Overview of charmonium decays and production from NRQCD

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Aim:

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Systematic study of heavy quark-antiquark systems from QCD

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- Charmonium ($c\bar{c}$)
- Bottomonium ($b\bar{b}$)
- B_c ($b\bar{c}$)
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- Also:
  - $\tilde{q}\bar{\tilde{q}}$, $\tilde{g}\bar{\tilde{g}}$
  - Double-heavy baryons (QQq, $Q = b, c$, $q = u, d, s$)
  - $\cdots$

Tool:

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Effective Field Theories (EFTs)

- Direct QCD calculations may be very complicated
- Construct a new theory (the effective theory) involving only the relevant degrees of freedom for the particular energy region of interest
  - Identify relevant degrees of freedom
  - Symmetries
  - Hierarchy of energy scales
- The EFT gives equivalent physical results in the region where it holds

Heavy Quarkonium

$Q\bar{Q}$ bound state , $m_Q >> \Lambda_{QCD}$, $\alpha_s(m_Q) << 1$

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- Non-relativistic system $\rightarrow$ multiscale problem
  - $m_Q >> m_Q v >> m_Q v^2$
  - $m_Q >> \Lambda_{QCD}$
- EFTs are useful (N. Brambilla, A. Pineda, JS and A. Vairo, Rev. Mod. Phys. 77, 1423 (2005))

NRQCD

W.E. Caswell and G.P. Lepage, Phys. Lett. **167B**, 437 (1986)
G. T. Bodwin, E. Braaten and G. P. Lepage, Phys. Rev. D **51** (1995) 1125

$$m_Q \quad >> \quad m_Q v \quad , \quad m_Q v^2 \quad , \quad \Lambda_{QCD}$$

$$\mathcal{L}_\psi = \psi^\dagger \left\{ iD_0 + \frac{1}{2m_Q}\mathbf{D}^2 + \frac{1}{8m_Q^3}\mathbf{D}^4 + \frac{c_F}{2m_Q}\boldsymbol{\sigma}.g\mathbf{B} + \right. \\ \left. + \frac{c_D}{8m_Q^2}\left(\mathbf{D}.g\mathbf{E} - g\mathbf{E}.\mathbf{D}\right) + i\,\frac{c_S}{8m_Q^2}\boldsymbol{\sigma}.\left(\mathbf{D}\times g\mathbf{E} - g\mathbf{E}\times\mathbf{D}\right) \right\}\psi$$

c_F, c_D and c_S are short distance matching coefficients which depend on m_Q and μ (factorization scale)

NRQCD(Cont.)

$$\mathcal{L}_{\psi\chi} = \frac{f_1(^1S_0)}{m_Q^2}O_1(^1S_0) + \frac{f_1(^3S_1)}{m_Q^2}O_1(^3S_1) + \\ + \frac{f_8(^1S_0)}{m_Q^2}O_8(^1S_0) + \frac{f_8(^3S_1)}{m_Q^2}O_8(^3S_1),$$

$$O_1(^1S_0) = \psi^\dagger\chi\,\chi^\dagger\psi \quad , \quad O_1(^3S_1) = \psi^\dagger\boldsymbol{\sigma}\chi\,\chi^\dagger\boldsymbol{\sigma}\psi,$$

$$O_8(^1S_0) = \psi^\dagger \mathrm{T}^a\chi\,\chi^\dagger \mathrm{T}^a\psi \quad , \quad O_8(^3S_1) = \psi^\dagger \mathrm{T}^a\boldsymbol{\sigma}\chi\,\chi^\dagger \mathrm{T}^a\boldsymbol{\sigma}\psi.$$

- The fs are short distance matching coefficients which depend on m_Q and μ (factorization scale)
- The fs contain imaginary parts

NRQCD(Cont.)

- Spectroscopy $\longrightarrow$ Lattice NRQCD
- Inclusive decays

$$\Gamma(\chi_Q(nJS) \to LH) = \frac{2}{m_Q^2}\Bigg(\mathrm{Im}\, f_1(^{2S+1}P_J) \times$$

$$\times \frac{\langle\chi_Q(nJS)|O_1(^{2S+1}P_J)|\chi_Q(nJS)\rangle}{m_Q^2}$$

$$+\mathrm{Im}\, f_8(^{2S+1}S_S)\langle\chi_Q(nJS)|O_8(^1S_0)|\chi_Q(nJS)\rangle\Bigg),$$

NRQCD(Cont.)

Current precision:

- $\alpha_s^3(m_Q)$ for $\text{Im}\, f$s of $d = 8$ operators (S- and P-wave) (A. Petrelli, M. Cacciari, M. Greco, F. Maltoni and M. L. Mangano, (98))
- $\alpha_s^4(m_Q)$ for $\text{Im}\, f_1(^3S_1)$ (P. B. Mackenzie and G. P. Lepage (81); J. M. Campbell, F. Maltoni, F. Tramontano (07))
- α^2 for $\text{Im}\, f_{\text{e.m.}}$ of $d = 10$ operators (S- and P-wave) (N. Brambilla, E. Mereghetti and A. Vairo, (06); G. T. Bodwin and A. Petrelli (02); J. P. Ma and Q. Wang (02))
- $\alpha_s^2(m_Q)$ for $\text{Im}\, f$s of $d = 10$ operators (S- and P-wave) (N. Brambilla, E. Mereghetti and A. Vairo (09); G. T. Bodwin and A. Petrelli (02); H. W. Huang, H. M. Hu and X. F. Zhang (97))
- $\alpha_s^3(m_Q)$ for $\text{Im}\, f$s of $d = 10$ (D-wave) (Z.-G. He, Y. Fan, K.-T. Chao (08,09); Y. Fan, Z.-G. He, Y.-Q. Ma, K.-T. Chao (09))

NRQCD(Cont.)

