中微子：质量与振荡

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二零二一年卓越创新中心中微子暑期学校，2021.08.20——28

Preliminary statistics: 65 years of ν-physics

1998.06

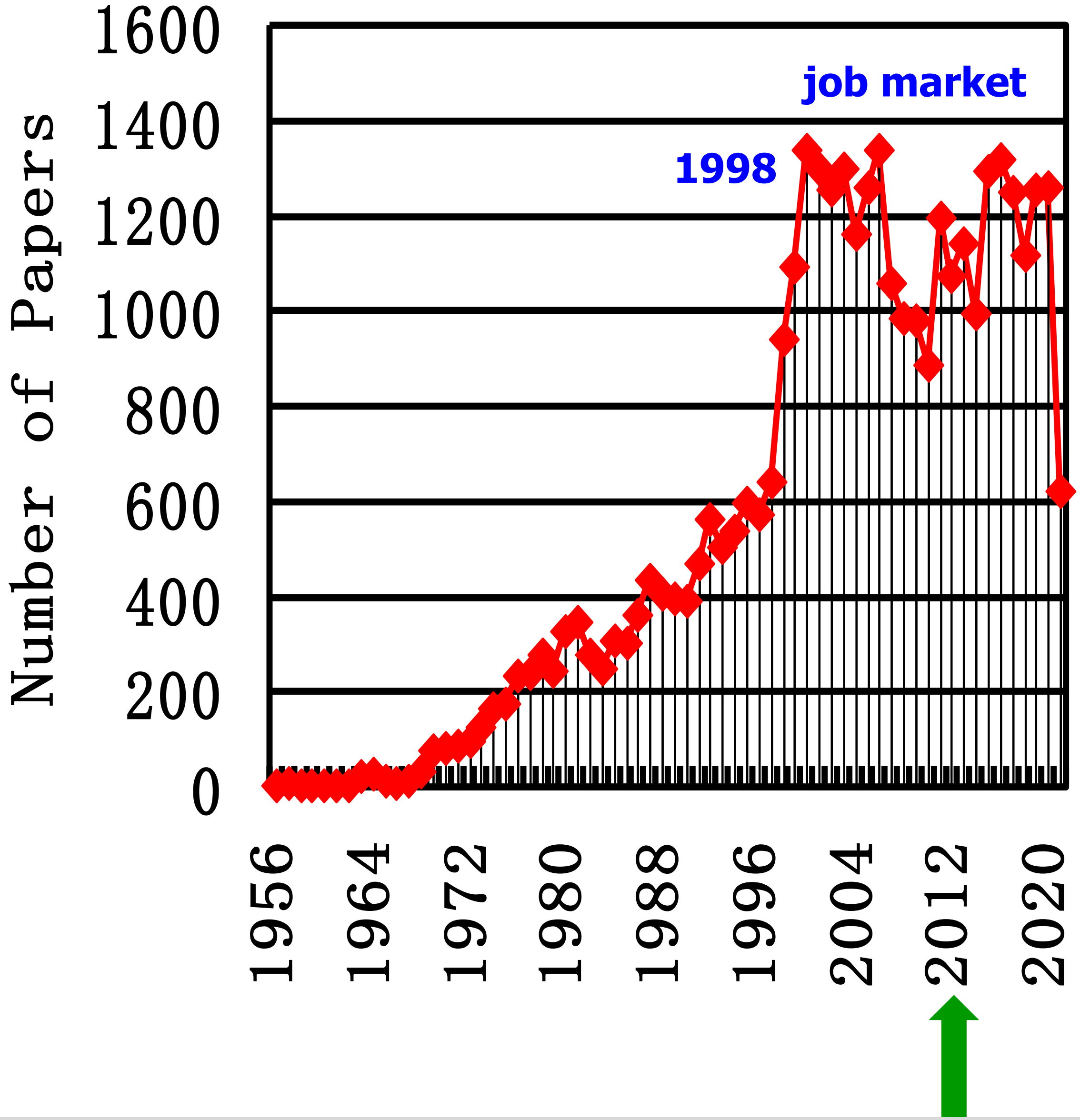
neutrino oscillation

Nobel 2015

1998.12

dark energy

Nobel 2011



NEUTRINOS

1933
β-decay EFT

1957
parity violation

1967
standard model

1973
neutral current

1998
ν-oscillations

INSPIRE: find title NEUTRINO and date XX **> 28 000 Papers**

We have known quite a lot ....

		Normal Ordering (best fit)		Inverted Ordering ($\Delta\chi^2 = 7.1$)	
		bfp $\pm 1\sigma$	3σ range	bfp $\pm 1\sigma$	3σ range
with SK atmospheric data	$\sin^2\theta_{12}$	$0.304^{+0.012}_{-0.012}$	$0.269 \to 0.343$	$0.304^{+0.013}_{-0.012}$	$0.269 \to 0.343$
	$\theta_{12}/^\circ$	$33.44^{+0.77}_{-0.74}$	$31.27 \to 35.86$	$33.45^{+0.78}_{-0.75}$	$31.27 \to 35.87$
	$\sin^2\theta_{23}$	$0.573^{+0.016}_{-0.020}$	$0.415 \to 0.616$	$0.575^{+0.016}_{-0.019}$	$0.419 \to 0.617$
	$\theta_{23}/^\circ$	$49.2^{+0.9}_{-1.2}$	$40.1 \to 51.7$	$49.3^{+0.9}_{-1.1}$	$40.3 \to 51.8$
	$\sin^2\theta_{13}$	$0.02219^{+0.00062}_{-0.00063}$	$0.02032 \to 0.02410$	$0.02238^{+0.00063}_{-0.00062}$	$0.02052 \to 0.02428$
	$\theta_{13}/^\circ$	$8.57^{+0.12}_{-0.12}$	$8.20 \to 8.93$	$8.60^{+0.12}_{-0.12}$	$8.24 \to 8.96$
	$\delta_{CP}/^\circ$	197^{+27}_{-24}	$120 \to 369$	282^{+26}_{-30}	$193 \to 352$
	$\frac{\Delta m^2_{21}}{10^{-5}\ eV^2}$	$7.42^{+0.21}_{-0.20}$	$6.82 \to 8.04$	$7.42^{+0.21}_{-0.20}$	$6.82 \to 8.04$
	$\frac{\Delta m^2_{3\ell}}{10^{-3}\ eV^2}$	$+2.517^{+0.026}_{-0.028}$	$+2.435 \to +2.598$	$-2.498^{+0.028}_{-0.028}$	$-2.581 \to -2.414$

A global fit by I. Esteban et al (2007.14792); also by F. Capozzi et al (2107.00532).

Experimental open questions: absolute neutrino mass, mass ordering; CP violation in oscillations; Majorana nature (phases); extra species...

Theoretical open questions: flavor structures of massive neutrinos, ...

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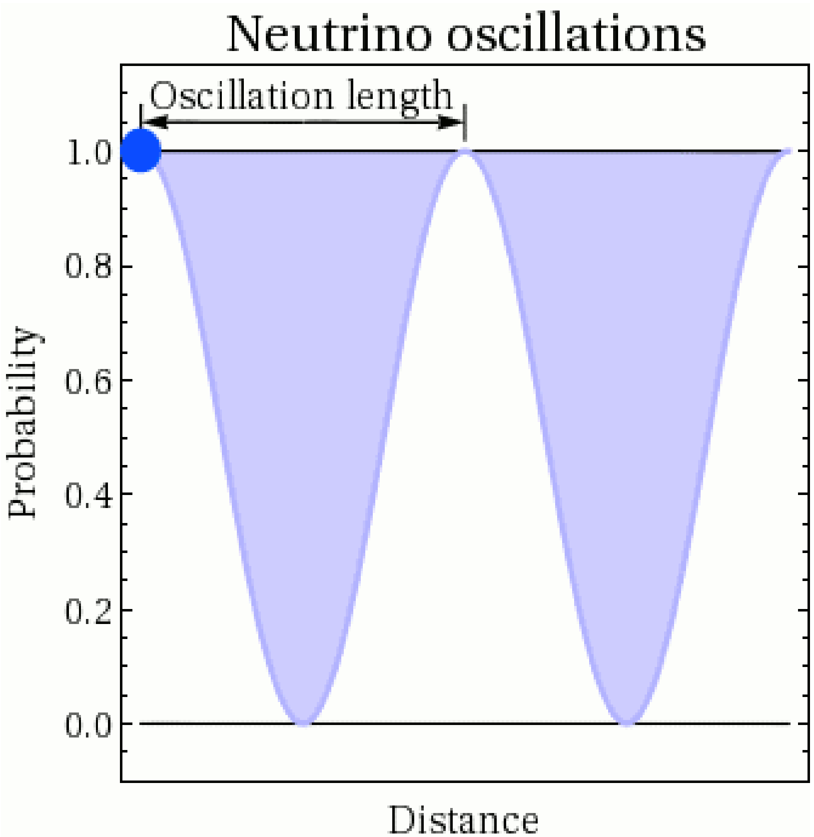


1.1. 振荡公式的推导（真空）

* 中微子的产生与探测，通过带电流相互作用：$-\mathcal{L}_{cc} = \frac{g}{\sqrt{2}} \overline{(e \ \ \mu \ \ \tau)_L} \gamma^\mu \begin{pmatrix}\nu_e\\ \nu_\mu\\ \nu_\tau\end{pmatrix}_L W^-_\mu + h.c.$

* 中微子质量态与相互作用态不匹配，互为线性叠加态（量子叠加）

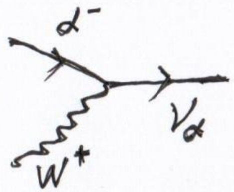
$$\begin{pmatrix}\nu_e\\ \nu_\mu\\ \nu_\tau\end{pmatrix}_L = \begin{pmatrix} U_{e1} & U_{e2} & U_{e3}\\ U_{\mu 1} & U_{\mu 2} & U_{\mu 3}\\ U_{\tau 1} & U_{\tau 2} & U_{\tau 3}\end{pmatrix} \begin{pmatrix}\nu_1\\ \nu_2\\ \nu_3\end{pmatrix}_L \Longrightarrow -\mathcal{L}_{cc} = \frac{g}{\sqrt{2}} \overline{(e \ \ \mu \ \ \tau)_L} \gamma^\mu U \begin{pmatrix}\nu_1\\ \nu_2\\ \nu_3\end{pmatrix}_L W^-_\mu + h.c.$$

相互作用态（flavor 态）　　PMNS 味混合矩阵　　质量态

质量态　　味混合　　质量态

* 产生 ν_α 中微子束流（$\alpha = e, \mu, \tau$）

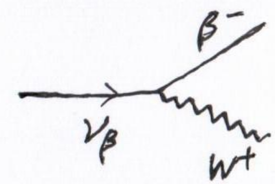
轻子数守恒



$$|\nu_\alpha\rangle = \sum_{i=1}^{3} U^*_{\alpha i} |\nu_i\rangle$$

* 探测 ν_β 中微子束流（$\beta = e, \mu, \tau$）

轻子数守恒



$$|\nu_\beta\rangle = \sum_{j=1}^{3} U^*_{\beta j} |\nu_j\rangle$$

传播

$x = 0, \ t = 0$　　　　$x = L, \ t = \frac{L}{c}$

中微子质量态在空间以极其接近光速的速度传播，可取"平面波近似"。

平面波近似：$e^{i(pL-E_it)/\hbar}$

ν_i中微子在起跑线处的动量相同

能量 $E_i=\sqrt{p^2c^2+m_i^2c^4}\approx pc+\frac{m_i^2c^3}{2p}$

中微子束流以极端相对论状态运动，$p\gg m_ic$，$t=\frac{L}{c}$，中微子束流的平均能量 $E=pc$。

$$e^{i(pL-E_it)/\hbar}=e^{-i\frac{m_i^2c^3}{2\hbar E}L}$$

中微子质量态 ν_i全空间直线跑出相位，使量子相干成为可能！

中微子源 $L=0$ —— 三缝相干过程 —— 探测器处 $L\neq 0$

$$|\nu_\alpha\rangle = U_{\alpha1}^*|\nu_1\rangle + U_{\alpha2}^*|\nu_2\rangle + U_{\alpha3}^*|\nu_3\rangle \;\longrightarrow\; U_{\alpha1}^*e^{-i\frac{m_1^2c^3}{2\hbar E}L}|\nu_1\rangle + U_{\alpha2}^*e^{-i\frac{m_2^2c^3}{2\hbar E}L}|\nu_2\rangle + U_{\alpha3}^*e^{-i\frac{m_3^2c^3}{2\hbar E}L}|\nu_3\rangle$$

形成 $|\nu_\beta\rangle=\sum_{j=1}^{3}U_{\beta j}^*|\nu_j\rangle$ 的概率 (flavor oscillation)

振荡振幅：$A(\nu_\alpha\to\nu_\beta)=\langle\nu_\beta|\nu_\alpha(L)\rangle=\left(\sum_{j=1}^{3}U_{\beta j}\langle\nu_j|\right)\left(\sum_{i=1}^{3}U_{\alpha i}^*e^{-i\frac{m_i^2c^3}{2\hbar E}L}|\nu_i\rangle\right)$

$$=\sum_{i=1}^{3}U_{\alpha i}^*U_{\beta i}e^{-i\frac{m_i^2c^3}{2\hbar E}L}$$

振荡概率：$P(\nu_\alpha\to\nu_\beta)=|A(\nu_\alpha\to\nu_\beta)|^2=\left|\sum_{i=1}^{3}U_{\alpha i}^*U_{\beta i}e^{-i\frac{m_i^2c^3}{2\hbar E}L}\right|^2$

$$=\delta_{\alpha\beta}-4\sum_{i<j}^{3}\mathrm{Re}(U_{\alpha i}U_{\beta j}U_{\alpha j}^*U_{\beta i}^*)\sin^2\frac{\Delta m_{ji}^2c^3}{4\hbar E}L+2\sum_{i<j}^{3}\mathrm{Im}(U_{\alpha i}U_{\beta j}U_{\alpha j}^*U_{\beta i}^*)\sin\frac{\Delta m_{ji}^2c^3}{2\hbar E}L$$

$\Delta m_{ji}^2\equiv m_j^2-m_i^2$

CP守恒项　　CP破坏项

$$P(\nu_\alpha\to\nu_\beta)=\delta_{\alpha\beta}-4\sum_{i<j}^{3}\mathrm{Re}(U_{\alpha i}U_{\beta j}U_{\alpha j}^{*}U_{\beta i}^{*})\sin^2\frac{\Delta m_{ji}^2c^3}{4\hbar E}L+2\sum_{i<j}^{3}\mathrm{Im}(U_{\alpha i}U_{\beta j}U_{\alpha j}^{*}U_{\beta i}^{*})\sin\frac{\Delta m_{ji}^2c^3}{2\hbar E}L$$

结果(1)：振荡项中的1.27因子来自单位换算。

取 $c\approx 2.998\times10^{5}\ \mathrm{km\cdot s^{-1}}$，$\hbar\approx 6.582\times10^{-25}\ \mathrm{GeV\cdot s}\approx 0.1973\ \mathrm{eV^{2}\cdot GeV^{-1}\cdot km}$

可得 $\frac{c^3}{4\hbar}\approx\frac{1}{4\times0.1973}\approx1.267\approx1.27$ （据说是Peter Minkowski最先做

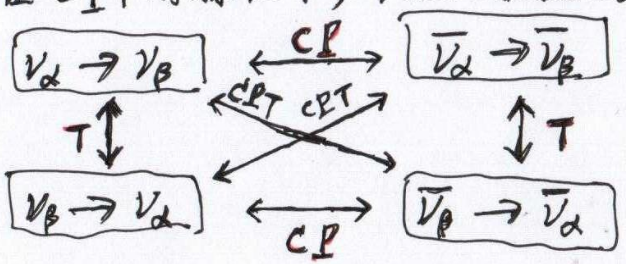
对应 Δm_{ji}^2 的单位 eV^2，L的单位km，E的单位GeV。

结果(2)：消失型(disappearance)实验，$\alpha=\beta$，CP破坏项消失。 $P(\bar\nu_\alpha\to\bar\nu_\alpha)=P(\nu_\alpha\to\nu_\alpha)$

$$\left.\begin{aligned}P(\nu_\alpha\to\nu_\alpha)=1&-4|U_{\alpha1}|^2|U_{\alpha2}|^2\sin^2\frac{\Delta m_{21}^2}{4E}L\\&-4|U_{\alpha1}|^2|U_{\alpha3}|^2\sin^2\frac{\Delta m_{31}^2}{4E}L\\&-4|U_{\alpha2}|^2|U_{\alpha3}|^2\sin^2\frac{\Delta m_{32}^2}{4E}L\end{aligned}\right\}$$ 先取自然单位制

结果(3)：出现型(appearance)实验，$\alpha\neq\beta$，原则上存在CP破坏效应。

在CPT对称性下，中微子振荡的CP、T和CPT变换如下：



$$P(\nu_\alpha\to\nu_\beta)-P(\bar\nu_\alpha\to\bar\nu_\beta)=P(\nu_\alpha\to\nu_\beta)-P(\nu_\beta\to\nu_\alpha)$$

$$=16J\sum_{\gamma}\epsilon_{\alpha\beta\gamma}\sin\frac{\Delta m_{21}^2}{4E}L\sin\frac{\Delta m_{31}^2}{4E}L\sin\frac{\Delta m_{32}^2}{4E}L$$

↑

Jarlskog不变量：$J=\sin\theta_{12}\cos\theta_{12}\sin\theta_{23}\cos\theta_{23}\sin\theta_{13}\cos^2\theta_{13}\sin\delta$

结果(4)：振荡长度(oscillation length)与刷洗距离(wash-out distance)

振荡项 $\sin^2\frac{\Delta m_{ji}^2}{4E}L$ $\Longrightarrow$ 定义 $\frac{\Delta m_{ji}^2}{4E}L_0=\pi$，则 $L_0=\frac{4\pi E}{\Delta m_{ji}^2}$ 为振荡长度 $\Longrightarrow$ $\sin^2\frac{\Delta m_{ji}^2}{4E}L=\sin^2\left(\pi\frac{L}{L_0}\right)$

※ 以反应堆中微子振荡为例(转算到现实单位制)：

$$\sin^2\frac{\Delta m_{ji}^2}{4E}L \longrightarrow \sin^2\left(1.27\frac{\Delta m_{ji}^2}{E}L\right)=\sin^2\left(\pi\frac{L}{L_0}\right)，则\ L_0=\frac{\pi E}{1.27\,\Delta m_{ji}^2}$$

取 $E\sim 4\times10^{-3}$ GeV为例，(反应堆情形)

- 当 $\Delta m_{21}^2=7.5\times10^{-5}\ \mathrm{eV}^2$ 时，$L_0^{(21)}\simeq 1.3\times10^2$ km
- 当 $\Delta m_{31}^2=2.4\times10^{-3}\ \mathrm{eV}^2$ 时，$L_0^{(31)}\simeq 4.1$ km

※ 一般结论：若 $L\ll L_0$，振荡不起来；若 $L\gg L_0$，振荡模式将被刷洗掉。

对于实验而言，中微子束流能量具有一定程度的分布(spread)，$E'=E+\frac{\Delta E}{2}$ 与 E 相比若产生相位差 π，则对振荡项求平均会导致其归零。

令 L_{wod} 为刷洗距离，L_0' 为对应 E' 的振荡长度，则 $L_0'=L_0\left(1+\frac{\Delta E}{2E}\right)$ 成立。

利用 $2\pi\frac{L_{wod}}{L_0'}=2\pi\frac{L_{wod}}{L_0}-\pi$，可推出 $L_{wod}=\frac{E}{\Delta E}L_0\gg L_0$.

