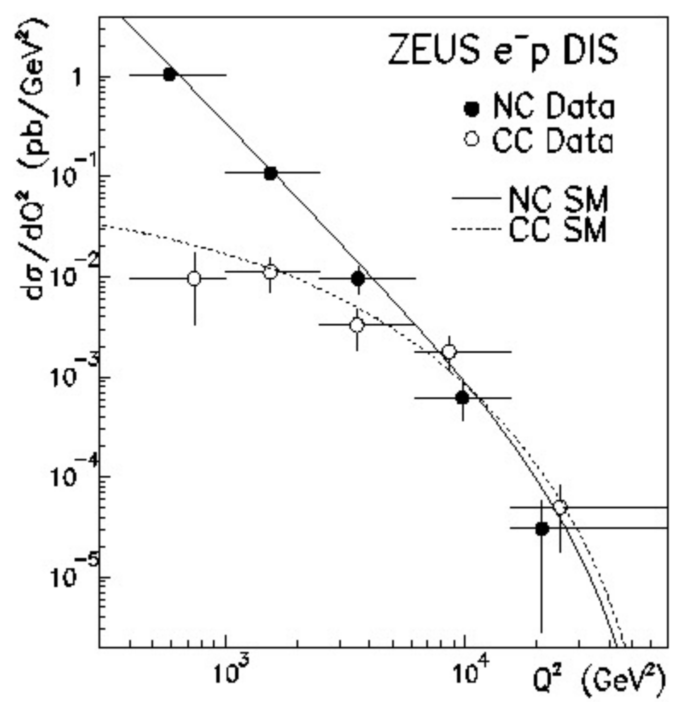


- Weak interactions are weak because of the massive W and Z boson exchanged

$$\frac{d\sigma}{dq^2} \propto \frac{1}{(q^2 - M^2)^2}$$

q is 4-momentum carried by exchange particle
M is mass of exchange particle

At HERA see W and Z propagator effects
- Also weak ~ EM strength

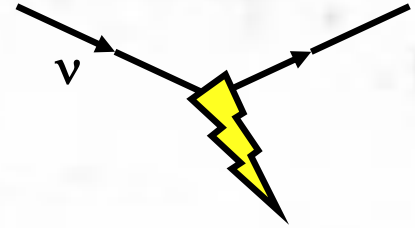


- Explains dimensions of Fermi “constant”

$$G_F = \frac{\sqrt{2}}{8}\left(\frac{g_W}{M_W}\right)^2$$

$$= 1.166 \times 10^{-5} / GeV^2 \quad (g_W \approx 0.7)$$

Neutrino-Electron Scattering

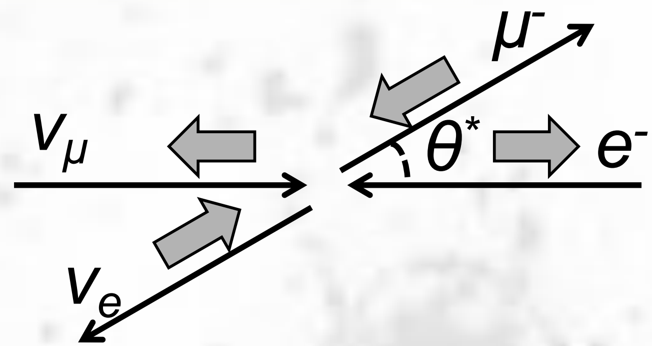


- Inverse μ–decay:

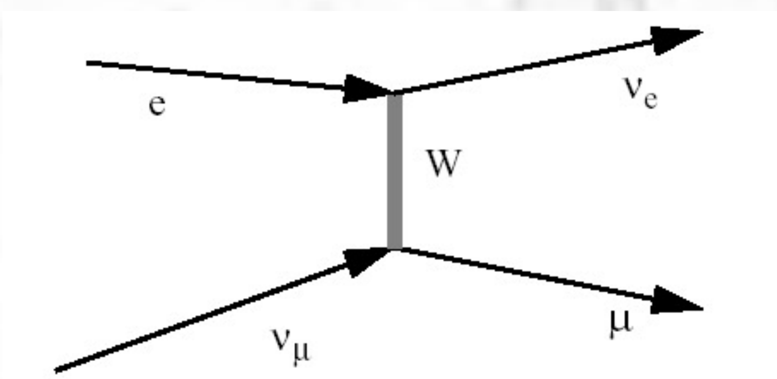
  $$\nu_\mu + e^- \to \mu^- + \nu_e$$

  - Total spin J=0

    (*Assuming massless muon*, helicity=chirality)

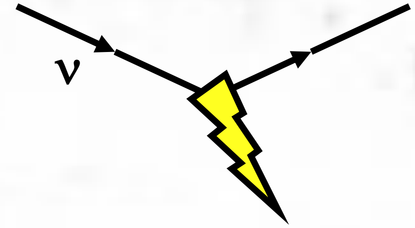


$$Q^2 \equiv -\left(\underline{e} - \underline{\nu}_e\right)^2$$



$$\sigma_{TOT} \propto \int_0^{Q^2_{\max}} dQ^2 \frac{1}{(Q^2 + M_W{}^2)^2}$$

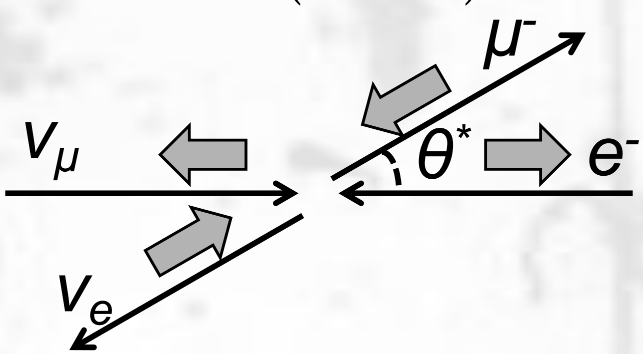
$$\approx \frac{Q^2_{\max}}{M_W{}^4}$$



Questions #2 and #3

What is Q^2_{max} for $\nu_\mu + e^- \rightarrow \mu^- + \nu_e$?

$$Q^2 \equiv -\left(\underline{e} - \underline{\nu}_e\right)^2$$



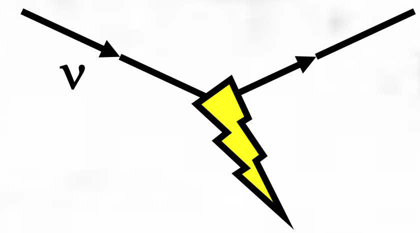
4-vector manipulation! Work in the center-of-mass frame and assume, ***for now****, that we can neglect the masses.*

Hint: there's only one variable (θ^*) in the 2→2 process. What choice of this variable gives the largest Q^2?

I said that

There is a small right-helicity component ∝ *m/E* but it can almost always be safely neglected for energies of interest in most applications

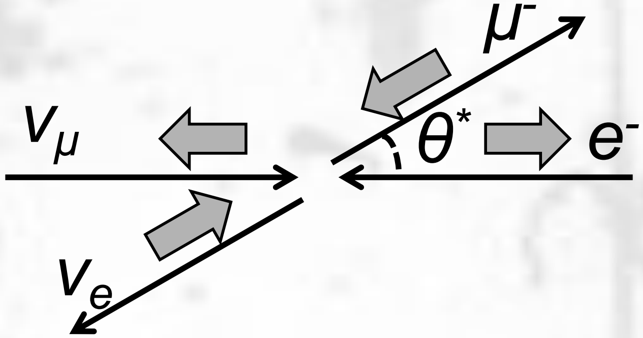
It's true if $E_\nu \gg m_\nu$. If $m_\nu \lesssim$1eV, why is this a good assumption? Can you think of any exceptions?



Answer #2

What is Q^2_{max} for $\nu_\mu + e^- \to \mu^- + \nu_e$?

$$Q^2 \equiv -\left(\underline{e} - \underline{\nu}_e\right)^2$$



$$\underline{e} \approx (E_\nu^*, 0, 0, -E_\nu^*)$$

$$\underline{\nu}_e \approx (E_\nu^*, -E_\nu^* \sin\theta^*, 0, -E_\nu^* \cos\theta^*)$$

4-vector manipulation! Work in the center-of-mass frame and assume, ***for now****, that we can neglect the masses.*

$$Q^2 = -\left(\underline{e}^2 + \underline{\nu}_e^{\,2} - 2\underline{e}\bullet\underline{\nu}_e\right)^2$$

$$\approx -\left[-2E_\nu^{*2}\left(1-\cos\theta^*\right)\right]$$

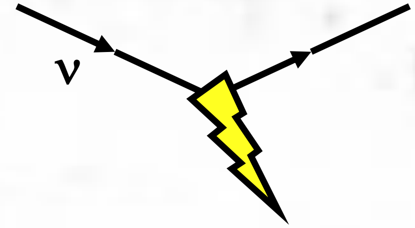
$$0 < Q^2 < \left(2E_\nu^*\right)^2 \approx \left(\underline{e} + \underline{\nu}_\mu\right)^2$$

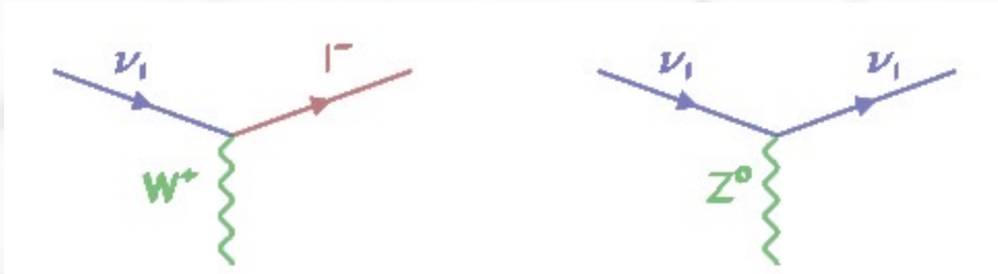
$$0 < Q^2 < s$$

Hint: there's only one variable (θ^*) in the 2→2 process. What choice of this variable gives the largest Q^2?

Mandelstam variable, E_{CM}^2

Answer #3





If neutrinos are produced in weak interactions, mass scales are ~1 MeV or greater and energies of resulting neutrinos are usually similar.

E.g., $n \rightarrow pe^-\overline{\nu_e}$ has a Q value of $m_n - m_p - m_e \approx$0.8 MeV

Exceptions are processes like "neutrino bremsstrahlung", $e^- \rightarrow e^-\nu\bar{\nu}$ in the field of a nucleus, which will be very rare.

Cosmic neutrinos from early universe have cooled to be non-relativistic.

I said that

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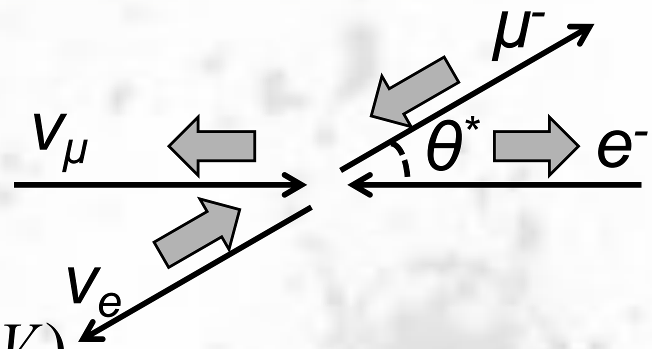
It's true if $E_\nu \gg m_\nu$. If $m_\nu \lesssim$1eV, why is this a good assumption? Can you think of any exceptions?

Neutrino-Electron (cont'd)

$$\sigma_{TOT} \propto Q^2_{\max} = s$$

$$\sigma_{TOT} = \frac{G_F^2 s}{\pi}$$

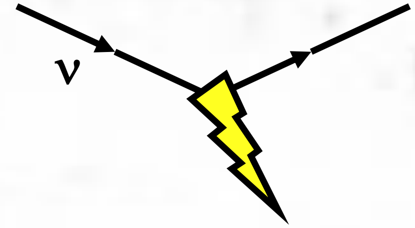
$$= 17.2 \times 10^{-42} cm^2 / GeV \cdot E_\nu (GeV)$$



- Why is it proportional to beam energy?

$$s = (\underline{p}_{\nu_\mu} + \underline{p}_e)^2 = m_e^2 + 2m_e E_\nu \text{ (e}^- \text{ rest frame)}$$

- Proportionality to energy is a generic feature of point-like scattering!
  - because $d\sigma/dQ^2$ is constant (at these energies)

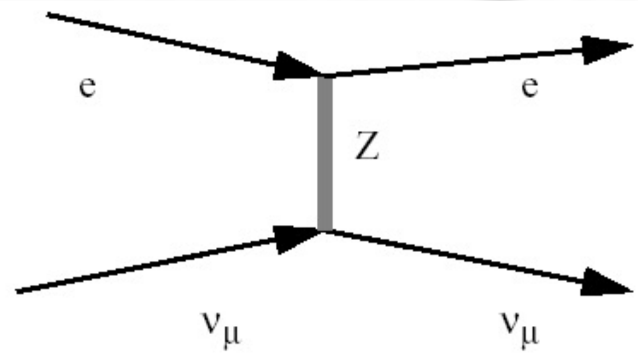


Neutrino-Electron (cont'd)

- Elastic scattering:

  $$\nu_\mu + e^- \rightarrow \nu_\mu + e^-$$

  - Recall, EW theory has coupling to left *or* right-handed electron
  - Total spin, J=0,1



Z Couplings	g_L	g_R
ν_e, ν_μ, ν_τ	1/2	0
e, μ, τ	$-1/2 + \sin^2\theta_W$	$\sin^2\theta_W$
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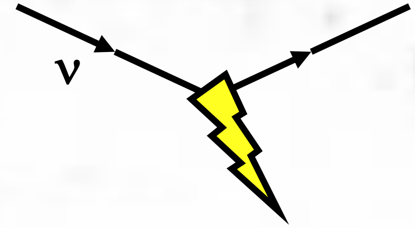
d, s, b	$-1/2 + 1/3 \sin^2\theta_W$	$1/3 \sin^2\theta_W$

- Electron-Z^0 coupling
  - Left-handed: $-1/2 + \sin^2\theta_W$

  $$\sigma \propto \frac{G_F^2 s}{\pi}\left(\frac{1}{4} - \sin^2\theta_W + \sin^4\theta_W\right)$$

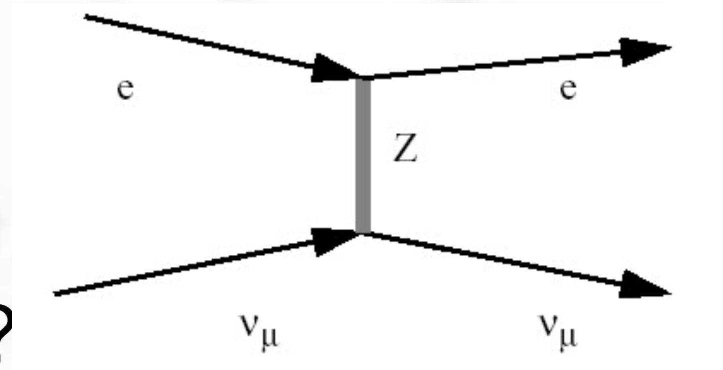
  - Right-handed: $\sin^2\theta_W$

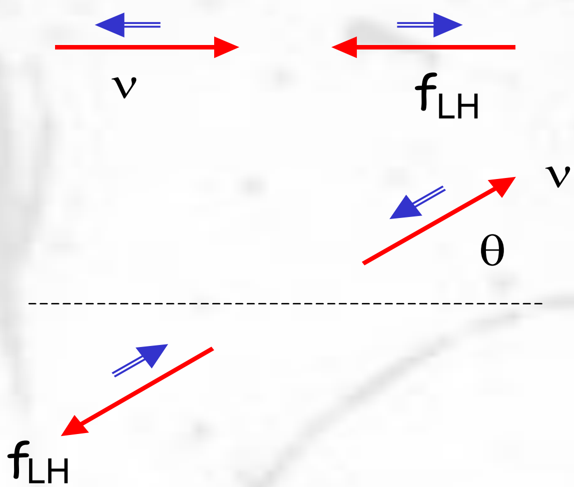
  $$\sigma \propto \frac{G_F^2 s}{\pi}\left(\sin^4\theta_W\right)$$



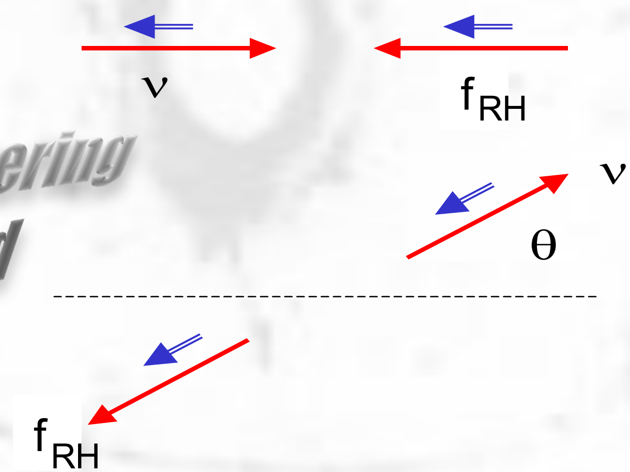
Neutrino-Electron (cont'd)

- What are relative contributions of scattering from left *and* right-handed electrons?





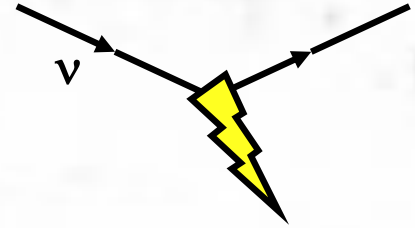
Backwards scattering is disfavored



$$\frac{d\sigma}{d\cos\theta} = \text{const}$$

$$\frac{d\sigma}{d\cos\theta} = \text{const} \times \left(\frac{1+\cos\theta}{2}\right)^2$$

Neutrino-Electron (cont'd)



- Electron-Z^0 coupling
  - (LH, V-A): $-1/2 + \sin^2\theta_W$
  - (RH, V+A): $\sin^2\theta_W$

$$\sigma \propto \frac{G_F^2 s}{\pi}\left(\frac{1}{4} - \sin^2\theta_W + \sin^4\theta_W\right)$$

$$\sigma \propto \frac{G_F^2 s}{\pi}\left(\sin^4\theta_W\right)$$

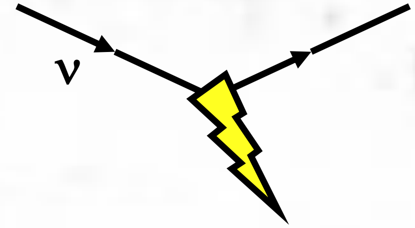
Let y denote inelasticity.
Recoil energy is related to
CM scattering angle by

$$y = \frac{E_e}{E_\nu} \approx 1 - \tfrac{1}{2}(1-\cos\theta)$$

$$\int dy \frac{d\sigma}{dy} = \begin{cases} \text{LH:} & \int dy = 1 \\ \text{RH:} & \int (1-y)^2 dy = 1/3 \end{cases}$$

$$\sigma_{TOT} = \frac{G_F^2 s}{\pi}\left(\frac{1}{4} - \sin^2\theta_W + \frac{4}{3}\sin^4\theta_W\right) = 1.4\times10^{-42}\, cm^2/GeV \cdot E_\nu(GeV)$$

Flavors and νe Scattering



The reaction

$$\nu_\mu + e^- \rightarrow \nu_\mu + e^-$$

has a much smaller cross-section than

$$\nu_e + e^- \rightarrow \nu_e + e^-$$

Why?

