

SM Predictions on D^0 oscillations and CPV



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in collaboration with

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Summary

The statement/textbook-wisdom

“CP-violation (in mixing) of the order of one per mille is an unambiguous signal for new physics”

is **currently not justified.**

More theoretical work has to be done to shrink the allowed region for CP-violation within in the SM

- * It was a hard fight to convince people!
— **6 Referee reports before published in JHEP** —
- * A.A.Petrov at CKM2010:
at most $\approx 10^{-3}$ in the SM; 10^{-2} is a “smoking gun” signature of NP



Outline

Introduction: D mixing

Theoretical approaches for D mixing

HQE for the D system?

- Naive look at lifetimes
- Mixing: $D = 6$
- Mixing: $D > 6$
- Mixing: New Physics

Outlook

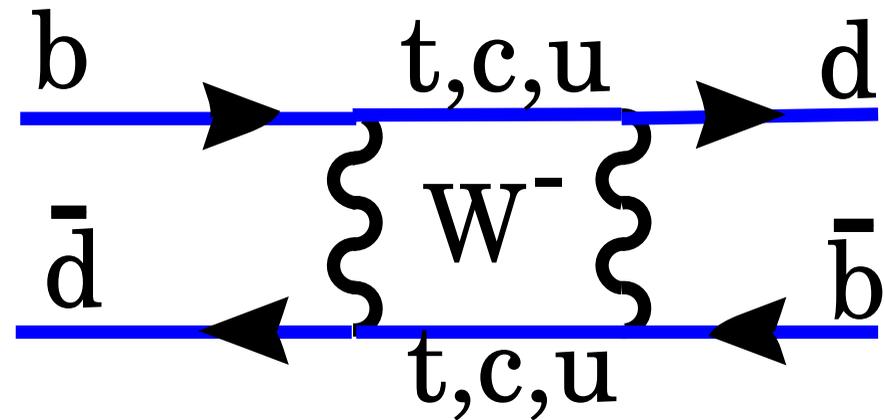
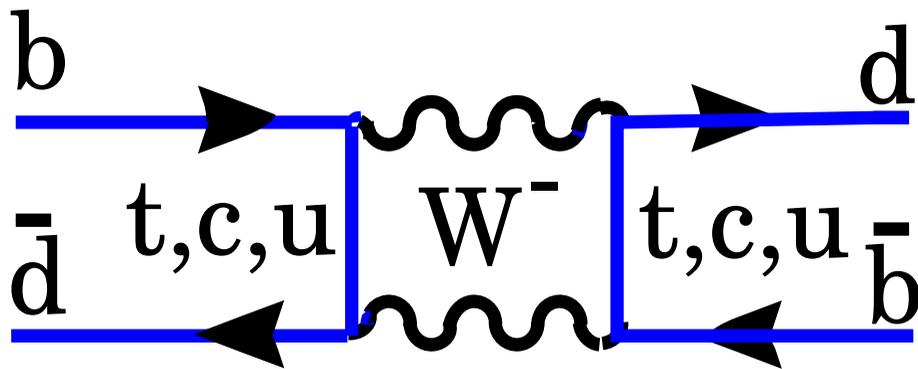


Mixing Formalism I

Time evolution of a decaying particle: $B(t) = \exp[-im_B t - \Gamma_B/2t]$
 can be written as

$$i \frac{d}{dt} \begin{pmatrix} |B(t)\rangle \\ |\bar{B}(t)\rangle \end{pmatrix} = \left(\hat{M} - \frac{i}{2} \hat{\Gamma} \right) \begin{pmatrix} |B(t)\rangle \\ |\bar{B}(t)\rangle \end{pmatrix}$$

BUT: In the neutral B -system transitions like $B_{d,s} \rightarrow \bar{B}_{d,s}$ are possible due to weak interaction: **Boxdiagrams**



\Rightarrow off-diagonal elements in \hat{M} , $\hat{\Gamma}$: M_{12} , Γ_{12} (complex)

Diagonalization of \hat{M} , $\hat{\Gamma}$ gives the physical eigenstates B_H and B_L with the masses M_H , M_L and the decay rates Γ_H , Γ_L red

CP-odd: $B_H := p B + q \bar{B}$, CP-even: $B_L := p B - q \bar{B}$ with $|p|^2 + |q|^2 = 1$



Mixing Formalism II

$|M_{12}|$, $|\Gamma_{12}|$ and $\phi = \arg(-M_{12}/\Gamma_{12})$ can be related to three observables:

- **Mass difference:** $\Delta M := M_H - M_L = 2|M_{12}| \left(1 + \frac{1}{8} \frac{|\Gamma_{12}|^2}{|M_{12}|^2} \sin^2 \phi + \dots \right)$

$|M_{12}|$: heavy internal particles: t, SUSY, ...

- **Decay rate difference:** $\Delta\Gamma := \Gamma_L - \Gamma_H = 2|\Gamma_{12}| \cos \phi \left(1 - \frac{1}{8} \frac{|\Gamma_{12}|^2}{|M_{12}|^2} \sin^2 \phi + \dots \right)$

$|\Gamma_{12}|$: light internal particles: u, c, ... (almost) no NP!!!

- **Flavor specific/semileptonic CP asymmetries:**

$\bar{B}_q \rightarrow f$ and $B_q \rightarrow \bar{f}$ forbidden

No direct CP violation: $|\langle f|B_q \rangle| = |\langle \bar{f}|\bar{B}_q \rangle|$

e.g. $B_s \rightarrow D_s^- \pi^+$ or $B_q \rightarrow X l \nu$ (semileptonic)

$$a_{sl} \equiv a_{fs} = \frac{\Gamma(\bar{B}_q(t) \rightarrow f) - \Gamma(B_q(t) \rightarrow \bar{f})}{\Gamma(\bar{B}_q(t) \rightarrow f) + \Gamma(B_q(t) \rightarrow \bar{f})} = -2 \left(\left| \frac{q}{p} \right| - 1 \right) = \text{Im} \frac{\Gamma_{12}}{M_{12}} = \frac{\Delta\Gamma}{\Delta M} \tan \phi$$



Introduction: D-mixing 1

- K^0 -mixing: 1955 Lederman (measured different lifetimes)
- B_d -mixing: 1987 DESY
- B_s -mixing: 2006 TeVatron