此时味转化仍可发生，但已无与距离相关的振荡！

$$P(\nu_\alpha\to\nu_\beta)=P(\bar\nu_\alpha\to\bar\nu_\beta)=\sum_{i=1}^{3}|U_{\alpha i}|^2|U_{\beta i}|^2$$

超高能宇宙线中微子

小练习：请利用第③页最上面的振荡公式证明此式。

举例：大气中微子振荡

两味近似振荡公式：

$$P(\nu_\mu \to \nu_\mu) \simeq 1 - \sin^2 2\theta_{23} \sin^2 \left(1.27 \frac{\Delta m^2_{32} L}{E}\right)$$

$\Delta m^2_{32} \simeq 2.4 \times 10^{-3}\ \mathrm{eV}^2$

L in unit of km

$E \sim \mathcal{O}(1)$ GeV

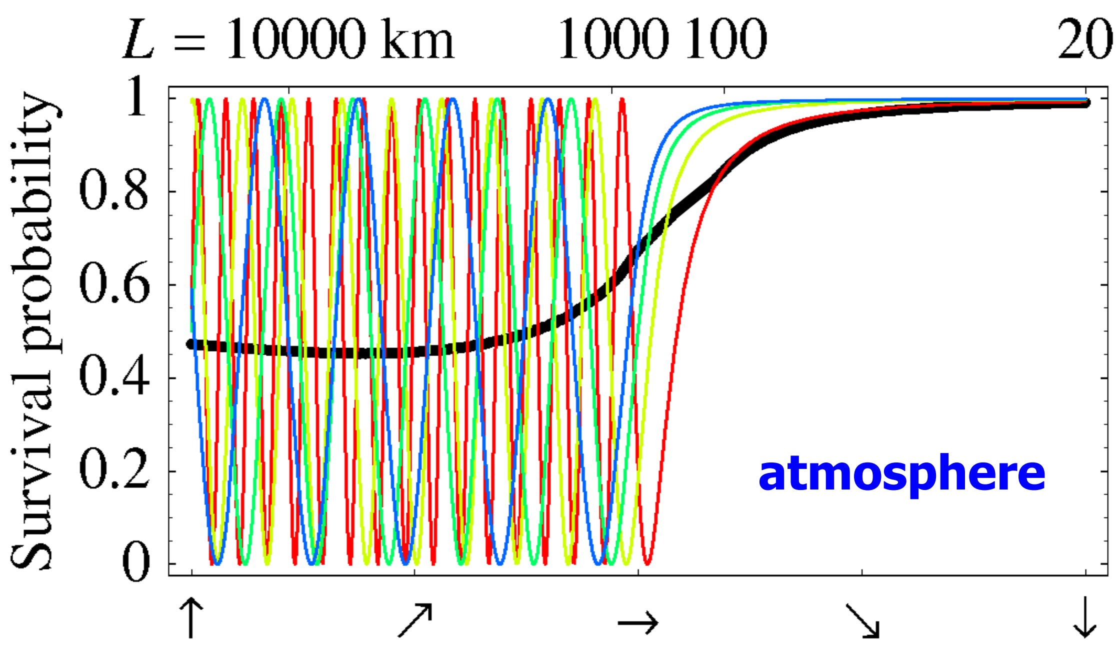
$\theta_{23} \simeq 45^\circ$

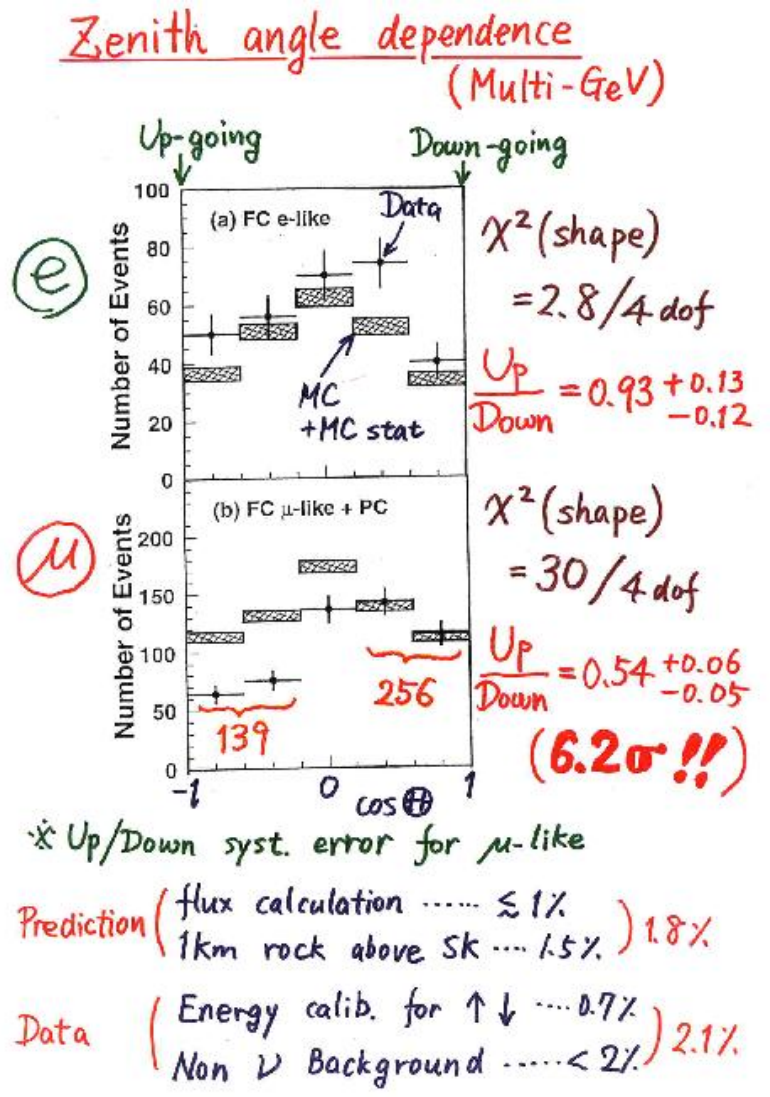
振荡长度~1000 km

T. Kajita

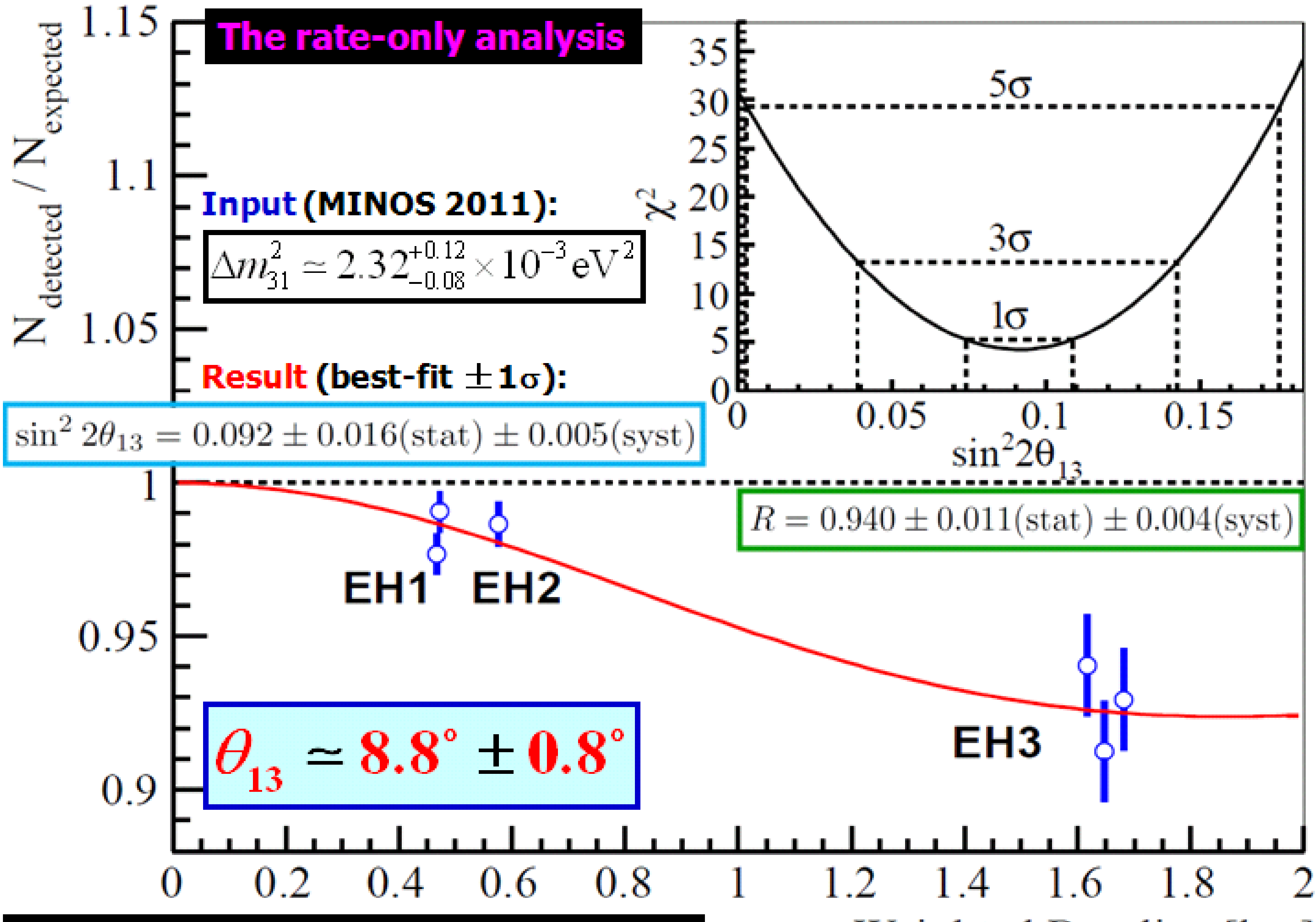
June 5, 1998

小练习：在**两味近似**下利用前页最下端的公式理解大气中微子振荡行为（提示：取PMNS的2×2参数化）。





1.2 大亚湾中微子失踪之谜



arXiv:1203.1669 (submitted to PRL)

1.2. 大亚湾中微子失踪之谜

2012年3月8日，大亚湾实验观测到约6%的反应堆$\bar{\nu}_e$在远点探测器失踪。它们转化为对探测器不敏感的$\bar{\nu}_\mu$与$\bar{\nu}_\tau$，但具体转化概率的计算颇为复杂。

2021年6月，在给国科大本科生出《粒子物理学基础》期末考题时，我终于找到一个简洁的、容易被理解的途径解答这一失踪之谜。

$$P(\bar{\nu}_e \to \bar{\nu}_\mu) = \frac{|U_{\mu 3}|^2}{1-|U_{e3}|^2}\left[1 - P(\bar{\nu}_e \to \bar{\nu}_e)\right] = \sin^2\theta_{23}\left[1 - P(\bar{\nu}_e \to \bar{\nu}_e)\right]$$

$$P(\bar{\nu}_e \to \bar{\nu}_\tau) = \frac{|U_{\tau 3}|^2}{1-|U_{e3}|^2}\left[1 - P(\bar{\nu}_e \to \bar{\nu}_e)\right] = \cos^2\theta_{23}\left[1 - P(\bar{\nu}_e \to \bar{\nu}_e)\right]$$

于是我陷入了沉思

证明如下： 从1.1的一般振荡公式出发：

$$P(\bar{\nu}_\alpha \to \bar{\nu}_\beta) = \delta_{\alpha\beta} - 4\sum_{i<j}^{3} \mathrm{Re}(U_{\alpha i}U_{\beta j}U^*_{\alpha j}U^*_{\beta i})\sin^2\frac{\Delta m^2_{ji}}{4E}L - 2\sum_{i<j}^{3}\mathrm{Im}(U_{\alpha i}U_{\beta j}U^*_{\alpha j}U^*_{\beta i})\sin\frac{\Delta m^2_{ji}}{2E}L$$

对于短基线的大亚湾$\bar{\nu}_e \to \bar{\nu}_e$振荡实验而言，即$L \sim 2\,\mathrm{km}$，对照 $\begin{cases} L_0^{(21)} \sim 1.3\times 10^2\ \mathrm{km} \\ L_0^{(31)} \sim L_0^{(32)} \sim 4.1\ \mathrm{km} \end{cases}$

可知 Δm^2_{21} 主导的振荡项几乎可忽略不计：

$$\sin^2\frac{\Delta m^2_{21}}{4E}L = \sin^2\left(\pi\frac{L}{L_0^{(21)}}\right) \sim 0。$$

因此取 $\Delta m^2_{21} \to 0$ 即 $\Delta m^2_{31} = \Delta m^2_{32}$ 是很好的近似！

于是

$$P(\bar{\nu}_e \to \bar{\nu}_\alpha) = \delta_{e\alpha} - 4\,\mathrm{Re}(U_{e1} \underline{U_{\alpha 3} U_{e3}^*} U_{\alpha 1}^*) \sin^2 \frac{\Delta m_{31}^2}{4E} L$$

$$- 4\,\mathrm{Re}(U_{e2} \underline{U_{\alpha 3} U_{e3}^*} U_{\alpha 2}^*) \sin^2 \frac{\Delta m_{32}^2}{4E} L$$

取 $\Delta m_{31}^2 = \Delta m_{32}^2$

$$= \delta_{e\alpha} - 4\,\mathrm{Re}[U_{\alpha 3} U_{e3}^* (U_{e1} U_{\alpha 1}^* + U_{e2} U_{\alpha 2}^*)] \sin^2 \frac{\Delta m_{31}^2}{4E} L$$

$$= \delta_{e\alpha} - 4\,\mathrm{Re}[U_{\alpha 3} U_{e3}^* (\delta_{e\alpha} - U_{e3} U_{\alpha 3}^*)] \sin^2 \frac{\Delta m_{31}^2}{4E} L$$

即

$$\begin{cases} P(\bar{\nu}_e \to \bar{\nu}_e) = 1 - 4|U_{e3}|^2 (1 - |U_{e3}|^2) \sin^2 \frac{\Delta m_{31}^2}{4E} L \\ P(\bar{\nu}_e \to \bar{\nu}_\mu) = 4|U_{e3}|^2 |U_{\mu 3}|^2 \sin^2 \frac{\Delta m_{31}^2}{4E} L \\ P(\bar{\nu}_e \to \bar{\nu}_\tau) = 4|U_{e3}|^2 |U_{\tau 3}|^2 \sin^2 \frac{\Delta m_{31}^2}{4E} L \end{cases}$$

针对每一确定的 L，代换掉振荡项。

得

$$\begin{cases} P(\bar{\nu}_e \to \bar{\nu}_\mu) = \frac{|U_{\mu 3}|^2}{1 - |U_{e3}|^2} [1 - P(\bar{\nu}_e \to \bar{\nu}_e)] = \sin^2\theta_{23} [1 - P(\bar{\nu}_e \to \bar{\nu}_e)] \approx 3.42\% \\ P(\bar{\nu}_e \to \bar{\nu}_\tau) = \frac{|U_{\tau 3}|^2}{1 - |U_{e3}|^2} [1 - P(\bar{\nu}_e \to \bar{\nu}_e)] = \cos^2\theta_{23} [1 - P(\bar{\nu}_e \to \bar{\nu}_e)] \approx 2.58\% \end{cases}$$

其中用到标准参数化：$|U_{e3}|^2 = s_{13}^2$，$|U_{\mu 3}|^2 = c_{13}^2 s_{23}^2$，$|U_{\tau 3}|^2 = c_{13}^2 c_{23}^2$。

取目前的最佳拟合值 $\theta_{23} \approx 49^\circ$，即 $\sin^2\theta_{23} \approx 0.57$，以及 $1 - P(\bar{\nu}_e \to \bar{\nu}_e) \approx 6\%$，可得上述近似数值估算结果。

1.3 江门中微子质量顺序：测量原理

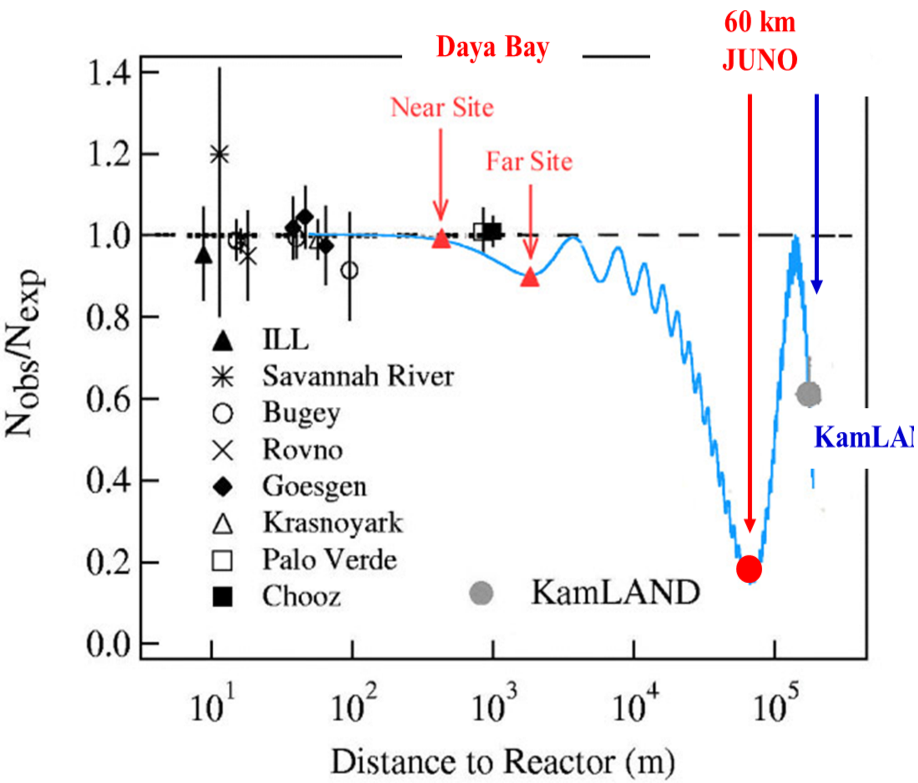
Accelerator/atmospheric: terrestrial matter effects play crucial roles.

$$\Delta m^2_{31} \mp 2\sqrt{2}\ G_F N_e E$$

T2K, NOνA, SK, DUNE, HK, ...

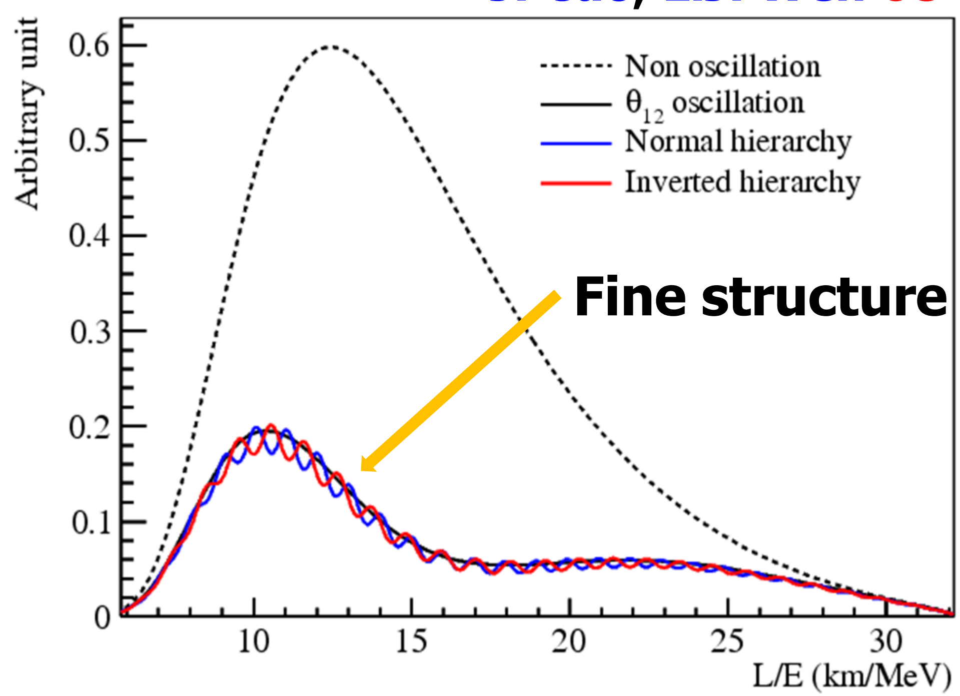
Normal mass ordering is favored over the inverted one at the 3σ level.

Reactor (JUNO): Optimum baseline at the valley of Δm^2_{21} oscillations, corrected by fine structures of Δm^2_{31} oscillations.



