$B \to \pi\pi$ decays from the perturbative QCD approach revisited

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Outline

Motivation

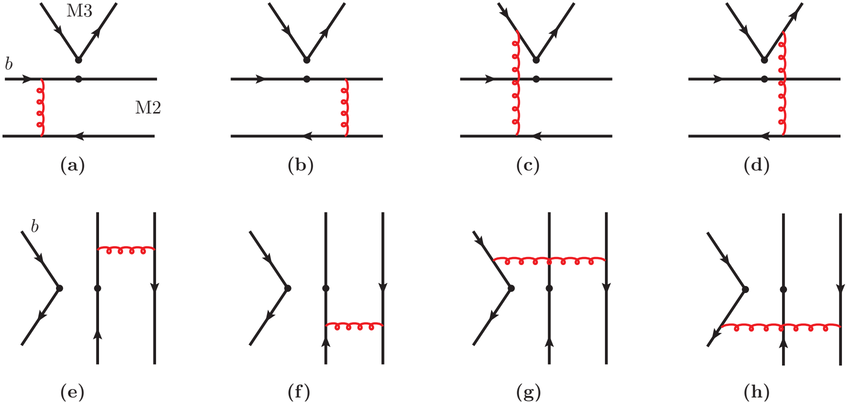
1. The High Luminosity phase of the Large Hadron Collider (HL-LHC) will improve the measurements of several crucial flavour observables for the extraction of CKM parameters.
2. Belle-II expects to reduce the experimental error of $|V_{ub}|$ measurement from 2.5% to 1.2% in semileptonic decays, and improve the measurements of charmless two-body B decays with much higher precision.

3. The NLO corrections are supplemented progressively to improve the reliability of the approach and to explain the more and more precise data.
4. Update the results of different channels in a consistent way with consistent input parameters so that they can be better analyzed later.

A renormalization theory is usual accompanied with a non-dimensional coupling constant, so in principle, infinity pairs of particles can be produced in a scattering process, in which the hard productions can be calculated well by the perturbative QCD theorem, and the soft couplings accompanied with nonperturbative information are absorbed into the universal wave functions.

$$
\begin{aligned}
\mathcal{M}(B \to M_2 M_3) &= H_r(M_W, t) \otimes H(t, \mu) \otimes \phi(x, P^+, b, \mu) \\
&= \sum_i C_i(M_W, t) \otimes H_i(t, b) \otimes \phi(x, b, 1/b) \\
&\quad \cdot \mathrm{Exp}\Big[- s(P^+, b) - \int_{1/b}^{t} \frac{d\bar{\mu}}{\bar{\mu}} \gamma_\phi(\alpha_s(\bar{\mu}))\Big]
\end{aligned}
$$

Framework—Feynman diagram



B meson nonlocal matrix element

$$\int d^4 z_1\, e^{i\bar{k}_1\cdot z_1}\langle 0|\bar{d}_\sigma(z_1)\, b_\beta(0)|\bar{B}^0(p_1)\rangle$$

$$= \frac{if_B}{4N_c}\left\{(\not p_1 + m_B)\gamma_5\left[\frac{\not n_+}{\sqrt{2}}\varphi_+(\bar{x}_1, b_1) + \left(\frac{\not n_-}{\sqrt{2}} - k_1^+\gamma_\perp^\nu \frac{\partial}{\partial \mathbf{k}_{1T}^\nu}\right)\varphi_-(\bar{x}_1, b_1)\right]\right\}_{\beta\sigma}$$

$$= \frac{-if_B}{4N_c}\left\{(\not p_1 + m_B)\gamma_5\left[\varphi_B(x_1, b_1) - \frac{\not n_+ - \not n_-}{\sqrt{2}}\bar{\varphi}_B(x_1, b_1)\right]\right\}_{\beta\sigma}$$

B meson DA

$$\phi_B(x,b) = N_B x^2(1-x)^2 e^{\left(-\frac{M_B^2 x^2}{2\omega_b^2} - \frac{1}{2}(\omega_b b)^2\right)}$$

Normalization

$$\int_0^1 dx\, \phi_B(x, b=0) = 1$$

N_B is 3417.18

Vacuum to pion matrix element

$$\begin{aligned} &\int d^4 z_2\, e^{i\bar{k}_2\cdot z_2} \left\langle \pi^+(p_2) \middle| \bar{u}_\delta(0)\, d_\alpha(z_2) \middle| 0 \right\rangle \\ = \quad & -\frac{i f_\pi}{4N_c} \Big(\gamma_5 \big[\, \not{p}_2\, \varphi_\pi(\bar{x}_2, b_2) + m_0^\pi\, \varphi_\pi^{\rm p}(\bar{x}_2, b_2) + m_0^\pi \left(\not{n}_+ \not{n}_- - 1 \right) \varphi_\pi^{\rm t}(\bar{x}_2, b_2) \big] \Big)_{\alpha\delta} \\ \longrightarrow \quad & \frac{-i f_\pi}{4N_c} \Big(\gamma_5 \big[\, \not{p}_2\, \varphi_\pi(x_2, b_2) + m_0^\pi\, \varphi_\pi^{\rm p}(x_2, b_2) - m_0^\pi \left(\not{n}_+ \not{n}_- - 1 \right) \varphi_\pi^{\rm t}(x_2, b_2) \big] \Big)_{\alpha\delta} \end{aligned}$$

