NRQCD Factorization and Quarkonium Production at Hadron-Hadron and ep Colliders

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- Summary

Factorization of the Inclusive Quarkonium Production Cross Section

- In heavy-quarkonium hard-scattering production, high-momentum scales appear: m and p_T.
- We would like to use NRQCD to separate the perturbative physics at these high-momentum scales from the low-momentum, nonperturbative effects in the heavy-quarkonium dynamics.
- The probability for a $Q\bar{Q}$ pair to evolve into a heavy quarkonium can be calculated as a vacuum-matrix element in NRQCD:

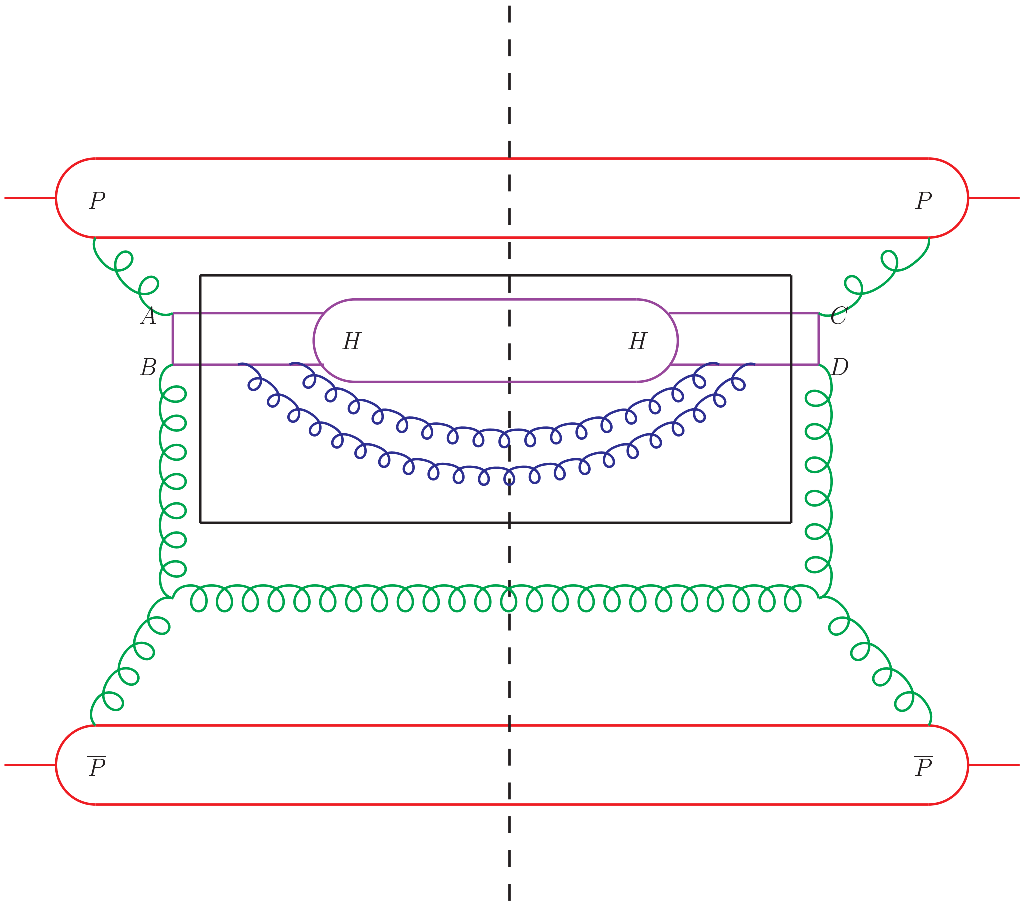
$$\mathcal{O}_n^H(\Lambda) = \langle 0|\chi^\dagger \kappa_n \psi \left(\sum_X |H + X\rangle\langle H + X| \right) \psi^\dagger \kappa'_n \chi|0\rangle.$$

- This is the matrix element of a four-fermion operator, but with a projection onto an intermediate state of the quarkonium H plus anything.
  - κ_n and κ'_n are combinations of Pauli and Color matrices.
- The production matrix elements are the crossed versions of quarkonium decay matrix elements.
  - Only the color-singlet production and decay matrix elements are simply related.

- Conjecture (GTB, Braaten, Lepage (1995)):
  The inclusive cross section for producing a quarkonium at large momentum transfer (p_T) can be written as hard-scattering cross section convolved with an NRQCD matrix element.

$$\sigma(H) = \sum_n F_n(\Lambda)\langle 0|\mathcal{O}_n^H(\Lambda)|0\rangle .$$

- The part of the diagram inside the box corresponds to an NRQCD matrix element.



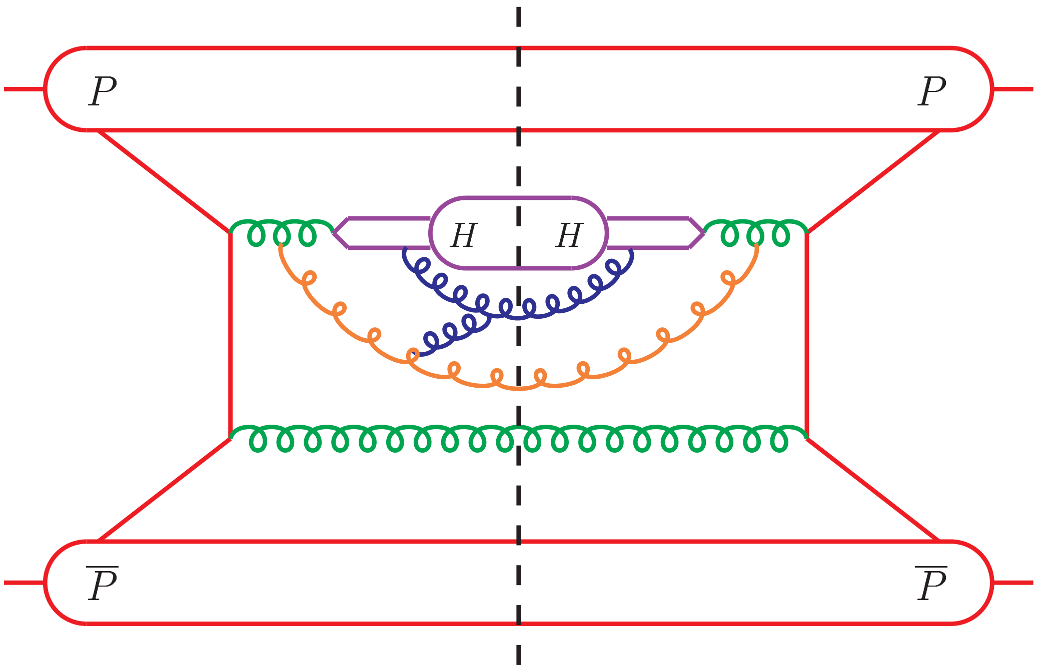
- The points $A(C)$ and $B(D)$ are within $\sim 1/m$ of each other.
  - Kinematics implies that the virtual Q is off shell by order m.
- The points $A(B)$ and $C(D)$ are within $1/p_T$ of each other.
  - The part of the diagram outside the box is insensitive to changes of momentum flow from $A(B)$ to $C(D)$ of order p_T.

- The “short-distance” coefficients $F_n(\Lambda)$ are essentially the process-dependent partonic cross sections to make a $Q\bar{Q}$ pair convolved with the parton distributions.
  - They have an expansion in powers of α_s.
- The operator matrix elements are universal (process independent).
  - Only the color-singlet production matrix elements are simply related to the decay matrix elements.
  - The matrix elements have a known scaling with v.
- The NRQCD factorization formula for production is a double expansion in powers of α_s and v.
- A key feature of NRQCD factorization:
  Quarkonium production can occur through color-octet, as well as color-singlet, $Q\bar{Q}$ states.
- If we drop all of the color-octet contributions and retain only the leading color-singlet contribution, then we have the color-singlet model (CSM).
  - Inconsistent for P-wave production: IR divergent.

Status of a Proof of Factorization

- A proof is complicated because individual gluons can dress the basic production process in ways that apparently violate factorization.
- A proof of factorization would involve a demonstration that diagrams in each order in α_s can be re-organized so that
  - All soft singularities cancel or can be absorbed into NRQCD matrix elements,
  - All collinear singularities and spectator interactions can be absorbed into parton distributions.
- Nayak, Qiu, Sterman (2005, 2006): The NRQCD matrix elements must be modified by the inclusion of eikonal lines to make them gauge invariant.
  - The eikonal lines are path integrals of the gauge field running from the creation and annihilation points to infinity.
  - Essential at two-loop order to allow certain soft contributions to be absorbed into the matrix elements.
  - Does not affect existing phenomenology, which is at tree order or one-loop order.
- Ma and Si (2005): In $e^+e^- \to$ quarkonium $+ X$ at one-loop order, all uncanceled IR divergences can be absorbed into NRQCD matrix elements without eikonal lines, as expected.

- Nayak, Qiu, Sterman (2005, 2006): A key difficulty in proving factorization to all orders is the treatment of gluons with momenta of order m_c in the quarkonium rest frame.



- If the orange gluon has momentum of order m_c, it can't be absorbed into the NRQCD matrix element as a quarkonium constituent.
- But the orange gluon can have non-vanishing soft exchanges with the quarkonium constituents.
- The orange gluon can be treated as the eikonal-line part of the NRQCD matrix element, provided that the answer does not depend on the direction of the eikonal line (universality of the matrix elements).

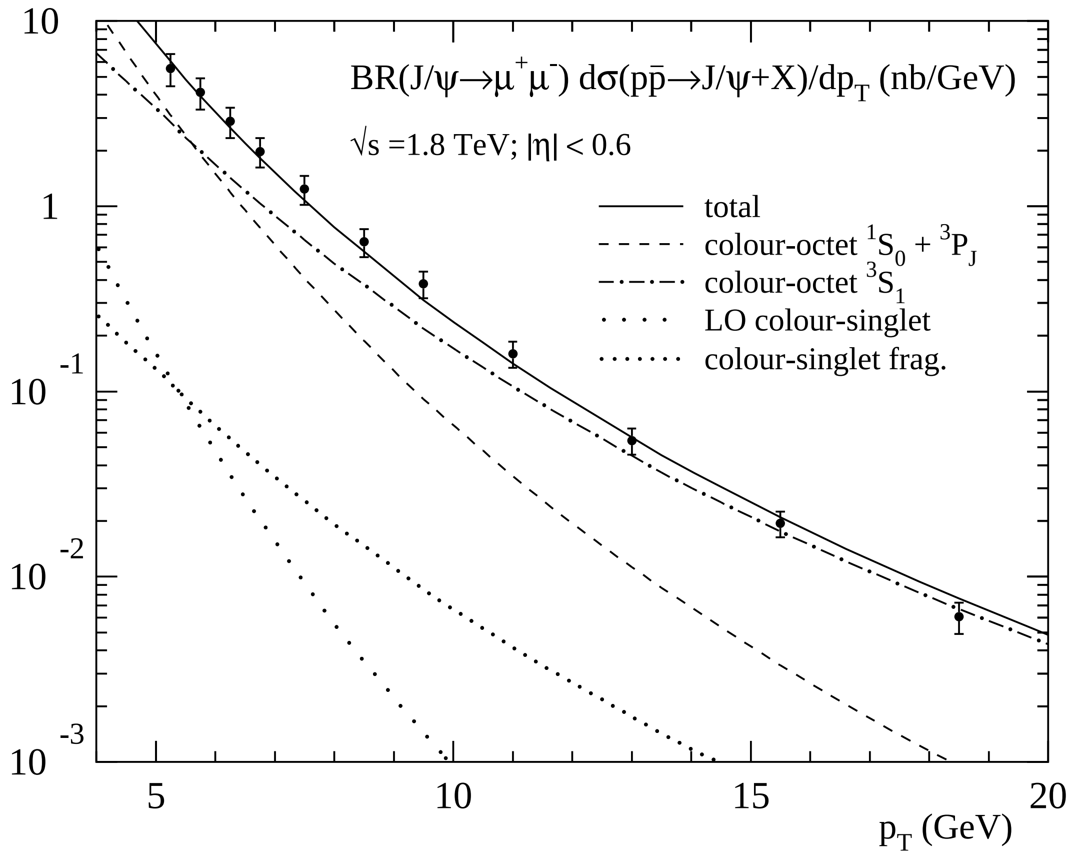
- Nayak, Qiu, Sterman (2005, 2006): At two-loop order, the eikonal lines contribute but a "miracle" occurs: The dependence on the direction of the eikonal line cancels.
- In general, factorization of the inclusive cross section beyond two-loop order is still an open question.
- An all-orders proof is essential because the α_s associated with soft gluons is not small.

- Nayak, Qiu, Sterman (2007, 2008): If an additional heavy quark is approximately co-moving with the $Q\bar{Q}$ pair that forms the quarkonium, there are soft color exchanges between the heavy quark and the $Q\bar{Q}$ pair.
  - This process does not fit into the NRQCD factorization picture.
    It requires production matrix elements that contain additional heavy quarks beyond the $Q\bar{Q}$ pair.
  - The process is nonperturbative: It can't be calculated reliably.
  - Can search for the process experimentally:
    The signature is additional heavy-meson production in a narrow cone ($\sim mv/p_T$) around the quarkonium.
  - This effect might be eliminated from the measured cross section through the use of an isolation cut.

Comparisons of NRQCD Factorization with Experiment

Quarkonium Production and Polarization at the Tevatron

Production Cross Section



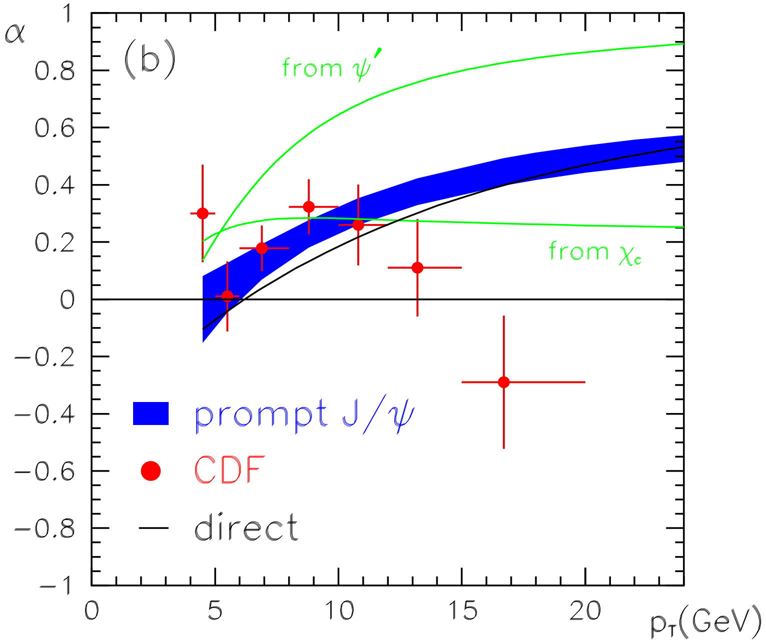
- The CDF (1997) data are more than an order of magnitude larger than the predictions of the color-singlet model.
- p_T distributions are consistent with NRQCD prediction (Krämer (2001)), but not with the LO color-singlet model.
- Color-octet matrix elements are determined from fits to the data.
- Good fits for J/ψ, $\psi(2S)$, χ_c, $\Upsilon(1S)$ production, as well.
- Use color-octet matrix elements from these fits to predict quarkonium production in other processes (test universality).

