Global Analysis of charmless two body B/Bs decays in PQCD Approach

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CONTENT

Motivation

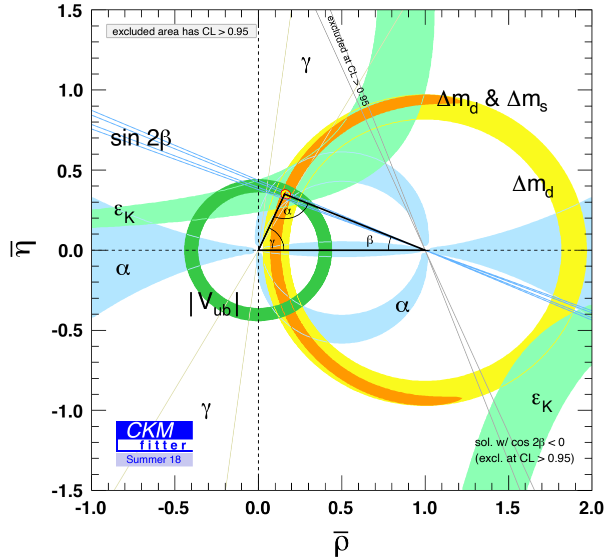
1999 —— 2008
SLAC(Babar)

2009 ——
CERN(LHCb)

KEKB(Belle)
1999 —— 2010

KEKB(Belle-II)
2018 ——

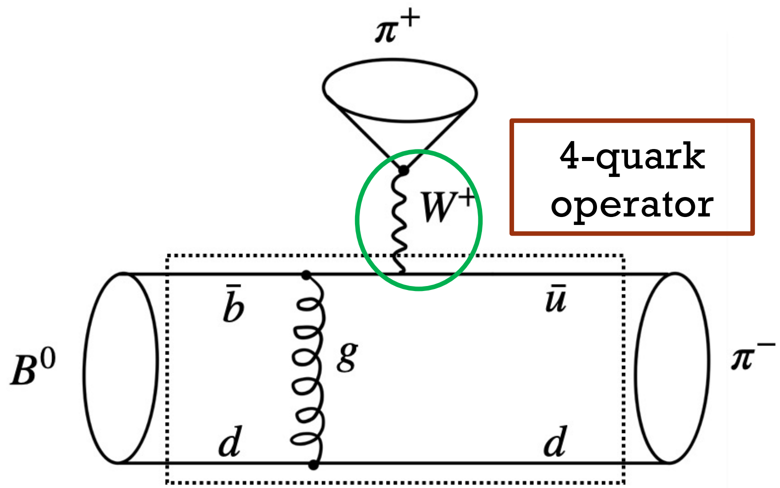
- **B meson decay is an ideal place to study CKM phase angle and CP violation.**
- **The precision test of Standard Model (SM) will help us to find new physics or search new particles indirectly**



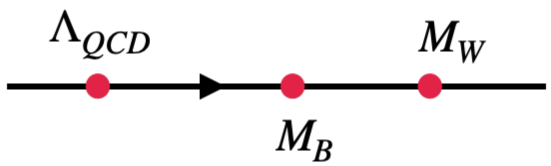
Motivation

Consider a B meson decay process

$$B^0 \to \pi^+ \pi^-$$



QCD factorization: separating the processes with different energy scales:



Perturbative!

$$\begin{aligned} \mathcal{A} &= \langle M_2 M_3 | \mathcal{H}_{eff} | B \rangle \\ &\sim \int d^4k_1 d^4k_2 d^4k_3 \ \mathrm{Tr}\left[C(t) \Phi_B(k_1) \Phi_{M_2}(k_2) \Phi_{M_3}(k_3) H(k_1, k_2, k_3, t) \right], \end{aligned}$$

Wilson coefficients

Non-perturbative, but UNIVERSAL DA

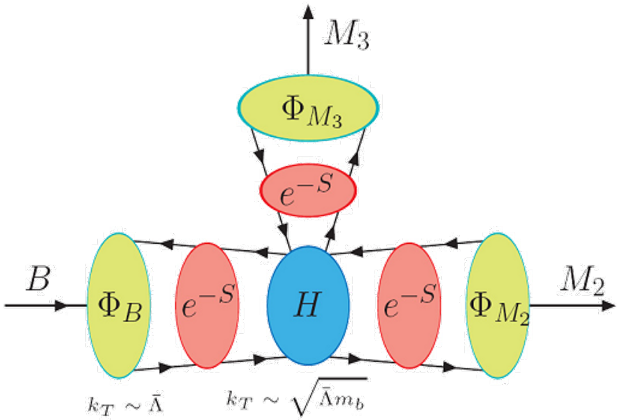
Motivation

k_T factorization:

The full amplitudes can be factorized as:

$$A = \phi_B \otimes C \otimes H \otimes J_t \otimes S \otimes \phi_{M_2} \otimes \phi_{M_3}$$

Hard Kernel	Wave Function	Sudakov Factor	Jet Function	Wilson Coefficients



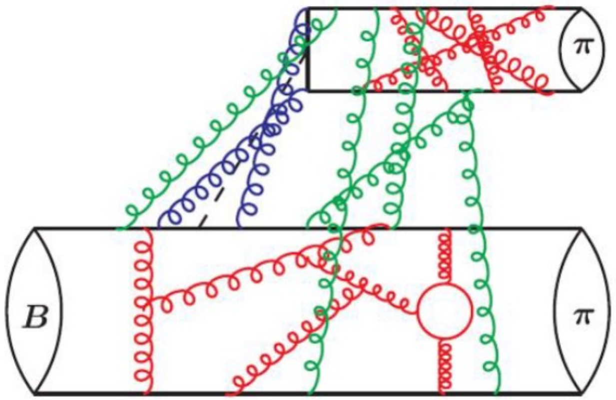
Sudakov Factor ↑ **k_T resummation**

Jet Function ↑ **Threshold resummation**

Other approaches :

- QCD factorization
- SCET factorization
- ……

Motivation



Complicated ! ! !

PHYSICAL REVIEW D **76,** 074018 (2007)

Charmless nonleptonic B_s decays to PP, PV, and VV final states in the perturbative QCD approach

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Charmless two-body decays:
B->PP: 34
B->VP: 62
B->VV: 34
Three body:
hundreds of process…

How to simplify ?

Auto calculation

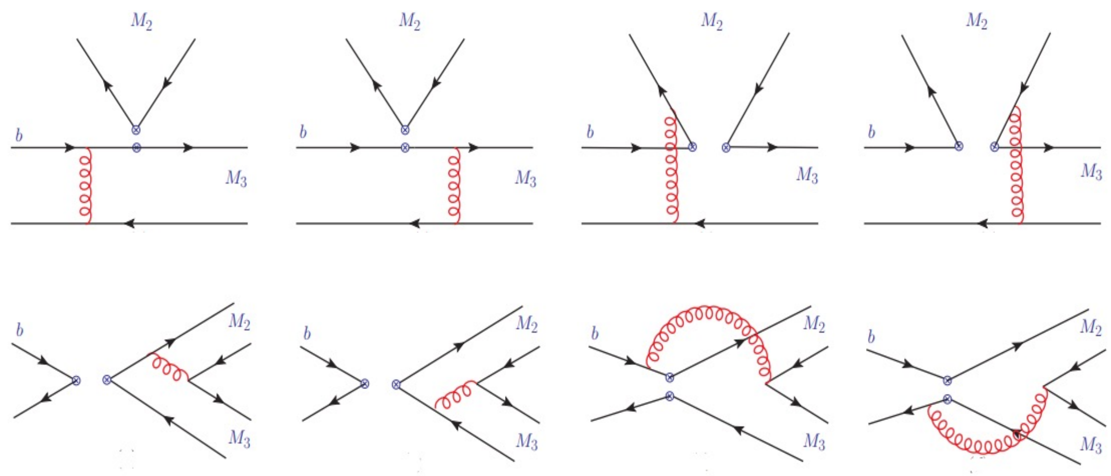
Factorizable | Non-factorizable

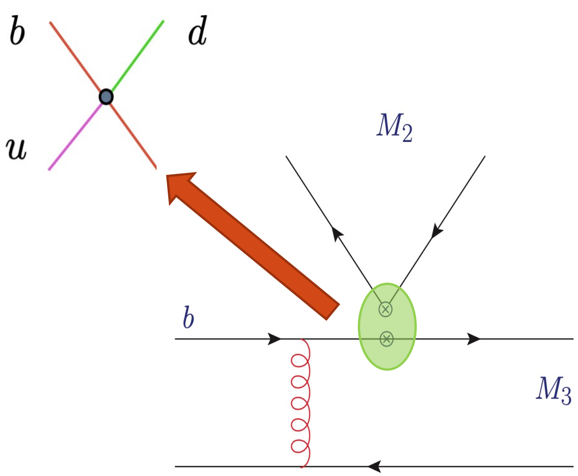
According to the Feynman diagram:

Emission

4 types

Annihilation





$$\mathcal{H}_{eff} = \frac{G_F}{\sqrt{2}}\Big\{\sum_{q=u,c} V_{qb}V^*_{qD}\left[C_1 O_1^q + C_2 O_2^q\right] - V_{tb}V^*_{tD}\left[\sum_{i=3}^{10} C_i O_i\right]\Big\} + \mathrm{H.c.},$$

3 types

According to the Fermi 4-quark interaction：

(V-A)(V-A)
(V-A)(V+A)
(S-P)(S+P)

$$O_1^q = (\bar{q}_\alpha b_\beta)_{V-A}(\bar{D}_\beta q_\alpha)_{V-A},$$

$C_1 O_1 \sim C_{10} O_{10}$

Auto calculation

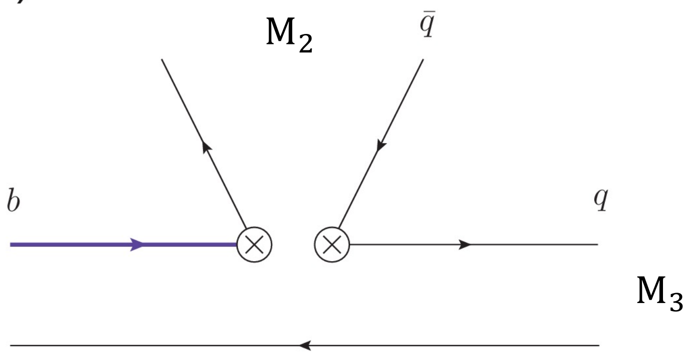
The decay amplitudes $\mathrm{B} \to M_2 M_3$

$$\langle M_2 M_3|\mathcal{H}_{eff}|B\rangle = \frac{G_F}{\sqrt{2}} V_{ub} V_{uq}^* \Big[\mathcal{A}_u(B \to M_2 M_3)\Big] - \frac{G_F}{\sqrt{2}} V_{tb} V_{tq}^* \Big[\mathcal{A}_t(B \to M_2 M_3)\Big],$$

SU(3) Flavor Structure:

$$B^- = (1,0,0), \quad \cdots\cdots \qquad \delta_u = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}, \quad \Lambda^d = \begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix}, \quad \cdots\cdots$$

$$M_{\pi^+} = M_{\rho^+} = \begin{array}{c} \begin{matrix} \bar{u} & \bar{d} & \bar{s} \end{matrix} \\ \begin{pmatrix} 0 & 0 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} \end{array} \begin{matrix} u \\ d \\ s \end{matrix} \qquad \cdots\cdots$$



$$A_u(B \to M_2 M_3) = [F_e(a_1) + M_e(C_1)]\, B M_3 \delta_u M_2 \Lambda_f + [F_e(a_2) + M_e(C_2)]\, B M_3 \Lambda_f Tr[\delta_u M_2]$$

$C_2 + 1/3C_1$

factorizable non-factorizable

Auto calculation

$$B^- = (1,0,0), \quad \overline{B} = (0,1,0), \quad \overline{B}_s^0 = (0,0,1),$$

$$M_{\pi^+} = M_{\rho^+} = \begin{pmatrix} 0 & 0 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}, \quad M_{K^+} = M_{K^{*+}} = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 1 & 0 & 0 \end{pmatrix}, \quad M_{K^0} = M_{K^{*0}} = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 1 & 0 \end{pmatrix},$$

$$\sqrt{2}M_{\pi^0} = \sqrt{2}M_{\rho^0} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & 0 \end{pmatrix}, \quad \sqrt{2}M_{\eta_q} = \sqrt{2}M_{\omega} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \end{pmatrix}, \quad M_{\eta_s} = M_{\phi} = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 1 \end{pmatrix},$$

$$M_{\pi^-} = M_{\rho^-} = M_{\pi^+}^T, \quad M_{K^-} = M_{K^{*-}} = M_{K^+}^T, \quad M_{\bar{K}^0} = M_{\bar{K}^{*0}} = M_{K^0}^T,$$

$$\delta_u = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}, \quad \Lambda_d = \begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix}, \quad \Lambda_s = \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix}, \quad e_Q = \begin{pmatrix} 1 & 0 & 0 \\ 0 & -\frac{1}{2} & 0 \\ 0 & 0 & -\frac{1}{2} \end{pmatrix}.$$