Leading twist DA

$$\varphi_P(x,\mu) = 6x\bar{x} \sum_{n=0} a_n^P(\mu)\, C_n^{3/2}(2x-1)$$

Normalization

$$\int_0^1 dx\, \varphi_\pi(x) = 1$$

Two-particle twist 3 DAs

$$
\begin{aligned}
\varphi_P^{\mathrm{p}}(x,\mu) &= 1+3\rho^P\Big(1-3a_1^P+6a_2^P\Big)(1+\ln x)-\frac{\rho^P}{2}\Big(3-27a_1^P+54a_2^P\Big)\,C_1^{1/2}(2x-1)\\
&+ 3\Big(10\eta_{3P}-\rho^P(a_1^P-5a_2^P)\Big)\,C_2^{1/2}(2x-1)+\Big(10\eta_{3P}\lambda_{3P}-\frac{9}{2}\rho^P a_2^P\Big)\\
&\quad C_3^{1/2}(2x-1)-3\eta_{3P}\omega_{3P}\,C_4^{1/2}(2x-1)\,,\\
\varphi_P^{\sigma}(x,\mu) &= 6x(1-x)\Big(1+\frac{\rho^P}{2}\Big(2-15a_1^P+30a_2^P\Big)+\rho^P\Big(3a_1^P-\frac{15}{2}a_2^P\Big)\,C_1^{3/2}(2x-1)\\
&+ \frac{1}{2}\Big(\eta_{3P}(10-\omega_{3P})+3\rho^P a_2^P\Big)\,C_2^{3/2}(2x-1)+\eta_{3P}\lambda_{3P}\,C_3^{3/2}(2x-1)\\
&+ 3\rho^P\Big(1-3a_1^P+6a_2^P\Big)\,\ln x\Big)
\end{aligned}
$$

Framework—Hard kernel

$$\begin{aligned}
\mathcal{F}_1^{B^0\to\pi^-}(Q^2) &= -\frac{8\pi\alpha_s m_B^2}{9} f_\pi f_B \int_0^1 dx_1\, dx_2 \int b_1\, db_1\, b_2\, db_2\, \mathrm{Exp}\left[-S_{B\pi}(x_i,\bar{x}_i,b_i,\mu)\right] S_t(x_1) S_t(x_3) \\
&\cdot \Big(h_e(x_3', x_1, b_3, b_1)\, r_\pi^0 \left(\varphi_+ + \varphi_- - \frac{m_b}{m_B}\varphi_+\right) \left(\varphi_\pi^p(x_3) - \varphi_\pi^t(x_3)\right) \\
&+ h_e(x_1, x_3, b_1, b_3) \left[x_1 \eta_3\, \varphi_+\, \varphi_\pi(x_3) - 2\, x_1\, r_\pi^0\, \varphi_+\, \varphi_\pi^p(x_3)\, S_t(x_3)\right] \Big) \\
\mathcal{F}_3^{B^0\to\pi^-}(Q^2) &= -\frac{8\pi\alpha_s m_B^2}{9} f_\pi f_B \int_0^1 dx_1\, dx_2 \int b_1\, db_1\, b_2\, db_2\, \mathrm{Exp}\left[-S_{B\pi}(x_i,\bar{x}_i,b_i,\mu)\right] S_t(x_1) \\
&\cdot \Big(h_e(x_3', x_1, b_3, bs_1) \Big[\left[\frac{m_b}{m_B}(\varphi_+ + \varphi_-) - (1 - x_3\eta_3)\,\varphi_+\right] \varphi_\pi(x_3) \\
&- r_\pi^0 \left[x_3(\varphi_+ + \varphi_-) + \frac{m_b}{\eta_3\, m_B}(\varphi_- - \varphi_+)\, S_t(x_3)\right] \varphi_\pi^p(x_3) \\
&+ r_\pi^0 \left[\left(-x_3 + \frac{2}{\eta_3} - \frac{m_b}{\eta_3\, m_B}\right)(\varphi_+ + \varphi_-)\, S_t(x_3)\right] \varphi_\pi^t(x_3) \Big] \\
&+ h_e(x_1, x_3, b_1, b_3) \left[-x_1\, \varphi_+\, \varphi_\pi(x_3) + 2\, r_\pi^0 \left(\varphi_- + \frac{x_1}{\eta_3}\varphi_+\right) \varphi_\pi^p(x_3)\, S_t(x_3)\right] \Big)
\end{aligned}$$

