Strong and radiative decays of X(3872) as a hadronic molecule with a negative parity

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Outline

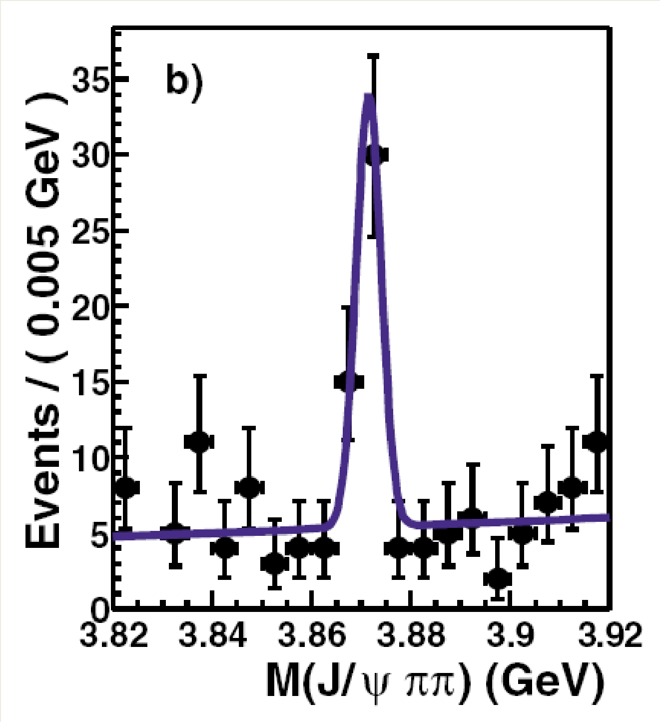
I. Introduction
II. Hadronic Molecular Structure of X(3872) with $J^{PC} = 2^{-+}$
III. Numerical Results and Discussions
IV. Conclusions

I. Introduction

X(3872) was first observed by Belle Collaboration:

Exclusive $B^{\pm} \to K^{\pm}\pi^{+}\pi^{-}J/\psi$ Decays

S. K. Choi et al. [Belle Collaboration], Phys. Rev. Lett.91, 262001 (2003) [arXiv: hep-ex/0309032].



Confirmed by the CDF, theD0 and the BaBar Collaborations.
For more experimental discussion, the following talk from Belle Collabortion.

Quantum numbers of X(3872):

$$X(3872) \to \gamma J/\psi$$

B. Aubert *et al*. [BABAR Collaboration],Phys. Rev. D 74, 071101 (2006) .
K. Abe *et al*. [Belle Collaboration], arXiv:hep-ex/0505037.

$$X(3872) \to \gamma\psi(2S)$$

B.Aubert *et al*. [BABAR Collaboration], Phys. Rev. Lett.102, 132001 (2009).

→ **C = +**

Analyze the invariant $\pi^+ \pi^-$ mass distribution and the total angular distribution of the $J/\psi\ \pi^+ \pi^-$ decay mode.

A.Abulencia et al. [CDF Collaboration], Phys. Rev. Lett. 96, 102002 (2006).
A.Abulencia *et al*. [CDF Collaboration], Phys.Rev. Lett. 98, 132002 (2007) .

→ **Possible spin-parity $J^P = 1^+, 2^-$**

Quantum numbers of X(3872): $J^P = 1^{++}, 2^{-+}$

Possible structures for X(3872)

- **Charmonium ?**

  S.Dubynskiy and M.B.Voloshin, Phys. Rev. D 77, 014013 (2008).
  Y.Jia, W.L.Sang and J.Xu, arXiv:1007.4541 [hep-ph].

- **Hybrid state**

  F.E.Close and S.Godfrey, Phys.Lett. B 574, 210 (2003) ;B.A.Li, Phys.Lett. B 605, 306 (2005)

- **Tetraquark state**

  J.Vijande, F.Fernandez and A.Valcarce, Int. J. Mod. Phys. A 20, 702 (2005)

- **Chrmonium-molecule mixing state**

  E.Braaten and M.Kusunoki, Phys. Rev. D 72, 054022 (2005)

  Y.b.Dong, A.Faessler, T.Gutsche and V.E.Lyubovitskij, Phys. Rev. D 77, 094013 (2008)
  S.Takeuchi,Prog.Theor.Phys.Suppl.168, 107 (2007).

- **Molecular state**

  N.A.Tornqvist , Y.b.Dong, M.B.Voloshin, E.S.Swanson, Yong-Liang Ma, ……
  The talk by Vijande this morning and Oset in the following section.

Here we regard X(3872) with $J^{PC} = 2^{-+}$ as DD* molecule.

Quantities used in the parameter fitting

$$\frac{\mathcal{B}(X \to \gamma\psi(2S))}{\mathcal{B}(X \to \gamma J/\psi)} = 3.4 \pm 1.4 \qquad \text{BaBar}$$

$$\frac{\mathcal{B}(X(3872) \to J/\psi\pi^+\pi^-\pi^0)}{\mathcal{B}(X(3872) \to J/\psi\pi^+\pi^-)} = 1.0 \pm 0.4(\text{stat.}) \pm 0.3(\text{syst.}) \qquad \text{Belle}$$

$$\frac{\mathcal{B}(X(3872) \to \gamma J/\psi)}{\mathcal{B}(X(3872) \to J/\psi\pi^+\pi^-)} = 0.14 \pm 0.05 \qquad \text{Belle}$$

$$0.33 \pm 0.12 \qquad \text{BaBar}$$

$$\frac{\mathcal{B}(X(3872) \to \gamma\psi(2S))}{\mathcal{B}(X(3872) \to J/\psi\pi^+\pi^-)} = 1.1 \pm 0.4, \qquad \text{BaBar}$$

$$m_X = m_{D^{*0}} + m_{D^0} - \Delta E$$

we take $\Delta E = 0.5, 1.0$ and 1.5 MeV

II. Hadronic Molecular Structure of X(3872) with $J^{PC} = 2^{-+}$

We write the wave function of $X(3872)$ as

$$|X(3872)\rangle = \frac{\cos\theta}{\sqrt{2}}|D^0\bar{D}^{*0}\rangle + \frac{\sin\theta}{\sqrt{2}}|D^+D^{*-}\rangle + \text{C.c.},$$