Matrix elements:

- From Data:
  - Color single matrix elements can be obtained from e.m. decays
  - Color octet ones from decays to light hadrons
    - E.g. (Maltoni (00))

$$\langle \chi_{cJ} | \mathcal{O}_8(^3S_1) | \chi_{cJ} \rangle = (4.3 \pm 0.9) \times 10^{-3}\, GeV^3$$

- From Theory:
  - Color single matrix elements are related to wave functions at the origin

NRQCD(Cont.)

Matrix elements:

- From Theory:
  - Color octet matrix elements:
    - RG plus assumption that they are zero at the $m_Q v$ scale (BBL (94); Y. Fan, Z.-G. He, Y.-Q. Ma, K.-T. Chao (09))
    - pNRQCD weak coupling regime (Garcia i Tormo, JS (04,07))

$$\langle J/\psi|\mathcal{O}_8(^1S_0)|J/\psi\rangle \sim 0.0012\, GeV^3\ ,\ \langle J/\psi|\mathcal{O}_8(^3P_0)|J/\psi\rangle \sim 0.0028\, GeV^5$$

    - pNRQCD strong coupling regime (N. Brambilla, D. Eiras, A. Pineda, JS, A. Vairo (01)): related to wave functions at the origin plus a few universal parameters.

NRQCD(Cont.)

Matrix elements:

- From Theory:
  - Color octet matrix elements:
    - Lattice NRQCD (G. T. Bodwin, D. K. Sinclair, S. Kim (96,97,01,05))

$$\langle \chi_{cJ} | \mathcal{O}_8(^3S_1) | \chi_{cJ} \rangle = (4.6 \pm 2.5) \times 10^{-3}\, GeV^3$$

NRQCD(Cont.)

Theory vs. Experiment

Ratio	PDG09	PDG00	LO	NLO
$\frac{\Gamma_{\chi_{c0}\to\gamma\gamma}}{\Gamma_{\chi_{c2}\to\gamma\gamma}}$	4.9 ± 0.9	13 ± 10	≈ 3.75	≈ 5.43
$\frac{\Gamma_{\chi_{c2}\to\mathrm{l.h.}} - \Gamma_{\chi_{c1}\to\mathrm{l.h.}}}{\Gamma_{\chi_{c0}\to\gamma\gamma}}$	440 ± 100	270 ± 20	≈ 347	≈ 383
$\frac{\Gamma_{\chi_{c0}\to\mathrm{l.h.}} - \Gamma_{\chi_{c1}\to\mathrm{l.h.}}}{\Gamma_{\chi_{c0}\to\gamma\gamma}}$	4000 ± 600	3500 ± 2500	≈ 1300	≈ 2781
$\frac{\Gamma_{\chi_{c0}\to\mathrm{l.h.}} - \Gamma_{\chi_{c2}\to\mathrm{l.h.}}}{\Gamma_{\chi_{c2}\to\mathrm{l.h.}} - \Gamma_{\chi_{c1}\to\mathrm{l.h.}}}$	8.0 ± 0.9	12.1 ± 3.2	≈ 2.75	≈ 6.63
$\frac{\Gamma_{\chi_{c0}\to\mathrm{l.h.}} - \Gamma_{\chi_{c1}\to\mathrm{l.h.}}}{\Gamma_{\chi_{c2}\to\mathrm{l.h.}} - \Gamma_{\chi_{c1}\to\mathrm{l.h.}}}$	9.0 ± 1.1	13.1 ± 3.3	≈ 3.75	≈ 7.63

$m_c =$ 1.5 GeV and $\alpha_s(2m_c) =$ 0.245 (G. T. Bodwin, based on A. Vairo (09))

NRQCD(Cont.)

Electromagnetic current:

$$\boldsymbol{j} = c_v(\mu)\psi^\dagger \boldsymbol{\sigma}\chi + \frac{d_v(\mu)}{6m_q^2}\psi^\dagger \boldsymbol{\sigma}\boldsymbol{D}^2\chi + \ldots,$$

- NLO (R. Barbieri, R. Gatto, R. Kogerler, Z. Kunszt (75))
- NNLO (M. Beneke, A. Signer, V.A. Smirnov; A. Czarnecki, K. Melnikov (97))
- NNNLO, diagrams with quark loops, light and heavy (P. Marquard, J.H. Piclum, D. Seidel, M. Steinhauser (06,09))
- NLO plus relativistic corrections to all orders (G. T. Bodwin, H. S. Chung, J. Lee, C. Yu (08))

NRQCD(Cont.)

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    - Integrate out energy scales $\sim m_Q v$ $\longrightarrow$ pNRQCD, A. Pineda and J. Soto, Nucl. Phys. Proc. Suppl. **64**, 428 (1998)
    - Mode decomposition $\longrightarrow$ vNRQCD, M. E. Luke, A. V. Manohar and I. Z. Rothstein, Phys. Rev. D **61**, 074025 (2000)