JUNO's idea

L. Zhan, Y.F. Wang, J. Cao, L.J. Wen 08



1.3. 江门中微子质量等级

反应堆 $\bar{\nu}_e \to \bar{\nu}_e$ 真空振荡公式：如何看出江门实验对 Δm^2_{31} 和 Δm^2_{32} 的符号的敏感性？

$$P(\bar{\nu}_e \to \bar{\nu}_e) = 1 - 4|U_{e1}|^2|U_{e2}|^2\sin^2\frac{\Delta m^2_{21}}{4E}L - 4|U_{e1}|^2|U_{e3}|^2\sin^2\frac{\Delta m^2_{31}}{4E}L - 4|U_{e2}|^2|U_{e3}|^2\sin^2\frac{\Delta m^2_{32}}{4E}L$$

关键点：$\Delta m^2_{21} \ll |\Delta m^2_{31}| \approx |\Delta m^2_{32}|$，因此无论正顺序（$m_1 < m_2 < m_3$）或倒顺序（$m_3 < m_1 < m_2$），$\Delta m^2_{31}$ 与 Δm^2_{32} 必同号！故而我们只需测量 $\Delta m^2_{31} + \Delta m^2_{32}$ 的符号！

选定：

$$\begin{cases} \Delta_- \equiv (\Delta m^2_{31} - \Delta m^2_{32})\dfrac{L}{4E} = \Delta m^2_{21}\dfrac{L}{4E} > 0 \\ \Delta_+ \equiv (\Delta m^2_{31} + \Delta m^2_{32})\dfrac{L}{4E} \quad \text{符号未定} \end{cases} \Longrightarrow \begin{cases} \dfrac{\Delta m^2_{31}}{4E}L = \dfrac{1}{2}(\Delta_+ + \Delta_-) \\ \dfrac{\Delta m^2_{32}}{4E}L = \dfrac{1}{2}(\Delta_+ - \Delta_-) \end{cases}$$

代入上式：

$$P(\bar{\nu}_e \to \bar{\nu}_e) = 1 - 4|U_{e1}|^2|U_{e2}|^2\sin^2\Delta_- - 4|U_{e3}|^2\left[|U_{e1}|^2\sin^2\frac{\Delta_+ + \Delta_-}{2} + |U_{e2}|^2\sin^2\frac{\Delta_+ - \Delta_-}{2}\right]$$

其中

$$\begin{aligned}
[\ \ ] &= \frac{1}{2}|U_{e1}|^2[1 - \cos(\Delta_+ + \Delta_-)] + \frac{1}{2}|U_{e2}|^2[1 - \cos(\Delta_+ - \Delta_-)] \\
&= \frac{1}{2}(|U_{e1}|^2 + |U_{e2}|^2) - \frac{1}{2}[|U_{e1}|^2\cos\Delta_+\cos\Delta_- - |U_{e1}|^2\sin\Delta_+\sin\Delta_- + |U_{e2}|^2\cos\Delta_+\cos\Delta_- + |U_{e2}|^2\sin\Delta_+\sin\Delta_-] \\
&= \frac{1}{2}(1 - |U_{e3}|^2)(1 - \cos\Delta_+\cos\Delta_-) + \frac{1}{2}(|U_{e1}|^2 - |U_{e2}|^2)\sin\Delta_+\sin\Delta_-
\end{aligned}$$

其中

$$\begin{aligned}
1 - \cos\Delta_+\cos\Delta_- &= 1 - \frac{1}{2}[\cos(\Delta_+ + \Delta_-) + \cos(\Delta_+ - \Delta_-)] \\
&= 1 - \frac{1}{2}\left[\cos 2\frac{\Delta m^2_{31}}{4E}L + \cos 2\frac{\Delta m^2_{32}}{4E}L\right] = \sin^2\frac{\Delta m^2_{31}}{4E}L + \sin^2\frac{\Delta m^2_{32}}{4E}L
\end{aligned}$$

代入，得 $P(\bar{\nu}_e \to \bar{\nu}_e) = 1 - 4|U_{e1}|^2|U_{e2}|^2 \sin^2\frac{\Delta m^2_{21}}{4E}L$

$$- 2|U_{e3}|^2(1-|U_{e3}|^2)\left[\sin^2\frac{\Delta m^2_{31}}{4E}L + \sin^2\frac{\Delta m^2_{32}}{4E}L\right]$$

$$- 2|U_{e3}|^2(|U_{e1}|^2-|U_{e2}|^2)\sin\frac{\Delta m^2_{21}}{4E}L\sin\frac{\Delta m^2_{31}+\Delta m^2_{32}}{4E}L$$

取标准参数化 $|U_{e1}| = c_{13}c_{12}$，$|U_{e2}| = c_{13}s_{12}$，$|U_{e3}| = s_{13}$，则

$$P(\bar{\nu}_e \to \bar{\nu}_e) = 1 - \sin^2 2\theta_{12}\cos^4\theta_{13}\sin^2\frac{\Delta m^2_{21}}{4E}L \quad \leftarrow \text{KamLAND}$$

$$- \frac{1}{2}\sin^2 2\theta_{13}\left[\sin^2\frac{\Delta m^2_{31}}{4E}L + \sin^2\frac{\Delta m^2_{32}}{4E}L\right] \quad \leftarrow \text{大亚湾}$$

邢 1808.02256

$$- \frac{1}{2}\cos 2\theta_{12}\sin^2 2\theta_{13}\sin\frac{\Delta m^2_{21}}{4E}L\sin\frac{\Delta m^2_{31}+\Delta m^2_{32}}{4E}L \quad \leftarrow \text{江门}$$

该结果具有美妙的 $\theta_{12} \to \theta_{12} - \pi/2$ 与 $m_1 \leftrightarrow m_2$ 变换对称性——一般性质（周顺，1612.03537）

问题(1)：如果存在轻的惰性中微子，可否类似测量质量顺序？

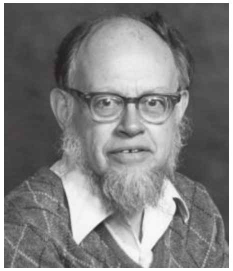
是的，可以！见 1808.02256 简要讨论。

问题(2)：地球物质效应是否应在江门实验中考虑？

是的！但它不影响测量质量顺序的原理和结果！（继续）

1.4 物质效应简介：第一篇论文

PHYSICAL REVIEW D VOLUME 17, NUMBER 9 1 MAY 1978



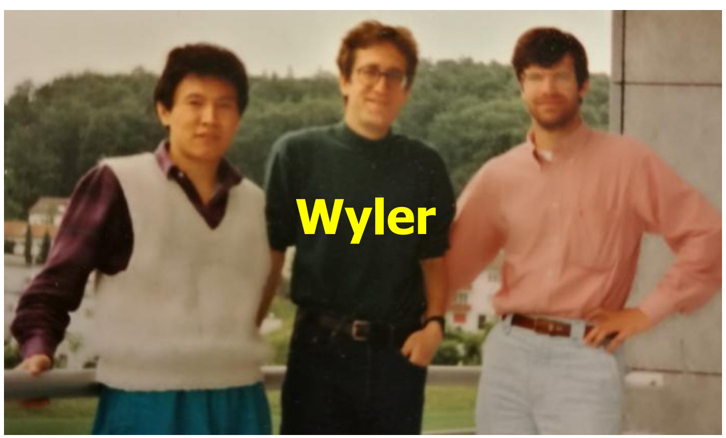
Neutrino oscillations in matter

L. Wolfenstein
Carnegie-Mellon University, Pittsburgh, Pennsylvania 15213
(Received 6 October 1977; revised manuscript received 5 December 1977)

The effect of coherent forward scattering must be taken into account when considering the oscillations of neutrinos traveling through matter. In particular, for the case of massless neutrinos for which vacuum oscillations cannot occur, oscillations can occur in matter if the neutral current has an off-diagonal piece connecting different neutrino types. Applications discussed are solar neutrinos and a proposed experiment involving transmission of neutrinos through 1000 km of rock.

[8]I am indebted to Dr. Daniel Wyler for pointing out the importance of the charged-current terms.

Ref. [8]



ACKNOWLEDGMENTS

I wish to thank E. Zavattini for asking the right question, and J. Ashkin, J. Russ, J. F. Donoghue, L. F. Li, S. Adler, and D. Wyler for discussions. This research was supported in part by the U. S Energy Research and Development Administration.

Lincoln Wolfenstein (2004): I think I have learnt as much from all my students as they have learnt from me.

1.4 物质效应简介：有效哈密顿量（1）

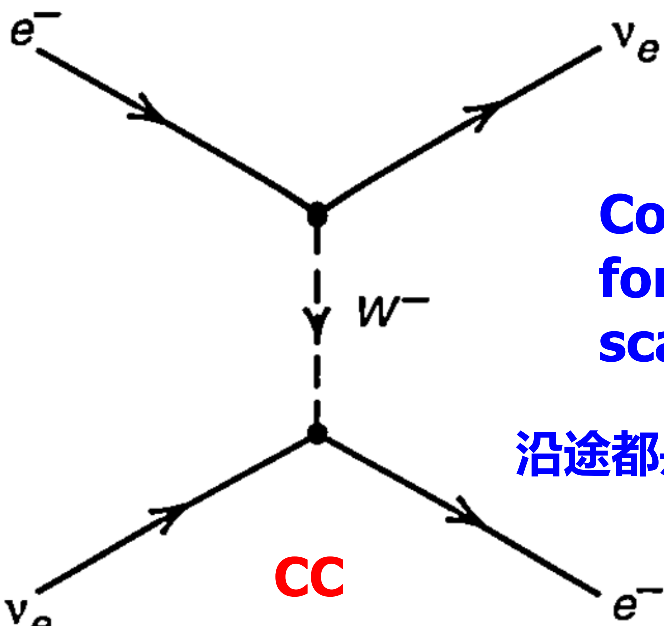
In vacuum the evolution of three neutrino **mass eigenstates** with time

$$\mathrm{i}\frac{\mathrm{d}}{\mathrm{d}t}\begin{pmatrix}\nu_1\\ \nu_2\\ \nu_3\end{pmatrix} = H_0\begin{pmatrix}\nu_1\\ \nu_2\\ \nu_3\end{pmatrix}, \quad H_0 = \frac{1}{2E}\begin{pmatrix}m_1^2 & & \\ & m_2^2 & \\ & & m_3^2\end{pmatrix}, \quad \begin{pmatrix}\nu_e\\ \nu_\mu\\ \nu_\tau\end{pmatrix} = U\begin{pmatrix}\nu_1\\ \nu_2\\ \nu_3\end{pmatrix}$$

In the flavor basis the evolution of three neutrino flavors is described by the Schroedinger-like equation:

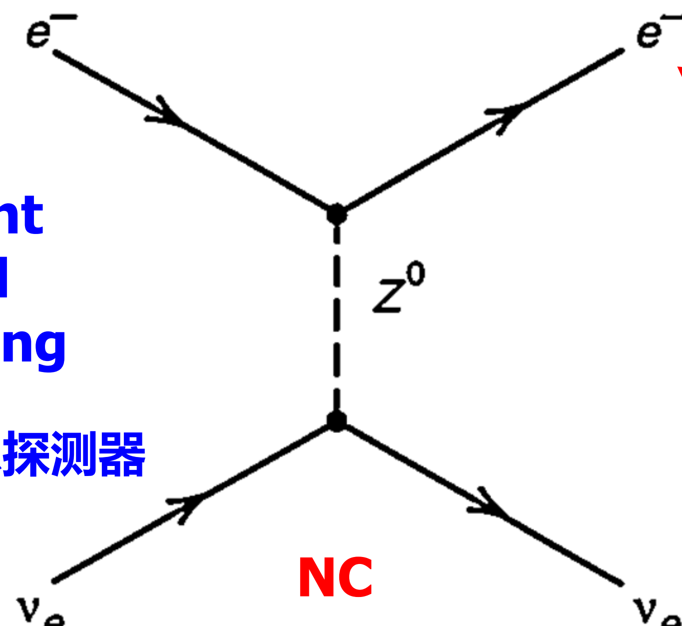
$$\mathrm{i}\frac{\mathrm{d}}{\mathrm{d}t}\begin{pmatrix}\nu_e\\ \nu_\mu\\ \nu_\tau\end{pmatrix} = UH_0U^\dagger\begin{pmatrix}\nu_e\\ \nu_\mu\\ \nu_\tau\end{pmatrix}$$

Propagating in a medium, neutrinos may have CC and NC interactions



Coherent forward scattering

沿途都是理想探测器



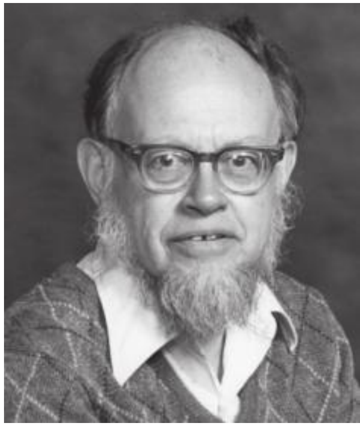
electron replaced by proton or neutron

1.4 物质效应简介：有效哈密顿量（2）

In this case the effective Hamiltonian with a matter potential is

$$H_{\rm m} = U H_0 U^\dagger + \begin{pmatrix} V_{\rm CC} & & \\ & 0 & \\ & & 0 \end{pmatrix} + \begin{pmatrix} V_{\rm NC} & & \\ & V_{\rm NC} & \\ & & V_{\rm NC} \end{pmatrix}$$

from electron **from neutron**



The NC contributions from electrons and protons cancel each other, since we stay with normal matter:

$$\boxed{N_e = N_p}$$

$$V_{\rm CC} = +\sqrt{2}\, G_{\rm F} N_e$$

$$V_{\rm NC}^n = -\frac{1}{\sqrt{2}} G_{\rm F} N_n$$

$$V_{\rm NC}^p = +\frac{1}{\sqrt{2}} G_{\rm F} N_p \left(1 - 4\sin^2\theta_{\rm w}\right)$$

$$V_{\rm NC}^e = -\frac{1}{\sqrt{2}} G_{\rm F} N_e \left(1 - 4\sin^2\theta_{\rm w}\right)$$

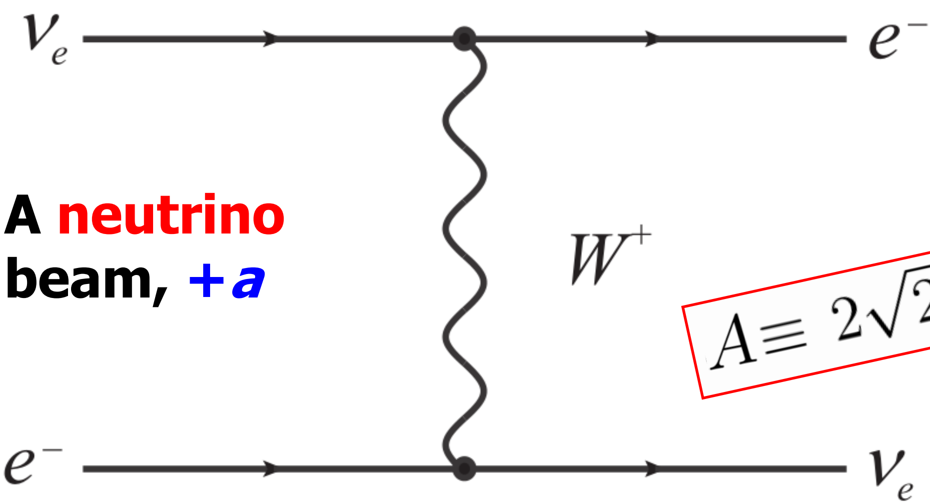
♣ **The NC term is universal for three neutrino flavors, and hence it can be neglected in the standard case.**

♣ **When an antineutrino beam is taken into consideration, the CC and NC terms flip their signs, and simultaneously the flavor mixing matrix *U* needs to be complex conjugated.**

♣ **The NC term should not be ignored if sterile neutrinos are included.**

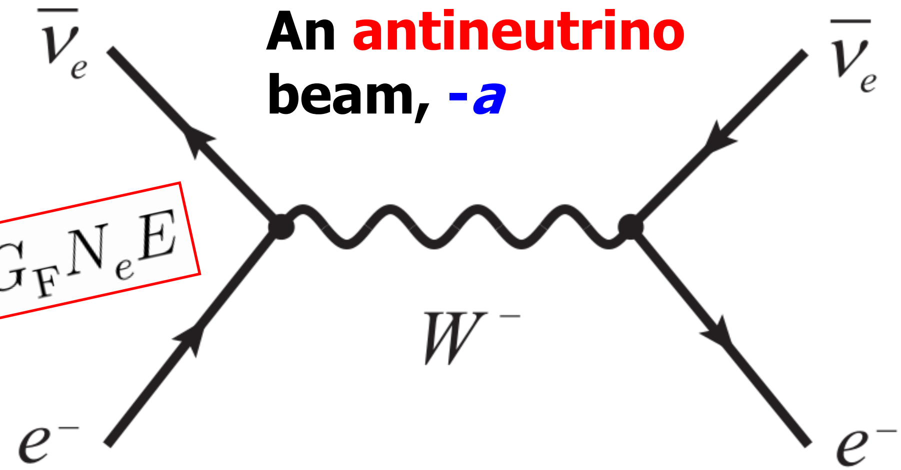
1.4 物质效应简介：有效哈密顿量（3）

★ **The effective Hamiltonian for neutrino oscillations in a medium ---- matter effects are from the CC-induced coherent forward scattering.**



A neutrino beam, +*a*

$$A \equiv 2\sqrt{2}\, G_F N_e E$$



An antineutrino beam, -*a*

scale-like

$$H_{\rm m} = \frac{1}{2E}\left[U\begin{pmatrix} m_1^2 & 0 & 0 \\ 0 & m_2^2 & 0 \\ 0 & 0 & m_3^2 \end{pmatrix}U^\dagger + \begin{pmatrix} A & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}\right] \equiv \frac{1}{2E}V\begin{pmatrix} \widetilde{m}_1^2 & 0 & 0 \\ 0 & \widetilde{m}_2^2 & 0 \\ 0 & 0 & \widetilde{m}_3^2 \end{pmatrix}V^\dagger$$

in vacuum **correction** **in matter**

★ **Using the effective quantities defined in matter, one may write out neutrino oscillation probabilities in the same form as in vacuum!**

1.4 物质效应简介：两味情形举例

The effective Hamiltonian for two-flavor neutrinos in vacuum/matter:

$$\mathcal{H}_{\rm v} = \frac{1}{2E} \begin{pmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{pmatrix} \begin{pmatrix} m_1^2 & 0 \\ 0 & m_2^2 \end{pmatrix} \begin{pmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{pmatrix}$$

$$\mathcal{H}_{\rm m} = \frac{1}{2E} \begin{pmatrix} \cos\tilde{\theta} & \sin\tilde{\theta} \\ -\sin\tilde{\theta} & \cos\tilde{\theta} \end{pmatrix} \begin{pmatrix} \tilde{m}_1^2 & 0 \\ 0 & \tilde{m}_2^2 \end{pmatrix} \begin{pmatrix} \cos\tilde{\theta} & -\sin\tilde{\theta} \\ \sin\tilde{\theta} & \cos\tilde{\theta} \end{pmatrix}$$

$$= \frac{1}{4E} \begin{pmatrix} m_1^2 + m_2^2 - \Delta m^2 \cos 2\theta + 4\sqrt{2} G_{\rm F} N_e E & \Delta m^2 \sin 2\theta \\ \Delta m^2 \sin 2\theta & m_1^2 + m_2^2 + \Delta m^2 \cos 2\theta \end{pmatrix}$$

The 2-flavor approximation is good for solar or atmospheric neutrinos

1.4 物质效应简介：有效混合角与质量平方差

Effective **neutrino mass-squared difference & mixing angle in matter:**

$$\Delta\tilde{m}^2 \sin 2\tilde{\theta} = \Delta m^2 \sin 2\theta$$

$$\tilde{m}_1^2 + \tilde{m}_2^2 + \Delta\tilde{m}^2 \cos 2\tilde{\theta} = m_1^2 + m_2^2 + \Delta m^2 \cos 2\theta$$

$$\tilde{m}_1^2 + \tilde{m}_2^2 - \Delta\tilde{m}^2 \cos 2\tilde{\theta} = m_1^2 + m_2^2 - \Delta m^2 \cos 2\theta + 4\sqrt{2} G_{\rm F} N_e E$$

$$P(\nu_e \to \nu_\mu)_{\rm v} = \sin^2 2\theta \sin^2\left(\frac{1.27\Delta m^2 L}{E}\right)$$

$$P(\nu_e \to \nu_\mu)_{\rm m} = \sin^2 2\tilde{\theta} \sin^2\left(\frac{1.27\Delta\tilde{m}^2 L}{E}\right)$$

The matter density changes for solar neutrinos to travel from the core to the surface

$$\begin{pmatrix} |\nu_e\rangle \\ |\nu_\mu\rangle \end{pmatrix} = \begin{pmatrix} \cos\tilde{\theta} & \sin\tilde{\theta} \\ -\sin\tilde{\theta} & \cos\tilde{\theta} \end{pmatrix} \begin{pmatrix} |\tilde{\nu}_1\rangle \\ |\tilde{\nu}_2\rangle \end{pmatrix}$$

$$\Delta\tilde{m}^2 = \sqrt{\left(\Delta m^2 \cos 2\theta - 2\sqrt{2}\, G_{\rm F} N_e E\right)^2 + \left(\Delta m^2 \sin 2\theta\right)^2}$$

$$\tan 2\tilde{\theta} = \frac{\Delta m^2 \sin 2\theta}{\Delta m^2 \cos 2\theta - 2\sqrt{2}\, G_{\rm F} N_e E}$$

exercise: discuss 2 extreme cases:

$$\tilde{\theta} = \frac{\pi}{2} \text{ or } \frac{\pi}{4}$$

1.4 物质效应简介：MSW共振效应

IL NUOVO CIMENTO VOL. 9 C, N. 1 Gennaio-Febbraio 1986

Resonant Amplification of ν Oscillations in Matter and Solar-Neutrino Spectroscopy.