D-mixing is now also experimentally established

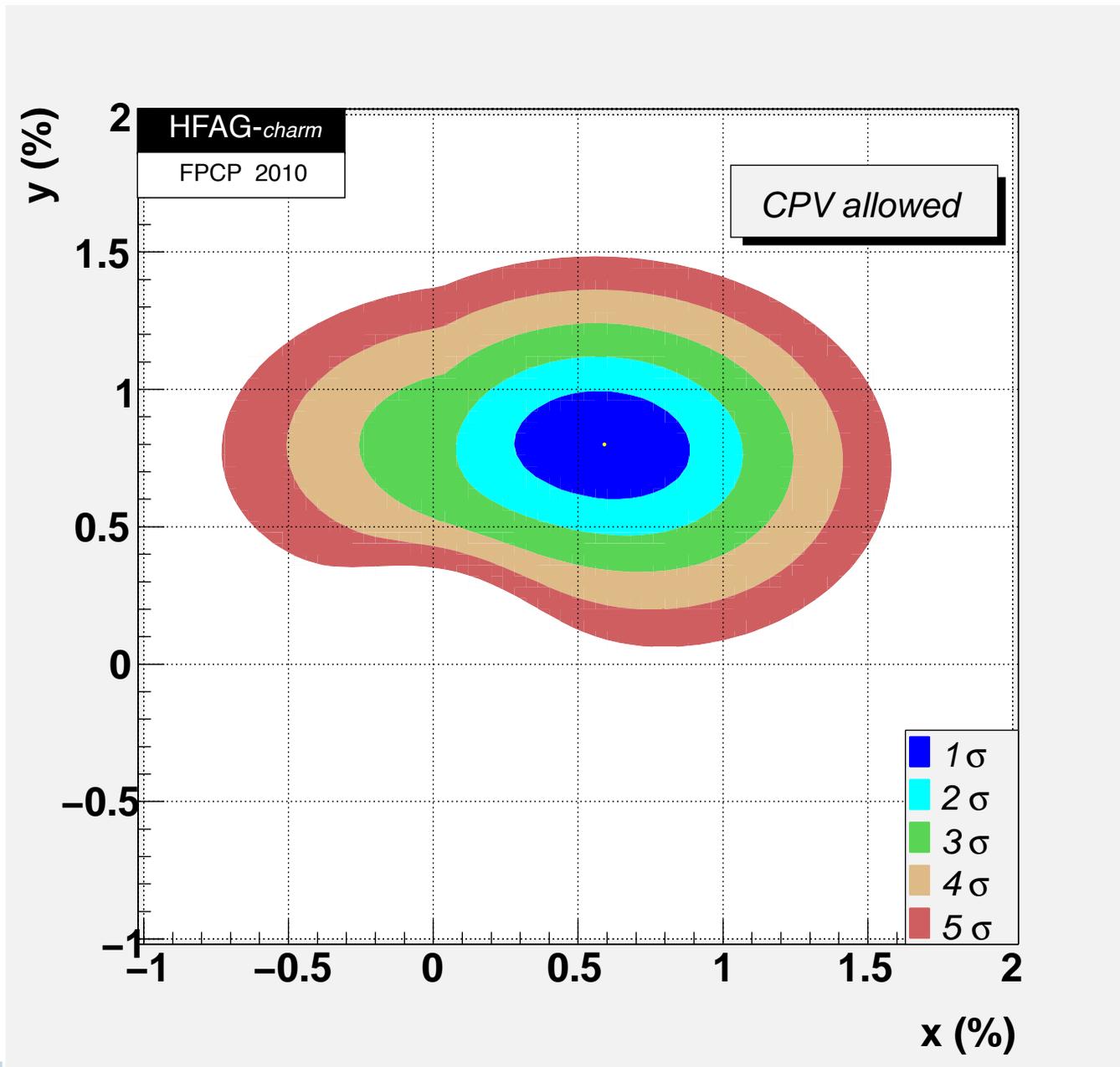
	1σ error	95% CL
$x := \frac{\Delta M}{\Gamma}$	$(0.59 \pm 0.20) \%$	$[0.19, 0.97] \%$
$y := \frac{\Delta \Gamma}{2\Gamma}$	$(0.83 \pm 0.13) \%$	$[0.54, 1.05] \%$

HFAG 2010
(BaBar, Belle,
CDF, CLEO)

- No single experiment above 5σ
- David Asner@CKM2010: The more precise, the less significant
- $\Rightarrow \Gamma_{12}/M_{12} \approx \mathcal{O}(1)$, i.e. not so nice formulas as in the B-case



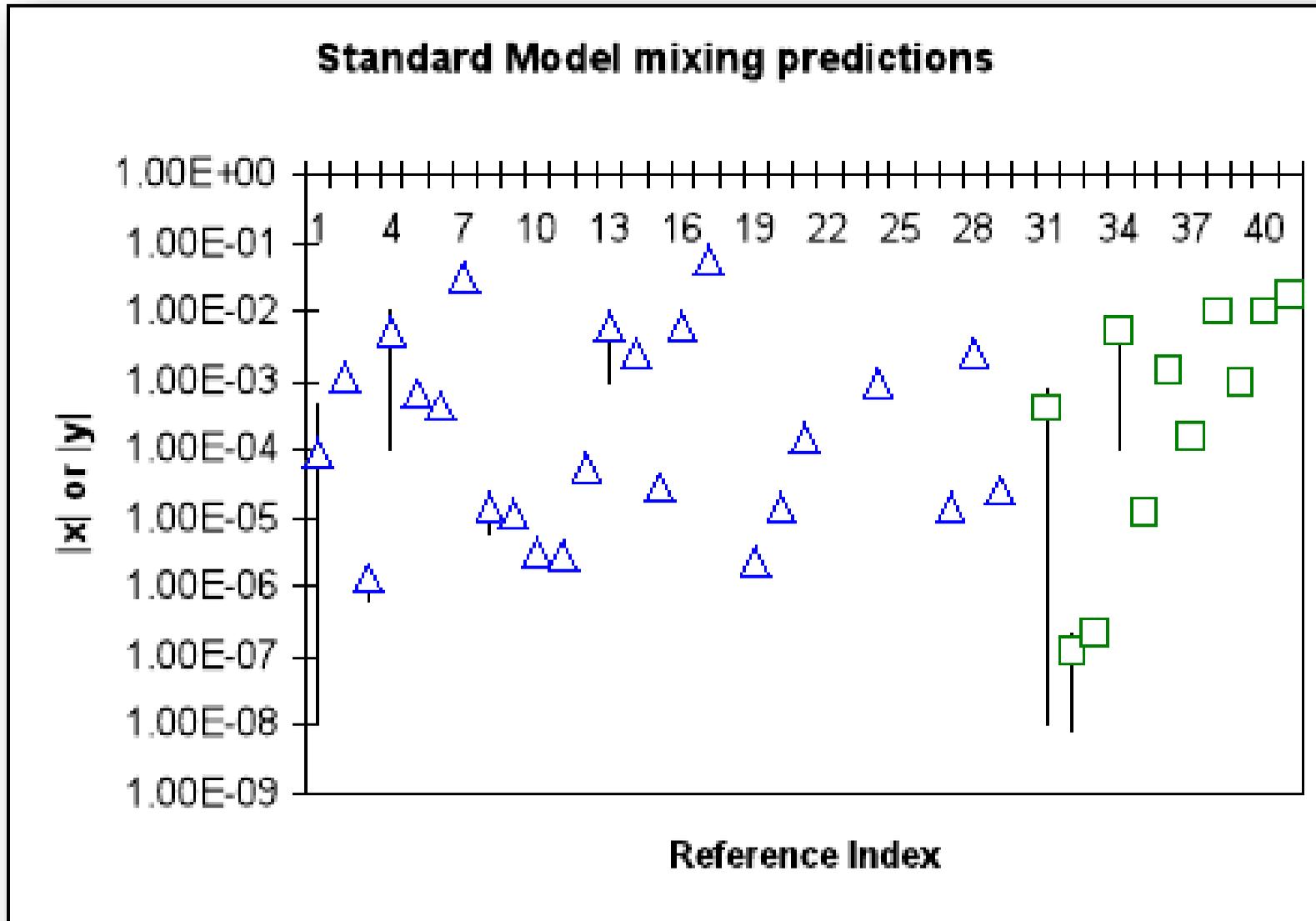
Introduction: D-mixing 2





Introduction: D-mixing 3

Theory fails? (grabbed from a talk of [Alexey Petrov](#))





Theory I

D mixing vs. B_s , B_d and K -mixing

1. **internal down-type quarks** in the box diagrams
2. the theory is **much more complicated!**

There are two approaches to describe the SM contribution to D-mixing

■ Exclusive Approach

Falk, Grossman, Ligeti, Petrov PRD65 (2002)

Falk, Grossman, Ligeti, Nir, Petrov PRD69 (2004)

■ Inclusive Approach

Georgi, PLB 297 (1992) Ohi, Ricciardi, Simmons, NPB 403 (1993)

Bigi, Uraltsev, NPB 592 (2001)

State of the art, but more an estimate than a calculation

⇒ x, y up to 1% not excluded

⇒ Essential no CPV in mixing — **unambiguous signal for NP!!!**



Theory II - Exclusive approach

y due to final states common to D and \bar{D}

$$y = \frac{1}{\Gamma} \sum_n \rho_n \langle \bar{D}^0 | \mathcal{H}_W^{\Delta C=1} | n \rangle \langle n | \mathcal{H}_W^{\Delta C=1} | D^0 \rangle$$

Much too complicated to calculate exclusive decay rates exactly!