pNRQCD

$\Lambda_{QCD} \lesssim m_Q v^2$: weak coupling regime

$$\mathcal{L}_{\rm pNRQCD} = \int d^3\mathbf{r}\ {\rm Tr} \Big\{ {\rm S}^\dagger \left(i\partial_0 - h_s(\mathbf{r}, \mathbf{p}, \mathbf{P_R}, \mathbf{S}_1, \mathbf{S}_2, \mu) \right) {\rm S} + $$
$$+ {\rm O}^\dagger \left(iD_0 - h_o(\mathbf{r}, \mathbf{p}, \mathbf{P_R}, \mathbf{S}_1, \mathbf{S}_2, \mu) \right) {\rm O} \Big\}$$
$$+ V_A(r,\mu) {\rm Tr} \left\{ {\rm O}^\dagger \mathbf{r} \cdot g\mathbf{E}\, {\rm S} + {\rm S}^\dagger \mathbf{r} \cdot g\mathbf{E}\, {\rm O} \right\} +$$
$$+ \frac{V_B(r,\mu)}{2} {\rm Tr} \left\{ {\rm O}^\dagger \mathbf{r} \cdot g\mathbf{E}\, {\rm O} + {\rm O}^\dagger {\rm O} \mathbf{r} \cdot g\mathbf{E} \right\}$$

h_s, h_o = quantum mechanical hamiltonians with scale dependent potentials calculable in pertubation theory in $\alpha_{\rm s}(m_Q v)$

pNRQCD (Cont.)

$\Lambda_{QCD} \lesssim m_Q v$: strong coupling regime

$$L_{\rm pNRQCD} = \int d^3\mathbf{R} \int d^3\mathbf{r}\; S^\dagger (i\partial_0 - h_s(\mathbf{x}_1, \mathbf{x}_2, \mathbf{p}_1, \mathbf{p}_2, \mathbf{S}_1, \mathbf{S}_2)) S,$$

$$h_s(\mathbf{x}_1, \mathbf{x}_2, \mathbf{p}_1, \mathbf{p}_2, \mathbf{S}_1, \mathbf{S}_2) = \frac{\mathbf{p}_1^2}{2m_Q} + \frac{\mathbf{p}_2^2}{2m_{Q'}} + V_s(\mathbf{x}_1, \mathbf{x}_2, \mathbf{p}_1, \mathbf{p}_2, \mathbf{S}_1, \mathbf{S}_2),$$

$$V_s = V_s^{(0)} + \frac{V_s^{(1,0)}}{m_Q} + \frac{V_s^{(0,1)}}{m_{Q'}} + \frac{V_s^{(2,0)}}{m_Q^2} + \frac{V_s^{(0,2)}}{m_{Q'}^2} + \frac{V_s^{(1,1)}}{m_Q m_{Q'}},$$

All V_ss can be, and most of them have been, calculated on the lattice (G. S. Bali, Klaus Schilling, A. Wachter (97); Y. Koma, M. Koma, H. Wittig (06); Y. Koma, M. Koma (06,09))

Lattice pNRQCD

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Example: the $1/m_Q$ potential (N. Brambilla, A. Pineda, JS, A. Vairo, Phys. Rev. D63:014023,2001)

$$V^{(1)}(r) \quad = \quad -\frac{1}{2}\int_0^\infty dt\, t\, \langle\!\langle g\mathbf{E}_1(t)\cdot g\mathbf{E}_1(0)\rangle\!\rangle_c,$$

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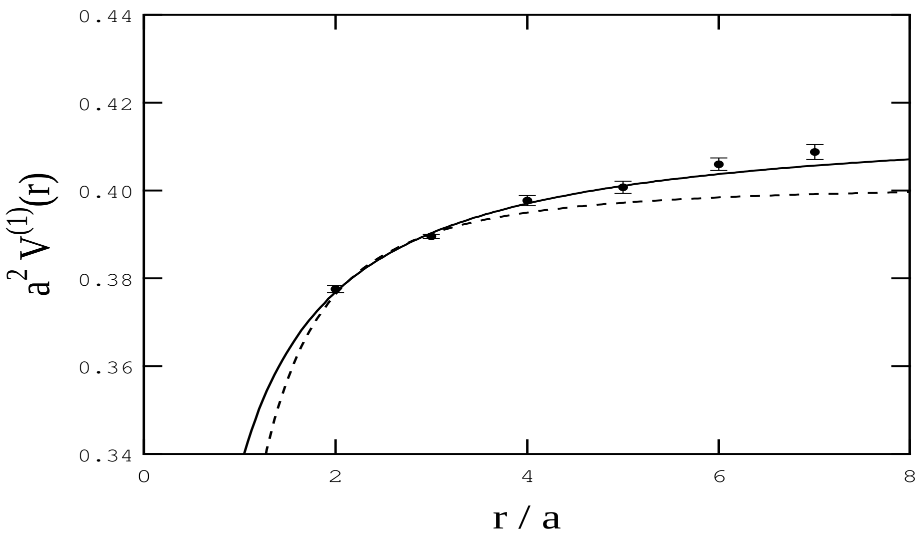

$$\text{Dashed}: \quad V^{(1)}(r) = \frac{a}{r^2} + c \quad , \qquad \text{Solid}: \quad V^{(1)}(r) = \frac{a}{r} + c$$