S. P. MIKHEYEV and A. YU. SMIRNOV

Institute for Nuclear Research of Academy of Sciences
60th October Anniversary prosp. 7a, Moscow 117 342,

(ricevuto il 3 Maggio 1985)

$$\tan 2\tilde{\theta} = \frac{\Delta m^2 \sin 2\theta}{\Delta m^2 \cos 2\theta - 2\sqrt{2}\, G_F N_e E}$$

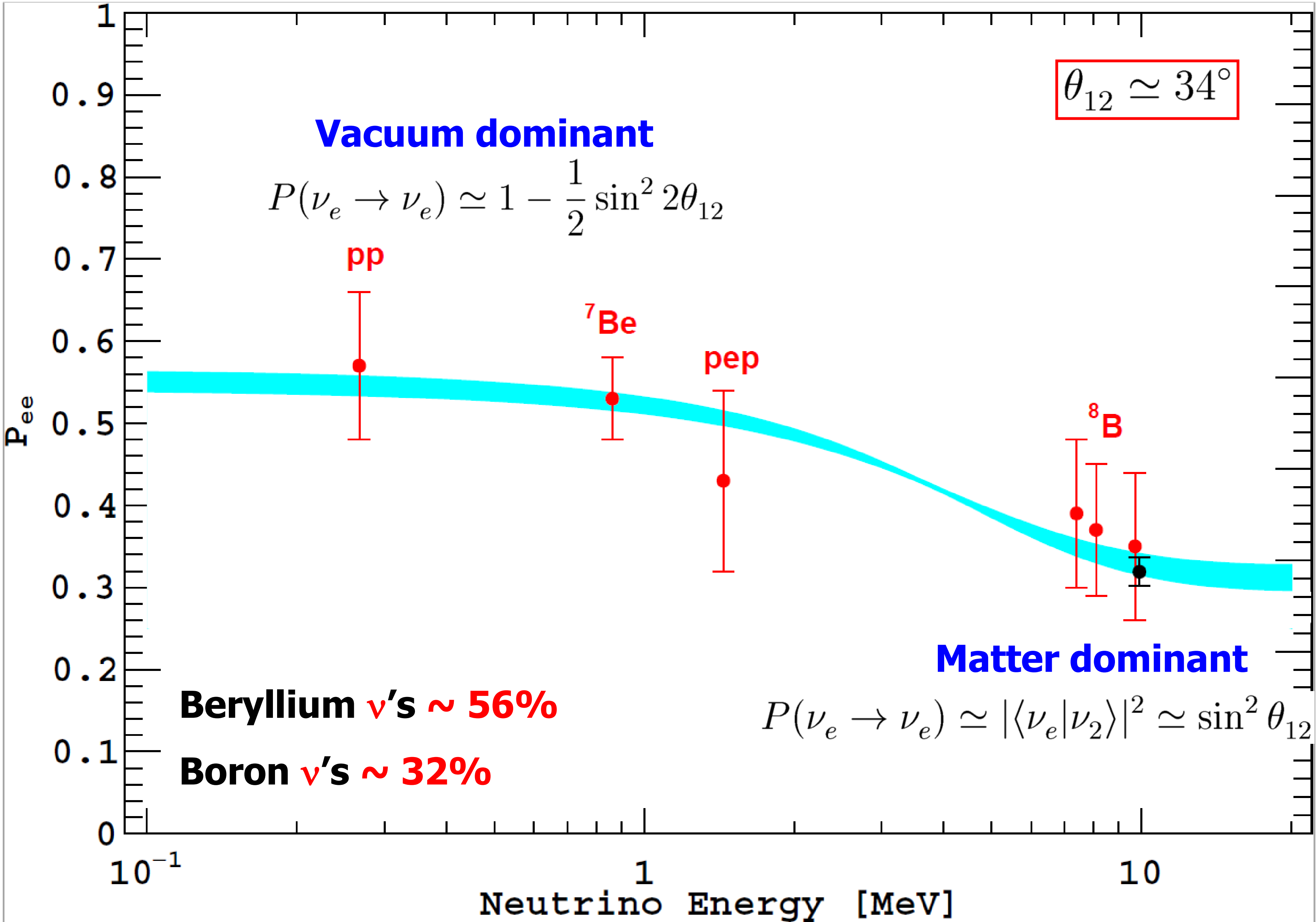
投稿Phys. Lett. B被拒！

G.T. Zatsepin帮忙在意大利期刊上发表（如果错了，会被忘记；如果对了，会很重要！）

Summary. — For small mixing angles θ the amplification of ν oscillations in matter has the resonance form (resonance in neutrino energy or matter density). In the Sun resonance effect results in nontrivial changing (suppression) of ν-flux for a wide range of neutrino parameters $\Delta m^2 = (3 \cdot 10^{-4} \div 10^{-8})\ (\text{eV})^2$, $\sin^2 2\theta > 10^{-4}$.

It is very hard to understand why Wolfenstein ignored the resonance.

太阳中微子与物质效应：实验观测



太阳中微子与物质效应：初步理解

In the two-flavor approximation, the effective Hamiltonian of solar neutrinos is:

$$N_e(0) \approx 6 \times 10^{25}\ \mathrm{cm}^{-3}$$

$$\mathcal{H}_{\rm eff} = \frac{\Delta m^2_{21}}{4E} \begin{bmatrix} -\cos 2\theta_{12} & \sin 2\theta_{12} \\ \sin 2\theta_{12} & \cos 2\theta_{12} \end{bmatrix} + \begin{bmatrix} \sqrt{2} G_{\rm F} N_e(r) & 0 \\ 0 & 0 \end{bmatrix}$$

$7.6 \times 10^{-5}\ \mathrm{eV}^2$

$0.75 \times 10^{-5}\ \mathrm{eV}^2 / \mathrm{MeV}\ (\text{at } r = 0)$

实验上没有探测到太阳中微子振荡行为？！

Be-7 ν's: *E* ~ 0.862 MeV. The vacuum term is dominant. The survival probability on the earth is (for theta_12 ~ 34°):

$$P(\nu_e \to \nu_e) \approx 1 - \frac{1}{2}\sin^2 2\theta_{12} \sim 0.56$$

B-8 ν's: *E* ~ 6 to 7 MeV. The matter term is dominant. The produced ν is roughly ν_e ~ ν_2 (for *V* >0). The ν-propagation from the center to the outer edge of the Sun is approximately adiabatic. That is why it keeps to be ν_2 on the way to the surface (for theta_12 ~ 34°):

$$|\nu_2\rangle \approx \sin\theta_{12}|\nu_e\rangle + \cos\theta_{12}|\nu_\mu\rangle$$

$$P(\nu_e \to \nu_e) = |\langle \nu_e|\nu_2\rangle|^2 = \sin^2\theta_{12} \approx 0.32$$

1.4 物质效应简介：江门实验

Chinese Physics C Vol. 40, No. 9 (2016) 091001

Terrestrial matter effects on reactor antineutrino oscillations at JUNO or RENO-50: how small is small? *

Yu-feng Li(李玉峰)[1)] Yi-fang Wang(王贻芳)[2)] Zhi-zhong Xing(邢志忠)[3)]

[1] Institute of High Energy Physics, Chinese Academy of Sciences, Beijing 100049, China
[2] School of Physical Sciences, University of Chinese Academy of Sciences, Beijing 100049, China

Abstract: We have carefully examined, in both analytical and numerical ways, how small the terrestrial matter effects can be in a given medium-baseline reactor antineutrino oscillation experiment like JUNO or RENO-50. Taking the forthcoming JUNO experiment as an example, we show that the inclusion of terrestrial matter effects may reduce the sensitivity of the neutrino mass ordering measurement by $\Delta\chi^2_{\rm MO} \simeq 0.6$, and a neglect of such effects may shift the best-fit values of the flavor mixing angle θ_{12} and the neutrino mass-squared difference Δ_{21} by about 1σ to 2σ in the future data analysis. In addition, a preliminary estimate indicates that a 2σ sensitivity of establishing the terrestrial matter effects can be achieved for about 10 years of data taking at JUNO with the help of a suitable near detector implementation.

物质效应很小，不影响质量平方差的符号；对混合角有约1%的影响，与江门灵敏度相当

$$\begin{aligned}
\Delta\widetilde{m}^2_{21} &\simeq \Delta m^2_{21} + A\cos 2\theta_{12} \; , \\
\Delta\widetilde{m}^2_{31} &\simeq \Delta m^2_{31} + \frac{1}{2}A\left(1+\cos 2\theta_{12}\right) \\
\Delta\widetilde{m}^2_{32} &\simeq \Delta m^2_{32} + \frac{1}{2}A\left(1-\cos 2\theta_{12}\right)
\end{aligned}$$

$$\begin{aligned}
\sin^2 2\widetilde{\theta}_{12} &\simeq \sin^2 2\theta_{12}\left(1 - \frac{2A}{\Delta m^2_{21}}\cos 2\theta_{12}\right) \\
\cos 2\widetilde{\theta}_{12} &\simeq \cos 2\theta_{12} + \frac{A}{\Delta m^2_{21}}\sin^2 2\theta_{12} \; ,
\end{aligned}$$

where $A = 2\sqrt{2}\, G_{\rm F} N_e E$ is the matter parameter and $A/\Delta m^2_{21} \simeq 1.05\times 10^{-2}\times E/\left(4\ {\rm MeV}\right)\times 7.5\times 10^{-5}\ {\rm eV}^2/\Delta m^2_{21}$ by taking $\rho \simeq 2.6\ {\rm g/cm}^3$ as a typical matter density of the Earth's crust [7] [3].

补充知识（1）：PMNS矩阵的标准参数化

The 3×3 unitary PMNS neutrino mixing matrix can be parametrized:

$$\begin{pmatrix} \nu_e \\ \nu_\mu \\ \nu_\tau \end{pmatrix}_{\mathrm{L}} = \begin{pmatrix} U_{e1} & U_{e2} & U_{e3} \\ U_{\mu 1} & U_{\mu 2} & U_{\mu 3} \\ U_{\tau 1} & U_{\tau 2} & U_{\tau 3} \end{pmatrix} \begin{pmatrix} \nu_1 \\ \nu_2 \\ \nu_3 \end{pmatrix}_{\mathrm{L}}$$

$$U = \begin{pmatrix} 1 & 0 & 0 \\ 0 & c_{23} & s_{23} \\ 0 & -s_{23} & c_{23} \end{pmatrix} \begin{pmatrix} c_{13} & 0 & s_{13}e^{-i\delta} \\ 0 & 1 & 0 \\ -s_{13}e^{i\delta} & 0 & c_{13} \end{pmatrix} \begin{pmatrix} c_{12} & s_{12} & 0 \\ -s_{12} & c_{12} & 0 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} e^{i\rho} & 0 & 0 \\ 0 & e^{i\sigma} & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

$$= \begin{pmatrix} c_{12}c_{13} & s_{12}c_{13} & s_{13}e^{-i\delta} \\ -s_{12}c_{23} - c_{12}s_{13}s_{23}e^{i\delta} & c_{12}c_{23} - s_{12}s_{13}s_{23}e^{i\delta} & c_{13}s_{23} \\ s_{12}s_{23} - c_{12}s_{13}c_{23}e^{i\delta} & -c_{12}s_{23} - s_{12}s_{13}c_{23}e^{i\delta} & c_{13}c_{23} \end{pmatrix} \begin{pmatrix} e^{i\rho} & 0 & 0 \\ 0 & e^{i\sigma} & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

- ♦ **(1,2) mixing sensitive to solar ν-oscillations;**
- ♦ **(1,3) mixing sensitive to short-baseline reactor anti-ν-oscillations;**
- ♦ **(2,3) mixing sensitive to atmospheric ν-oscillations.**

- ♦ **Dirac phase δ sensitive to long-baseline accelerator ν-oscillations;**
- ♦ **Majorana phases ρ and σ sensitive to lepton number violation.**

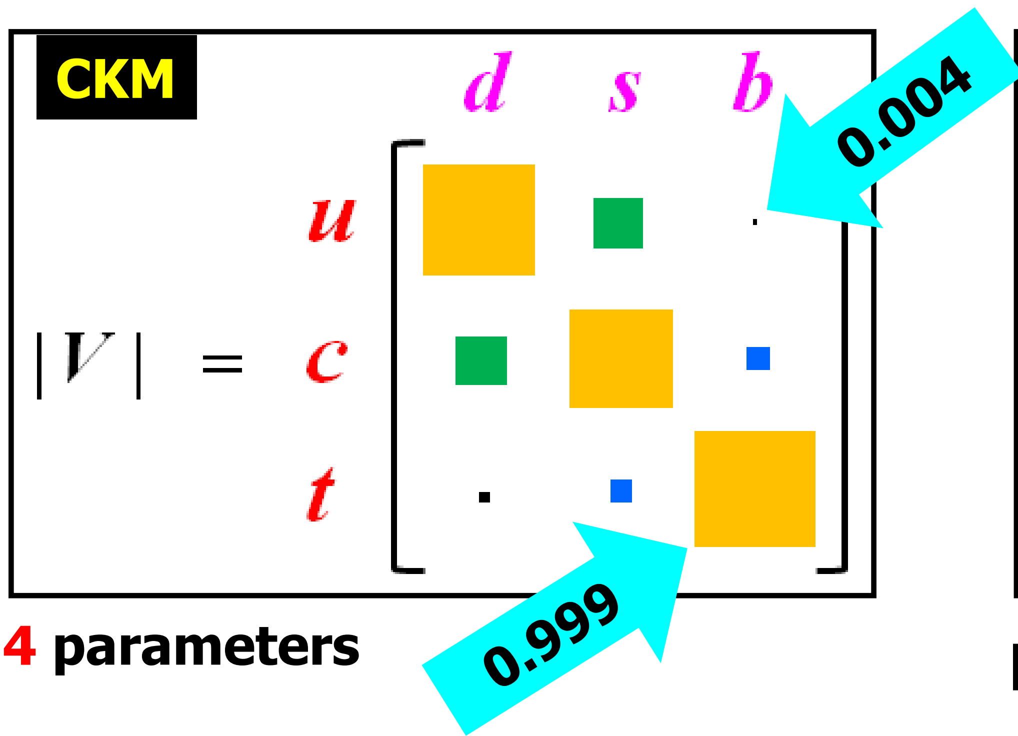
补充知识（2）：PMNS与CKM矩阵的比较

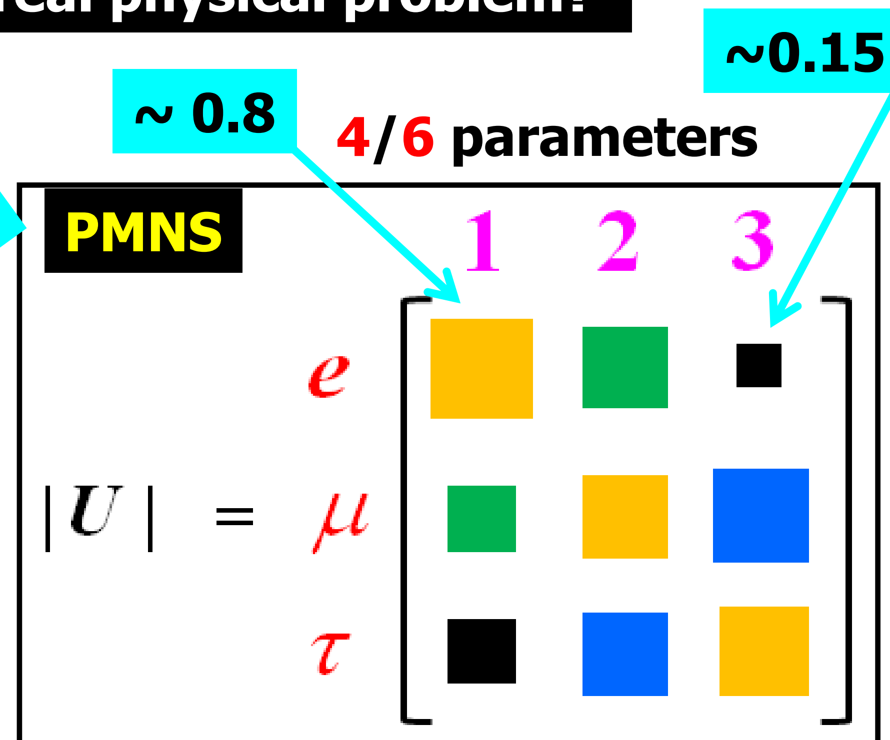
$$-\mathcal{L}_{cc} = \frac{g}{\sqrt{2}}\left[\overline{(u \quad c \quad t)_L}\,\gamma^\mu\, V \begin{pmatrix} d \\ s \\ b \end{pmatrix}_L W^+_\mu + \overline{(e \quad \mu \quad \tau)_L}\,\gamma^\mu\, U \begin{pmatrix} \nu_1 \\ \nu_2 \\ \nu_3 \end{pmatrix}_L W^-_\mu\right] + \text{h.c.}$$

CKM (V) **PMNS** (U)

Why is the neutrino mass ordering a real physical problem?

Quark mixing: hierarchy!

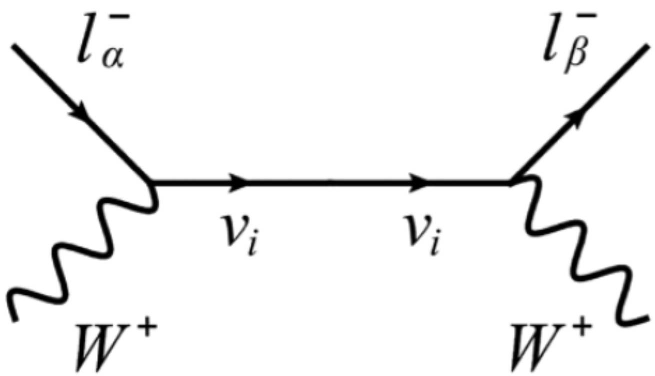


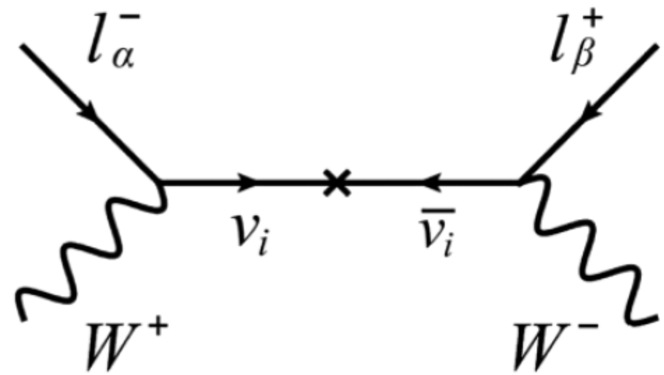


Lepton mixing: anarchy?

补充知识（3）：中微子—反中微子振荡？

Comparison: neutrino-neutrino and neutrino-antineutrino oscillation experiments.





neutrino → neutrino

$$A = \sum_{k=1}^{3} V_{\alpha k}^{*} V_{\beta k} e^{-iE_k t}$$

Feasible and successful today!

neutrino → antineutrino

$$A = \frac{1}{E} \sum_{k=1}^{3} V_{\alpha k} V_{\beta k} m_k e^{-iE_k t}$$

Unfeasible, a hope tomorrow?

