Neutrino cosmology

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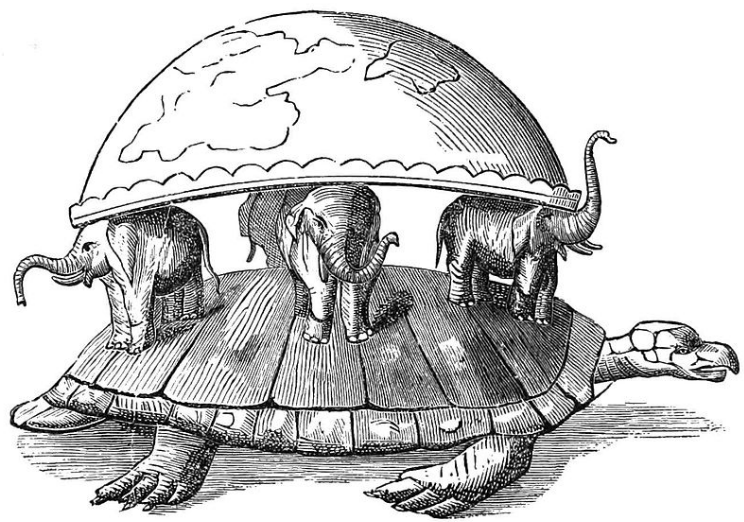


CCEPP Summer School on Neutrino Physics, August 20 – 28, 2021

The grand lecture plan...

Lecture 1: Neutrinos in homogeneous cosmology

Lecture 2: Neutrinos in inhomogeneous cosmology

1. Neutrinos and structure formation
2. Signatures of neutrino dark matter and neutrino mass constraints
3. Future prospects

Lecture 2: Neutrinos in inhomogeneous cosmology

1. Neutrinos and structure formation

2. Signatures of neutrino dark matter and neutrino mass constraints

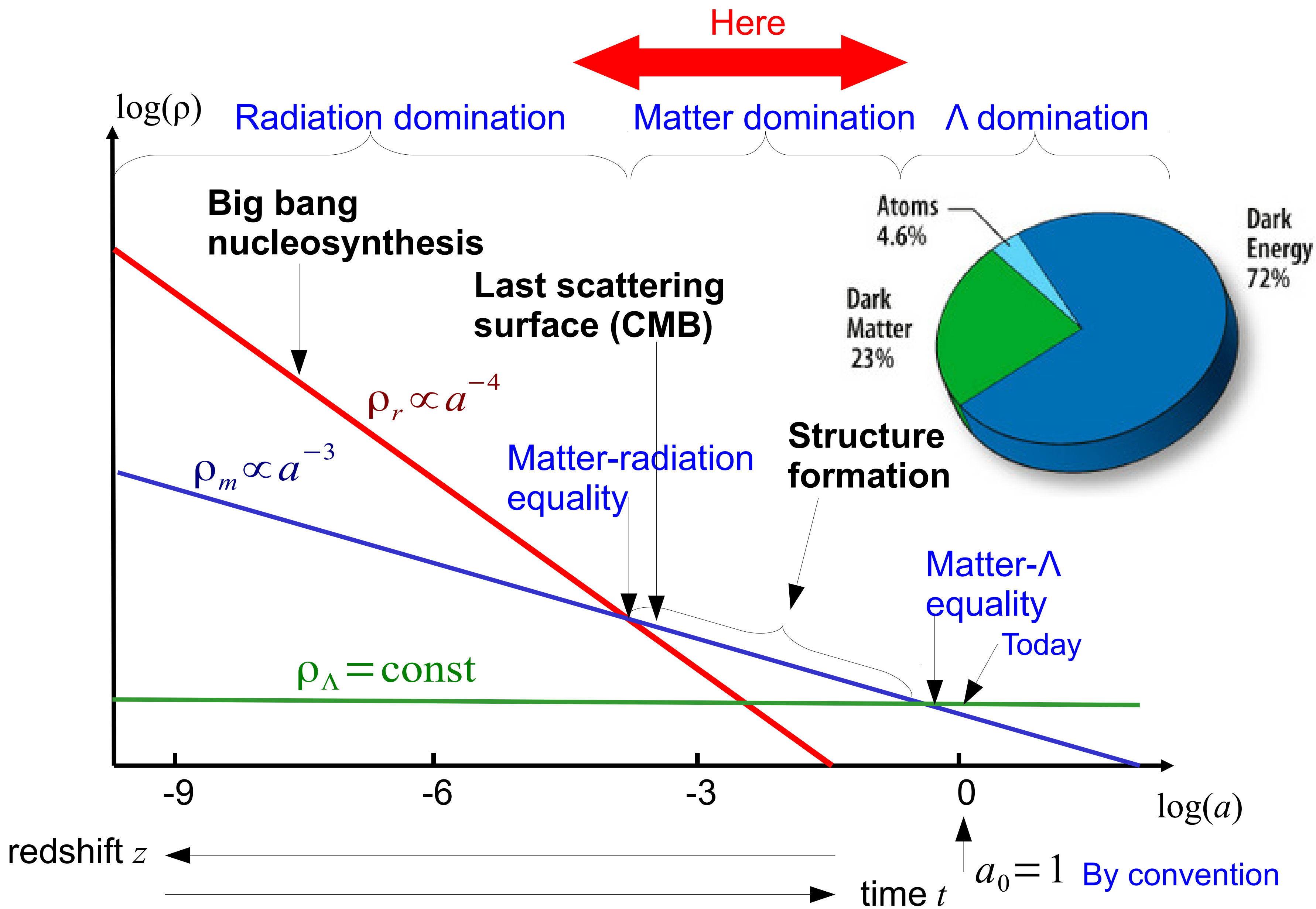
3. Future prospects

Useful references...

- **Lecture notes**
  - E. Bertschinger, *Cosmological dynamics*, astro-ph/9503125.

- **Reviews**
  - J. Lesgourgues & S. Pastor, *Massive neutrinos and cosmology*, Phys. Rep. **429** (2006) 307 [astro-ph/0603494].
  - Y. Y. Y. Wong, *Neutrino mass in cosmology: status and prospects*, Ann. Rev. Nucl. Part. Sci. **61** (2011) 69 [arXiv:1111.1436].

- **Textbooks**
  - J. Lesgourgues, G. Mangano, G. Miele & S. Pastor, *Neutrino cosmology*



Here
log(ρ)
Radiation domination
Matter domination
Λ domination
Big bang
nucleosynthesis
Last scattering
surface (CMB)
Atoms
4.6%
Dark
Energy
72%
Dark
Matter
23%
$\rho_r \propto a^{-4}$
$\rho_m \propto a^{-3}$
Matter-radiation
equality
Structure
formation
Matter-Λ
equality
Today
$\rho_\Lambda = \mathrm{const}$
-9
-6
-3
0
log(a)
redshift z
time t
$a_0 = 1$
By convention

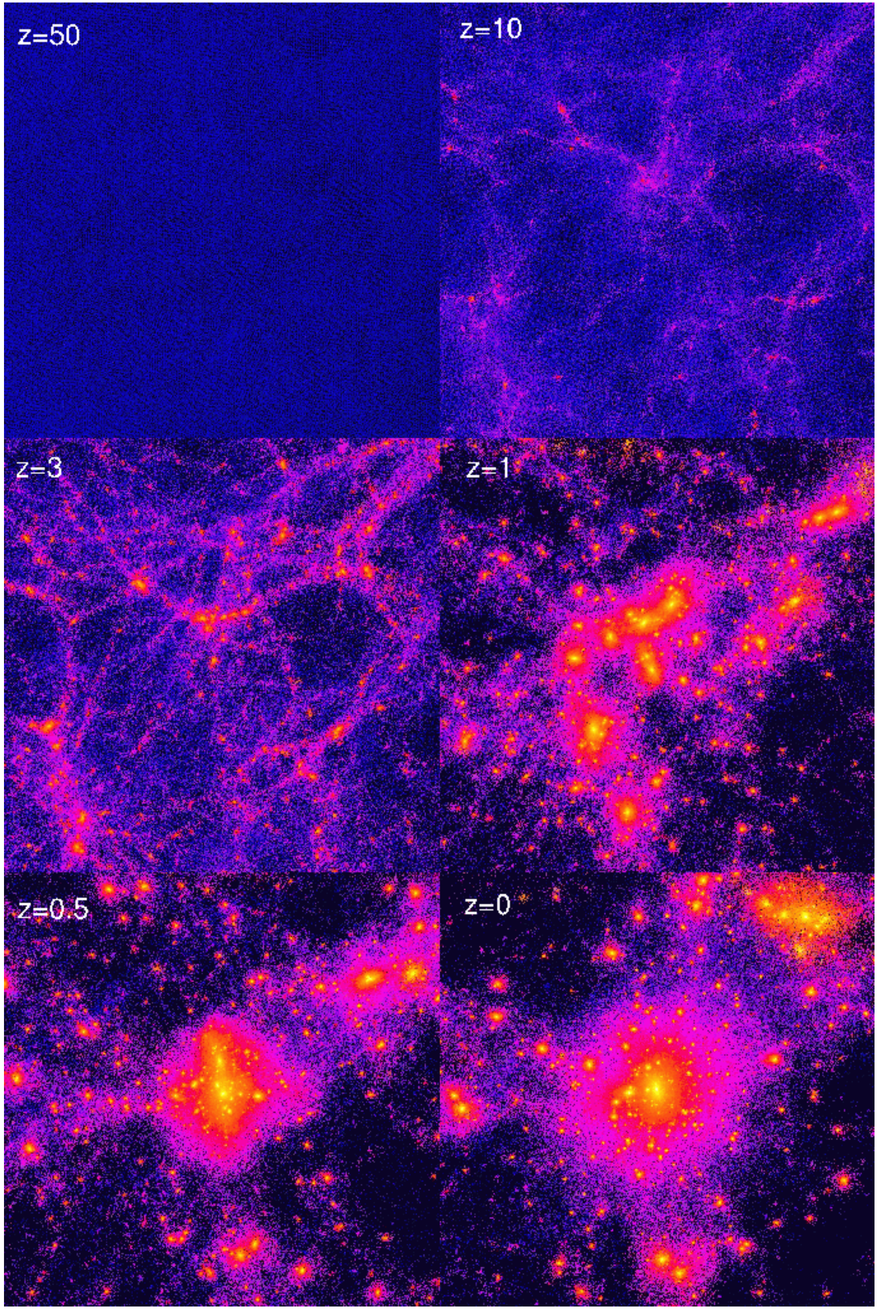
1. Neutrinos and structure formation...

How structures form...

- The early universe is filled with an almost homogeneous matter density field with tiny **random fluctuations**:

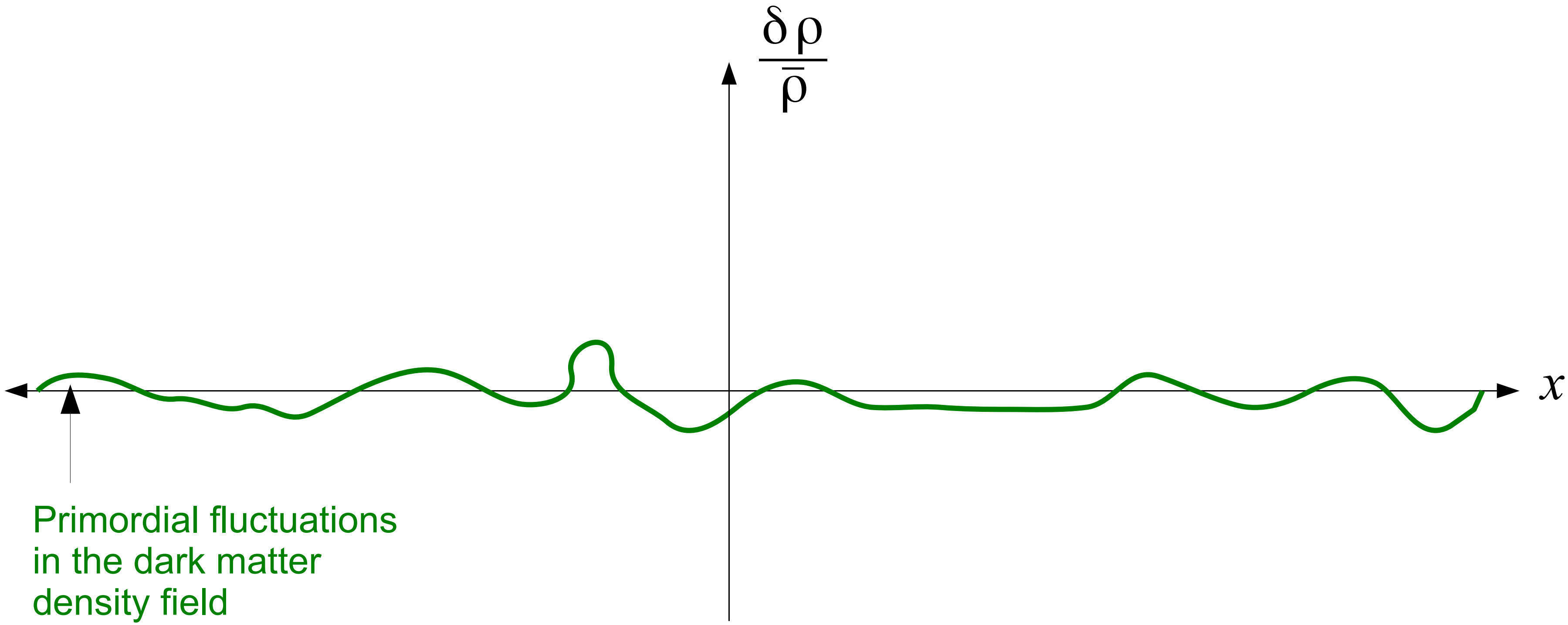
$$\delta \equiv \frac{\delta \rho}{\bar{\rho}}$$

- Perturbations “grow” via **gravitational instability**, and eventually form galaxies and galaxy clusters, etc.
- Leading theory for the origin of small fluctuations is inflation. (Quantum fluctuations on the inflaton field.)



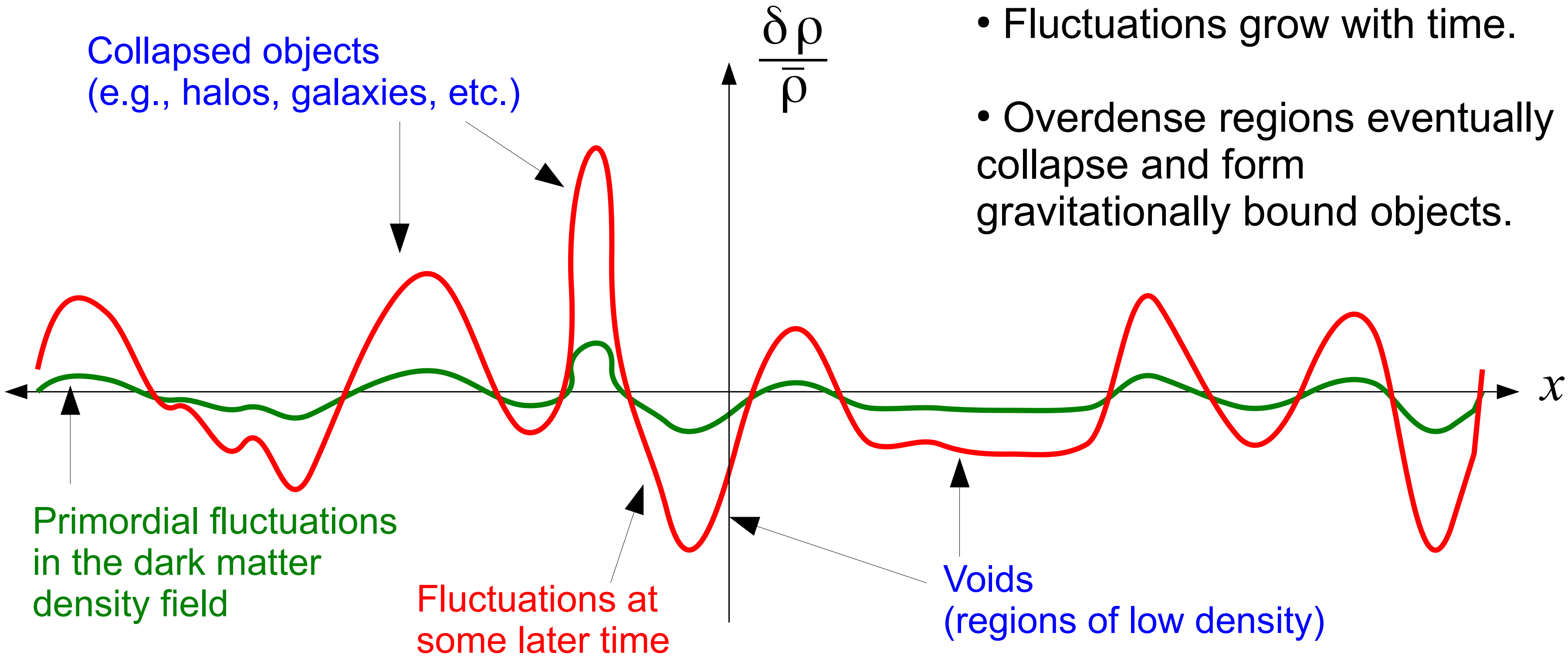
How structures form...

• Primordial fluctuations seeded by, e.g., inflation.



How structures form...

- Initial fluctuations seeded by, e.g., inflation.
- Fluctuations grow with time.
- Overdense regions eventually collapse and form gravitationally bound objects.



Neutrino dark matter...

- Standard hot big bang predicts a relic neutrino background:
  - **Temperature**:

$$T_{\nu,0} = \left(\frac{4}{11}\right)^{1/3} T_{\mathrm{CMB},0} = 1.95\,\mathrm{K}$$

  - **Number density** (per flavour):

$$n_{\nu,0} = \frac{6}{4}\frac{\zeta(3)}{\pi^2} T^3_{\nu,0} = 112\ \mathrm{cm}^{-3}$$

  - **Energy density** (per flavour):

If the relic neutrinos are nonrelativistic today (i.e., $m_\nu > 0.1$ meV)

$$\Omega_{\nu,0} = \frac{m_\nu}{94\,h^2\,\mathrm{eV}}$$

Observations indicate

$$\Omega_{\mathrm{DM}} \sim 0.25$$

Can it be explained by neutrino dark matter?

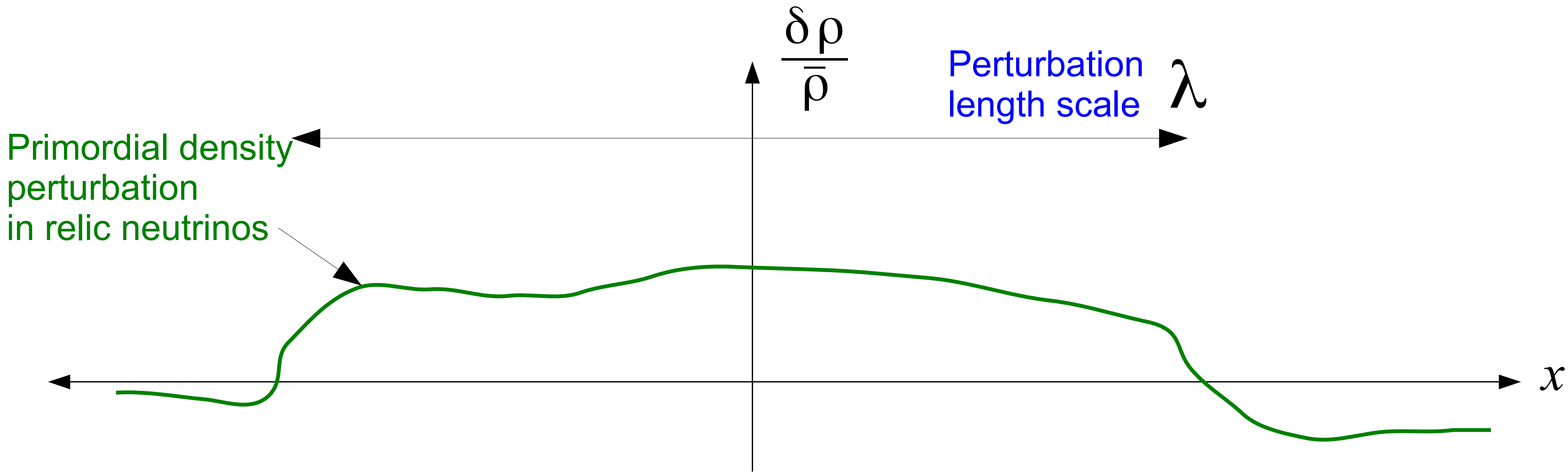
Neutrino dark matter...

- **Answer**: No
- **Reason**:
  - The obvious one: a neutrino mass of ~ 10 eV is needed (not allowed by current tritium β-decay experiments).
  - The deeper one: relic neutrinos come with large thermal motion, with a characteristic **thermal speed**

$$v_{\mathrm{thermal}} = \frac{T_\nu}{m_\nu} \simeq 50.4(1+z)\left(\frac{\mathrm{eV}}{m_\nu}\right) \ \mathrm{km\ s}^{-1}$$

→ Thermal motion counters the effect of gravitational instability.
→ Neutrino gas **does not collapse** because neutrinos fly away!

Suppose relic neutrinos make up **all** of the dark matter...