Y. Koma, M. Koma, H. Wittig, Phys. Rev. Lett. 97:122003,2006

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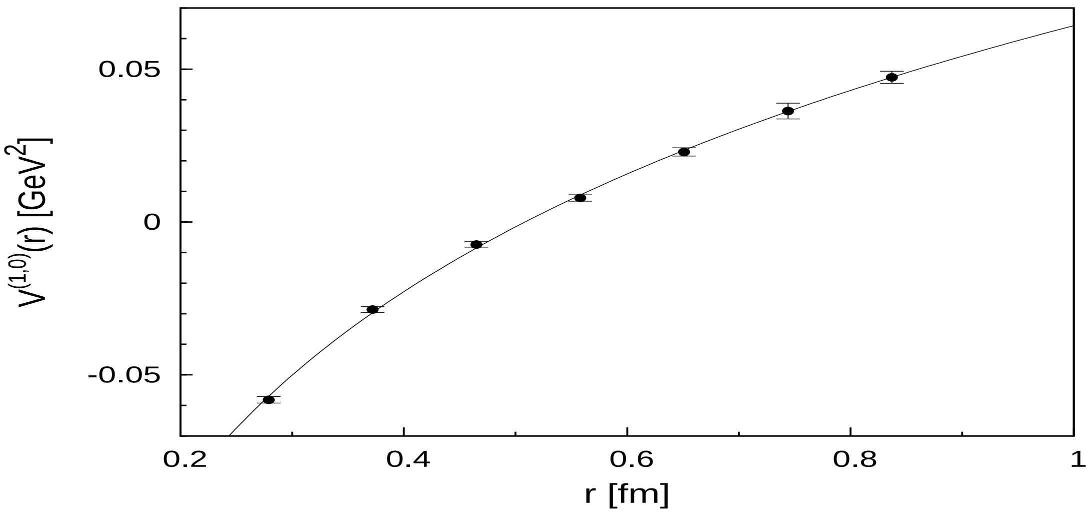
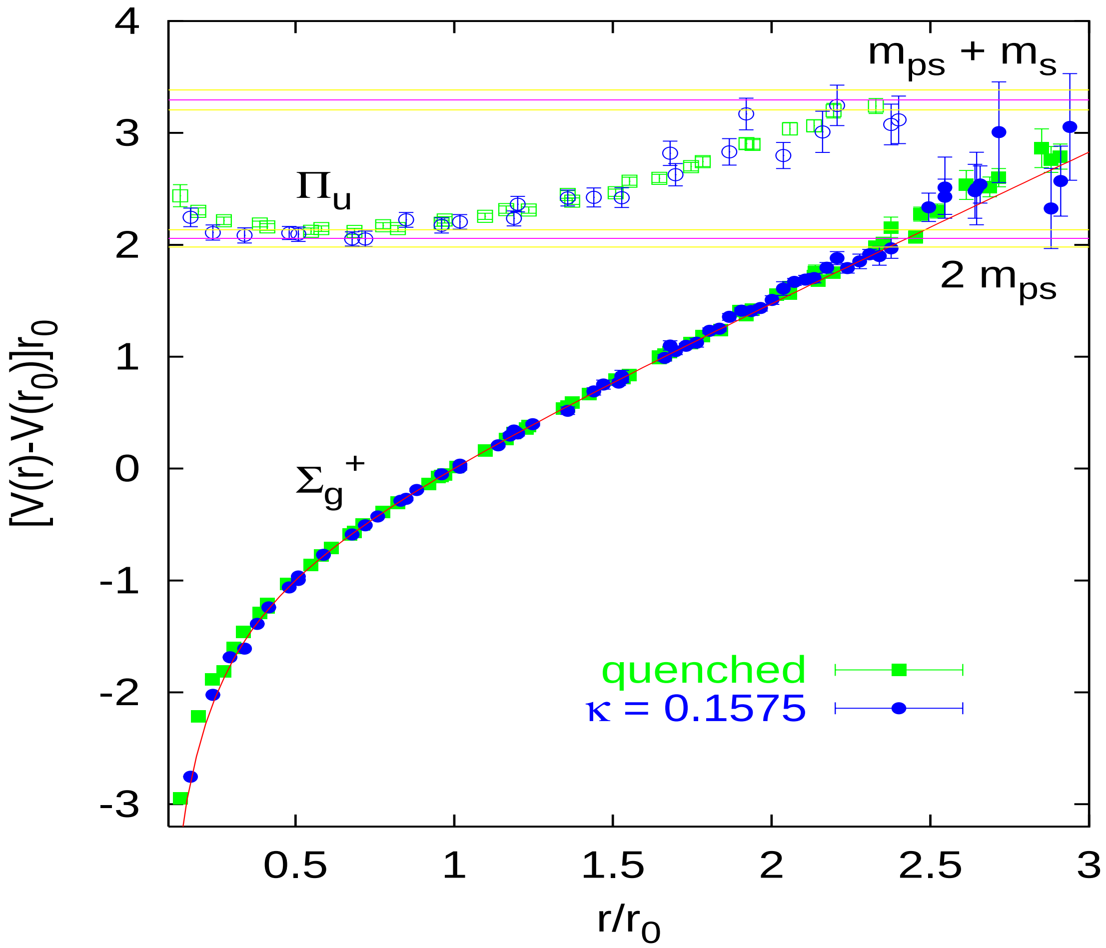


pNRQCD (Cont.)



G.S. Bali at al. (TXL Collaboration), Phys. Rev. **D62**,(2000):054503

pNRQCD : weak coupling regime

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- Fixed order calculations:
  - Complete NNNLO calculation for the spectrum $(m_Q v^2 \gg \Lambda_{QCD})$(B. A. Kniehl, A. A. Penin, V. A. Smirnov and M. Steinhauser (02), ; A. A. Penin and M. Steinhauser (02), ; A. V. Smirnov, V. A. Smirnov, M. Steinhauser (10))
  - NNNLO contributions to the wave function at the origin $(m_Q v^2 \gg \Lambda_{QCD})$ (Beneke, Kiyo (08,05), Beneke, Kiyo, Penin, Schuler (07), Beneke, Kiyo, Schuler (07,05))