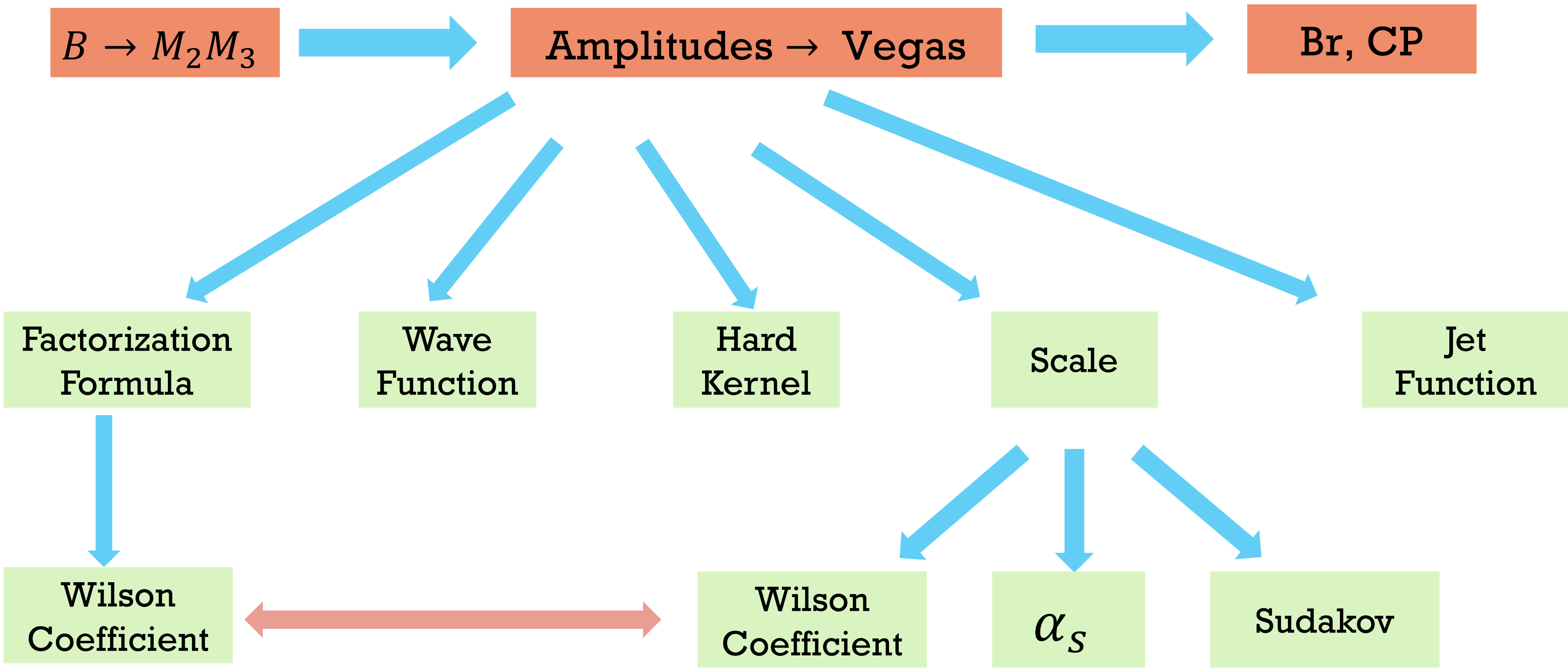
Auto calculation

The total amplitudes：

$$M = \frac{G_F}{\sqrt{2}} V_{ub} V_{uq}^* \Big[\mathcal{A}_u(B \to M_2 M_3) \Big] - \frac{G_F}{\sqrt{2}} V_{tb} V_{tq}^* \Big[\mathcal{A}_t(B \to M_2 M_3) \Big],$$