Polarization

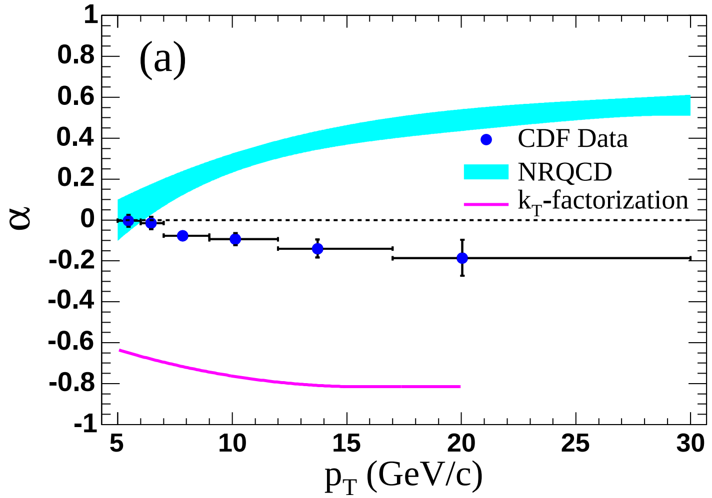
- Touted as a "smoking gun" for the color-octet mechanism.
- In LO quarkonium production at large p_T ($p_T \gtrsim 4m_c$ for J/ψ), gluon fragmentation via the color-octet mechanism dominates ($\langle \mathcal{O}_8(^3S_1) \rangle$).
- At large p_T, the gluon is nearly on mass shell, and, so, is transversely polarized.
- In color-octet gluon fragmentation, most of the gluon's polarization is transferred to the quarkonium (Cho, Wise (1994)).
  - Spin-flip interactions are suppressed as v^2.
  - Verified in a lattice calculation of decay matrix elements (GTB, Lee, Sinclair (2005)).
- Radiative corrections dilute this (Beneke, Rothstein (1995); Beneke, Krämer (1996)).

J/ψ Polarization

Run I:



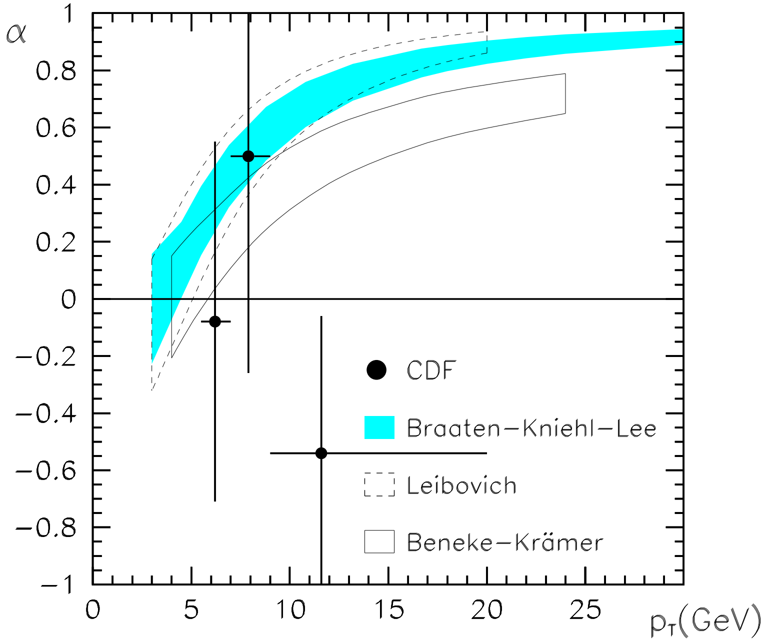
Run II:



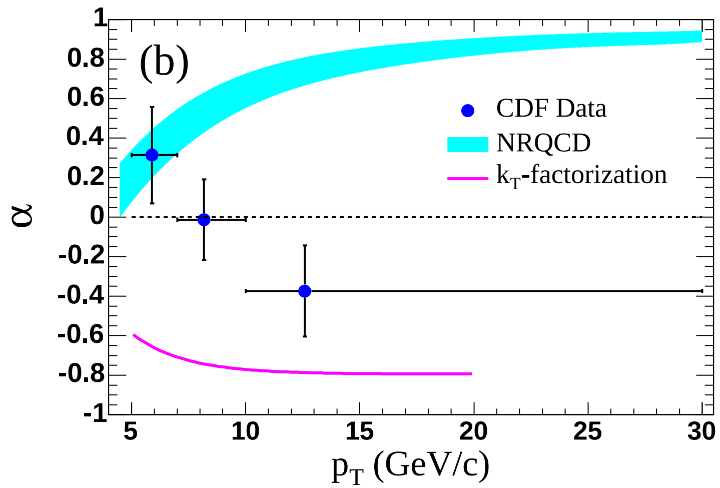
- $d\sigma/d(\cos\theta) \propto 1 + \alpha\cos^2\theta$.
  - $\alpha = 1$ is completely transverse;
  - $\alpha = -1$ is completely longitudinal.
- NRQCD prediction from Braaten, Kniehl, Lee (1999).
  - Feeddown from χ_c states is about 30% of the J/ψ sample and dilutes the polarization.
  - Feeddown from $\psi(2S)$ is about 10% of the J/ψ sample and is largely transversely polarized.
- Run I results are marginally compatible with the NRQCD prediction.
- Run II results are inconsistent with the NRQCD prediction.
- Also inconsistent with the Run I results.
  CDF was unable to track down the source of the Run I-Run II discrepancy.

$\psi(2S)$ Polarization

Run: I



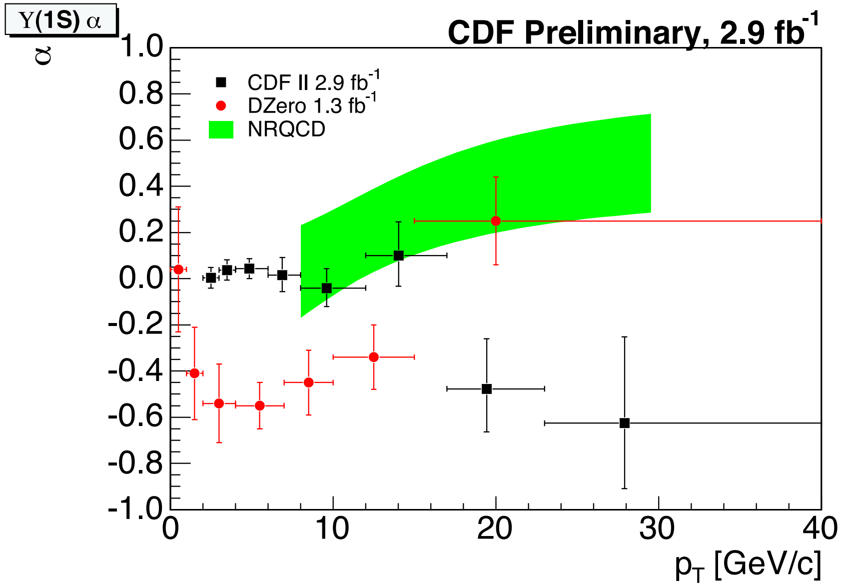
Run: II



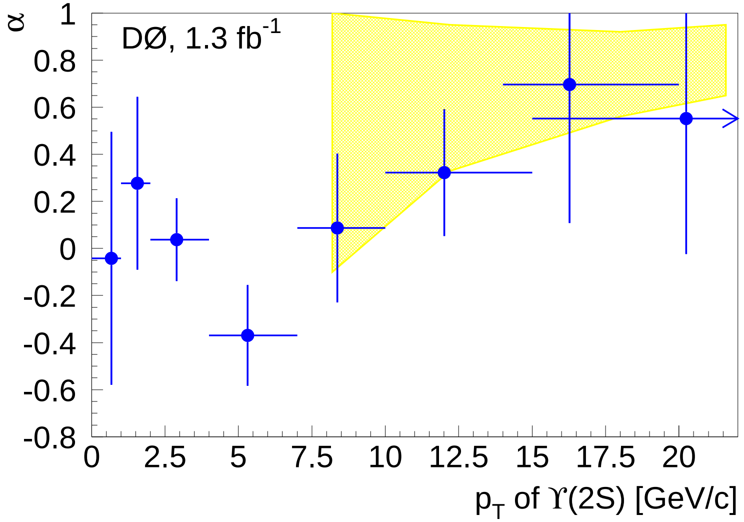
- The Run II data are incompatible with the NRQCD prediction.

Υ Polarization

$\Upsilon(1S)$ Polarization:



$\Upsilon(2S)$ Polarization:

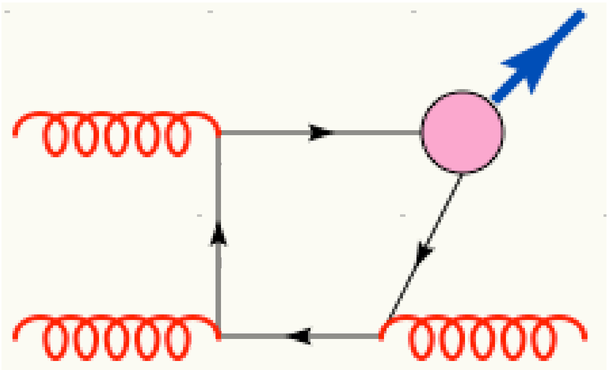


- In the $\Upsilon(1S)$ case, the D0 results (red) are incompatible with the CDF results (black).
- Both the CDF and D0 results are incompatible with the NRQCD prediction of Braaten and Lee (2000) (green), but in different regions of p_T.
- In the $\Upsilon(2S)$ case, the theoretical and experimental error bars are too large to make a stringent test.

Recent Theoretical Developments

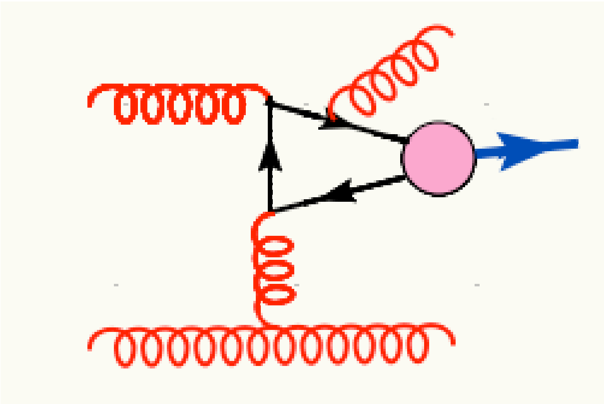
- Campbell, Maltoni, Tramontano(2007); Artoisenet, Lansberg, Maltoni (2007): Higher-order corrections to color-singlet quarkonium production at the Tevatron are unexpectedly large.
- At high p_T, higher powers of α_s can be offset by a less rapid fall-off with p_T.

LO:

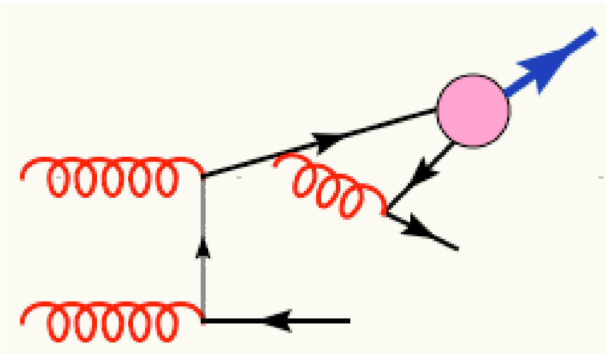


$$\sim \alpha_s^3 \frac{(2m_c)^4}{p_T^8}$$

NLO:

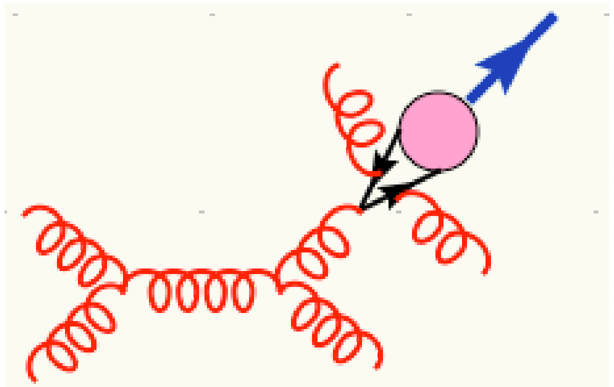


$$\sim \alpha_s^4 \frac{(2m_c)^2}{p_T^6}$$



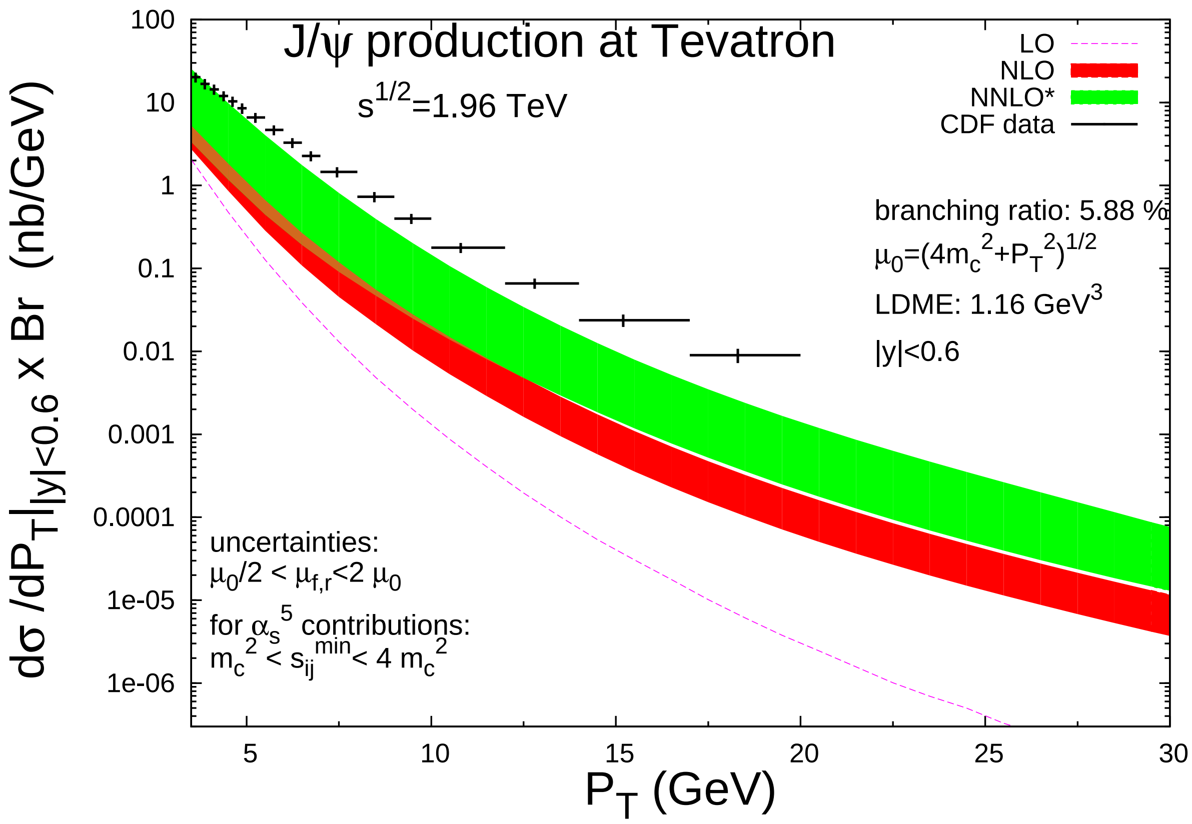
$$\sim \alpha_s^4 \frac{1}{p_T^4}$$

NNLO:



$$\sim \alpha_s^5 \frac{1}{p_T^4}$$

New Results for color-singlet J/ψ Production

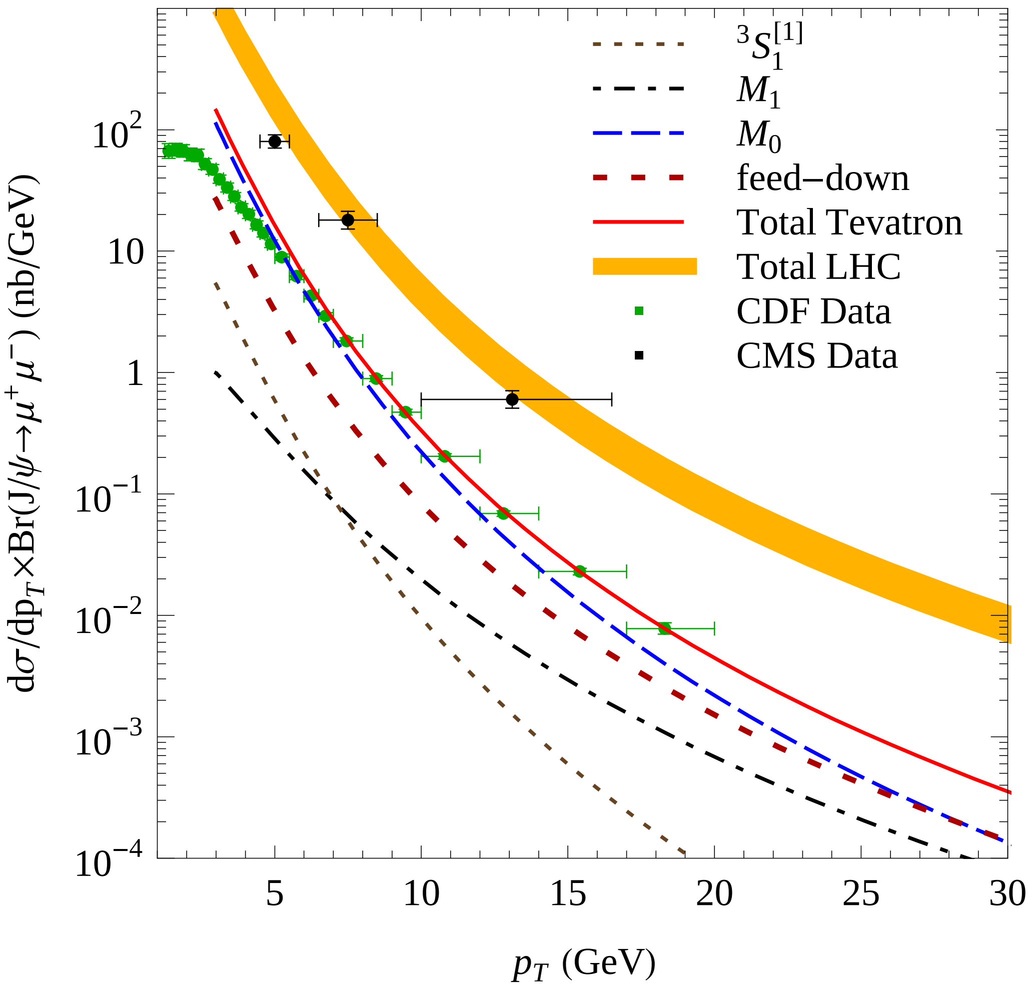


- Plot from Pierre Artoisenet, based on work by Artoisenet, Campbell, Lansberg, Maltoni, Tramontano (in progress)
- The NNLO* calculation is an estimate based on real-emission contributions only.
- The data still seem to require a color-octet contribution, but its size may be reduced from previous estimates. Affects the matrix elements used to compute all other processes.