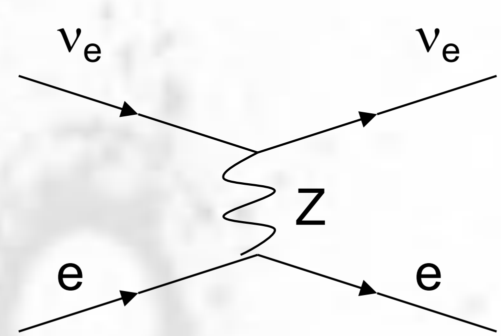
Flavors and ve Scattering

The reaction

$$\nu_\mu + e^- \rightarrow \nu_\mu + e^-$$

has a much smaller cross-section than

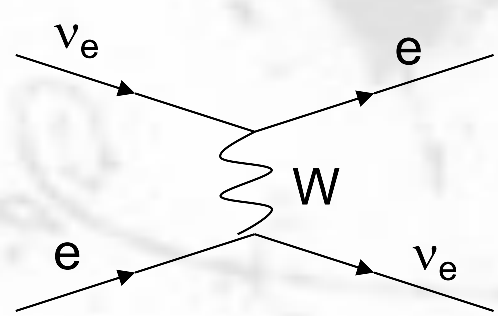
$$\nu_e + e^- \rightarrow \nu_e + e^-$$



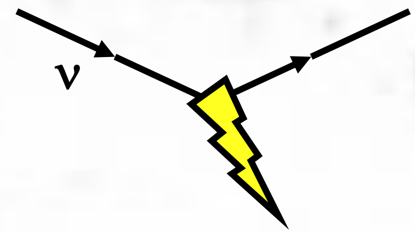
Why?

$$\nu_e + e^- \rightarrow \nu_e + e^-$$

has a second contributing reaction, charged current



Flavors and νe Scattering



Let's show that this increases the rate

(Recall from the previous pages…

$$\sigma_{TOT} = \int dy \frac{d\sigma}{dy} = \int dy \left[\frac{d\sigma^{LH}}{dy} + \frac{d\sigma^{RH}}{dy} \right] = \sigma_{TOT}^{LH} + \tfrac{1}{3}\sigma_{TOT}^{RH}$$

$$\sigma_{TOT}^{LH} \propto \left| \text{total coupling}_{e^-}^{\text{LH}} \right|^2$$

For electron…	LH coupling	RH coupling
Weak NC	$-1/2+\sin^2\theta_W$	$\sin^2\theta_W$
Weak CC	+1	0

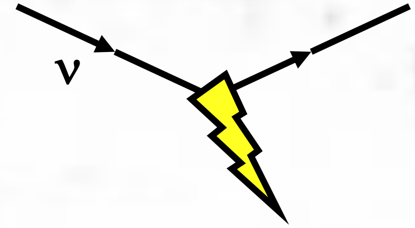
)

We have to show the interference between CC and NC increases instead of decreases the rate.

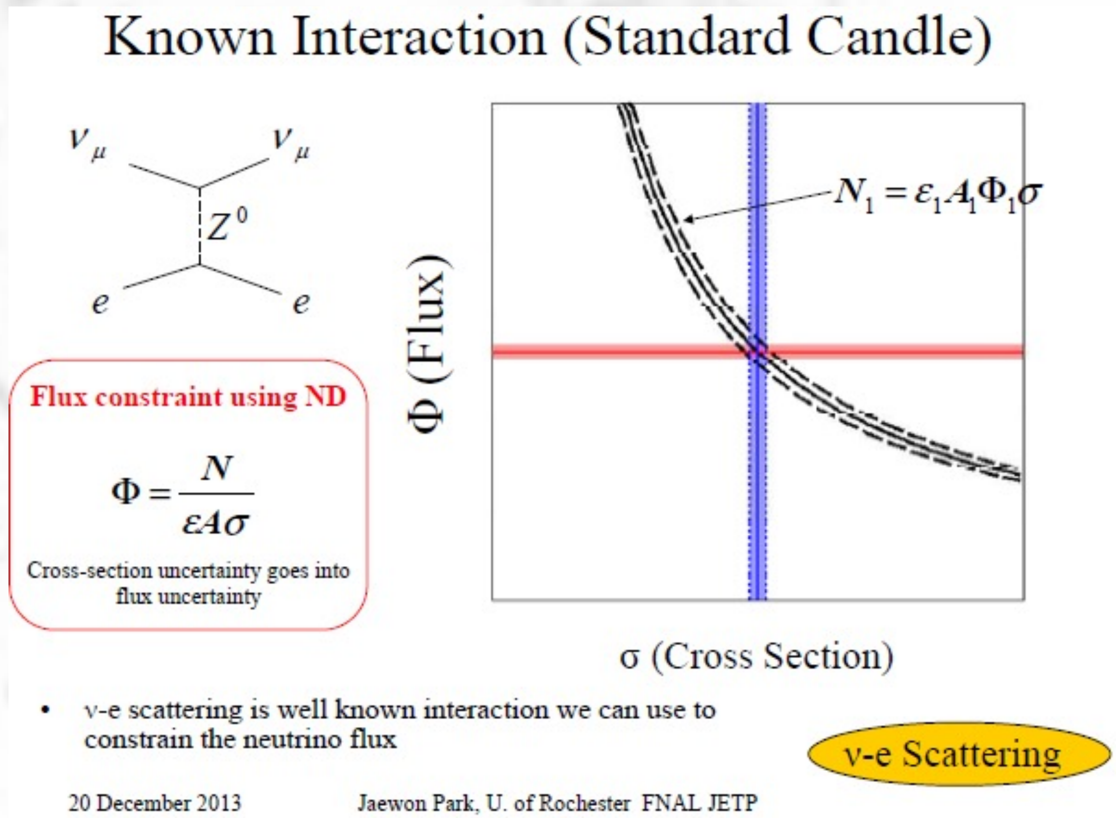
The total RH coupling is unchanged by addition of CC because there is no RH weak CC coupling

There are two LH couplings: NC coupling is $-1/2+\sin^2\theta_W \approx -1/4$ and the CC coupling is +1. We add the associated amplitudes… and get $+1/2+\sin^2\theta_W \approx 3/4$

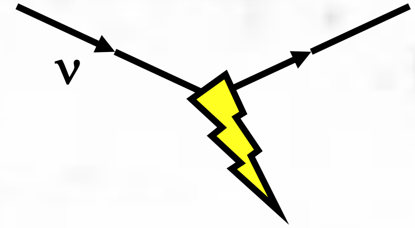
Who Cares about ν-e Elastic Scattering?



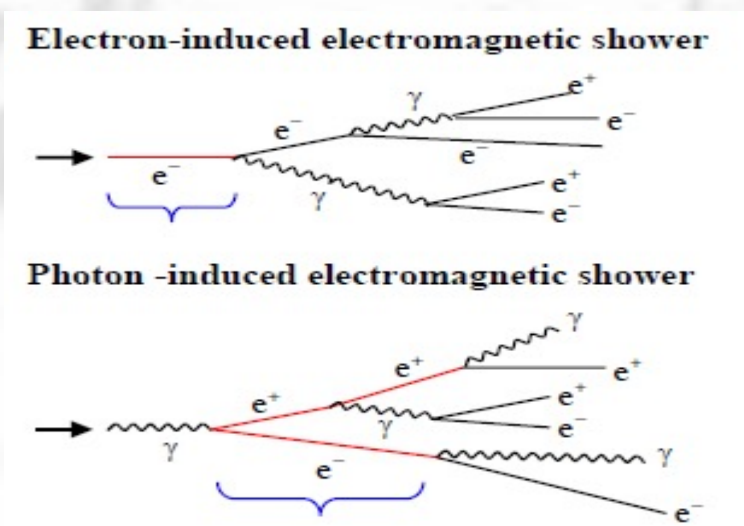
- I just spent ~10^{-6} of your life span telling you about a reaction whose rate is 500×10^{-6} of the leading reaction for accelerator neutrinos
  - Was this a good deal?
  - I'll argue yes… maybe…
- This reaction, as we will see, is nearly unique in being predicted to a fraction of a % precision



Who Cares… *(cont'd)*



- Not easy to measure at high energies. Reaction is rare and the detector is filled with photons from π^0 decays, easily confused with electrons
  - But electrons from $\nu + e^- \rightarrow \nu + e^-$ are very forward (because of small Q^2_{max}) and electromagnetic showers from photons & electrons are subtly different



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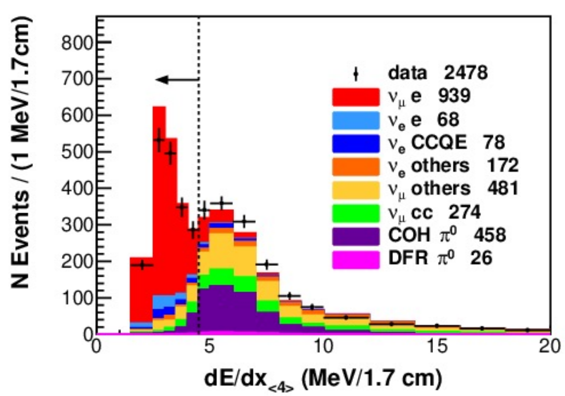
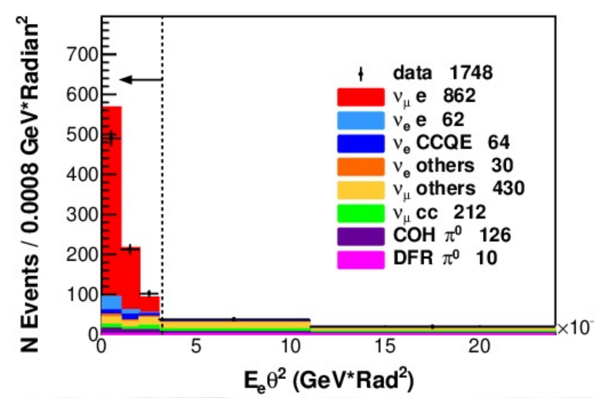
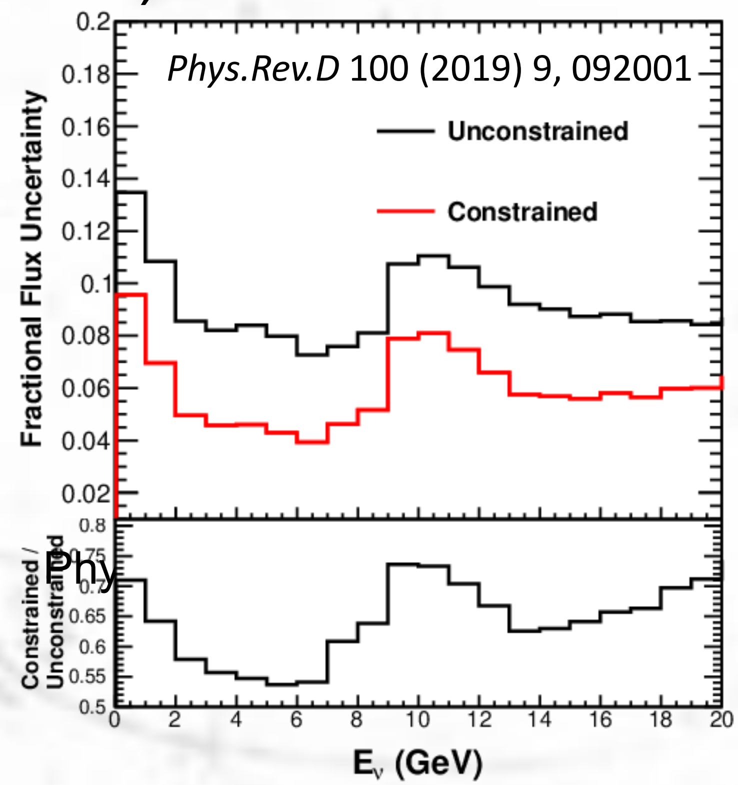




Who Cares… *(cont'd)*

- In this example (from MINERvA data) the number of events is small, but impact is still significant.
  - ~10%→4%
- And for LBNF beams for DUNE, another order of magnitude in events makes this the leading method for measuring neutrino flux.



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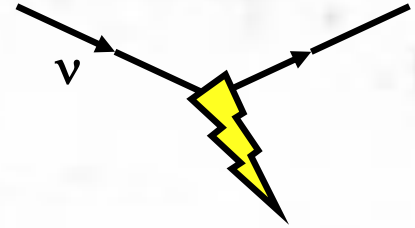
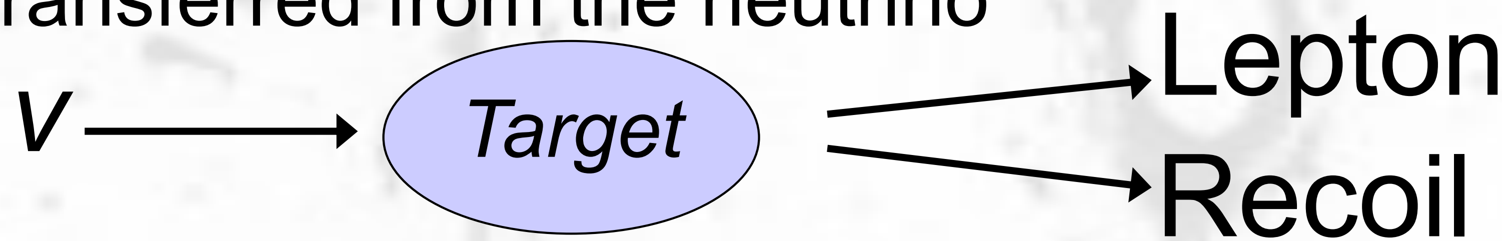
Final state mass effects

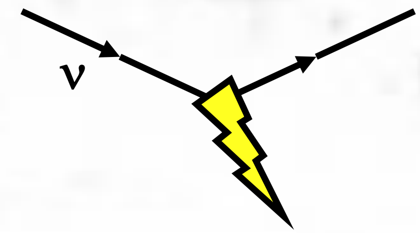


- As always, we detect neutrino interactions only in the final state.
- Creation of that final state may require energy to be transferred from the neutrino



  - In charged-current reactions, where the final state lepton is charged, this lepton has mass
  - The recoil may be a higher mass object than the initial state, or it may be in an excited state

Thresholds and Processes



Process	Considerations	Threshold (typical)
$vN \to vN$ (elastic)	Target nucleus is free and recoil is very small	none
$v_e n \to e^- p$	In some nuclei (mostly metastable ones), this reaction is exothermic if proton not ejected	None for free neutron & some other nuclei.
$ve \to ve$ (elastic)	Most targets have atomic electrons	~ 10eV – 100 keV
anti-$v_e p \to e^- n$	$m_n > m_p$ & m_e. Typically more to make recoil from stable nucleus.	1.8 MeV (free p). More in nuclei.
$v_\ell n \to \ell^- p$ (quasielastic)	Final state nucleon is ejected from nucleus. Massive lepton	~ 10s MeV for v_e +~100 MeV for v_μ
$v_\ell N \to \ell^- X$ (inelastic)	Must create additional hadrons. Massive lepton.	~ 200 MeV for v_e +~100 MeV for v_μ

- Energy of neutrinos determines available reactions, and therefore experimental technique

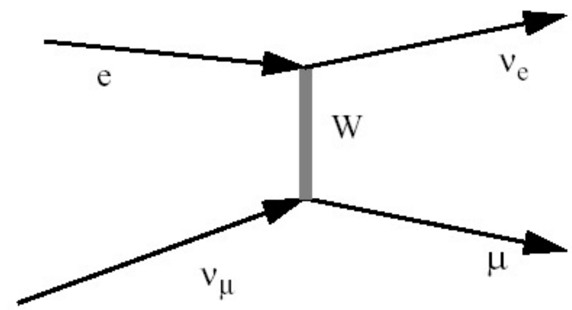
Lepton Mass Effects

- Let's return to Inverse μ–decay:

  $\nu_\mu + e^- \to \mu^- + \nu_e$

  - What changes in the presence of final state mass?
    - o pure CC so always left-handed
    - o BUT there must be finite Q^2 to create muon in final state!

$$Q^2_{\min} = m^2_\mu$$



$$\sigma_{TOT} \propto \int_{Q^2_{\min}}^{Q^2_{\max}} dQ^2 \frac{1}{(Q^2 + M_W{}^2)^2}$$

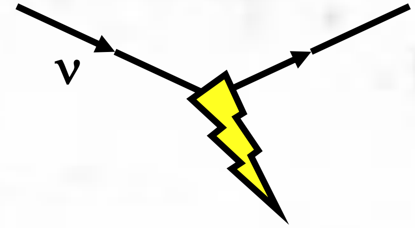
$$\approx \frac{Q^2_{\max} - Q^2_{\min}}{M_W{}^4}$$

$$\sigma_{TOT} = \frac{G_F^2 (s - m_\mu^2)}{\pi}$$

$$= \left[\sigma_{TOT}^{\text{(massless)}}\right]\left(1 - \frac{m_\mu^2}{s}\right)$$

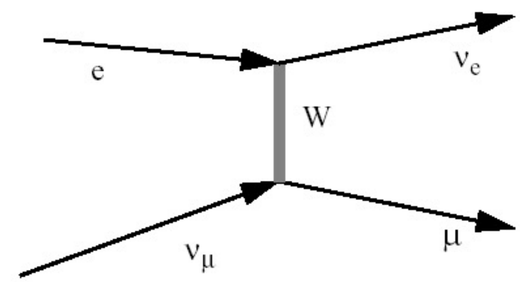
  - see a suppression scaling with **(mass/CM energy)²**
    - o This can be generalized…

Question #4: Lepton Mass Effect



- **Which is closest to the minimum beam energy in which the reaction**

$$\nu_\mu + e^- \to \mu^- + \nu_e$$



can be observed?

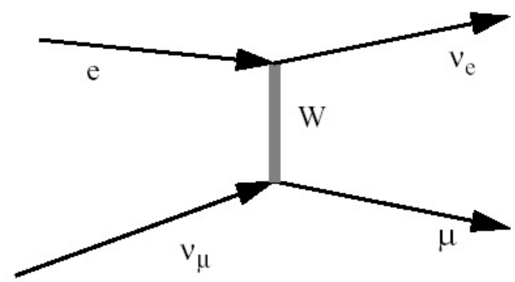
(a) 100 MeV **(b) 1 GeV** **(c) 10 GeV**

(It might help you to remember that $Q^2_{\min} = m^2_\mu$
or you might just want to think about the total CM energy required to produce the particles in the final state.)

Answer #4: Lepton Mass Effect

- **Which is closest to the minimum beam energy in which the reaction**

$$\nu_\mu + e^- \rightarrow \mu^- + \nu_e$$



can be observed?

(a) 100 MeV **(b) 1 GeV** **(c) 10 GeV**

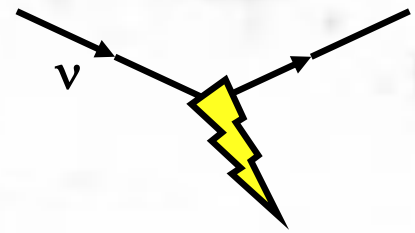
$$Q^2_{\min} = m_\mu^2$$

$$Q^2 < s = (\underline{p}_e + \underline{p}_\nu)^2$$

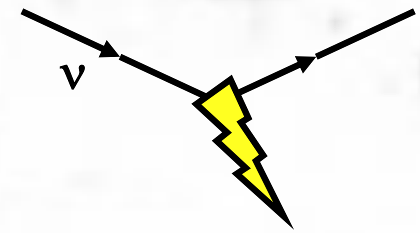
$$= (m_e + E_\nu, 0, 0, \sqrt{E_\nu{}^2 - m_\nu{}^2})^2 \approx m_e^2 + 2m_e E_\nu$$

$$\therefore E_\nu > \frac{m_\mu^2}{2m_e} \approx 10.9 \text{ GeV}$$

Summary and next type of point scattering…

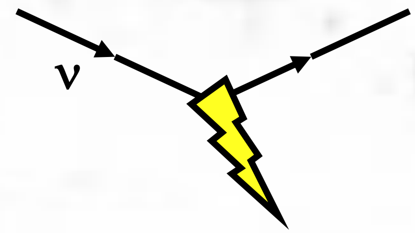


- We calculated νe^-!
- In point-like weak interactions, key features are:
  - $d\sigma/dQ^2$ is ≈ constant.
    - o Integrating gives $\sigma \propto E_\nu$
  - LH coupling enters w/ $d\sigma/dy \propto 1$, RH w/ $d\sigma/dy \propto (1-y)^2$
    - o Integrating these gives 1 and 1/3, respectively
  - Lepton mass effect gives minimum Q^2
    - o Integrating gives correction factor in σ of $(1-Q^2_{min}/s)$
- High energy point-like ν-quark scattering ("deep inelastic scattering") and *what's in between*…

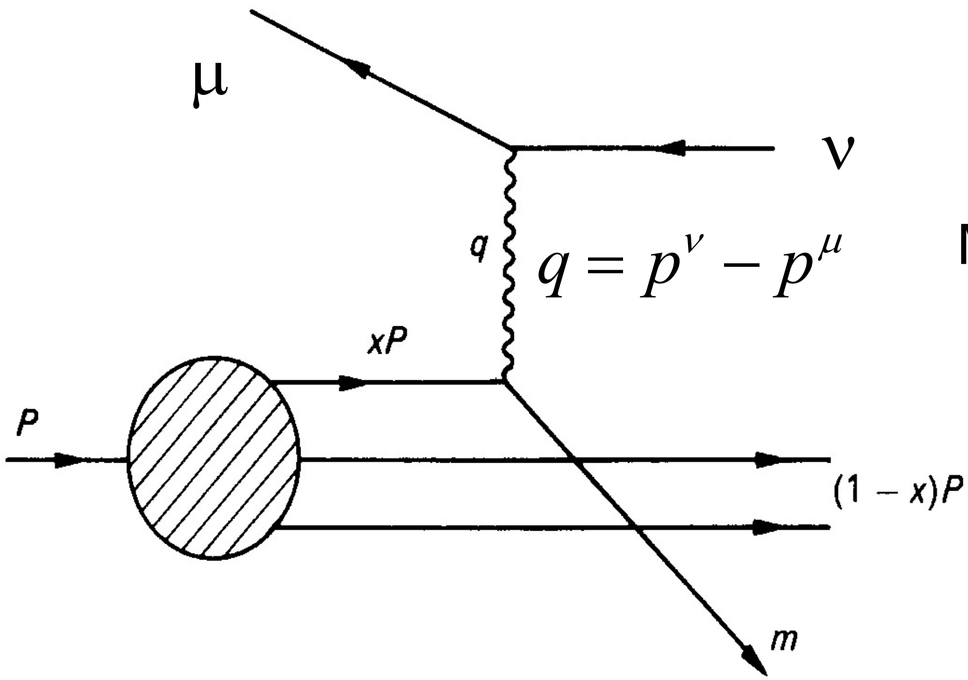


Neutrino-Nucleon Deep Inelastic Scattering

High Energy Limit and Quark-Parton model of DIS



In "infinite momentum frame", xP is four momentum of partons inside the nucleon



Neutrino scatters off a parton (a quark) inside the nucleon

Mass of target quark

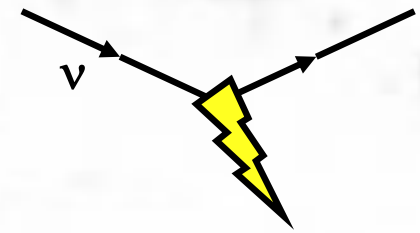
$$m_q^{\;2} = x^2 P^2 = x^2 M_T^2$$

Mass of final state quark

$$m_{q'}^{\;2} = (xP + q)^2$$

$$x = \frac{Q^2}{2P \cdot q} = \frac{Q^2}{2M_T \nu}$$

So why is cross-section so large?



