Revisiting the four-quark operator matrix elements for the lifetime of Λ_b

Zhen-Xing Zhao
Inner Mongolia University

PQCD Group meeting
2021.07.12

arXiv:2101.11874

Outline

- Introduction
- QCD sum rule calculation

  (Hadronic side, QCD side)
- Numerical results
- Conclusions

Introduction

Lifetime of Ω_c

$$\tau(\Omega_c^0) = 268 \pm 24 \pm 10 \pm 2 \text{ fs}$$

PhysRevLett.121.092003

$$\tau(\Omega_c^0) = 69 \pm 12 \text{ fs}$$

PDG2018

Lifetime of Λ_b

$$\tau(\Lambda_b) = (1.14 \pm 0.08)\ \text{ps}$$ PDG1996

$$\tau(B^0) = (1.519 \pm 0.004)\ \text{ps}$$ PDG2020

$$\tau(\Lambda_b^0)/\tau(B^0) = 0.75 \pm 0.05$$

Deviated from 1 at level of 20% -- too large!

$$\tau(\Lambda_b) = (1.471 \pm 0.009)\ \text{ps}, \qquad \tau(\Lambda_b^0)/\tau(B^0) = 0.964 \pm 0.007$$

PDG2020

HQE in 2014

Expt:

$$\tau(\Lambda_b) = (1.471 \pm 0.009)\ \text{ps}, \qquad \tau(\Lambda_b^0)/\tau(B^0) = 0.964 \pm 0.007$$

PDG2020

Theory:

$$\frac{\tau(\Lambda_b)}{\tau(B_d)}^{\text{HQE 2014}} = 1 - (0.8 \pm 0.5)\%_{1/m_b^2} - (4.2 \pm 3.3)\%^{\Lambda_b}_{1/m_b^3} - (0.0 \pm 0.5)\%^{B_d}_{1/m_b^3} - (1.6 \pm 1.2)\%_{1/m_b^4}$$

$$= 0.935 \pm 0.054$$

Lenz
Int. J. Mod. Phys. A 30,1543005 (2015)

large!

HQE can nicely explain the lifetime of Λ_b

Heavy Quark Expansion

$$\mathcal{T} = i \int d^4x \; T[\mathcal{L}_W(x)\mathcal{L}_W^\dagger(0)]$$

$$\Gamma(H_Q) = \frac{2\ \mathrm{Im}\langle H_Q|\mathcal{T}|H_Q\rangle}{2M_H}$$

Optical theorem

OPE

$$2\ \mathrm{Im}\mathcal{T} = \frac{G_F^2 m_Q^5}{192\pi^3}\xi\left(c_{3,Q}\bar{Q}Q + \frac{c_{5,Q}}{m_Q^2}\bar{Q}\sigma\cdot GQ + \frac{c_{6,Q}}{m_Q^3}T_6 + \frac{c_{7,Q}}{m_Q^4}T_7 + \cdots\right)$$

CKM

Uncertainty is large!

Parameterize the baryon matrix elements

$$\langle\Lambda_b|(\bar{b}q)_{V-A}(\bar{q}b)_{V-A}|\Lambda_b\rangle = f_{B_q}^2 m_{B_q} m_{\Lambda_b} L_1,$$
$$\langle\Lambda_b|(\bar{b}q)_{S-P}(\bar{q}b)_{S+P}|\Lambda_b\rangle = f_{B_q}^2 m_{B_q} m_{\Lambda_b} L_2,$$
$$\langle\Lambda_b|(\bar{b}b)_{V-A}(\bar{q}q)_{V-A}|\Lambda_b\rangle = f_{B_q}^2 m_{B_q} m_{\Lambda_b} L_3,$$
$$\langle\Lambda_b|(\bar{b}^\alpha q^\beta)_{S-P}(\bar{q}^\beta b^\alpha)_{S+P}|\Lambda_b\rangle = f_{B_q}^2 m_{B_q} m_{\Lambda_b} L_4,$$

$$L_1 = (-1/6)r, \quad L_2 = (1/12)r, \quad L_3 = (1/6)\tilde{B}\ r, \quad L_4 = (-1/12)\tilde{B}\ r$$

$$r, \qquad \tilde{B}$$

Theoretical results

TABLE I: L_1 predicted by different theoretical methods. This table is copied from [6].