- Estimate only SU(3) violating phase space effects (mild assumptions about \vec{p} -dependence of matrix elements) = **calculable source of SU(3) breaking**
- **Assume** hadronic matrix elements are SU(3) invariant
- **Assume** CP invariance of D decays
- **Assume** no cancellations with other sources of SU(3) breaking
- **Assume** no cancellations between different SU(3) multiplets

⇒ individual effects of 1% possible: $y^{Exp} \approx 1\% \not\approx NP$

- "our analysis does not amount to a SM calculation of y "



Theory III - “Phenomenological” approach

See talk of Hai-Yang Cheng:

“There is no QCD based theory for hadronic decays because $1/m_c$ is large
⇒ rely more on data than theory

$$x \approx 10^{-3} \quad y \approx \text{few} \times 10^{-3}$$

Cheng, Chiang PRD81,114020

Our approach:

**Do not give up yet
Try to push QCD to its limits**



Theory IV - Inclusive approach

Systematic expansion of the decay rate in powers of m_b^{-1} yields

$$\Gamma = \Gamma_0 + \frac{\Lambda^2}{m_b^2} \Gamma_2 + \frac{\Lambda^3}{m_b^3} \Gamma_3 + \dots$$

Voloshin, Uraltsev, Khoze, Shifman, Vainshtein

Γ_0 : Decay of a free quark \Rightarrow **all b-hadrons have the same lifetime**

Γ_2 : First corrections due to kinetic and chromomagnetic operator

Γ_3 : Weak annihilation and Pauli interference

Distinguish between different spectators \Rightarrow **Lifetime differences** $\frac{\tau_1}{\tau_2}, \Delta\Gamma$

numerically enhanced by phase space factor $16\pi^2$

The use of the HQE for the D-system is questionable!

- Λ/m_c might be too large ($\Lambda \neq \Lambda_{QCD}$!)
- $\alpha_s(m_c)$ might be too large



Conclusion for the B-system

Investigation of $\tau(B^+)/\tau(B_d)$ and $\tau(B_s)/\tau(B_d)$

HQE seems to work very well!

But: still a lot of work to do!

- Lattice determination of non-perturbative parameters
- Perturbative determination of all contributions to baryon lifetimes
- ...

⇒ **Use HQE in the search for new physics in B mixing** CKMfitter; UTfit;...
SM is excluded by 3.8σ A.L., Nierste, CKMfitter 1008.1593

Does it also work for the D-system?



Try HQE for the D-system

! This is just a naive estimate - a quantitative analysis has to be done!

$$\text{Exp.: } \frac{\tau(D^+)}{\tau(D^0)} = \frac{1040 \text{ fs}}{410 \text{ fs}} \approx 2.5 \qquad \frac{\tau(D_s^+)}{\tau(D^0)} = \frac{500 \text{ fs}}{410 \text{ fs}} \approx 1.2$$

■ HQE for D-system

- ◆ D^0 : weak annihilation (=WA)
- ◆ D^+, D_s^+ : Pauli interference (=PI); $\text{PI}(D_s^+) = (V_{us}/V_{ud})^2 \text{PI}(D^+)$

■ HQE for B-system

- ◆ B_d, B_s : WA, similar CKM structure, differences due to phase space
- ◆ B^+ : PI (larger than WA)

$$\Gamma(D_x) = \Gamma(c) + \delta\Gamma(D_x)$$

The experimental constraints are full-filled for

$$\frac{\delta\Gamma(D^+)}{\Gamma(c)} \approx -53\%, \qquad \frac{\delta\Gamma(D^0)}{\Gamma(c)} \approx +19\%$$

This looks reasonable: $(m_b/m_c)^3 \approx 20\dots30$



Definitions for D-mixing

$$y := \frac{\Delta\Gamma}{2\Gamma_{D^0}}, \quad x := \frac{\Delta M}{\Gamma_{D^0}}.$$

Connection to box diagrams:

$$\begin{aligned}(\Delta M)^2 - \frac{1}{4}(\Delta\Gamma)^2 &= 4|M_{12}|^2 - |\Gamma_{12}|^2, \\ \Delta M\Delta\Gamma &= 4|M_{12}||\Gamma_{12}|\cos(\phi).\end{aligned}$$

with $\phi := \arg[-M_{12}/\Gamma_{12}]$

If $|\Gamma_{12}/M_{12}| \ll 1$, as in the case of the B_s system ($\approx 5 \cdot 10^{-3}$) or if $\phi \ll 1$, one gets the famous approximate formulae

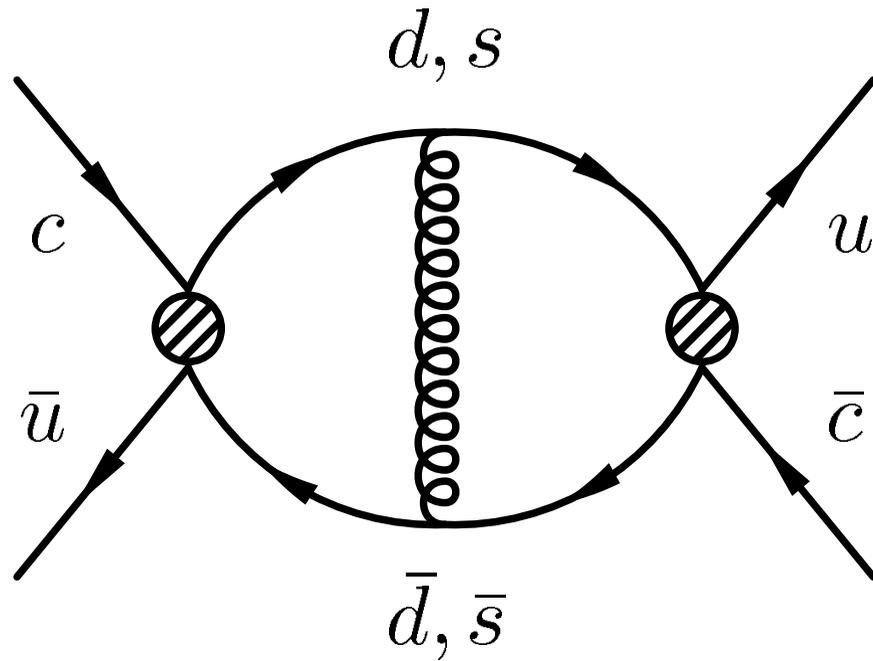
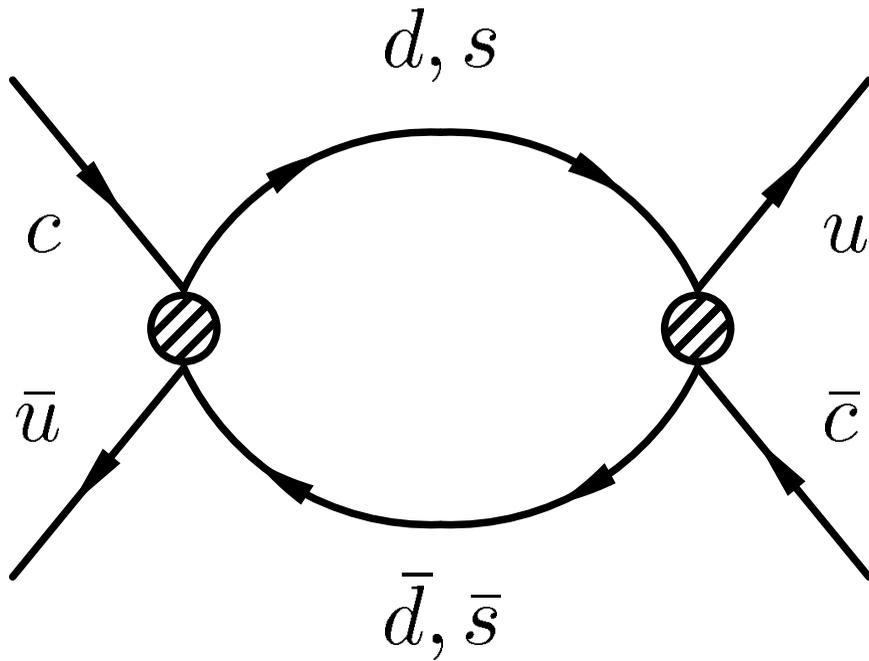
$$\Delta M = 2|M_{12}|, \quad \Delta\Gamma = 2|\Gamma_{12}|\cos\phi.$$

In the D-system $|\Gamma_{12}/M_{12}| \approx 1$ possible — Solve Eigenvalue equation exactly
Estimate: $\Delta\Gamma \leq 2|\Gamma_{12}|$