New Results for Color-Octet J/ψ and $\psi(2S)$ Production

- Gong, Li, and Wang (2008, 2010):
  NLO corrections to the S-wave channels are small.
  - K factors at the Tevatron are about 1.235 for the ${}^1S_0$ channel and 1.139 for the ${}^3S_1$ channel.
- First Complete NLO Calculations: Ma, Wang, and Chao (2010); Butenschön and Kniehl (2010)
  - NLO corrections for all of the color-octet (and color-singlet) channels of leading order in v.
    Color-octet: ${}^1S_0$, ${}^3S_1$, ${}^3P_J$.
  - Confirm that the NLO corrections to the S-wave channels are small.
  - Very large K factor ~ -10 for the ${}^3P_J$ channel.
    A $1/p_T^4$ contribution appears for the first time in NLO.
  - The results of Ma, Wang, and Chao and Butenschön and Kniehl for the short-distance cross sections agree.

Ma, Wang, and Chao (2010):



- Matrix elements were fit to the CDF (2005, 2009) Run II data.
- Feeddown from the $\psi(2s)$ was taken into account by using the CDF (2005, 2009) Run II data.
- Feeddown from the χ_{cJ} states was taken into account by using the NLO prediction of Ma, Wang, and Chao (2010) for χ_{cJ} production.
  - Uses a color-octet matrix element that is obtained by fitting to the CDF (2007) measurements of $R_{\chi_c} = \sigma_{\chi_{c2}}/\sigma_{\chi_{c1}}$.
  - The predicted χ_{cJ} fraction increases with increasing p_T, while the χ_{cJ} fraction measured by CDF (1997) in Run I decreases with increasing p_T.
- The fits were used to predict the CMS (2010) data.

- Only the linear combinations

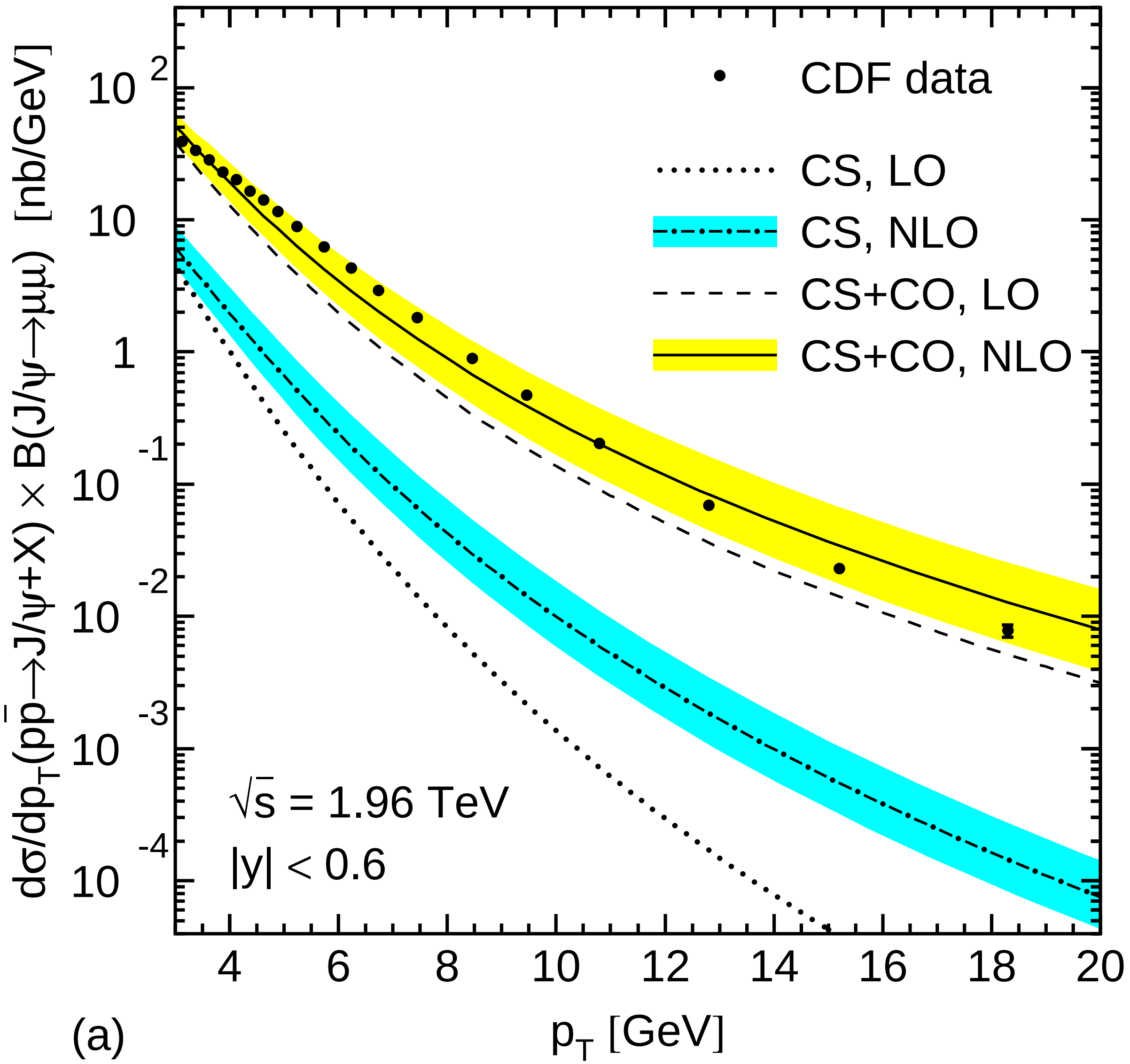
$$M_{0,r_0} = \langle O^\psi({}^1S_0^{[8]})\rangle + (r_0/m_c^2)\langle O^\psi({}^3P_0^{[8]})\rangle = (7.4 \pm 1.9) \times 10^{-2}\ \text{GeV}^3$$
$$M_{1,r_1} = \langle O^\psi({}^3S_1^{[8]})\rangle + (r_1/m_c^2)\langle O^\psi({}^3P_0^{[8]})\rangle = (0.05 \pm 0.02) \times 10^{-2}\ \text{GeV}^3$$

could be fit unambiguously.

$r_0 = 3.9$ and $r_1 = -0.56$ for J/ψ production at the Tevatron.

- The small size of M_{1,r_1} suggests that $\langle O^\psi({}^3S_1^{[8]})\rangle$ is small.
  - Assumes that there is not an accidental cancellation between the $\langle O^\psi({}^3S_1^{[8]})\rangle$ and $\langle O^\psi({}^3P_0^{[8]})\rangle$.
  - Might explain the absence of transverse J/ψ polarization in the Tevatron data.

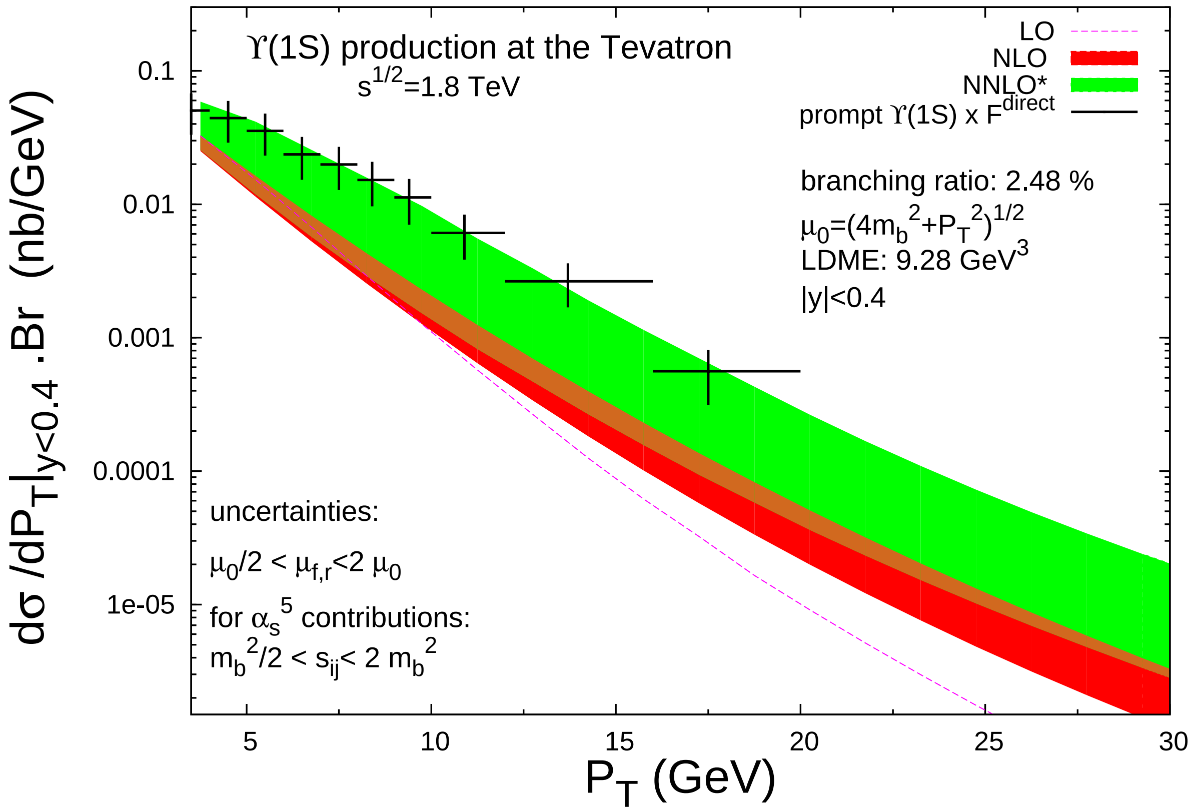
Butenschön and Kniehl (2010):



- NRQCD matrix elements were extracted in a fit that made use of both the CDF (2005) Run II data and the H1 (2002, 2005) HERA I and HERA II data.
- All three NRQCD matrix elements were determined in the fit.
- A cut $p_T > 3$ was applied to the CDF data.
- No corrections were made for feed-down.
- This fit describes shape of the CDF data less well than the fit of Ma, Wang, and Chao.
  - There may be some tension between the theory and the combined CDF and H1 data.
- The results were used to predict cross sections at PHENIX and CMS.

- The Kniehl and Butenschön matrix elements are not very different from those from LO extractions.
- In comparison to the values in the Ma, Wang, and Chao fit
  - M_{0,r_0} is about a factor 4 smaller,
  - M_{1,r_1} is about a factor 11 larger.
- Since the short-distance cross section agree, the differences between the matrix elements must arise from the differences in the fitting procedures.
- Discussions are underway to resolve this issue.
- The importance of the $\langle O^{\psi}({}^3S_1^{[8]})\rangle$ contribution may be decided by the outcome.

New Results for color-singlet Υ Production



- Plot from Pierre Artoisenet, based on work by Artoisenet, Campbell, Lansberg, Maltoni, Tramontano (2008)
- NLO results confirmed by Gong and Wang (2007).

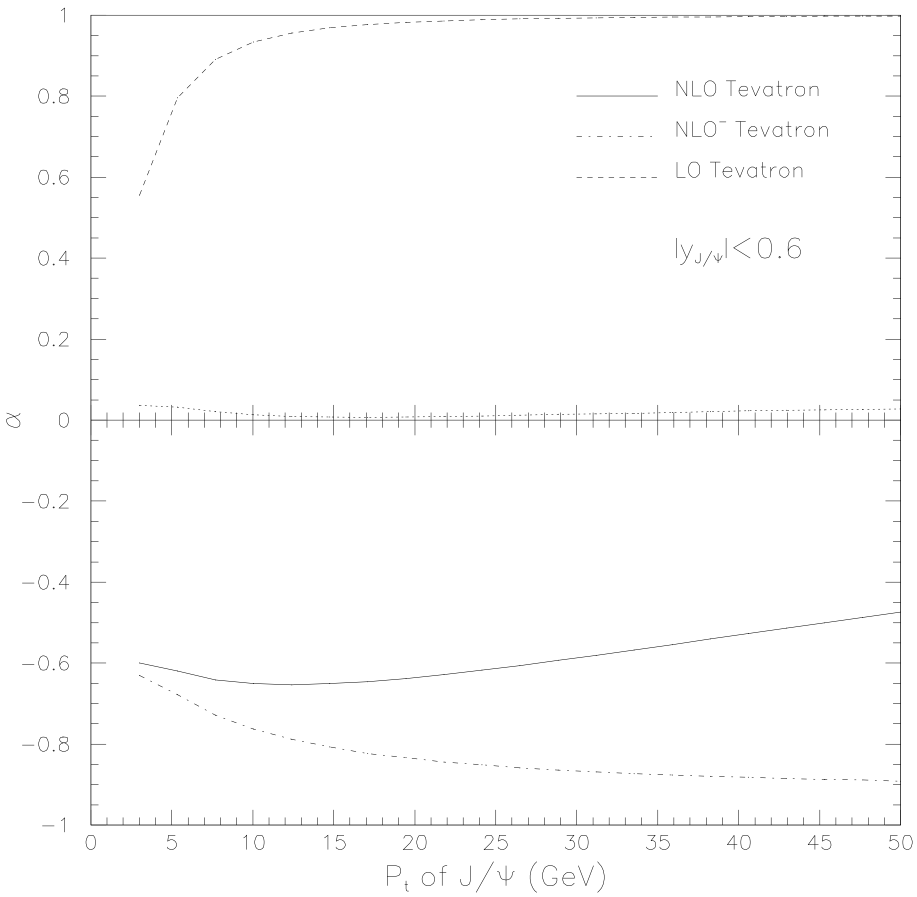
- The data could be explained by color-singlet production alone.
- There is still room for a substantial amount of color-octet production.
- Color-octet production is suppressed as v^4.
  Should be smaller for Υ $(v^2 \approx 0.1)$ than for J/ψ $(v^2 \approx 0.3)$.