B meson

$$p_1 = \left(\frac{m_B}{\sqrt{2}}, \frac{m_B}{\sqrt{2}}, \mathbf{0_T} \right)$$
$$k_1 = \left(x_1 \frac{m_B}{\sqrt{2}}, 0, \mathbf{k_{1T}} \right)$$

Pseudoscalar mesons

$$p_2 = \left(\eta_2 \frac{m_B}{\sqrt{2}}, \frac{r_P^2}{\eta_2} \frac{m_B}{\sqrt{2}}, \mathbf{0_T} \right) \quad k_2 = \left(x_2 \eta_2 \frac{m_B}{\sqrt{2}}, 0, \mathbf{k_{2T}} \right)$$
$$p_3 = \left(\frac{r_P^2}{\eta_3} \frac{m_B}{\sqrt{2}}, \eta_3 \frac{m_B}{\sqrt{2}}, \mathbf{0_T} \right) \quad k_3 = \left(0, x_3 \eta_3 \frac{m_B}{\sqrt{2}}, \mathbf{k_{3T}} \right)$$

Framework—Inputs

Parameters	B Meson
ω_B (GeV)	0.35 ± 0.04
f_B (GeV)	0.19
m_b (GeV)	4.98
m_B (GeV)	5.28

$$\alpha_s(\mu) = \frac{2\,\pi}{\beta_1 \log(\mu/\Lambda^{(n_f)})} \left[1 - \frac{\beta_2}{\beta_1^2} \frac{\log(2\log(\mu/\Lambda^{(n_f)}))}{2\log(\mu/\Lambda^{(n_f)})}\right]$$

$$\beta_1 = \frac{33 - 2\,n_f}{3} \qquad \beta_2 = 102 - 10\,n_f - 2\,C_F\,n_f$$

$$\Lambda^{(n_f)} = \Lambda^{(n_f)}_{\overline{\mathrm{MS}}} = \mathrm{Which}[\,n_f \leq 3\,, 0.332\,, 3 < n_f < 5\,, 0.292\,, 4 < n_f < 6\,, 0.210\,]$$

Framework—Inputs

Meson	$\pi^\pm/\pi^0$	$K^\pm/K^0$	η_q	η_s
m (GeV)	0.140/0.135	0.494/0.498	0.104	0.705
f (GeV)	0.130	0.156	0.125	0.177
m_0 (GeV)	1.913	1.892	1.087	1.990
a_1	0	0.076 ± 0.004	0	0
a_2	0.270 ± 0.047	0.221 ± 0.082	0.250 ± 0.150	0.250 ± 0.150

Meson	$\rho^\pm/\rho^0$	$K^{*\pm}/K^0$	ω	ϕ
m (GeV)	0.775	0.892	0.783	1.019
$f^\parallel$ (GeV)	0.210/0.213	0.204	0.197	0.233
$f^\perp$ (GeV)	0.144/0.146	0.159	0.162	0.191
$a_1^\parallel$	0	0.060 ± 0.040	0	0
$a_1^\perp$	0	0.040 ± 0.030	0	0
$a_2^\parallel$	0.180 ± 0.037	0.160 ± 0.090	0.150 ± 0.120	0.230 ± 0.080
$a_2^\perp$	0.137 ± 0.030	0.100 ± 0.080	0.140 ± 0.120	0.140 ± 0.070

Branching ratio

$$Br(B \to P_2 P_3) = \frac{|p_c|}{8\pi m_B^2 \Gamma_B} \overline{\Sigma} |\mathcal{A}(B \to P_2 P_3)|^2$$

The direct and mixing-induced CP asymmetries

$$C_f = \frac{1 - |\zeta|^2}{1 + |\zeta|^2} \qquad S_f = \frac{2\,\mathrm{Im}[\zeta]}{1 + |\zeta|^2}$$

$\zeta = \bar{\mathcal{M}}(\bar{B}^0 \to \bar{f}) / \mathcal{M}(B^0 \to f)$

Numerics—Branching ratio

Ya-Lan Zhang,et al. $Phys. Rev. D\, 90, 014029$ (in unit of 10^{-6})

Channel	LO	NLOWC	+VC	+QL	+MP	NLO	QCDF	Data
$B^0 \to \pi^+\pi^-$	7.46	6.65	6.91	7.02	6.87	$7.67^{+3.47}_{-2.64}$	8.9	5.10±0.19
$B^+ \to \pi^+\pi^0$	3.54	4.23	3.54	—	—	$4.27^{+1.85}_{-1.47}$	6.0	$5.48^{+0.35}_{-0.34}$
$B^0 \to \pi^0\pi^0$	0.12	0.24	0.27	0.29	0.21	$0.23^{+0.19}_{-0.15}$	0.3	$1.91^{+0.22}_{-0.23}$