$$\begin{aligned}
A_u(B \to M_2 M_3) &= \Big[F_e^{LL}(a_1) + M_e^{LL}(C_1) \Big] B M_3 \delta_u M_2 \Lambda_f + \Big[F_e^{LL}(a_2) + M_e^{LL}(C_2) \Big] B M_3 \Lambda_f Tr[\delta_u M_2] \\
&+ \Big[F_{ann}^{LL}(a_1) + M_{ann}^{LL}(C_1) \Big] B \delta_u M_3 M_2 \Lambda_f + \Big[F_{ann}^{LL}(a_2) + M_{ann}^{LL}(C_2) \Big] B \Lambda_f Tr[\delta_u M_3 M_2], \\
A_t(B \to M_2 M_3) &= \Big[F_e^{LL}(a_3) + F_e^{LR}(a_5) + M_e^{LL}(C_4) + M_e^{SP}(C_6) \Big] B M_3 \Lambda_f Tr[M_2] \\
&+ \Big[F_e^{LL}(a_4) + F_e^{SP}(a_6) + M_e^{LL}(C_3) + M_e^{LR}(C_5) \Big] B M_3 M_2 \Lambda_f \\
&+ \Big[F_e^{LR}(a_7) + F_e^{LL}(a_9) + M_e^{SP}(C_8) + M_e^{LL}(C_{10}) \Big] B M_3 \Lambda_f Tr[e_Q M_2] \\
&+ \Big[F_e^{SP}(a_8) + F_e^{LL}(a_{10}) + M_e^{LR}(C_7) + M_e^{LL}(C_9) \Big] B M_3 e_Q M_2 \Lambda_f \\
&+ \Big[F_{ann}^{LL}(a_3) + F_{ann}^{LR}(a_5) + M_{ann}^{LL}(C_4) + M_{ann}^{SP}(C_6) \Big] B \Lambda_f Tr[M_3 M_2] \\
&+ \Big[F_{ann}^{LL}(a_4) + F_{ann}^{SP}(a_6) + M_{ann}^{LL}(C_3) + M_{ann}^{LR}(C_5) \Big] B M_3 M_2 \Lambda_f \\
&+ \Big[F_{ann}^{LR}(a_7) + F_{ann}^{LL}(a_9) + M_{ann}^{SP}(C_8) + M_{ann}^{LL}(C_{10}) \Big] B \Lambda_f Tr[e_Q M_3 M_2] \\
&+ \Big[F_{ann}^{SP}(a_8) + F_{ann}^{LL}(a_{10}) + M_{ann}^{LR}(C_7) + M_{ann}^{LL}(C_9) \Big] B e_Q M_3 M_2 \Lambda_f,
\end{aligned}$$

Auto calculation



C++ , gsl

Bessel function
Vegas

Global fit

Fit parameters :

CKM phase angle $\boldsymbol{\gamma}$

Gegenbauer moments:

$$\underbrace{(a_{2\pi}, a_{4\pi}, a_{1K}, a_{2K}, a_{4K})}_{twist2 \quad (pseudoscalar\ meson)}, \qquad \underbrace{(a_{2\rho}, a_{1K*}, a_{2K*})}_{twist2 \quad (vector\ meson)}$$

$$\underbrace{(a^{P}_{2\pi}, a^{T}_{2\pi}, a^{P}_{2K}, a^{T}_{2K})}_{twist3 \quad (pseudoscalar\ meson)},$$

Global fit

- **A specific process of two-body B meson decay ——(20 mins each)**

How to fit Gegenbauer moments ?

Distribution amplitude of Pseudo-scalar meson(twist-2):

$$\phi_P(x) = \frac{f_P}{2\sqrt{2N_c}} 6x(1-x)\left[1 + a_1 C_1^{3/2}(1-2x) + a_2 C_2^{3/2}(1-2x) + a_4 C_4^{3/2}(1-2x)\right]$$

The contribution of Gegenbauer moments can be decomposed:

$$\langle M_2 (a_2 + a_4) M_3 (a_2) | \mathcal{H}_{eff} | B \rangle$$
$$= a_{2,M_2} a_{2,M_3} \langle M_2 M_3 | \mathcal{H}_{eff} | B \rangle_{22} + a_{4,M_2} a_{2,M_3} \langle M_2 M_3 | \mathcal{H}_{eff} | B \rangle_{42}$$

Then the full amplitude: $A = \sum_{n,m=1} a_n \cdot a_m \cdot \underline{M_{nm}}$