New Result for Color-Octet Υ Production

- (Gong, Wang, Zhang (2008, 2010)):
  NLO corrections to the S-wave channels are small.
  - K factors at the Tevatron are about 1.313 for the ${}^1S_0$ channel and 1.379 for the ${}^3S_1$ channel.

New Results for Polarization

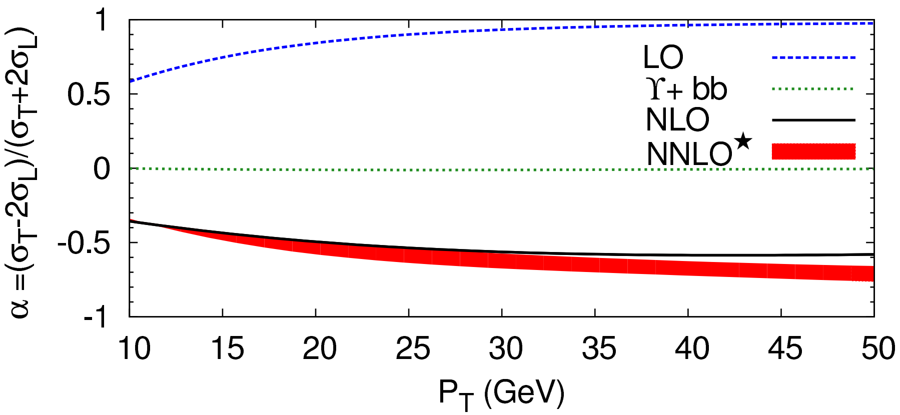
- Gong and Wang (2008) find that color-singlet J/ψ polarization at the Tevatron changes from transverse to longitudinal when NLO corrections are included.



- NLO$^-$ excludes $gg \to J/\psi c\bar{c}$.
- Unlabeled line is contribution of $gg \to J/\psi c\bar{c}$.

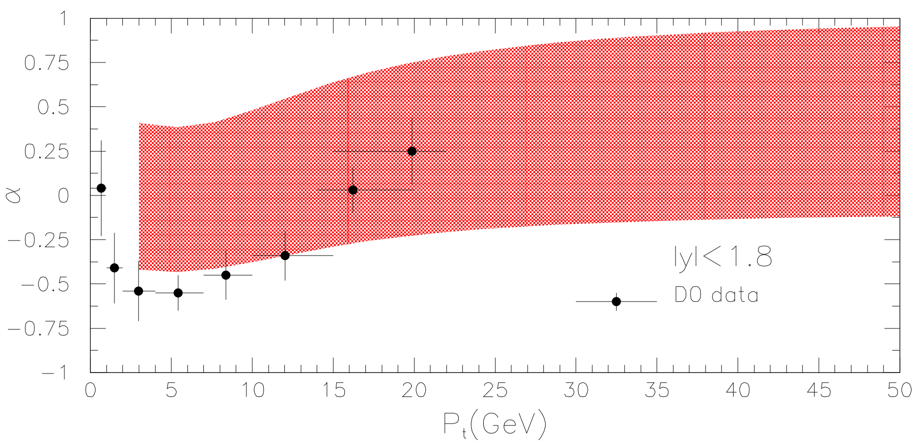
- Gong, Li, and Wang (2008): NLO corrections to the color-octet contribution to the J/ψ polarization have very little effect.

- Artoisenet, Campbell, Lansberg, Maltoni, Tramontano (2008) find that color-singlet Υ polarization at the Tevatron changes from transverse to longitudinal when NLO and NNLO* corrections are included.



- NLO result confirmed by Gong and Wang (2008).

- The NLO calculation of Gong, Wang, and Zhang (2010) for the color-octet $^1S_0$ and $^3S_1$ channels does not shift the theoretical prediction significantly.



- There are large uncertainties because of the feeddown from the χ_{bJ} states.

Fragmentation-Function Approach to Quarkonium Production

- The large renormalization-scale uncertainties in the higher-order calculations are a major obstacle to meaningful comparisons with experiment.
- Kang, Qiu, and Sterman have proposed a method for bringing the higher-order corrections under better control.
- Based on writing the cross section in terms of partonic production processes convolved with fragmentation functions in which a single parton or a $Q\bar{Q}$ pair fragment into a quarkonium.
- The leading and first subleading behaviors in p_T already appear in the Born-level diagrams for
  - $A + B \rightarrow$ fragmenting parton $+ X$,
  - $A + B \rightarrow Q\bar{Q} + X$.
- In the fragmentation functions, an important class of higher-order corrections is resummed by making use of evolution equations.
- If NRQCD factorization holds, then the fragmentation functions can be written as a sum of NRQCD matrix elements times perturbatively calculable short-distance coefficients.
- See Qiu's talk at QWG2010 (conferences.fnal.gov/QWG2010).

Discussion

- The NNLO* corrections greatly increase the color-singlet contributions to the J/ψ and Υ cross sections, but the uncertainties are very large.
- A reduced color-octet contribution plus significant longitudinal polarization from the color-singlet contribution might help to bring theory into agreement with the CDF data for J/ψ polarization.
- The J/ψ production data still seem to require a color-octet contribution.
- NLO calculations might change our ideas about the relative proportions of the color-octet channels.
- NLO corrections to the color-octet ${}^3P_J$ channels might also have a substantial effect on polarization.
- Interpretation of the Tevatron J/ψ data, both polarized and unpolarized, is complicated by feed-down from the $\psi(2S)$ and χ_{cJ} states.
- High-statistics, high-p_T measurements of the cross section and polarization for direct production of the J/ψ, χ_{cJ}, and $\psi(2S)$ states would be of great help.
- A color-octet contribution is not required or excluded by the Υ production data.
- The discrepancies between the CDF and D0 Υ polarization data must be resolved before any meaningful comparisons can be made with theory.

χ_{cJ} Production

- $$R_{\chi c} = \sigma_{\chi_{c2}}/\sigma_{\chi_{c1}}.$$

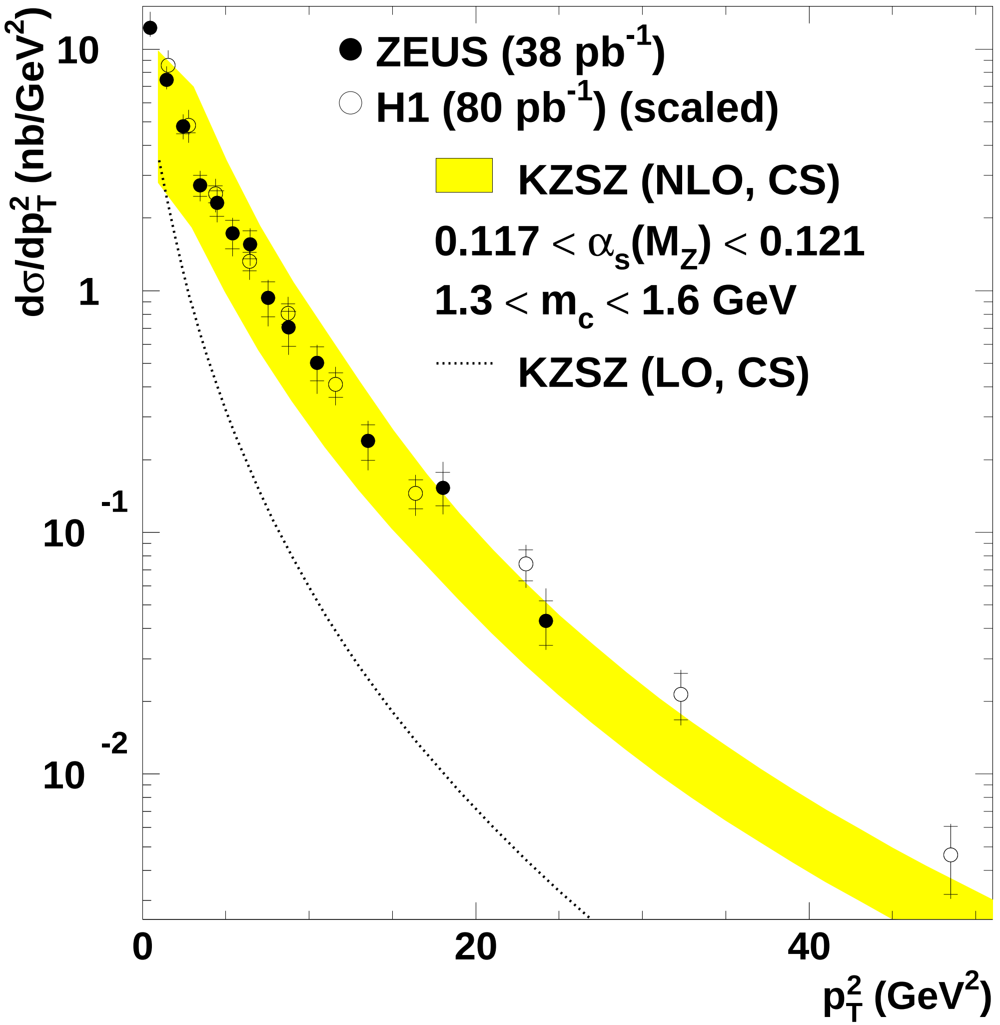
- In NRQCD factorization in LO, R_{χ_c} is dominated by color-octet contributions at large p_T. It is predicted large p_T to be
$$R_{\chi c} = 5/3.$$

- CDF (2007): At large p_T
$$R_{\chi c} \approx 0.75.$$

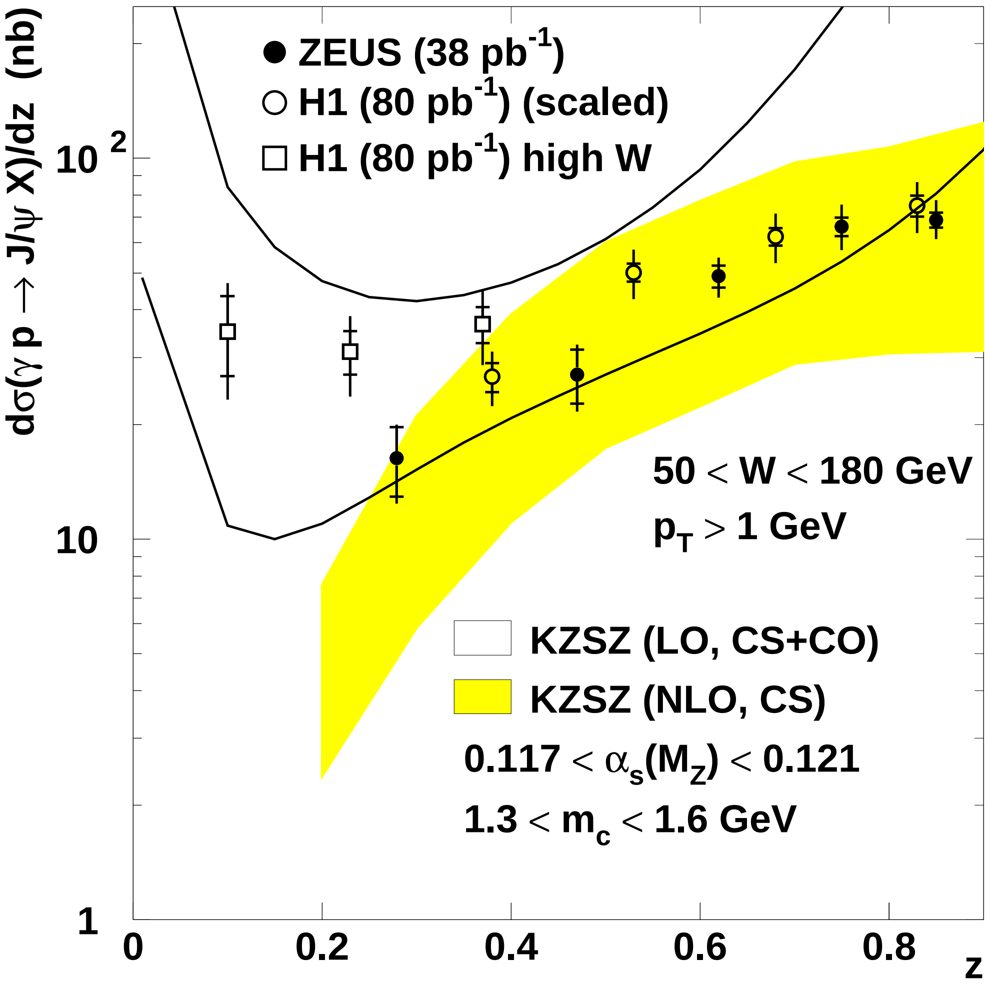
- Ma, Wang, Chao (2010): NLO corrections to R_{χ_c} are large at large p_T.
  - Using the NLO results, they are able to fit the p_T distribution of R_{χ_c}, using plausible values of the color-octet NRQCD matrix elements.
  - The fit predicts that feeddown from the χ_{cJ} states to the J/ψ, may be as large as 30% of the J/ψ rate at $p_T = 20$ GeV.
  - The predicted χ_{cJ} fraction increases with increasing p_T, while the χ_{cJ} fraction measured by CDF (1997) in Run I decreases with increasing p_T.

Inelastic J/ψ Photoproduction Cross Section at HERA

- It had been believed that NLO color-singlet calculations leave little room for a color-octet contribution.



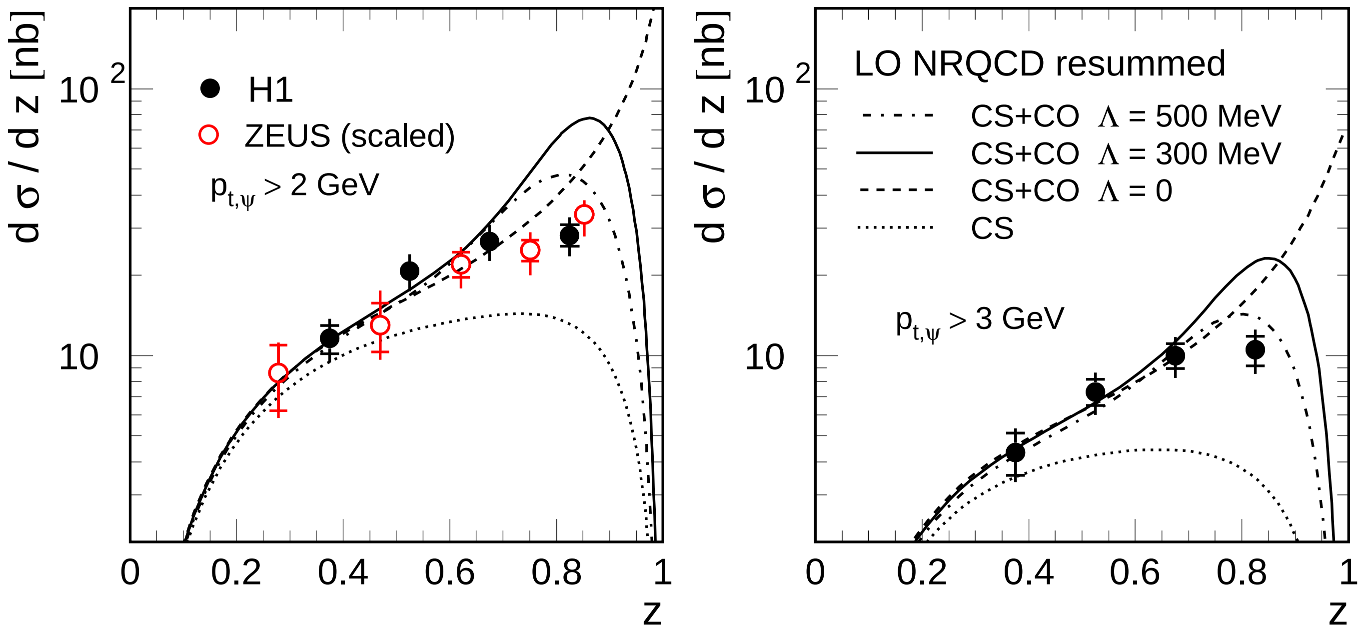
- NLO corrections increase the color-singlet contribution substantially. (Krämer, Zunft, Steegborn, Zerwas (1994); Krämer (1995))
- NLO corrections include $\gamma + g \to (c\bar{c}) + gg$.
- At large p_T, this process goes as $\alpha_s^3 m_c^2/p_T^6$, instead of $\alpha_s^2 m_c^4/p_T^8$.
- Are NNLO corrections also important?