- **Collapse time scale**:

$$\Delta t_{\text{collapse}} \equiv \left(4\pi G \bar{\rho} a\right)^{-1/2}$$

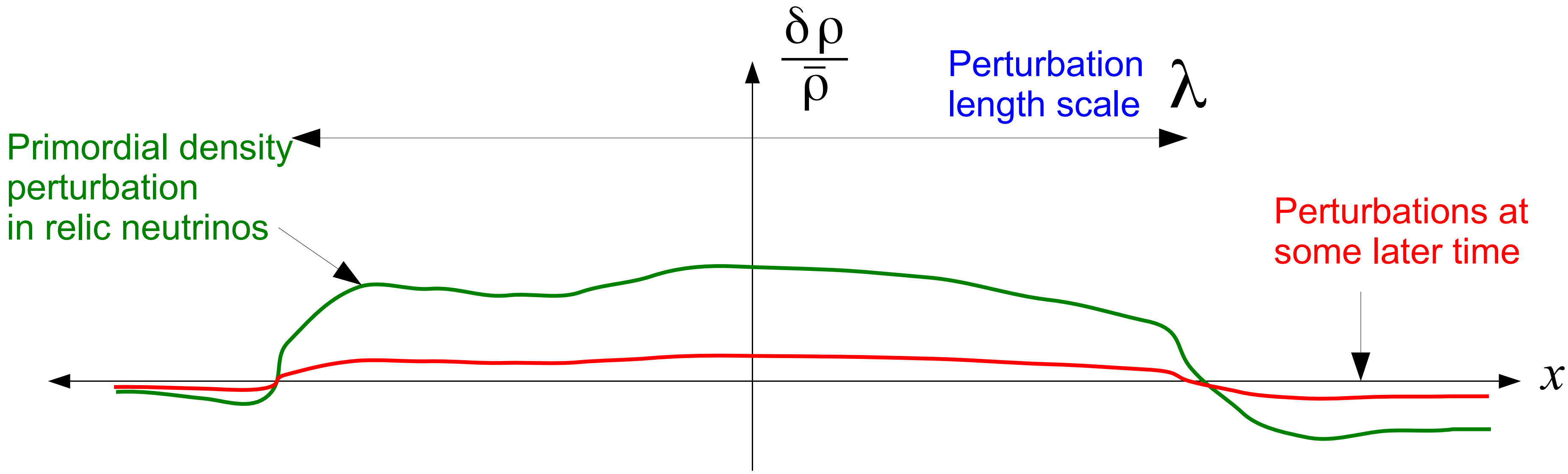
How long does it take for the overdense region to collapse to a point

Escape time scale:

$$\Delta t_{\text{escape}} \equiv \frac{\lambda}{v_{\text{thermal}}}$$

How long does it take for the neutrinos to fly out of the region

Suppose relic neutrinos make up **all** of the dark matter...



- **Collapse time scale**:

$$\Delta t_{\text{collapse}} \equiv (4\pi G \bar{\rho} a)^{-1/2}$$

Escape time scale:

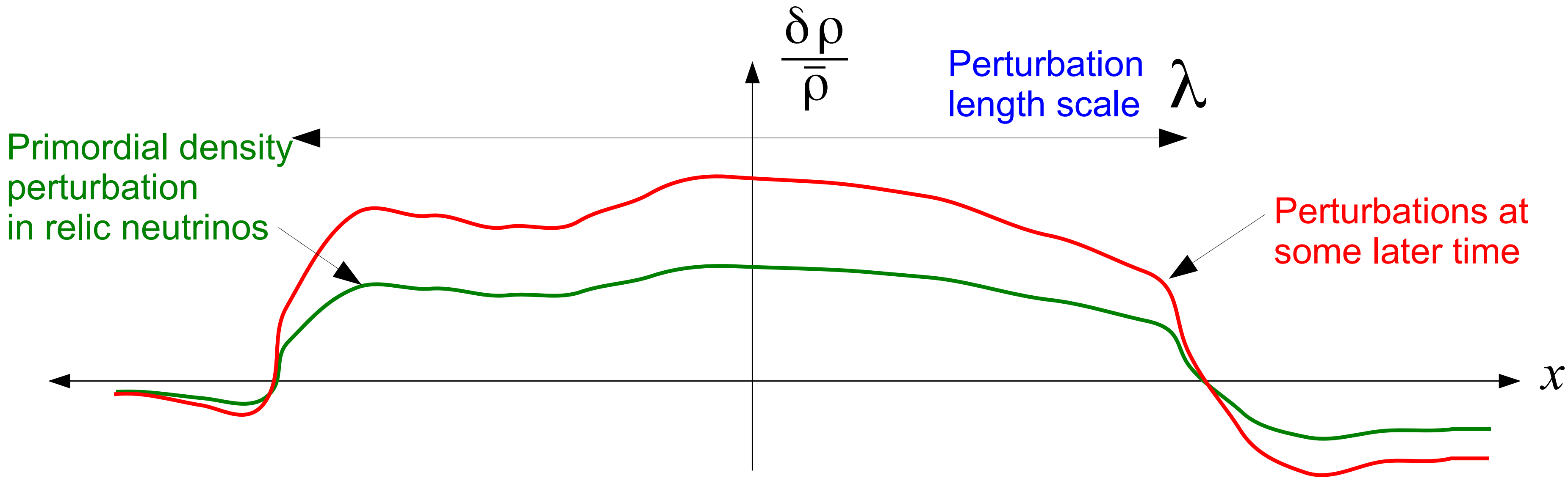
$$\Delta t_{\text{escape}} \equiv \frac{\lambda}{v_{\text{thermal}}}$$

Limit 1: Erasure
Collapse happens **slower** than escape

$$\Delta t_{\text{collapse}} \gg \Delta t_{\text{escape}}$$

→ Neutrinos fly away before gravity can capture them.
→ Perturbation is erased.

Suppose relic neutrinos make up **all** of the dark matter...



- **Collapse time scale**:

$$\Delta t_{\text{collapse}} \equiv (4\pi G \bar{\rho} a)^{-1/2}$$

Escape time scale:

$$\Delta t_{\text{escape}} \equiv \frac{\lambda}{v_{\text{thermal}}}$$

Limit 2: Growth
Collapse happens **faster** than escape

$$\Delta t_{\text{collapse}} \ll \Delta t_{\text{escape}}$$

→ Density perturbation collapses before neutrinos can fly away.
→ Perturbation grows.

Suppose relic neutrinos make up **all** of the dark matter...

- **Growth or erasure**? Define the free-streaming scale at redshift z:

$$\lambda_{FS}(z) \equiv v_{\mathrm{thermal}} \Delta t_{\mathrm{collapse}}$$
$$= 0.41\, \Omega_{m,0}^{-1/2} (1+z)^{1/2} \left(\frac{\mathrm{eV}}{m_\nu}\right) h^{-1}\, \mathrm{Mpc}$$

Equivalent to
$\Delta t_{\mathrm{collapse}} = \Delta t_{\mathrm{escape}}$

→ Unless density perturbations are regenerated by other means, at any redshift z, structures of length scale $\lambda < \lambda_{FS}(z)$ **cannot be formed** out of relic neutrinos.

Suppose relic neutrinos make up **all** of the dark matter...

- The maximum free-streaming scale is that at the time when neutrinos become nonrelativistic:

$$\lambda_{\mathrm{FS,max}} \equiv \lambda_{\mathrm{FS}}(z_{\mathrm{nr}}) = 31.8\,\Omega_{m,0}^{-1/2}\left(\frac{\mathrm{eV}}{m_\nu}\right)^{1/2} h^{-1}\,\mathrm{Mpc}$$

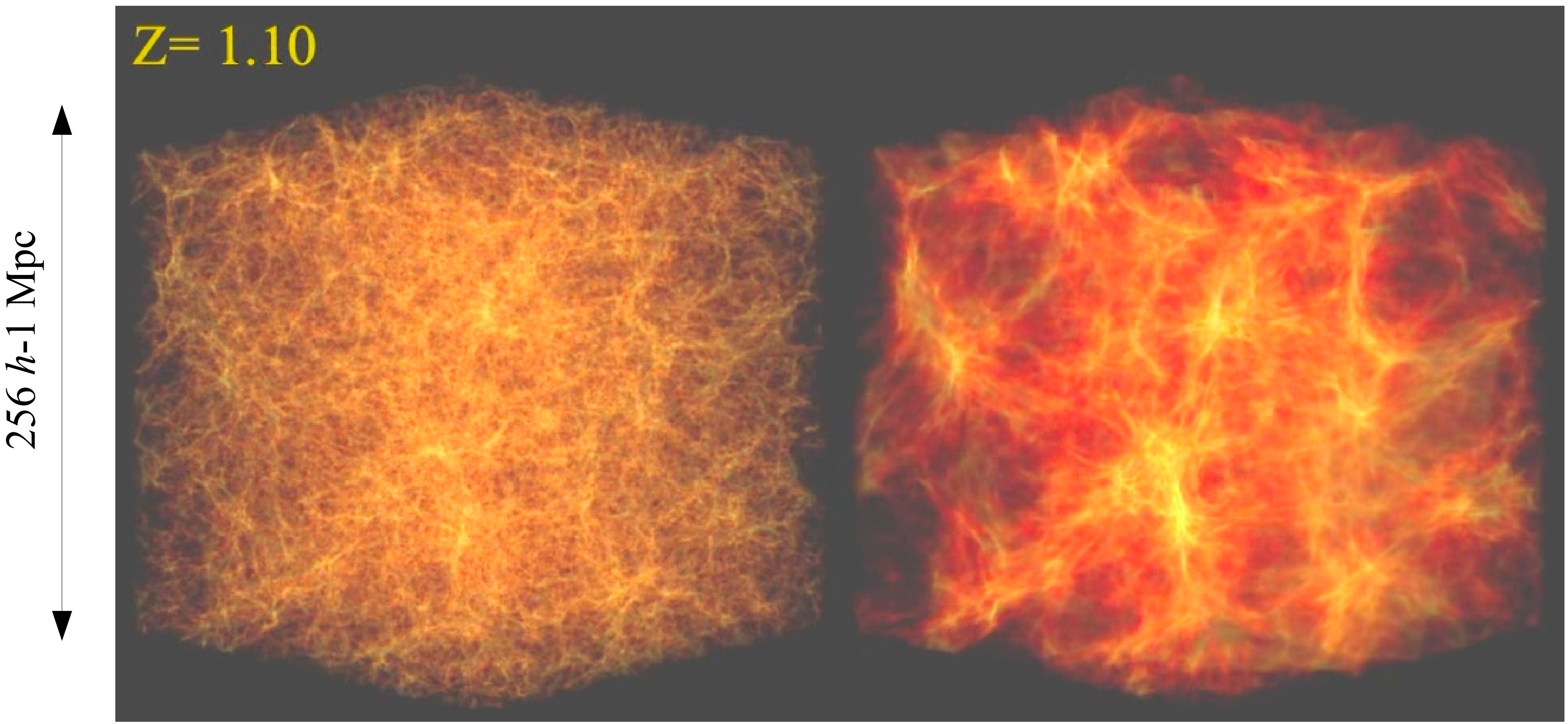
Using

$$1+z_{\mathrm{nr}} \simeq \frac{m_\nu}{T_{\nu,0}}$$

→ $\lambda_{\mathrm{FS,max}}$ corresponds to the maximum size of objects that **could not have been formed** in a neutrino dark matter-only universe.

→ If a 10 eV-mass neutrino was the dark matter, $\lambda_{\mathrm{FS,max}} \sim 25$ Mpc, we would not have galaxies ($\lambda \sim 10$ kpc) and galaxies clusters ($\lambda \sim 1$ Mpc)!

Cold dark matter Massive neutrinos 7 eV



Simulations by Troels Haugbølle

Why study neutrino dark matter then?

- **Because it must be there.**

- Neutrino oscillations indicate that at least one neutrino mass eigenstate has a **mass of > 0.05 eV**.

→ Predictions for cosmology:

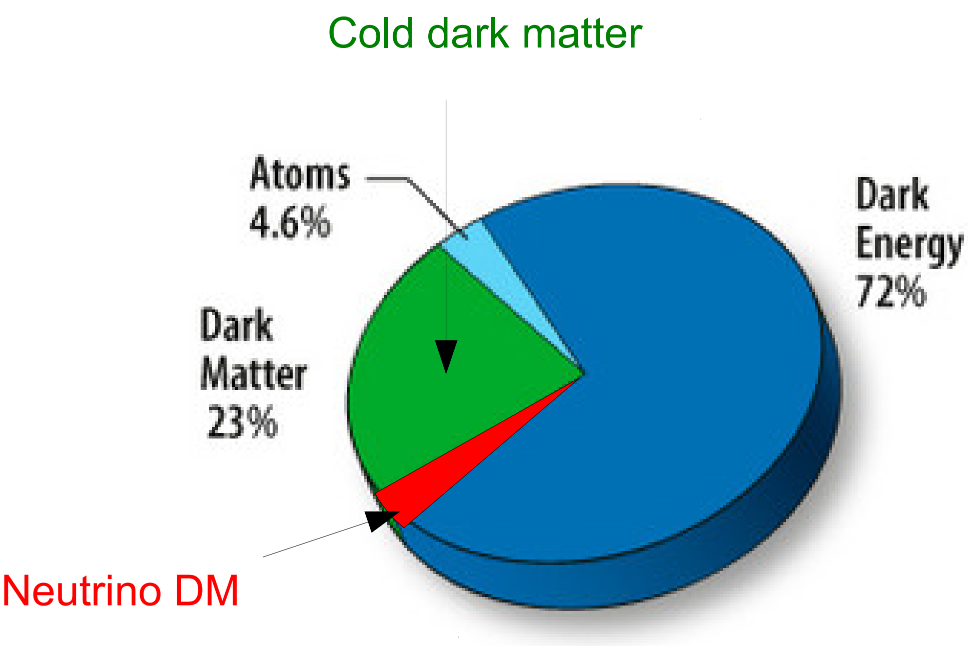
$$\Omega_\nu = \sum \frac{m_\nu}{94\, h^2 \mathrm{eV}} > 0.1\%$$

→ Although only a subdominant DM component, the free-streaming behaviour of neutrino DM still leaves an **imprint** on large-scale structures.

→ Can be used to establish Ω_ν and hence the neutrino mass.

The concordance framework...

- We work within the **ΛCDM framework** extended with a subdominant component of massive neutrino dark matter.
  - Flat geometry.
  - Initial conditions from standard single-field slow-roll inflation.



2. Signatures of subdominant neutrino DM and neutrino mass constraints...

Subdominant neutrino DM and large-scale structure...

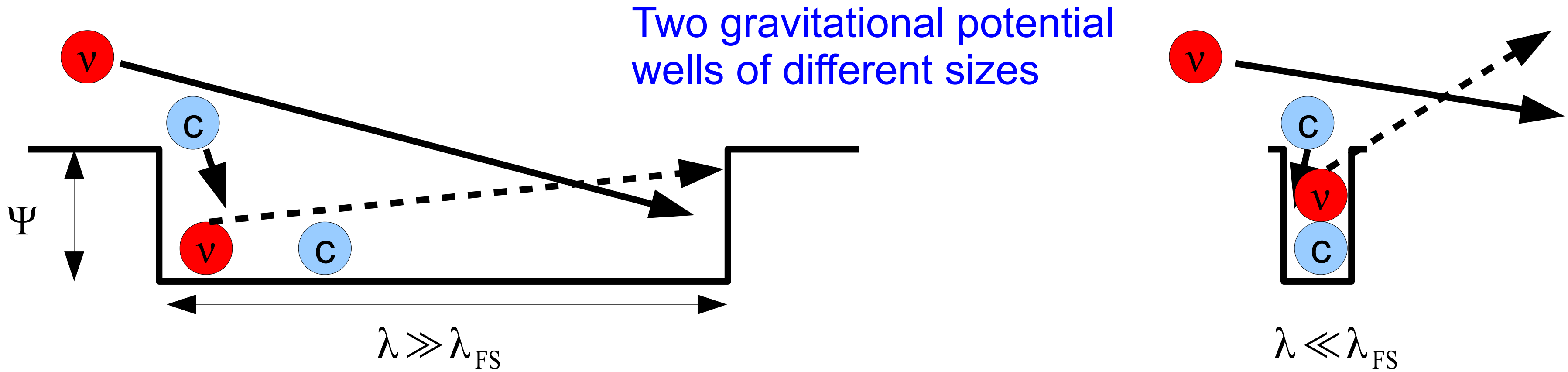
- The presence of CDM acts as a source of density perturbations.

  → Density perturbations on length scales smaller than the neutrino free-streaming scale λ_{FS} are **not completely erased**.

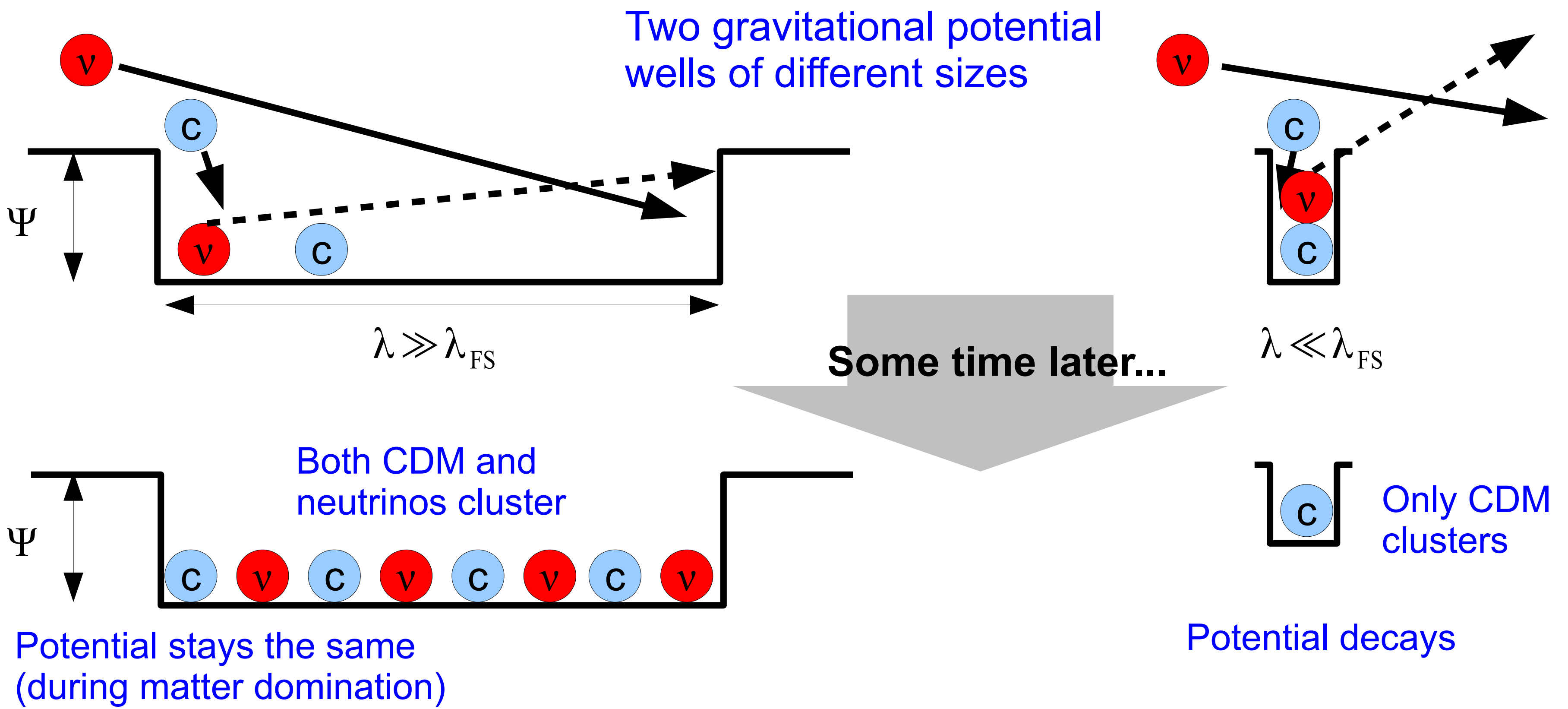
- However, thermal motion of the relic neutrinos still makes neutrino clustering difficult.

  → Expect a suppression in the abundance of structures on scales below λ_{FS} through free-streaming-induced potential decay.

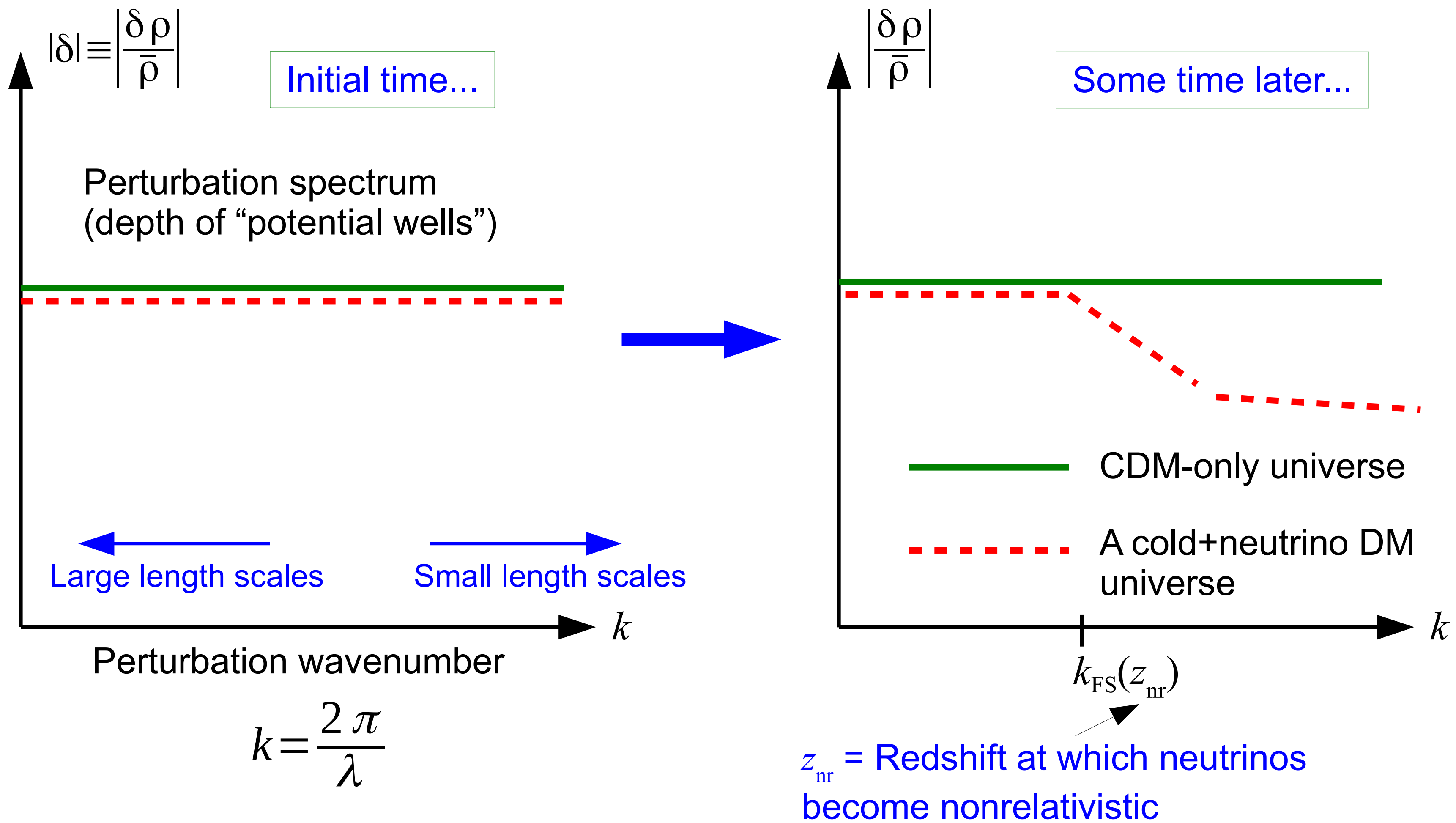
Free-streaming-induced potential decay...