In isospin eigenstate

$$|X(3872)\rangle = \cos\phi|X(3872)\rangle_{I=0} + \sin\phi|X(3872)\rangle_{I=1},$$

with $\cos\theta = (\cos\phi + \sin\phi)/\sqrt{2}$ and $\sin\theta = (\cos\phi - \sin\phi)/\sqrt{2}$

$$|X(3872)\rangle_{I=0} = \frac{1}{2}\Big(|D^0\bar{D}^{*0}\rangle + |D^+D^{*-}\rangle\Big) + \text{C.c.},$$

$$|X(3872)\rangle_{I=1} = \frac{1}{2}\Big(|D^0\bar{D}^{*0}\rangle - |D^+D^{*-}\rangle\Big) + \text{C.c.}$$

$$\mathcal{L}_X = \frac{i}{\sqrt{2}} X^{\mu\nu}(x) \int dx_1 dx_2 \Phi_X((x_1 - x_2)^2)\delta(x - \omega_v x_1 - \omega_p x_2)$$

$$\times \Big\{ g_X^N \Big[C_{\mu\nu}^N(x_1, x_2) + C_{\nu\mu}^N(x_1, x_2) - \frac{1}{4} g_{\mu\nu} C_\alpha^{N;\alpha}(x_1, x_2) \Big]$$

$$+ g_X^C \Big[C_{\mu\nu}^C(x_1, x_2) + C_{\nu\mu}^C(x_1, x_2) - \frac{1}{4} g_{\mu\nu} C_\alpha^{C;\alpha}(x_1, x_2) \Big] \Big\},$$

$$\omega_v = \frac{m_{D^*}}{m_{D^*} + m_D}, \quad \omega_p = \frac{m_D}{m_{D^*} + m_D},$$

$$\Phi_X(y^2) = \int \frac{d^4p}{(2\pi)^4} \tilde{\Phi}_X(p^2) e^{-ip\cdot y},$$

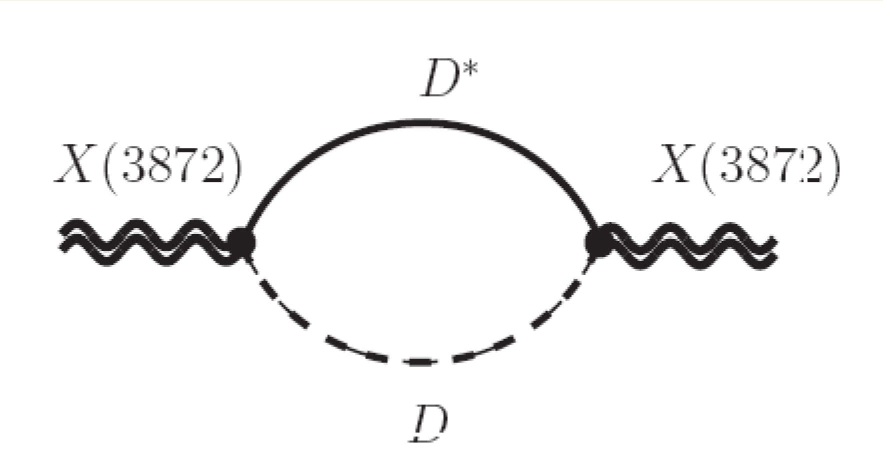
$$\tilde{\Phi}_X(p^2) = \exp(p^2/\Lambda_X^2)$$

$$C_{\mu\nu}^N(x_1, x_2) = \bar{D}_\mu^{*0}(x_1)\partial_\nu D^0(x_2) + D_\nu^{*0}(x_1)\partial_\mu \bar{D}^0(x_2)$$

Compositeness condition : $Z_X = 0$

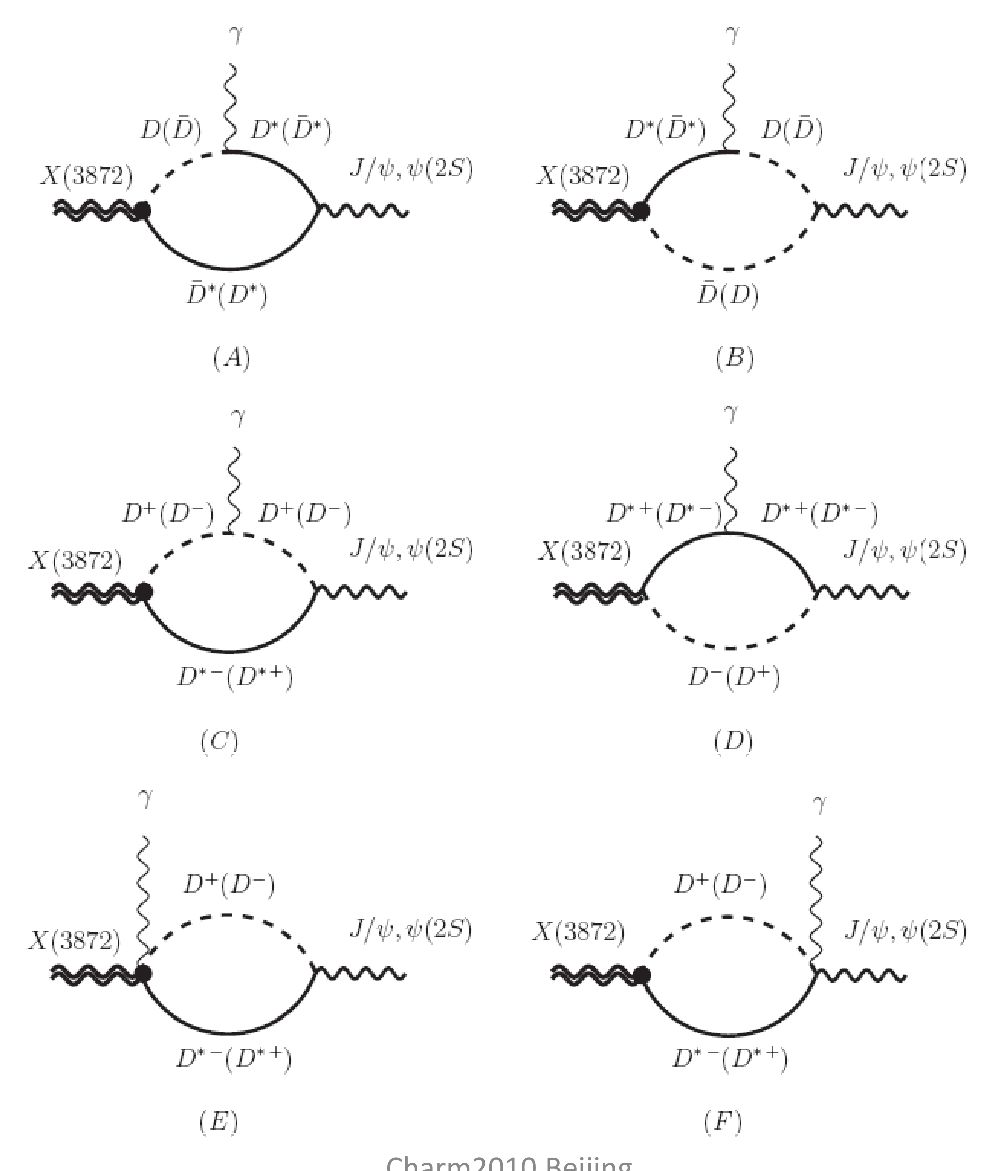
$$Z_X = 1 - g_X^2 \frac{d}{dp^2}\Sigma_X(p^2)\Big|_{p^2=m_X^2}$$