- LO NRQCD calculations by Cacciari, Krämer (1996); Amundson, Fleming, Maksymyk (1996); Ko, Lee, Song (1996); Kniehl, Krämer (1997).
- NLO color-singlet calculations by (Krämer, Zunft, Steegborn, Zerwas (1994); Krämer (1995))

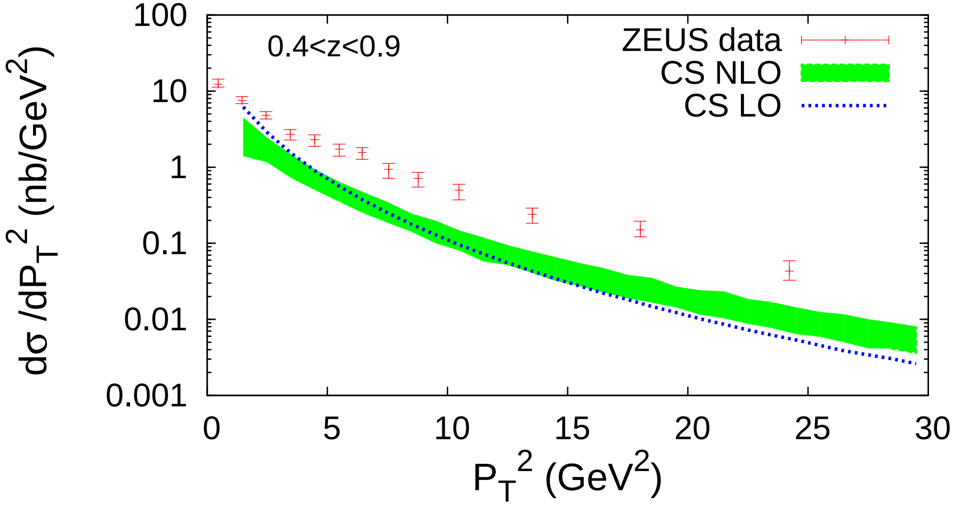
- The α_s expansion breaks down near $z = 1$.
  - Resummation of multiple soft-gluon emission is needed. (Beneke, Schuler, Wolf (2000))

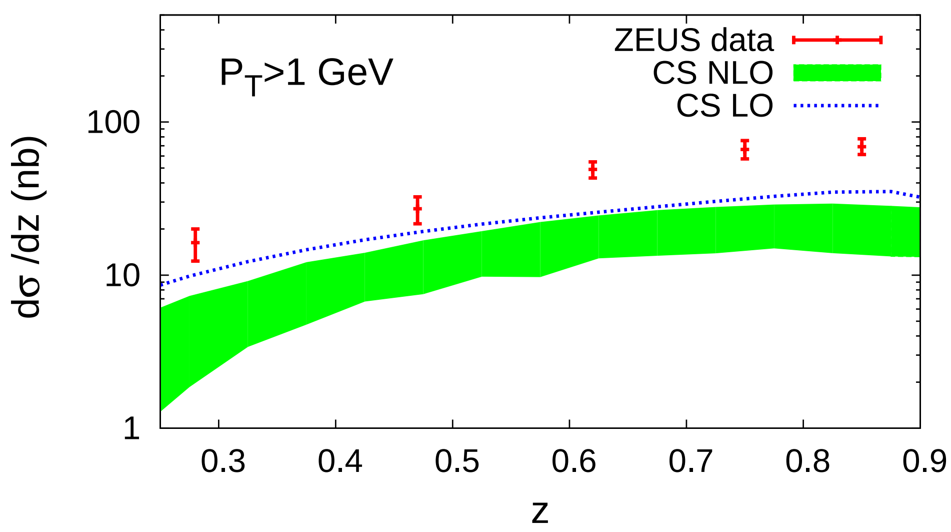
- The v expansion breaks down near $z = 1$.
  - Resummation of the v expansion leads to a nonperturbative shape function. (Beneke, Rothstein, Wise (1997))
- Inclusion of a shape function with reasonable choices of parameters leads to an improved fit.



Recent Theoretical Developments

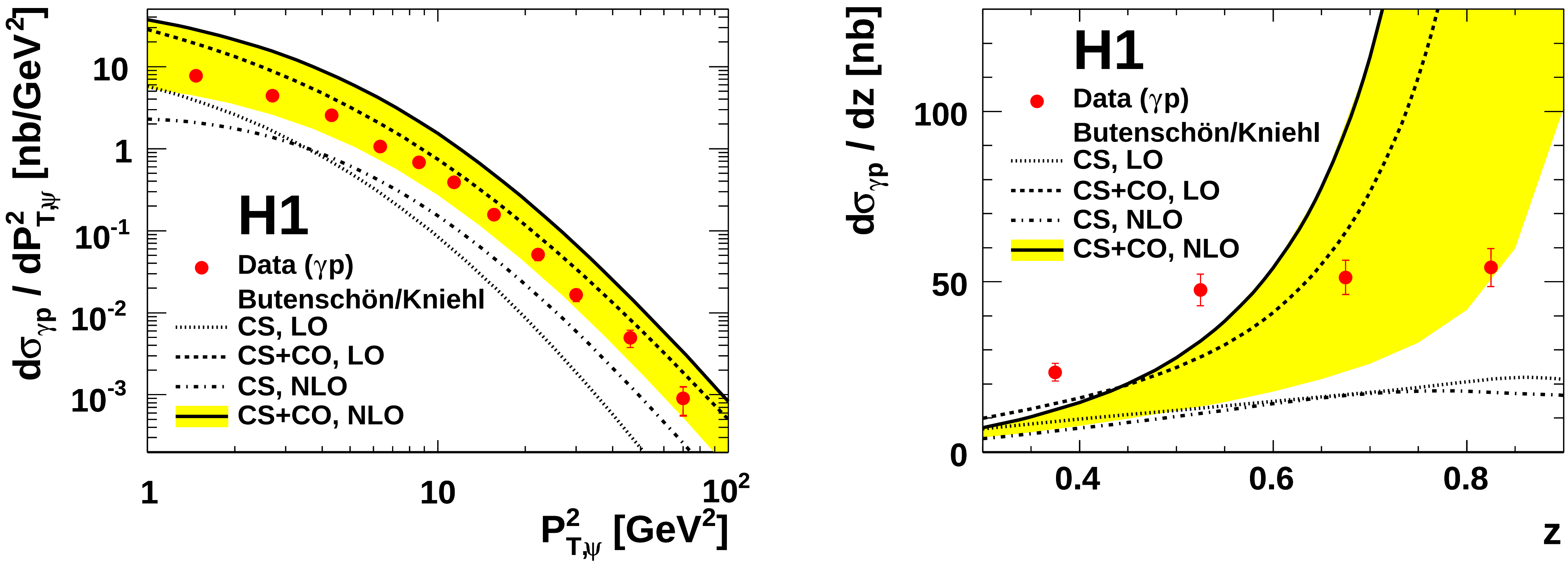
- Artoisenet, Campbell, Maltoni, Tramontano (2009): A new calculation of NLO color-singlet contribution
  - Confirms the analytic results of previous calculations.
  - But a more reasonable choice of renormalization/factorization scale ($\sqrt{4m_c^2 + p_T^2}$ instead of $m_c/\sqrt{2}$) yields much smaller numerical results for cross sections.





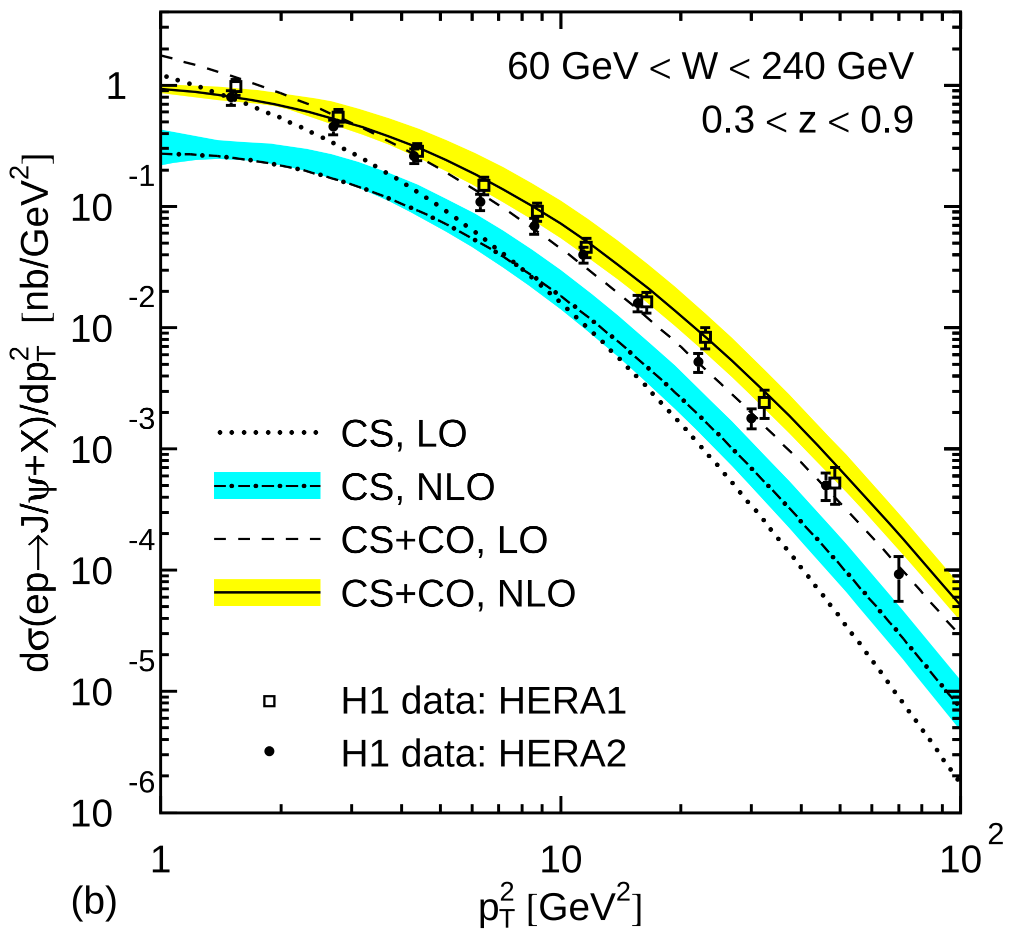
- Leaves room for a color-octet contribution.

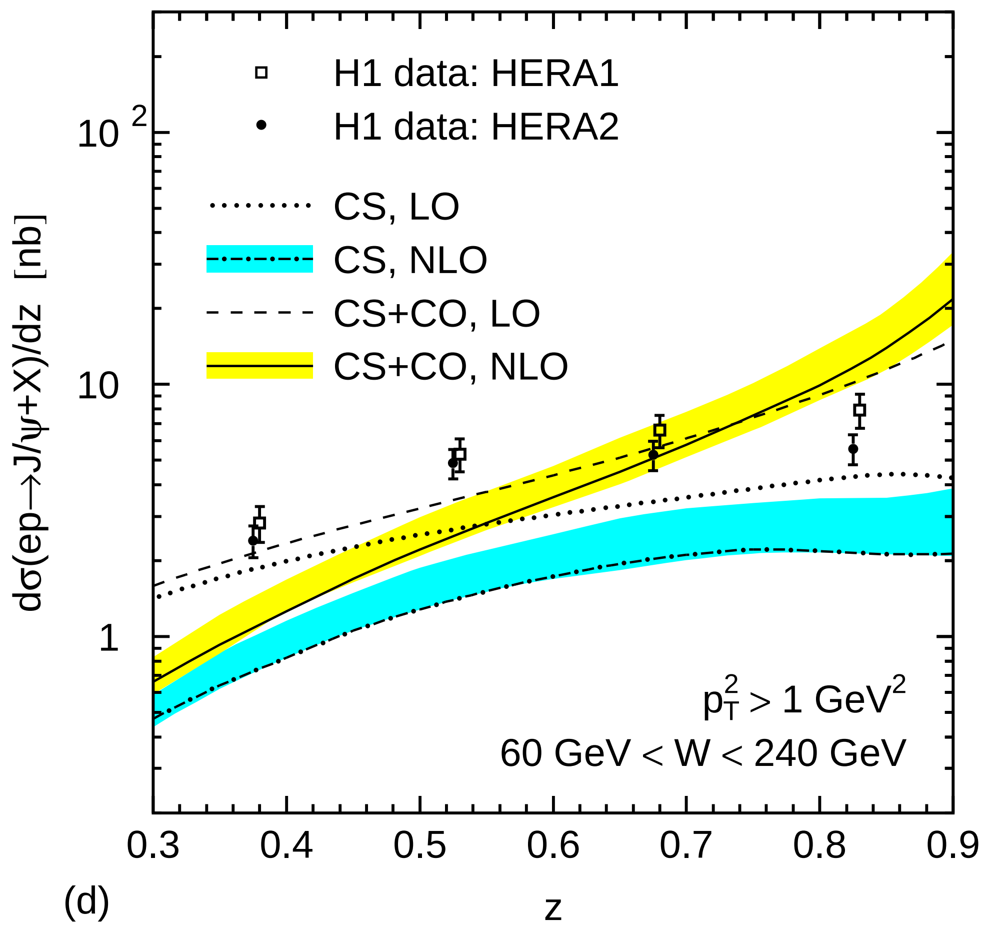
- Butenschön and Kniehl (2009) have carried out a complete NLO calculation of photoproduction in NRQCD factorization, including both color-octet and color-singlet contributions.
- The color-octet matrix elements are from the fit to the Tevatron data by Kniehl and Kramer (1998), which uses LO plus approximate NLO calculations.



- The comparison of the H1 (2009) data with the Butenschön and Kniehl calculation strongly favors NRQCD factorization over the color-singlet model.
- The discrepancy at low z is probably due to the omission of resolved contributions.
- The discrepancy at high z would probably be fixed by resummation.

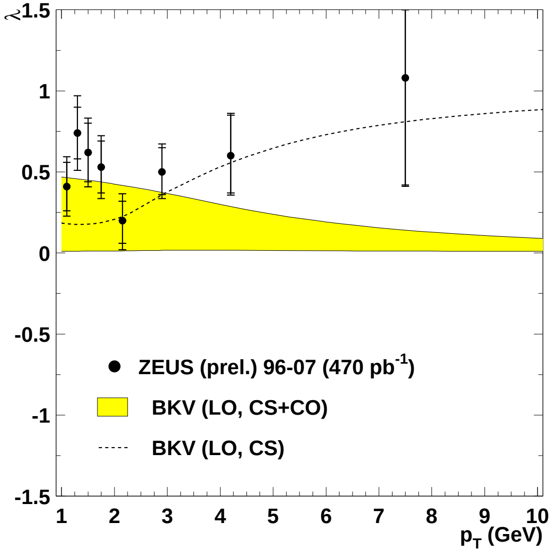
- The simultaneous NLO fit to the CDF (2005) and H1 (2009) data yields slightly higher central values and smaller uncertainties.