This work

Channel	LO	+VC	+QL	+MP	NLOx23	NLOx23p	Data
$B^0 \to \pi^+\pi^-$	8.99	4.81	4.80	4.95	4.99	5.60	5.12±0.19
$B^+ \to \pi^+\pi^0$	4.36	2.90	—	—	2.77	—	5.5±0.4
$B^0 \to \pi^0\pi^0$	0.130	0.116	0.121	0.190	0.190	0.216	1.59±0.26

Numerics—Branching ratio

- with $\mathcal{O}(x_1)$, $\mathcal{O}(\frac{m_b}{m_B})$ corrections

Channel	LO	+VC	+QL	+MP	NLOx23	NLOx23p	Data
$B^0 \to \pi^+\pi^-$	8.81	4.63	4.70	4.79	4.50	5.09	5.12±0.19
$B^+ \to \pi^+\pi^0$	4.30	2.65	—	—	2.50	—	5.5±0.4
$B^0 \to \pi^0\pi^0$	0.123	0.108	0.112	0.154	0.150	0.137	1.59±0.26

- with also $\varphi_+ \neq \varphi_-$ correction in progress

$$\varphi_+(x,b) = N_B\, x^2(1-x)^2 \times \mathrm{Exp}\left(-\frac{1}{2}(\frac{xm_B}{\omega_B})^2 - \frac{\omega^2 b^2}{2}\right)$$

$$\varphi_-(x,b) = N_B \frac{\omega_B^2}{m_B^4}\left\{\mathrm{Exp}\left[-\frac{1}{2}\left(\frac{xm_B}{\omega_B}\right)^2\right] \times \left(m_B^2(1-x)^2 + 2\omega_B^2\right) + \sqrt{2\pi}\mathrm{Erf}\left(\frac{xm_B}{\sqrt{2}\omega_B}\right)\right\} \times \mathrm{Exp}\left[-\frac{1}{2}(\omega_B b)^2\right] + C$$

Takeshi Kurimoto. $Phys. Rev. D$ 74, 014027

Numerics—CPV

Ya-Lan Zhang,et al. $Phys. Rev. D\, 90, 014029$

Channel	LO	+VC	+QL	+MP	NLO	Data$_{2014}$
$C_{\pi^+\pi^-}$	-0.10	0.61	0.69	0.74	$0.78^{+0.05}_{-0.08}$	—
$S_{\pi^+\pi^-}$	-0.02	0.67	0.41	0.50	$0.47^{+0.02}_{-0.11}$	—
$C_{\pi^0\pi^0}$	0.27	0.26	0.13	0.12	$0.12^{+0.04}_{-0.06}$	+0.31±0.05

This work

Channel	LO	+VC	+QL	+MP	NLOx23	NLOx23p	Data
$C_{\pi^+\pi^-}$	0.25	0.31	0.32	0.29	0.29	0.61	-0.32±0.04
$S_{\pi^+\pi^-}$	-0.19	0.18	-0.07	0.35	0.35	0.39	-0.65±0.04
$C_{\pi^0\pi^0}$	0.02	0.03	-0.31	-0.88	-0.88	-0.84	-0.33±0.22

Operators (in unit of 10^{-4})

Channel	T	C	E	P	P_{em}
$B^0 \to \pi^0\pi^0{}_{LO}$	—	3.74-1.47i	-3.85+1.91i	5.61+1.17i	0.57+1.24i
$B^0 \to \pi^0\pi^0{}_{NLO}$	—	1.61+3.36i	-2.35+1.29i	3.99-0.81i	0.39+0.90i
$B^0 \to \pi^+\pi^-{}_{LO}$	39.2-15.5i	—	-3.40+1.56i	5.63+1.18i	-0.08i
$B^0 \to \pi^+\pi^-{}_{NLO}$	28.6-10.1i	—	-2.08+0.88i	3.99-0.81i	-0.09
$B^0 \to \pi^+\pi^0{}_{LO}$	39.2-15.5i	-3.75+1.48i	0.32-0.45i	—	-0.57-1.32i
$B^0 \to \pi^+\pi^0{}_{NLO}$	28.6-10.1i	-1.61-3.36i	0.26-0.40i	—	-0.33-0.97i

Summary

1. The most precise PQCD prediction of $B \to \pi\pi$.
2. Updated study with new inputs, with sizable changes of the result.
3. Go on to complete other 18 decay channels of $B \to PP$ processes.

THANK YOU