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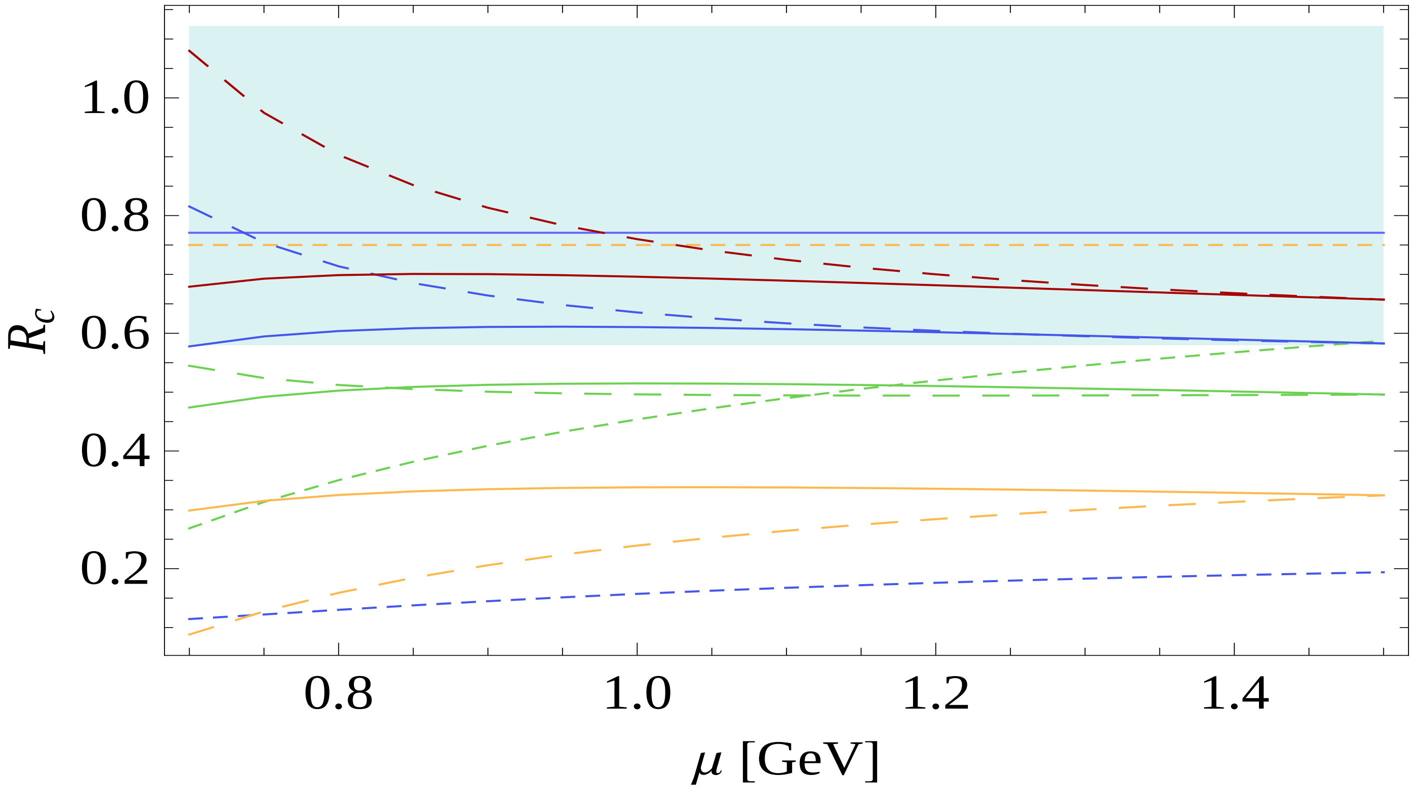
- Renormalization Group resummations ($\ln(v)$):
  - Emphasized in vNRQCD (M. E. Luke, A. V. Manohar and I. Z. Rothstein (99))
  - Implementation in pNRQCD (A. Pineda and JS (00);A. Pineda (01))

pNRQCD : weak coupling regime (Cont.)

- Hyperfine splitting at NNLL accuracy (Kniehl, Penin, Pineda, Smirnov, and Steinhauser (03))
  - $m_{J/\psi} - m_{\eta_c(1S)} \sim 112 \pm 40 \, (\delta\alpha_{\rm s}) \, {\rm MeV}$
    [Exp: $116.6 \pm 1.0 \, , {\rm MeV}$]
  - $m_{\Upsilon(1S)} - m_{\eta_b(1S)} = 39 \pm 11 \, ({\rm th}) \, ^{+9}_{-8} \, (\delta\alpha_{\rm s}) \, {\rm MeV}$
    [Exp: $69.6 \pm 2.9 \, {\rm MeV}$]
- Ratio of vector/pseudoscalar electromagnetic widths at NNLL accuracy
  - $Q\bar{Q}$ propagator with static potential $\mathcal{O}(\alpha_{\rm s}(\mu))$ (A. A. Penin, A. Pineda, V. A. Smirnov and M. Steinhauser (04))
  - $Q\bar{Q}$ propagator with static potential up to $\mathcal{O}(\alpha_{\rm s}^4(1/r))$ (Y. Kiyo, A. Pineda, A. Signer (10))

pNRQCD : weak coupling regime (Cont.)