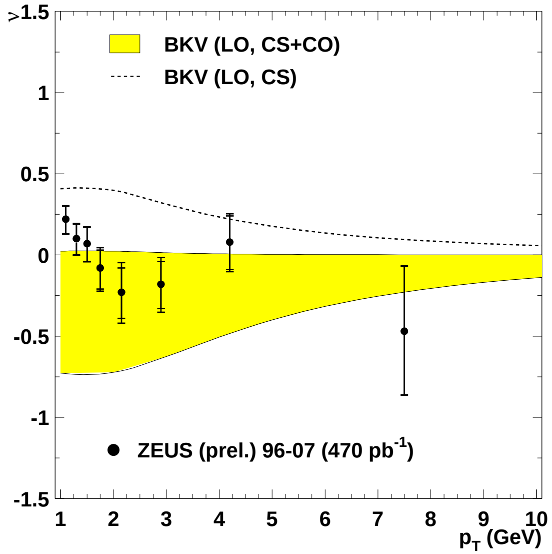




Polarization in Inelastic J/ψ Photoproduction at HERA

$$\frac{d\Gamma(J/\psi \to l^+ l^-)}{d\Omega} \propto 1 + \lambda \cos^2\theta + \mu \sin(2\theta)\cos\phi + \frac{\nu}{2}\sin^2\theta\cos(2\phi)$$



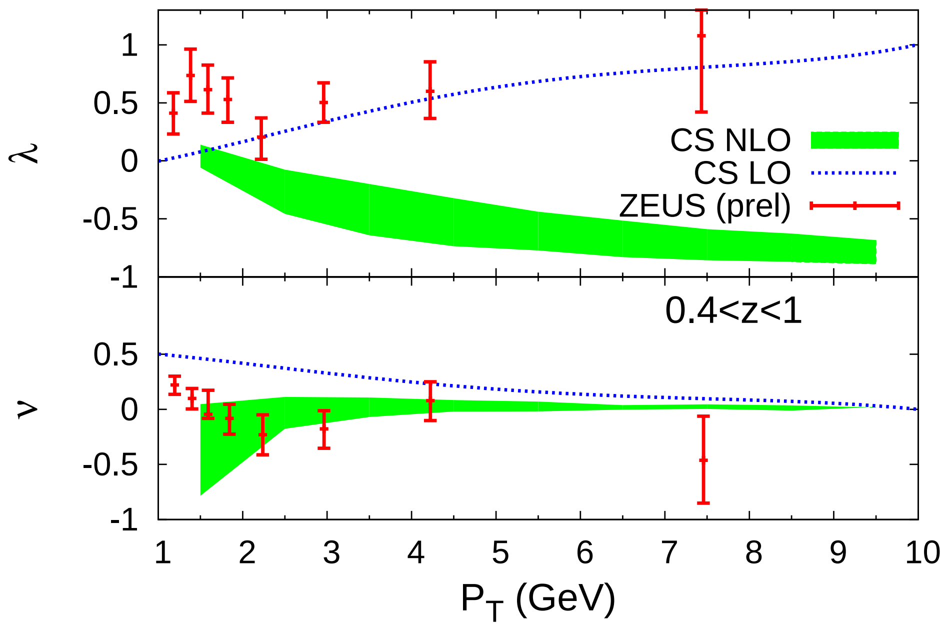


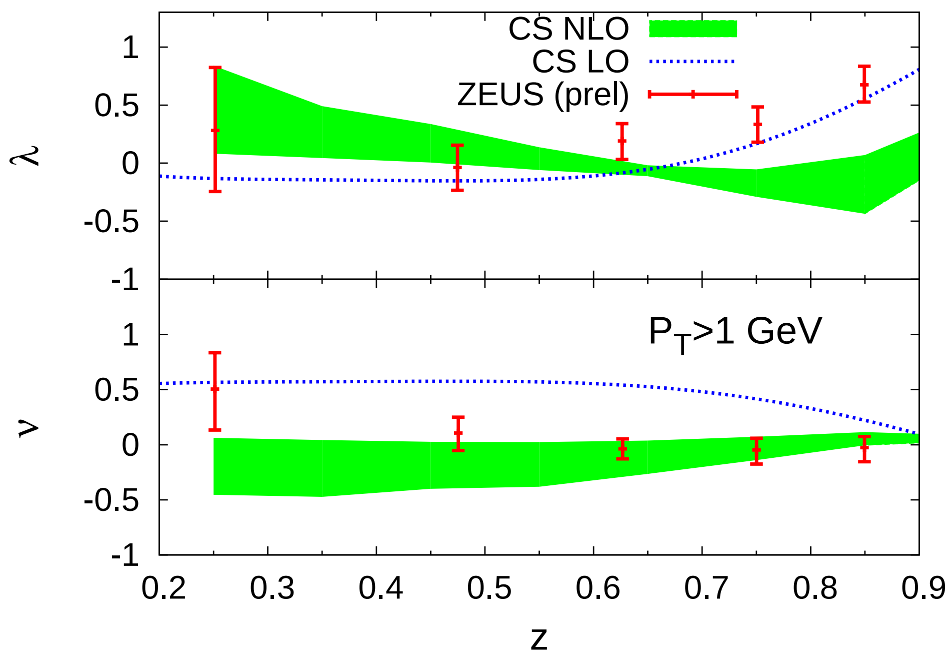
- LO NRQCD calculation by Beneke, Krämer, Vanttinen (1997) using Beneke-Krämer (1996) matrix elements from fits to the Tevatron data.
- θ and ϕ are the polar and azimuthal angles of the l^+ 3-momentum with respect to the helicity frame, which is defined in the J/ψ rest frame.
  - The z axis is the direction of the J/ψ momentum in the lab frame.
  - ϕ is defined with respect to the scattering plane.
- The data for λ at high p_T slightly favor the color-singlet prediction.
- The data for ν at high p_T slightly favor the color-singlet+color-octet prediction.

Recent Theoretical Developments

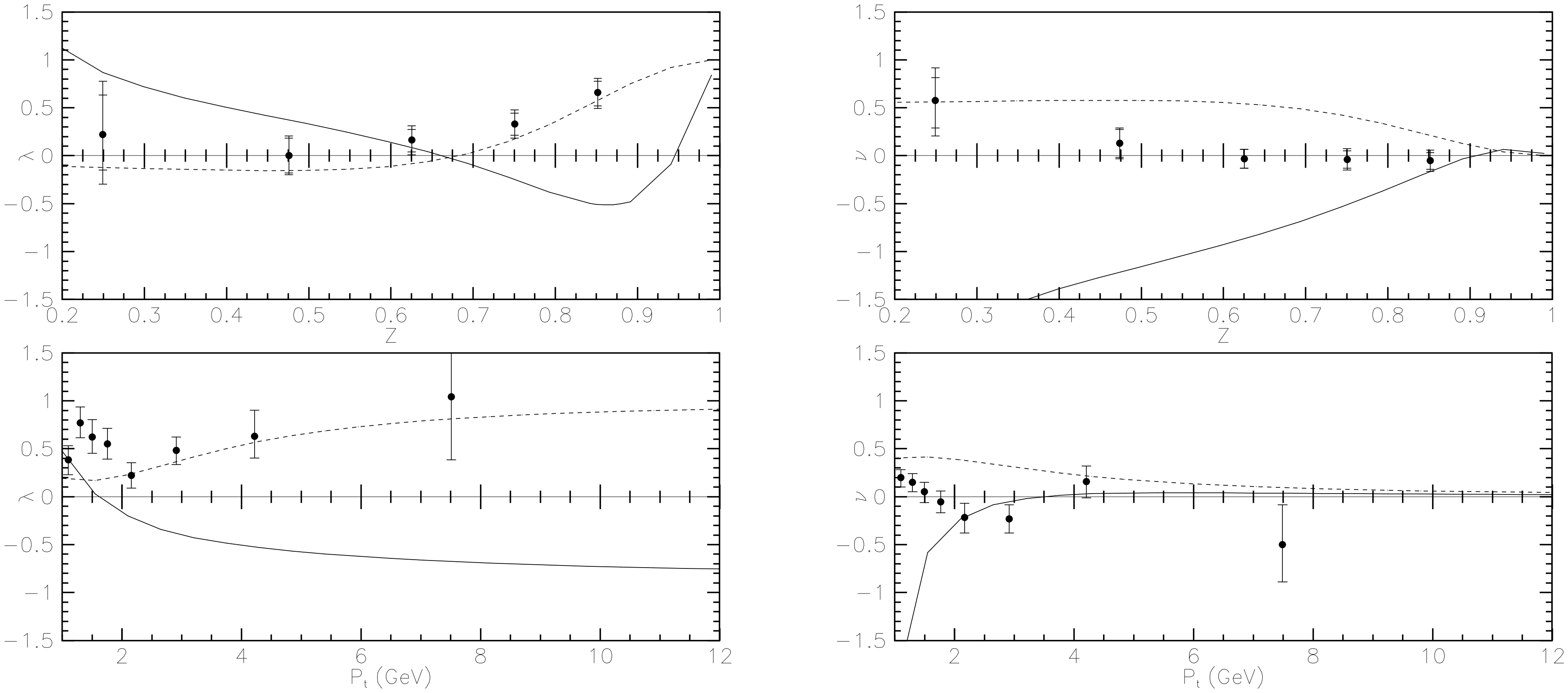
- NLO calculations have a significant effect on the color-singlet polarization predictions.
  - LO calculation: Beneke, Krämer, Vanttinen (1997).
  - NLO calculations: Artoisenet, Campbell, Maltoni, Tramontano (2009); Chang, Li, Wang (2009).

Artoisenet, Campbell, Maltoni, Tramontano:





Chang, Li, Wang:



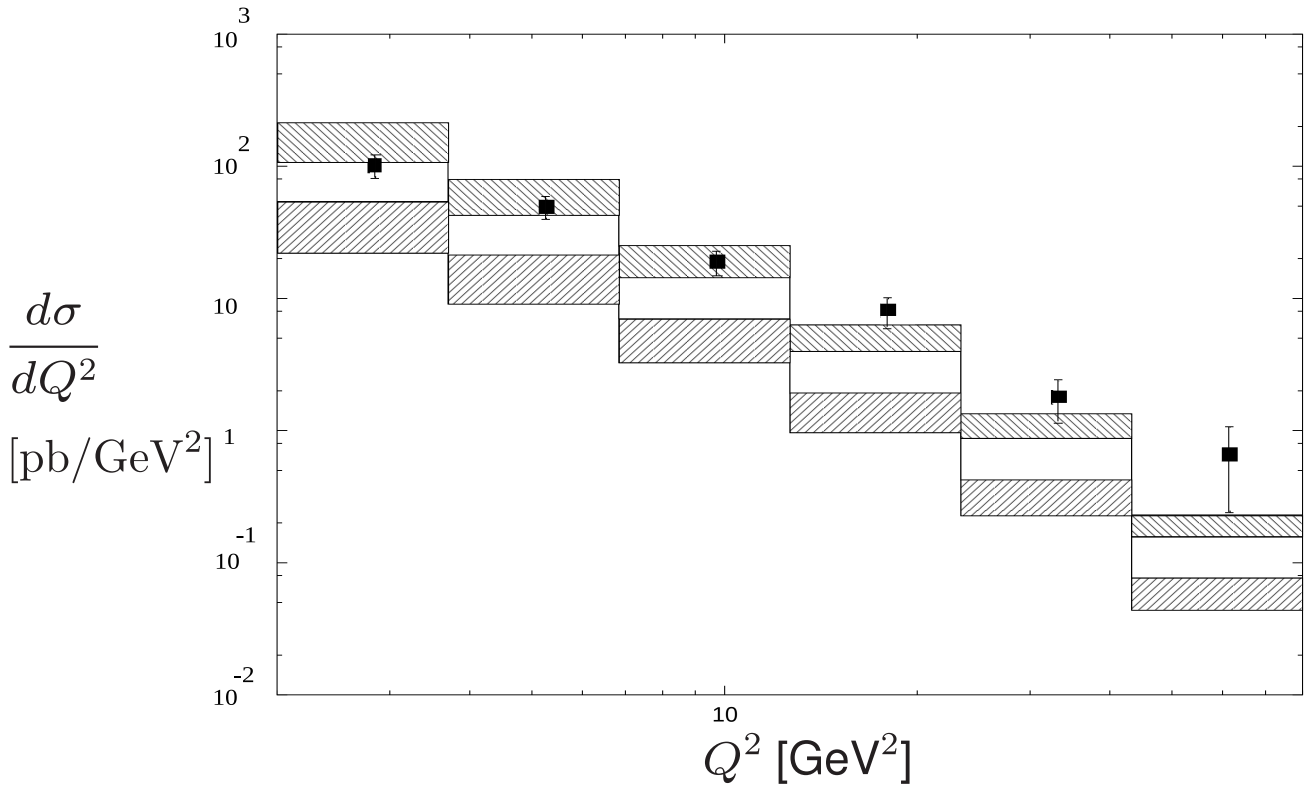
Dotted lines are LO. Solid lines are NLO.

- The NLO calculations agree, except that Chang, Li, and Wang find a more negative result for ν vs. z than Artoisenet, Campbell, Maltoni, Tramontano.
- The NLO color-singlet contribution alone cannot explain the data for λ at large p_T or large z.
- Would a color-octet contribution bring theory into agreement with data?

J/ψ Production in DIS at HERA

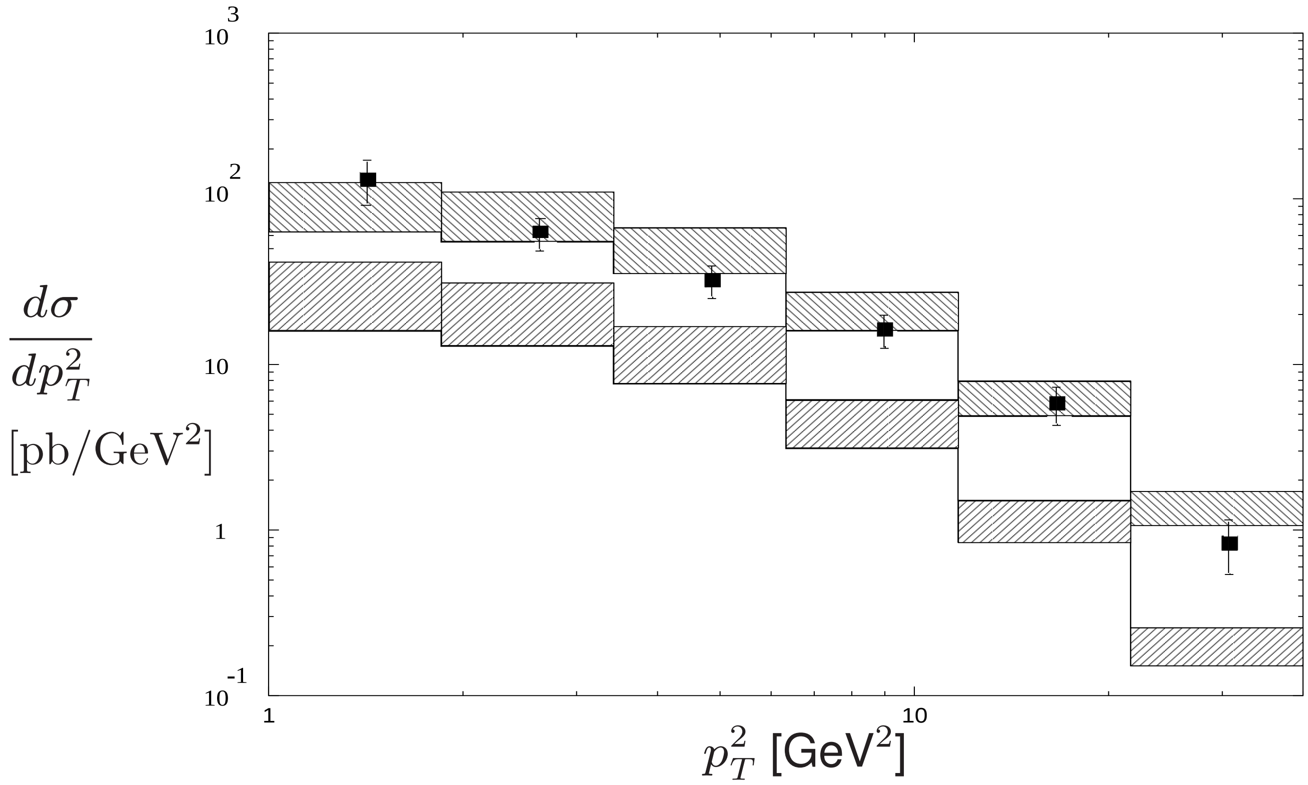
- Note that NLO calculations are not yet available for this process.
- The NRQCD prediction (Kniehl, Zwirner (2001)) uses matrix elements extracted from the Tevatron data (Braaten-Kniehl-Lee (1999)).

- The H1 (1998) data plotted as a function of Q^2 favor the NRQCD prediction over the color-singlet-model prediction. (Q is the virtual-photon momentum.)