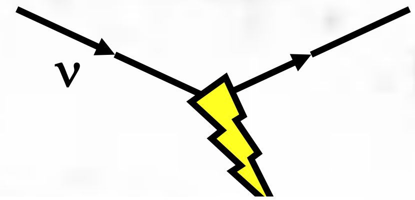
- (at least compared to νe^- scattering!)
- Recall that for neutrino beam and target at rest

$$\sigma_{TOT} \approx \frac{G_F^2}{\pi} \int_0^{Q^2_{\max} \equiv s} dQ^2 = \frac{G_F^2 s}{\pi}$$

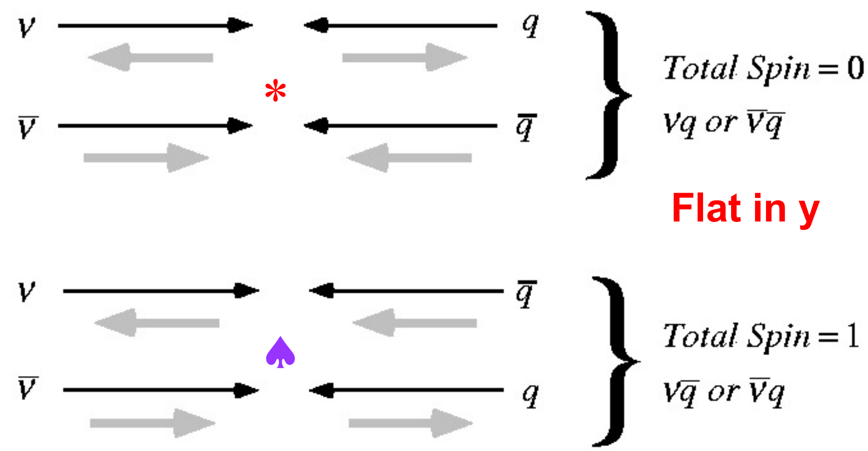
$$s = m_e^{\,2} + 2 m_e E_\nu$$

- But we just learned for DIS that effective mass of each target quark is $m_q = x m_{\text{nucleon}}$
- So much larger target mass means larger σ_{TOT}

Helicity, Charge in CC ν-q Interaction



- Massless limit for simplicity
- Total spin determines inelasticity distribution
  - Familiar from neutrino-electron scattering



Flat in y

point-like scattering implies linear with energy

$1/4(1+\cos\theta^*)^2 = (1-y)^2$

$\int(1-y)^2dy=1/3$

$$\frac{d\sigma^{\nu p}}{dxdy} = \frac{G_F^2 s}{\pi}\left(x\overset{*}{d}(x) + x\overset{\spadesuit}{\bar{u}}(x)(1-y)^2\right)$$

$$\frac{d\sigma^{\bar{\nu} p}}{dxdy} = \frac{G_F^2 s}{\pi}\left(x\overset{*}{\bar{d}}(x) + x\overset{\spadesuit}{u}(x)(1-y)^2\right)$$

but what is this "u(x)" and "d(x)"?

- Neutrino/Anti-neutrino CC each produce particular Δq in scattering

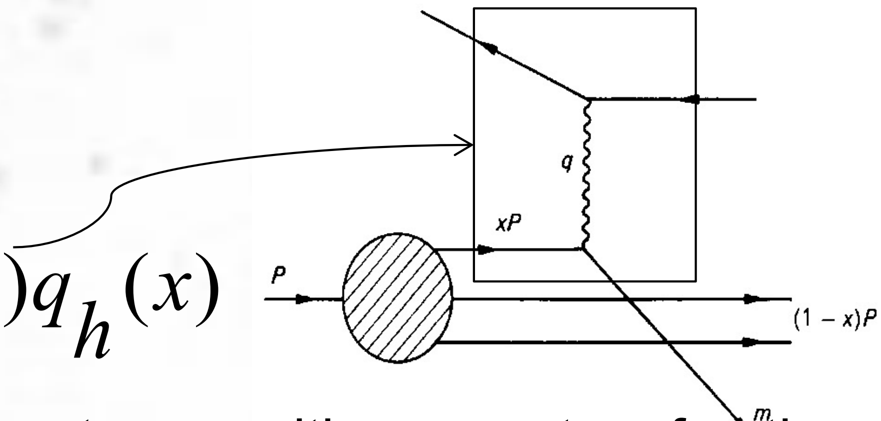
$$\nu d \rightarrow \mu^- u$$

$$\bar{\nu} u \rightarrow \mu^+ d$$

Factorization and Partons

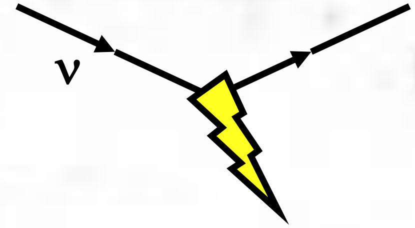
- Factorization Theorem of QCD allows cross-sections for hadronic processes to be written as:

$$\sigma(l + h \rightarrow l + X)$$
$$= \sum_q \int dx \sigma(l + q(x) \rightarrow l + X) q_h(x)$$



  - $q_h(x)$ is the probability of finding a parton, q, with momentum fraction x inside the hadron, h. It is called a parton distribution function (PDF).
  - PDFs are universal
  - PDFs are not (yet) calculable from first principles in QCD
- "Scaling": parton distributions are largely independent of Q^2 scale, and depend on fractional momentum, x.

Complication: Charged Current to Neutral Current



- We previously saw how to generalize from charged current to neutral current in νe^- scattering
  - Right handed current couples to target (but not neutrino)
  - Complicated couplings
  - For neutral current case, scattering from all flavors of quarks because there is no charge carried by boson

$$\frac{d\sigma^{\nu p,CC}}{dxdy} = \frac{G_F^2 s}{\pi} x\left(d(x) + \bar{u}(x)(1-y)^2\right)$$

$$\frac{d\sigma^{\nu p,NC}}{dxdy} = \frac{G_F^2 s}{\pi}\begin{pmatrix} x d_L^2 d(x) + d_R^2 \bar{d}(x) + u_L^2 u(x) + u_R^2 \bar{u}(x) \\ +(1-y)^2\left(d_R^2 d(x) + d_L^2 \bar{d}(x) + u_R^2 u(x) + u_L^2 \bar{u}(x)\right) \end{pmatrix}$$

Simplification: Isoscalar Targets

- Heavy nuclei are roughly neutron-proton isoscalar
  - OK, more neutrons than protons, but it's closer to 1:1 than 2:1 or 0:1
- Isospin symmetry implies $u_p = d_n, d_p = u_n$

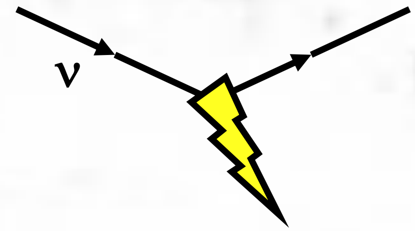
$$\frac{d\sigma^{\nu N,CC}}{dxdy} = \frac{G_F^2 s}{\pi} x\left(u(x) + d(x) + \left(\bar{u}(x) + \bar{d}(x)\right)(1-y)^2\right)$$

$$= \frac{G_F^2 s}{\pi} x\left(q(x) + \bar{q}(x)(1-y)^2\right)$$

$$\frac{d\sigma^{\bar{\nu} N,CC}}{dxdy} = \frac{G_F^2 s}{\pi} x\left(\bar{u}(x) + \bar{d}(x) + \left(u(x) + d(x)\right)(1-y)^2\right)$$

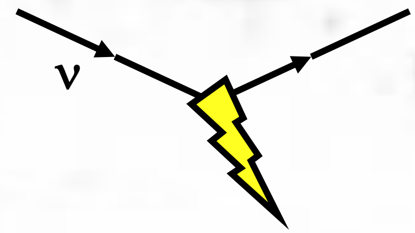
$$= \frac{G_F^2 s}{\pi} x\left(\bar{q}(x) + q(x)(1-y)^2\right)$$

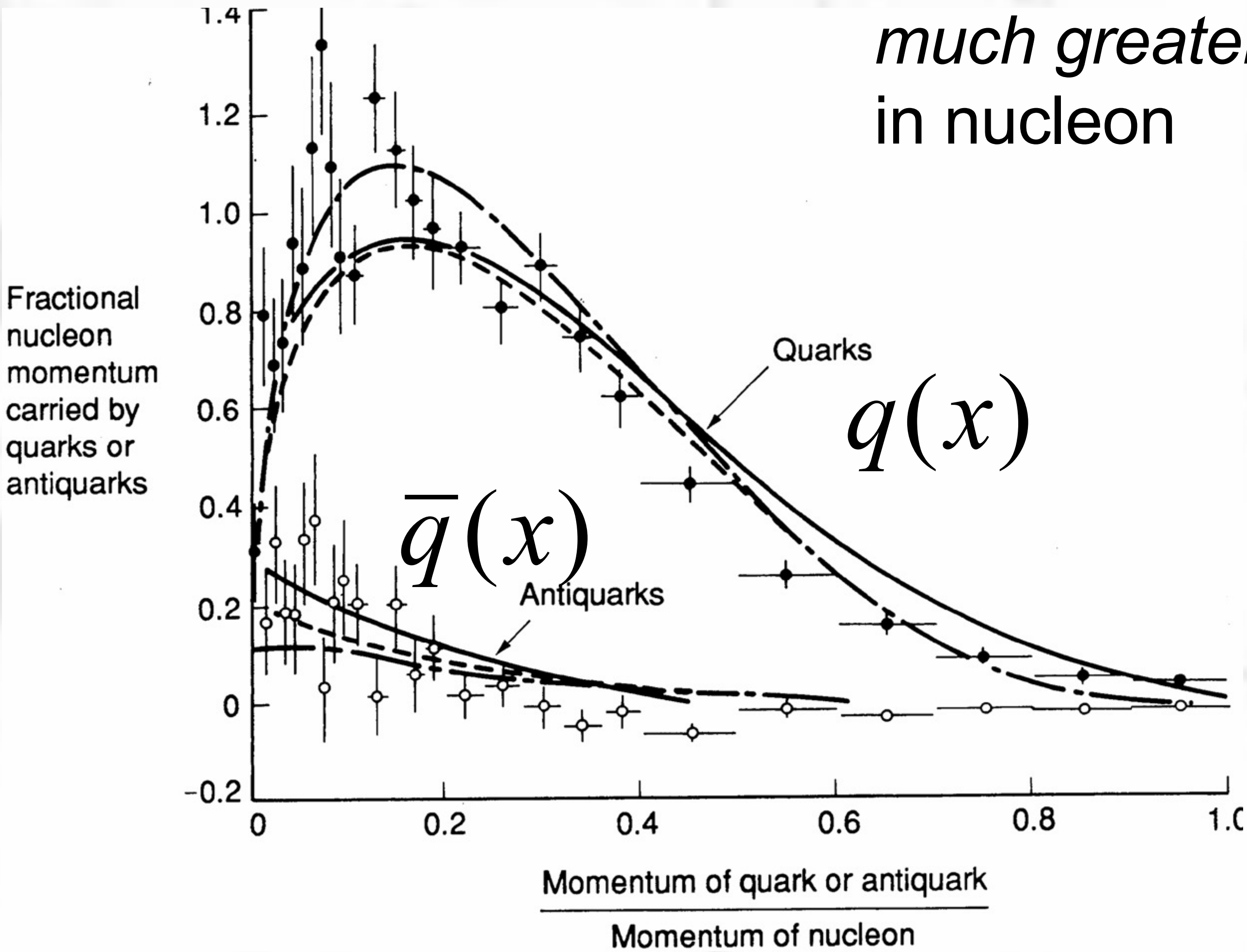
Brief Summary of Neutrino-Quark Scattering so Far



- $x \equiv Q^2/2M_T\nu$ is the fraction of the nucleon 4-momentum carried by a quark in the infinite momentum frame
  - Effective mass for struck quark, $M_q = \sqrt{(x\underline{P})^2} = xM_T$
  - Parton distribution functions, q(x), incorporate information about the "flux" of quarks inside the hadron
- Quark and anti-quark scattering spin:
  - νq and $\overline{\nu q}$ are spin 0, isotropic
  - $\nu\bar{q}$ and $\nu\bar{q}$ are spin 1, backscattering is suppressed
- Neutrinos and anti-neutrinos pick out definite quark and anti-quark flavors (charge conservation)
  - Isoscalar targets re-average over flavors

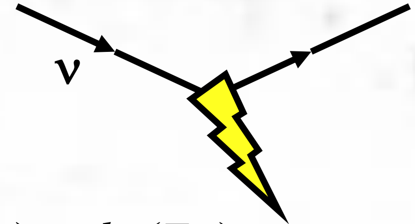
Momentum of Quarks & Antiquarks





- Momentum carried by quarks *much greater* than anti-quarks in nucleon

y distribution in Neutrino CC DIS



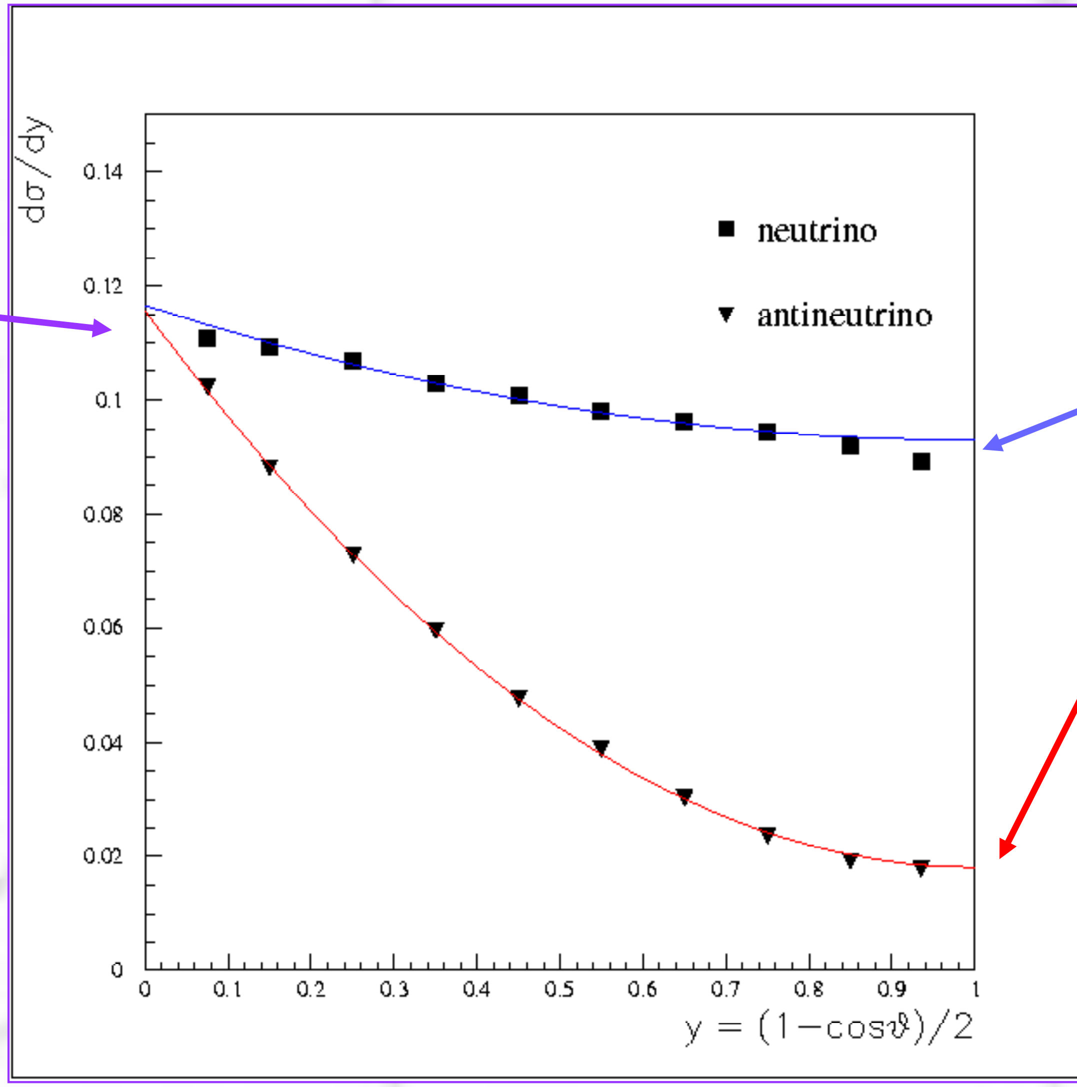
$$\frac{d\sigma(\nu q)}{dxdy} = \frac{d\sigma(\bar{\nu}\bar{q})}{dxdy} \propto 1$$

$$\frac{d\sigma(\nu\bar{q})}{dxdy} = \frac{d\sigma(\bar{\nu}q)}{dxdy} \propto (1-y)^2$$

At y=0:

Quarks & anti-quarks

Neutrino and anti-neutrino identical



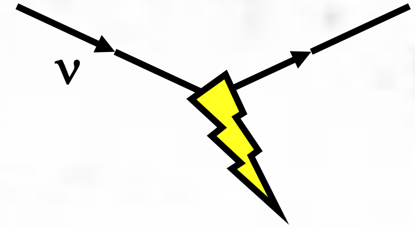
At y=1:

Neutrinos see only quarks.

Anti-neutrinos see only anti-quarks

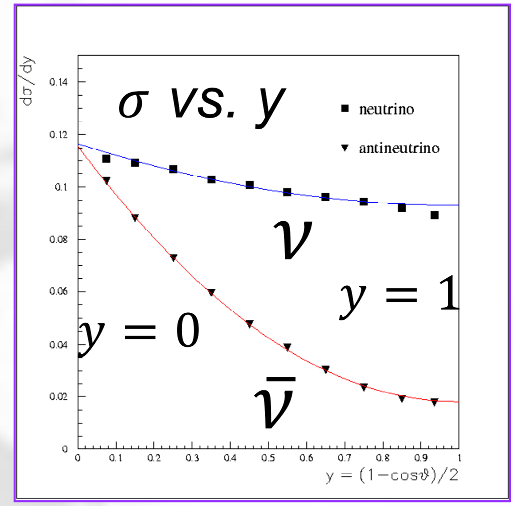
Averaged over protons and neutrons, $\sigma^{\bar{\nu}} = \frac{1}{2}\sigma^{\nu}$

Question #5



What is the ratio of anti-quark to quark momentum in the nucleon?