Database of different Gegenbauer moments

PP: 9*9

PV: 9*4*2

Global fit

Python Lsq-fit

Least-Squares Fitting (lsq)

- Minimizes the summed χ^2 of residuals
- The summed square of residuals is defined as :

$$\chi^2 = \sum_{i=1}^{n} \frac{(y_i - \hat{y}_i)^2}{\sigma_y^2}$$

Channel

Experimental data

Fitted data

Experimental error

Bayesian analysis

- To stabilize a complicate non-linear lsq fit, one can use bayesian analysis.
- The modified residuals is defined by:

$$\chi_m^2 = \chi^2 + \chi_{prior}^2, \quad \chi_{prior}^2 = \sum_n \frac{(a_n - \tilde{a}_n)^2}{\tilde{\sigma}_{a_n}^2},$$

fit parameter (Gegenbauer)

Refers to QCDSR[1]

$\tilde{a}_n \pm \tilde{\sigma}_{a_n}^2$ is chosen by physical background at reasonable range.

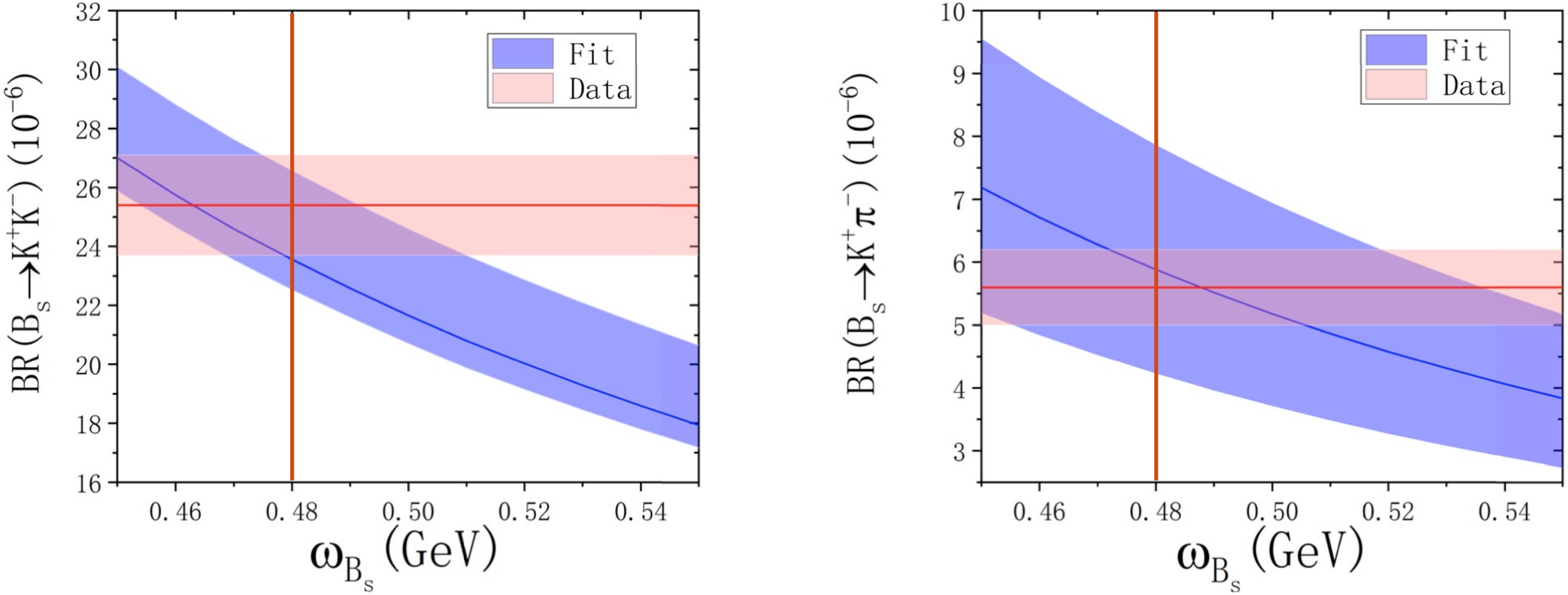
[1]. Ball P, Zwicky R, Phys. Rev. D 71, 014015 (2006)

Global fit

ω_b **in two body** B_s **decay depends on specific process, can not extracted by fitting database**

Method of extracting ω_{b_s} in two body B_s decay:

We compare the experimental data with PQCD prediction of running ω_{b_s}.