$$\Gamma(\eta_b(1S) \to \gamma\gamma) = 0.659 \pm 0.089(\mathrm{th.})^{+0.019}_{-0.018}(\delta\alpha_s) \pm 0.015(\mathrm{exp.})\ \mathrm{keV}\,,$$
$$\to 0.54 \pm 0.15\,\mathrm{keV}$$

pNRQCD : strong coupling regime

- Factorization formulas for NRQCD matrix elements can be worked out (N. Brambilla, D. Eiras, A. Pineda, JS and A. Vairo, Phys. Rev. Lett. **88**, 012003 (2002); Phys. Rev. D **67**, 034018 (2003))

$$\langle \Upsilon(n)|O_8(^1S_0)|\Upsilon(n)\rangle = C_A \frac{|R_n(0)|^2}{2\pi}\left(-\frac{(C_A/2 - C_f)c_F^2 \mathcal{B}_1}{3m_Q^2}\right)$$

  - $R_n(0)$, wave function at the origin
  - $\mathcal{B}_1 \sim \Lambda_{QCD}^2$, independent of n
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- New predictions can be put forward

$$\frac{\Gamma(\chi_{b0}(1P)\to \mathrm{LH})}{\Gamma(\chi_{b1}(1P)\to \mathrm{LH})} = \frac{\Gamma(\chi_{b0}(2P)\to \mathrm{LH})}{\Gamma(\chi_{b1}(2P)\to \mathrm{LH})} = 8.0 \pm 1.3,$$

pNRQCD: strong coupling regime (Cont.)

- The ratio of photon spectra in inclusive radiative decays (X. Garcia i Tormo, JS, Phys.Rev.Lett.96:111801,2006)

$$\frac{\frac{d\Gamma_n}{dz}}{\frac{d\Gamma_r}{dz}} = \frac{\langle\mathcal{O}_1(^3S_1)\rangle_n}{\langle\mathcal{O}_1(^3S_1)\rangle_r}\left(1+\frac{C_1'\left[^3S_1\right](z)}{C_1\left[^3S_1\right](z)}\frac{1}{m_Q}(E_n-E_r)\right)$$

$$\frac{\langle\mathcal{O}_1(^3S_1)\rangle_n}{\langle\mathcal{O}_1(^3S_1)\rangle_r} = \frac{\Gamma\left(\Upsilon(n)\to e^+e^-\right)}{\Gamma\left(\Upsilon(r)\to e^+e^-\right)}\left[1-\frac{\mathrm{Im}g_{ee}\left(^3S_1\right)}{\mathrm{Im}f_{ee}\left(^3S_1\right)}\frac{E_n-E_r}{m_Q}\right]$$

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  - CLEO data for $\Upsilon(n) \to \gamma X$, $n = 1, 2, 3$ (D. Besson *et al.* [CLEO Collaboration], (05)) clearly suggest that $n = 2, 3$ are in the strong coupling regime.

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- Proposal': the leptonic widths as well (J. L. Domenech Garret, M. A. Sanchis Lozano (08)), reach the same conclusions

Beyond inclusive decays

- Transitions
  - NRQCD OK, but provides little info.
  - pNRQCD OK:
    - Weak coupling: detailed study of magnetic dipole transitions (N. Brambilla, Y. Jia, A. Vairo (05))

      $\Gamma(J/\psi \to \eta_c \gamma) = 1.5 \pm 1.0\,\mathrm{keV}$ [Exp. 1.7 ± 0.4 keV]
  - Hadronic EFTs incorporating heavy quark and chiral symmetry (R.Casalbuoni, A. Deandrea, N.Di Bartolomeo, F.Feruglio, R.Gatto, G. Nardulli (96), F. De Fazio (08))
- Decays to heavy-light meson pairs
  - NRQCD OK, but provides little info.
  - pNRQCD must be augmented.

Beyond inclusive decays

- Semi-inclusive and exclusive decays
  - NRQCD must be augmented: there are gluons of energy $\sim m_Q$ in the final state, which have been integrated out in NRQCD
    - Incorporate QCD factorization formulas, fragmentation functions, light cone distribution amplitudes,...
    - SCET (C. W. Bauer, S. Fleming, M. E. Luke (00)) is supposed to do it in an EFT framework

NRQCD+SCET

Let me focus on $\Upsilon(1S) \to X\gamma$

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- Long standing discrepancies between QCD and experiment in the upper end point region of the spectrum $z = E_\gamma/2M \to 1$

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- Long standing discrepancies between QCD and experiment in the upper end point region of the spectrum $z = E_\gamma/2M \to 1$

The upper end-point region:

- NRQCD factorization breaks down (the scales $M(1-z)$ and $M\sqrt{1-z}$ play a rôle)
- Collinear degrees of freedom become relevant
- Shape functions must be introduced
- Color octet contributions become LO

NRQCD+SCET (Cont.)

Combining NRQCD and SCET:

NRQCD+SCET (Cont.)

Combining NRQCD and SCET:

- Factorization formulas have been proved (S. Fleming and A. K. Leibovich, Phys. Rev. Lett. **90** (03) 032001 ; Phys. Rev. D **67** (03) 074035)

NRQCD+SCET (Cont.)

Combining NRQCD and SCET:

- Factorization formulas have been proved (S. Fleming and A. K. Leibovich, Phys. Rev. Lett. **90** (03) 032001 ; Phys. Rev. D **67** (03) 074035)
- Large (Sudakov) logs have been resummed
  - Color octet (C. W. Bauer, C. W. Chiang, S. Fleming, A. K. Leibovich and I. Low, Phys. Rev. D **64** (01) 114014)
  - Color singlet (S. Fleming and A. K. Leibovich, Phys. Rev. D **70** (04) 094016)

NRQCD+SCET (Cont.)

Factorization formula:

$$\frac{d\Gamma^e}{dz} = \sum_{\omega} H(M,\omega,\mu) \int dk^+ S(k^+,\mu) \mathrm{Im} J_\omega(k^+ + M(1-z),\mu)$$

NRQCD+SCET (Cont.)