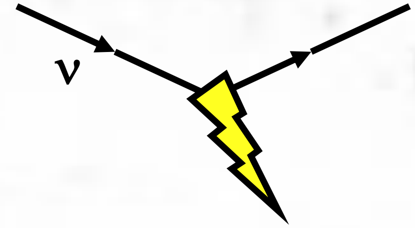
$$\sigma_{CC}^{\bar{\nu}N} \approx \frac{1}{2}\sigma_{CC}^{\nu N} \qquad \frac{d\sigma(\nu q)}{dx} = \frac{d\sigma(\bar{\nu}\bar{q})}{dx} = 3\frac{d\sigma(\nu\bar{q})}{dx} = 3\frac{d\sigma(\bar{\nu}q)}{dx}$$



Cross-section is proportional to total parton momentum (x summed over all quarks or antiquarks). Given the above, you can see that if there were no antiquarks, the cross-section for neutrinos would be three times higher than for antineutrinos.

(a) $\bar{q}/q \sim 1$ **(b)** $\bar{q}/q \sim 1/5$ **(c)** $\bar{q}/q \sim 1/25$

Answer #5: Neutrino and Anti-Neutrino $\sigma^{\nu N}$



- **Given:** $\sigma_{CC}^{\bar{\nu}N} \approx \frac{1}{2}\sigma_{CC}^{\nu N}$ **in the DIS regime (CC)**

and $$\frac{d\sigma(\nu q)}{dx} = \frac{d\sigma(\bar{\nu}\bar{q})}{dx} = 3\frac{d\sigma(\nu\bar{q})}{dx} = 3\frac{d\sigma(\bar{\nu}q)}{dx}$$

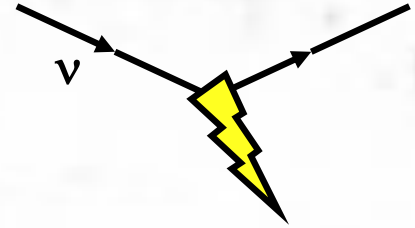
$$\sigma_\nu = \int_{q,\bar{q}} dx\left(\frac{d\sigma(\nu q)}{dx} + \frac{d\sigma(\nu\bar{q})}{dx}\right)$$

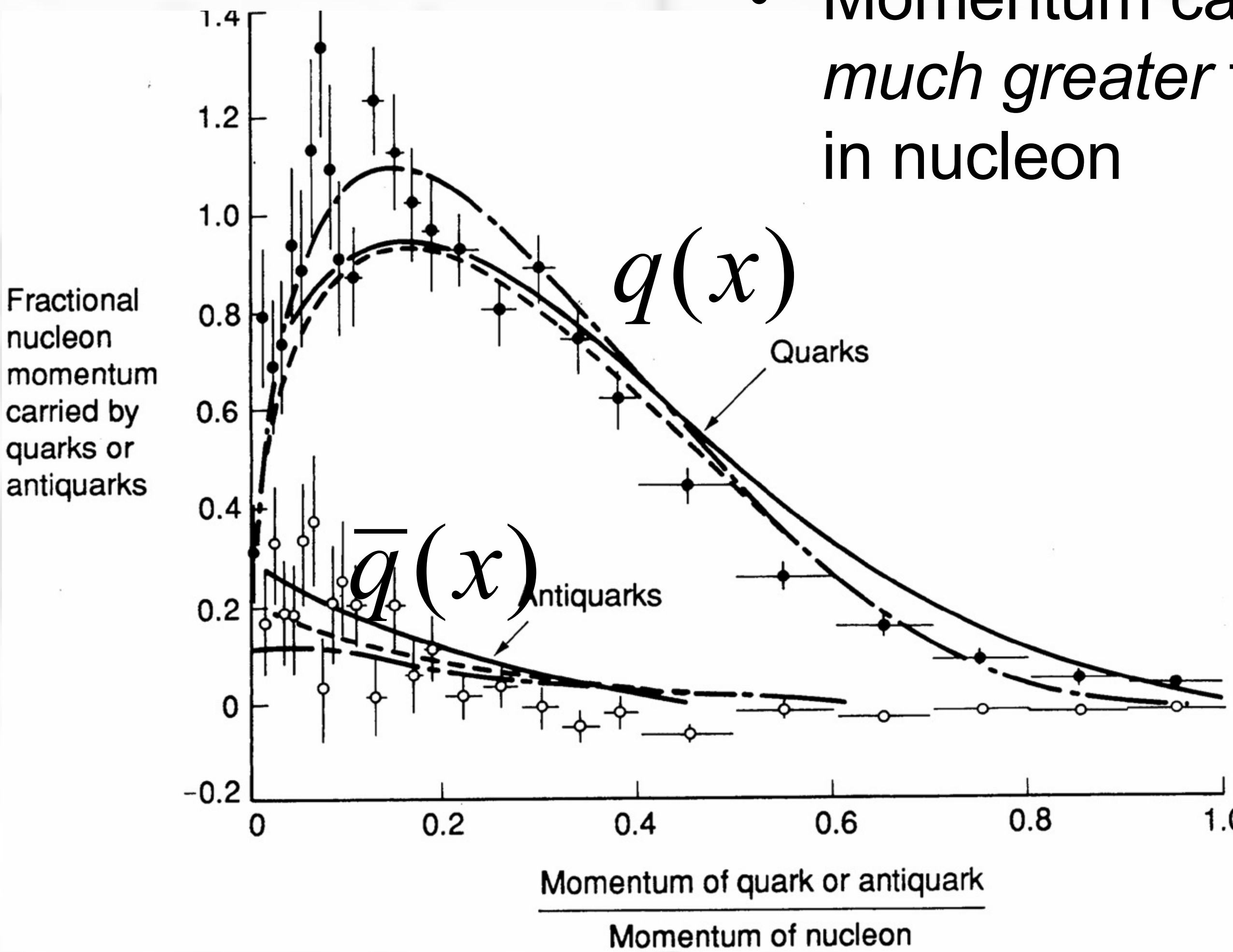
$$\sigma_{\bar{\nu}} = \int_{q,\bar{q}} dx\left(\frac{d\sigma(\bar{\nu}q)}{dx} + \frac{d\sigma(\bar{\nu}\bar{q})}{dx}\right) = \int_{q,\bar{q}} dx\left(\frac{d\sigma(\nu q)}{3dx} + \frac{3d\sigma(\nu\bar{q})}{dx}\right)$$

$$\therefore \int_{q,\bar{q}} dx\left(\frac{d\sigma(\nu q)}{dx} + \frac{d\sigma(\nu\bar{q})}{dx}\right) = 2\int_{q,\bar{q}} dx\left(\frac{d\sigma(\nu q)}{3dx} + \frac{3d\sigma(\nu\bar{q})}{dx}\right)$$

$$\frac{1}{3}\int_q dx\,\frac{d\sigma(\nu q)}{dx} = 5\int_{\bar{q}} dx\,\frac{d\sigma(\nu\bar{q})}{dx} = \frac{5}{3}\int_{\bar{q}} dx\,\frac{d\sigma(\bar{\nu}\bar{q})}{dx}$$

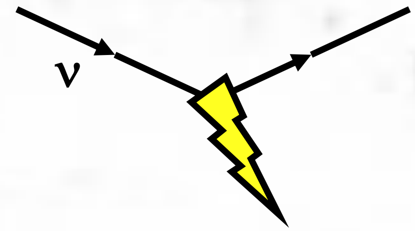
Momentum of Quarks & Antiquarks





- Momentum carried by quarks *much greater* than anti-quarks in nucleon

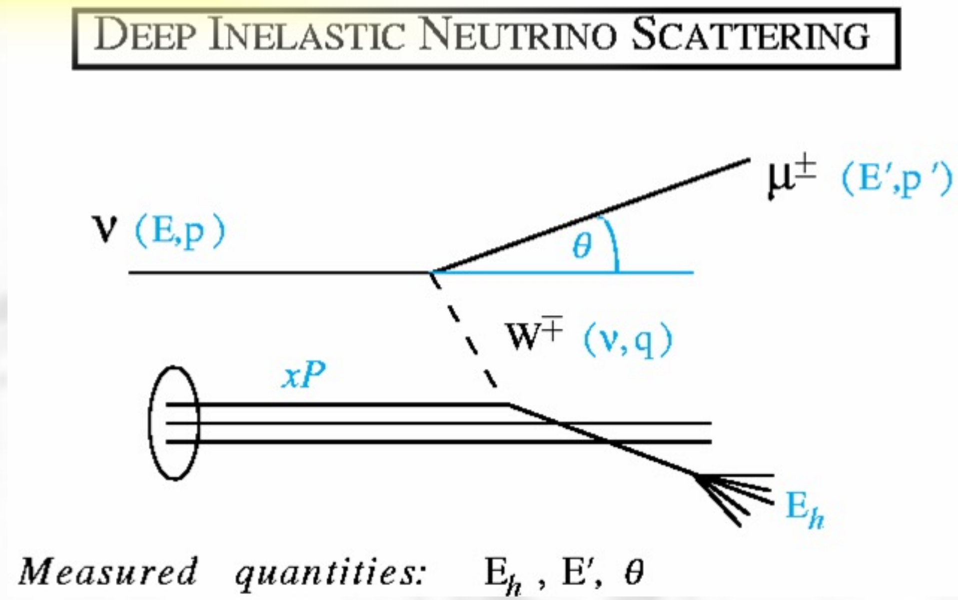
Deep Inelastic Scattering: Conclusions and Summary



- Neutrino-quark scattering is elastic scattering!
  - complicated by fact that quarks live in nucleons
  - *and, as we will discuss later, nucleons in nuclei!*
- But with those caveats, this is another scattering cross-section we can "calculate"
- Supplemental material (won't cover):
  - structure functions
  - scaling violations of partons
    (more partons with lower momentum at higher Q^2)
  - mass effects for tau neutrino interactions and production of charm quarks
  - ultra-high energy neutrinos

SUPPLEMENT: From Parton Distributions to Structure Functions (and back again)

Scattering Variables



Scattering variables given in terms of invariants

- More general than just deep inelastic (neutrino-quark) scattering, although interpretation may change.

4-momentum Transfer^2: $Q^2 = -q^2 = -\left(p' - p\right)^2 \approx \left(4EE' \sin^2(\theta/2)\right)_{Lab}$

Energy Transfer: $\nu = \left(q \cdot P\right)/M_T = \left(E - E'\right)_{Lab} = \left(E_h - M_T\right)_{Lab}$

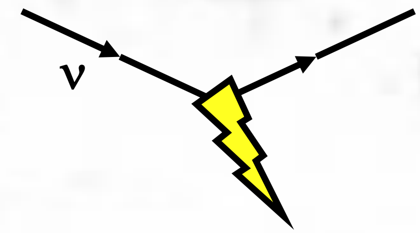
Inelasticity: $y = \left(q \cdot P\right)/\left(p \cdot P\right) = \left(E_h - M_T\right)/\left(E_h + E'\right)_{Lab}$

Fractional Momentum of Struck Quark: $x = -q^2/2\left(p \cdot q\right) = Q^2/2M_T\nu$

Recoil Mass^2 : $W^2 = (q+P)^2 = M_T{}^2 + 2M_T\nu - Q^2$

CM Energy^2 : $s = (p+P)^2 = M_T{}^2 + \dfrac{Q^2}{xy}$

Structure Functions (SFs)



- A model-independent picture of these interactions can also be formed in terms of nucleon "structure functions"
  - All Lorentz-invariant terms included
  - Approximate zero lepton mass (small correction)

$$\frac{d\sigma^{\nu,\bar{\nu}}}{dxdy} \propto \left[y^2 2xF_1(x,Q^2) + \left(2 - 2y - \frac{M_T xy}{E} \right) F_2(x,Q^2) \pm y(2-y) xF_3(x,Q^2) \right]$$

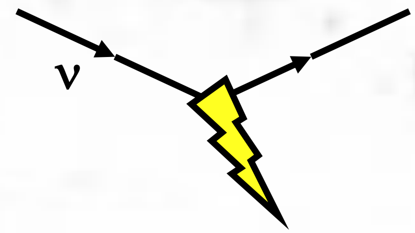
- For massless free spin-1/2 partons, one simplification…
  - Callan-Gross relationship, $2xF_1=F_2$
  - Implies intermediate bosons are completely transverse

Can parameterize longitudinal cross-section by R_L.

Callan-Gross violations result from M_T, NLO pQCD, $g \to qq$

$$R_L = \frac{\sigma_L}{\sigma_T} = \frac{F_2}{2xF_1}\left(1 + \frac{4M_T^2 x^2}{Q^2}\right)$$

SFs to PDFs



- Can relate SFs to PDFs in naïve quark-parton model by matching y dependence
  - Assuming Callan-Gross, massless targets and partons…
  - F_3: $2y-y^2=(1-y)^2-1$, $2xF_1=F_2$: $2-2y+y^2=(1-y)^2+1$

$$2xF_1^{\nu p,CC} = x\left[d_p(x)+\overline{u_p}(x)+s_p(x)+\overline{c_p}(x)\right]$$

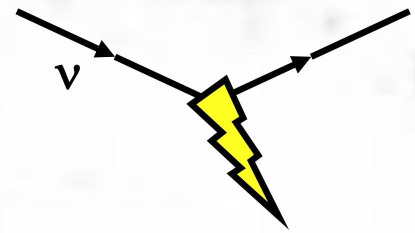
$$xF_3^{\nu p,CC} = x\left[d_p(x)-\overline{u_p}(x)+s_p(x)-\overline{c_p}(x)\right]$$

- In analogy with neutrino-electron scattering, CC only involves left-handed quarks
- However, NC involves both chiralities (V-A and V+A)
  - Also couplings from EW Unification
  - And no selection by quark charge

$$2xF_1^{\nu p,NC} = x\left[(u_L^2+u_R^2)\left(u_p(x)+\overline{u_p}(x)+c_p(x)+\overline{c_p}(x)\right)+(d_L^2+d_R^2)\left(d_p(x)+\overline{d_p}(x)+s_p(x)+\overline{s_p}(x)\right)\right]$$

$$xF_3^{\nu p,NC} = x\left[(u_L^2-u_R^2)\left(u_p(x)-\overline{u_p}(x)+c_p(x)-\overline{c_p}(x)\right)+(d_L^2-d_R^2)\left(d_p(x)-\overline{d_p}(x)+s_p(x)-\overline{s_p}(x)\right)\right]$$

Isoscalar Targets



- Heavy nuclei are roughly neutron-proton isoscalar
- Isospin symmetry implies $u_p = d_n, d_p = u_n$
- Structure Functions have a particularly simple interpretation in quark-parton model for this case…

$$\frac{d^2\sigma^{\nu(\bar{\nu})N}}{dxdy} = \frac{G_F^2 s}{2\pi}\left\{ \left(1+(1-y)^2\right) F_2(x) \pm \left(1-(1-y)^2\right) xF_3^{\nu(\bar{\nu})}(x) \right\}$$

$$2xF_1^{\nu(\bar{\nu})N,CC}(x) = x(u(x)+d(x)+\bar{u}(x)+\bar{d}(x)+s(x)+\bar{s}(x)+c(x)+\bar{c}(x)) = xq(x)+x\bar{q}(x)$$

$$xF_3^{\nu(\bar{\nu})N,CC}(x) = xu_{Val}(x)+xd_{Val}(x) \pm 2x(s(x)-c(x))$$

$$\text{where } u_{Val}(x) = u(x) - \bar{u}(x)$$

From SFs to PDFs

- As you all know, there is a large industry in determining Parton Distributions for hadron collider simulations.
  - to the point where some of my colleagues on collider experiments might think of parton distributions as an annoying piece of FORTRAN code in their software package
- The purpose, of course, is to use factorization to predict cross-sections for various processes
  - combining deep inelastic scattering data from various sources together allows us to “measure” parton distributions
  - which then are applied to predict hadron-hadron processes at colliders, and can also be used in predictions for neutrino scattering, as we shall see.

From SFs to PDFs (cont'd)

- We just learned that…

$$2xF_1^{\nu(\bar{\nu})N,CC}(x) = xq(x) + x\bar{q}(x)$$

$$xF_3^{\nu(\bar{\nu})N,CC}(x) = xu_{Val}(x) + xd_{Val}(x) \pm 2x(s(x) - c(x))$$

$$\text{where } u_{Val}(x) = u(x) - \bar{u}(x)$$

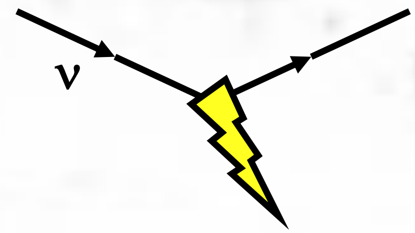
- In charged-lepton DIS

$$2xF_1^{\gamma p}(x) = \left(\tfrac{2}{3}\right)^2 \sum_{\text{up type quarks}} q(x) + \bar{q}(x) + \left(\tfrac{1}{3}\right)^2 \sum_{\text{down type quarks}} q(x) + \bar{q}(x)$$

- So you begin to see how one can combine neutrino and charged lepton DIS and separate
  - the quark sea from valence quarks
  - up quarks from down quarks

SUPPLEMENT: Scaling Violations of Partons

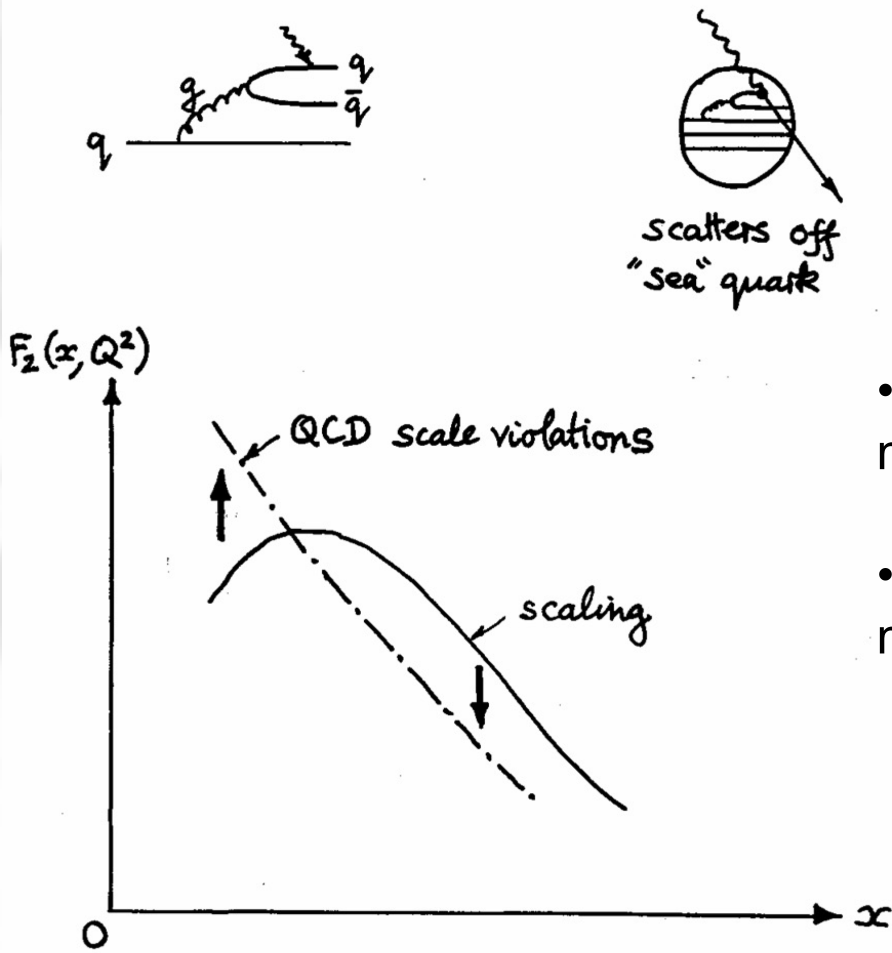
Strong Interactions among Partons



Q^2 Scaling fails due to these interactions

$$\frac{\partial q(x,Q^2)}{\partial \log Q^2} = \frac{\alpha_s(Q^2)}{2\pi}\int_x^1 \frac{dy}{y}$$

$$\left[P_{qq}\left(\frac{x}{y}\right) q(y,Q^2) + P_{qg}\left(\frac{x}{y}\right) g(y,Q^2) \right]$$