	L_1	$\tilde{B}$		
Central	$-0.103(10)$	1	2014 Spectroscopy update [7]	Rosner
Upper	$-0.22(4)$	$1.21(34)$	1999 Exploratory Lattice [8]	
	$-0.22(5)$	1	1999 QCDSR v1 [9]	Huang,Liu,Zhu
	$-0.60(15)$	1	1999 QCDSR v2 [9]	
Lower	$-0.033(17)$	1	1996 QCDSR [10]	Colangelo,De Fazio
	≈ -0.03	1	1979 Bag model [11]	Guberina
	≈ -0.08	1	1979 NRQM [11]	

QCD sum rule calculation

Hadronic level

Hadronic level

$$\Pi(p_1,p_2) = i^2 \int d^4x d^4y \; e^{-ip_1\cdot x + ip_2\cdot y} \langle 0|T\{J(y)\Gamma_6(0)\bar{J}(x)\}|0\rangle$$

$$\Pi^{\text{had}}(p_1,p_2) = \lambda_H^2 \frac{(\not{p}_2 + M)(a + b\gamma_5)(\not{p}_1 + M)}{(p_2^2 - M^2)(p_1^2 - M^2)} + \cdots$$

2 * 2 * 2

$$\langle \Lambda_b(q',s')|\Gamma_6|\Lambda_b(q,s)\rangle = \bar{u}(q',s')(a + b\gamma_5)u(q,s)$$

Hadronic level

$$
\begin{aligned}
\Pi^{\mathrm{had}}(p_1,p_2) &= \lambda_+\lambda_+ \frac{(\not{p}_2 + M_+)(a^{++} + b^{++}\gamma_5)(\not{p}_1 + M_+)}{(p_2^2 - M_+^2)(p_1^2 - M_+^2)} \\
&+ \lambda_+\lambda_- \frac{(\not{p}_2 + M_+)(a^{+-} + b^{+-}\gamma_5)(\not{p}_1 - M_-)}{(p_2^2 - M_+^2)(p_1^2 - M_-^2)} \\
&+ \lambda_-\lambda_+ \frac{(\not{p}_2 - M_-)(a^{-+} + b^{-+}\gamma_5)(\not{p}_1 + M_+)}{(p_2^2 - M_-^2)(p_1^2 - M_+^2)} \\
&+ \lambda_-\lambda_- \frac{(\not{p}_2 - M_-)(a^{--} + b^{--}\gamma_5)(\not{p}_1 - M_-)}{(p_2^2 - M_-^2)(p_1^2 - M_-^2)} \\
&+ \cdots
\end{aligned}
$$

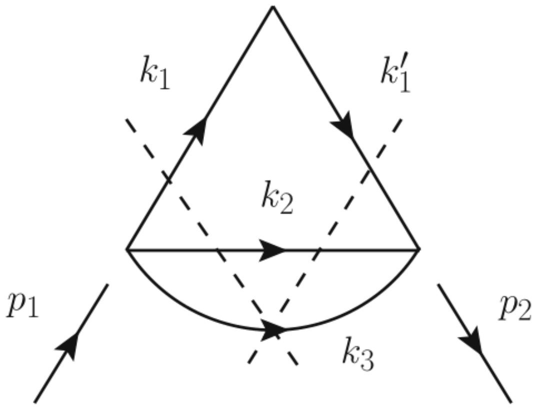
8 parameters -- 8 Dirac structures

$$
\begin{aligned}
\langle \Lambda_b(q,s)|\Gamma_6|\Lambda_b(q,s)\rangle &= \bar{u}(q,s)(a + b\gamma_5)u(q,s) \\
&= 2\; a\; m_{\Lambda_b}
\end{aligned}
$$

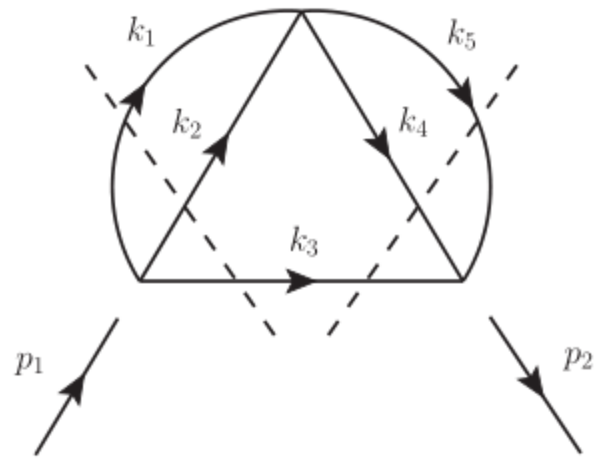
QCD sum rule calculation

QCD level

Dim-0



FF of $\Lambda_b \to \Lambda_c$
pert



dim-6 matrix elements
pert

dim-3 & dim-5 ~
m_u/m_d

a^{++}

$$a^{++} = \frac{\{M_-^2, M_-, M_-, 1\}.\{\mathcal{B}A_1, \mathcal{B}A_2, \mathcal{B}A_3, \mathcal{B}A_4\}}{\lambda_+^2 (M_+ + M_-)^2} \exp\left(\frac{2M_+^2}{T^2}\right)$$

How can we get these A_i?