Free-streaming-induced potential decay...

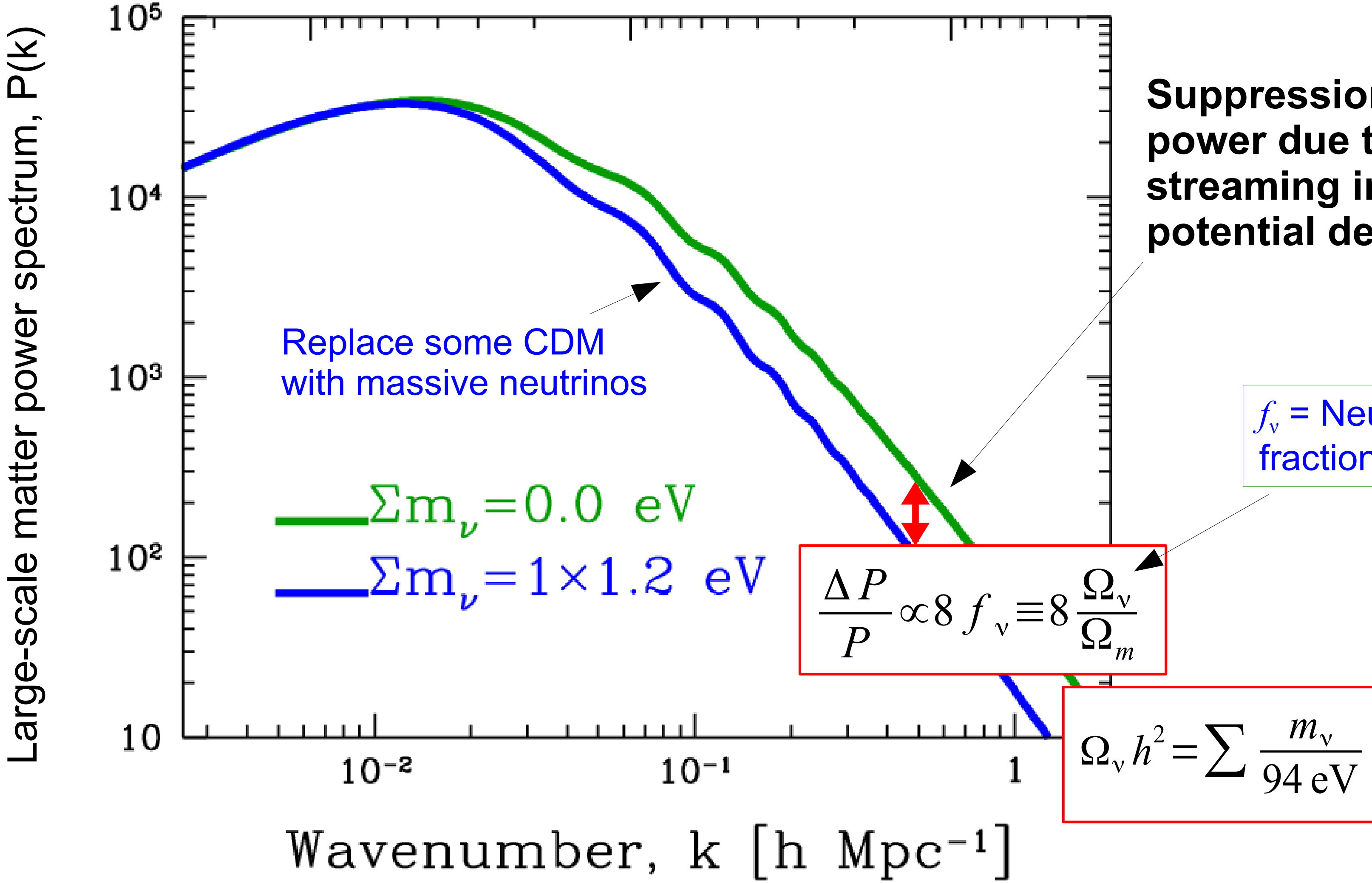


→ **Cosmological neutrino mass measurement** is based on observing this free-streaming induced potential decay at $\lambda << \lambda_{FS}$.

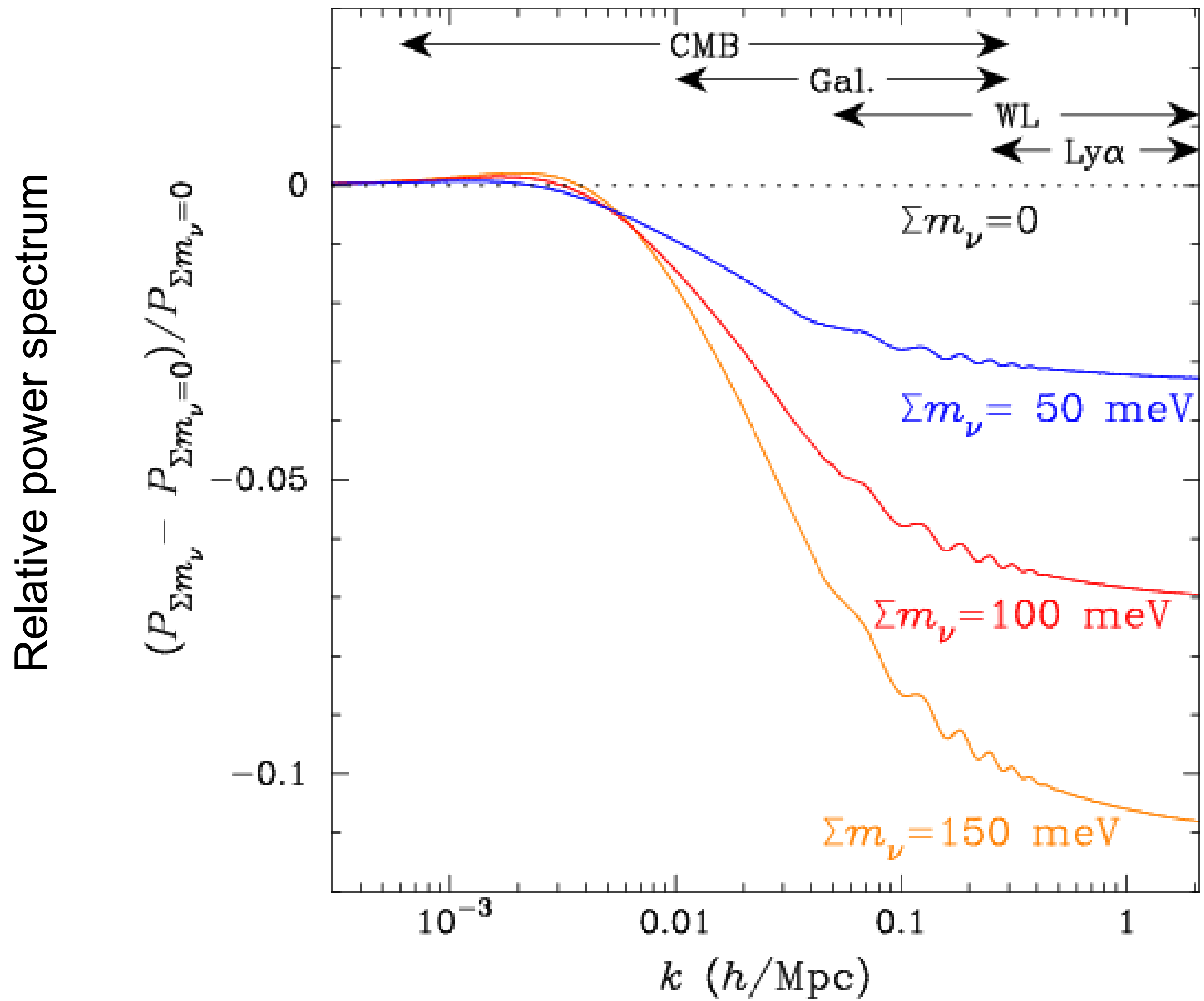


The presence of neutrino dark matter induces a step-like feature in the **spectrum** of gravitational potential wells

Large-scale matter power spectrum...

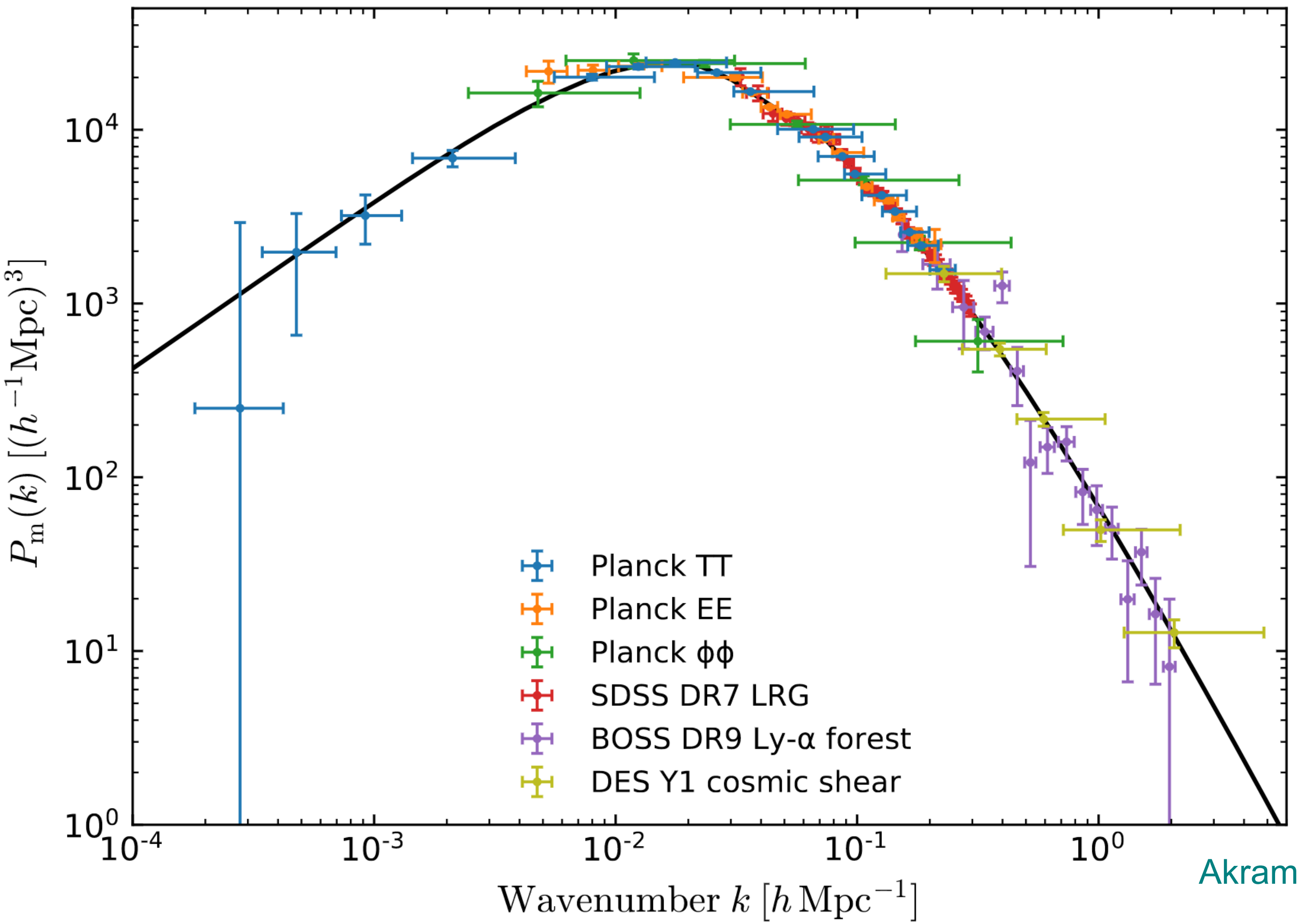


Large-scale matter power spectrum...



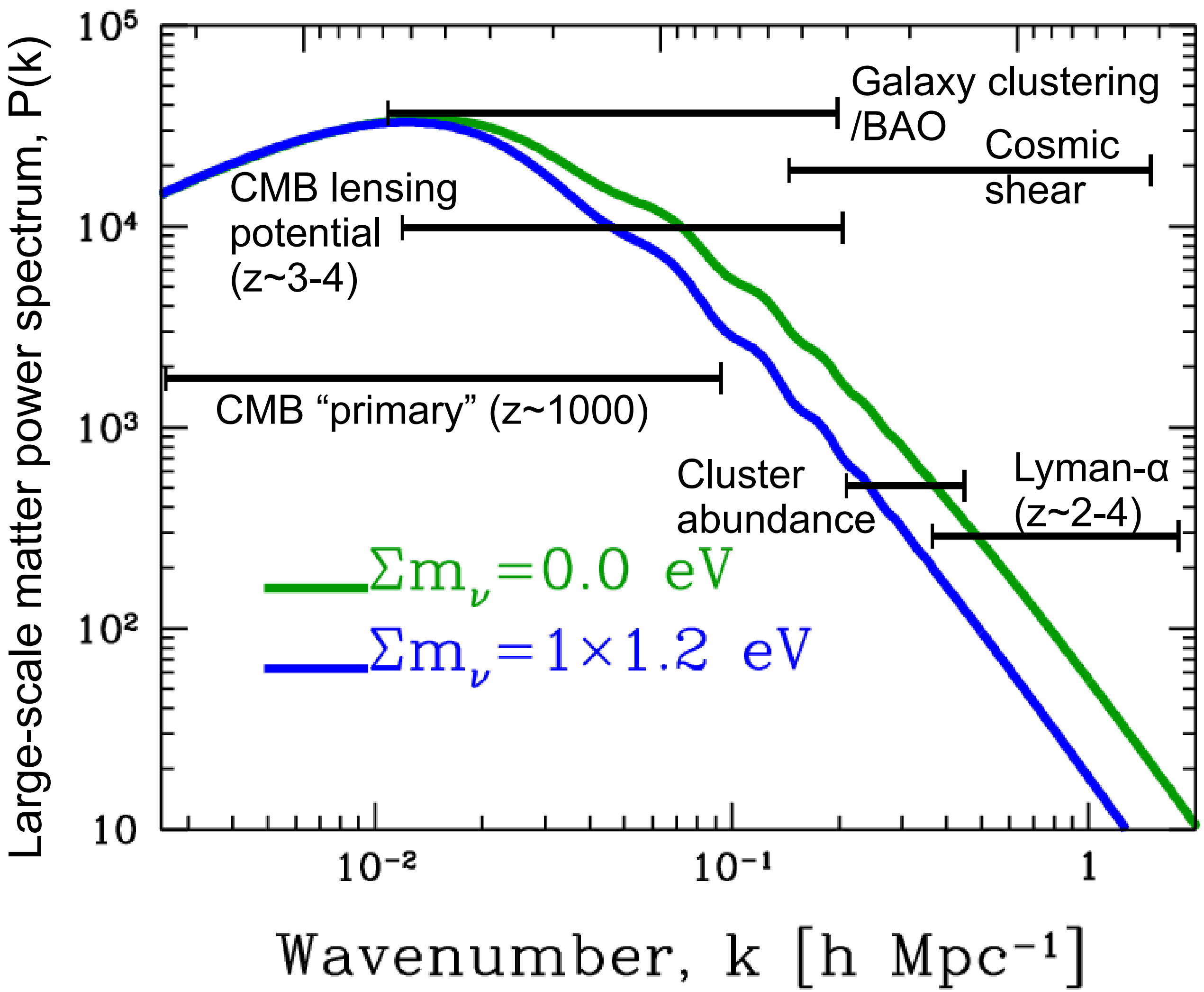
Who can measure it?

Large-scale power spectrum measurements circa 2018

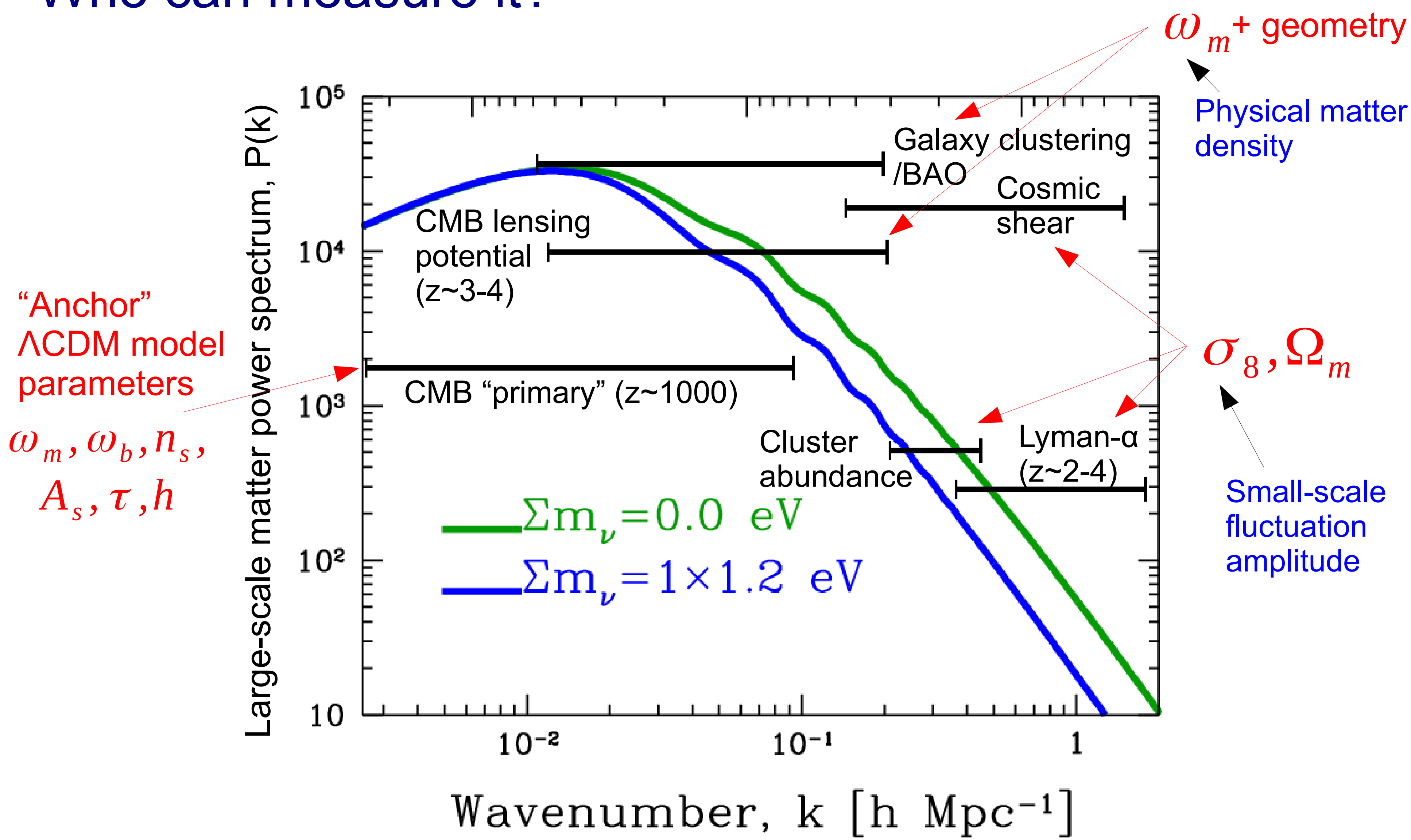


Akrami et al. 2018

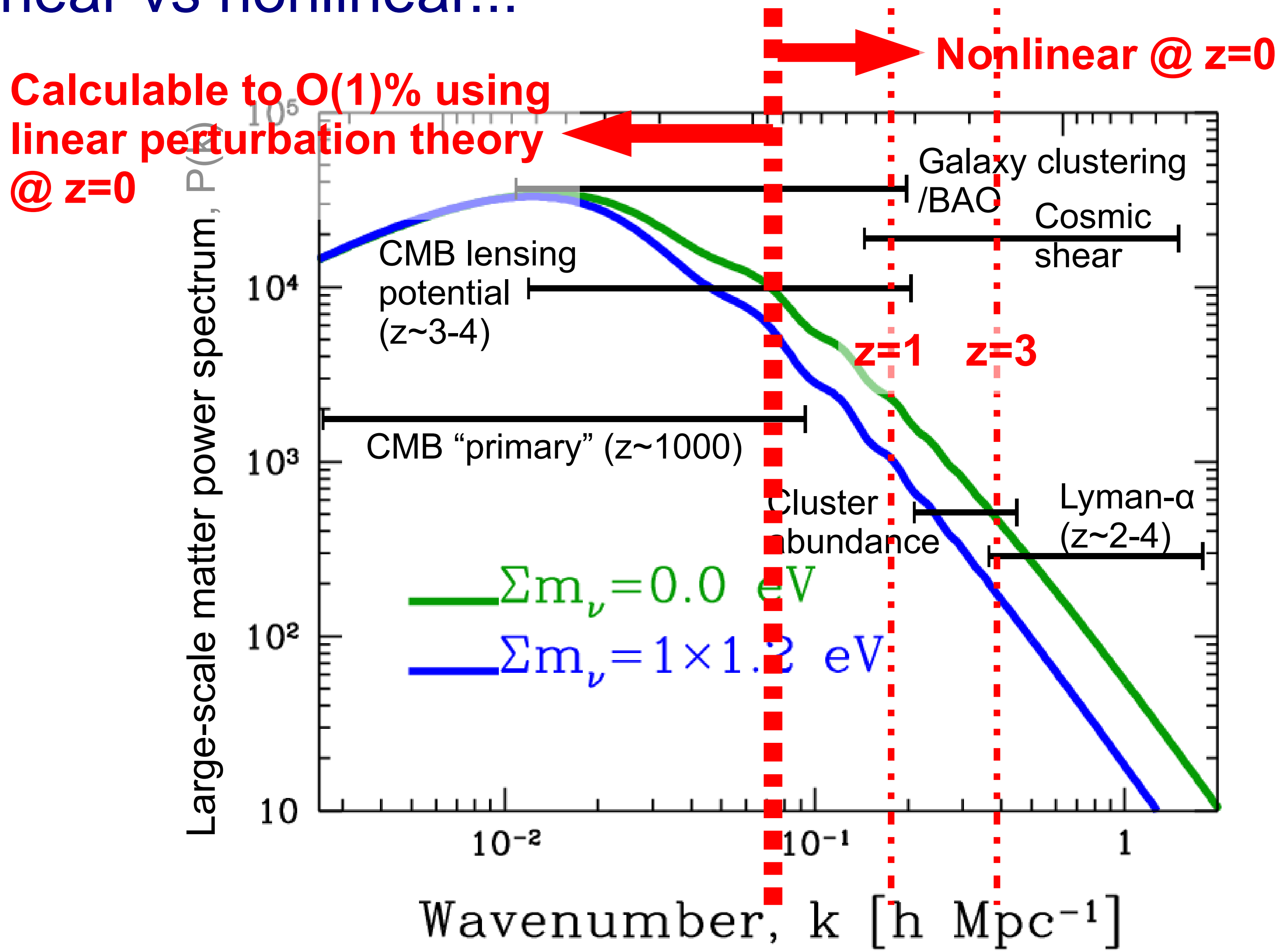
Who can measure it?