SM predictions for Γ_{12} in D-mixing I

$$\Gamma_{12} = -(\lambda_s^2 \Gamma_{ss} + 2\lambda_s \lambda_d \Gamma_{sd} + \lambda_d^2 \Gamma_{dd})$$



$$\lambda_d = V_{cd}V_{ud}^* = -c_{12}c_{23}c_{13}s_{12} - c_{12}^2 c_{13}s_{23}s_{13}e^{i\delta_{13}} = \mathcal{O}(\lambda^1 + i\lambda^5),$$

$$\lambda_s = V_{cs}V_{us}^* = +c_{12}c_{23}c_{13}s_{12} - s_{12}^2 c_{13}s_{23}s_{13}e^{i\delta_{13}} = \mathcal{O}(\lambda^1 + i\lambda^7),$$

$$\lambda_b = V_{cb}V_{ub}^* = c_{13}s_{23}s_{13}e^{i\delta_{13}} = \mathcal{O}(\lambda^5 + i\lambda^5),$$



SM predictions for Γ_{12} in D-mixing II

Common folklore $\lambda_b \approx 0$ (looks reasonable!)

$$\text{Unitarity: } \lambda_d + \lambda_s = 0 \Rightarrow \Gamma_{12} = -\lambda_s^2 (\Gamma_{ss} - 2\Gamma_{sd} + \Gamma_{dd})$$

■ Γ_{12} vanishes in the $SU(3)_F$ limit

Use the results for B_s -mixing from [Beneke, Buchalla, \(Greub\), A.L., Nierste 1998; 2003;](#)
[Ciuchini, Franco, Lubicz, Mescia, Tarantino 2003, A.L., Nierste 2006](#)

$$\Gamma_{ss} - 2\Gamma_{sd} + \Gamma_{dd} \approx 1.2 \frac{m_s^4}{m_c^4} - 59 \frac{m_s^6}{m_c^6}$$

[Golowich, Petrov 2005, Bobrowski, A.L., Riedl, Rohrwild 2009](#)

■ Γ_{12} is real to a very high accuracy

$$\lambda_s^2 = \mathcal{O}(\lambda^2 + i\lambda^8) \Rightarrow \text{Arg}(\lambda_s^2) \approx \frac{1}{\lambda^6} \approx 10^{-4}$$

■ Overall result much too small

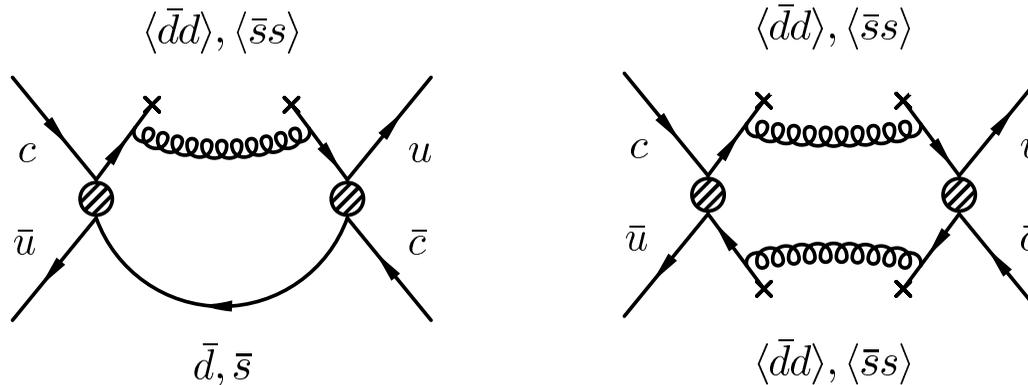
$$y \approx \mathcal{O}(10^{-6})$$

!!! Huge cancellations \Rightarrow be careful with approximations !!!



SM predictions for Γ_{12} in D-mixing III

Idea: higher orders in HQE might be dominant if GIM is less pronounced



naive expectation for a single diagram:

y_D	no GIM	with GIM	
$D = 6, 7$	$2 \cdot 10^{-2}$	$1 \cdot 10^{-6}$	Calculation
$D = 9$	$2 \cdot 10^{-2} \dots 5 \cdot 10^{-4}$???	Dimensional Estimate
$D = 12$	$2 \cdot 10^{-2} \dots 1 \cdot 10^{-5}$???	Dimensional Estimate

? Can one obtain $y_D^{Exp.}$?

?How big can ϕ be?



SM predictions for Γ_{12} in D-mixing IV

Our dimensional estimates

- Determine Γ_{12} : Imaginary part of 1-loop
- Estimate D = 9:
 - ◆ Quark condensate: $\langle \bar{s}s \rangle / m_c^3$
 - ◆ $4\pi\alpha_s$ relative to LO diagram
 - ◆ GIM : $(m_s/m_c)^3$ and m_s/m_c

Suppressed by about $2 \cdot 10^{-5}$, $3 \cdot 10^{-3}$ compared to D=6 diagram

D=6 GIM suppressed by about $5 \cdot 10^{-5} \Rightarrow$ **! IMPORTANT !**

Dimensional estimate in Bigi, Uraltsev 2001

- Determine M_{12} : 0-loop
- Estimate D = 9: Quark condensate: $\mu_{hadron.}^3 / m_c^3$ soft GIM : $m_s / \mu_{hadr.}$
- Estimate Γ_{12} via dispersion integral over M_{12}

Difference: $\frac{\langle \bar{s}s \rangle m_s}{m_c^4}$ vs. $\frac{m_s \mu_{hadron.}^2}{m_c^3}$ or better $\langle \bar{q}q \rangle \approx (0.24\text{GeV})^3$ vs. $\mu_{hadr.} \approx 1 \text{ GeV}$

\Rightarrow BU/BBLNP $\approx 80 \Rightarrow$ Calculation has to decide!



SM predictions for Γ_{12} in D-mixing V

Our Research Program

1. Redo $D=6$ without any approximations
Bobrowski, A.L, Riedl, Rohrwild, JHEP 2010
2. Calculate $D \geq 9$
Bobrowski, A.L. 2010; Bobrowski, Braun, A.L., Nierste, Prill unpublished
3. Calculate $D \geq 12$
4. Calculate M_{12}
5. Calculate lifetimes of D mesons
6. **Give a much more reliable range for the SM values of the possible size of CP violation in D mixing**



The failure of common folklore

D= 6,7 without folklore!!!! Bobrowski, A.L., Riedl, Rohrwild 2009, 2010

Unitarity: $\lambda_d + \lambda_s + \lambda_b = 0$

$$\Gamma_{12} = -\lambda_s^2 (\Gamma_{ss} - 2\Gamma_{sd} + \Gamma_{dd}) + 2\lambda_s \lambda_b (\Gamma_{sd} - \Gamma_{dd}) - \lambda_b^2 \Gamma_{dd}$$

$$\Gamma_{sd}^{D=6,7} = 1.8696 - 2.7616 \frac{m_s^2}{m_c^2} - 7.4906 \frac{m_s^4}{m_c^4} + \dots$$

$$\Gamma_{dd}^{D=6,7} = 1.8696$$

$$\Gamma_{12} \propto \lambda_s^2 \frac{m_s^6}{m_c^6} + 2\lambda_s \lambda_b \frac{m_s^2}{m_c^2} - \lambda_b^2 1$$

$$\begin{aligned} 10^7 \Gamma_{12}^{D=6,7} &= -14.6 + 0.0009i(\text{1st term}) - 6.7 - 16i(\text{2nd term}) + 0.3 - 0.3i(\text{3rd term}) \\ &= -21.1 - 16.0i = (11\dots39) e^{-i(0.5\dots2.6)}. \end{aligned}$$

■ not zero in $SU(3)_F$ limit

■ large phase ($\mathcal{O}(1)$) possible!!!

■ $y_D \in [0.5, 1.9] \cdot 10^{-6} \Rightarrow$ still much smaller than experiment ($8 \cdot 10^{-3}$)



SM predictions for Γ_{12} in D-mixing VII

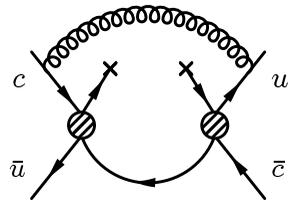
What does this mean?