Factorization formula:

$$\frac{d\Gamma^e}{dz} = \sum_{\omega} H(M,\omega,\mu) \int dk^+ S(k^+,\mu) \mathrm{Im} J_\omega(k^+ + M(1-z),\mu)$$

Combining pNRQCD and SCET

NRQCD+SCET (Cont.)

Factorization formula:

$$\frac{d\Gamma^e}{dz} = \sum_\omega H(M,\omega,\mu) \int dk^+ S(k^+,\mu) \mathrm{Im} J_\omega(k^+ + M(1-z),\mu)$$

Combining pNRQCD and SCET

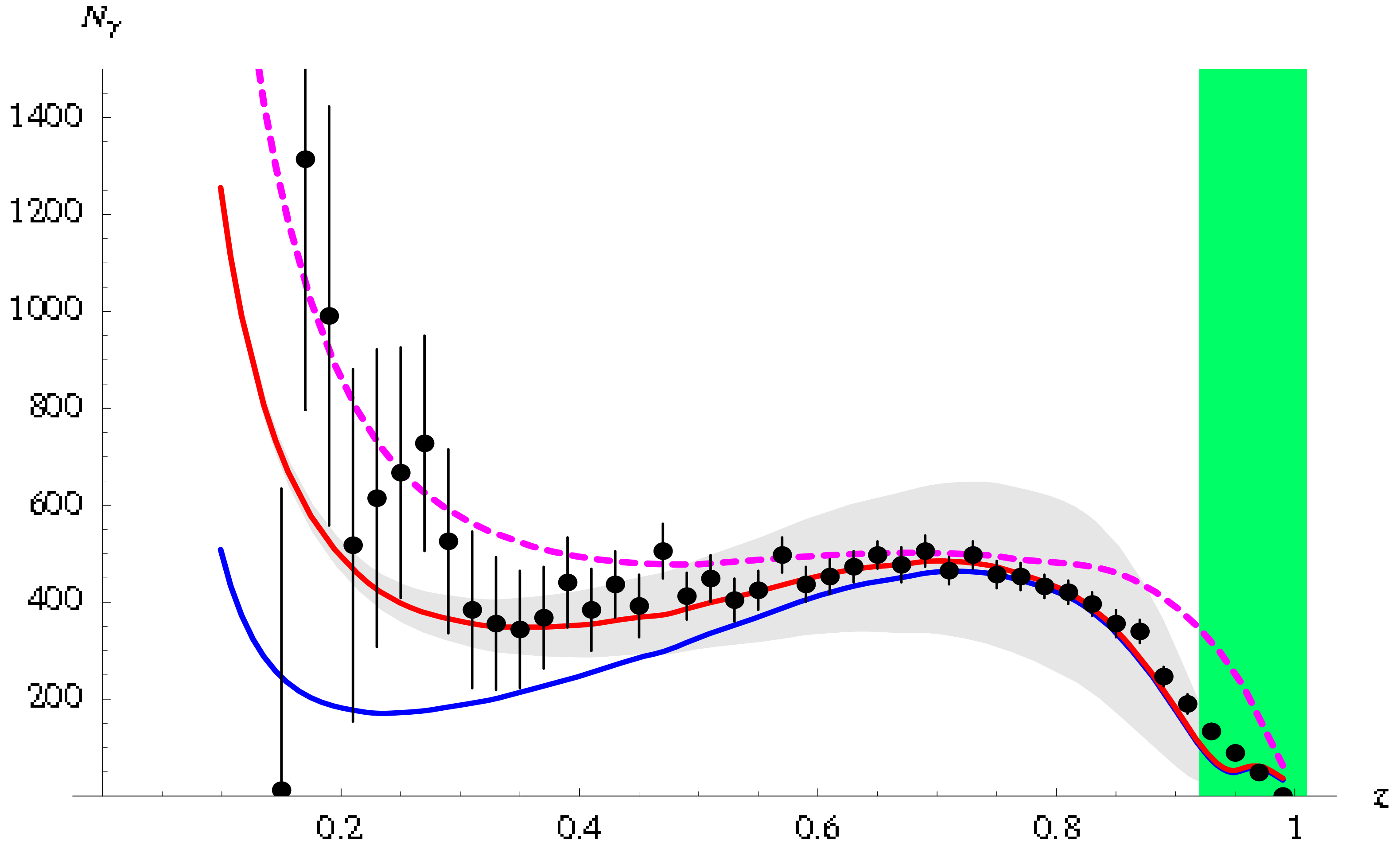
- Color octet shape functions have been calculated (X. Garcia i Tormo and J. Soto, Phys. Rev. D **69** (2004) 114006)

  The calculation is reliable for:

  - $z \lesssim 0.9$ (from $1GeV \lesssim M(1-z)$)
  - $z \gtrsim 0.7$ (from $(1-z) \ll 1$)

NRQCD+SCET (Cont.)