$$\Pi_X^{\mu\nu;\alpha\beta}(p^2) = \frac{1}{2}(g^{\mu\alpha}g^{\nu\beta} + g^{\mu\beta}g^{\nu\alpha})g_X^2\Sigma_X(p^2) + \cdots$$

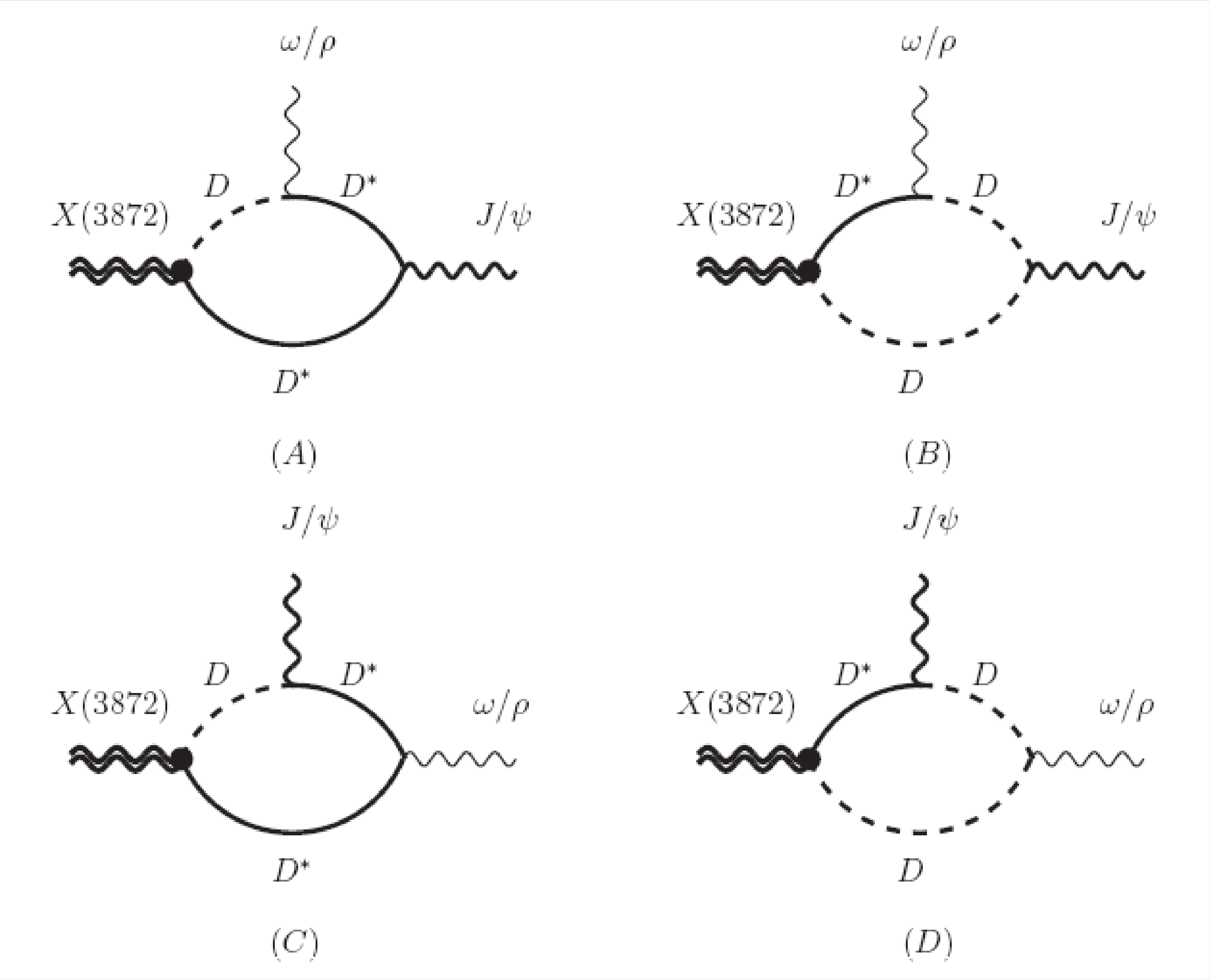


For other interaction, we use the phenomenological Lagrangian

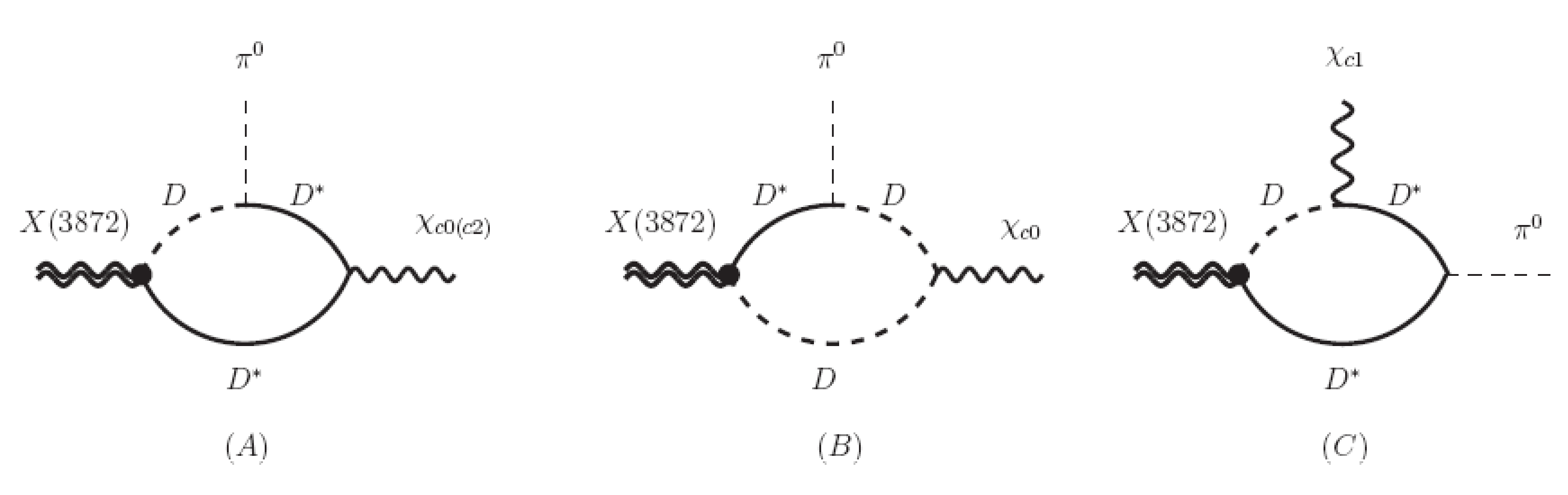
A. Radiative $X(3872) \to \gamma J/\psi$ and $X(3872) \to \gamma\psi(2S)$ decays



B. Strong $X(3872) \to J/\psi h$ decays



C. Strong $X(3872) \to \chi_{cJ}\pi^0$ decays



III. Numerical Results and Discussions

TABLE I: Fitted parameters θ and Λ_x and the corresponding branching ratio.

ϕ	Λ_x (GeV)	ΔE(MeV)	$\frac{\mathcal{B}(X\to\gamma\psi(2S))}{\mathcal{B}(X\to\gamma J/\psi)}$	$\frac{\mathcal{B}(X\to J/\psi\pi^+\pi^-\pi^0)}{\mathcal{B}(X\to J/\psi\pi^+\pi^-)}$	$\frac{\mathcal{B}(X\to\gamma J/\psi)}{\mathcal{B}(X\to J/\psi\pi^+\pi^-)}$	$\frac{\mathcal{B}(X\to\gamma\psi(2S))}{\mathcal{B}(X\to J/\psi\pi^+\pi^-)}$
12°	2.6	0.5	2.089	0.483	0.218	0.455
		1.0	2.101	0.461	0.214	0.450
		1.5	2.113	0.439	0.210	0.444
11°	2.7	0.5	2.628	0.591	0.211	0.555
		1.0	2.640	0.566	0.208	0.549
		1.5	2.647	0.542	0.206	0.545
10°	$2.7-2.8$	0.5	$2.158-3.007$	$0.748-0.740$	$0.320-0.233$	$0.690-0.700$
		1.0	$2.165-3.014$	$0.716-0.705$	$0.316-0.229$	$0.684-0.690$
		1.5	$2.168-3.017$	$0.685-0.673$	$0.312-0.226$	$0.676-0.682$
		0.5	$2.430-3.486$	$0.964-1.035$	$0.369-0.283$	$0.897-0.987$
9°	$2.8-3.0$	1.0	$2.433-3.504$	$0.917-0.971$	$0.363-0.274$	$0.883-0.960$
		1.5	$2.433-3.516$	$0.874-0.914$	$0.358-0.266$	$0.871-0.935$
data			3.4 ± 1.4 [8]	$1.0\pm0.4\pm0.3$ [7]	0.14 ± 0.05 [7]	1.1 ± 0.4 [8]
					0.33 ± 0.12 [8]	

TABLE II: Effective coupling constants g_X^N and g_X^C from the fitted parameters ϕ and Λ_X.

ϕ	Λ_X(GeV)	ΔE(MeV)	g_X^N	g_X^C
		0.5	16.29 − 15.84	12.64 − 12.40
9°	2.8 − 3.0	1.0	16.38 − 15.94	12.68 − 12.44