Sensitivity to CP-violating phase(s):

δ

δ ρ σ

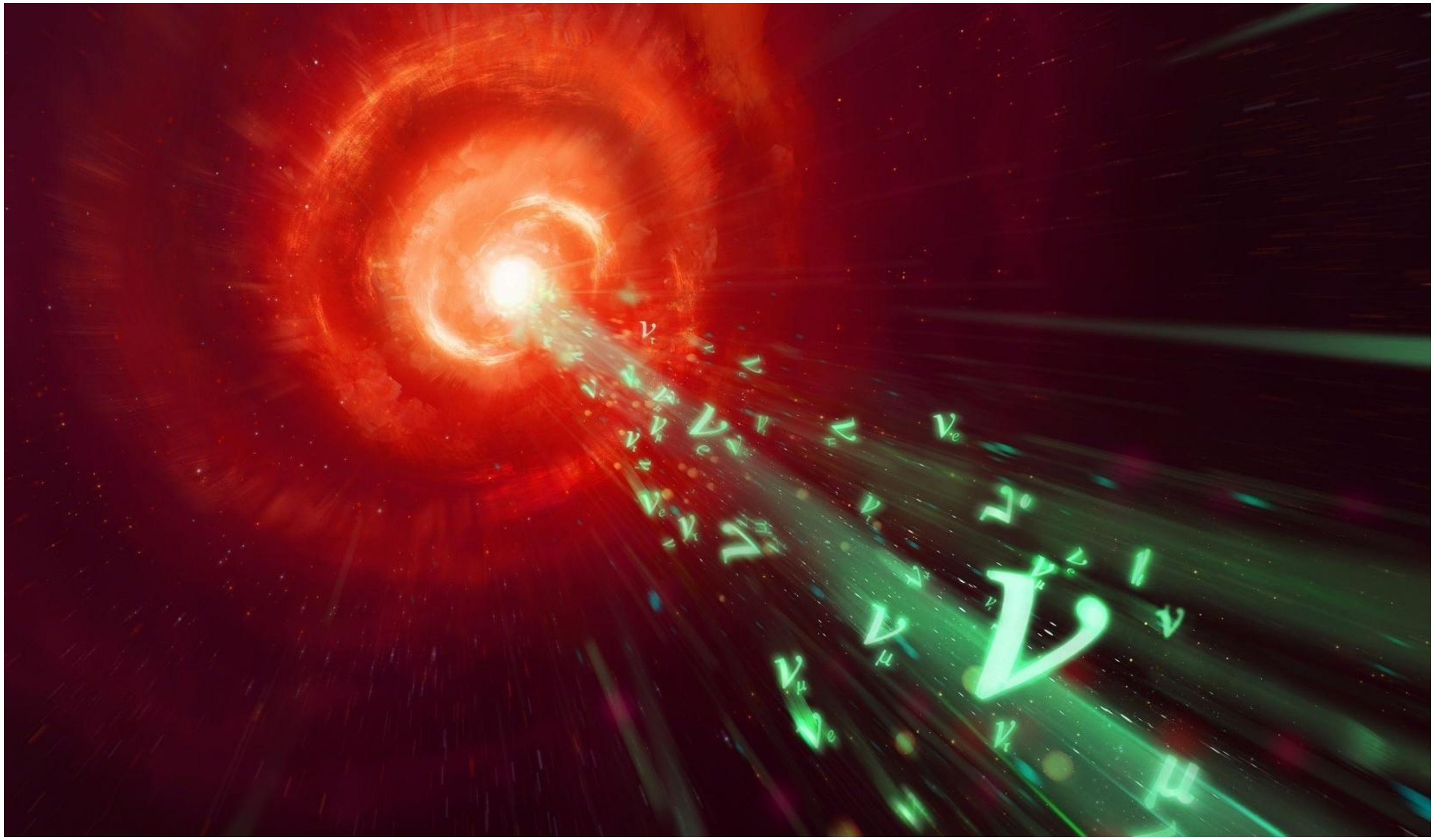
第二部分：中微子质量起源

2.1 中微子汤川相互作用

2.2 跷跷板机制

2.3 轻子味与轻子数破坏

2.4 宇宙重子数不对称之谜



2.1 Weinberg's paper in 1967

VOLUME 19, NUMBER 21 PHYSICAL REVIEW LETTERS 20 NOVEMBER 1967

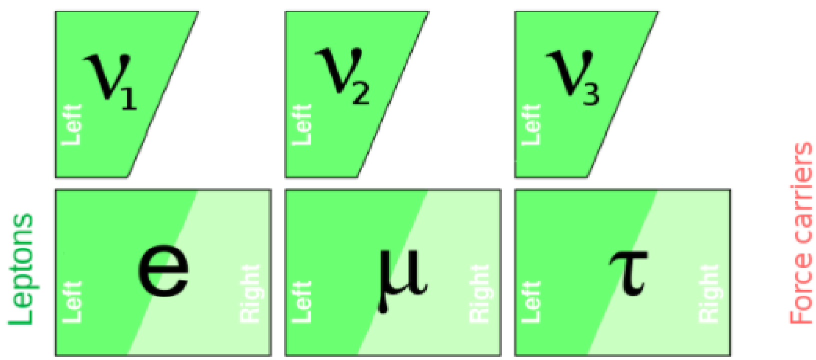
三无产品

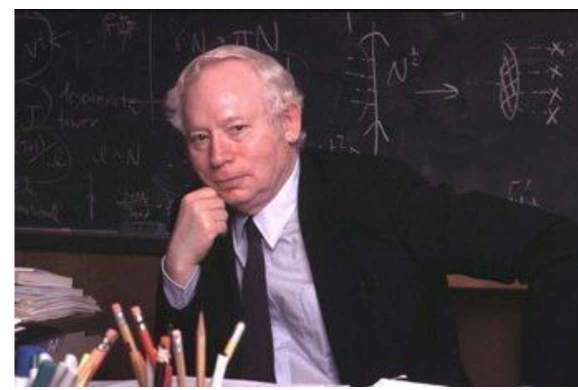
A MODEL OF LEPTONS*

Steven Weinberg†
Laboratory for Nuclear Science and Physics Department,
Massachusetts Institute of Technology, Cambridge, Massachusetts
(Received 17 October 1967)

Theoretical ingredients: it's got what it matters（五脏俱全）

Particle content: no neutrino mass, no quarks, no flavor mixing & CPV





My style is usually not to propose specific models that will lead to specific experimental predictions, but rather to interpret in a broad way what is going on and make very general remarks, like with the development of the point of view associated with effective field theory ---- Weinberg 2021@CERN Courier

2.1 Possible ways to go out

All ν's are massless in the SM, a result of the model's simple structure:

---- SU(2)_L×U(1)_Y gauge symmetry and Lorentz invariance;

Fundamentals of the model, mandatory for consistency of a QFT.

---- Economical particle content:

No right-handed neutrinos, and only one Higgs doublet.

---- Mandatory renormalizability:

No dimension ≥ 5 operators.

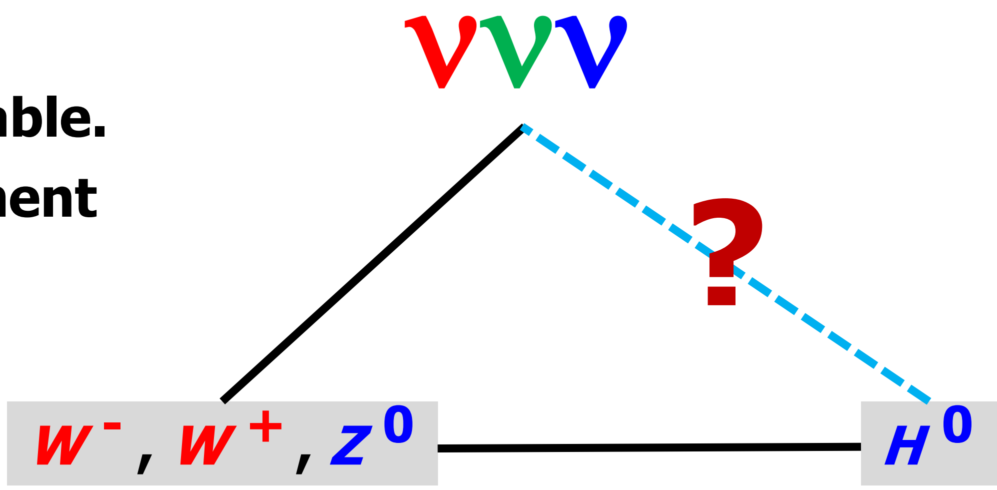
To generate ν-masses, which of the above constraints can be relaxed?

--- The *gauge symmetry* and *Lorentz invariance* should not be abandoned.

--- The particle content is extendable.

--- The renormalizability requirement can be abandoned.

Open a new window for going beyond the SM.



2.1 Objecting to Weinberg's razor

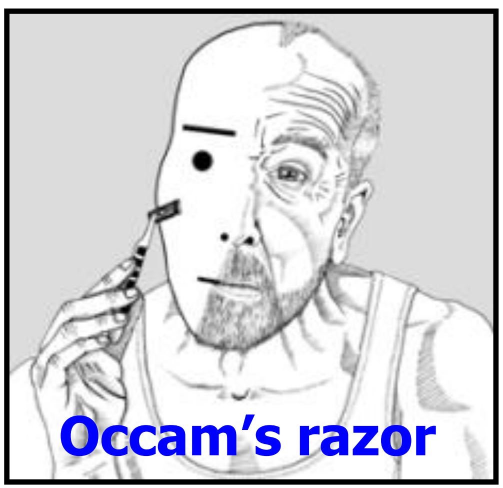
Albert Einstein: Everything should be made as simple as possible, but not simpler!

maximal P violation

$$\begin{pmatrix} u_{\rm L} \\ d_{\rm L} \end{pmatrix} \quad \begin{matrix} \longleftrightarrow & u_{\rm R} \\ \longleftrightarrow & d_{\rm R} \end{matrix}$$

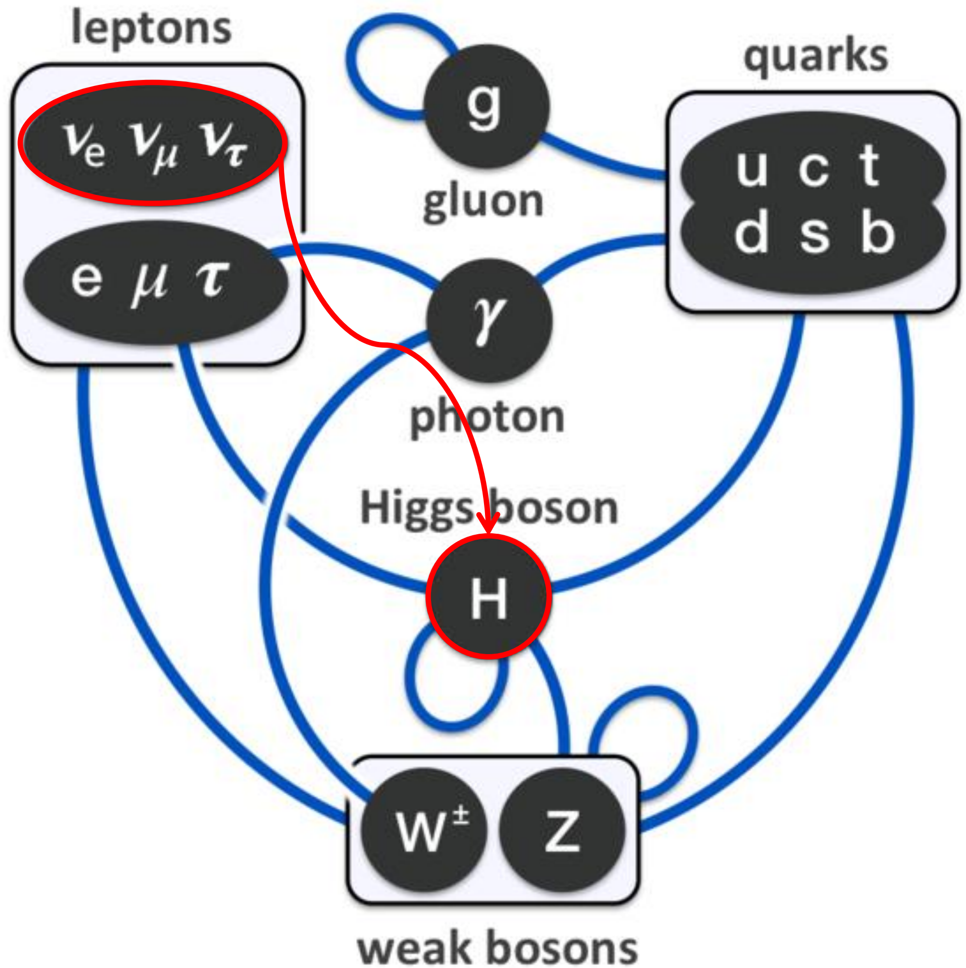
$$\begin{pmatrix} \nu_{e{\rm L}} \\ e_{\rm L} \end{pmatrix} \quad \begin{matrix} \longleftrightarrow & \bigcirc \\ \longleftrightarrow & e_{\rm R} \end{matrix}$$

- **Theoretically unnatural**
- **Experimentally natural**



cut off 7 physical parameters

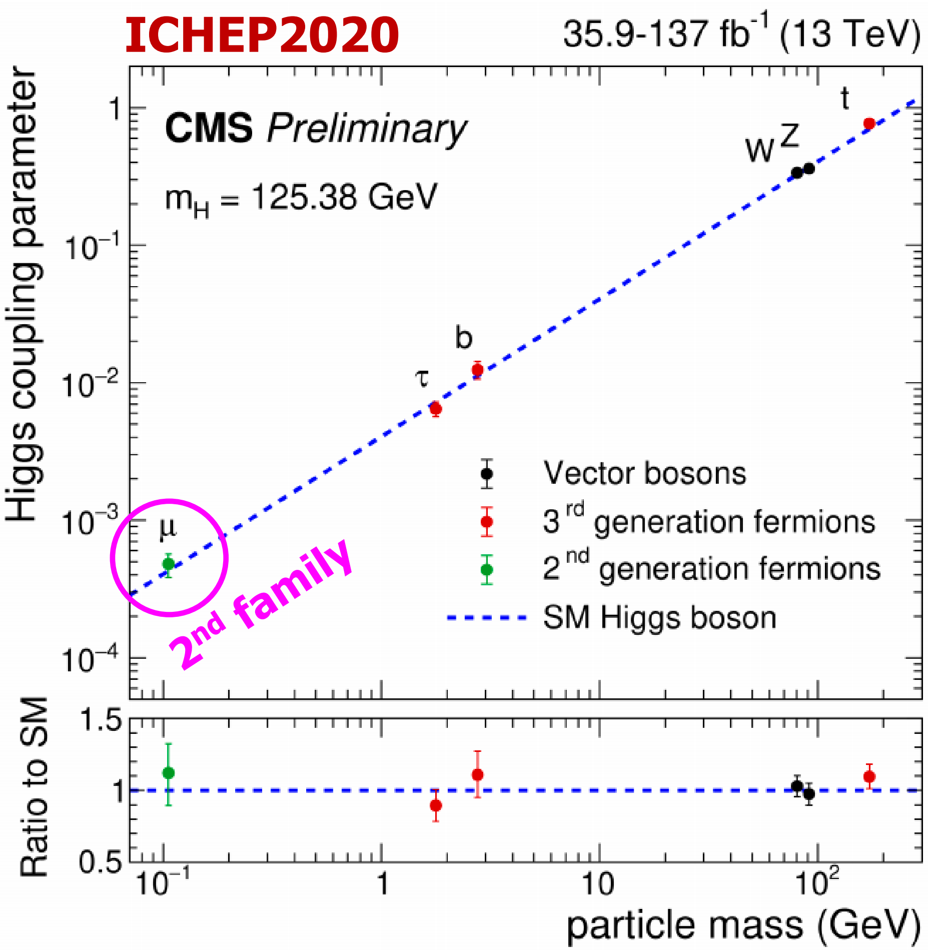
结构存在自然性问题



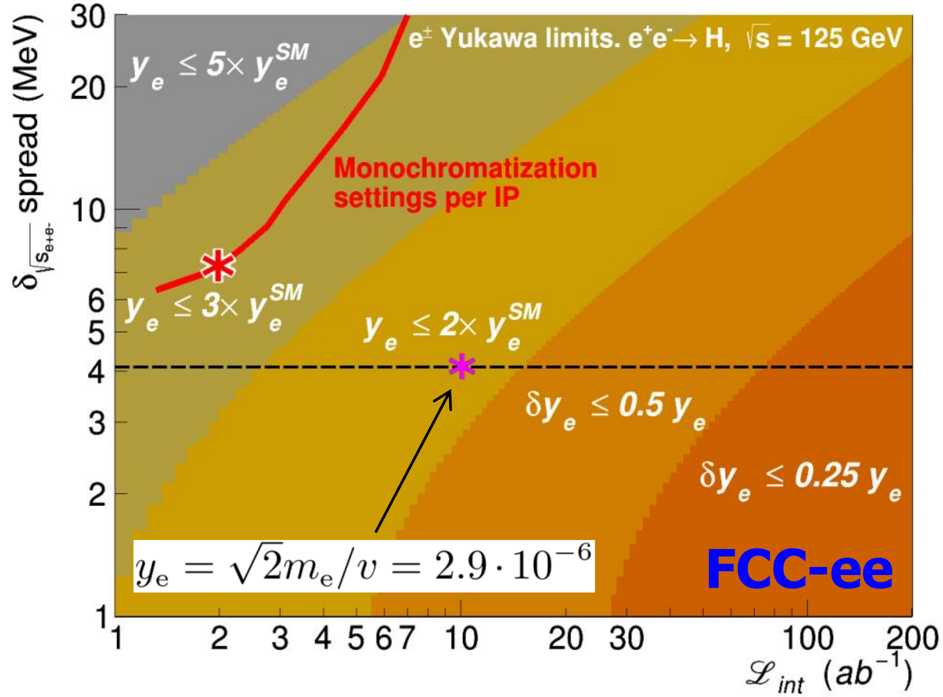
★ **The least cost to generate ν-mass is a Yukawa coupling**

★ **In this way testability will be completely lost, and theoretically uneasy**

2.1 How hard to confirm Yukawa interactions?



The ambition to measure the electron Yukawa coupling through resonant s-channel Higgs production (D. d'Enterria et al, arXiv: 2107.02686)



Three remarks:

★ **Fermion masses**: primarily stem from **tree-level Yukawa** interactions in SM.

★ **Neutrino Yukawa interactions**: no hope to **directly** test them in any manner.

★ **Flavor mixing**: a mismatch between the **Yukawa** and **CC gauge** interactions, should originate **at the same time** as fermion masses.

2.2 Majorana is more natural

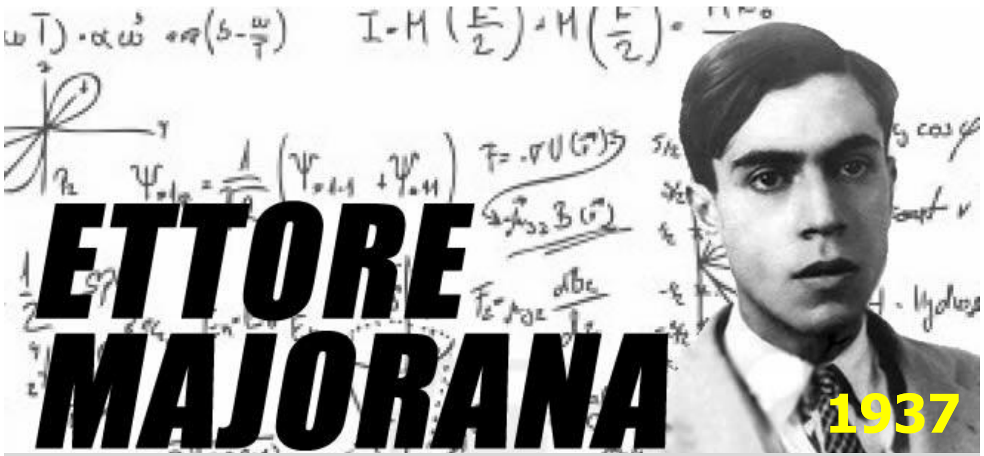
★ **The simplest way to extend the SM is to introduce the right-handed neutrino fields and write out a Dirac mass term.**

Dirac mass $\overline{\ell_{\rm L}} Y_\nu \widetilde{H} N_{\rm R} \longrightarrow M_{\rm D} = Y_\nu \langle H \rangle$

Murray Gell-Mann: everything not forbidden is compulsory!