Who can measure it?



Linear vs nonlinear...

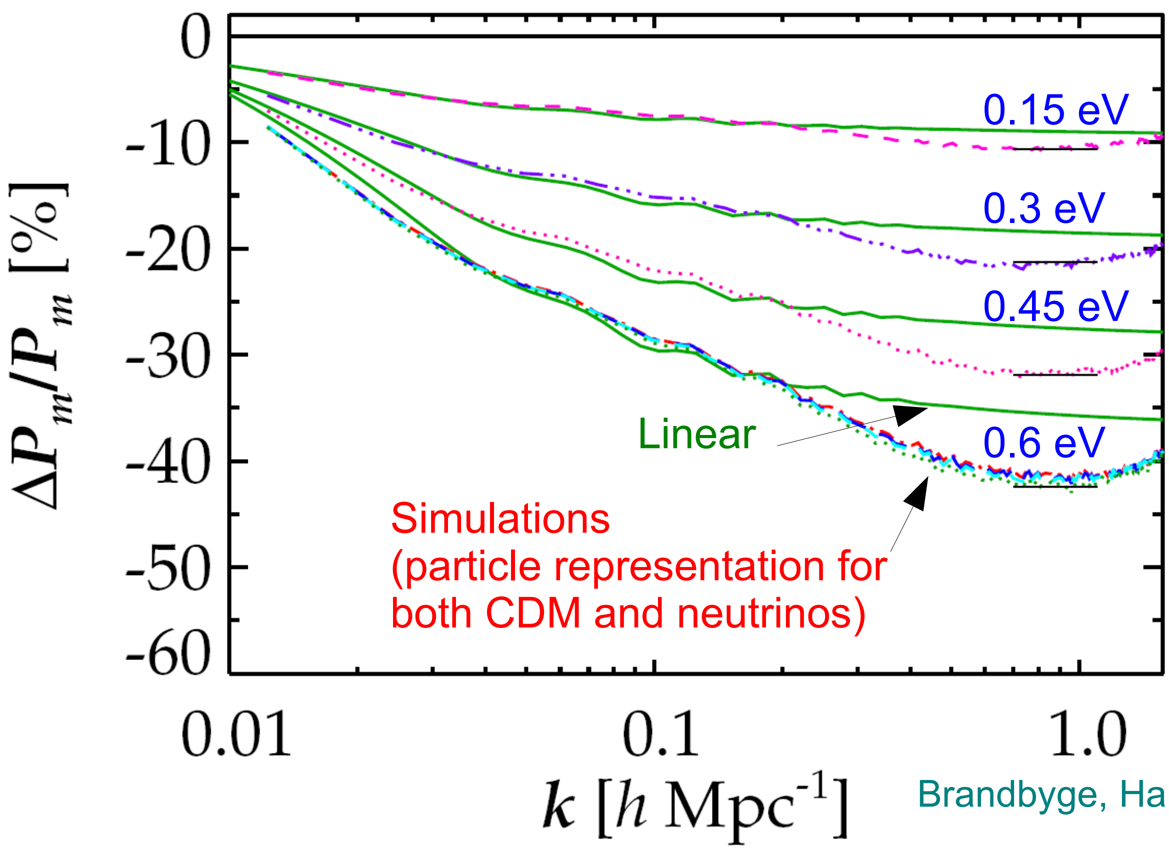


Types and degrees of nonlinearity...

	Nonlinear DM (collisionless)	Baryons @ k < O(1) Mpc^{-1}	Nonlinear tracer bias	Empirical proxy
BAO	Mild	No	Mild	No
Cosmic shear	Yes	No	No	No
Galaxy power spectrum	Yes	No	Yes	No
Cluster abundance	Yes	No	No	Cluster mass vs X-ray temp or richness
Lyman alpha	Yes	Yes	No	No
Calculable from 1st principles?	**Fairly easy**	**No**	**No**	**No**

“Fairly easily” calculable nonlinearities...

Collisionless DM (gravity-only) nonlinearities



Linear perturbation theory:

$$\frac{\Delta P_m}{P_m} \sim 8 \frac{\Omega_\nu}{\Omega_m}$$

With nonlinear corrections:

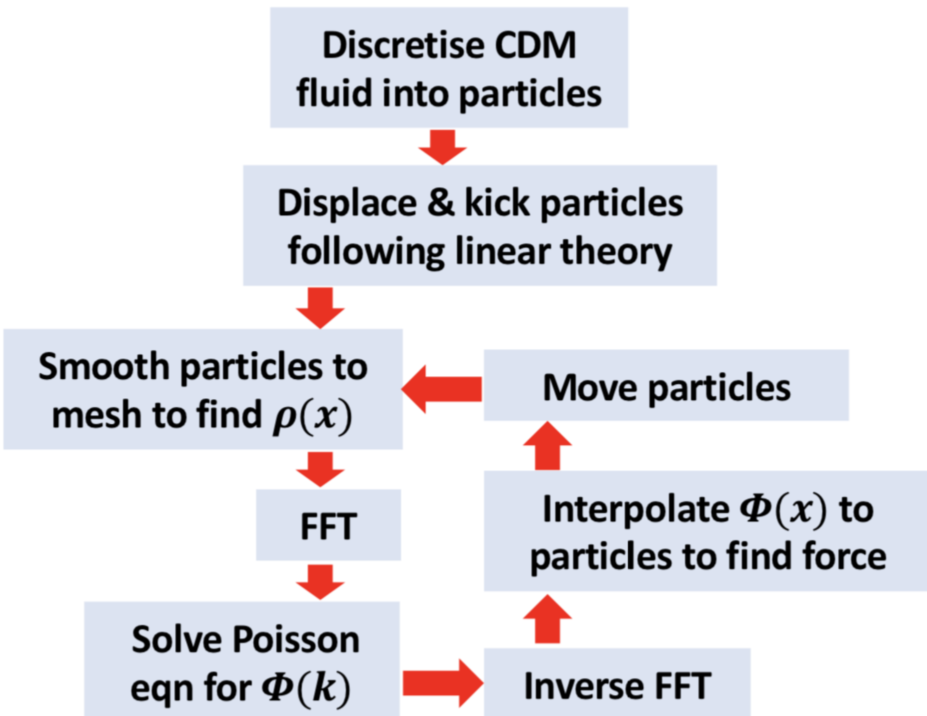
$$\frac{\Delta P_m}{P_m} \sim \underline{9.8} \frac{\Omega_\nu}{\Omega_m}$$

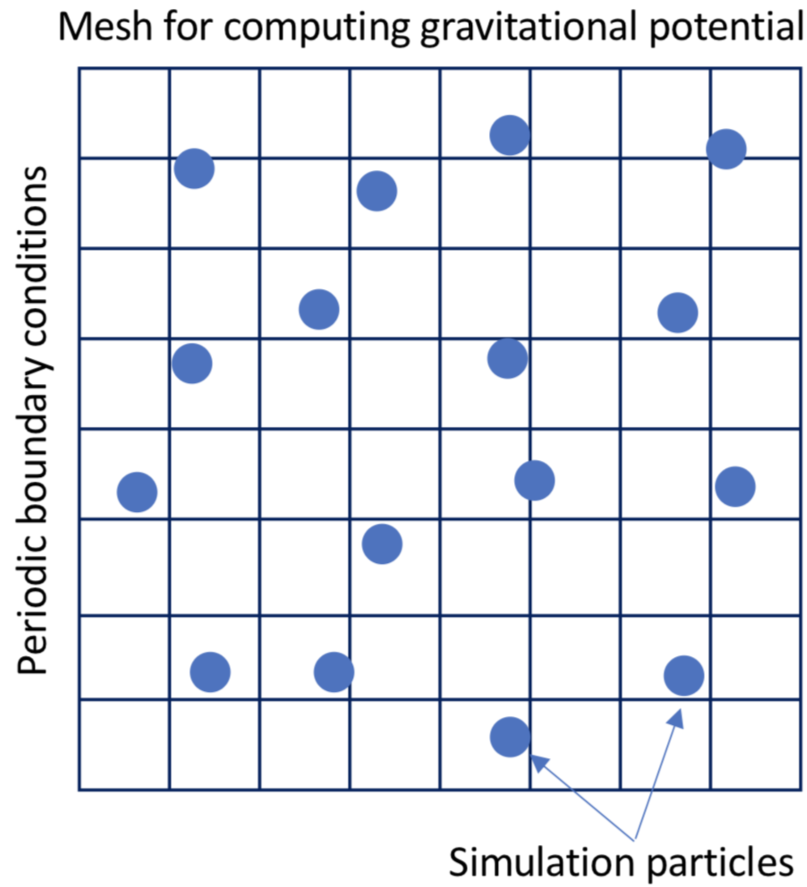
Brandbyge, Hannestad, Haugbolle & Thomsen 2008

N-body simulation of CDM...

A standard method for compute non-linear CDM dynamics.

- A basic particle-mesh (PM) simulation:



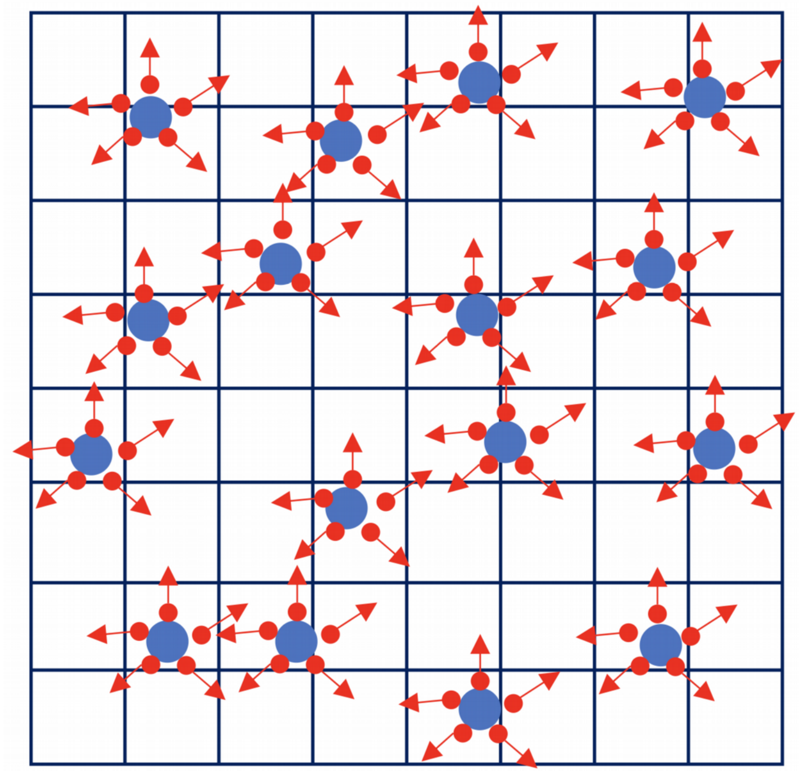


N-body simulation of CDM+neutrinos?

In principle, we can represent the cosmic neutrino background with a few neutrino particles per CDM particles, sampled from the Fermi-Dirac distribution, to model neutrino free-streaming.

- In practice, it is notoriously difficult to get reliable results from this type of simulations because of **shot-noise** and **long run-time**.

  → A lot of recent literature exploring alternative ways to represent the neutrino background.

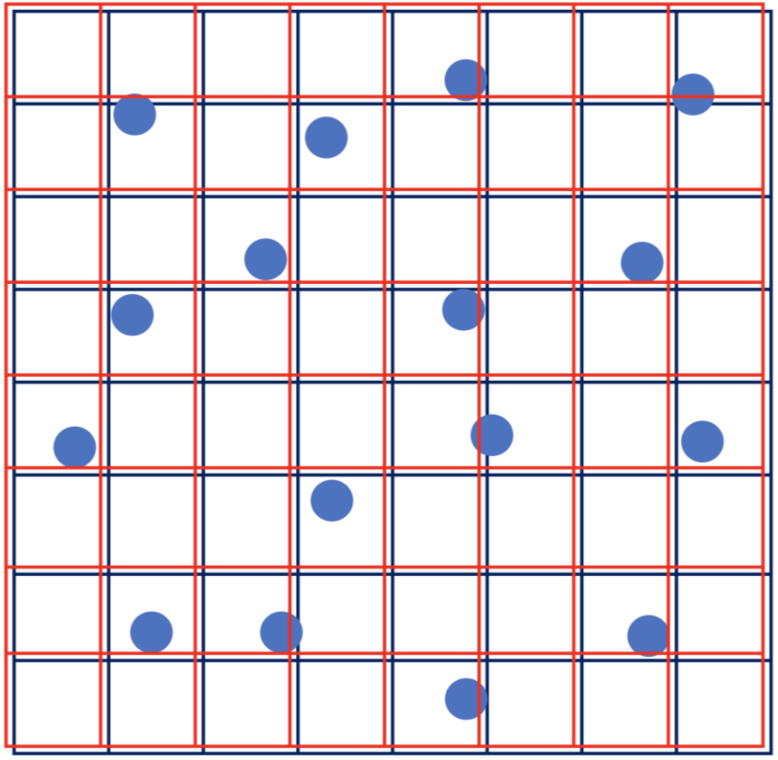


Grid-based neutrino simulations...

Brandbyge & Hannestad 2010
Ali-Haimoud & Bird 2012
Dakin et al. 2019
Chen, Upadhye & Y[3]W 2020a,b

Abandon neutrino particles and work with the mesh instead!

- N-body CDM particles plus solve a set of fluid equations for neutrinos on the mesh.
- Avoid FD sampling noise & fast propagation
- Free-streaming = generally less neutrino clustering than CDM clustering → neutrinos *a prior* **amenable to perturbative treatment**



Our two methods: Chen, Upadhye & Y[3]W 2020a,b

- **SuperEasy linear response**
- **Multi-fluid linear response**

https://github.com/joechenUNSW/gadget4-nu_lr

Take home message...

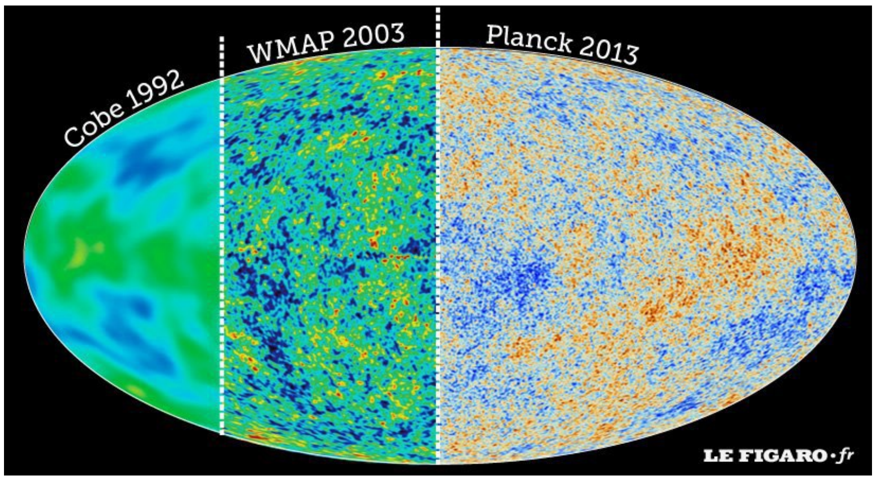
- Signatures of massive neutrinos on the large-scale matter distribution on **linear scales** are well understood.
  - However, precision cosmology is moving to the **nonlinear scales**.
  - To maximise the potential of future cosmological observations to measure/constrain neutrino masses, we need to have %-level accurate predictions on nonlinear scales.
- We have devised two perturbative+N-body methods for this purpose.
  - **SuperEasy linear response:** Simple, low-resource
  - **Multi-Fluid linear response:** Clear pathway to include nonlinear corrections
- Implementation in Gadget-4. Check them out at:

https://github.com/joechenUNSW/gadget4-nu_lr

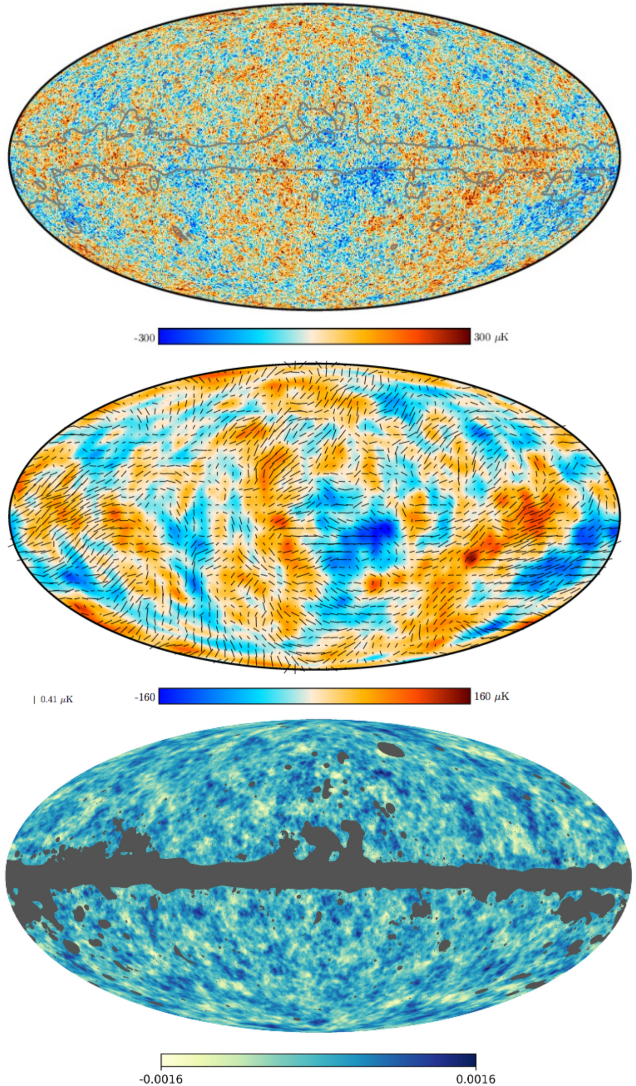
1a. Neutrino masses and Planck 2018

ESA Planck mission...

State-of-the-art measurements of the temperature and polarisation fluctuations in the **cosmic microwave background**. (Latest results 2018.)