1. Standard argument for “arg Γ_{12} is negligible” is wrong
2. Can there be a sizeable phase in D-mixing?
 - Phase of Γ_{12} is unphysical
 - Phase of M_{12}/Γ_{12} is physical \Rightarrow **determine also M_{12}**
3. $\Gamma_{12}^{D=6,7}$ has a large phase, but $y^{D=6,7} \ll y^{Exp.}$
 - **Georgi 1992; Ohl, Ricciardi, Simmons 1993; Bigi, Uraltsev 2001**
Higher orders in the HQE might be dominant: $y^{D \geq 9} = y^{Exp.}$ not excluded
 - **Bobrowski, A.L., Riedl, Rohrwild 2009, 2010**
If estimate of Bigi/Uraltsev is correct + our findings for D=6:
 $y^{Theorie} = y^{Exp.}$ and 5 per mille CP-violation not excluded
 - **Bobrowski, A.L. 2010; Bobrowski, Braun, A.L., Nierste, Prill in progress**
Do the real calculation for $D \geq 9$

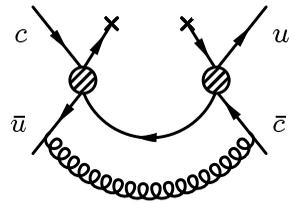


SM predictions for Γ_{12} in D-mixing VIII

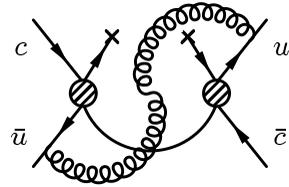
Determination of $D=9,10,\dots$ in factorization approximation



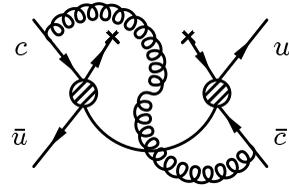
D₁



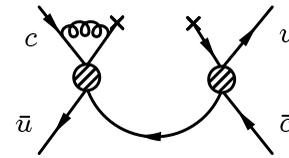
D₂



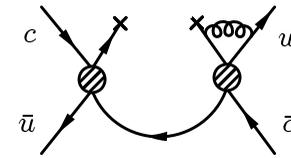
D_{3a}



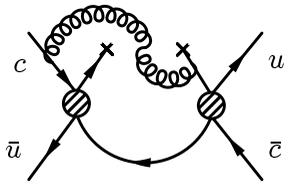
D_{3b}



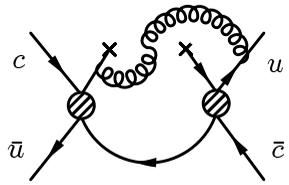
D_{4a}



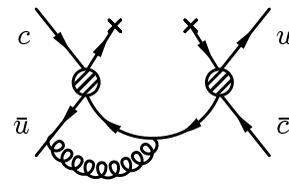
D_{4b}



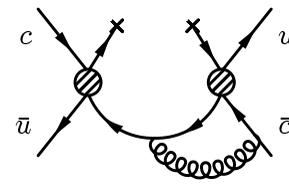
D_{5a}



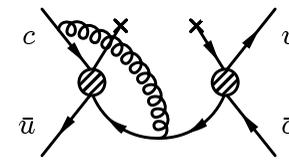
D_{5b}



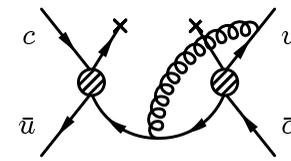
D_{6a}



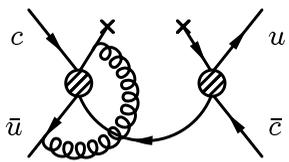
D_{6b}



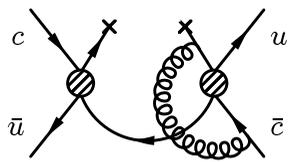
D_{7a}



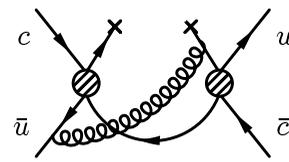
D_{7b}



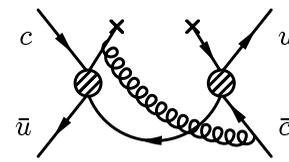
D_{8a}



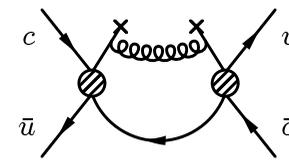
D_{8b}



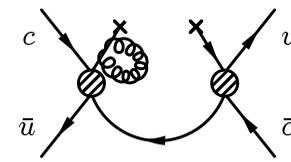
D_{9a}



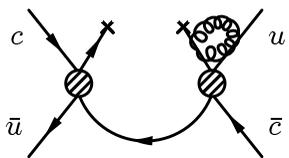
D_{9b}



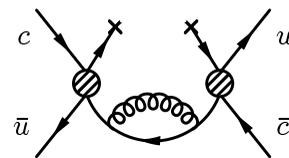
D₁₀



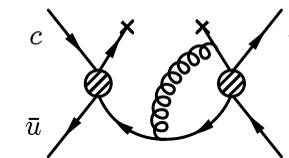
D_{11a}



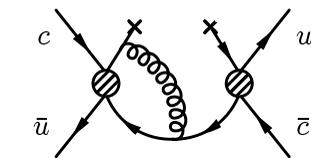
D_{11b}



D₁₂



D_{13a}



D_{13b}



SM predictions for Γ_{12} in D-mixing IX

Determination of $D=9,10,\dots$ in factorization approximation

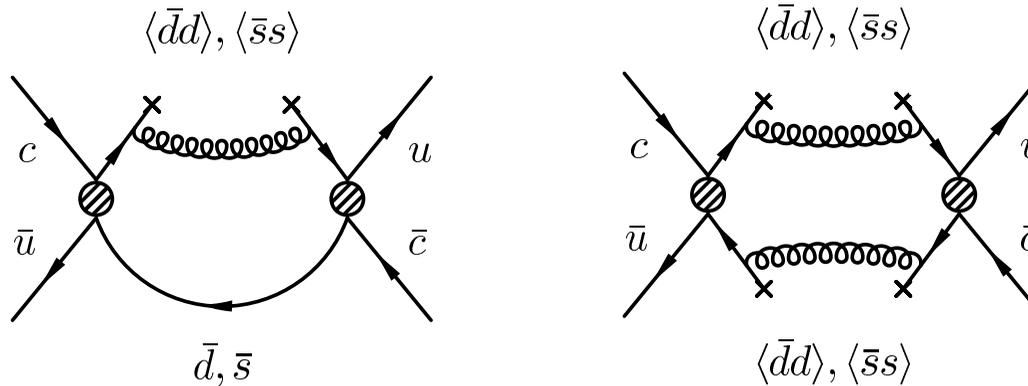
- Factorization approximation, expected to hold up to $1/N_c$
- Enhancement of $\mathcal{O}(15)$ compared to leading term
Large effect, but not as large as estimated by Bigi, Uraltsev
- GIM cancellation reduced to: $\propto m_s^3$

$$\Gamma_{12} \propto \lambda_s^2 \cdot \frac{m_s^6}{m_c^6} + 2\lambda_s \lambda_b \cdot \frac{m_s^2}{m_c^2} + \lambda_b^2 \cdot 1$$
$$\rightarrow \Gamma_{12} \propto \lambda_s^2 \cdot \frac{m_s^3}{m_c^3} + 2\lambda_s \lambda_b \cdot \frac{m_s^2}{m_c^2} + \lambda_b^2 \cdot 1$$



SM predictions for Γ_{12} in D-mixing III

Idea: higher orders in HQE might be dominant if GIM is less pronounced



naive expectation for a single diagram:

y_D	no GIM	with GIM	CP violation	
$D = 6, 7$	$2 \cdot 10^{-2}$	$1 \cdot 10^{-6}$	$\mathcal{O}(1)$	Calculation
$D = 9$	$2 \cdot 10^{-2} \dots 5 \cdot 10^{-4}$	$1.5 \cdot 10^{-5}$	$\mathcal{O}(5\%)$	Calculation
$D = 12$	$2 \cdot 10^{-2} \dots 1 \cdot 10^{-5}$???		Dimensional Estimate

? Can one obtain $y_D^{Exp.}$?

?How big can ϕ be?