		1.5	16.46 − 16.04	12.72 − 12.48

$$g_{XD^0D^{*0}} = 7.13 \text{ GeV}\sqrt{Z_{D^0D^{*0}}} \quad \text{(nonlocal)},$$

$$g_{XD^\pm D^{*\mp}} = 11.39 \text{ GeV}\sqrt{Z_{D^\pm D^{*\mp}}} \quad \text{(nonlocal)},$$

Y.Dong, *et al.* , Phys. Rev. D79, 094013 (2009).

Our results for the coupling constants are bigger than the corresponding ones in axial vector case.

Interpretation: In our case, X(3872) has quantum numbers $\mathbf{J^{PC}=2^{-+}}$ so the coupling between X(3872) and its constituents DD* is in P-wave. Compared with the case that X(3872) has quantum numbers $\mathbf{J^{PC}=1^{++}}$, it needs stronger attractive interaction to compensate the repulsive interaction induced by angular momentum.

Our result of the small mixing angle implies that the isospin singlet component is dominant.

Wave function:

$$|X(3872)\rangle = 0.988 \times |X(3872)\rangle_{I=0} + 0.156 \times |X(3872)\rangle_{I=1}.$$

~~From this one might think that decay $X \to J/\psi\pi^+\pi^-$ would be strongly suppressed compared to $X \to J/\psi\pi^+\pi^-\pi^0$ decay.~~

$$iM^{\alpha}(X \to J/\psi V) = iG_{X\psi V}\,\epsilon^{\mu\sigma\alpha\beta}p_{\sigma}q^{\nu}\epsilon_{\mu\nu}(p)\epsilon^{*}_{\beta}(q_2).$$

$$\frac{\mathcal{B}(X \to J/\psi\pi^+\pi^-\pi^0)}{\mathcal{B}(X \to J/\psi\pi^+\pi^-)} = \frac{|G_{X\psi\omega}|^2}{|G_{X\psi\rho}|^2}\frac{\int d\Pi(X \to J/\psi\pi^+\pi^-\pi^0)}{\int d\Pi(X \to J/\psi\pi^+\pi^-)},$$

≈ 1.0

Consistent with data!

$$\int d\Pi(X \to J/\psi\pi^+\pi^-\pi^0)/\int d\Pi(X \to J/\psi\pi^+\pi^-) \simeq 0.0127$$

$$\frac{G_{X\psi\omega}}{G_{X\psi\rho}} \simeq \frac{g_X^N + g_X^C}{g_X^N - g_X^C}\cdot\frac{g_{DD\omega}}{g_{DD\rho}} = 8.9$$

TABLE III: Fitted parameters ϕ and Λ_x and the corresponding branching ratio related to the data.