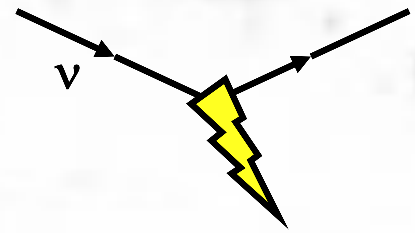
•Pqq(x/y) = probability of finding a quark with momentum x within a quark with momentum y

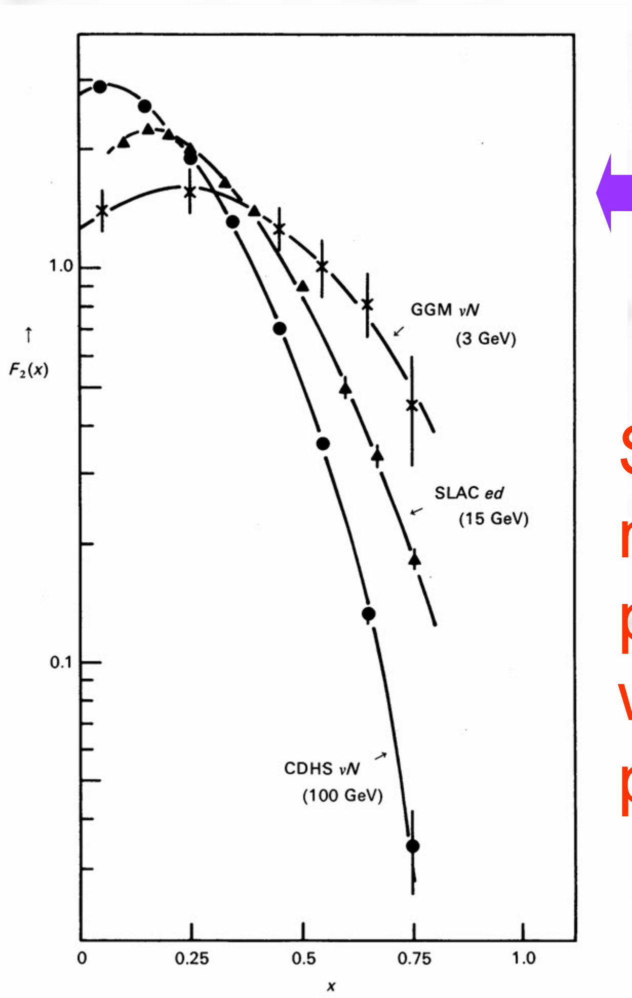
•Pqq(x/y) = probability of finding a q with momentum x within a gluon with momentum y

$$P_{qq}(z) = \frac{4}{3}\frac{1+z^2}{(1-z)} + 2\delta(1-z)$$

$$P_{gq}(z) = \frac{1}{2}\left[z^2 + (1-z)^2\right]$$

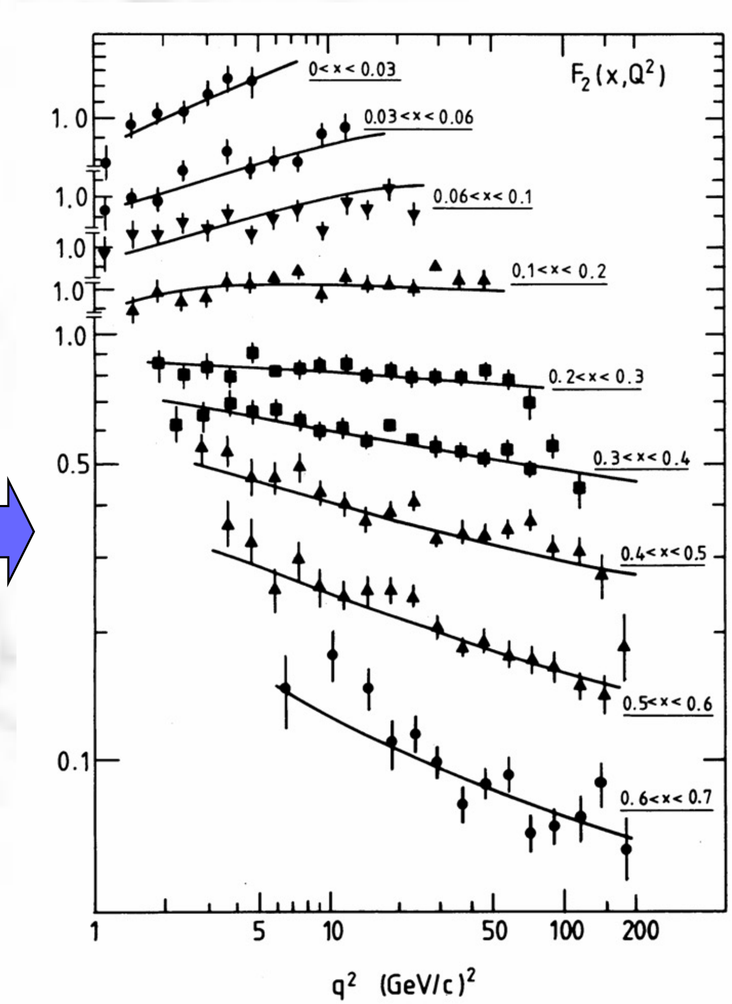
Scaling from QCD

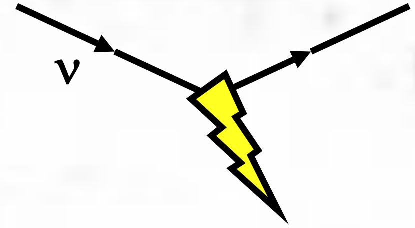




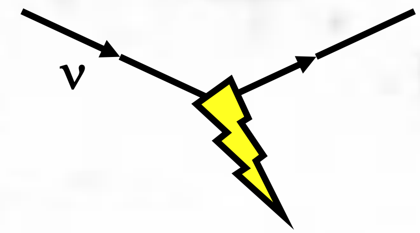
Observed quark distributions vary with Q^2

Scaling well modeled by perturbative QCD with a single free parameter (α_s)



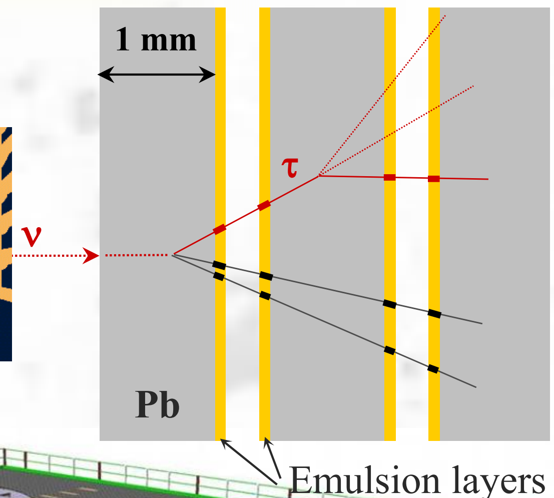


SUPPLEMENT: Massive Leptons (Taus) and Quarks (Charm) in DIS

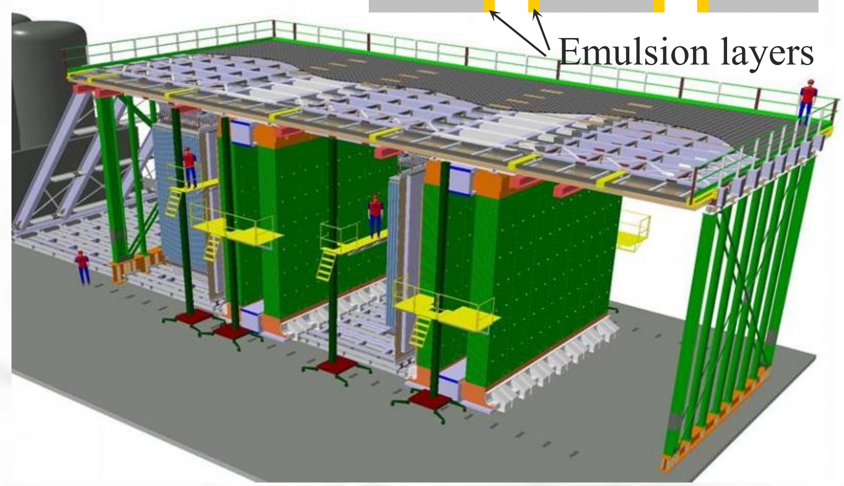


Opera at CNGS

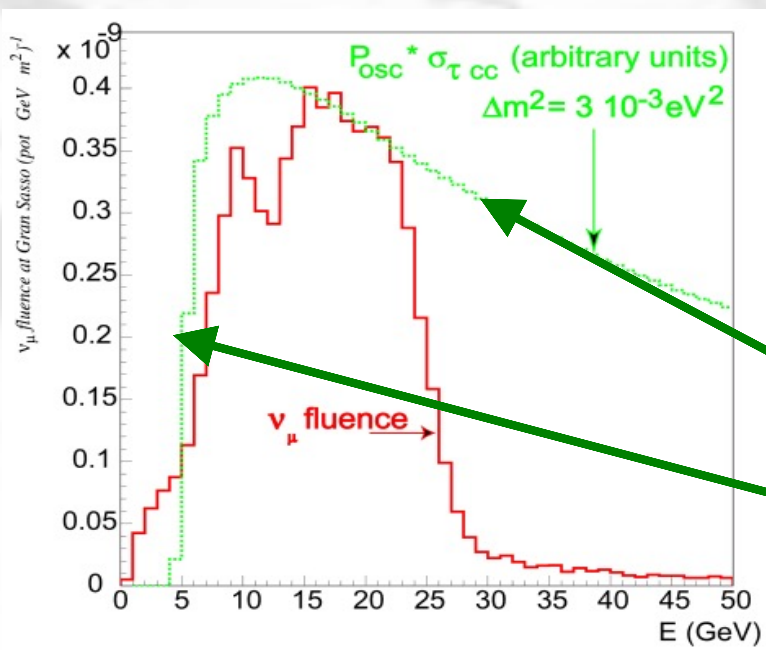
Goal: ν_τ appearance
- 0.15 MWatt source
- high energy ν_μ beam
- 732 km baseline
- handfuls of events/yr



1.8kTon

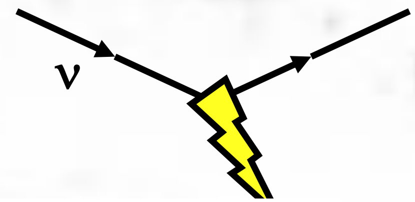


figures courtesy D. Autiero



oscillation probability

but what is this effect?



Lepton Mass Effects in DIS

- Recall that final state mass effects enter as corrections:

$$1-\frac{m^2_{\text{lepton}}}{s_{\text{point-like}}} \rightarrow 1-\frac{m^2_{\text{lepton}}}{x s_{\text{nucleon}}}$$

  - *relevant center-of-mass energy is that of the "point-like" neutrino-parton system*
  - *this is high energy approximation*

- For ν_τ charged-current, there is a threshold of

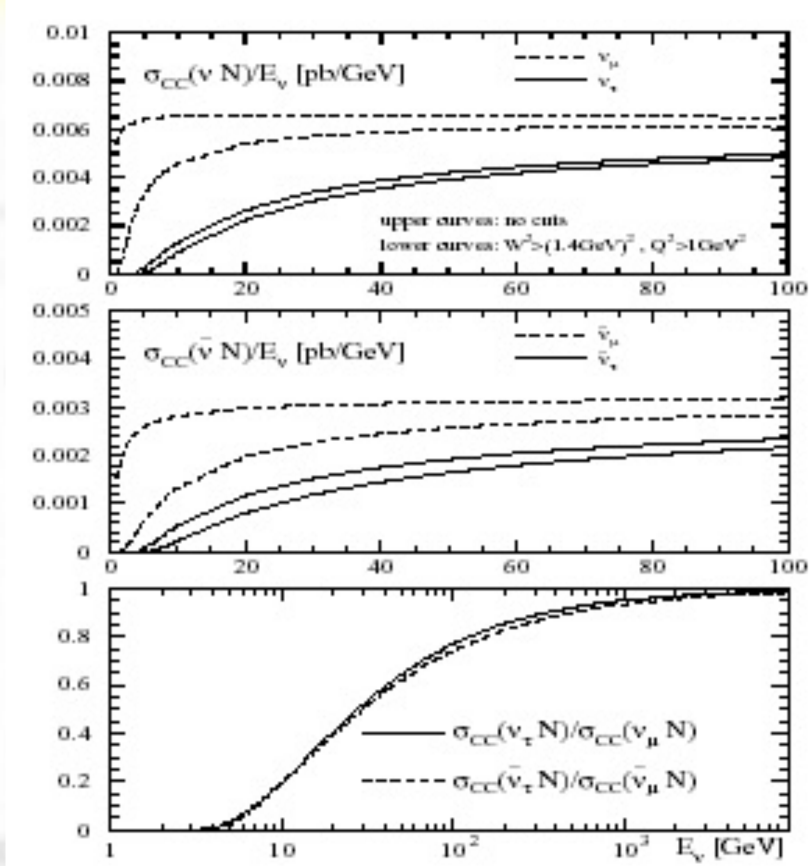
$$s_{\min} = (m_{\text{nucleon}} + m_\tau)^2$$

where

$$s_{initial} = m^2_{\text{nucleon}} + 2E_\nu m_{\text{nucleon}}$$

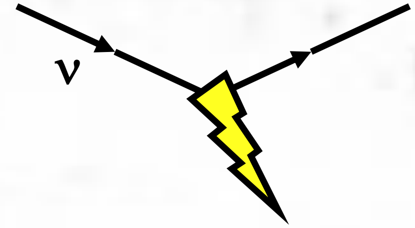
$$\therefore E_\nu > \frac{m_\tau^2 + 2m_\tau m_{\text{nucleon}}}{2m_{\text{nucleon}}} \approx 3.5 \text{ GeV}$$

"m_{nucleon}" is M_T elsewhere, but don't want to confuse with m_τ...



(Kretzer and Reno)

- This is threshold for partons with *entire* nucleon momentum
  - effects big at higher E_ν also

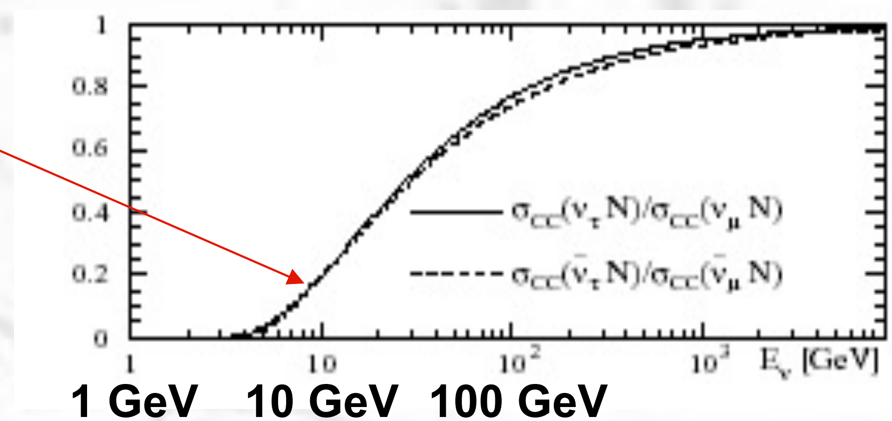


Lecture Question: What if Taus were Lighter?

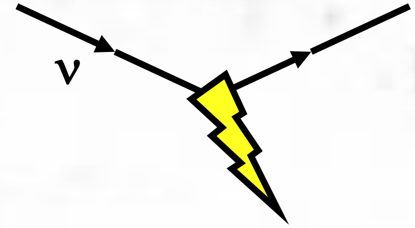
- Imagine we lived in a universe where the tau mass was not 1.777 GeV, but was 0.888 GeV
- By how much would the tau appearance cross-section for an 8 GeV tau neutrino increase at OPERA?

mass suppression: $1-\frac{m^2_{\text{lepton}}}{xs_{\text{nucleon}}}$

$$s_{\text{nucleon}} = m^2_{\text{nucleon}} + 2E_\nu m_{\text{nucleon}}$$



(a) $\frac{\sigma_{\text{Light Tau}}}{\sigma_{\text{Reality}}} \sim 1.4$ **(b)** $\frac{\sigma_{\text{Light Tau}}}{\sigma_{\text{Reality}}} \sim 2$ **(c)** $\frac{\sigma_{\text{Light Tau}}}{\sigma_{\text{Reality}}} \sim 3$



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mass suppression: $$1-\frac{m_{\text{lepton}}^2}{x s_{\text{nucleon}}}$$

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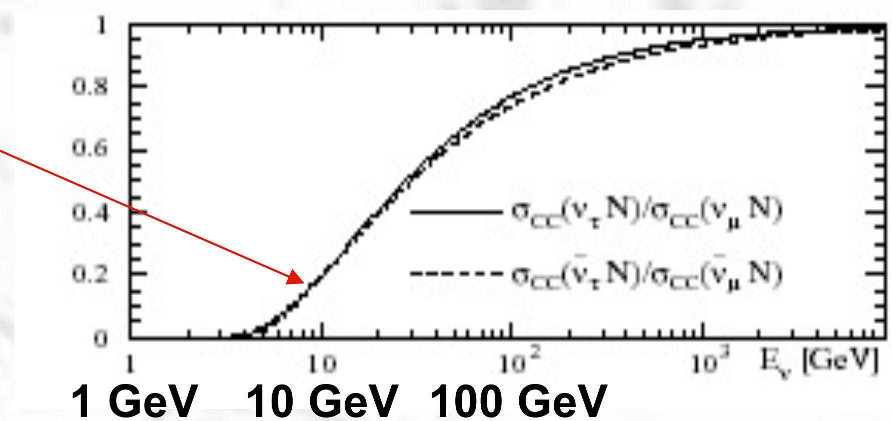

(a) $$\frac{\sigma_{\text{Light Tau}}}{\sigma_{\text{Reality}}} \sim 1.4$$

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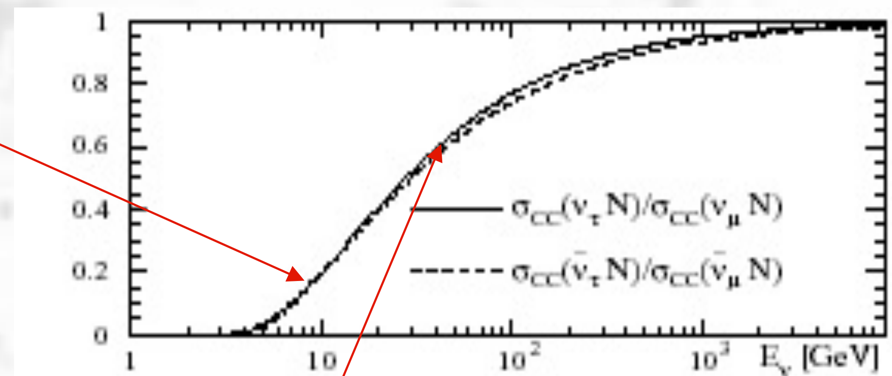