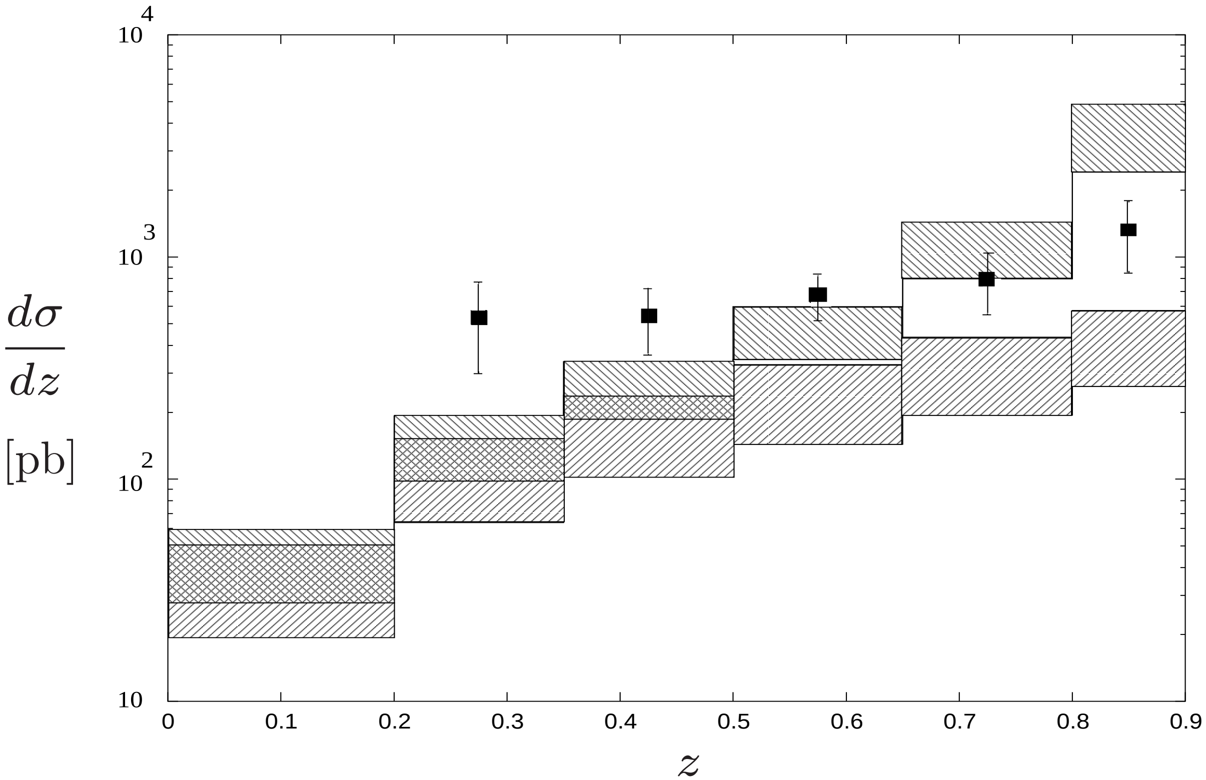
H1 data vs. leading-order NRQCD (upper) and Color-Singlet Model (lower).

- The H1 (1998) data plotted as a function of P_T^2 favor the NRQCD prediction over the color-singlet-model prediction. p_T is the transverse momentum of the J/ψ.



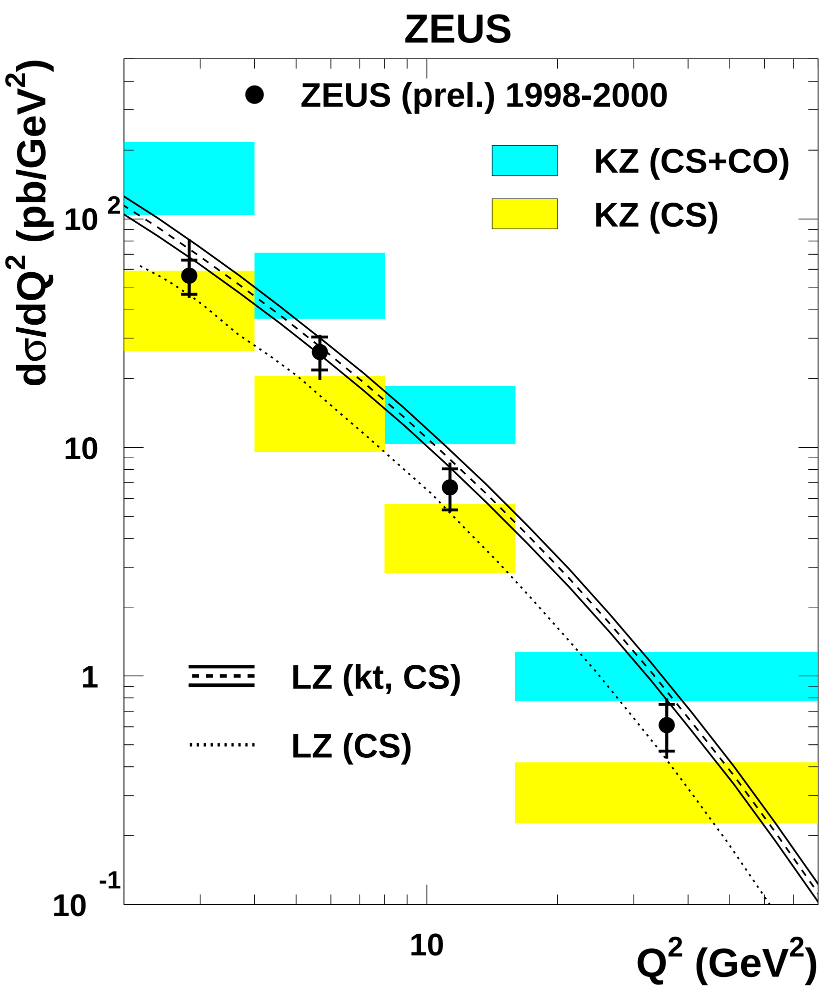
H1 data vs. leading-order NRQCD (upper) and Color-Singlet Model (lower).

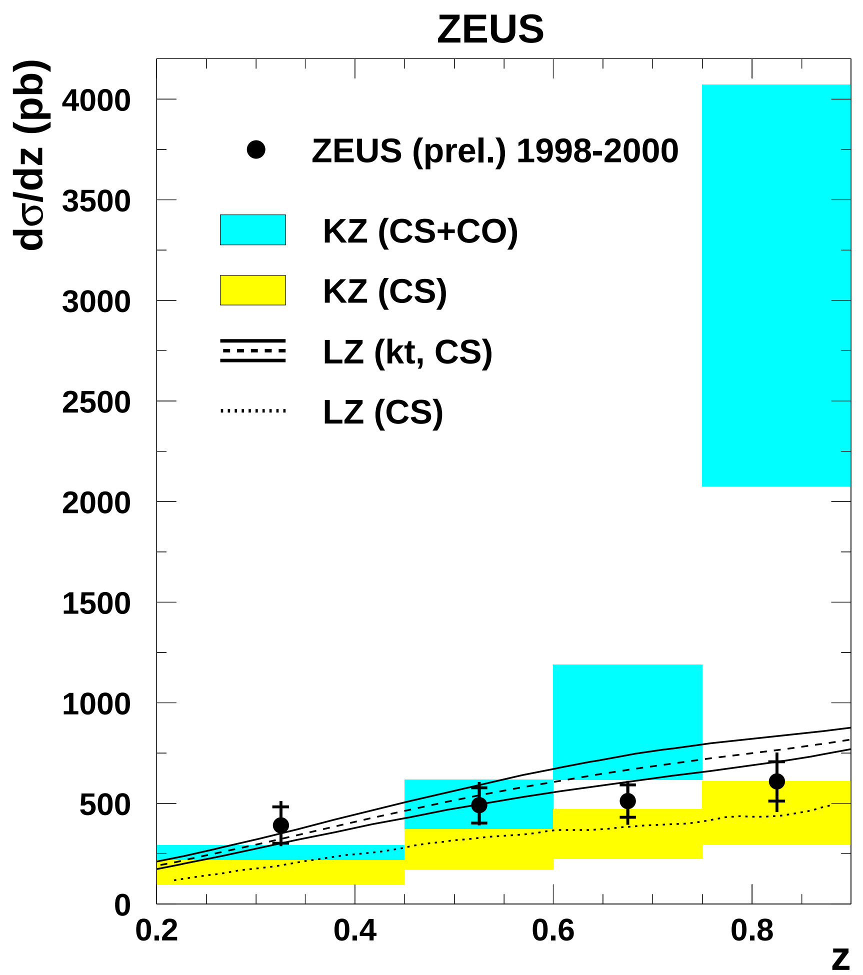
- The H1 (1998) data plotted as a function of z do not agree well with either the NRQCD prediction or the color-singlet-model prediction. (z is the energy fraction of the J/ψ.)
- The data do not show the expected color-octet rise at $z = 1$.
  Resummations of the α_s and v expansions are needed.



H1 data vs. leading-order NRQCD (upper) and Color-Singlet Model (lower).

- The ZEUS data are systematically lower than the H1 (1998) data and agree less well with the NRQCD prediction (but have larger error bars).
- The data plotted as a function of z do not show the expected color-octet rise at $z = 1$.



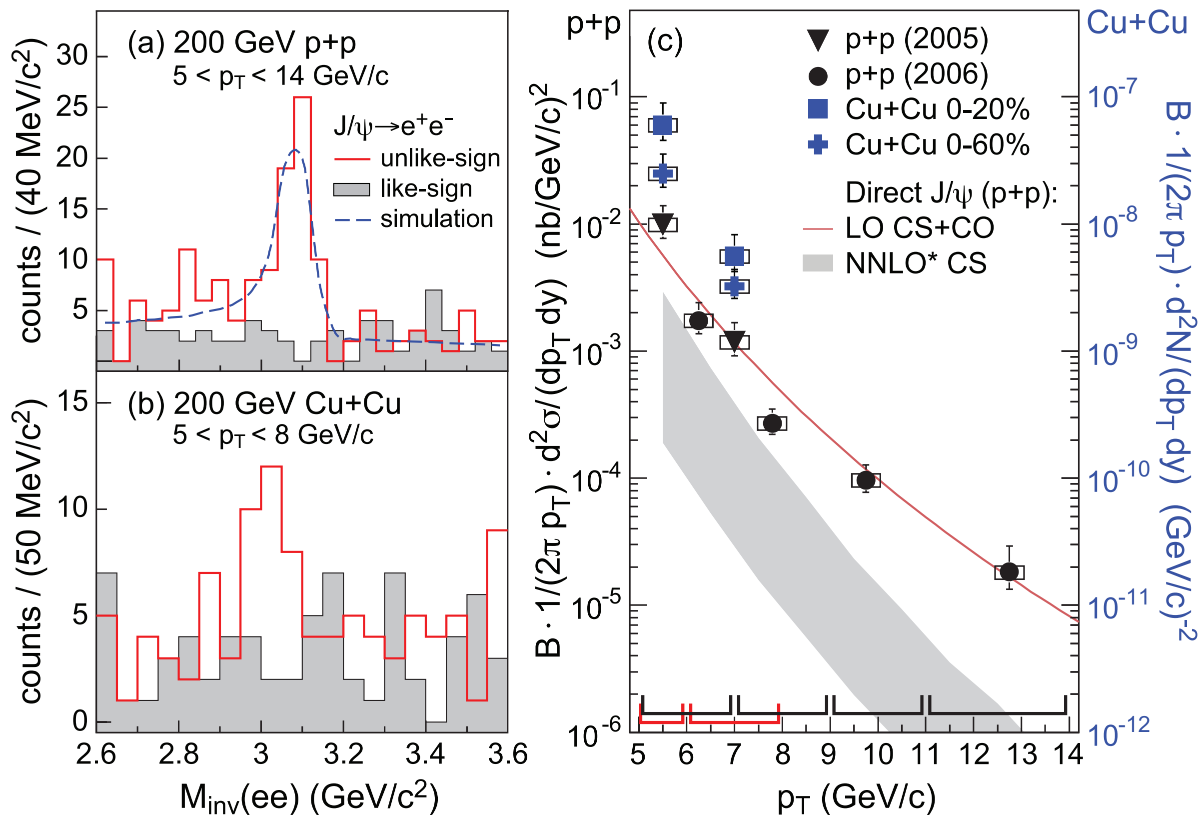


Curves from A.V. Lipatov and N.P. Zotov.

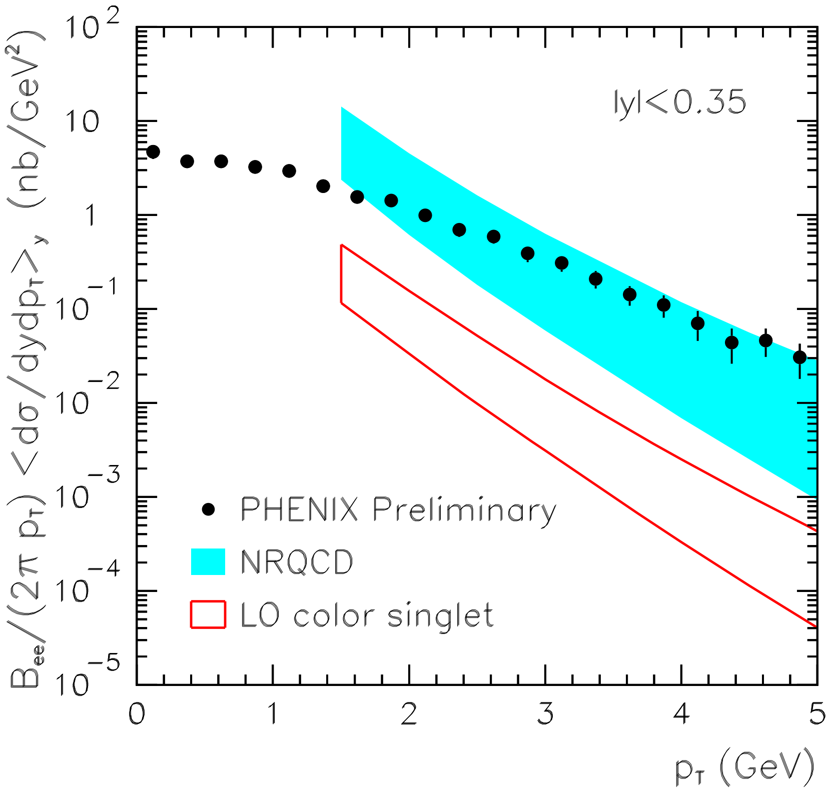
J/ψ Production at RHIC

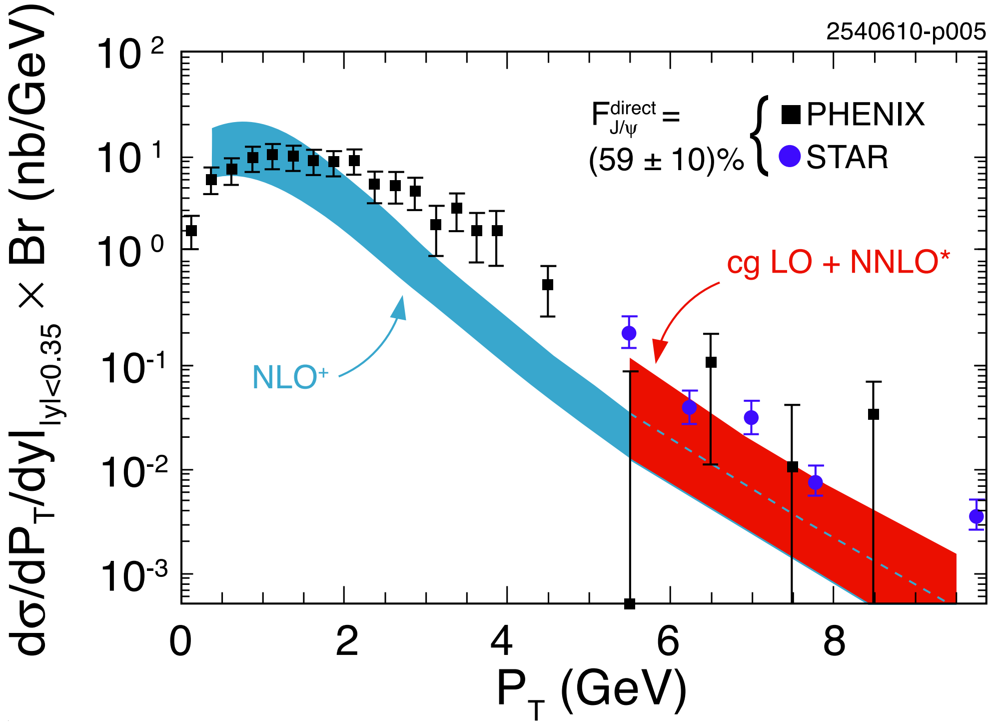
Production Cross Section

- The STAR collaboration has measured the J/ψ p_T distributions in $p + p$ and Cu+Cu collisions:



- Nayak, Liu, Cooper (2003): An LO NRQCD calculation (color-singlet plus color-octet contributions) fits the data well.
  - Does not include feeddown from $\psi(2S)$, χ_c, or B decays. (Estimated to be a factor 1.5.)