Outlook

Careful investigation of the HQE terms

- **Brand-New:** Standard argument for negligible phase in Γ_{12} seems not to work
- **New :** Γ_{12} sensitive to NP **Petrov et al**
- **Text-Book-Wisdom:** Overall value much too small

Finish HQE estimates (incl. higher orders) of D-mixing and lifetimes

If y^{Theory} stays small: Interesting options:

- a) HQE does not work in the D-system
- b) **Actual exp. value for y is very small ($> 5 \sigma$)**
 \Rightarrow Theoreticians dream: Real prediction \neq post-diction
- c) **New physics is acting in the D-system**
 - c1) **SU(3) suppression is much less pronounced**
 - c2) **unitarity of 3x3 CKM matrix is violated**



New physics in D-mixing I

Contrary to expectation: Γ_{12} is sensitive to new physics!!!

$$\Gamma_{12} = -\lambda_s^2 (\Gamma_{ss} - 2\Gamma_{sd} + \Gamma_{dd}) + 2\lambda_s \lambda_b (\Gamma_{sd} - \Gamma_{dd}) - \lambda_b^2 \Gamma_{dd}$$

Γ_{12} is small, because

1. $\Gamma_{ss} - 2\Gamma_{sd} + \Gamma_{dd}$ is small
2. λ_b is small

\Rightarrow 2 possibilities for enhancements

1. Enhance $\Gamma_{ss} - 2\Gamma_{sd} + \Gamma_{dd}$
see talk by Alexey Petrov
2. “Enhance λ_b ”
see next slides



New physics in D-mixing II

The most simple (boring?) extension of the SM: fourth generation SM4

- Obvious effect: New particles (b' , t') in the box diagrams for M_{12}
- Often not seen: huge cancellations possible - $\delta V_{td,ts,tb}$ vs. (b' , t')

$$\Delta_{B_s} = \frac{M_{12,SM4}^{B_s}}{M_{12,SM3}^{B_s}} = 1 + \frac{M_{12,SM4}^{tt,B_s} - M_{12,SM3}^{B_s}}{M_{12,SM3}^{B_s}} + \frac{M_{12,SM4}^{tt'+t't',B_s}}{M_{12,SM3}^{B_s}}.$$

Check allowed parameter range for V_{CKM4} : e.g. possible ($V_{tb} = 0.93$)

$$\Delta_{B_s} = 1 + (1.2044 - 0.6715i) + (-1.3434 - 0.0354i) = 1.11 \cdot e^{-i39^\circ},$$

- Overseen: Large Effects in Γ_{12} in D-mixing possible

$$\Gamma_{12} = -\lambda_s^2 (\Gamma_{ss} - 2\Gamma_{sd} + \Gamma_{dd}) + 2\lambda_s (\lambda_b + \lambda_{b'}) (\Gamma_{sd} - \Gamma_{dd}) - (\lambda_b + \lambda_{b'})^2 \Gamma_{dd}$$

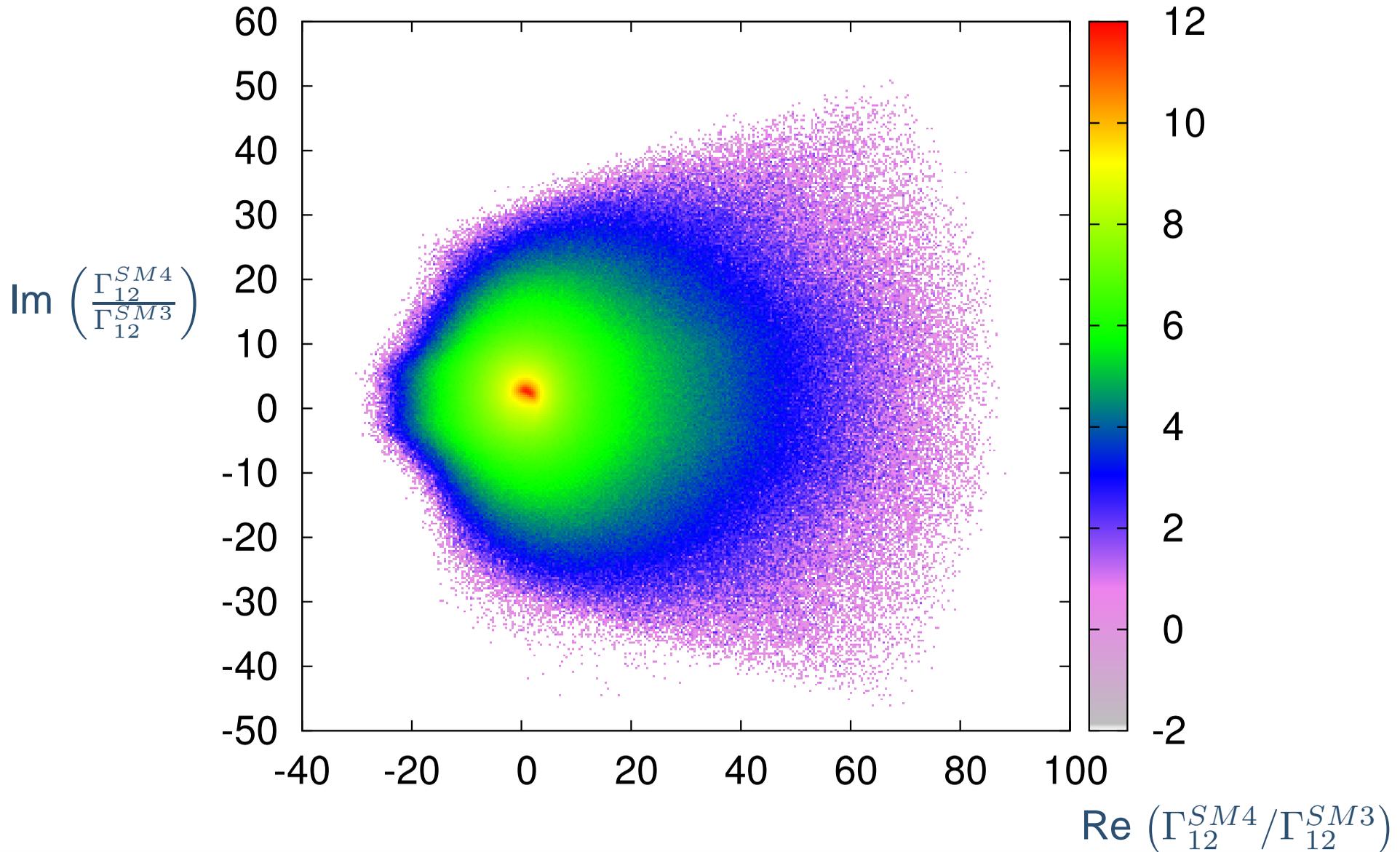
$\lambda_b \propto \lambda^{5\dots 6}$ - still possible $\lambda_{b'} \propto \lambda^3$ (arXiv:0902.4883)

see also Melic et al, Kou et al., Soni et. al, Hou et al. ...