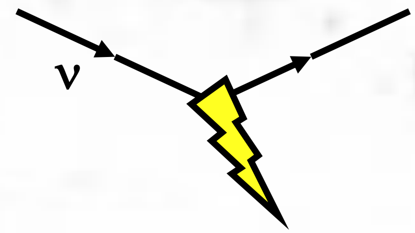
Numerator goes down by factor of four. Equivalent to denominator increasing by factor of four and tau mass unchanged…

$$s_{\text{nucleon}} = m_{\text{nucleon}}^2 + 2E_\nu m_{\text{nucleon}}$$

energy term dominates…
so set energy a factor of four higher

$$\frac{\sigma_{\text{Light Tau}}}{\sigma_{\text{Reality}}} \sim 3$$

Opera at CNGS



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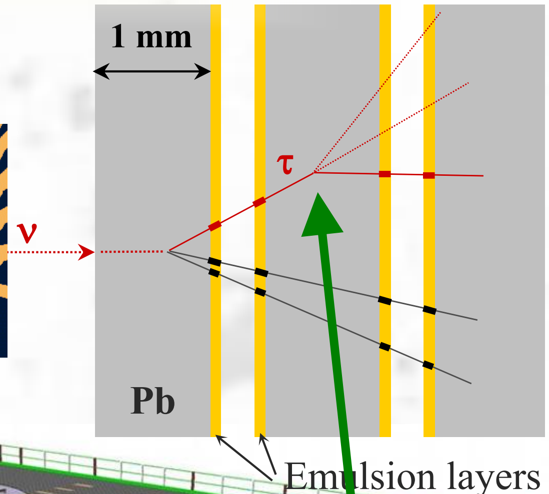
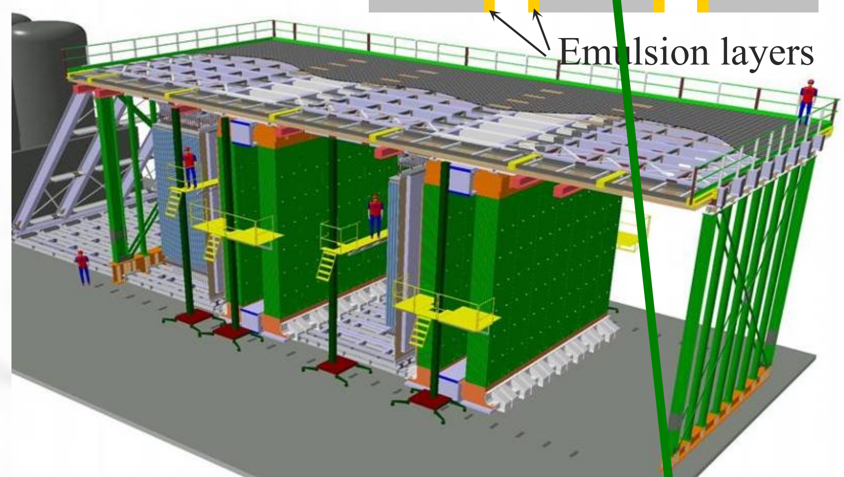
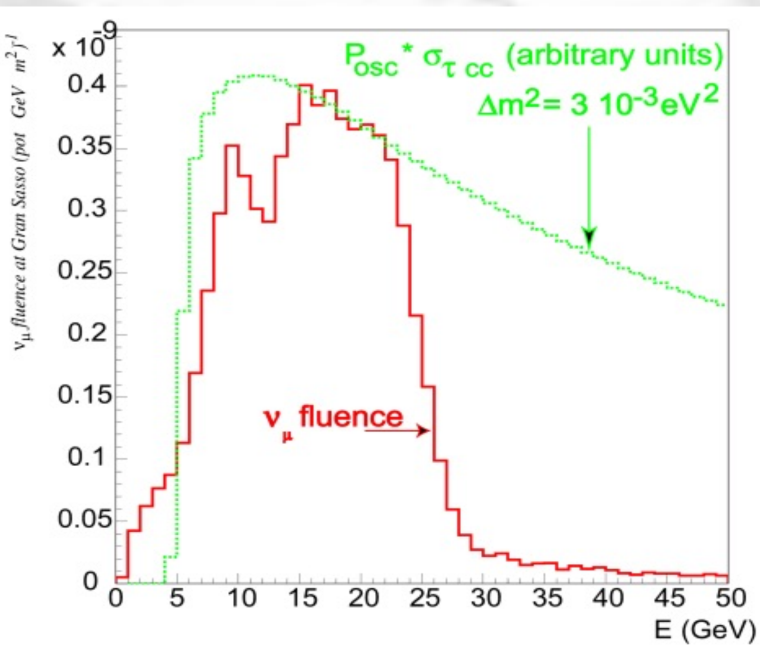


1.8kTon

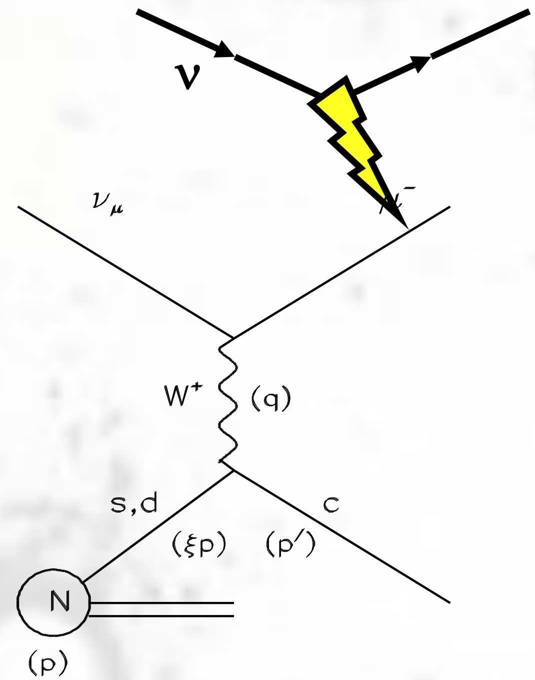


figures courtesy D. Autiero



what else is copiously produced in neutrino interactions with $c\tau \sim 100\mu m$ and decays to hadrons?

Heavy Quark Production



- Production of heavy quarks modifies kinematics of our earlier definition of x.
  - Charm is heavier than proton; hints that its mass is not a negligible effect…

$$(q+\zeta p)^2 = p'^2 = m_c^{\;2}$$

$$q^2 + 2\zeta p \bullet q + \zeta^2 M^2 = m_c^{\;2}$$

$$\text{Therefore } \zeta \cong \frac{-q^2 + m_c^{\;2}}{2p\bullet q}$$

Note different definition of fractional momentum

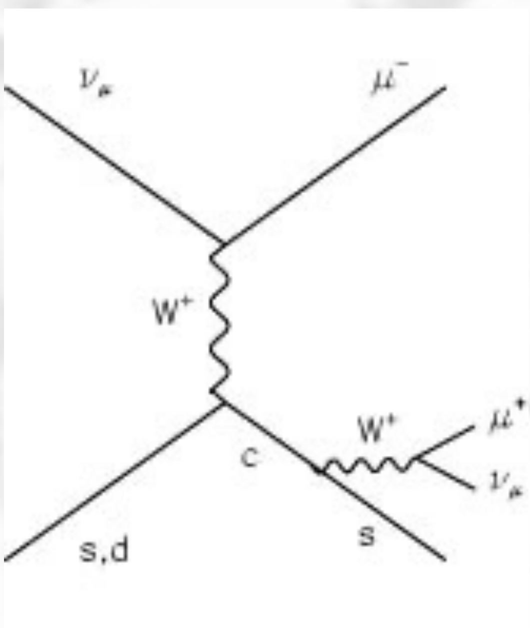
$$\zeta \cong \frac{Q^2 + m_c^{\;2}}{2M\upsilon} = \frac{Q^2 + m_c^{\;2}}{Q^2 / x}$$

$$\zeta \cong x\left(1 + \frac{m_c^{\;2}}{Q^2}\right)$$

"slow rescaling" leads to kinematic suppression of charm production

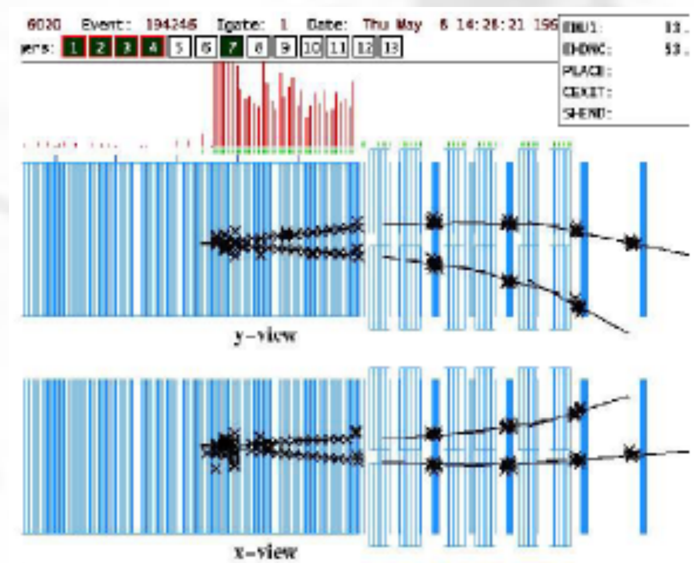
Neutrino Dilepton Events

- Neutrino induced charm production has been extensively studied
  - Emulsion/Bubble Chambers (low statistics, 10s of events). Reconstruct the charm final state, but limited by target mass.
  - "Dimuon events" (high statistics, 1000s of events)



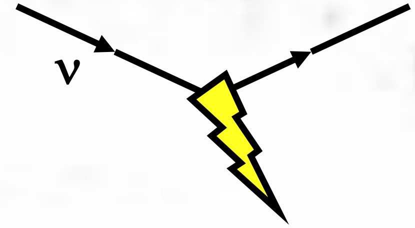
$$\nu_\mu + \begin{pmatrix} d \\ s \end{pmatrix} \to \mu^- + c + X, \qquad c \to \mu^+ + \nu_\mu + X'$$

$$\bar{\nu}_\mu + \begin{pmatrix} \bar{d} \\ \bar{s} \end{pmatrix} \to \mu^+ + \bar{c} + X, \qquad \bar{c} \to \mu^- + \bar{\nu}_\mu + X'$$

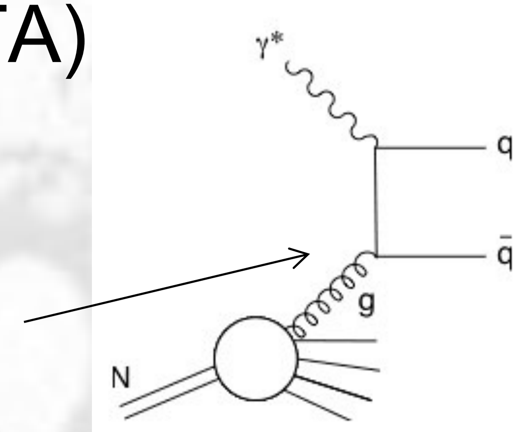


SUPPLEMENT: Ultra-High Energy Cross-Sections

Ultra-High Energies

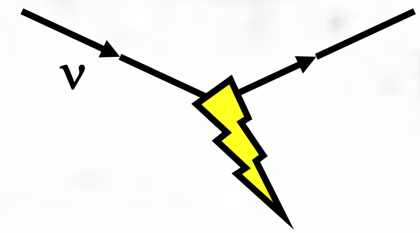


- At energies relevant for UHE Cosmic Ray studies (e.g., IceCube, Antares, ANITA)
  - ν-parton cross-section is dominated by high Q^2, since $d\sigma/dQ^2$ is constant
    - o at high Q^2, gluon radiation and splitting lead to more sea quarks at fewer high x partons (see supplemental material: scaling violations)
    - o see a rise in σ/E_ν from growth of sea at low x
    - o neutrino & anti-neutrino cross-sections nearly equal
  - *Until* $Q^2 » M_W^2$, then propagator term starts decreasing and cross-section stops growing linearly with energy



$$\frac{d\sigma}{dq^2} \propto \frac{1}{(q^2 - M^2)^2}$$

UHE Question



At what energy does σ stop increasing $\propto E_\nu$?

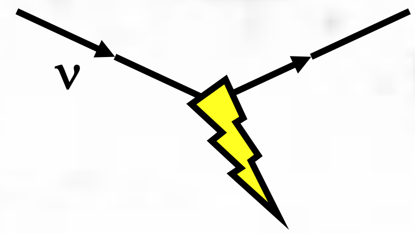
- When $Q^2 \gg M_W^2$, propagator term starts decreasing and cross-section becomes constant
- To within a few orders of magnitude, at what beam energy for a nucleon target at rest will this happen?

$$\frac{d\sigma}{dq^2} \propto \frac{1}{(q^2 - M^2)^2}$$

$$s_{\text{nucleon}} = m^2_{\text{nucleon}} + 2E_\nu m_{\text{nucleon}}$$

(a) $E_\nu \sim 10\,\text{TeV}$ **(b)** $E_\nu \sim 10{,}000\,\text{TeV}$ **(c)** $E_\nu \sim 10{,}000{,}000\,\text{TeV}$

UHE Answer
Energy when σ no longer ∝ E_ν?



- When $Q^2 \gg M_W^2$, propagator term starts decreasing and cross-section becomes constant

$$\frac{d\sigma}{dq^2} \propto \frac{1}{(q^2 - M^2)^2}$$

- At what beam energy for a target at rest will this happen?

$$Q^2 < s_{\text{nucleon}} = m_{\text{nucleon}}^2 + 2E_\nu m_{\text{nucleon}}$$

$$Q^2 < s_{\text{nucleon}} \approx 2E_\nu m_{\text{nucleon}}$$

$$\frac{M_W^2}{2m_{\text{nucleon}}} < E_\nu$$

$$\therefore E_\nu \gtrsim \frac{(80.4)^2\ \text{GeV}^2}{2(.938)\text{GeV}} \sim 3000\text{GeV}$$

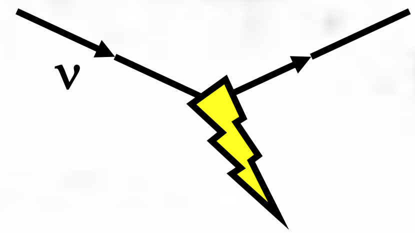
Q^2 limit is s.
So won't start to plateau until $s > M_W^2$

However…

In reality, that is only correct for a parton at x=1. Typical quark x is much less, say ~0.03

$$\frac{M_W^2}{2m_{\text{nucleon}}x} < E_\nu$$

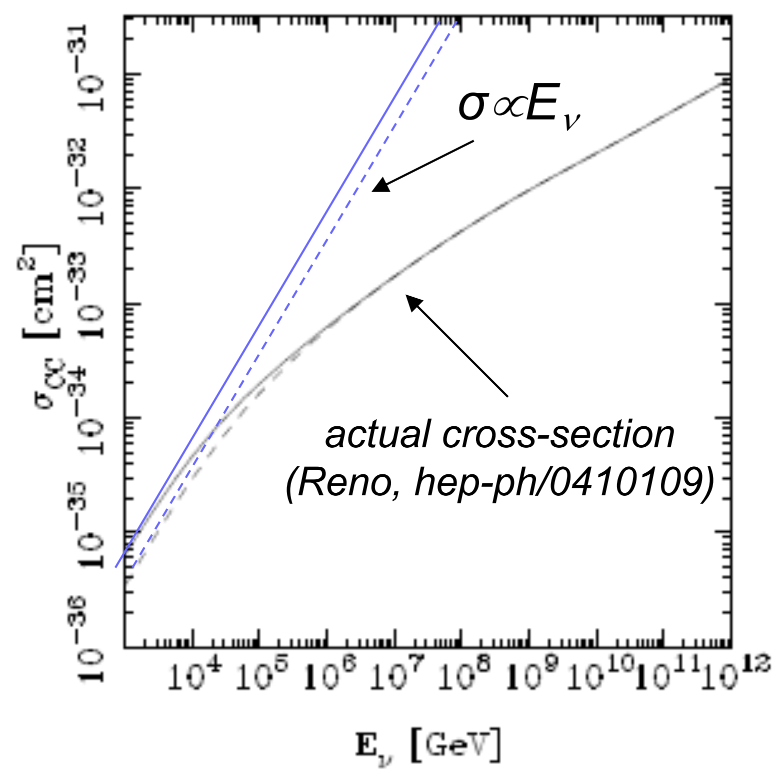
$$\therefore E_\nu \gtrsim \frac{3000\text{GeV}}{0.03} \sim 100\text{TeV}$$



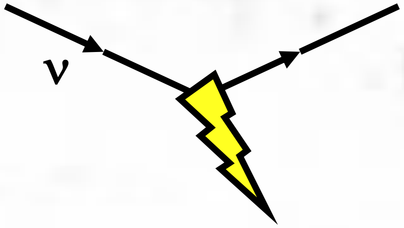
Ultra-High Energies

- ν-parton cross-section is dominated by high Q^2, since $d\sigma/dQ^2$ is constant
  - at high Q^2, scaling violations have made most of nucleon momentum carried by sea quarks
  - see a rise in σ/E_ν from growth of sea at low x
  - neutrino & anti-neutrino cross-sections nearly equal
- *Until* $Q^2 \gg M_W^2$, then propagator term starts decreasing and cross-section becomes constant

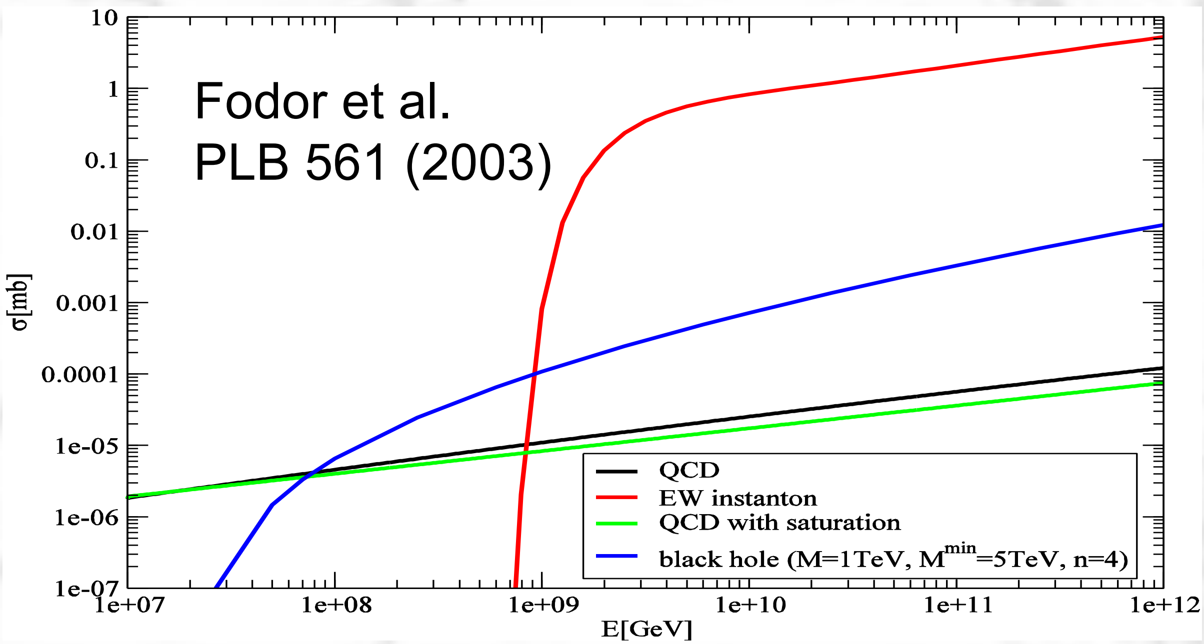
$$\frac{d\sigma}{dq^2} \propto \frac{1}{(q^2 - M^2)^2}$$

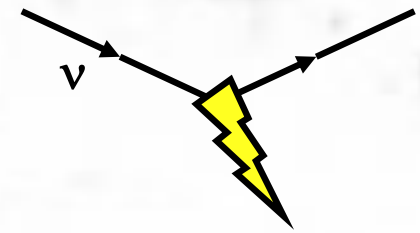


Example: Ultra-High Energies



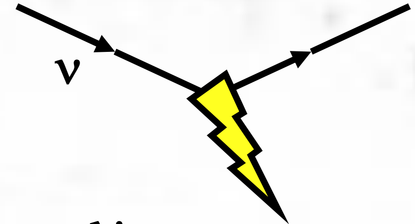
- At UHE, can we reach thresholds of non-SM processes?
  - E.g., structure of quark or leptons, black holes from extra dimensions, etc.
  - Then no one knows what to expect…





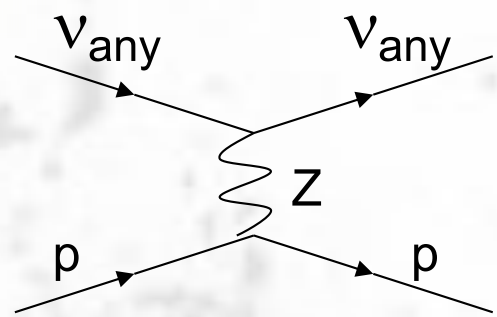
Targets with Structure: Nucleons

Enough about pointlike targets…



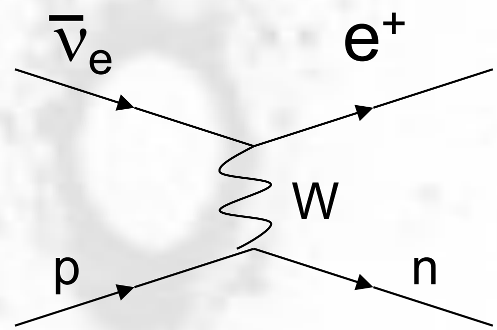
- Imagine now a nucleon target
  - Neutrino-proton elastic scattering:

$$\nu_e + p \rightarrow \nu_e + p$$



  - "Inverse beta-decay" (IBD):