$B_s \to K^+K^-$
$B_s \to K^+\pi^-$
$B_s \to K^0\bar{K}^0$
$B_s \to \pi^+\pi^-$

Global fit

Channel:19
Data:22

channel	data		fit	
	branching ratio(10^{-6})	A_{CP}(%)	branching ratio(10^{-6})	A_{CP}(%)
$B^0 \to \bar{K}^0 K^0$	1.21 ± 0.16	-60 ± 70	1.23 ± 0.08	0 ± 0
$B^0 \to \bar{K}^0 \pi^0$	9.90 ± 0.50	0 ± 13	8.98 ± 0.19	-4.02 ± 0.48
$B^0 \to K^- \pi^+$	19.6 ± 0.50	-8.3 ± 0.6	20.3 ± 0.36	-8.34 ± 0.36
$B^0 \to \pi^- \pi^+$	5.12 ± 0.19	32 ± 4	5.24 ± 0.17	23.2 ± 2.1
$B^0 \to \rho^0 \bar{K}^0$	3.40 ± 1.10	4 ± 20	3.06 ± 0.37	2.853 ± 0.068
$B^0 \to \pi^0 \bar{K}^{*0}$	3.30 ± 0.60	-15 ± 13	1.73 ± 0.10	-6.02 ± 0.6
$B^0 \to \pi^- \rho^+ / \pi^+ \rho^-$	23.0 ± 2.30	$13 \pm 6 / -8 \pm 8$	23.33 ± 0.8	$-24.3 \pm 1/8.1 \pm 1.1$
$B^- \to K^0 K^-$	1.31 ± 0.17	4 ± 14	1.47 ± 0.09	22.5 ± 2.7
$B^- \to \pi^0 K^-$	12.9 ± 0.50	3.7 ± 2.1	12.99 ± 0.23	-6.44 ± 0.6
$B^- \to \bar{K}^0 \pi^-$	23.7 ± 0.80	-1.7 ± 1.6	23.15 ± 0.42	-2.84 ± 0.24
$B^- \to \rho^- \pi^0$	10.9 ± 1.40	2 ± 11	8.73 ± 0.25	24.2 ± 2.3
$B^- \to \pi^0 K^{*-}$	6.80 ± 0.90	-39 ± 21	3.51 ± 0.19	-33.5 ± 1.7
$B^- \to K^- K^{*0}$	0.59 ± 0.08	12 ± 10	0.476 ± 0.022	22.5 ± 1.3
$B_s \to K^- K^+$	26.6 ± 2.20	-14 ± 11	24.8 ± 1.50	-8.1 ± 2.3
$B_s \to \pi^- \pi^+$	0.7 ± 0.1	$-$	0.798 ± 0.092	-1.62 ± 0.39
$B_s \to K^0 \bar{K}^0$	20.0 ± 6.00	0 ± 0	26.2 ± 1.60	0 ± 0
$B_s \to \pi^- K^+$	5.80 ± 0.70	22.1 ± 1.5	5.69 ± 0.64	22.1 ± 1.2
$B_s \to K^+ K^{*-} / K^- K^{*+}$	19.0 ± 5.0	$-$	15.28 ± 0.90	$-33.8 \pm 1.3/53.5 \pm 2.4$
$B_s \to K^0 \bar{K}^{*0} / \bar{K}^0 K^{*0}$	20.0 ± 6.00	$-$	15.06 ± 0.96	0 ± 0

over 5 sigma error data

Global fit

CKM phase angle γ**:** 75.1 ± 2.9

Experimental γ**:** $72.1^{+4.1}_{-4.5}$

Our fitted Gegenbauer moments of pseudoscalar meson and vector meson:

Gegenbauer moments	$a_{1\pi}$	$a_{2\pi}$	$a_{4\pi}$	$a_{2\pi}^P$	$a_{2\pi}^T$	$a_{1\rho}^{\parallel}$	$a_{2\rho}^{\parallel}$
Our Result	—	**0.644±0.075**	**-0.41 ±0.09**	**1.08 ±0.15**	**-0.48±0.33**	—	**0.16±0.08**
Sum Rule	—	**0.25±0.15**	**-0.015±0.025**	—	—	—	**0.15±0.07**