Majorana mass $\frac{1}{2}\overline{N_{\rm R}^c} M_{\rm R} N_{\rm R}$

It is lepton-number-violating.



mass state: antineutrino=neutrino

In the SM, L and B are violated by instantons, only B – L is conserved.

$$-\mathcal{L}_{\nu+N} = \overline{\nu_{\rm L}} M_{\rm D} N_{\rm R} + \frac{1}{2}\overline{(N_{\rm R})^c} M_{\rm R} N_{\rm R} + {\rm h.c.} = \frac{1}{2}\overline{[\nu_{\rm L} \quad (N_{\rm R})^c]} \begin{pmatrix} 0 & M_{\rm D} \\ M_{\rm D}^T & M_{\rm R} \end{pmatrix} \begin{bmatrix} (\nu_{\rm L})^c \\ N_{\rm R} \end{bmatrix} + {\rm h.c.}$$

P. Minkowski 1977,
T. Yanagida 1979…

$$M_\nu \simeq -M_{\rm D} M_{\rm R}^{-1} M_{\rm D}^T = -\langle H \rangle^2 Y_\nu M_{\rm R}^{-1} Y_\nu^T$$

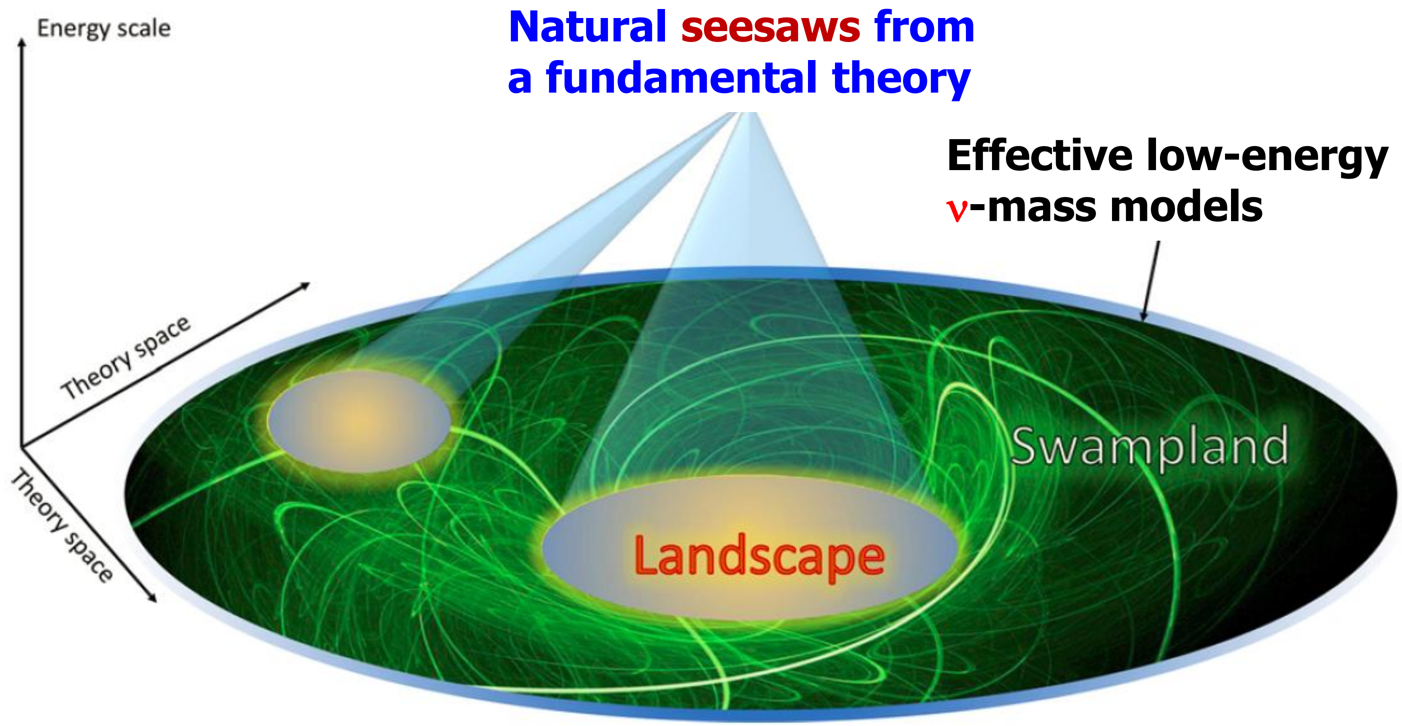
★ **Such a seesaw picture is consistent with the unique operator proposed by Weinberg (1979)**

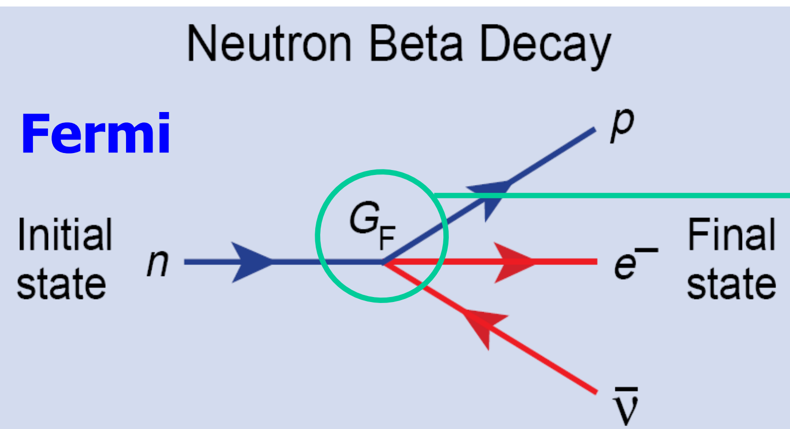
$$\mathcal{O}_{\rm w} = \frac{\overline{\ell_{\rm L}} \widetilde{H} \widetilde{H}^T \ell_{\rm L}^c}{\Lambda}$$

2.2 Seesaw is arguably in the landscape

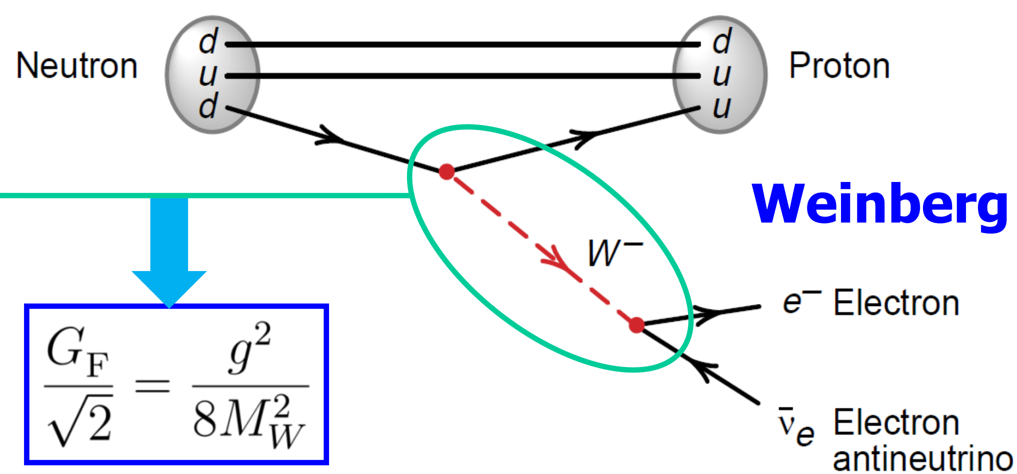
The swampland conjecture

Cumrun Vafa 2005





$G_F \simeq 1.166 \times 10^{-5}\ \mathrm{GeV}^{-2}$



$$\frac{G_F}{\sqrt{2}} = \frac{g^2}{8M_W^2}$$

gauge seesaw

$g \simeq 0.65 \quad M_W \simeq 80.4\ \mathrm{GeV}$

2.2 Majorana nature and exact seesaw

Diagonalize the 6×6 Majorana neutrino mass matrix by a 6×6 unitary matrix:

$$\begin{pmatrix} U & R \\ S & U' \end{pmatrix}^\dagger \begin{pmatrix} 0 & M_{\rm D} \\ M_{\rm D}^T & M_{\rm R} \end{pmatrix} \begin{pmatrix} U & R \\ S & U' \end{pmatrix}^* = \begin{pmatrix} D_\nu & 0 \\ 0 & D_N \end{pmatrix}$$

$$D_\nu \equiv {\rm Diag}\{m_1, m_2, m_3\},\ D_N \equiv {\rm Diag}\{M_1, M_2, M_3\}$$

$$\overline{(N_{\rm R})^c} M_{\rm D}^T (\nu_{\rm L})^c = \left[(N_{\rm R})^T \mathcal{C} M_{\rm D}^T \mathcal{C} \overline{\nu_{\rm L}}^T \right]^T = \overline{\nu_{\rm L}} M_{\rm D} N_{\rm R}$$

Majorana mass states:

$$\nu' = \begin{bmatrix} \nu'_{\rm L} \\ (N'_{\rm R})^c \end{bmatrix} + \begin{bmatrix} (\nu'_{\rm L})^c \\ N'_{\rm R} \end{bmatrix} = \begin{pmatrix} \nu_1 \\ \nu_2 \\ \nu_3 \\ N_1 \\ N_2 \\ N_3 \end{pmatrix}$$

$$(\nu')^c = \nu'$$

The exact *seesaw* relation between light and heavy Majorana neutrinos

$$U D_\nu U^T = -R D_N R^T$$

Three flavor states are linear combinations of six mass states (LFV):

$$\begin{pmatrix} \nu_e \\ \nu_\mu \\ \nu_\tau \end{pmatrix}_{\rm L} = U \begin{pmatrix} \nu_1 \\ \nu_2 \\ \nu_3 \end{pmatrix}_{\rm L} + R \begin{pmatrix} N_1 \\ N_2 \\ N_3 \end{pmatrix}_{\rm L}$$

$$U U^\dagger + R R^\dagger = I$$

★ The standard weak charged-current interactions:

$$-\mathcal{L}_{\rm cc} = \frac{g}{\sqrt{2}} \overline{(e \ \ \mu \ \ \tau)_{\rm L}} \gamma^\mu \left[U \begin{pmatrix} \nu_1 \\ \nu_2 \\ \nu_3 \end{pmatrix}_{\rm L} + R \begin{pmatrix} N_1 \\ N_2 \\ N_3 \end{pmatrix}_{\rm L} \right] W^-_\mu + {\rm h.c.}$$

$$\ell_{\rm L}(x) \to e^{{\rm i}\phi} \ell_{\rm L}(x)$$
$$\nu'_{\rm L}(x) \to e^{{\rm i}\phi} \nu'_{\rm L}(x)$$

LNV

$$-\mathcal{L}'_\nu = \frac{1}{2} \overline{\nu'_{\rm L}} D_\nu (\nu'_{\rm L})^c + {\rm h.c.}$$

U **= light ν mixing;** ***R*** **= light-heavy ν mixing.**

2.3 Active-sterile neutrino mixing

- **An Euler-like parametrization of active-sterile ν mixing (ZZX, 2008)**

active: ν_e, ν_μ, ν_τ; sterile: ν_x, ν_y, ν_z; mass eigenstate: $\nu_1, \dots, \nu_6$

$$\begin{pmatrix}\nu_e\\ \nu_\mu\\ \nu_\tau\\ \nu_x\\ \nu_y\\ \nu_z\end{pmatrix} = \mathcal{U}\begin{pmatrix}\nu_1\\ \nu_2\\ \nu_3\\ \nu_4\\ \nu_5\\ \nu_6\end{pmatrix} = \underbrace{\begin{pmatrix} I & 0\\ 0 & U_0'\end{pmatrix}}_{\text{sterile}}\begin{pmatrix} A & R\\ S & B\end{pmatrix}\underbrace{\begin{pmatrix} U_0 & 0\\ 0 & I\end{pmatrix}}_{\text{active}}\begin{pmatrix}\nu_1\\ \nu_2\\ \nu_3\\ \nu_4\\ \nu_5\\ \nu_6\end{pmatrix}$$

for explicit expression

e-prints: 0709.2220 and 1110.0083

sterile → $O_{56}O_{46}O_{45}$: **3 angles, 3 phases**

active → $O_{23}O_{13}O_{12}$: **3 angles, 3 phases**

interplay → $O_{36}O_{26}O_{16}O_{35}O_{25}O_{15}O_{34}O_{24}O_{14}$ ← **the portal**

9 mixing angles, 9 CP-violating phases

- **The PMNS active neutrino mixing matrix:** $U = AU_0$ **(weak cc).**
- **The active-sterile neutrino mixing matrix:** R **(weak cc).**
- **The PMNS sterile neutrino mixing matrix:** $U' = U_0' B$ **(sterile).**

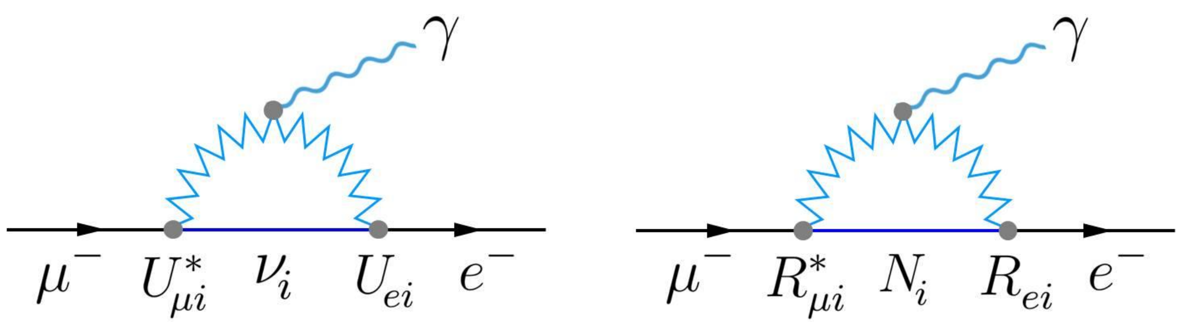
2.3 Lepton flavor violation

♦ **Neutrino oscillations are the only established evidence of νLFV:**

在真空中，不同类型的振荡实验所对应的中微子或反中微子种类、典型能量、基线长度，以及对中微子质量平方差的敏感区域。

中微子源	中微子类型	典型能量	基线长度	质量平方差
太阳	ν_e	~ 1 MeV	$\sim 1.5\times10^8$ km	$\sim 10^{-11}$ eV^2
大气	ν_e、ν_μ、$\overline{\nu}_e$、$\overline{\nu}_\mu$	~ 1 GeV	$\sim 10^4$ km	$\sim 10^{-4}$ eV^2
反应堆（短基线）	$\overline{\nu}_e$	~ 1 MeV	~ 1 km	$\sim 10^{-3}$ eV^2
反应堆（长基线）	$\overline{\nu}_e$	~ 1 MeV	$\sim 10^2$ km	$\sim 10^{-4}$ eV^2
加速器（短基线）	ν_μ、$\overline{\nu}_\mu$	~ 1 GeV	~ 1 km	~ 1 eV^2
加速器（长基线）	ν_μ、$\overline{\nu}_\mu$	~ 1 GeV	$\sim 10^3$ km	$\sim 10^{-3}$ eV^2

♦ **Charged-lepton flavor violation (cLFV) has never been observed:**



An example

The PMNS unitarity is slightly violated due to active-sterile ν mixing.

2.3 Radiative decays of charged leptons

$$\xi\left(\beta^- \to \alpha^- + \gamma\right) \equiv \frac{\Gamma\left(\beta^- \to \alpha^- + \gamma\right)}{\Gamma\left(\beta^- \to \alpha^- + \overline{\nu}_\alpha + \nu_\beta\right)}$$

$$\simeq \frac{3\alpha_{\rm em}}{2\pi}\left|\sum_{i=1}^{3} U_{\alpha i}U^*_{\beta i} G_\gamma\left(\frac{m_i^2}{M_W^2}\right) + \sum_{i=1}^{n} R_{\alpha i}R^*_{\beta i} G_\gamma\left(\frac{M_i^2}{M_W^2}\right)\right|^2$$

- **In the limit of $M_i \gg M_W$, we are left with**

$$\xi\left(\beta^- \to \alpha^- + \gamma\right) \simeq \frac{3\alpha_{\rm em}}{2\pi}\left|\sum_{i=1}^{3} U_{\alpha i}U^*_{\beta i}\left(-\frac{5}{6} + \frac{1}{4}\cdot\frac{m_i^2}{M_W^2}\right) - \frac{1}{3}\sum_{i=1}^{n} R_{\alpha i}R^*_{\beta i}\right|^2$$

$UU^\dagger + RR^\dagger = I$

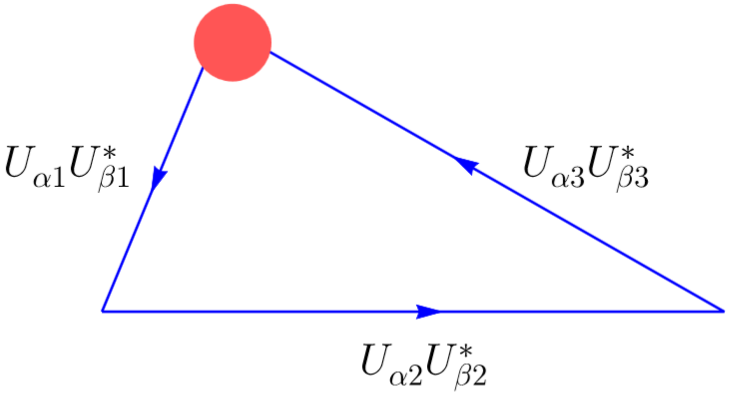
$$= \frac{3\alpha_{\rm em}}{8\pi}\left|\sum_{i=1}^{3} U_{\alpha i}U^*_{\beta i}\left(1 - \frac{1}{2}\cdot\frac{m_i^2}{M_W^2}\right)\right|^2 ,$$

- **Switching off heavy degrees of freedom, we are left with**

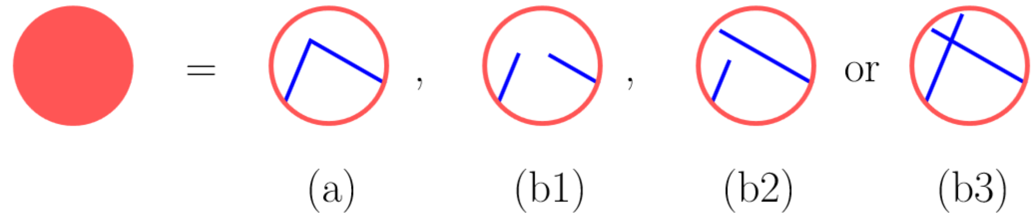
$$\xi\left(\beta^- \to \alpha^- + \gamma\right) \simeq \frac{3\alpha_{\rm em}}{32\pi}\left|\sum_{i=1}^{3} U_{\alpha i}U^*_{\beta i}\frac{m_i^2}{M_W^2}\right|^2 = \frac{3\alpha_{\rm em}}{32\pi}\left|\sum_{i=2}^{3} U_{\alpha i}U^*_{\beta i}\frac{\Delta m_{i1}^2}{M_W^2}\right|^2 \lesssim \mathcal{O}(10^{-54})$$

2.3 Example: unitarity polygons and cLFV

- **A natural *seesaw* will lead to slight violation of the PMNS unitarity!**



a few typical topologies of the apex:



$$U_{\alpha 1}U^*_{\beta 1} + U_{\alpha 2}U^*_{\beta 2} + U_{\alpha 3}U^*_{\beta 3} = -\sum_{i=1}^{n} R_{\alpha i}R^*_{\beta i}$$

$$(\alpha, \beta) = (e, \mu),\ (\mu, \tau) \text{ or } (\tau, e)$$

$$\mathcal{B}\left(\mu^- \to e^- + \gamma\right) < 4.2 \times 10^{-13}$$

$$\mathcal{B}\left(\tau^- \to e^- + \gamma\right) < 3.3 \times 10^{-8}$$

$$\mathcal{B}\left(\tau^- \to \mu^- + \gamma\right) < 4.4 \times 10^{-8}$$

cLFV Constraints on the PMNS unitarity:

$$\left|\sum_{i=1}^{3} U_{ei}U^*_{\mu i}\right| = \left|\sum_{i=1}^{n} R_{ei}R^*_{\mu i}\right| < 2.20 \times 10^{-5}$$