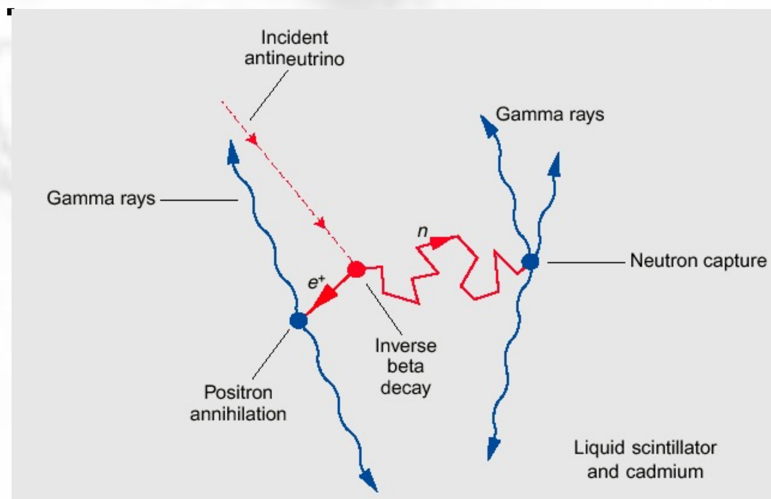
$$\overline{\nu}_e + p \rightarrow e^+ + n$$

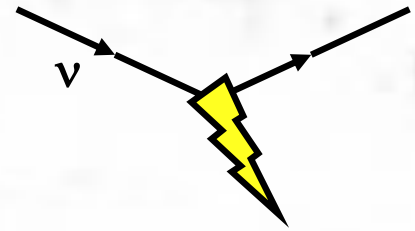


  - and "stimulated" beta decay:

$$\nu_e + n \rightarrow e^- + p$$

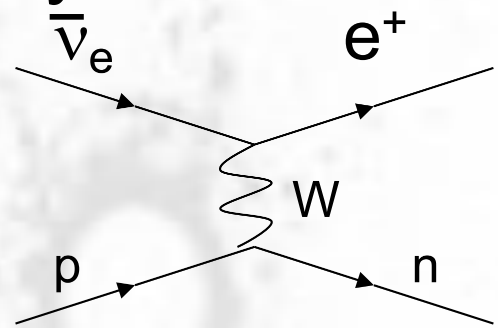
  - Recall that IBD was the Reines and Cowan discovery signal





Final State Mass Effects

- In IBD, $\overline{\nu}_e + p \rightarrow e^+ + n$, have to pay a mass penalty *twice*
  - *M_n-M_p≈1.3 MeV, M_e≈0.5 MeV*



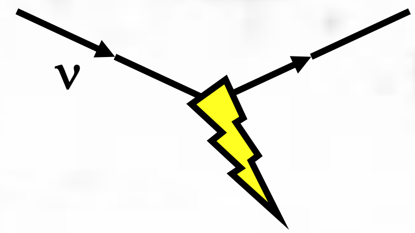
- What is the threshold?
  - kinematics are simple, at least to zeroth order in M_e/M_n → heavy nucleon kinetic energy is zero

$$s_{\text{initial}} = (\underline{p}_\nu + \underline{p}_p)^2 = M_p^2 + 2M_p E_\nu \text{ (proton rest frame)}$$

$$s_{\text{final}} = (\underline{p}_e + \underline{p}_n)^2 \approx M_n^2 + m_e^2 + 2M_n\left(E_\nu - \left(M_n - M_p\right)\right)$$

- Solving… $E_\nu^{\min} \approx \dfrac{(M_n + m_e)^2 - M_p{}^2}{2M_p} \approx 1.806 \text{ MeV}$

Final State Mass Effects (cont'd)



- Define δE as $E_\nu - E_\nu^{min}$, then

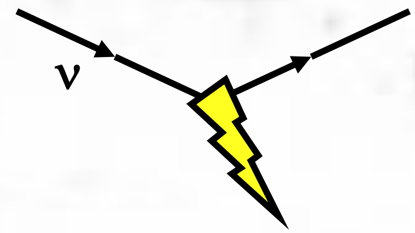
$$\begin{aligned} s_{\text{initial}} &= M_p^2 + 2M_p\left(\delta E + E_\nu^{\text{min}}\right) \\ &= M_p^2 + 2\delta E \times M_p + \left(M_n + m_e\right)^2 - M_p^2 \\ &= 2\delta E \times M_p + \left(M_n + m_e\right)^2 \end{aligned}$$

- Remember the suppression generally goes as

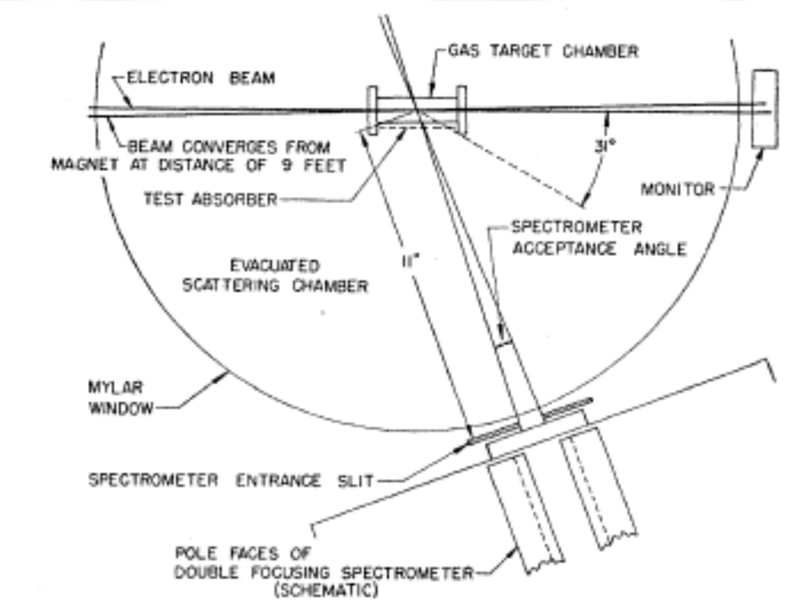
$$\xi_{\text{mass}} = 1 - \frac{m_{\text{final}}{}^2}{\text{s}} = 1 - \frac{\left(M_n + m_e\right)^2}{\left(M_n + m_e\right)^2 + 2M_p \times \delta E}$$

$$= \frac{2M_p \times \delta E}{\left(M_n + m_e\right)^2 + 2M_p \times \delta E} \approx \begin{cases} \delta E \times \dfrac{2M_p}{\left(M_n + m_e\right)^2} & \text{low energy} \\ 1 - \dfrac{\left(M_n + m_e\right)^2}{2M_p{}^2} \dfrac{M_p}{\delta E} & \text{high energy} \end{cases}$$

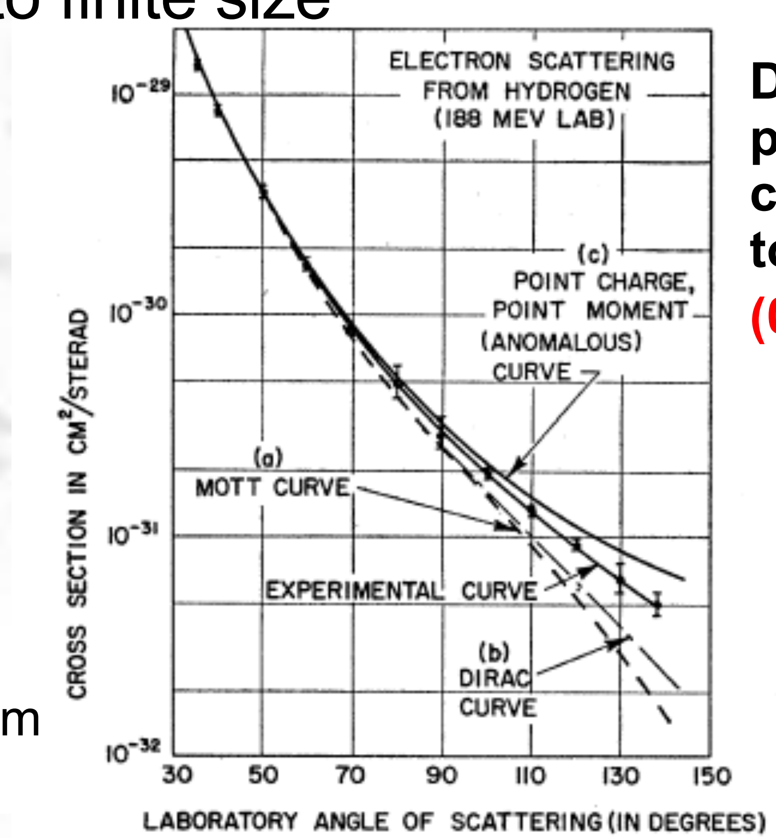
Proton Structure



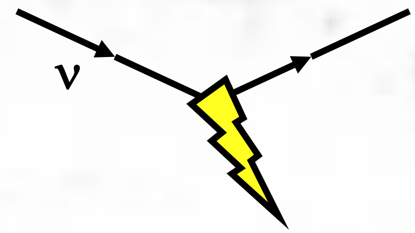
- How is a proton different from an electron?
  - anomalous magnetic moment, $\kappa \equiv \frac{g-2}{2} \neq 0$
  - "form factors" related to finite size



McAllister and Hofstadter 1956
188 MeV and 236 MeV electron beam from linear accelerator at Stanford



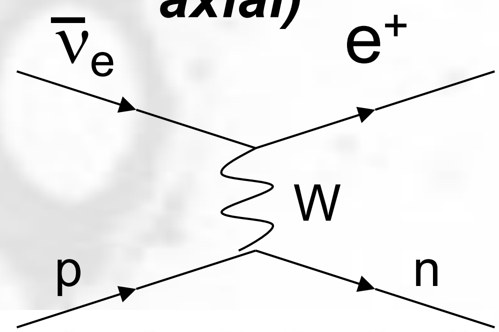
Determined proton RMS charge radius to be (0.7±0.2) x10^{-13} cm



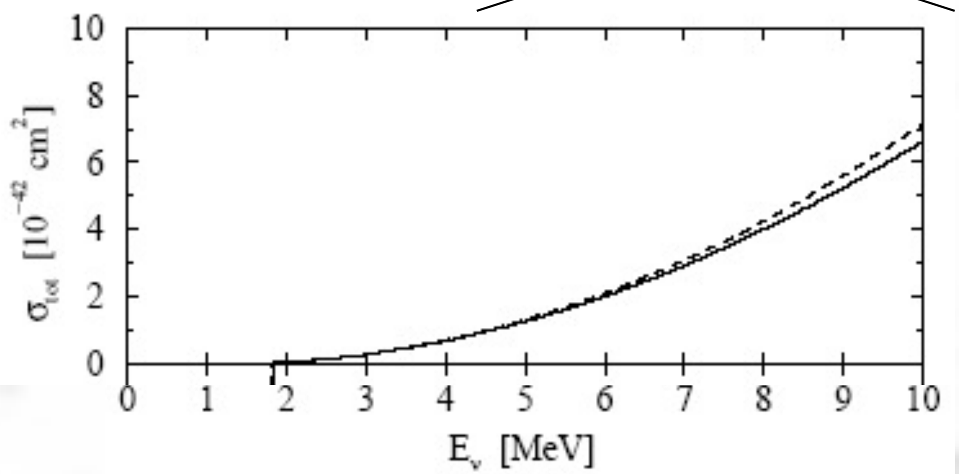
Putting it all together…

$$\sigma_{TOT} = \frac{G_F^2 s}{\pi} \times \cos^2 \vartheta_{\text{Cabibbo}} \times \left(\xi_{\text{mass}}\right) \times \left(g_V{}^2 + 3g_A{}^2\right)$$

quark mixing! ***final state mass suppression*** ***proton form factors (vector, axial)***

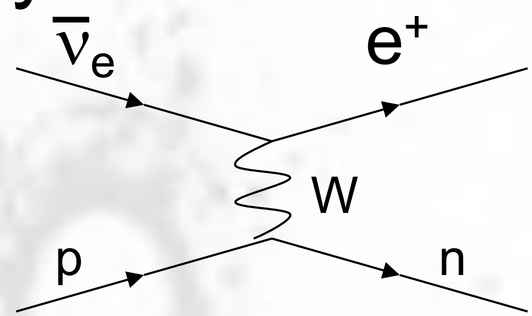


- mass suppression is proportional to δE at low E_ν, so quadratic near threshold
- vector and axial-vector form factors (for IBD usually referred to as f and g, respectively) $g_V, g_A \approx 1, 1.26$.
  - FFs, $\theta_{Cabibbo}$, best known from τ_n



More about IBD Kinematics

- In IBD, $\bar{\nu}_e + p \rightarrow e^+ + n$, have to pay a mass penalty *twice*
  - *M_n-M_p≈1.3 MeV, M_e≈0.5 MeV*



- Kinematics are simple, at least to zeroth order in M_e/M_n → heavy nucleon kinetic energy is zero

$$s_{\text{initial}} = (\underline{p}_\nu + \underline{p}_p)^2 = M_p^2 + 2M_pE_\nu \text{ (proton rest frame)}$$

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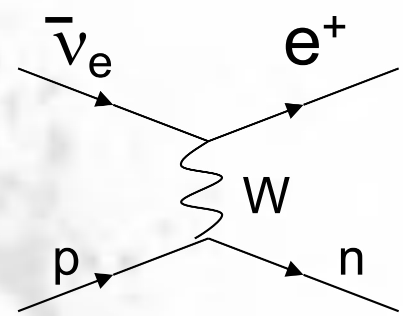
- We can derive other interesting features by going to beyond zeroth order in M_e/M_n …

More IBD Kinematics

- In IBD, $\bar{\nu}_e + p \rightarrow e^+ + n$, angle and energy must be related, since $2 \rightarrow 2$ process

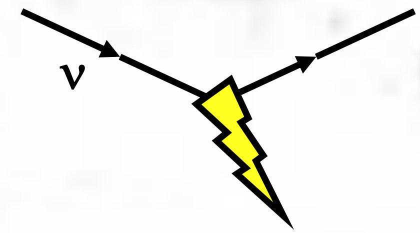


  - Heavy neutron takes all necessary momentum, but not energy! $T = {p^2}/{2M}$

$$\cos\theta_e = \frac{M_n^2 - M_p^2 - M_e^2 + 2E_e\left(E_\nu + M_p\right) - 2E_\nu M_p}{2E_e E_\nu \sqrt{1 - {M_e^2}/{E_e^2}}}$$

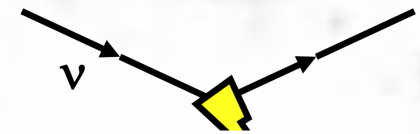
  - Note large numbers in numerator that have to balance carefully if $E_\nu \ll M_p$. A very narrow range of electron energies for a given neutrino energy (~½% at 4 MeV)

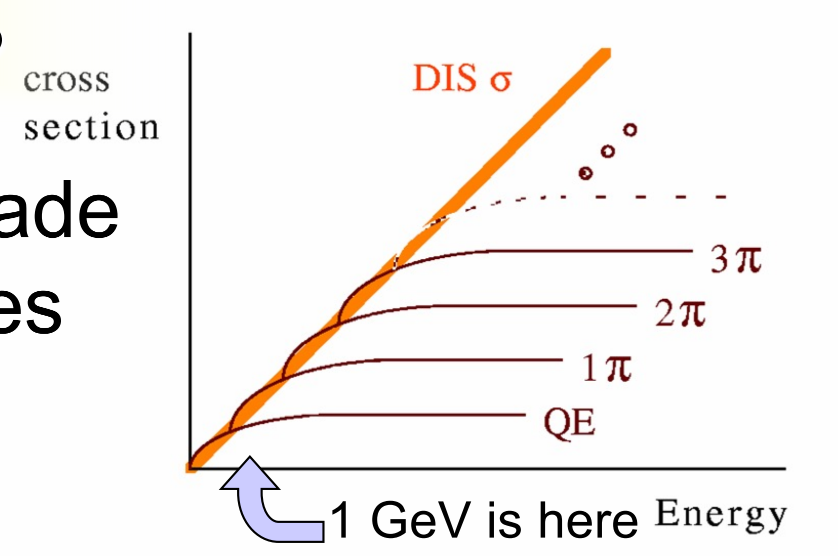
$$\langle E_e \rangle = \frac{2E_\nu M_p - M_n^2 + M_p^2 + M_e^2}{2\left(E_\nu + M_p\right)} \approx E_\nu - 1.3\ \text{MeV}$$



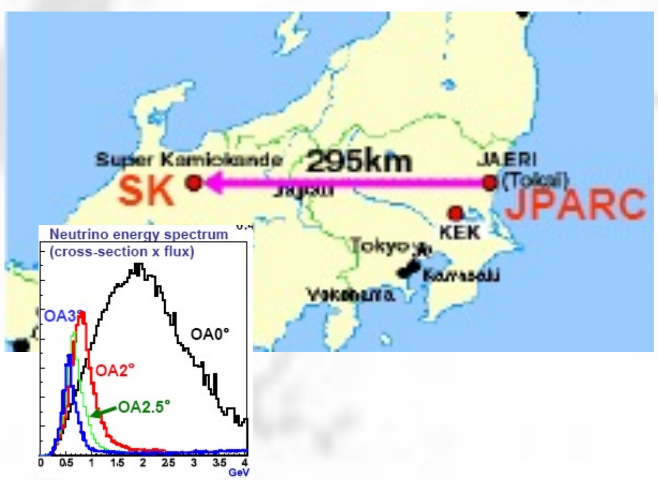
Motivation for Understanding GeV Cross-Sections

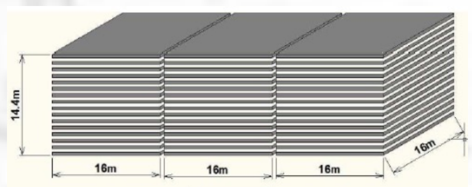
What's special about it? Why do we care?

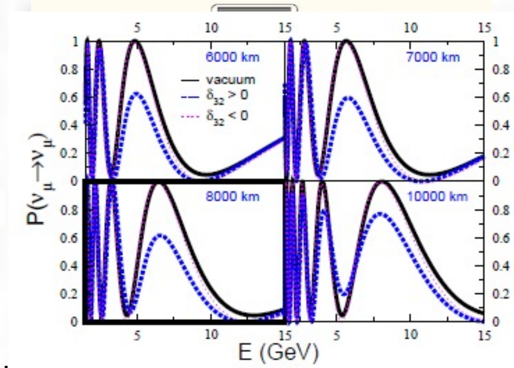




- Our calculation of DIS made no reference to final states
  - But at 1-few GeV, the final state has few particles
  - Final states & threshold effects matter
- Why is 1-few GeV important? Examples from T2K, ICAL





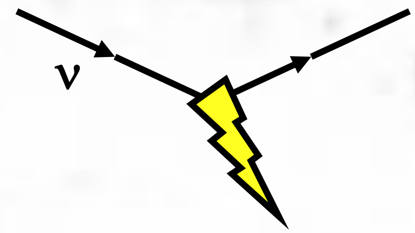


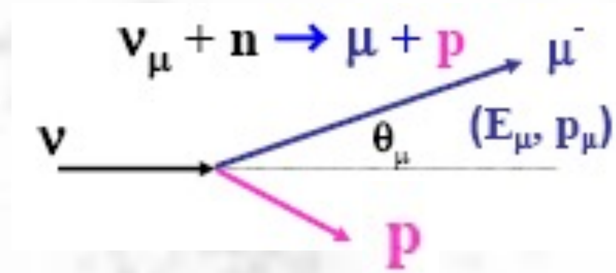
Goals:

1. $\nu_\mu \to \nu_e$
2. ν_μ disappearance

E_ν is 0.4-2.0 GeV (T2K) or 3-10 GeV (INO ICAL)

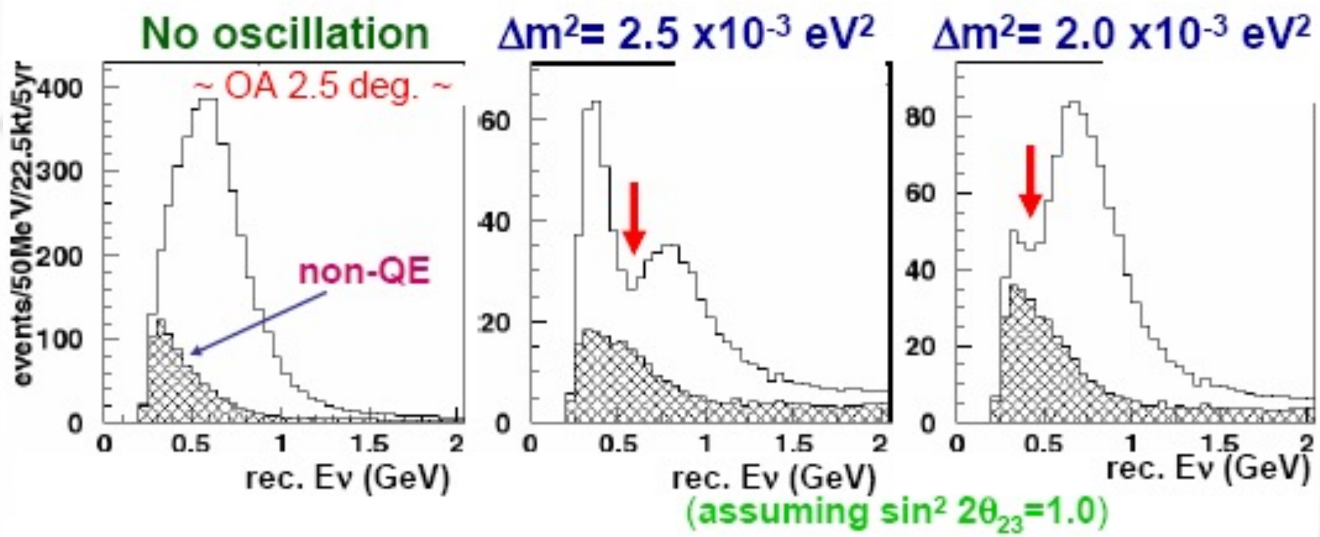
How do cross-sections effect oscillation analysis?