ϕ	Λ_x (GeV)	ΔE (MeV)	$\Gamma(X \to \gamma J/\psi)$ (KeV)	$\Gamma(X \to \gamma\psi(2S))$ (KeV)	$\Gamma(X \to J/\psi\pi^+\pi^-\pi^0)$ (KeV)	$\Gamma(X \to J/\psi\pi^+\pi^-)$ (KeV)
		0.5	$2.085 - 1.872$	$5.066 - 6.525$	$5.447 - 6.842$	$5.648 - 6.612$
9°	$2.8 - 3.0$	1.0	$2.082 - 1.864$	$5.065 - 6.533$	$5.253 - 6.606$	$5.730 - 6.802$
		1.5	$2.079 - 1.859$	$5.058 - 6.537$	$5.067 - 6.380$	$5.801 - 6.977$

From this we see the partial widths are of order of KeV.

These partial widths have been computed in the case that X(3872) has positive parity.

- For the strong decays, it was found that the decay widths are around 50KeV which are both about one order larger than our present results.

- But for radiative decays, the results depend on model closely.

1. X(3872) as a bound state of mesons, the radiative decay widths are of order KeV, at the same order as our present results.
2. X(3872) is a mixing state of molecule (without the charged DD* components) and charmonium component, the decay width is found to be of 100-200KeV, much larger than that computed from the hadronic molecule assumption.
3. admixture model of molecule with the charged DD* components and the charmonium components, the similar results as ours were yield.

The inclusion of other components as was done in the case that X(3872) has quantum numbers $J^{PC}=1^{++}$ may change our results.

- Include J/ψω and J/ψρ in the wave function. This may increase the magnitudes of the strong decay widths and the results depend on the probability of J/ψω and J/ψρ in X(3872) even in the case that only the long distance effect is considered. In this sense, if the strong decay widths for tensor X(3872) are observed bigger than our present results, one may conclude that the tensor X(3872) cannot be a pure DD*molecule and other component should be included.

- Regard X(3872) as a mixing state of charmonium and DD*. One may borrow the lesson from the X(3872) with1^{++} case to naively expect that this change of the wave function of X(3872) may increase the magnitude of radiative decay width of X(3872).

- In case other constituent is included, the probabilities of the relevant components should be fitted from data again so the magnitudes of the partial widths should be studied in detail.

The precise measurement of the strong decay widths can provide some clues on the structure of X(3872).

Predictions of the $X(3872) \to \chi_{cJ}\pi$ decays

TABLE IV: Partial widths for $X(3872) \to \chi_{cJ}\pi^0$ decays.

ϕ	Λ_X (GeV)	ΔE (MeV)	$\Gamma(X \to \chi_{c0}\pi^0)$ (KeV)	$\Gamma(X \to \chi_{c1}\pi^0)$ (KeV)	$\Gamma(X \to \chi_{c2}\pi^0)$ (KeV)
		0.5	22.41 – 21.97	0.294 – 0.276	207.5 – 8.335
9°	2.8 – 3.0	1.0	22.60 – 22.40	0.296 – 0.281	211.8 – 8.618
		1.5	22.78 – 22.76	0.296 – 0.285	215.9 – 8.880

From Table. IV, one can yield the following ratio of the partial widths

$$\Gamma(X \to \chi_{c0}\pi^0) : \Gamma(X \to \chi_{c1}\pi^0) \simeq 1 : 0.013,$$

which indicates that, compared with the decay $X \to \chi_{c0}\pi^0$, $X \to \chi_{c1}\pi^0$ is strongly suppressed. From the expressions for the partial widths, using the expressions for the coupling constants, one naively has the ratio

$$\frac{\Gamma(X \to \chi_{c0}\pi^0)}{\Gamma(X \to \chi_{c1}\pi^0)} \simeq \frac{8}{3m_X^2}\left(\frac{4\sqrt{2}}{\sqrt{3}}(m_{D^*} + 3m_D)\right)^2 \simeq \frac{8}{3m_X^2}\left(\frac{8\sqrt{2}}{\sqrt{3}}m_X\right)^2 \simeq 110,$$

Similarly, the introduction of other components may change our numerical results.

- In case of the charmonium component is included, the magnitudes for the partial widths might be changed but the ratio for the partial widths must be kept since the charmonium component is a definitely isospin singlet so it does not contribute to the isospin violating decays .

- The same conclusion can bedrown if J/ψω are constituents of X(3872).

- However complication arises if X(3872) has a J/ψρ component as was done . This is because the component J/ψρ gives a contribution to the isospin violating decays.

From this, in the case $X(3872)$ with $J^{PC} = 2^{-+}$, the strong suppression of the decay $X(3872) \to \chi_{c1}\pi^0$ compared with the decay $X(3872) \to \chi_{c0}\pi^0$ may signal the pure DD^* molecular structure of $X(3872)$.

IV. Conclusion

By regarding the hidden charm state X(3872) as a DD* bound state, we studied its radiative and strong decays in the effective Lagrangian approach.

We find, with an approximate probability of 97.6% isospin singlet component and a 2.4% isospin triplet component, the ratios $\mathcal{B}(X \to \gamma\psi(2S))/\mathcal{B}(X \to \gamma J/\psi)$ by Babar, $\mathcal{B}(X \to J/\psi\pi^+\pi^-\pi^0)/\mathcal{B}(X \to J/\psi\pi^+\pi^-)$ by Belle and $\mathcal{B}(X \to \gamma J/\psi)/\mathcal{B}(X \to J/\psi\pi^+\pi^-)$ by Babar can be explained consistently, but the ratio $\mathcal{B}(X \to \gamma J/\psi)/\mathcal{B}(X \to J/\psi\pi^+\pi^-)$ by Belle cannot be reproduced.