Eur. Phys. J. C 80, 568 (2020)
Eur. Phys. J. C 80, 1181 (2020)

Numerical results

Inputs

$$\mu = m_b$$

$$m_b(m_b) = 4.18 \pm 0.03 \text{ GeV}$$

$$\lambda_+ = 0.0432 \pm 0.0022 \text{ GeV}^3$$

Figure and table

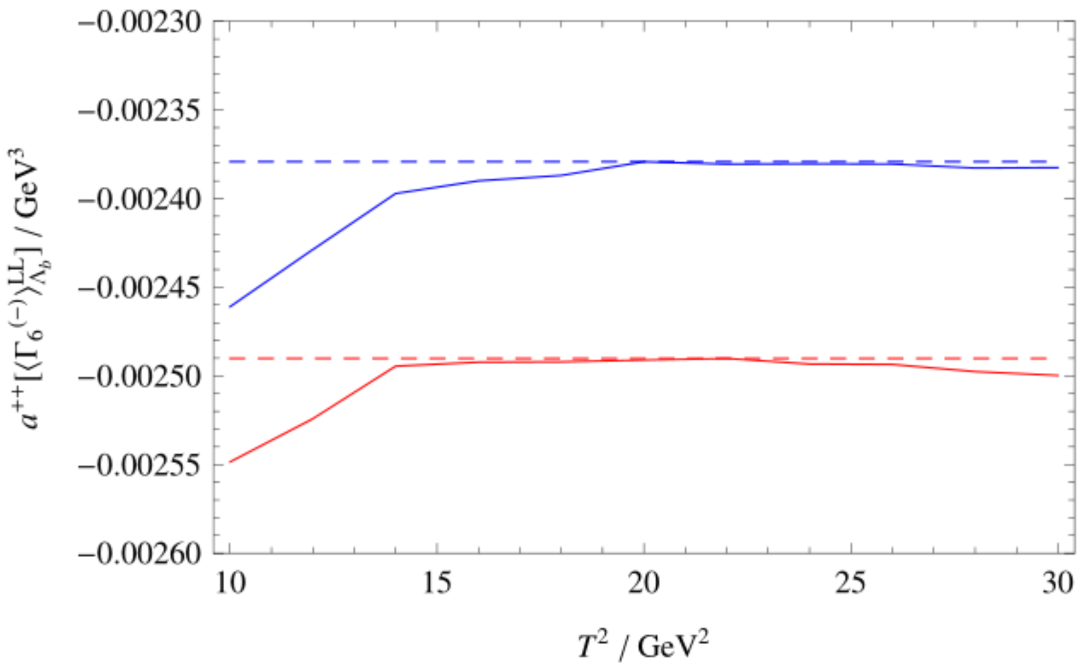


TABLE II: The predictions of $a^{++}[\langle\Gamma_6^{(-)}\rangle^{\mathrm{LL}}_{\Lambda_b}]$.

	s_0/GeV^2	T^2/GeV^2	$a^{++}[\langle\Gamma_6^{(-)}\rangle^{\mathrm{LL}}_{\Lambda_b}]/(10^{-4}\ \mathrm{GeV}^3)$
Optimal	6.01^2	20	−23.8
Suboptimal	6.02^2	22	−24.9

Results

$$a^{++}[\langle\Gamma_6^{(-)}\rangle_{\Lambda_b}^{\rm LL}] = (-23.8 \pm 1.1 \pm 3.4 \pm 2.2) \times 10^{-4}\ {\rm GeV}^3$$

$$L_1 = -0.0260 \pm 0.0012 \pm 0.0037 \pm 0.0025$$

- QCDSR parameters s_0 and T^2
- Bottom quark mass m_b
- Pole residue λ_+

Theoretical results

TABLE I: L_1 predicted by different theoretical methods. This table is copied from [6].

	L_1	$\tilde{B}$		
Central →	$-0.103(10)$	1	2014 Spectroscopy update [7]	Rosner
Upper →	$-0.22(4)$	$1.21(34)$	1999 Exploratory Lattice [8]	
	$-0.22(5)$	1	1999 QCDSR v1 [9]	Huang,Liu,Zhu
	$-0.60(15)$	1	1999 QCDSR v2 [9]	
Lower →	$-0.033(17)$	1	1996 QCDSR [10]	Colangelo,De Fazio
	≈ -0.03	1	1979 Bag model [11]	Guberina
	≈ -0.08	1	1979 NRQM [11]	

Discussions

- $s_0 = (6.01\ GeV)^2$ vs $s_0 = (5.95\ GeV)^2$ (for two-point correlation function of Lb)
- $T^2 = 20\ GeV^2$, which is ~ μ_b^2
- $\tilde{B} = 1$ is strictly true for dim-0,3,5
- LL correction for L1 is about 13%

Summary and outlook

Summary

- QCDSR to investigate the four-quark operator matrix elements: L1 and L3 (or L1 and $\tilde{B}$)
- At the QCD level, dim-0, 3, 5 are considered, and dim-3,5 ~ mu/md
- LL corrections are considered
- Our results are close to the lower bound of existing theoretical predictions for L1
- Our results can lead to those of Colangelo

Outlook

- More complete version for bottom baryons: $\Lambda_b, \Xi_b, \Omega_b$
- Charmed baryons
- Doubly charmed baryons…

- Dim-6 matrix elements
- Dim-7 matrix elements

Thank you for your attention!