The full spectrum:



Production

- Except for the case of e.m. production at threshold, production processes involve gluons of energy $\sim m_Q$
- NRQCD must be supplemented with additional factorization formulas
- Proposal for inclusive production (BBL (04))

$$\sigma(H) = \sum_n \frac{F_n(\mu)}{m_Q^{d_n-4}} \langle 0|\mathcal{O}_n^H(\mu)|0\rangle,$$

$F_n(\mu)$ short distance matching coefficient

$$\mathcal{O}_n^H = \chi^\dagger \mathcal{K}_n \psi \left(\sum_X \sum_{m_J} |H+X\rangle\langle H+X| \right) \psi^\dagger \mathcal{K}'_n \chi$$

Production

- See Bodwin's talk for:
  - Status of factorization proofs
  - Production at hadron colliders
  - Production at electron-proton colliders
- Next:
  - Production at $e^+ e^-$ colliders
  - Charmonium production in bottomonium decays
- Remark:
  - Standard factorization formulas give results in terms of universal functions (FF, LCDA,...)
  - NRQCD factorization formulas give results in terms of universal numbers (NRQCD matrix elements)

Inclusive double $c\bar{c}$ production at Belle

Belle (02)

$$\sigma(e^+e^- \to J/\psi + c\bar{c} + X)/(e^+e^- \to J/\psi + X) = 0.59^{+0.15}_{0.13} \pm 0.12. \quad [\mathrm{Th.} : \sim 0.1]$$

Th. : pQCD+CS Cho, Leibovich (96); Baek, Ko, Lee, Song (97); Yuan, Qiao, Chao (97)). Belle (09) confirms this figure

- Theoretical progress in $\sigma(e^+e^- \to J/\psi + c\bar{c} + X)$
  - NLO calculation of the color-singlet contribution large ($K \sim 1.8$) (Zhang and Chao (07); Gong and Wang (09))
  - Two photon contribution small (Liu, He, Chao (03))
  - Direct relativistic corrections (He, Fan, Chao (07))
  - LO color octet contributions (Liu, He, Chao (04))
  - NLO color octet contributions large ($K \sim 1.9$) (Zhang, Ma, Wang, Chao (09))

Inclusive double $c\bar{c}$ production at Belle

- Theoretical progress in $\sigma(e^+e^- \to J/\psi + X(\mathrm{non}\, c\bar{c}))$
  - LO color-octet contribution (Wang (03))
  - NLO color-singlet contribution (Ma, Zhang, Chao (08); Gong, Wang (09))
  - Relativistic corrections to CS (He, Fan, Chao (09))

Putting all together

$$\sigma(e^+e^- \to J/\psi + c\bar{c} + X)/(e^+e^- \to J/\psi + X)|_{\mathrm{Th.}} \sim 0.50$$

- No obvious discrepancy anymore, but theoretical uncertainties large

Exclusive double charmonium production at B-factories

$$\text{Belle(04)} \quad \sigma(e^+e^- \to J/\psi + \eta_c) \times B_{>2} = 25.6 \pm 2.8 \pm 3.4\,\text{fb.}$$

$$\text{Babar(05)} \quad \sigma(e^+e^- \to J/\psi + \eta_c) \times B_{>2} = 17.6 \pm 2.8^{+1.5}_{-2.1}\,\text{fb.}$$

Th.: $2.3 - 5.5$ fb. (Liu, He, Chao (02); Braaten, Lee (03); Hagiwara, Kou, Qiao (03)), LO NRQCD in α_s and v (color octet suppressed by v^4)

- Theoretical progress:
  - NLO in α_s is large ($K \sim 2$) (Zhang, Gao, Chao (05); Gong, Wang (07))
  - NRQCD higher order (in v) matrix elements determined through a potential model (Bodwin, Kang, Lee (06))
  - Relativistic corrections + NLO in α_s seem to be OK with Exp. (Bodwin, Chung, Kang, Kim, Lee, Yu (06); He, Fan, Chao (07))
  - Detailed error analysis (Bodwin, Chung, Kang, Lee, Yu (07))

$$\sigma(e^+e^- \to J/ + \eta_c) = 17.6^{+8.1}_{-6.7}\,\text{fb.}$$

Bottomonium decays to charmonium

- $\Upsilon(1S) \to J/\psi + X$

  $\text{CLEO(04)} \quad B(\Upsilon(1S) \to J/\psi + X) = (6.4 \pm 0.4 \pm 0.6) \times 10^{-4}$

  - LO color-octet (Cheung, Keung, Yuan (96); Napsuciale (97))
    $B \sim 2.5 - 2.1 \times 10^{-4}$
  - LO color-singlet ($\Upsilon(1S) \to J/\psi + c\bar{c} + g$) (Lie, Xie, Wang (00))
    $B \sim 5.9 \times 10^{-4}$, recently corrected to $B \sim 2.3 - 8.3 \times 10^{-5}$ (He, Wang (09)), e.m. contribution also small.
  - NLO color-singlet ($\Upsilon(1S) \to J/\psi + gg(gggg)$) (He, Wang (09))
    $B \sim 3.2 - 0.9 \times 10^{-4}$
  - Spectrum at $z \to 1$, LO color-singlet + SCET (Leibovich, Liu (07)); LO e.m. and LO color-octet + SCET (Liu (09))

Bottomonium decays to charmonium

- $\eta_b(1S) \to \eta_c\,,\ \chi_c + X$, LO (He, Li (09))
- $\Upsilon(1S) \to c + \bar{c} + X$, LO, invariant mass distribution (Chung, Kim, Lee (08))
- $\Upsilon(1S) \to c + \bar{c}$, two charm jets, LO+ NLO in α_s, CS+CO (Zhang, Chao (08))
- $\eta_b(1S) \to J/\psi J/\psi$, NLO in α_s (v^0), (Gong, Jia, Wang (08)), proposed as a discovery mode (Jia (06); Braaten, Fleming, Leibivich (01)). Analysis LC formalism vs. NRQCD (Sun, Hao, Qiao (10))
- $\eta_b, \chi_b \to D\bar{D}$, combine NRQCD, pNRQCD and SCET (Azevedo, Long, Mereghetti (09))

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Thank You