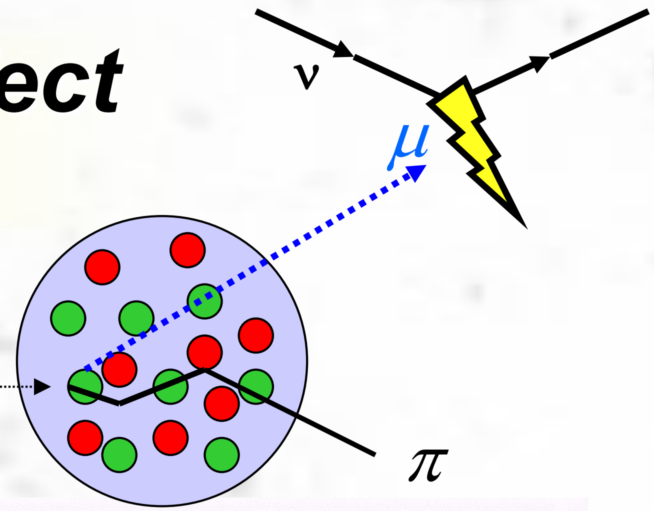


- ν_μ disappearance (low energy)
  - at Super-K reconstruct these events by muon angle and momentum (proton below Cerenkov threshold in H_2O)
  - other final states with more particles below threshold ("non-QE") will disrupt this reconstruction
- T2K must know these events at few % level to do disappearance analysis to measure Δm^2_{23}, θ_{23}

(fig. courtesy Y. Hayato)

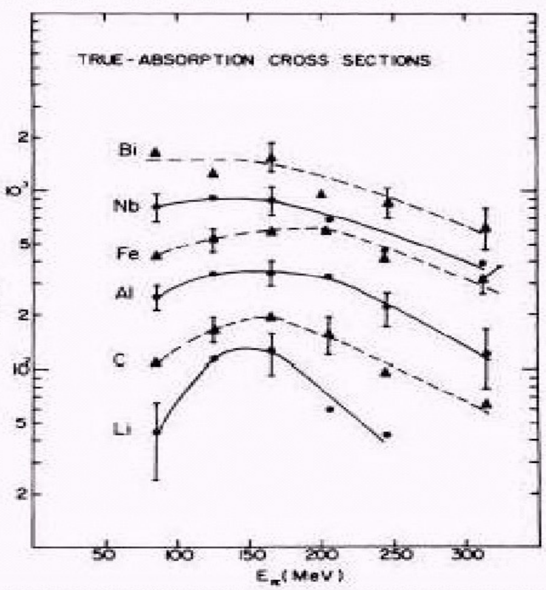


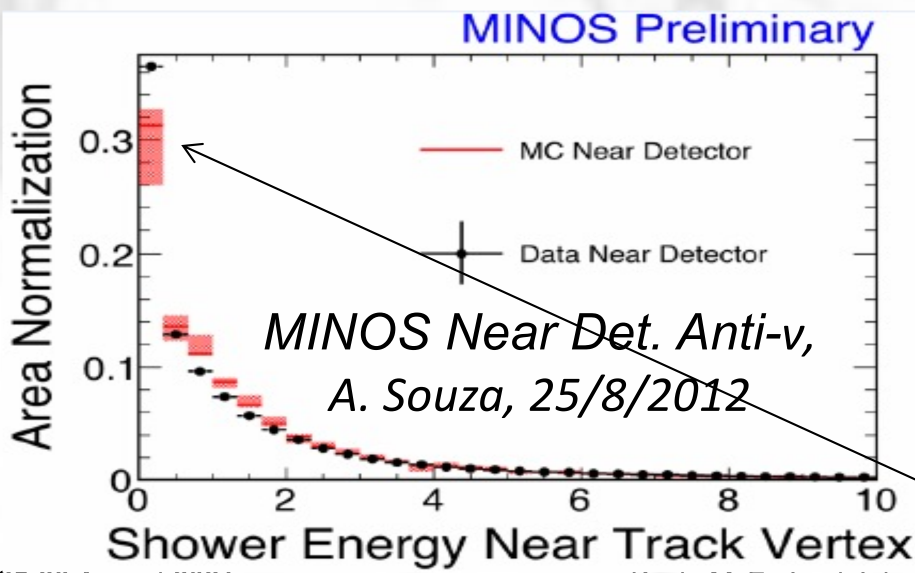
How do cross-sections effect oscillation analysis?



- ν_μ disappearance (high energy)
- Visible Energy in a calorimeter is NOT the ν energy transferred to the hadronic system
  - ➢ π absorption, π re-scattering, final state rest mass effect the calorimetric response
    - ➢ Can use external data to constrain

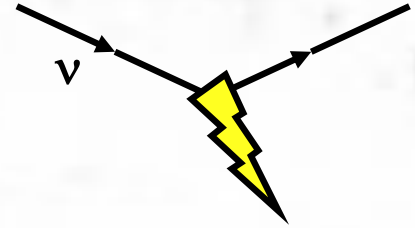
D. Ashery et al, PRC **23**, 1993



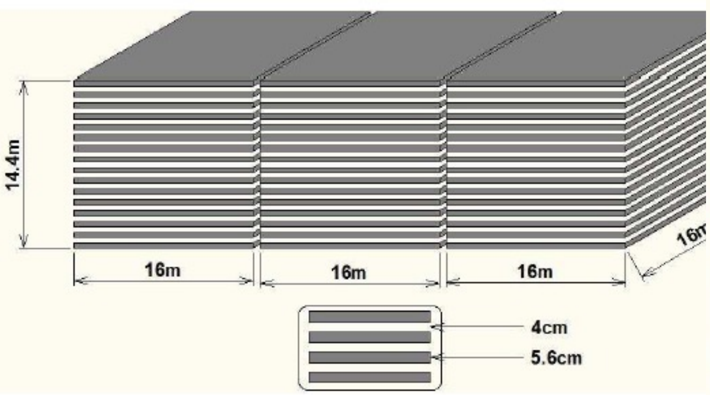


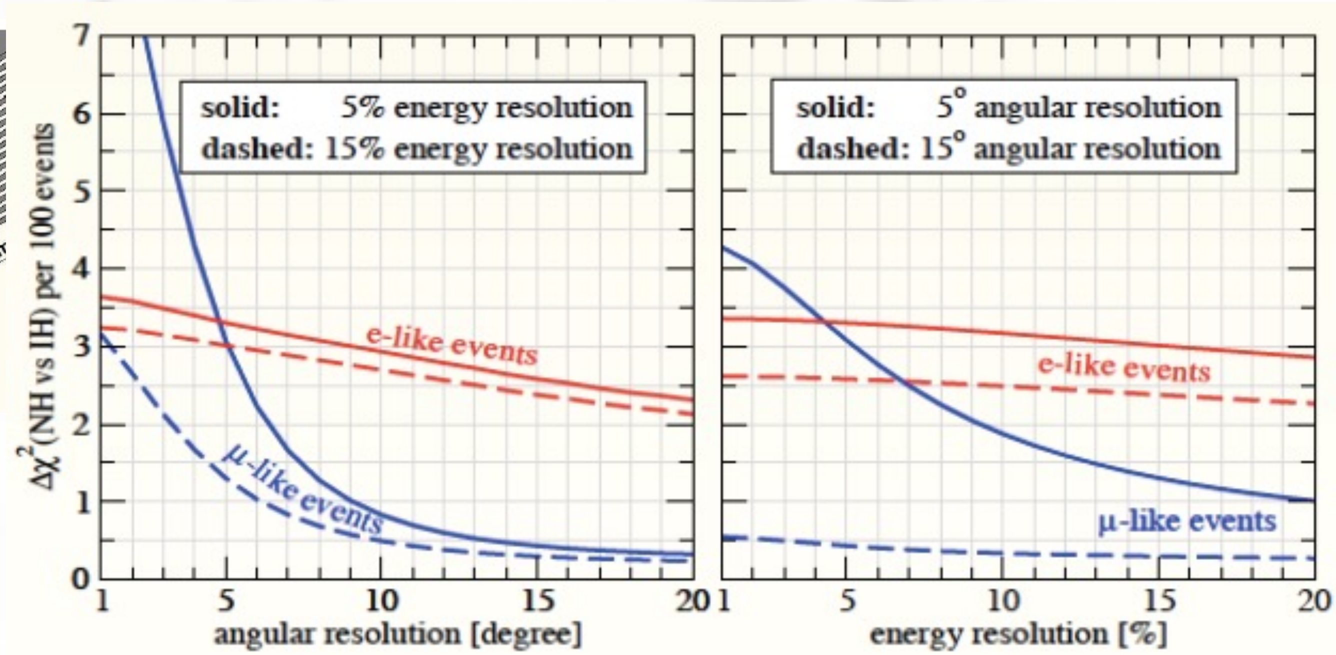
- ➢ *At very high energies, particle multiplicities are high and these effects will average out*
- ➢ *Low energy is more difficult*

How do cross-sections effect oscillation analysis?



- In the case of INO ICAL, need good energy and angle resolution to separate normal and inverted hierarchy
  - Best sensitivity requires survival probability in both E_ν and L

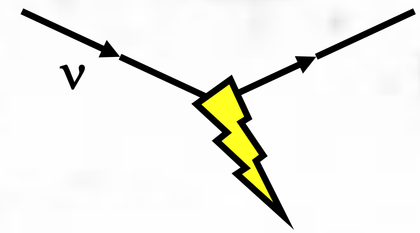




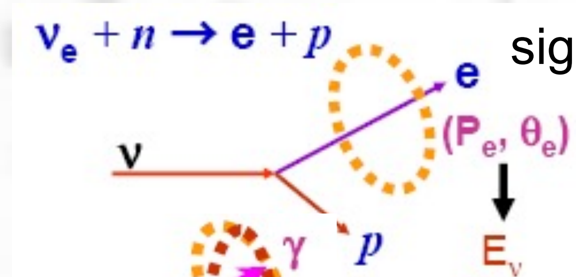
- Interaction models are understanding of detector response both needed to optimize resolution

Petcov, Schwetz, hep-ph/0511277

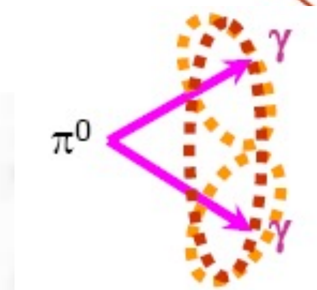
How do cross-sections effect oscillation analysis?



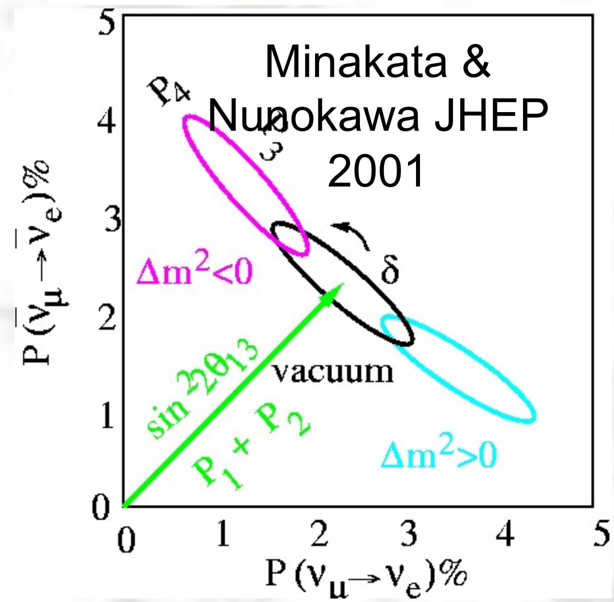
- ν_e appearance
  - different problem: signal rate is very low so even rare backgrounds contribute!
- Remember the end goal of electron neutrino appearance experiments
- Want to compare two signals with two different sets of backgrounds and signal reactions
  - with sub-percent precision
  - Requires precise knowledge of background and signal reactions

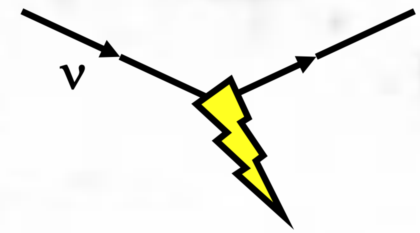


signal



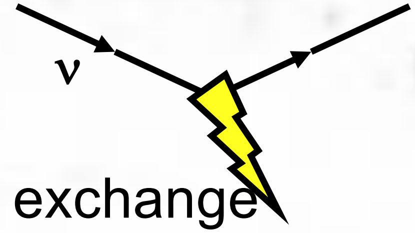
π^0 background from E_ν>peak



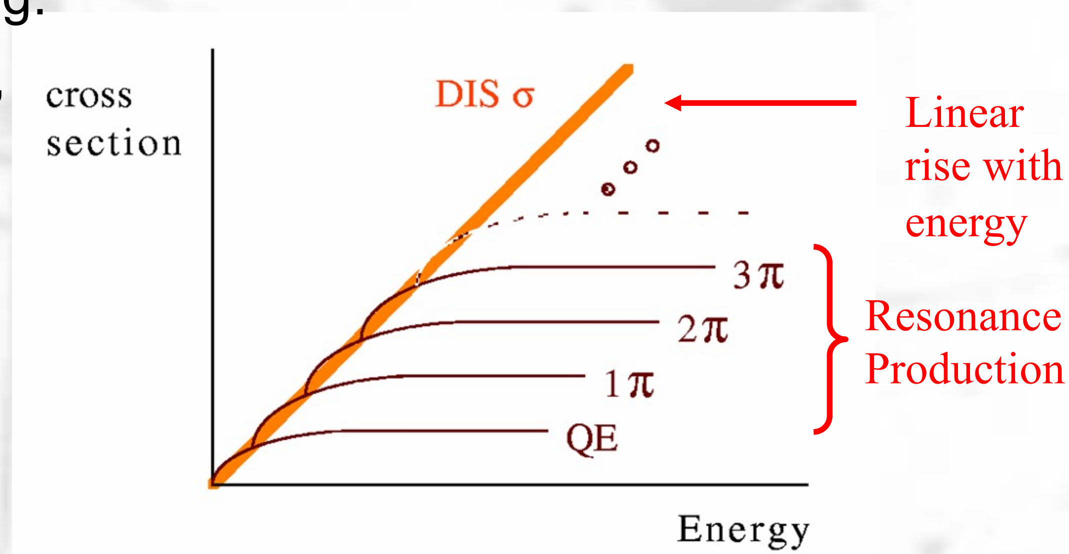


Models for GeV Cross-Sections

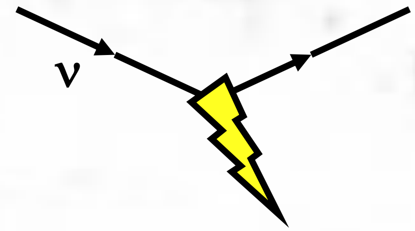
Neutrino-Nucleon Scattering



- Charged - Current: $W^{\pm}$ exchange
  - CC Elastic Scattering *(sometimes called "quasi-elastic" since neutron targets are only found in nuclei)*
    (Target changes but no break up)
    $\nu_\mu + n \rightarrow \mu^- + p$
  - Baryon Resonance Production:
    (Target goes to excited state)
    $\nu_\mu + n \rightarrow \mu^- + p + \pi^0$ $(N^*$ or $\Delta)$
    $n + \pi^+$
  - Deep-Inelastic Scattering:
    (Nucleon broken up)
    $\nu_\mu + \text{quark} \rightarrow \mu^- + \text{quark}'$
- Neutral - Current: Z^0 exchange
  - Elastic Scattering:
    (Target unchanged)
    $\nu_\mu + N \rightarrow \nu_\mu + N$
  - Baryon Resonance Production:
    (Target goes to excited state)
    $\nu_\mu + N \rightarrow \nu_\mu + N + \pi$ $(N^*$ or $\Delta)$
  - Deep-Inelastic Scattering
    (Nucleon broken up)
    $\nu_\mu + \text{quark} \rightarrow \nu_\mu + \text{quark}$

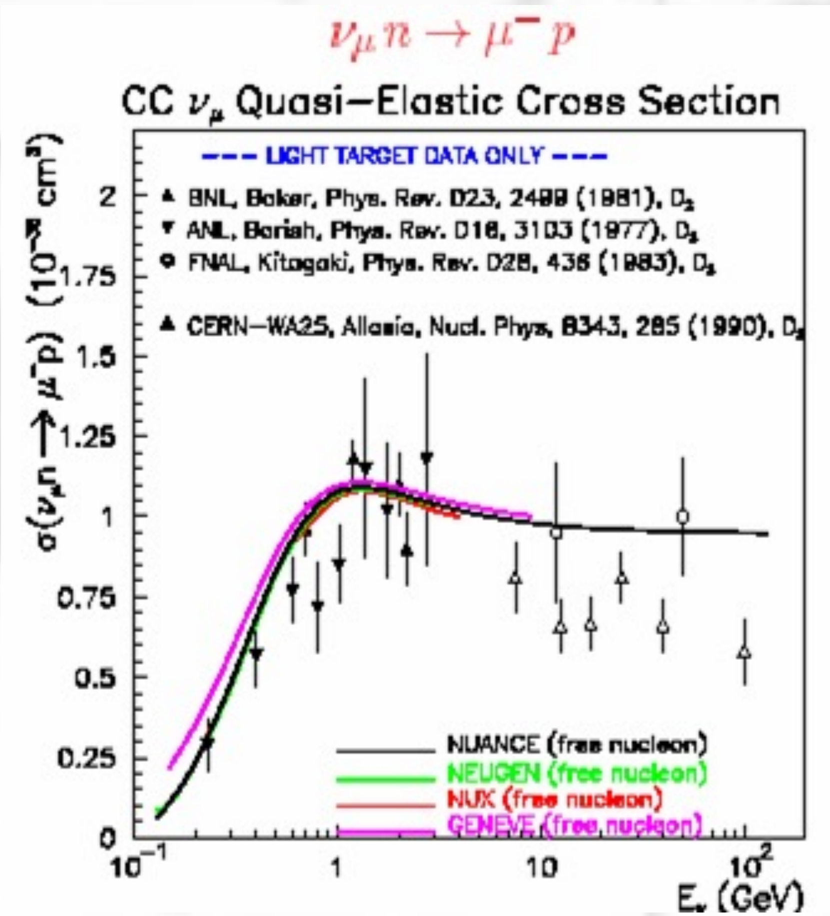


(Quasi-)Elastic Scattering



- Elastic scattering leaves a single nucleon in the final state
  - CC quasielastic *("quasi" since neutrons are in nuclei)* is easier to observe

$$\nu n \to l^- p$$
$$\bar{\nu} p \to l^+ n$$
$$\overset{(-)}{\nu} N \to \overset{(-)}{\nu} N$$



- State of data on "free-ish" neutrons (D_2) is marginal
  - No free neutrons implies nuclear corrections
  - Low energy statistics poor
- Cross-section is calculable
  - But depends on incalculable form-factors of the nucleon
- Theoretically and experimentally constant at high energy
  - 1 GeV^2 is ~ a limit in Q^2