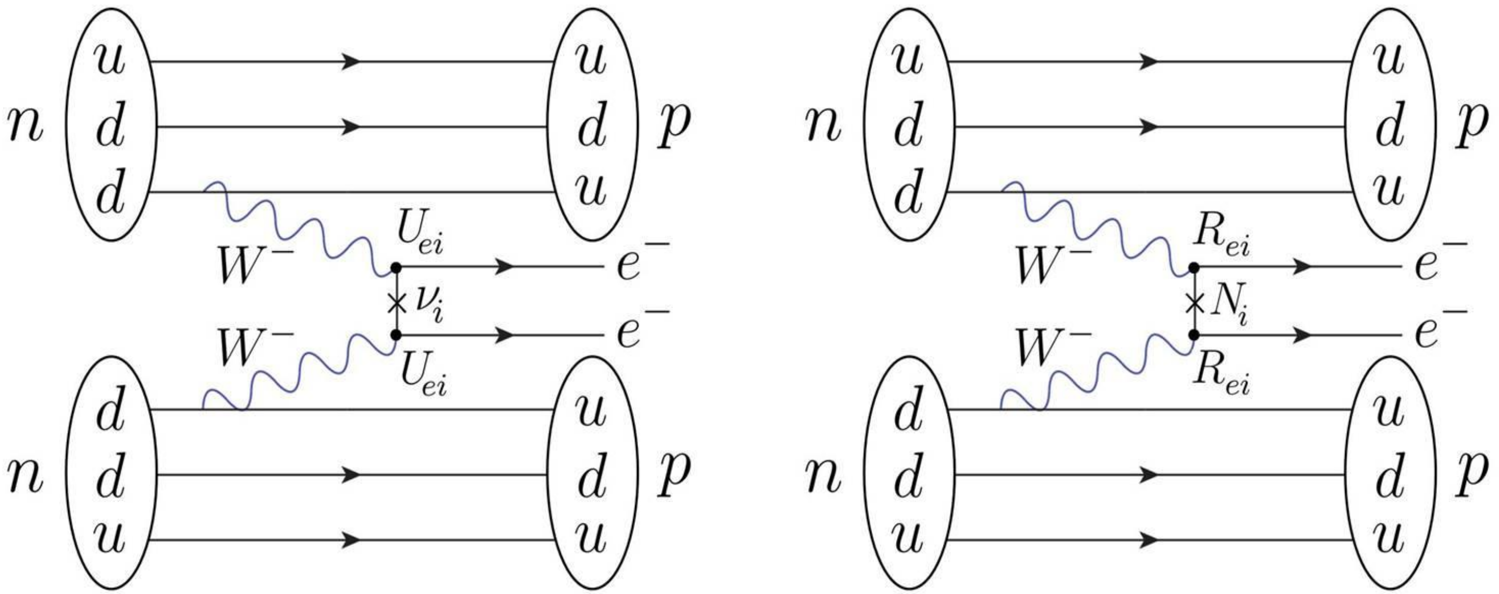
$$\left|\sum_{i=1}^{3} U_{ei}U^*_{\tau i}\right| = \left|\sum_{i=1}^{n} R_{ei}R^*_{\tau i}\right| < 1.46 \times 10^{-2}$$

$$\left|\sum_{i=1}^{3} U_{\mu i}U^*_{\tau i}\right| = \left|\sum_{i=1}^{n} R_{\mu i}R^*_{\tau i}\right| < 1.70 \times 10^{-2}$$

- **ZZX, D. Zhang, 2009.09717**
- **D. Zhang, S. Zhou, 2102.04954**

2.3 Lepton-number-violating 0ν2β decays

★ **Lepton number violation (neutrinoless double-beta decays):**



https://margheritamorotti.com/ettore-majorana

★ **In most cases the contribution of heavy Majorana neutrinos to *0ν2β* is negligible in the canonical type-one seesaw.**
ZZX, arXiv:0907.3014; W. Rodejohann, 0912.3388.

$$U D_\nu U^T = -R D_N R^T$$

$$\Gamma_{0\nu 2\beta} \propto \left| \sum_{i=1}^{3} m_i U_{ei}^2 - M_A^2 \sum_{i=1}^{3} \frac{R_{ei}^2}{M_i} \mathcal{F}(A, M_i) \right|^2 = \left| \sum_{i=1}^{3} M_i R_{ei}^2 \left[1 + \frac{M_A^2}{M_i^2} \mathcal{F}(A, M_i) \right] \right|^2$$

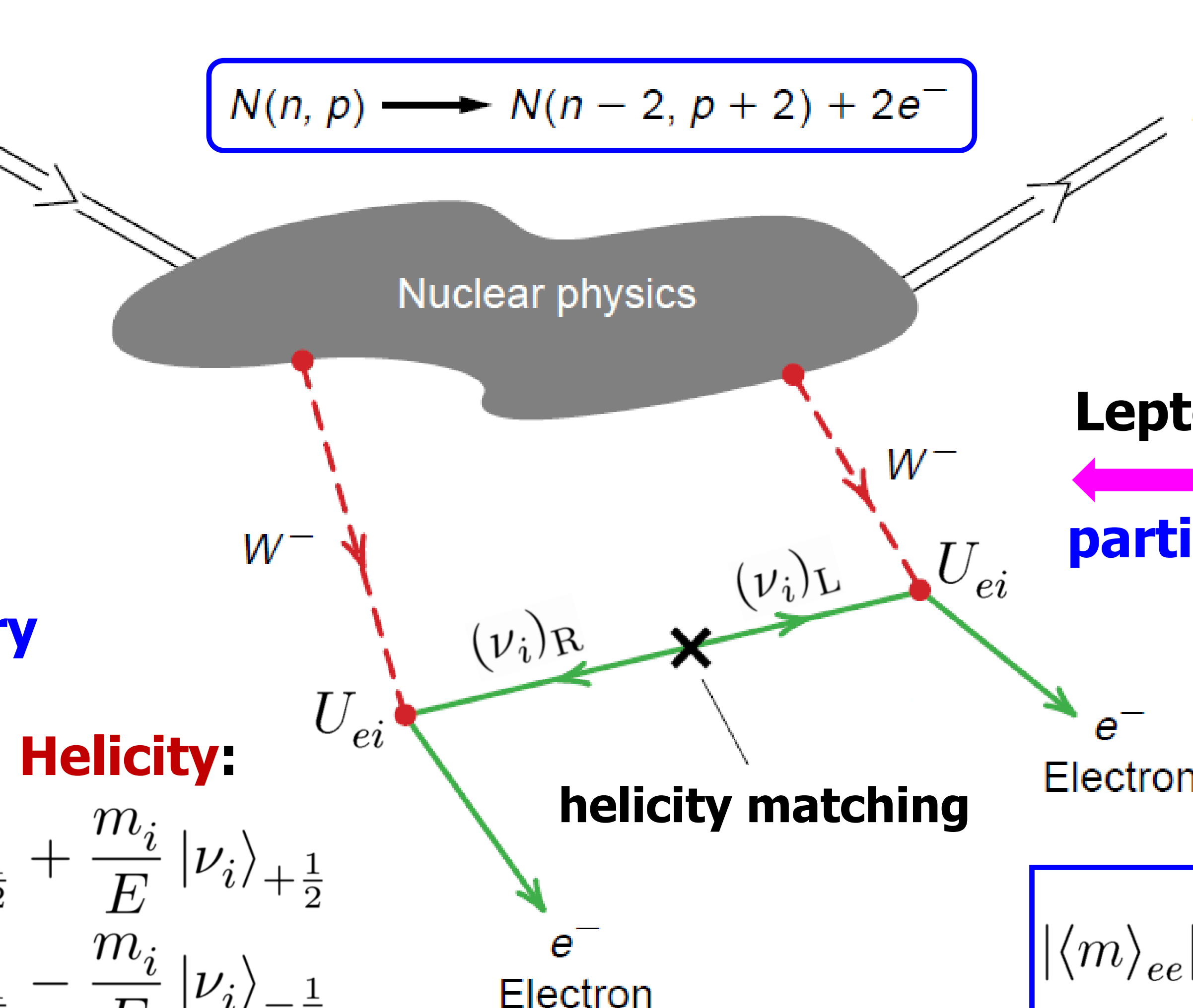
★ **There're many different lepton-number-violating scenarios for 0ν2β.**

2.3 The 0ν2β effective neutrino mass

★ A *0ν2β* decay may occur if massive ν's have the **Majorana** nature, as first pointed out by **Furry** in 1939.

$$T^{0\nu}_{1/2} = \left(G^{0\nu}\right)^{-1} \left|M^{0\nu}\right|^{-2} \left|\langle m\rangle_{ee}\right|^{-2}$$

Wendell Furry



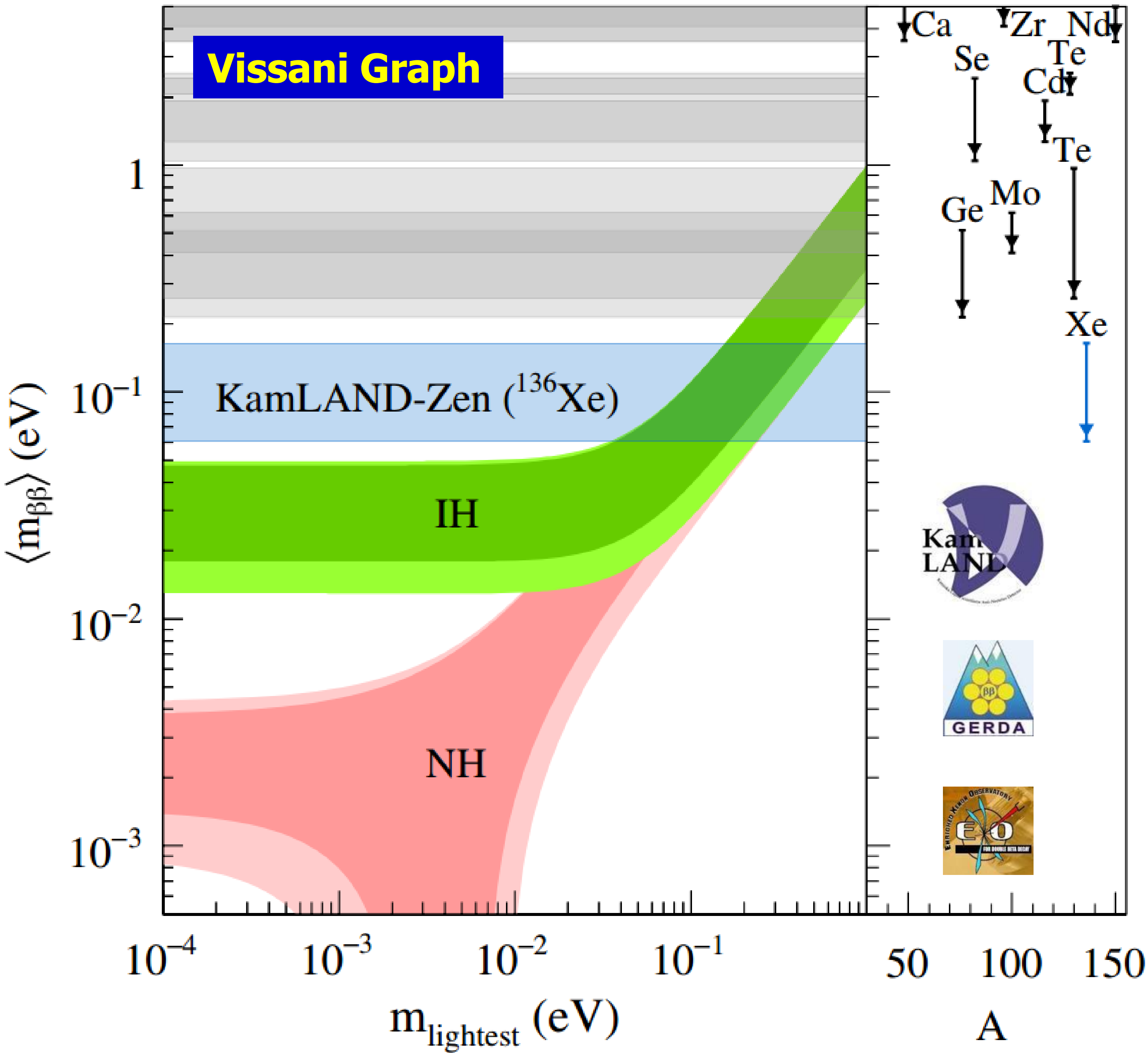
Lepton number violation ← particle physics

$$|\langle m\rangle_{ee}| = \left|\sum_i m_i U_{ei}^2\right|$$

Chirality and Helicity:

$$|\nu_i\rangle_{\rm L} \propto |\nu_i\rangle_{-\frac{1}{2}} + \frac{m_i}{E}|\nu_i\rangle_{+\frac{1}{2}}$$

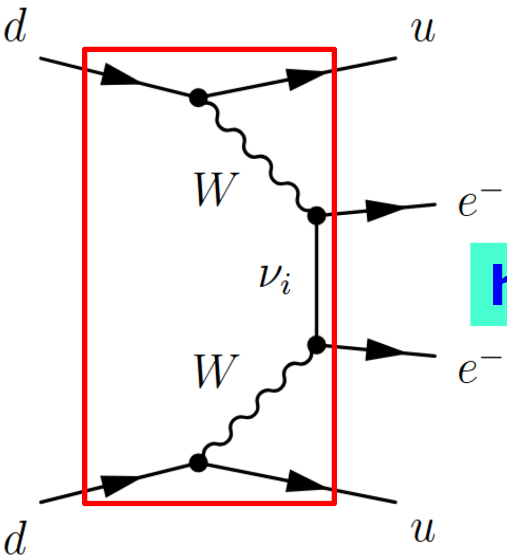
$$|\nu_i\rangle_{\rm R} \propto |\nu_i\rangle_{+\frac{1}{2}} - \frac{m_i}{E}|\nu_i\rangle_{-\frac{1}{2}}$$

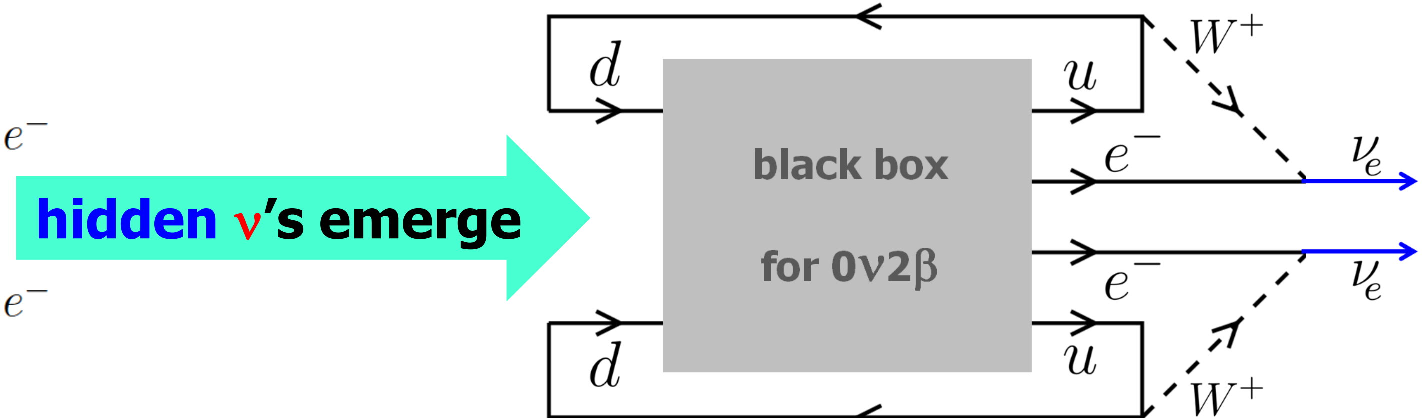


Vissani Graph
KamLAND-Zen (^{136}Xe)
IH
NH
Ca
Zr
Nd
Se
Te
Cd
Te
Ge
Mo
Xe
Kam
LAND
GERDA
1
10^{-1}
10^{-2}
10^{-3}
$\langle m_{\beta\beta} \rangle$ (eV)
10^{-4}
10^{-3}
10^{-2}
10^{-1}
$m_{lightest}$ (eV)
50
100
150
A

2.3 The Schechter-Valle theorem

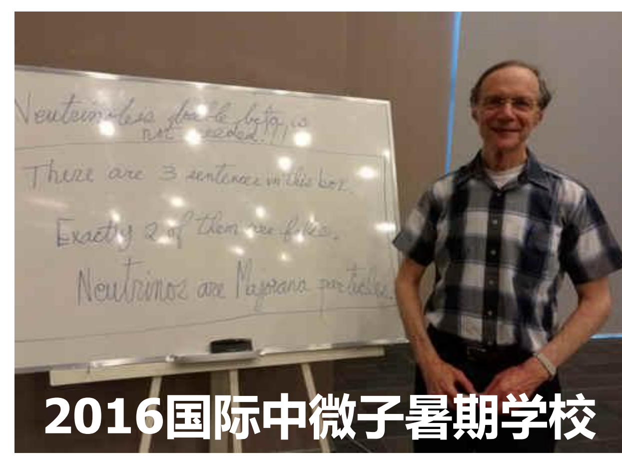
★ **Joseph Schechter** and **Jose Valle** suggested a theorem in **June 1982**: if a **0ν2β** decay happens, there must be an effective **Majorana** mass term. The reverse is also true.





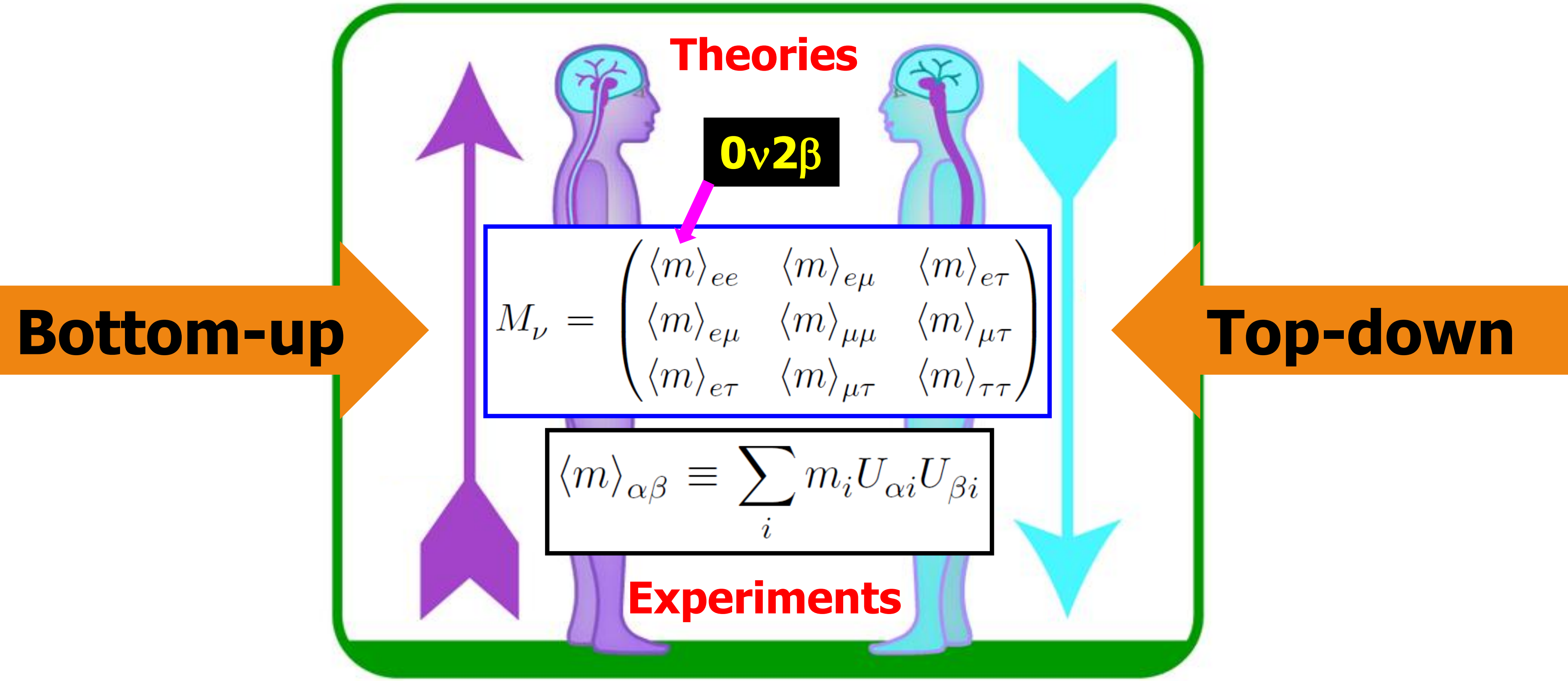
Boris Kayser 关于 Majorana 的神逻辑

- **There are three sentences in this box.**
- **Exactly two of them are false.**
- **Neutrinos are Majorana particles.**



2.3 How about the other effective masses?

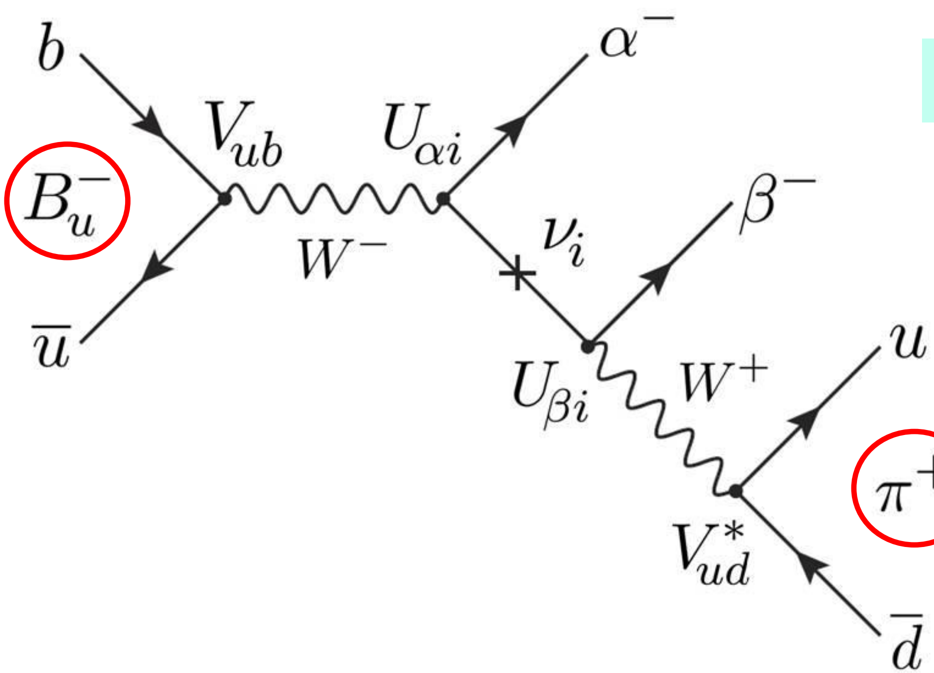
Without information on the nature of massive neutrinos (Majorana or not) and all the CP-violating phases, one will have no way to establish a full theory of ν masses and flavor mixing. Give 0ν2β a chance!