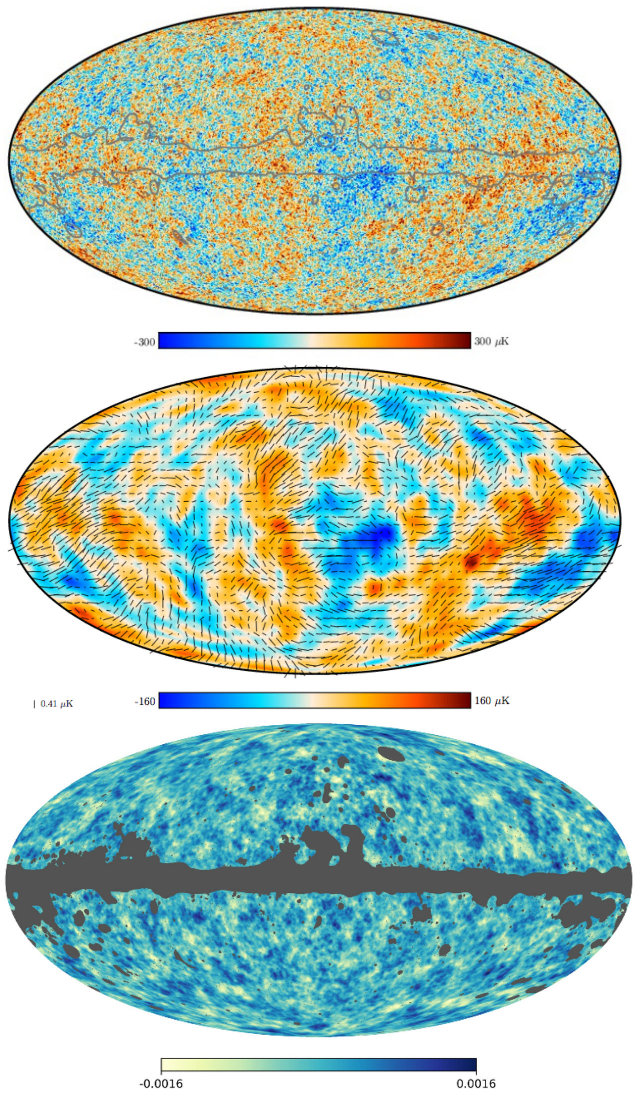
Three CMB observables...



Temperature:

- Neutrino mass signatures.
- Cosmic-variance-limited to ℓ ~ 2000 since 2013 (i.e., nothing more to be done here)

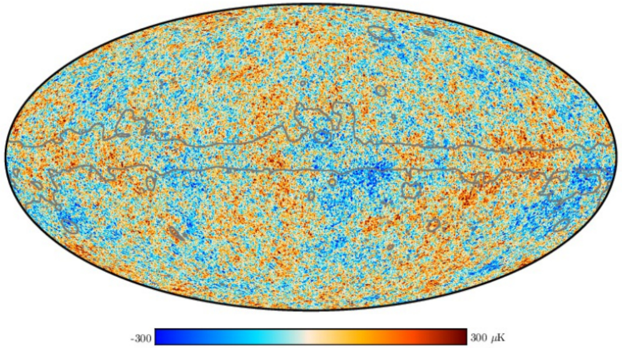
Three CMB observables...

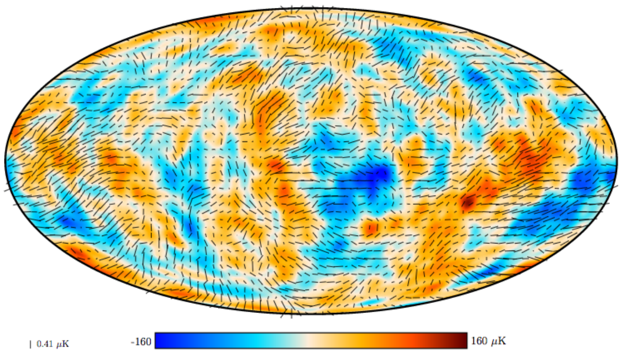


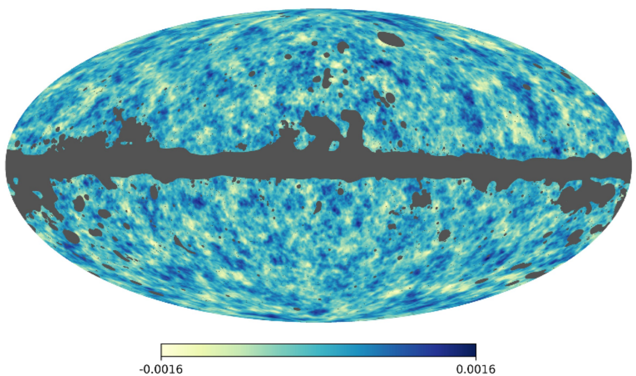
Polarisation:

- No independent neutrino mass signature.
- Low multipoles lifts A_s-τ degeneracy, which helps to tighten other parameter constraints.
- **Planck 2018 vs 2015**: improved measurement and modelling of the likelihood functions.

Three CMB observables...



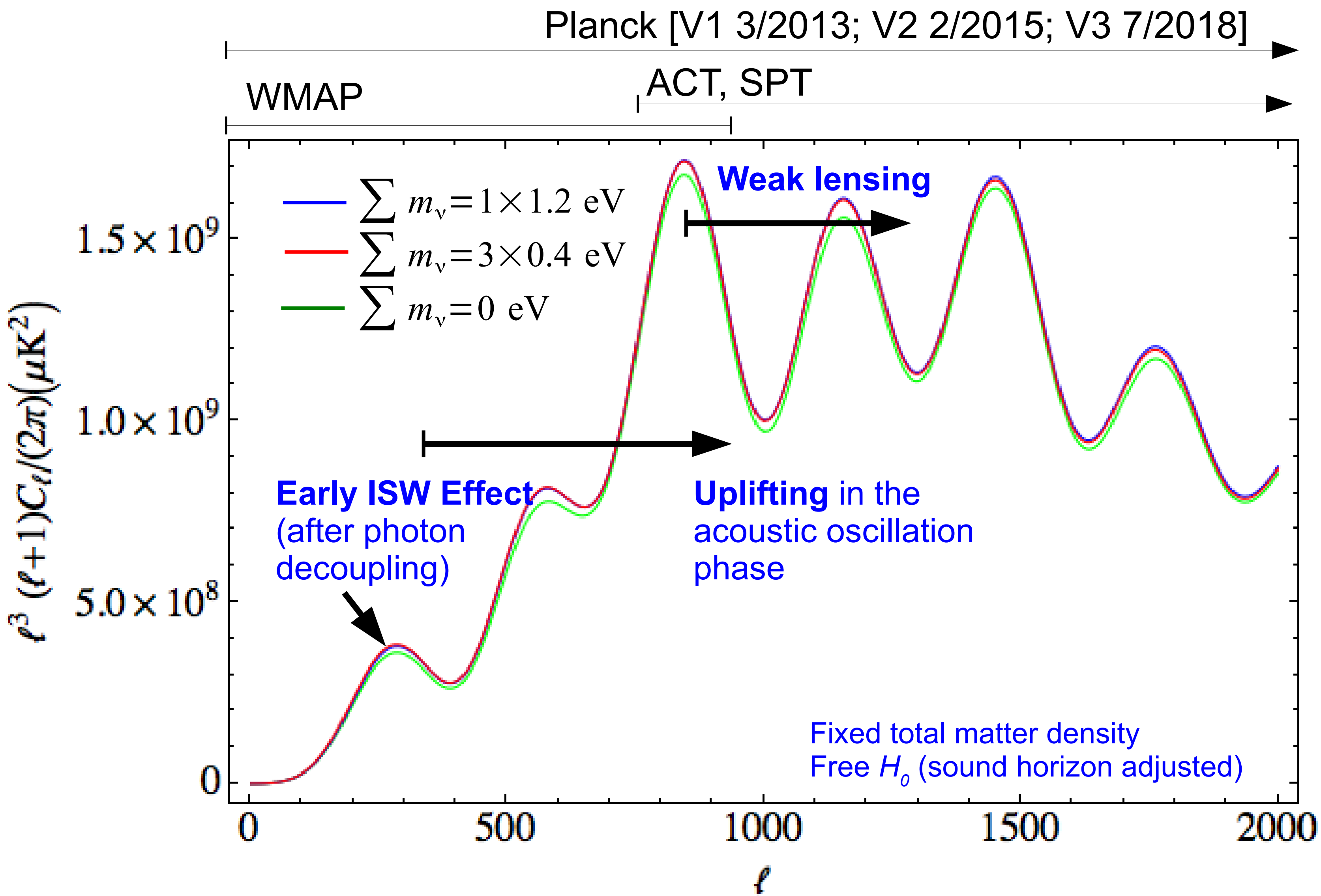




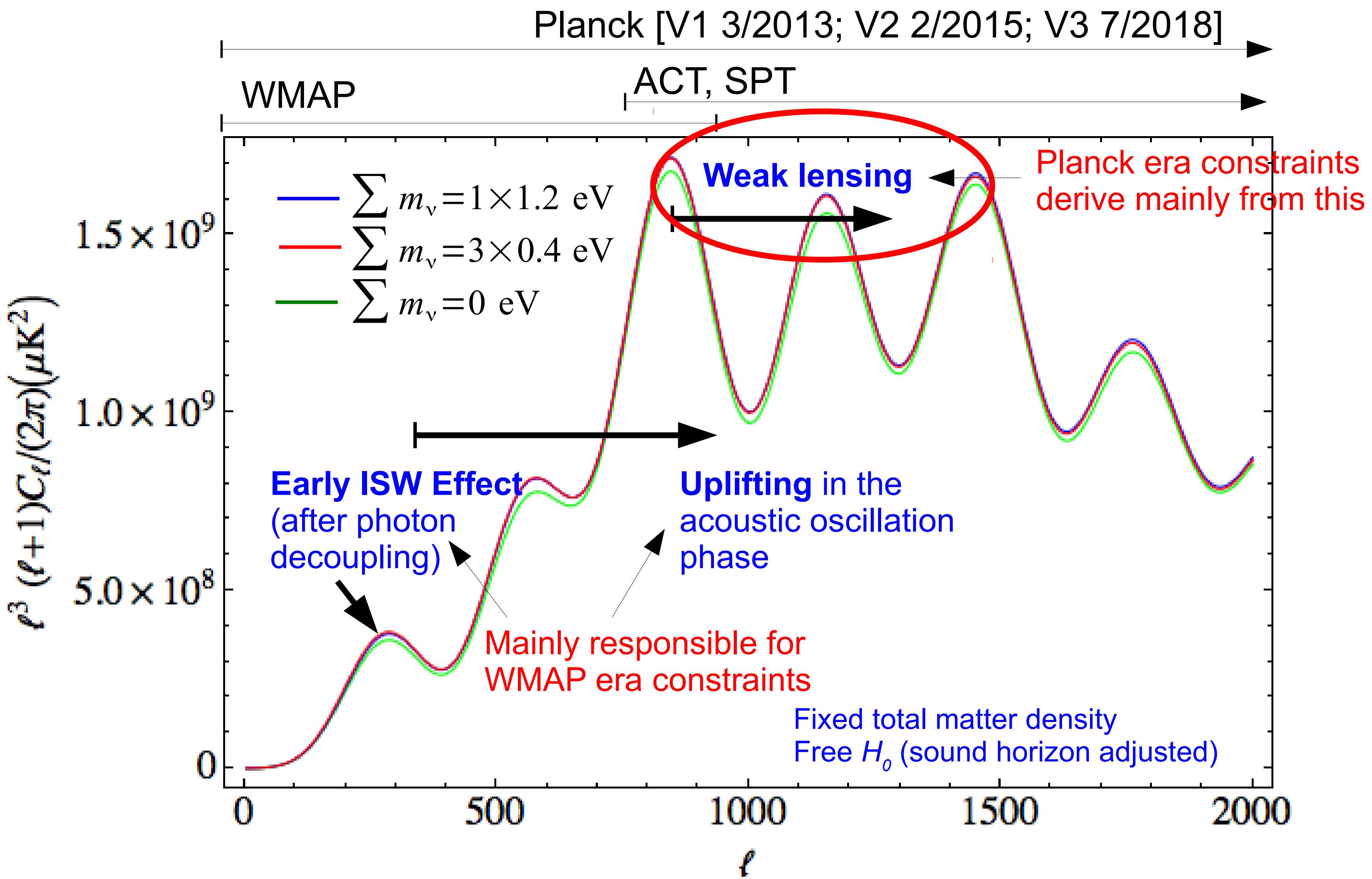
Lensing potential:

- Secondary observable reconstructed from temperature (present) and/or polarisation (future) maps.
- Contains independent neutrino mass signatures.

Neutrino mass and the CMB temperature...

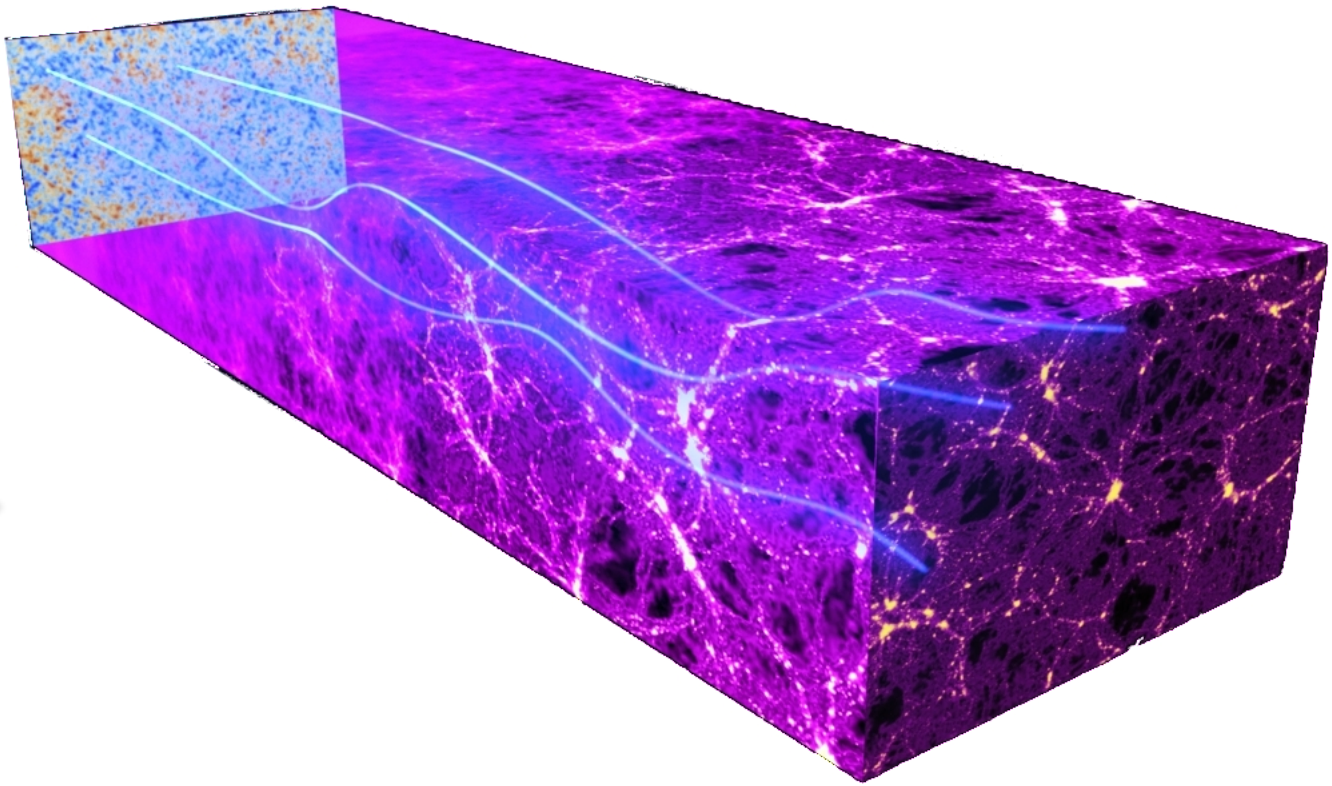


Neutrino mass and the CMB temperature...

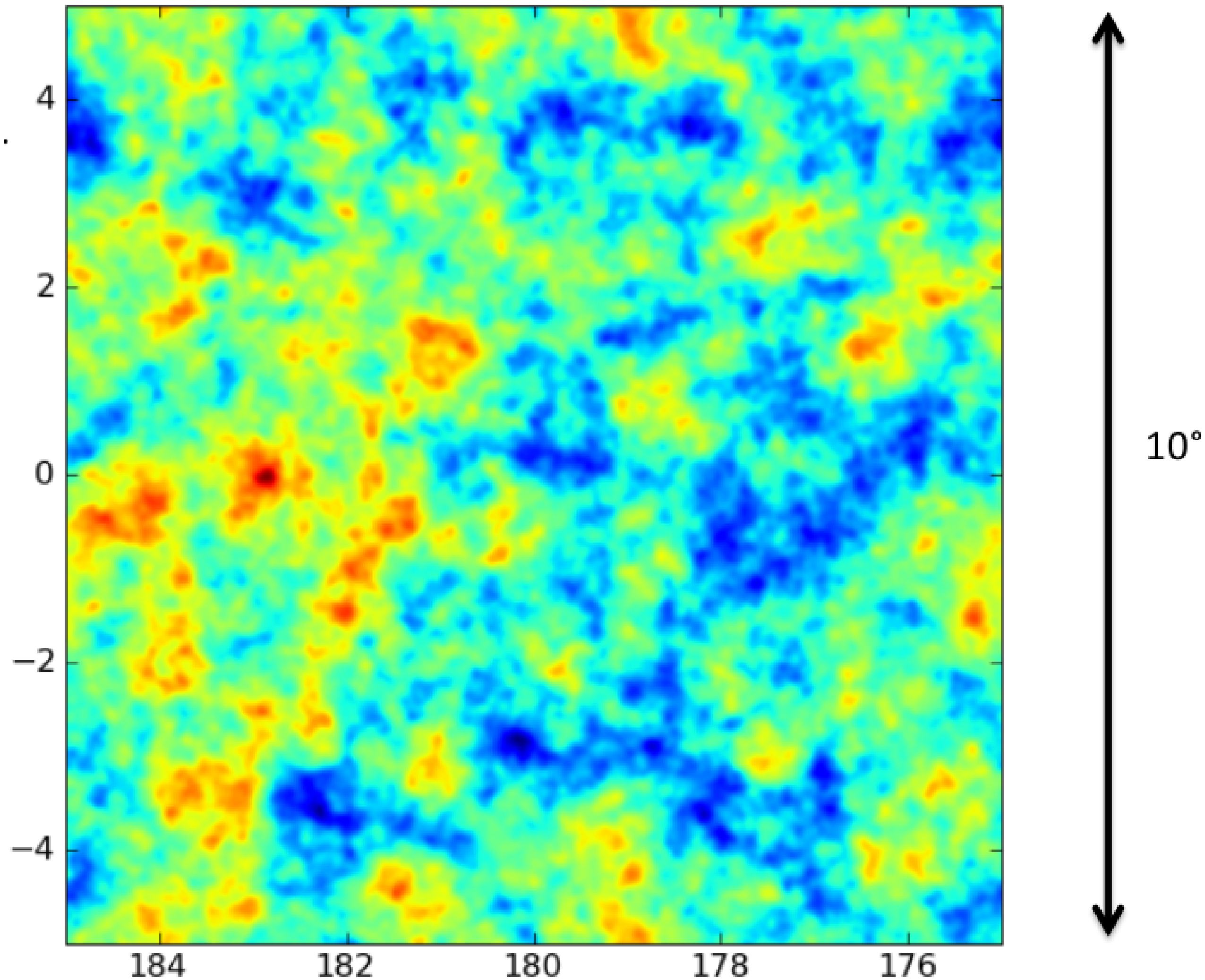


Weak lensing of the CMB...

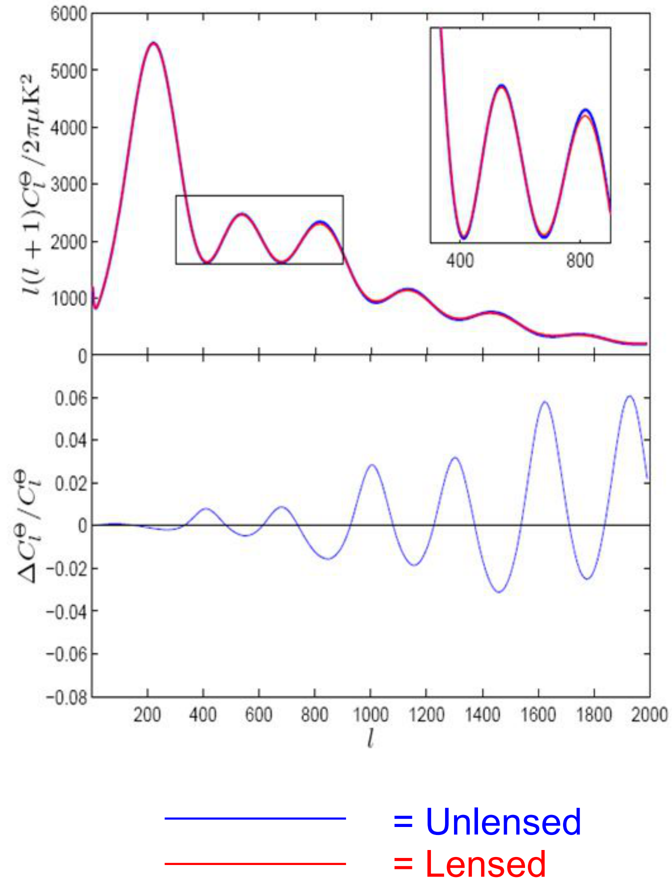
CMB photons are deflected by the intervening matter distribution, leading to a slightly **distorted image** of the last scattering surface.