New Physics in D-Mixing III

Bobrowski, A.L., Riedl, Rohrwild; 0904.3971





Inclusive Decays I*

Theoretical determination of observables

$$\frac{1}{\tau} = \sum_X \Gamma(B \rightarrow X),$$

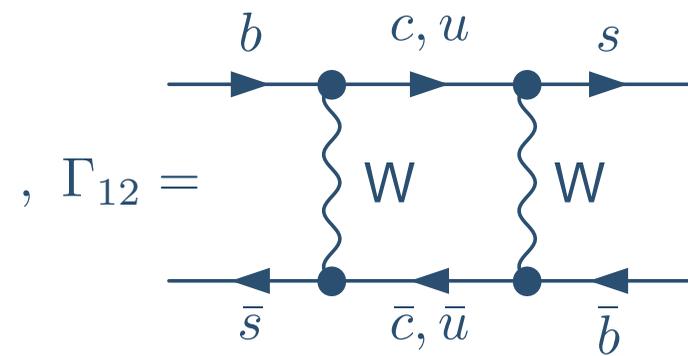
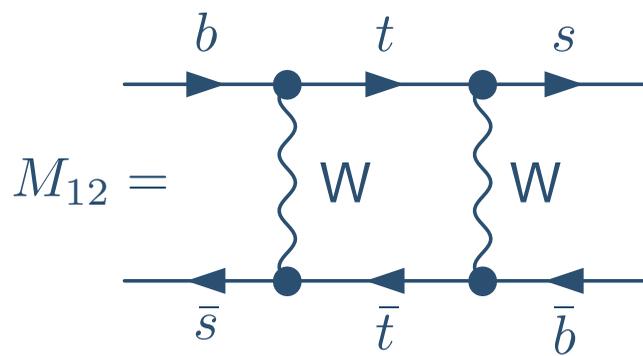
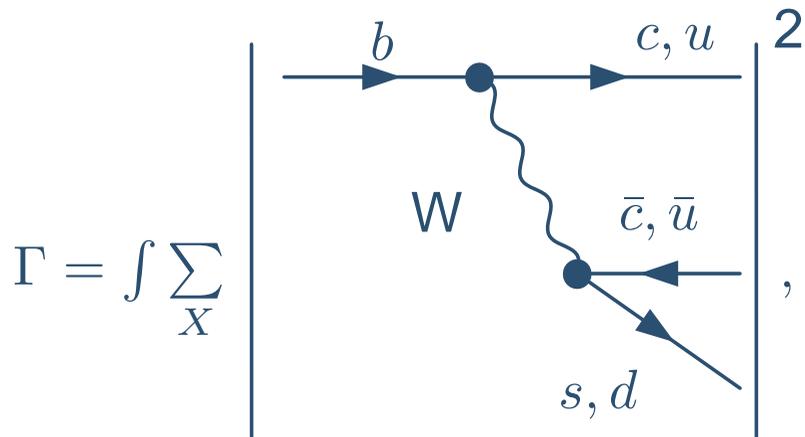
$$\Delta M = 2|M_{12}|,$$

$$\Delta\Gamma = 2|\Gamma_{12}| \cos(\phi),$$

$$a_{sl} = \Im \left(\frac{\Gamma_{12}}{M_{12}} \right),$$

$$\phi = \arg \left(-\frac{M_{12}}{\Gamma_{12}} \right).$$

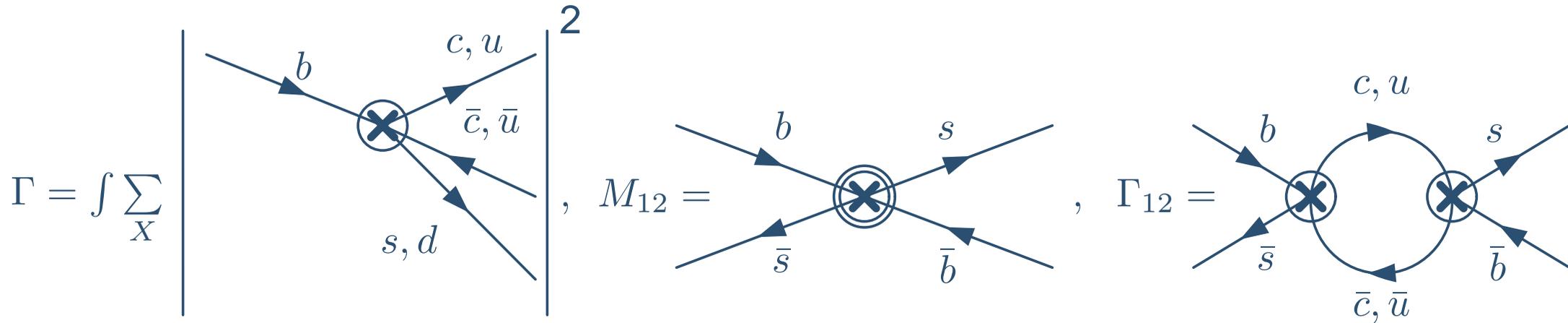
These quantities correspond to the following SM diagrams



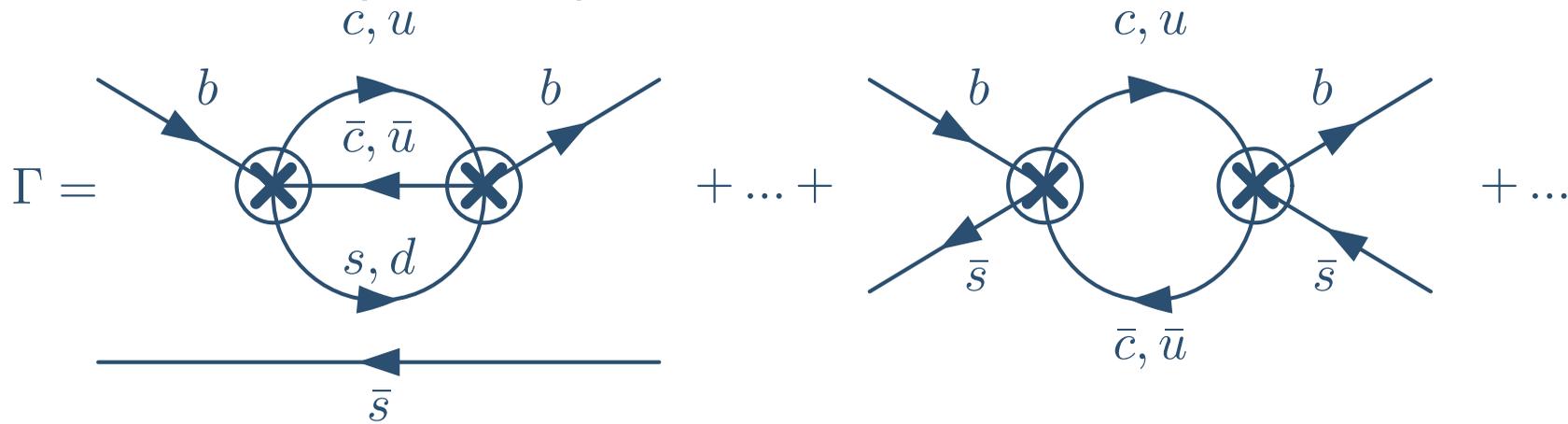


Inclusive Decays II*

Use the fact: $m_t, M_W \gg m_b$ - integrate out heavy particles



Rewrite Γ with the help of the optical theorem



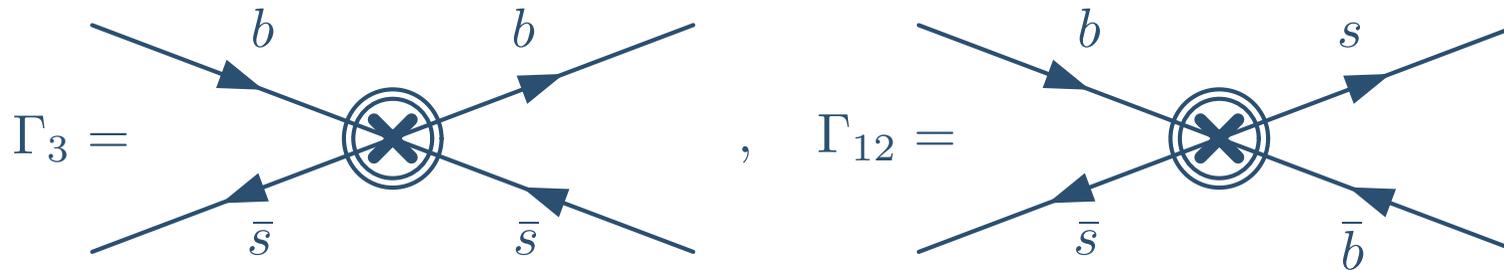
Γ_0

Γ_3



Inclusive Decays III*

Use the fact: $m_b \gg \Lambda_{QCD}$ for Γ_0 , Γ_3 and Γ_{12} - also local operators



- Γ , M_{12} and Γ_{12} are expressed in terms of local $\Delta B = 0, 2$ -operators
- Determination of Γ_3 and Γ_{12} almost identical
- **OPE II might be questionable - quark hadron duality**
 - \Rightarrow test reliability of OPE II via lifetimes (no NP effects expected)
 - \Rightarrow calculate corrections in all possible “directions”, to get a feeling for the convergence



Heavy Quark Expansion*

Systematic expansion of the decay rate in powers of m_b^{-1} yields

$$\Gamma = \Gamma_0 + \frac{\Lambda^2}{m_b^2} \Gamma_2 + \frac{\Lambda^3}{m_b^3} \Gamma_3 + \dots$$