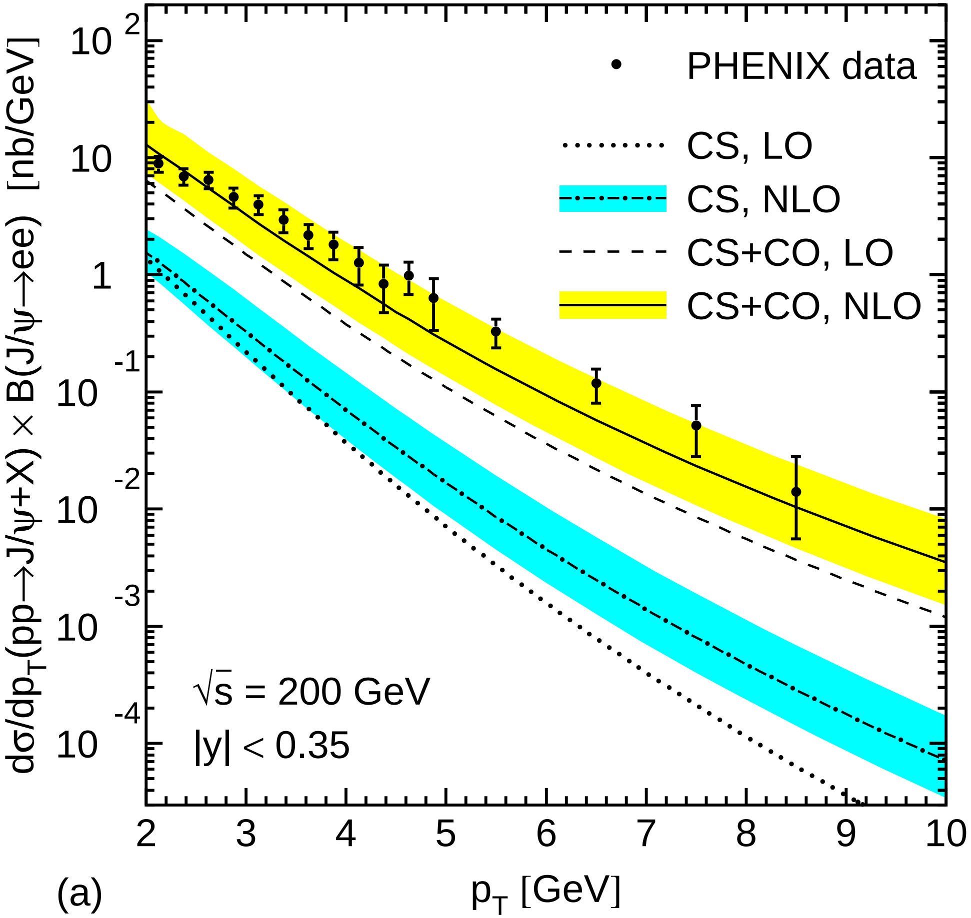




- Chung, Yu, Kim, Lee (2010): An LO NRQCD calculation, including feeddown, fits the PHENIX (2009) data well.
- The color-singlet contribution is well below the PHENIX (2009) data.

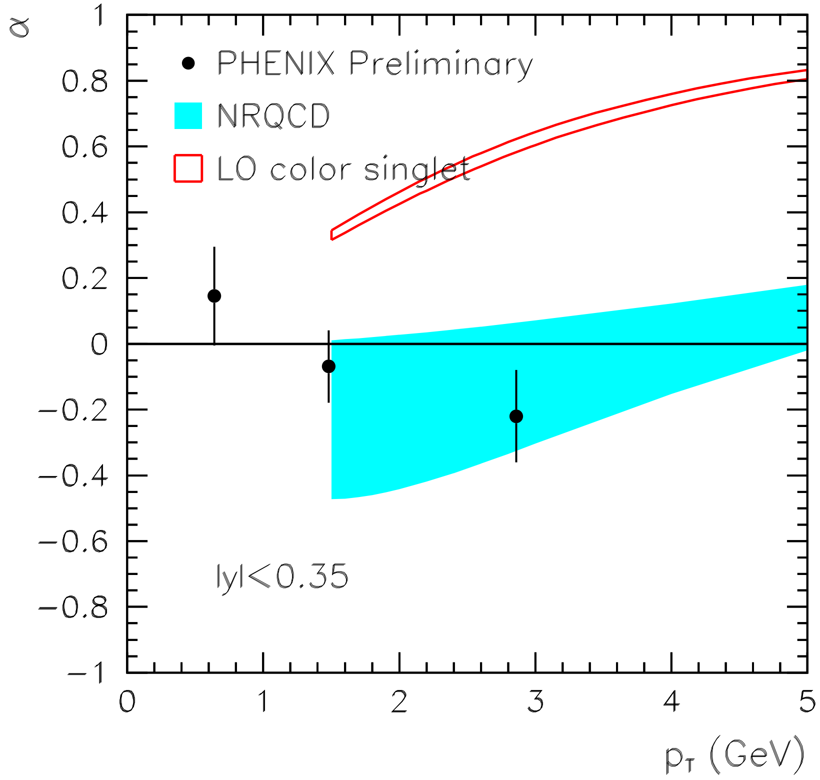
- Lansberg (2010): NLO corrections increase the size of the color-singlet contribution substantially.
- The color-singlet contribution still lies below the PHENIX (2006) and STAR (2009) data at large p_T.

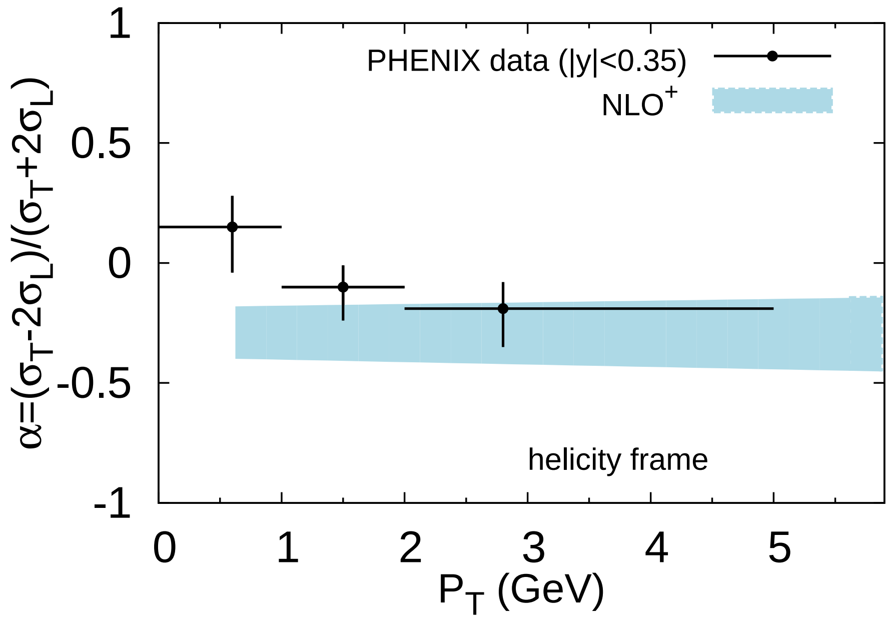
- The NLO NRQCD calculation of Kniehl and Butenschön (2010), with NRQCD matrix elements fit to the CDF (2005) and H1 (2002, 2010) data, agrees well with the PHENIX data:



- The NLO color-singlet contribution is well below the PHENIX data.

Polarization

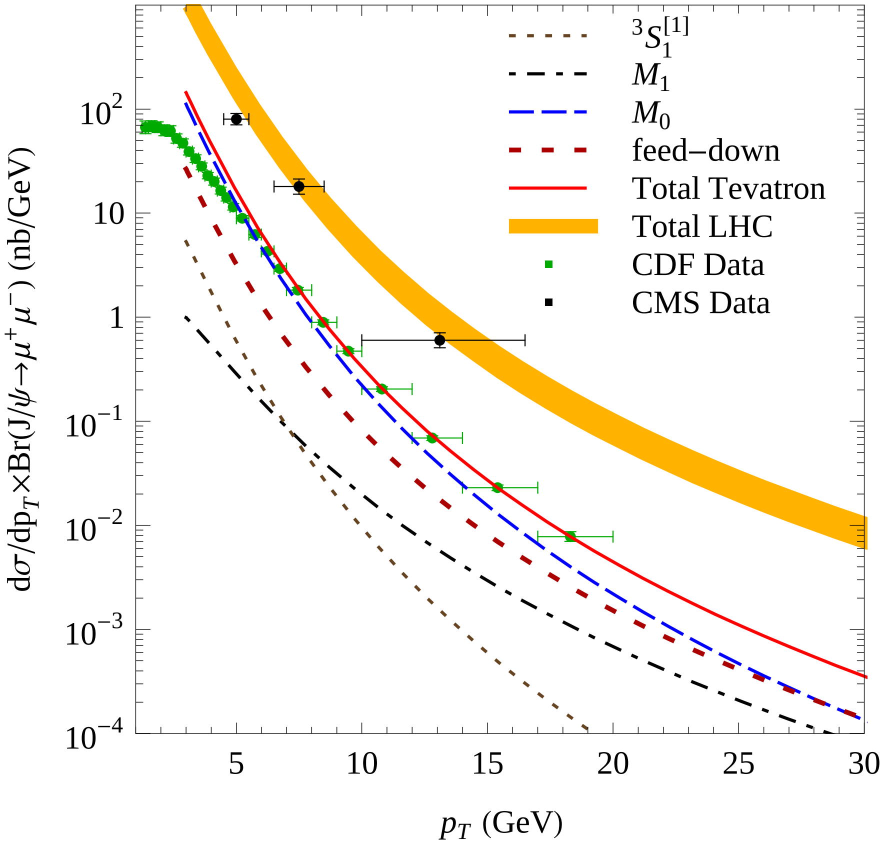




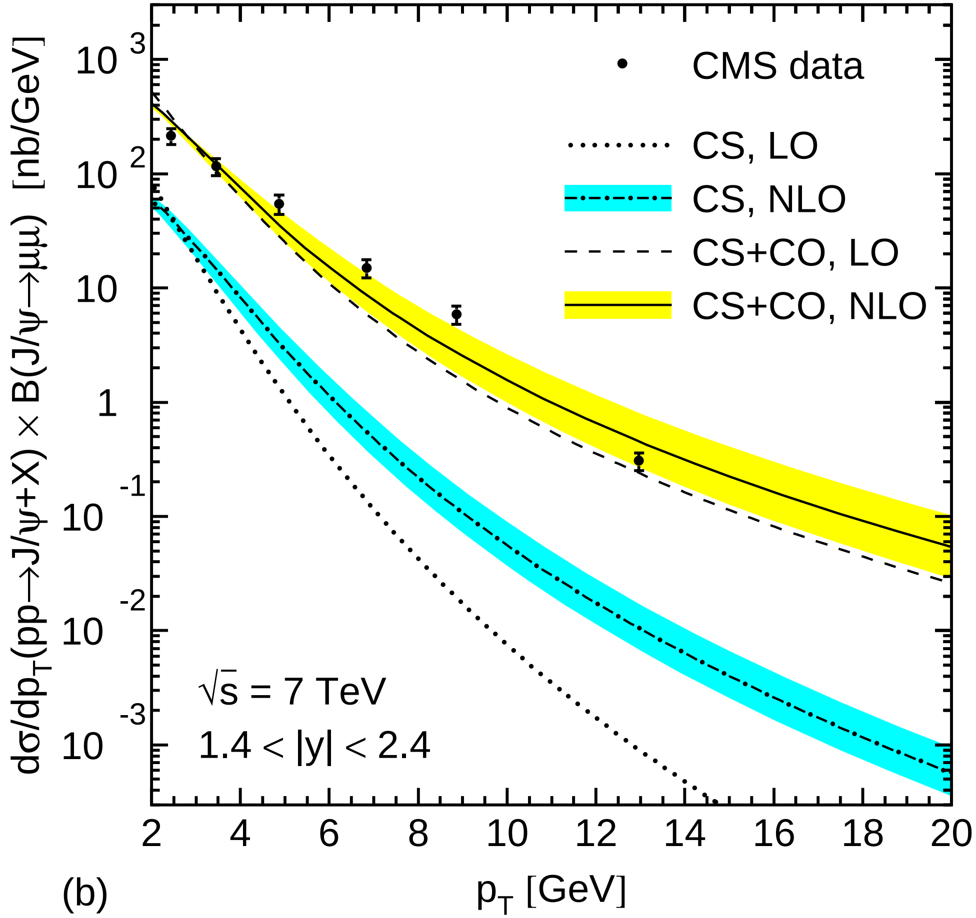
- Chung, Yu, Kim, Lee (2010): An LO NRQCD calculation, including feeddown, fits the PHENIX (2009) data well.
- The color-singlet contribution in LO is in poor agreement with the PHENIX data.
- Lansberg (2010): The NLO corrections to the color-singlet contribution make it virtually indistinguishable from the color-octet contribution.

J/ψ Production at the LHC

- The NLO predictions of Ma, Wang and Chao (2010) and Kniehl and Butenschön (2010) agree well with the CMS (2010) data:



Ma, Wang, and Chao (2010)
$|y| < 2.4$



Kniehl and Butenschön (2010)
$1.4 < |y| < 2.4$

- Only the calculation of Ma, Wang, and Chao (2010) includes the effects of feeddown.

- Atlas, LHCb, and ALICE have also measured J/ψ production, but, so far, $d\sigma/dp_T$ for prompt production is not available.
- We look forward to comparisons between NLO theory and measurements from these experiments.

Summary

- The effective field theory NRQCD is a convenient formalism for separating physics at the scale of the heavy-quark mass from physics at the scale of quarkonium bound-state dynamics.
- The NRQCD factorization approach provides a systematic method for calculating quarkonium decay and production rates as double expansions in powers of α_s and v.
- NRQCD factorization for inclusive production rates has not yet been established.
- There is now a proof of NRQCD factorization for exclusive charmonium production in B-meson decays and e^+e^- annihilation.
- NRQCD factorization has enjoyed a number of successes:
  - inclusive P-wave quarkonium decays,
  - quarkonium production at the Tevatron,
  - inelastic J/ψ photoproduction at HERA,
  - exclusive double-charmonium production at Belle and BABAR,
  - quarkonium production in DIS at HERA,
  - J/ψ production at RHIC,
  - $\gamma\gamma \to J/\psi + X$ at LEP,
  - J/ψ production at the LHC.

- Theory and experiment are no longer in obvious conflict for $\sigma(e^+e^- \to J/\psi + c\bar{c} + X)/\sigma(e^+e^- \to J/\psi + X)$ at Belle, but only if the color-octet matrix elements are much smaller than previously believed.
- The situation is still ambiguous for polarization in inelastic J/ψ photoproduction at HERA.
- The disagreement between theory and experiment for quarkonium polarization at the Tevatron presents a serious challenge.
- In a number of cases, corrections of higher order in α_s and v and resummations near kinematic endpoints have proven to be essential to obtain reliable theoretical predictions.
- NLO calculations are the new state of the art for quarkonium production cross sections and polarizations.
- NNLO calculations of color-singlet quarkonium production at the Tevatron may reduce the importance of color-octet contributions and could possibly resolve some puzzles, but theoretical uncertainties are not yet under good control.
- Measurements of direct-production cross sections and polarizations would be of great help in understanding production mechanisms.