Unlensed CMB

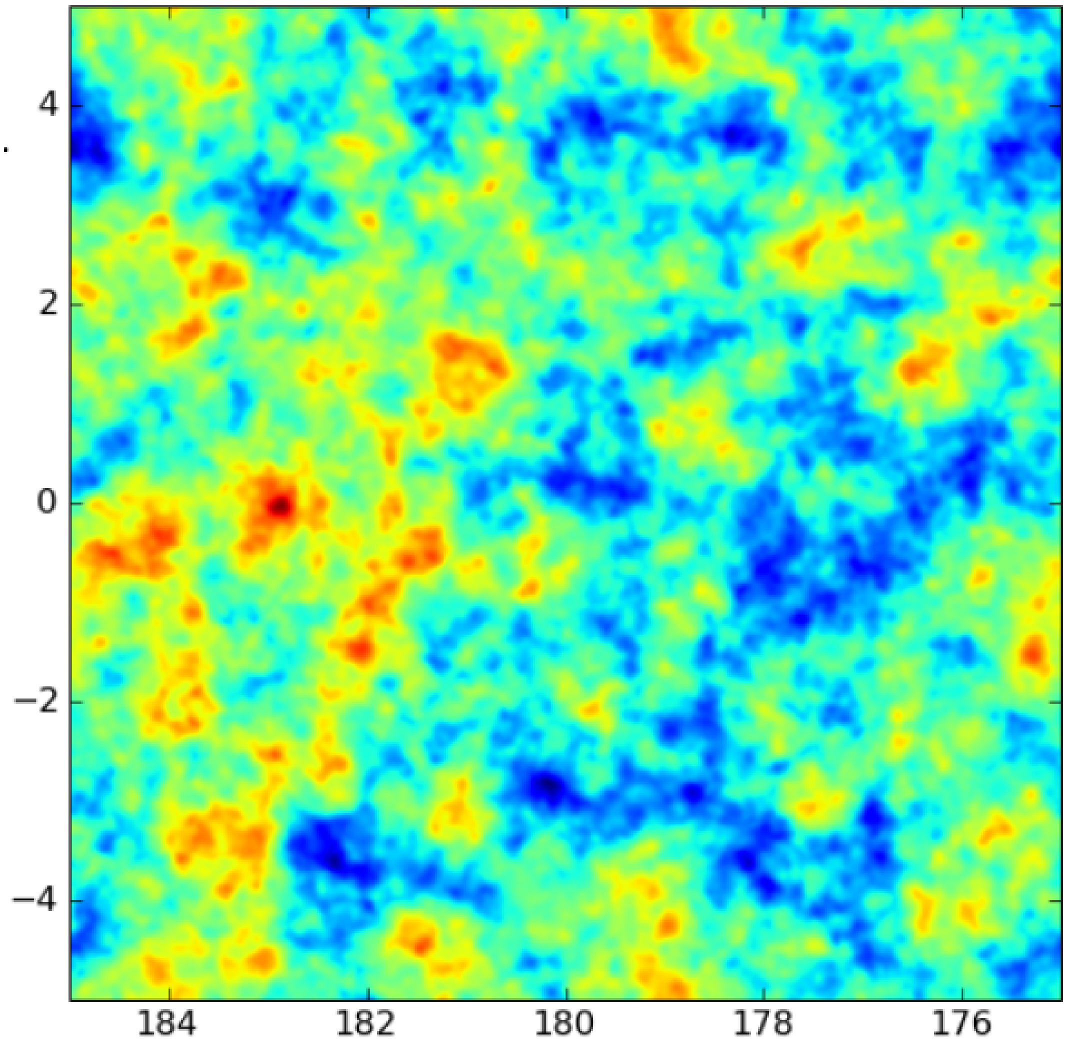


Smearing of the TT power spectrum at ℓ > 500



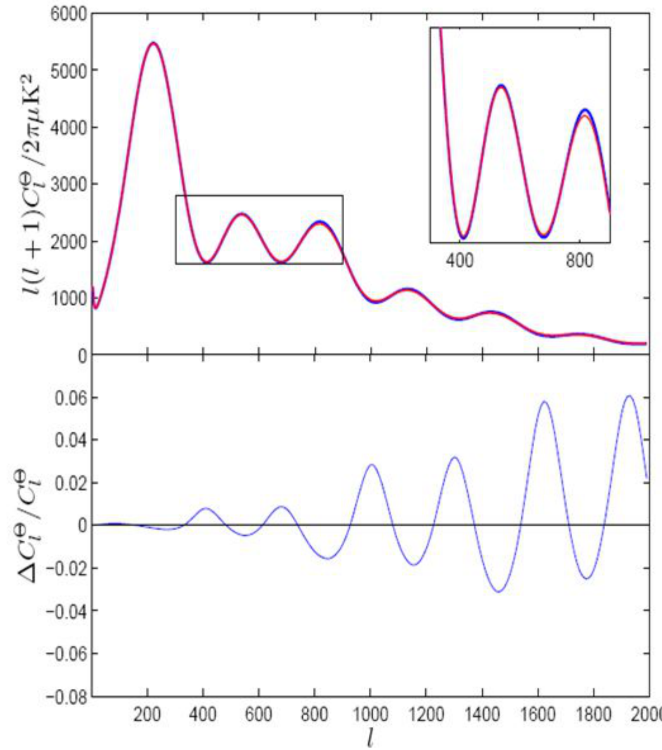
From Blake Sherwin

Lensed CMB



10°

Smearing of the TT power spectrum at ℓ > 500



= Unlensed
= Lensed

From Blake Sherwin

Constraints on the neutrino mass sum…

ΛCDM+neutrino mass 7-parameter fit; 95% C.L. on $\sum m_\nu$ in [eV].

Low-ℓ polarisation only

Planck2018 TT+lowE	**0.54**
2015 numbers	0.72

Plus high-ℓ polarisation

Two different high-ℓ likelihood functions

Planck2018 TT +lowE+TE+EE	**0.26**
Planck2018 TT +lowE+TE+EE [CamSpec]	**0.38**
2015 numbers	0.49

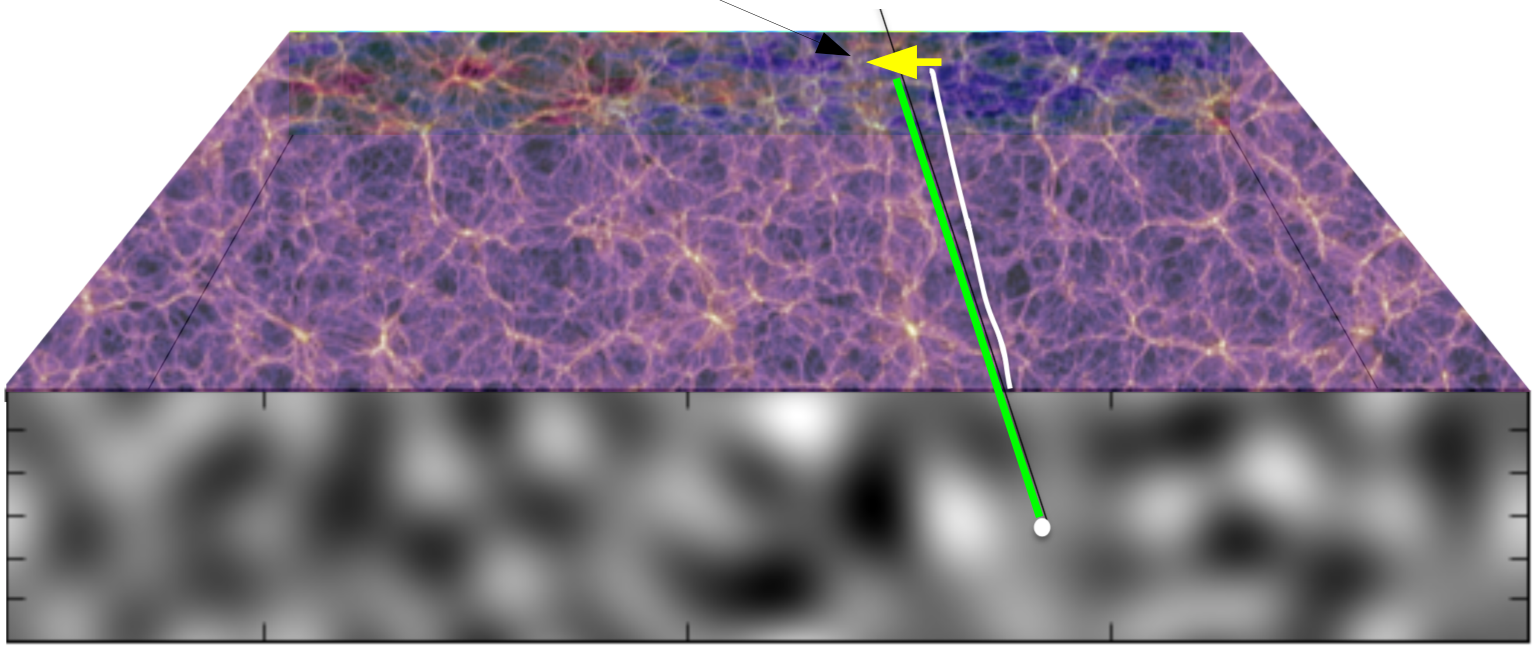
Aghanim et al. [Planck] 2018
Ade et al. [Planck] 2015

Weak lensing again: Lensing potential...

The amount of lensing deflection in any direction depends on the projected matter density in that direction.

Projected matter density ~ $\nabla \cdot d(\hat{\mathbf{n}}) = \int_0^{r_{\mathrm{CMB}}} dr W(r) \delta(\hat{\mathbf{n}}, r)$

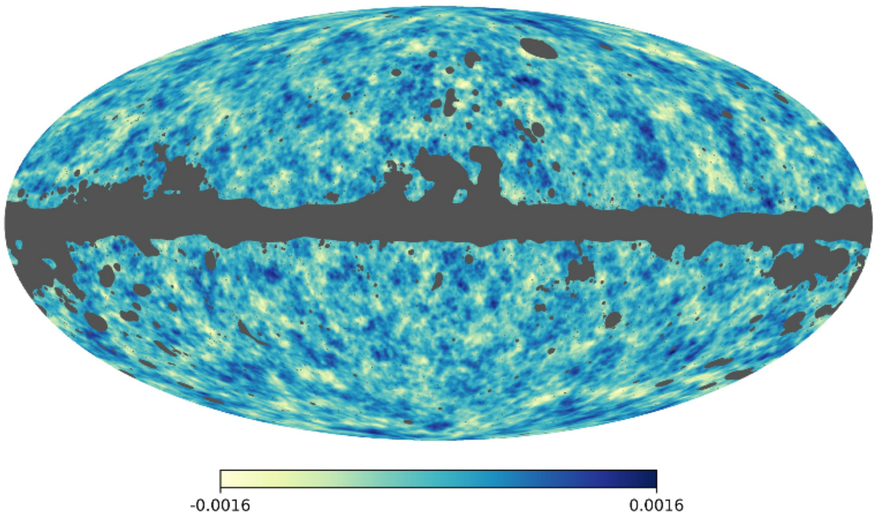
lensing — geometry — density



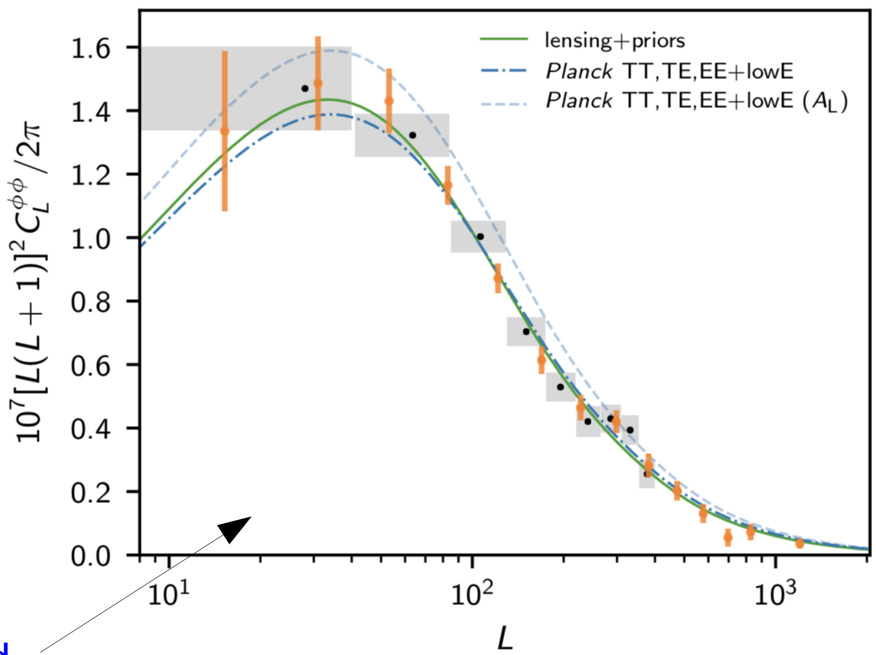
From Blake Sherwin

Weak lensing again: Lensing potential...

Projected matter density (or, equivalently, the lensing potential) reconstructed from the CMB temperature 4-point correlation function.



Lensing potential power spectrum



Line-of-sight integral of the 3D matter power spectrum weighted by geometric factors; dominated by contributions at z~3-4

Akrami et al. [Planck] 2018

Constraints on the neutrino mass sum…

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		+Lensing
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2015 numbers	0.72	0.68

Plus high-ℓ polarisation

Two different high-ℓ likelihood functions

Planck2018 TT +lowE+TE+EE	**0.26**	**0.24**
Planck2018 TT +lowE+TE+EE [CamSpec]	**0.38**	**0.27**
2015 numbers	0.49	0.59

Aghanim et al. [Planck] 2018
Ade et al. [Planck] 2015

Constraints on the neutrino mass sum…

ΛCDM+neutrino mass 7-parameter fit; 95% C.L. on $\sum m_\nu$ in [eV].

Low-ℓ polarisation only

		+Lensing	+BAO (non-CMB)
Planck2018 TT+lowE	**0.54**	**0.44**	**0.16**
2015 numbers	0.72	0.68	0.21

Plus high-ℓ polarisation

Planck2018 TT +lowE+TE+EE	**0.26**	**0.24**	**0.13**
Planck2018 TT +lowE+TE+EE [CamSpec]	**0.38**	**0.27**	n/a
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Two different high-ℓ likelihood functions

Aghanim et al. [Planck] 2018
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Plus high-ℓ polarisation

Two different high-ℓ likelihood functions

Planck2018 TT +lowE+TE+EE	**0.26**	**0.24**	**0.13**	**0.12**
Planck2018 TT +lowE+TE+EE [CamSpec]	**0.38**	**0.27**	n/a	**0.13**
2015 numbers	0.49	0.59	0.17	n/a

Planck2015 TT+lowP+Lyα $\sum m_\nu < 0.13 \text{ eV}$

Palanque-Delabrouille et al. 2015

Aghanim et al. [Planck] 2018
Ade et al. [Planck] 2015

Take home message...

- The tightest post-Planck 2018 cosmological bound on the neutrino mass sum from a 7-parameter fit remains at around 0.13 eV (95% C.L.).

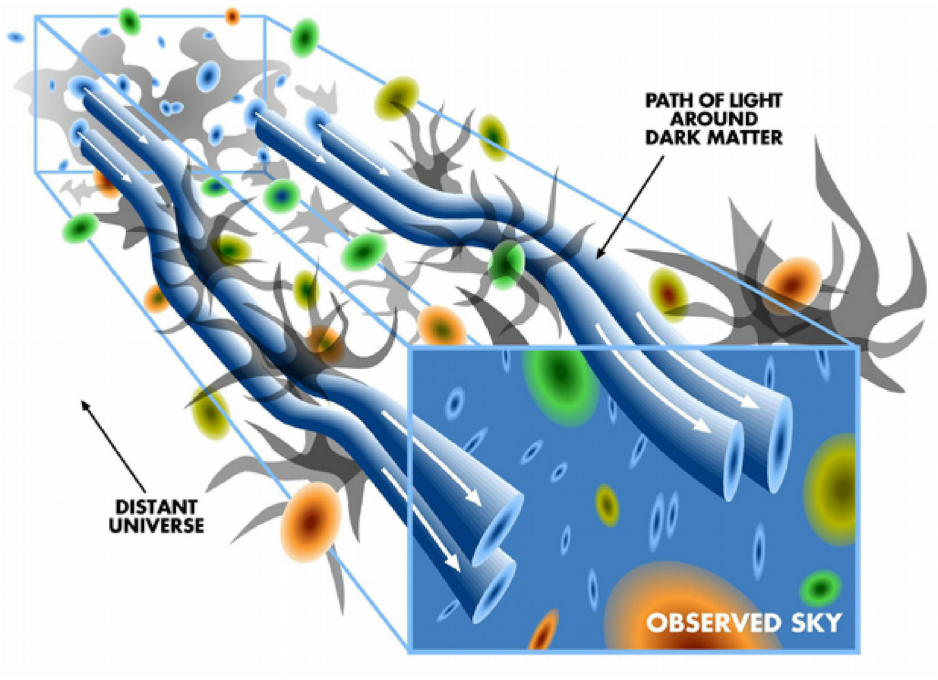
- It is however arguably far more robust than the existing Lyman-alpha bound formally of the same value.
  - Quasi-linear observables calculable from linear theory.

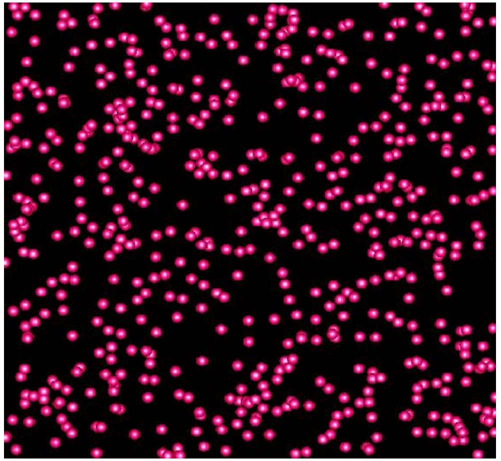
3. Future prospects…

Weak lensing of galaxies/Cosmic shear...

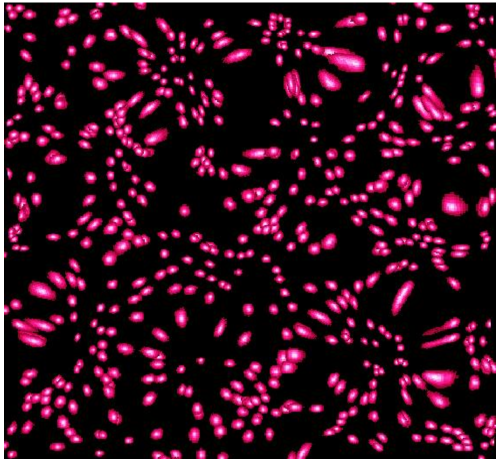
Distortion (magnification or stretching) of distant galaxy images by **foreground matter**.

- Sensitive to both luminous and dark matter (no bias problem).