Gegenbauer moments	a_{1K}	a_{2K}	a_{4K}	a_{2K}^P	a_{2K}^T	$a_{1K^*}^{\parallel}$	$a_{2K^*}^{\parallel}$
Our Result	**0.331±0.082**	**0.28±0.10**	**-0.398±0.073**	—	—	—	**0.137±0.029**
Sum Rule	**0.06±0.03**	**0.25±0.15**	—	—	—	**0.03±0.02**	**0.11±0.09**

[1]. Ball P, Zwicky R, Phys. Rev. D 71, 014015 (2006)

Global fit

Other processes predicted by our results:

channel	data		fit		PQCD
	branching ratio(10^{-6})	$A_{CP}(\%)$	branching ratio(10^{-6})	$A_{CP}(\%)$	branching ratio
$B^0 \to K^+K^-$	0.078 ± 0.015	$-$	0.155 ± 0.027	52.0 ± 15.0	
$B^0 \to \pi^+K^{*-}$	7.5 ± 0.4	-27 ± 4	4.93 ± 0.28	-52.0 ± 2.1	5.1 [58]
$B^0 \to \pi^0\rho^0$	2.0 ± 0.5	-27 ± 24	0.026 ± 0.0022	-47 ± 21	0.15 [59]
$B^0 \to K^-\rho^+$	7.0 ± 0.9	20 ± 11	4.41 ± 0.6	48.3 ± 4.9	4.7 [58]
$B^- \to \rho^-\bar{K}^0$	7.3 ± 1.2	-3 ± 15	3.39 ± 0.55	3.18 ± 0.55	3.6 [58]
$B^- \to \rho^0K^-$	3.7 ± 0.5	37 ± 1	2.24 ± 0.41	69.7 ± 3.0	2.5 [58]
$B^- \to \pi^-\bar{K}^{*0}$	10.1 ± 0.8	-4 ± 9	5.17 ± 0.23	-0.61 ± 0.19	5.5 [58]
$B^- \to \pi^-\rho^0$	8.3 ± 1.2	0.009 ± 0.019	4.61 ± 0.36	-35.3 ± 1.8	~ 5.39[63]
$B_s \to \pi^-K^{*+}$	2.9 ± 1.1	$-$	9.53 ± 0.24	-25.5 ± 1.0	7.6[52]

These channels are not predicted well in leading order by PQCD approach.

[52] A. Ali, G. Kramer, Y. Li, C. D. Lu, Y. L. Shen, W. Wang and Y. M. Wang, Phys. Rev. D 76, 074018 (2007)
[58] H. n. Li and S. Mishima, Phys. Rev. D 74, 094020 (2006)
[59] Z. Rui, X. Gao and C. D. Lu, Eur. Phys. J. C 72, 1923 (2012)
[63] C. D. Lu and M. Z. Yang, Eur. Phys. J. C 23, 275-287 (2002)

Summary

- We establish a database of B/Bs charmless two body decays by an automated program. Branching ratios and CPVs of different processes can be easily derived by this database.

- Based on this database, we fit the CKM phase angle γ and Gegenbauer moments of light mesons.

- With the fitted parameters, we give the PQCD predictions of branching ratios and CPVs of several processes. Some processes can not be predicted well at leading order.

Thank You !

Backup

$$A(\bar{B}_s^0 \to \pi^- K^+) = \frac{G_F}{\sqrt{2}} V_{ub} V_{ud}^* \{ f_\pi F_{B_s \to K}^{LL}[a_1] + M_{B_s \to K}^{LL}[C_1] \} - \frac{G_F}{\sqrt{2}} V_{tb} V_{td}^* \Big\{ f_\pi F_{B_s \to K}^{LL}[a_4 + a_{10}] + f_\pi F_{B_s \to K}^{SP}[a_6 + a_8]$$
$$+ M_{B_s \to K}^{LL}[C_3 + C_9] + f_{B_s} F_{\text{ann}}^{LL}\left[a_4 - \frac{1}{2} a_{10}\right] + f_{B_s} F_{\text{ann}}^{SP}\left[a_6 - \frac{1}{2} a_8\right] + M_{\text{ann}}^{LL}\left[C_3 - \frac{1}{2} C_9\right]$$
$$+ M_{\text{ann}}^{LR}\left[C_5 - \frac{1}{2} C_7\right] \Big\},$$