We would like to stress that, in case of $X(3872)$ as a tensor meson, few percent of isospin one component of $X(3872)$ in the wave function can accommodate the large isospin violating partial width since the phase space of the decay $X(3872) \to J/\psi\pi^+\pi^-\pi^0$ is about one percent of that of the decay $X(3872) \to J/\psi\pi^+\pi^-$.

We calculated the strong decays $X(3872) \to J/\psi\pi^+\pi^-\pi^0$, $X(3872) \to J/\psi\pi^+\pi^-$, $X(3872) \to \chi_{cJ}\pi^0$ $(J = 0, 1, 2)$ and radiative decays $X(3872) \to \gamma J/\psi, \gamma\psi(2S)$. For the strong decays $X(3872) \to J/\psi\pi^+\pi^-$ and $X(3872) \to J/\psi\pi^+\pi^-\pi^0$, our results are several KeV which are both around one order smaller than the case $X(3872)$ with $J^{PC} = 1^{++}$. For radiative decays, our results are both of order KeV. They are at the same order as those in the case that $X(3872)$ with $J^{PC} = 1^{++}$ is a bound state of mesons, while much smaller than those obtained in the case that $X(3872)$ is a mixing state of molecule and charmonium component.

For the $X(3872) \to \chi_{cJ}\pi^0$ decays, we found that, compared with the decay $X(3872) \to \chi_{c0}\pi^0$, the $X(3872) \to \chi_{c1}\pi^0$ decay is strongly suppressed. The experimental observation of this suppression may be a signal of the pure DD^* molecular structure of $X(3872)$.

Concerning the lessons from the study of $X(3872)$ with $J^{PC} = 1^{++}$, we naively expect that the inclusion of other constituent in the wave function may change our numerical results of the relevant partial widths. Explicitly, the inclusion of $J/\psi\omega$ or/and $J/\psi\rho$ components may increase some partial widths for the strong decays while the inclusion of $c\bar{c}$ component may increase the partial width for radiative decay. As was noted, our above calculation cannot yield the ratio $\mathcal{B}(X \to \gamma J/\psi)/\mathcal{B}(X \to J/\psi\pi^+\pi^-)$ by Belle although we reproduced this ratio by Babar. If the Belle value is preferred in the future experiment, in the molecular interpretation, we should include other constituents.

In conclusion, from our study, comparison of these values with the future experiments will shed a light on the nature of $X(3872)$.

$$\mathcal{L}_{D^*D^*V} = ig_{D^*D^*\psi_n}[\bar{D}^*_\mu(D^*_\nu \overleftrightarrow{\partial}^\mu \psi^\nu_n) + D^*_\mu(\psi_{n;\nu}\overleftrightarrow{\partial}^\mu \bar{D}^{*\nu}) + \psi_{n;\mu}(\bar{D}^*_\nu \overleftrightarrow{\partial}^\mu D^{*\nu})]$$

$$+ig_{D^*D^*\omega}[\bar{D}^*_\mu(D^*_\nu \overleftrightarrow{\partial}^\mu \omega^\nu) + D^*_\mu(\omega_\nu \overleftrightarrow{\partial}^\mu \bar{D}^{*\nu}) + \omega_\mu(\bar{D}^*_\nu \overleftrightarrow{\partial}^\mu D^{*\nu})]$$

$$+ig_{D^*D^*\rho}[\bar{D}^*_\mu(D^*_\nu \overleftrightarrow{\partial}^\mu \vec{\tau}\cdot\vec{\rho}^\nu) + D^*_\mu(\vec{\tau}\cdot\vec{\rho}_\nu \overleftrightarrow{\partial}^\mu \bar{D}^{*\nu}) + \vec{\tau}\cdot\vec{\rho}_\mu(\bar{D}^*_\nu \overleftrightarrow{\partial}^\mu D^{*\nu})],$$

$$\mathcal{L}_{DDV} = -ig_{DD\psi_n}\psi_{n;\mu}(\bar{D}\overleftrightarrow{\partial}^\mu D) - ig_{DD\omega}\omega_\mu(\bar{D}\overleftrightarrow{\partial}^\mu D) - ig_{DD\rho}\vec{\rho}_\mu\cdot(\bar{D}\vec{\tau}\overleftrightarrow{\partial}^\mu D),$$

$$\mathcal{L}_{D^*DV} = g_{D^*D\psi_n}\epsilon^{\mu\nu\alpha\beta}\,\psi_{n;\mu\nu}\bar{D}^*_{\alpha\beta}D$$

$$+g_{D^*D\omega}\epsilon^{\mu\nu\alpha\beta}\,\omega_{\mu\nu}\bar{D}^*_{\alpha\beta}D + g_{D^*D\rho}\epsilon^{\mu\nu\alpha\beta}\,\rho_{\mu\nu}\cdot\bar{D}^*_{\alpha\beta}\vec{\tau}D + \mathrm{H.c.},$$

$$\mathcal{L}_{D^*D\pi} = \frac{ig_{D^*D\pi}}{2\sqrt{2}}\bar{D}^*_\mu(\vec{\tau}\cdot\vec{\pi}\overleftrightarrow{\partial}^\mu D) + \mathrm{H.c.},$$

$$\mathcal{L}_{D^*D^*\pi} = \frac{g_{D^*D^*\pi}}{2\sqrt{2}}\epsilon^{\mu\nu\alpha\beta}\bar{D}^*_{\mu\nu}\vec{\tau}\cdot\vec{\pi}D^*_{\alpha\beta},$$

$$\mathcal{L}_{\chi_{c0}DD} = g_{\chi_{c0}DD}\chi_{c0}\bar{D}D,$$

$$\mathcal{L}_{\chi_{c0}D^*D^*} = g_{\chi_{c0}D^*D^*}\chi_{c0}\bar{D}^*_\mu D^{*\mu},$$

$$\mathcal{L}_{\chi_{c1}D^*D} = ig_{\chi_{c1}D^*D}\chi_{c1;\mu}\bar{D}D^{*\mu} + \mathrm{H.c.},$$

$$\mathcal{L}_{\chi_{c2}D^*D^*} = g_{\chi_{c2}D^*D^*}\chi_{c2;\mu\nu}[\bar{D}^{*\mu}D^{*\nu} + \bar{D}^{*\nu}D^{*\mu}],$$