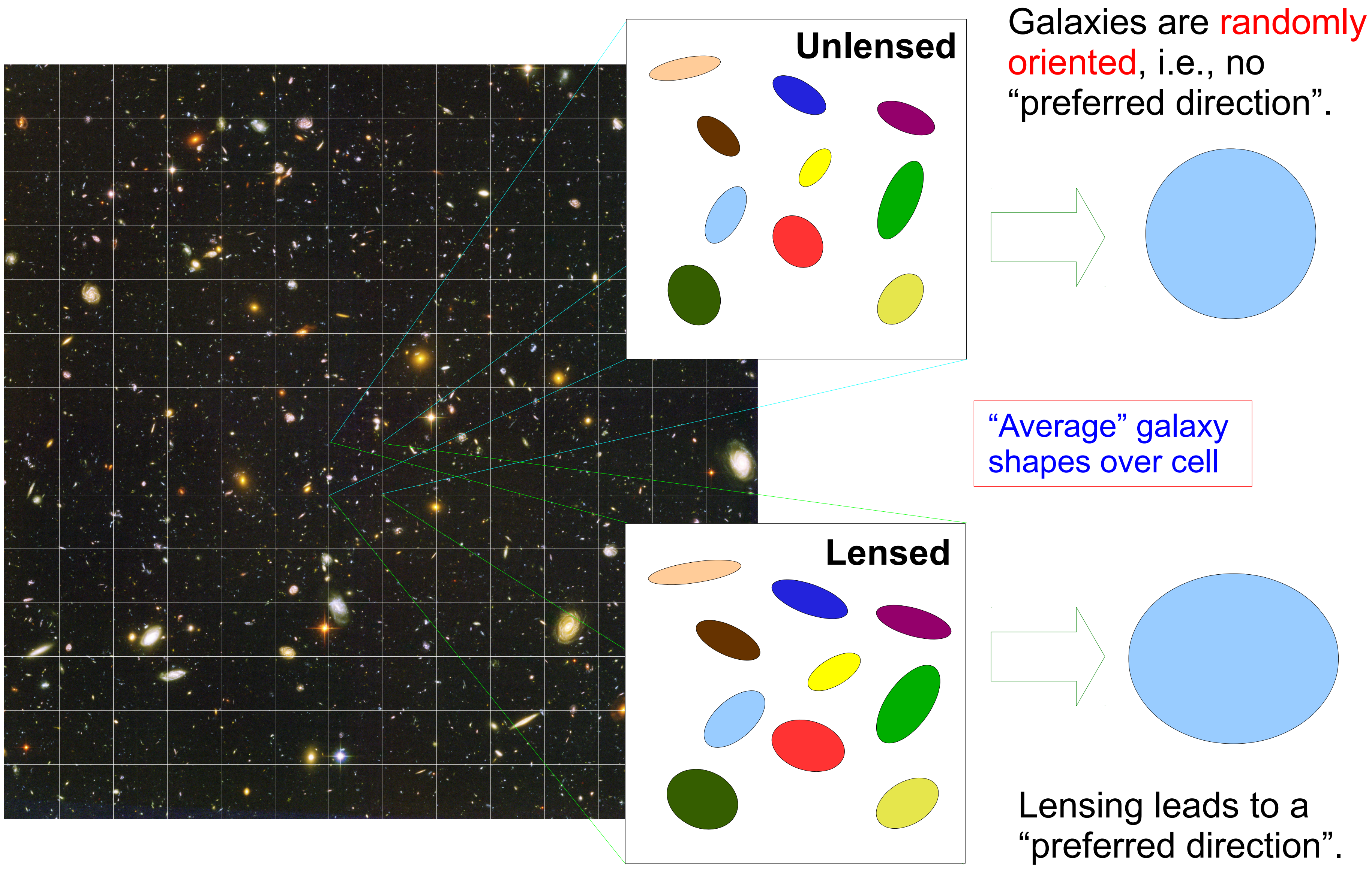




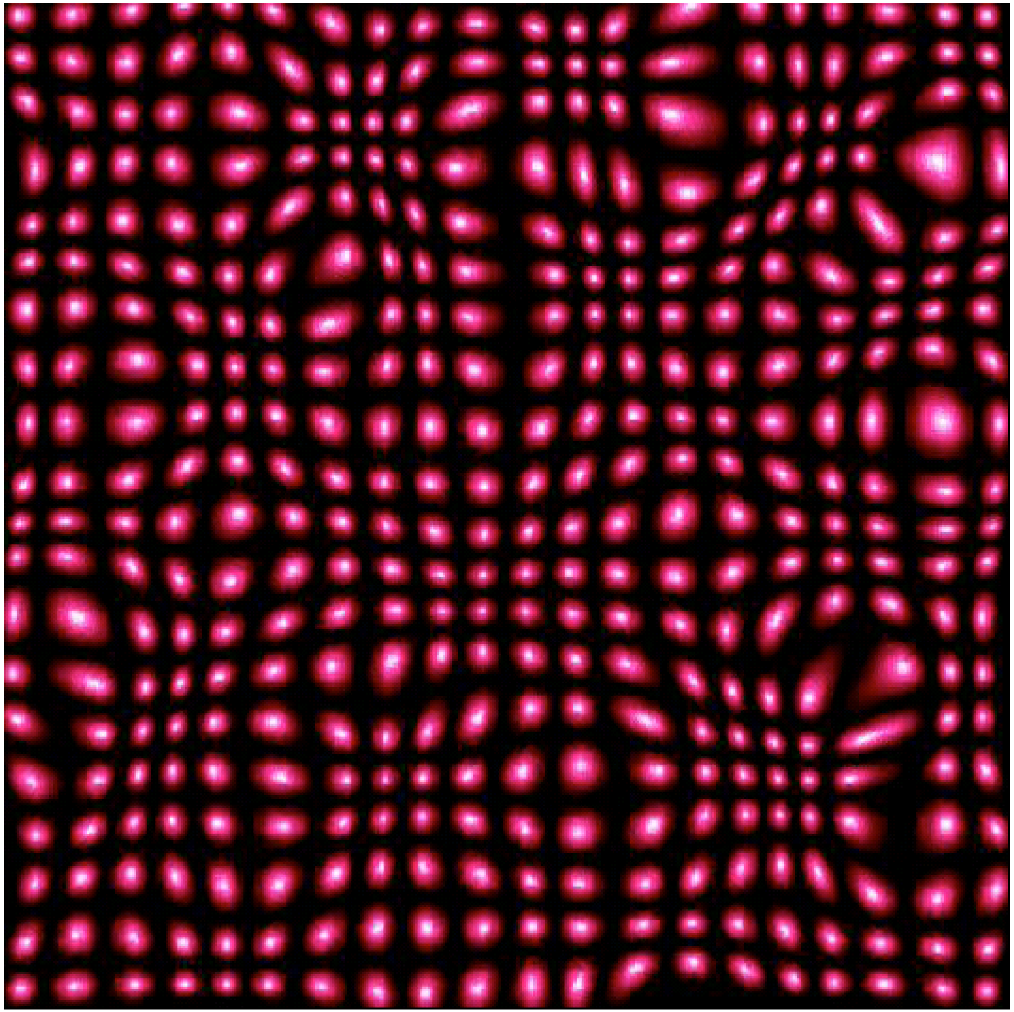
Unlensed



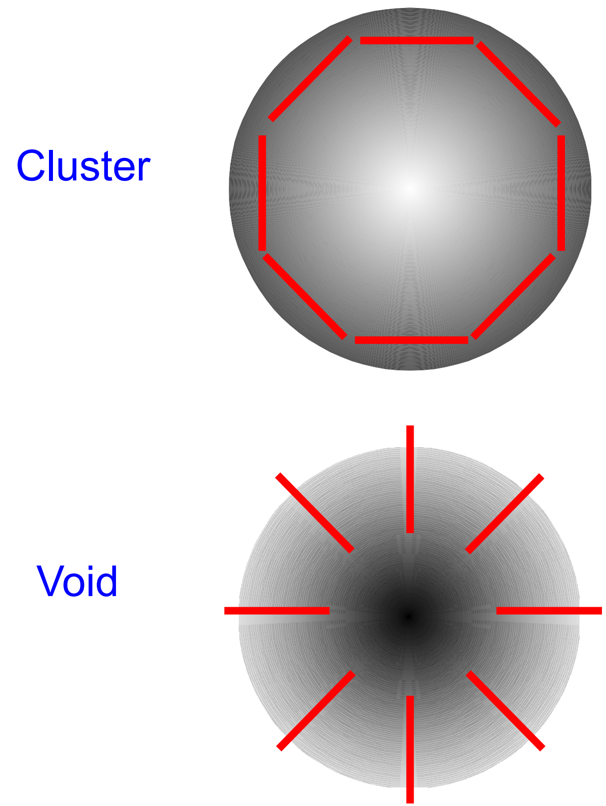
Lensed


Unlensed
Galaxies are randomly oriented, i.e., no "preferred direction".
"Average" galaxy shapes over cell
Lensed
Lensing leads to a "preferred direction".

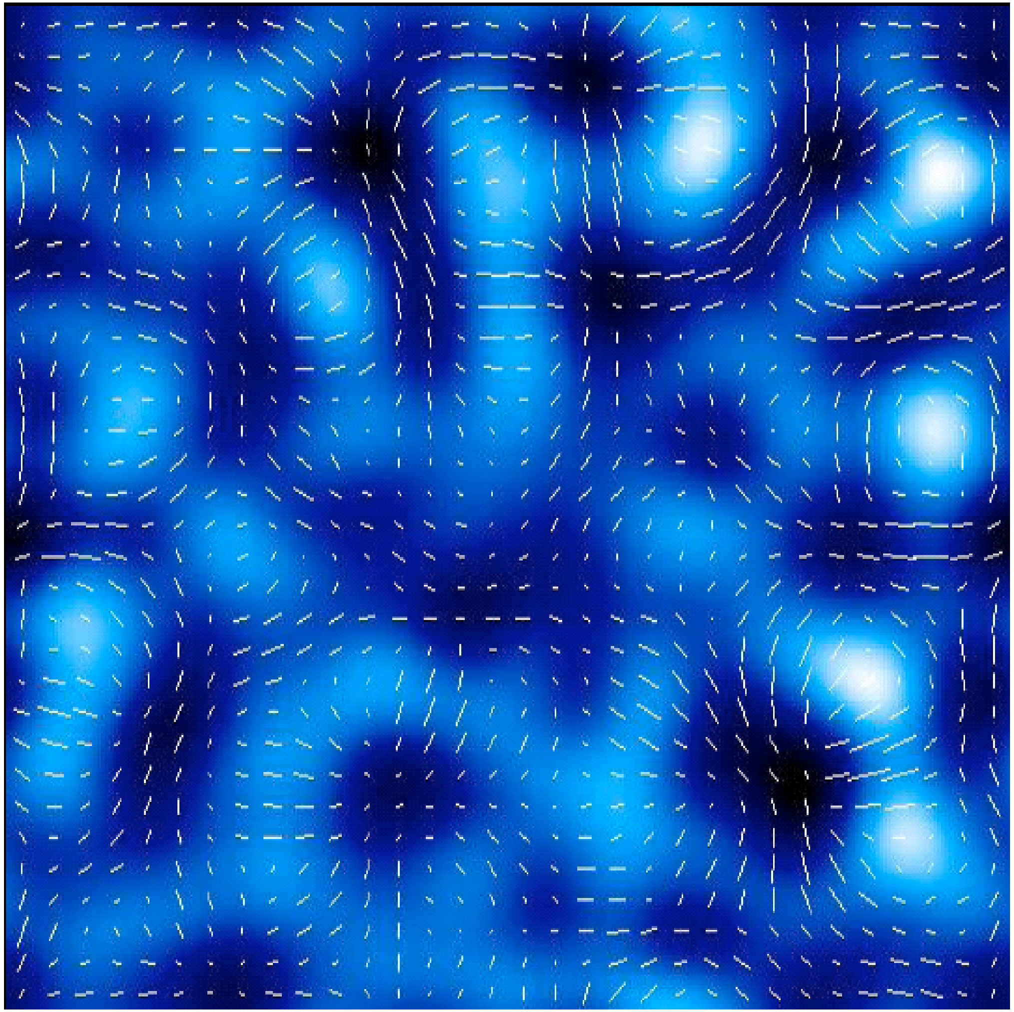
Shear map



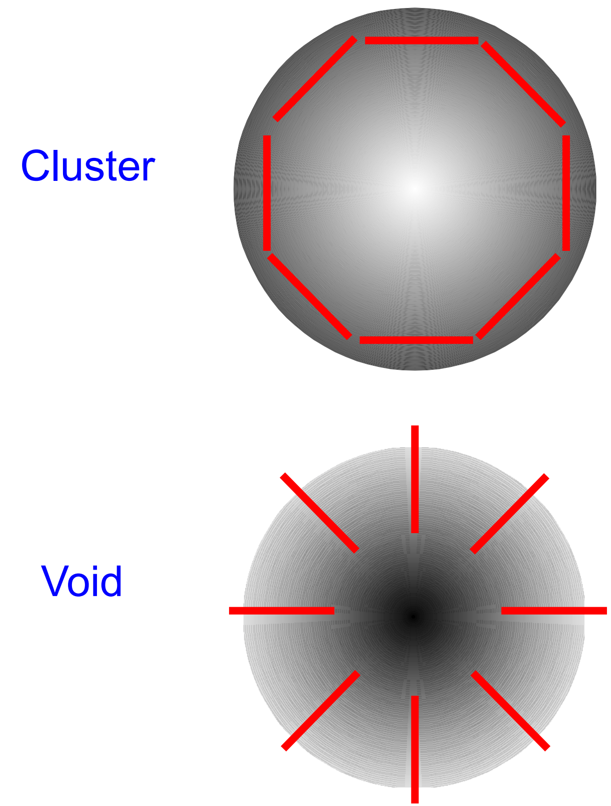
Weak lensing theory predicts:



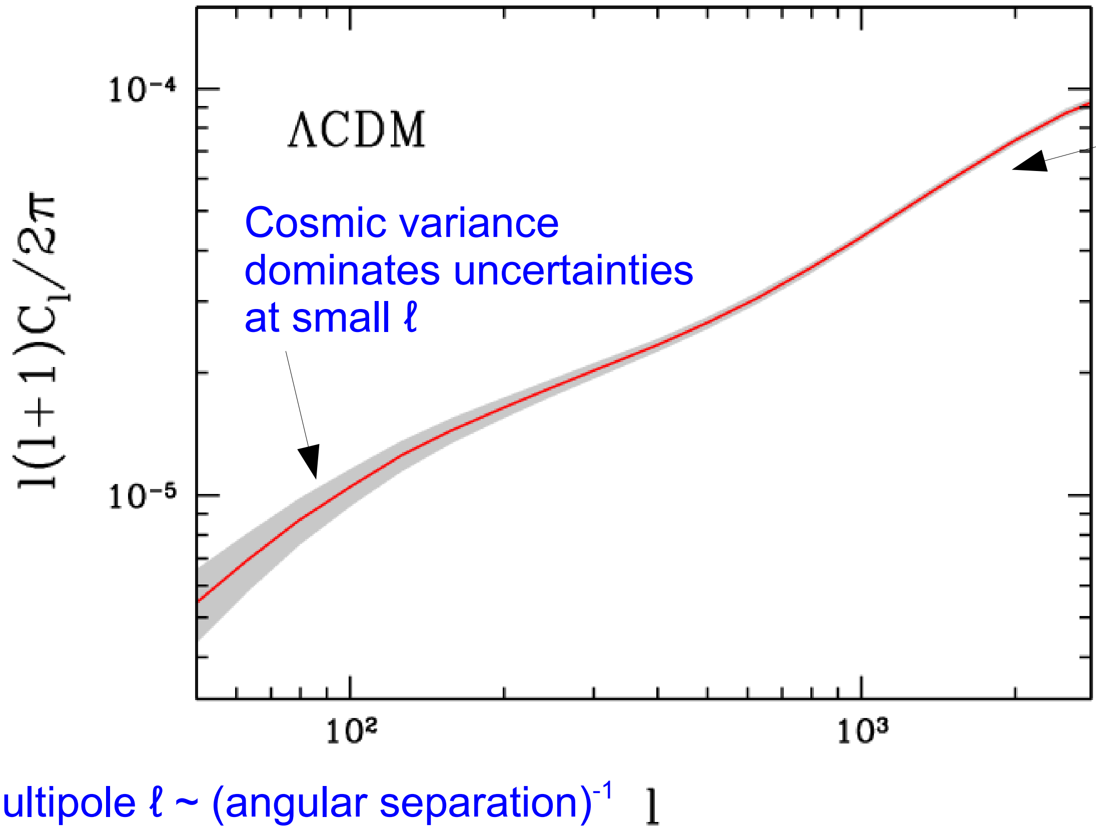
Shear map → Convergence map (projected mass)



Weak lensing theory predicts:



Convergence (or shear) power spectrum:



Cosmic variance dominates uncertainties at small ℓ

Shot noise dominates at large ℓ

$$\Delta C_\ell = \frac{\langle \gamma^2 \rangle}{n^2}$$

γ = Intrinsic ellipticity

n = source galaxy surface density

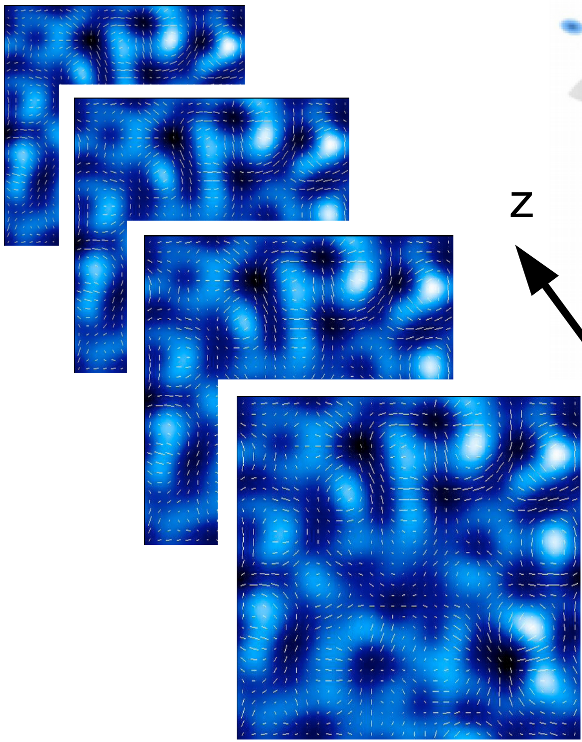
multipole ℓ ~ (angular separation)$^{-1}$

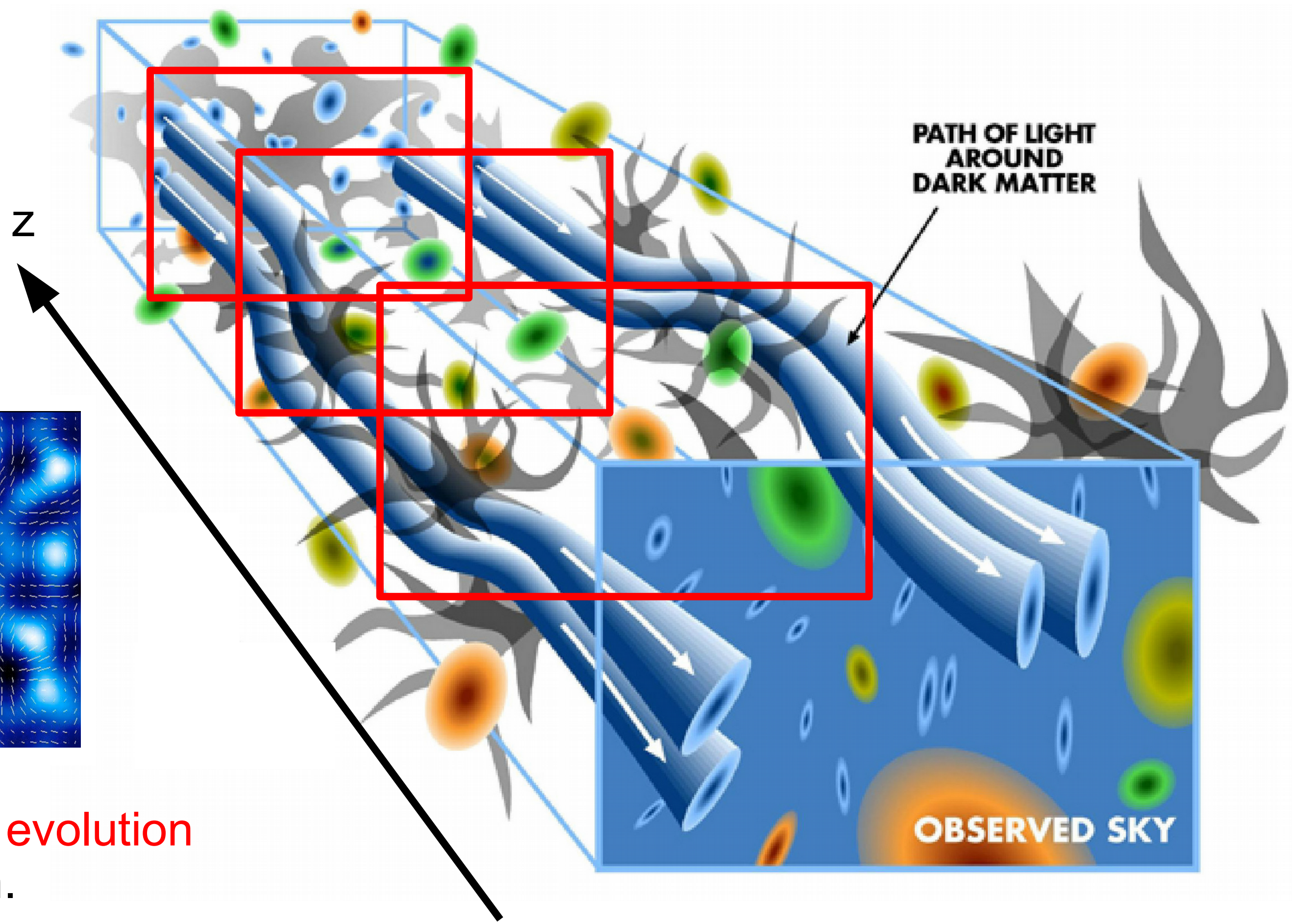
$$C_l \propto \int_0^\infty d\chi\, a^{-2} \left[\int_\chi^\infty d\chi' \, n_{\rm gal}(\chi') \frac{D(\chi'-\chi)}{D(\chi')} \right]^2 P_{\rm matter}(k = l/D(\chi))$$

(Limber limit)

Tomography = bin galaxy images by redshift

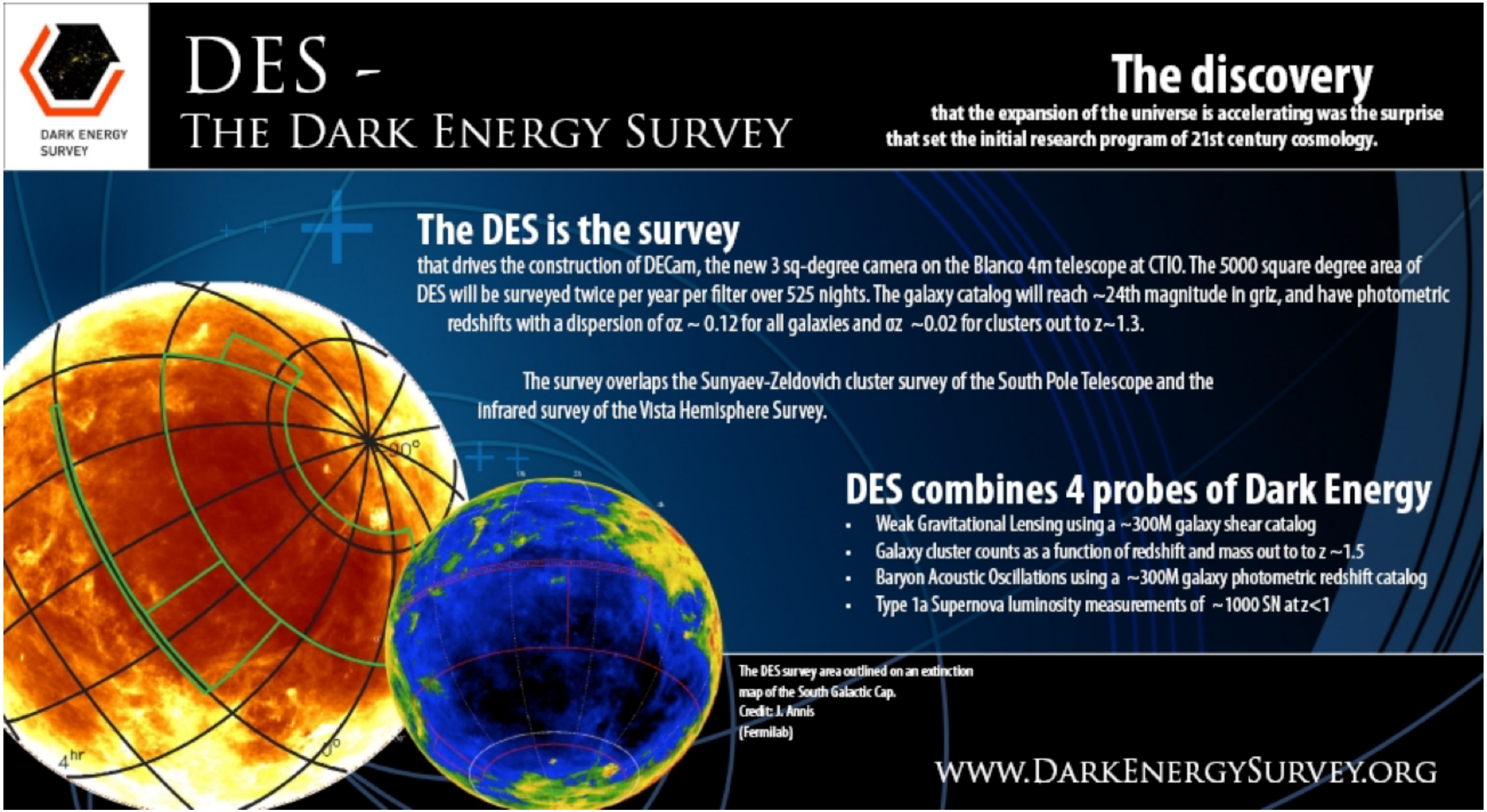
- Photometric redshifts for ~ 1 billion galaxies in Euclid survey.