Voloshin, Uraltsev, Khoze, Shifman, Vainshtein

Γ_0 : Decay of a free quark \Rightarrow **all b-hadrons have the same lifetime**

Γ_2 : First corrections due to kinetic and chromomagnetic operator

Γ_3 : Weak annihilation and Pauli interference

Distinguish between different spectators \Rightarrow **Lifetime differences**
numerically enhanced by phase space factor $16\pi^2$



State of the art*

Meson vs Meson

$$\frac{\tau_1}{\tau_2} = 1 + \frac{\Lambda^3}{m_b^3} \left(\Gamma_3^{(0)} + \frac{\alpha_s}{4\pi} \Gamma_3^{(1)} + \dots \right) + \frac{\Lambda^4}{m_b^4} \left(\Gamma_4^{(0)} + \dots \right) + \dots$$

Baryon vs Meson

$$\frac{\tau_1}{\tau_2} = 1 + \frac{\Lambda^2}{m_b^2} \left(\Gamma_2^{(0)} + \dots \right) + \frac{\Lambda^3}{m_b^3} \left(\Gamma_3^{(0)} + \frac{\alpha_s}{4\pi} \Gamma_3^{(1)} + \dots \right) + \frac{\Lambda^4}{m_b^4} \left(\Gamma_4^{(0)} + \dots \right) + \dots$$

Neutral Mesons

$$\frac{\Delta\Gamma}{\Gamma} = \frac{\Lambda^3}{m_b^3} \left(\Gamma_3^{(0)} + \frac{\alpha_s}{4\pi} \Gamma_3^{(1)} + \dots \right) + \frac{\Lambda^4}{m_b^4} \left(\Gamma_4^{(0)} + \dots \right) + \dots$$

$$\Gamma_i^{(j)} = C_i^{(j)} \cdot \langle Q_i^{(j)} \rangle \propto f^2 \cdot B_i^{(j)} \cdot C_i^{(j)}$$

Perturbative corrections

- $C_3^{(0)}$: '79...'92
- $C_4^{(0)}$: '96...'03
- $C_3^{(1)}$: '98...'03; incomplete for Λ_b
- $C_5^{(0)}$: '03...'06

non-perturbative corrections

- $\langle Q_3 \rangle$: prel. $n_f = 2 + 1$ for B-mixing
only one determination for τ_{B^+}/τ_{B_d}
only prel. studies for Λ_b
- $\langle Q_4 \rangle$: mostly VIA
- $\langle Q_5 \rangle$: only naive estimates



Strategy*

1. **Test reliability of the theoretical framework via lifetimes**
— no NP effects expected —
2. **Currently no precise prediction of Γ_{12} and M_{12} possible**
— compared to $\Delta M^{\text{Exp.}}$ —
3. **Cleaner SM prediction of Γ_{12}/M_{12} possible**
— many non-pert. uncertainties cancel —
4. **Search for NP in Γ_{12}/M_{12} (and M_{12} - combined analysis)**



Test 1: τ_{B^+}/τ_{B_d} in NLO-QCD I*

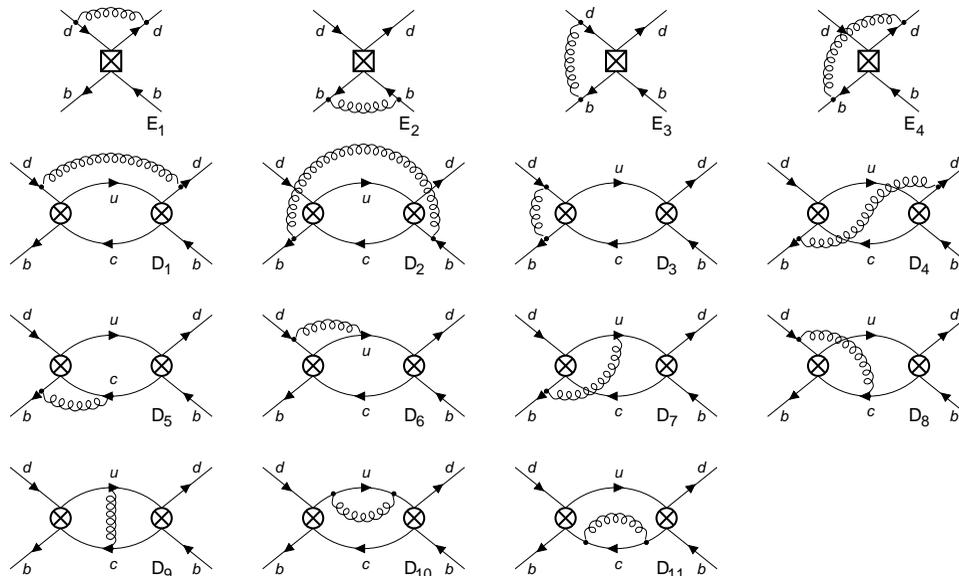
$$\frac{\tau_1}{\tau_2} = 1 + \left(\frac{\Lambda}{m_b}\right)^3 \left(\Gamma_3^{(0)} + \frac{\alpha_s}{4\pi}\Gamma_3^{(1)} + \dots\right) + \left(\frac{\Lambda}{m_b}\right)^4 \left(\Gamma_4^{(0)} + \dots\right) + \dots$$

$\Gamma_3^{(0)}$: Shifman, Voloshin; Uraltsev; Bigi, Vainshtein; Neubert, Sachrajda

$\Gamma_4^{(0)}$: Gabbiani, Onishchenko, Petrov; Greub, A.L., Nierste (unpublished)

$\Gamma_3^{(1)}$: Beneke, Buchalla, Greub, A.L., Nierste; Ciuchini, Franco, Lubicz, Mescia, Tarantino

lattice : Di Pierro, Sachrajda, Michael; Becirevic





Test 1: τ_{B^+}/τ_{B_d} in NLO-QCD II*

$$\begin{aligned} \frac{\tau(B^+)}{\tau(B_d^0)} - 1 &= \tau(B^+) [\Gamma(B_d^0) - \Gamma(B^+)] \\ &= 0.0325 \frac{\tau(B^+)}{1.653 \text{ ps}} \left(\frac{|V_{cb}|}{0.04} \right)^2 \left(\frac{m_b}{4.8 \text{ GeV}} \right)^2 \left(\frac{f_B}{200 \text{ MeV}} \right)^2 \\ &\quad \left[(1.0 \pm 0.2) B_1 + (0.1 \pm 0.1) B_2 - (18.4 \pm 0.9) \epsilon_1 + (4.0 \pm 0.2) \epsilon_2 \right] + \delta_{1/m} \end{aligned}$$

$(B_1, B_2, \epsilon_1, \epsilon_2) = (1.10 \pm 0.20, 0.99 \pm 0.10, -0.02 \pm 0.02, 0.03 \pm 0.01)$ '01: Becirevic

$$\left[\frac{\tau(B^+)}{\tau(B_d^0)} \right]_{\text{LO}} = 1.047 \pm 0.049$$

$$\left[\frac{\tau(B^+)}{\tau(B_d^0)} \right]_{\text{NLO}} = 1.063 \pm 0.027$$

NLO-QCD: '02: Beneke, Buchalla, A.L, Greub, Nierste; Ciuchini, Franco, Lubicz, Mescia, Tarantino

$1/m_b$: '03: Gabbiani, Onishchenko, Petrov; Greub, A.L, Nierste (unpublished): $\text{tiny} \leq 0.005$

$$\text{HFAG 09: } \left[\frac{\tau(B^+)}{\tau(B_d^0)} \right] = 1.071 \pm 0.009$$



Test 2: The lifetime ratio $\tau_{B_s}/\tau_{B_d}^*$

$$\frac{\tau(B_s)}{\tau(B_d)} = 1.00 \pm 0.01$$

Neubert, Sachrajda; Beneke, Buchalla, Dunietz; Bigi, Blok, Shifman, Uraltsev, Vainshtein; U. Nierste, Y.-Y. Keum; M. Ciuchini, E. Franco, V. Lubicz, F. Mescia

Weak annihilation contributions for B_d and B_s have almost the same size.

Lifetime differences only due to small difference in phase space and by $SU(3)_F$ violations of the hadronic parameters.

NLO penguin contributions to τ_{B_s}/τ_{B_d} give a comparable effect – > search for new physics

$$\text{HFAG 09: } \left[\frac{\tau(B_s^0)}{\tau(B_d^0)} \right] = 0.965 \pm 0.017$$