The coupling constants $g_{D^*D^*V}$ and g_{DDV} were determined with the help of the VMD, i.e.,

$$g_{D^*D^*\rho} = g_{DD\rho} = -2.52 , \quad g_{D^*D^*\omega} = g_{DD\omega} = -2.84 , \quad g_{D^*D^*\psi} = g_{DD\psi} = 7.64 .$$

Using the VMD and the partial widths $\Gamma(J/\psi \to e^+e^-) = 5.55$ KeV and $\Gamma(\psi(2S) \to e^+e^-) = 2.38$ KeV, we have

$$\frac{g_{D^*D^*\psi'}}{g_{D^*D^*\psi}} = \frac{g_{DD\psi'}}{g_{DD\psi}} = \frac{m_{\psi'}}{m_\psi}\sqrt{\frac{\Gamma(\psi \to e^+e^-)}{\Gamma(\psi' \to e^+e^-)}} = 1.82,$$

which leads to $g_{D^*D^*\psi'} = g_{DD\psi'} = 13.90$.

From the HHChPT including vector mesons within the hidden local symmetry method, one can yield the following relation for the coupling constants

$$g_{D^*D\omega} = g_{D^*D\rho} = \frac{\lambda}{2} g \sqrt{\frac{m_D}{m_{D^*}}},$$

where g is the universal coupling constant introduced in the hidden local symmetry method with $g = 5.8 \pm 0.91$. And, λ gives the coupling of the light vector meson with the heavy states and $\lambda = -0.41\ \mathrm{GeV}^{-1}$. Finally, we get $g_{D^*D\omega} = g_{D^*D\rho} = -1.23\ \mathrm{GeV}^{-1}$.

We fix the coupling constant $g_{D^*D\pi}$ from the partial width for the decay of $D^* \to D\pi$ which leads to $g_{D^*D\pi} = 17.9$. $g_{D^*D^*\pi}$ is related to $g_{D^*D\pi}$ via HHChPT, i.e.,

$$g_{D^*D^*\pi} = \frac{g_{D^*D\pi}}{4\sqrt{m_D m_{D^*}}} = 2.31 \text{ GeV}^{-1}.$$

From the HHChPT for the interaction of charmonium with open charm mesons one can yield the following relations for the coupling constants

$$\begin{aligned}
g_{D^*D\psi_n} &= -\frac{1}{2} g_{DD\psi_n} \frac{\sqrt{m_{D^*}}}{m_{\psi_n}\sqrt{m_D}}, \\
g_{\chi_{c0}DD} &= 3\frac{m_D}{m_{D^*}} g_{\chi_{c0}D^*D^*} = -2\sqrt{3} g_1 m_D \sqrt{m_{\chi_{c0}}}, \\
g_{\chi_{c1}D^*D} &= \sqrt{2} g_1 \sqrt{m_{\chi_{c1}} m_D m_{D^*}}, \\
g_{\chi_{c2}D^*D^*} &= 2 g_1 m_{D^*} \sqrt{m_{\chi_{c2}}},
\end{aligned}$$

with the expression for g_1 as

$$g_1 = -\sqrt{\frac{m_{\chi_{c0}}}{3}} \frac{1}{f_{\chi_{c0}}},$$

where $f_{\chi_{c0}}$ is defined via the relation $\langle 0|\bar{c}c|\chi_{c0}(p)\rangle = f_{\chi_{c0}} m_{\chi_{c0}}$ and the QCD sum rules yield $f_{\chi_{c0}} = 510$. So that we have $g_1 = -2.09 \text{ GeV}^{-1/2}$.

$$\mathcal{L}_{\rm em} = \mathcal{L}_{\rm em}^{\rm NL} + \mathcal{L}_{\rm kin}^{\rm gauge} + \mathcal{L}_{D^* D\psi_n\gamma} + \mathcal{L}_{D^* D\gamma}.$$

$$C_{\mu\nu}^{\rm C;\,gauge}(x_1,x_2) = e^{ieI(x_1,x_2;P)} D_\mu^{*\,-}(x_1)[\partial_\nu - ieA_\nu(x_2)]D^+(x_2) + e^{-ieI(x_1,x_2;P)} D_\nu^{*\,+}(x_1)[\partial_\mu + ieA_\mu(x_2)]D^-(x_2)$$

$$I(x,y;P) = \int_y^x dz_\mu A^\mu(z)\,.$$

$$\mathcal{L}_{\rm kin}^{\rm gauge} = ieA_\mu(D^- \overleftrightarrow{\partial}^\mu D^+) + ieA_\mu[-D_\alpha^{*\,-}\overleftrightarrow{\partial}^\mu D^{*\,+\,\alpha} + \frac{1}{2}D_\alpha^{*\,-}\overleftrightarrow{\partial}^\alpha D^{*\,+\mu} + \frac{1}{2}D^{*\,-\,\mu}\overleftrightarrow{\partial}_\alpha D^{*\,+\,\alpha}]$$

$$\mathcal{L}_{D^* D\gamma} = eg_{D^{*+}D^+\gamma}\epsilon^{\mu\nu\alpha\beta}D^+ D_{\mu\nu}^{*\,-}F_{\alpha\beta} - eg_{D^{*0}D^0\gamma}\epsilon^{\mu\nu\alpha\beta}D^0 \bar{D}_{\mu\nu}^{*\,0}F_{\alpha\beta}$$