Tomography probes the evolution
of the matter distribution.

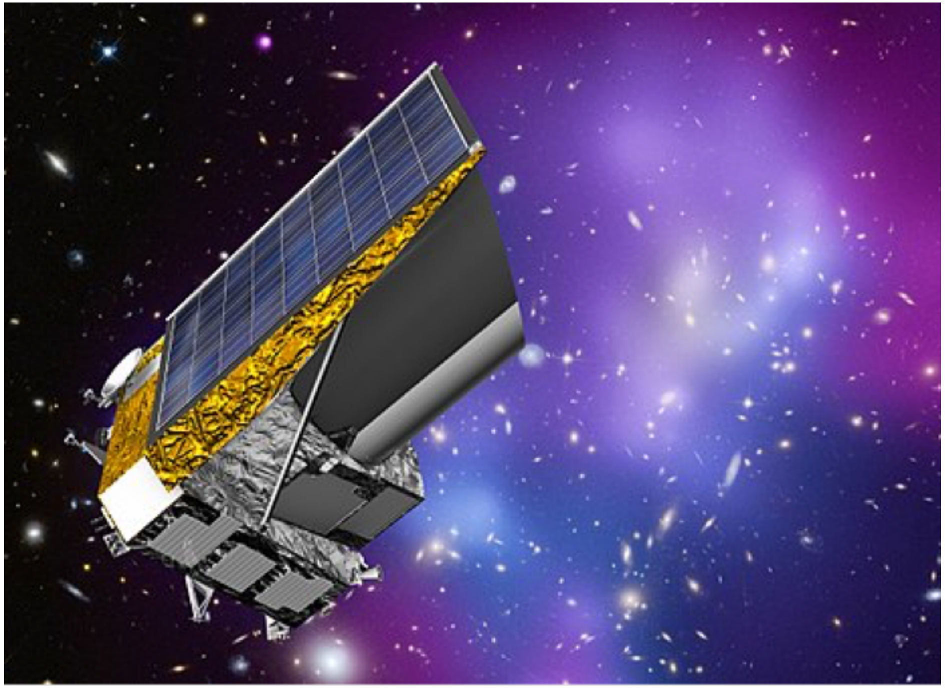
DES is happening right now...



ESA Euclid mission selected for implementation...

Launch planned for 2022.

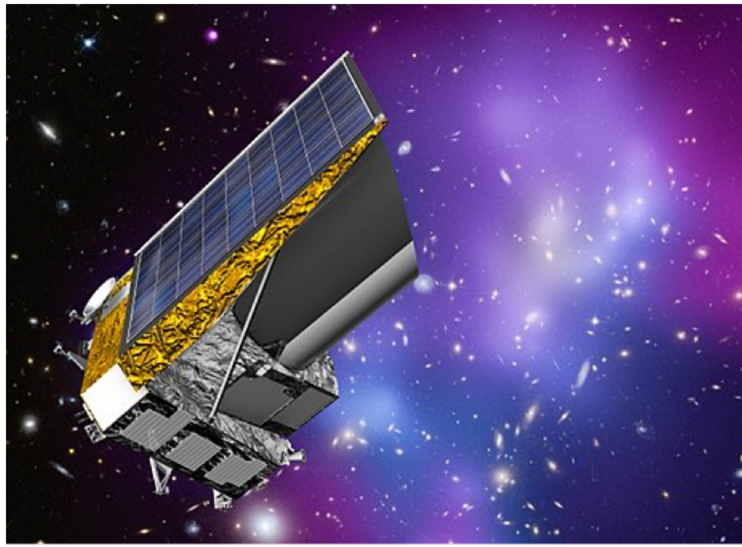
- 6-year lifetime
- 15000 deg^2 (>1/3 of the sky)
- Galaxies and clusters out to z~2
  - Photo-z for 1 billion galaxies
  - Spectro-z for 50 million galaxies
- Optimised for weak gravitational lensing (cosmic shear)



ESA Euclid mission selected for implementation...

Type Ia supernovae

Galaxy clusters
(cluster mass function)



Baryon acoustic oscillations (BAO)

Galaxy distribution
(photo-z and spectro-z)

Cosmic shear
(weak gravitational lensing of galaxies)

Cosmic shear with Euclid...

A **7-parameter forecast**:

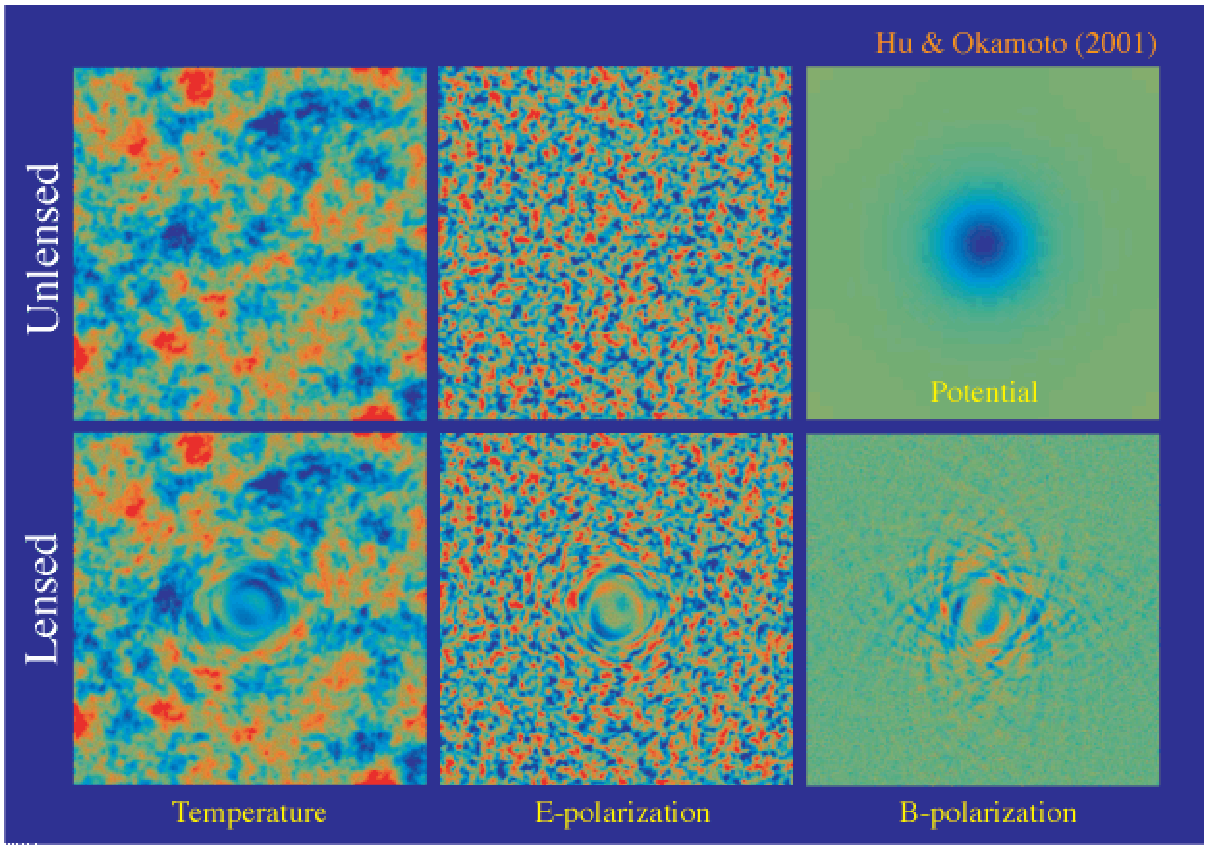
Data	$10^3 \times \sigma(\omega_{\rm dm})$	$100 \times \sigma(h)$	$\sigma(\sum m_\nu)/\mathrm{eV}$
c	2.02	1.427	0.143
cs	0.423	0.295	0.025
cg	0.583	0.317	0.016
$\mathrm{cg_l}$	0.828	0.448	0.019
$\mathrm{cg_b}$	0.723	0.488	0.039
$\mathrm{cg_{bl}}$	1.165	0.780	0.059
csg	0.201	0.083	0.011
csgx	0.181	0.071	0.011
$\mathrm{csg_b}$	0.385	0.268	0.023
$\mathrm{csg_bx}$	0.354	0.244	0.022

Hamann, Hannestad & Y^{3}W 2012

c = CMB (Planck); g = galaxy clustering
s = cosmic shear; x = shear-galaxy cross

Lensing of the CMB polarisation...

Weak gravitational lensing leads to a small **transfer of power** from the E-mode polarisation to the B-mode.

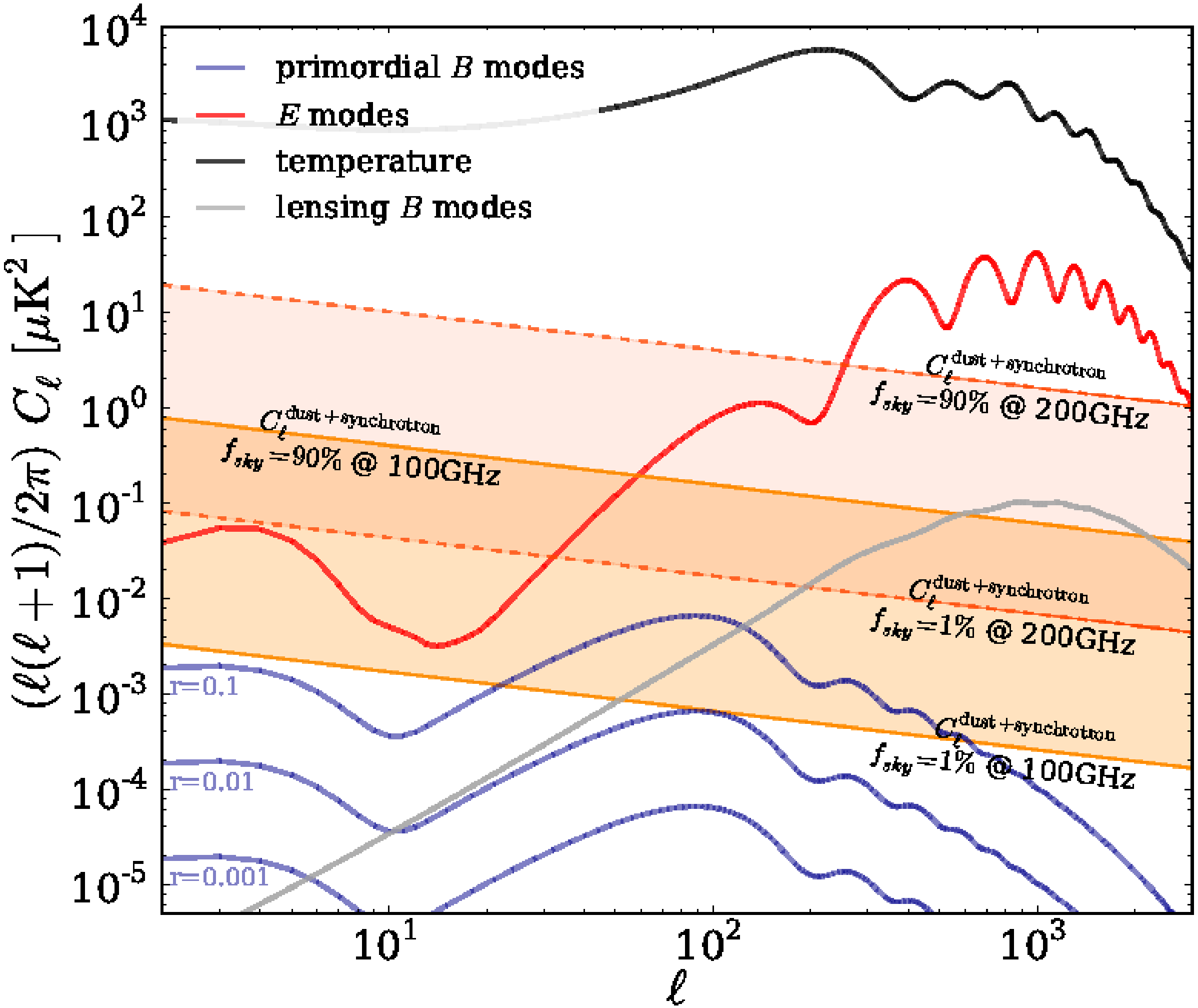


A hugely exaggerated example

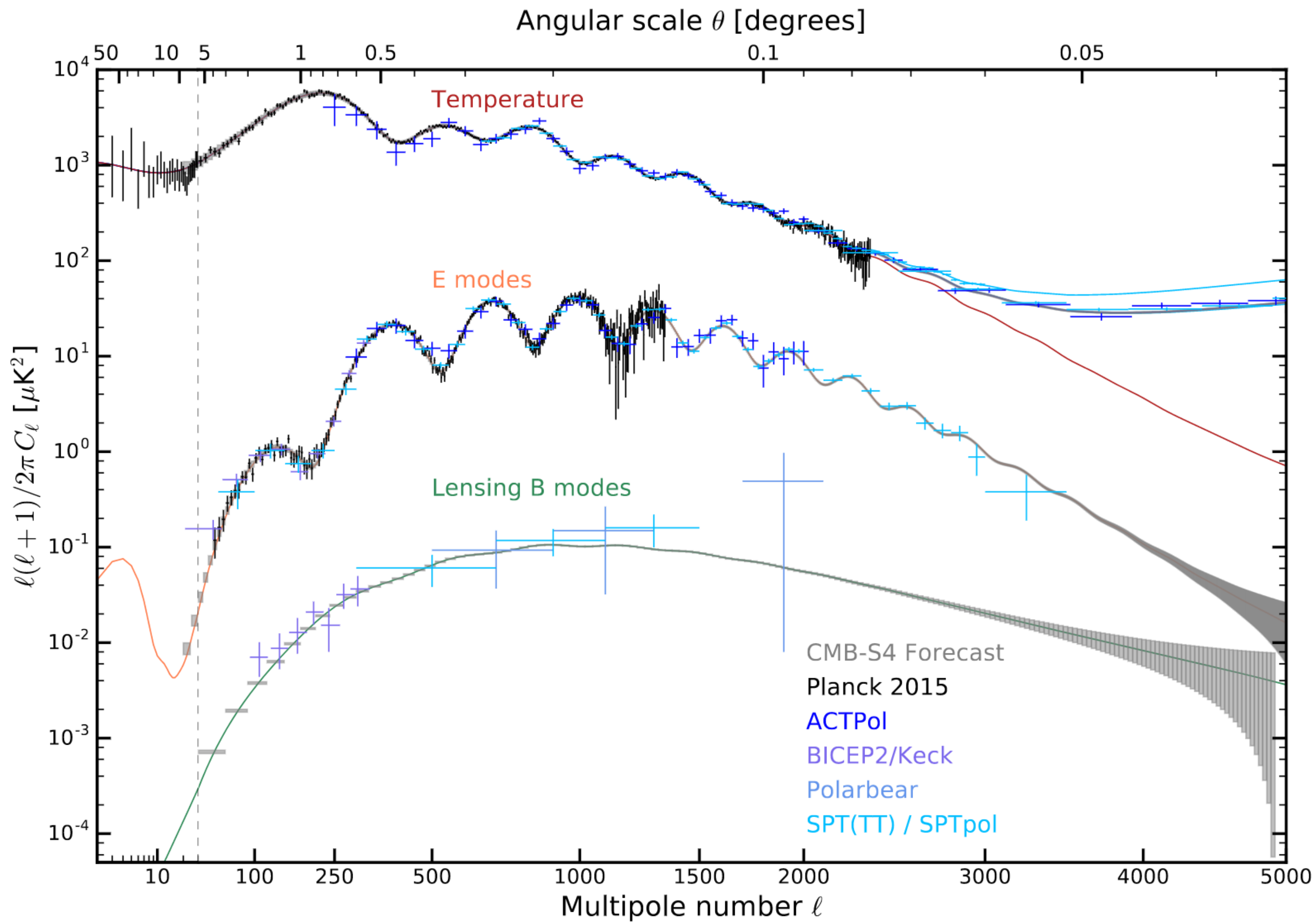
Lensing of the CMB polarisation...

Lensing signal = dominant B-mode signal at large multipoles especially in the absence of primordial gravitational waves

- Noise for primordial gravitational wave detection
- Great for neutrino cosmology



Lensing of the CMB polarisation...



CMB S4 science book

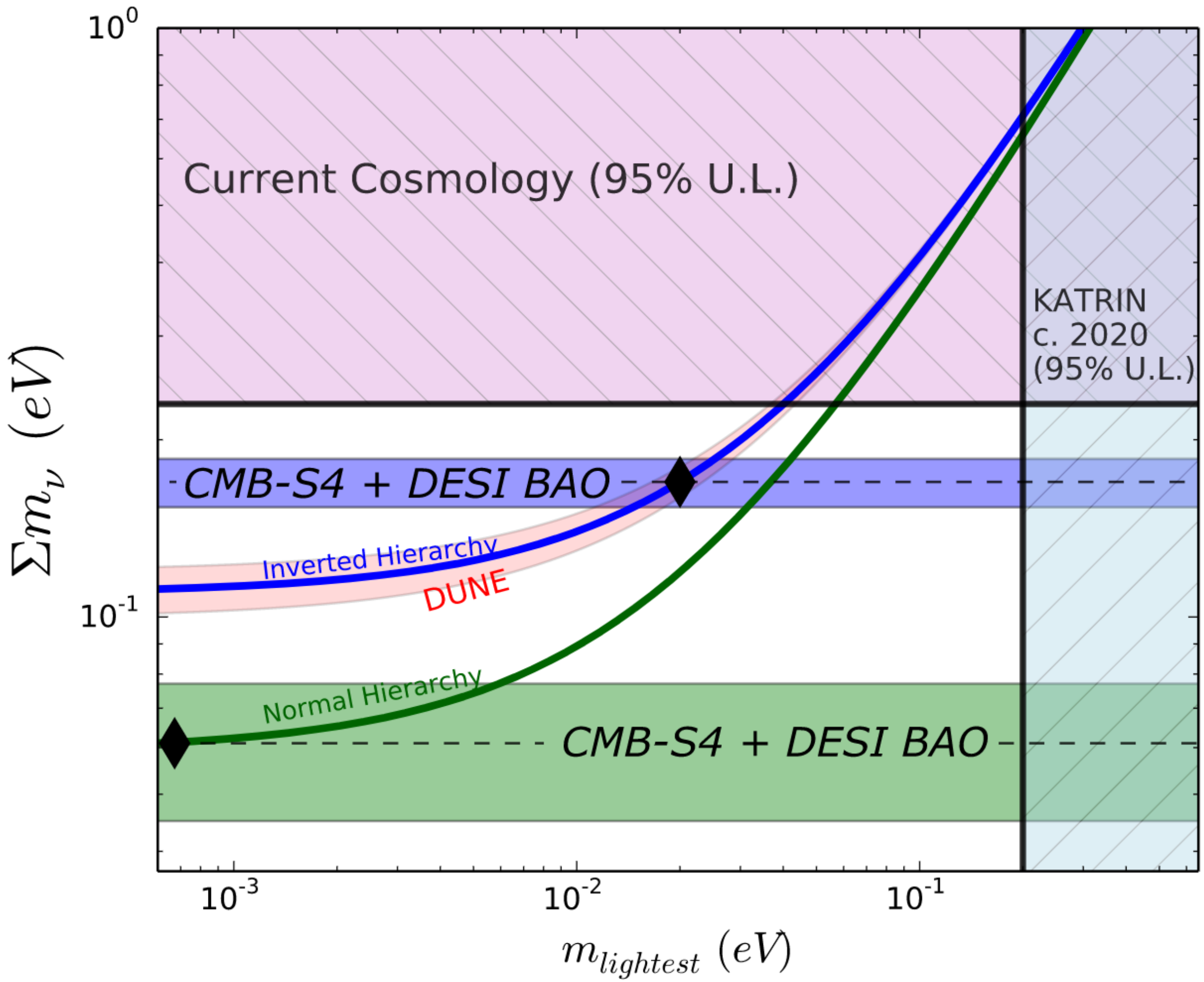
Lensing of the CMB polarisation with CMB-S4...

- Ground-based CMB probe planned for the 2020s.

- Potential **1σ sensitivity** to neutrino masses:

$$\sigma\left(\sum m_\nu\right) = 0.015\ \mathrm{eV}$$

CMB S4 science book



Take-home message...

- The cosmic microwave background anisotropies and the large-scale structure distribution can be used to probe neutrino physics.

- **Existing data** already place strong constraints on the neutrino mass.

- **Future probes** exploiting weak gravitational lensing of CMB polarisation (e.g., CMB S4) and cosmic shear (e.g., Euclid) can potentially tighter the bound 10-fold.