Within about 10 years, after both the neutrino mass ordering and the Dirac CP-violating phase are measured, one has to try all the possible ways to determine the absolute mass scale and two Majorana phases.

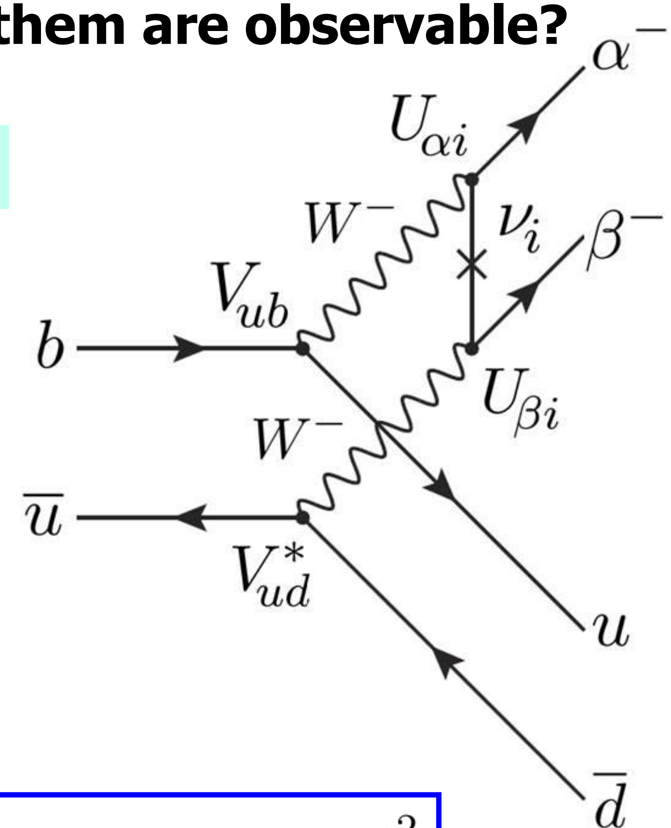
2.3 Hopeless to see other effective ν masses?

There are many LNV processes, but none of them are observable?



Example

+



decay rates suppressed

$$\Gamma(B_u^- \to \pi^+ \alpha^- \beta^-) \propto \left|\langle m\rangle_{\alpha\beta}\right|^2 = \left|\sum_{i=1}^{3} \left(m_i U_{\alpha i} U_{\beta i}\right)\right|^2$$

$$\mathcal{B}(B_u^- \to \pi^+ e^- e^-) < 2.3 \times 10^{-8} \ (\mathrm{CL} = 90\%)$$

$$\mathcal{B}(B_u^- \to \pi^+ e^- \mu^-) < 1.5 \times 10^{-7} \ (\mathrm{CL} = 90\%)$$

$$\mathcal{B}(B_u^- \to \pi^+ \mu^- \mu^-) < 4.0 \times 10^{-9} \ (\mathrm{CL} = 95\%)$$

History tells us: the fool didn’t know it’s impossible, so he did it and sometimes succeeded…

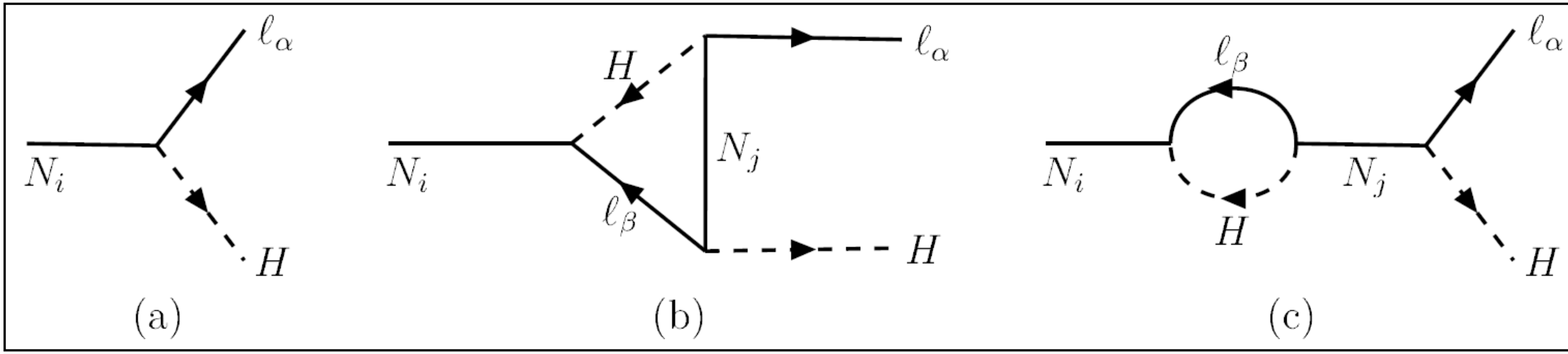
2.4 A role of heavy neutrino in the Universe?

★ **About 230 million years ago, the earliest dinosaurs appeared on the Earth, and they mysteriously disappeared about 65 million years ago.**

★ **Heavy Majorana neutrinos, if they once existed, might have had the same experience in the early Universe: their disappearance led to the appearance of a baryonic matter world (M. Fukugita, T. Yanagida 1986).**

2.4 Thermal leptogenesis

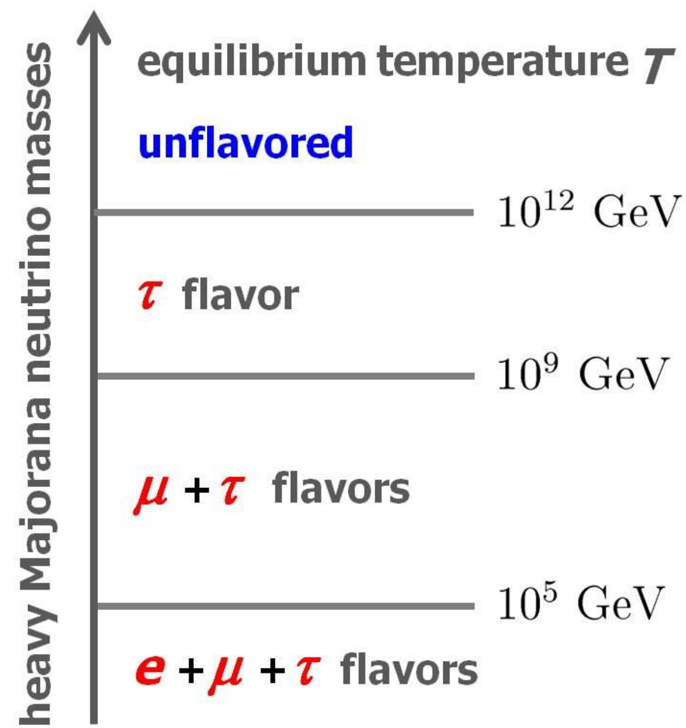
★ **Lepton-number-violating & CP-violating decays of heavy neutrinos:**



★ **Given** $M_3 > M_2 \gg M_1 = T \gtrsim 10^{12}$ GeV **, the CP-violating asymmetry responsible for unflavored leptogenesis is**

$$\varepsilon_1 \equiv \frac{\sum_\alpha \left[\Gamma\left(N_1 \to \ell_\alpha + H\right) - \Gamma\left(N_1 \to \overline{\ell_\alpha} + \overline{H}\right)\right]}{\sum_\alpha \left[\Gamma\left(N_1 \to \ell_\alpha + H\right) + \Gamma\left(N_1 \to \overline{\ell_\alpha} + \overline{H}\right)\right]}$$

$$\simeq -\frac{3M_1}{16\pi\left(Y_\nu^\dagger Y_\nu\right)_{11}} \sum_i \left[\frac{\mathrm{Im}\left(Y_\nu^\dagger Y_\nu\right)^2_{1i}}{M_i}\right]$$



2.4 Is this CPV related to low-energy CPV?

★ **Yes, but in general this relation is NOT direct and transparent.**

$$\varepsilon_1 \simeq -\frac{3M_1}{16\pi\left(Y_\nu^\dagger Y_\nu\right)_{11}} \sum_i \left[\frac{\mathrm{Im}\left(Y_\nu^\dagger Y_\nu\right)^2_{1i}}{M_i}\right]$$

in the early Universe

⬍

$$M_\nu \simeq -M_{\rm D} M_{\rm R}^{-1} M_{\rm D}^T = -\langle H \rangle^2 Y_\nu M_{\rm R}^{-1} Y_\nu^T$$

at the seesaw scale

neutrino masses + flavor mixing (renormalization-group equations)

★ **A direct relation is possible in some very specific models.**

$$P(\nu_\alpha \to \nu_\beta) = \delta_{\alpha\beta} - 4\sum_{i<j}\left[\mathrm{Re}\left(U_{\alpha i}U_{\beta j}U^*_{\alpha j}U^*_{\beta i}\right)\sin^2\frac{\Delta m^2_{ji}L}{4E}\right]$$
$$+2\sum_{i<j}\left[\mathrm{Im}\left(U_{\alpha i}U_{\beta j}U^*_{\alpha j}U^*_{\beta i}\right)\sin\frac{\Delta m^2_{ji}L}{2E}\right]$$

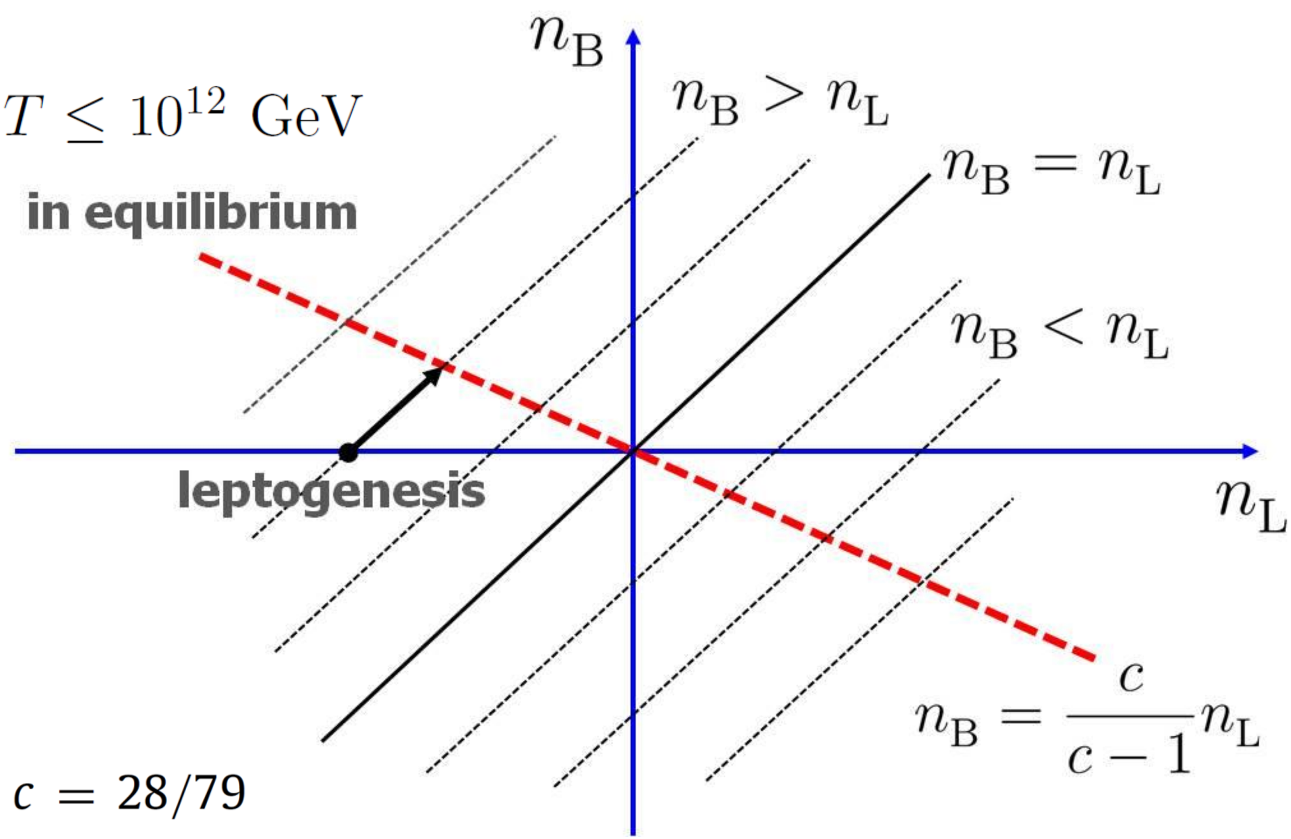
at low energies

2.4 Baryon number asymmetry

B − L-conserving *sphaleron* interaction → baryon number asymmetry.

$$n_B\big|_{\text{equilibrium}} = c\,(n_B - n_L)\big|_{\text{equilibrium}} = -c\,n_L\big|_{\text{initial}}$$
$$n_{\overline{B}}\big|_{\text{equilibrium}} = c\,(n_{\overline{B}} - n_{\overline{L}})\big|_{\text{equilibrium}} = -c\,n_{\overline{L}}\big|_{\text{initial}}$$

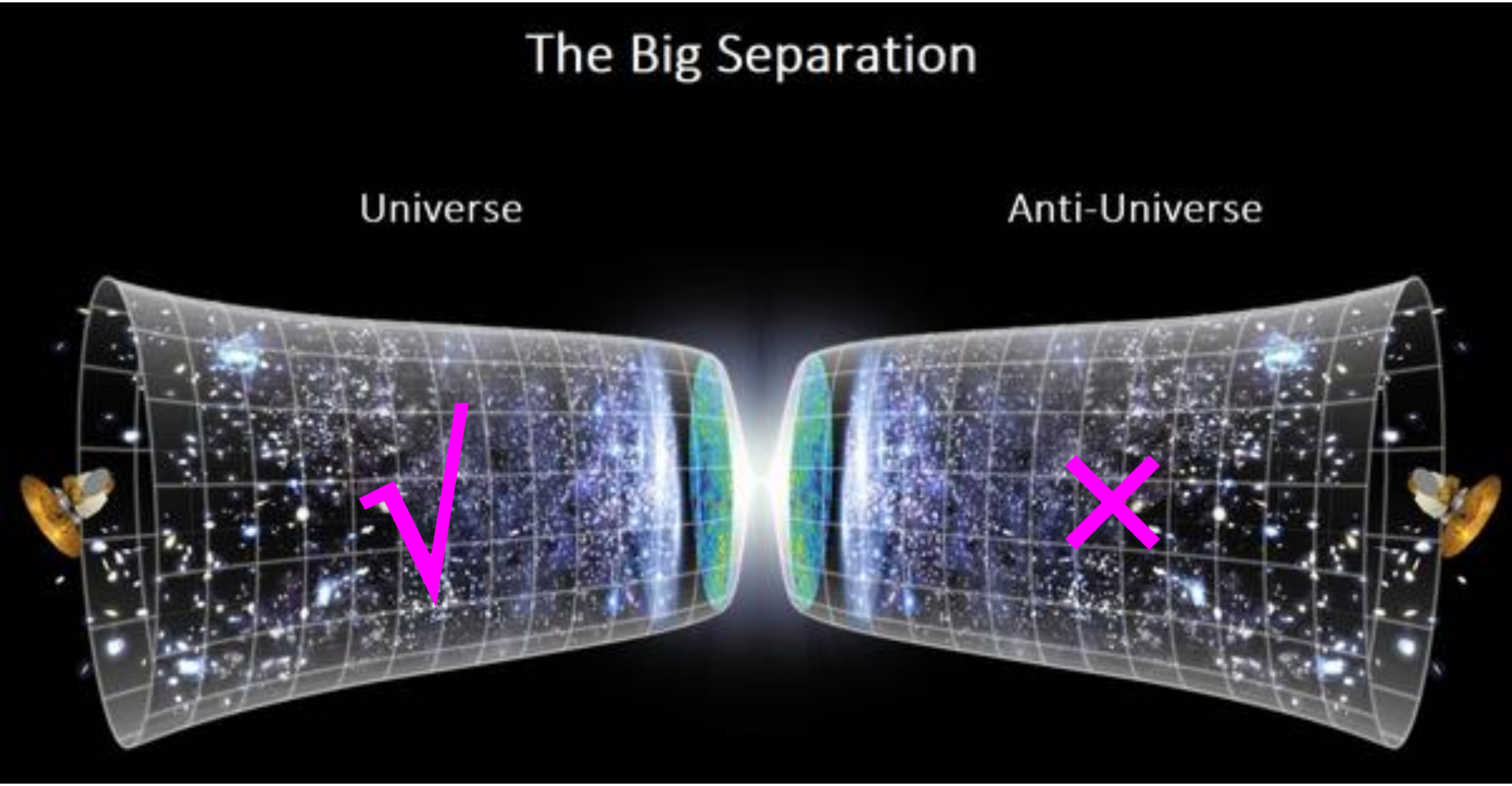
$10^2\ \text{GeV} \le T \le 10^{12}\ \text{GeV}$



$c = 28/79$

小练习：请尝试验证该图中红色虚线的斜率（提示：黑色斜线的斜率为+1）。

2.4 How to test leptogenesis?



Hitoshi Murayama's archaeological arguments (2002):

- **Electroweak baryogenesis is proved to be wrong;**
- **CP violation is observed in neutrino oscillations;**
- **Neutrinos are verified to be the Majorana fermion.**

Concluding remarks

You may use *any* degrees of freedom you like to describe a physical system, but if you use the wrong ones, you will be sorry —— S. Weinberg 83.

sub-eV active neutrinos

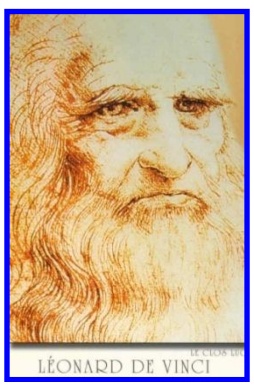
sub-eV sterile neutrinos

keV sterile neutrinos

TeV Majorana neutrinos

≥ EeV Majorana neutrinos

the portal to sterile world



Bottom-Up Way

Although nature commences with reason and ends in experience, it is necessary for us to do the opposite, that is, to commence with experience and from this to proceed to investigate the reason