A. Radiative $X(3872) \to \gamma J/\psi$ and $X(3872) \to \gamma\psi(2S)$ decays

$$\begin{aligned} iM_{XV\gamma} &= ieM^{\mu\nu\alpha\beta}_{XV\gamma}\epsilon_{\mu\nu}(p)\epsilon^*_\alpha(q_1)\epsilon^*_\beta(q_2) \\ &= ie\Big\{\alpha^{(1)}_{XV\gamma}\Big[\epsilon^{\mu\sigma\alpha\rho}g^{\nu\beta}p_\sigma q_\rho + (\mu\leftrightarrow\nu)\Big] + \alpha^{(2)}_{XV\gamma}\Big[\epsilon^{\mu\sigma\rho\beta}p_\sigma q_\rho[g^{\nu\alpha} - \frac{(p+q)^\nu q^\alpha}{q\cdot(p+q)}] + (\mu\leftrightarrow\nu)\Big] \\ &+\alpha^{(3)}_{XV\gamma}\Big[\epsilon^{\mu\sigma\alpha\beta}(p+q)_\sigma q^\nu + (\mu\leftrightarrow\nu)\Big] + \alpha^{(4)}_{XV\gamma}\Big[\epsilon^{\mu\sigma\alpha\rho}p_\sigma q_\rho q^\nu q^\beta + (\mu\leftrightarrow\nu)\Big] \\ &+\alpha^{(5)}_{XV\gamma}\Big[\epsilon^{\sigma\rho\alpha\beta}p_\sigma q_\rho q^\mu q^\nu\Big]\Big\}\epsilon_{\mu\nu}(p)\epsilon^*_\alpha(q_1)\epsilon^*_\beta(q_2), \end{aligned} \quad (30)$$

$$\Gamma_{XV\gamma} = \frac{\alpha_{\rm em}}{10m_X^2}\Big[\sum_{i\le j=5}\alpha^{(i)}_{XV\gamma}\alpha^{(j)}_{XV\gamma}C^{ij}_{XV\gamma}\Big]P^*_\gamma,$$

B. Strong $X(3872) \to J/\psi h$ decays

$$iM(X \to J/\psi h) \;=\; iM^{\alpha}(X \to J/\psi V)\frac{1}{q_1^2 - m_V^2 + im_V\Gamma_V}(g_{\alpha\sigma} - \frac{q_{1;\alpha}q_{1;\sigma}}{m_V^2})M^{\sigma}(V \to h),$$

$$iM(V \to h) \;=\; i\epsilon_{\sigma}M^{\sigma}(V \to h),$$

$$\begin{aligned}
iM^{\alpha}(X \to J/\psi V) \;&=\; iM^{\mu\nu\alpha\beta}_{X\psi V}(m_X^2, m_{J/\psi}^2, q^2)\epsilon_{\mu\nu}(p)\epsilon^*_{\beta}(q_2) \\
&=\; i\Big\{G^{(1)}_{X\psi V}\Big[\epsilon^{\mu\sigma\alpha\rho}g^{\nu\beta}p_{\sigma}q_{\rho} + (\mu \leftrightarrow \nu)\Big] + G^{(2)}_{X\psi V}\Big[\epsilon^{\mu\sigma\rho\beta}g^{\nu\alpha}p_{\sigma}q_{\rho} + (\mu \leftrightarrow \nu)\Big] \\
&\quad + G^{(3)}_{X\psi V}\Big[\epsilon^{\mu\sigma\alpha\beta}p_{\sigma}q^{\nu} + (\mu \leftrightarrow \nu)\Big] + G^{(4)}_{X\psi V}\Big[\epsilon^{\mu\sigma\alpha\beta}q_{\sigma}q^{\nu} + (\mu \leftrightarrow \nu)\Big] \\
&\quad + G^{(5)}_{X\psi V}\Big[\epsilon^{\mu\sigma\alpha\rho}p_{\sigma}q_{\rho}q^{\nu}q^{\beta} + (\mu \leftrightarrow \nu)\Big] + G^{(6)}_{X\psi V}\Big[\epsilon^{\mu\sigma\rho\beta}p_{\sigma}q_{\rho}q^{\nu}q^{\alpha} + (\mu \leftrightarrow \nu)\Big] \\
&\quad + G^{(7)}_{X\psi V}\Big[\epsilon^{\sigma\rho\alpha\beta}p_{\sigma}q_{\rho}q^{\mu}q^{\nu}\Big]\Big\}\epsilon_{\mu\nu}(p)\epsilon^*_{\beta}(q_2),
\end{aligned}$$

$$\frac{d\Gamma}{dq_1}(X \to J/\psi h) \;=\; \frac{1}{80\pi^2 m_X^3}\frac{q_1\lambda^{1/2}(m_X^2, m_{\psi}^2, m_V^2)}{(q_1^2 - m_V^2)^2 + m_V^2\Gamma_V^2}\Big(\sum_{i\leq j=1}^{7} C_{ij}G^{(i)}_{X\psi V}G^{(j)}_{X\psi V}\Big)\Big(\int d\Pi F(V \to h)\Big),$$

C. Strong $X(3872) \to \chi_{cJ}\pi^0$ decays

$$iM(X \to \chi_{c0}\pi^0) = iM^{\mu\nu}_{X\chi_{c0}\pi^0}\epsilon_{\mu\nu}(p) = iG_{X\chi_{c0}\pi^0}q^\mu q^\nu \epsilon_{\mu\nu}(p),$$

$$iM(X \to \chi_{c1}\pi^0) = iM^{\mu\nu\alpha}_{X\chi_{c1}\pi^0}\epsilon_{\mu\nu}(p)\epsilon^*_\alpha(q) = iG_{X\chi_{c1}\pi^0}\Big[\epsilon^{\mu\alpha\theta\tau}q^\nu + \epsilon^{\nu\alpha\theta\tau}q^\mu\Big]p_\theta q_\tau \epsilon_{\mu\nu}(p)\epsilon^*_\alpha(q),$$

$$\begin{aligned} iM(X \to \chi_{c2}\pi^0) &= iM^{\mu\nu\alpha}_{X\chi_{c2}\pi^0}\epsilon_{\mu\nu}(p)\epsilon^*_{\alpha\beta}(q) \\ &= i\Big\{G^{(1)}_{X\chi_{c2}\pi^0}q^\mu q^\nu p^\alpha p^\beta + G^{(2)}_{X\chi_{c2}\pi^0}(g^{\mu\alpha}g^{\nu\beta} + g^{\nu\alpha}g^{\mu\beta}) \\ &\quad + G^{(3)}_{X\chi_{c2}\pi^0}\Big[(g^{\mu\alpha}q^\nu + g^{\nu\alpha}q^\mu)p^\beta + (\alpha \leftrightarrow \beta)\Big]\Big\}\epsilon_{\mu\nu}(p)\epsilon^*_{\alpha\beta}(q), \end{aligned}$$

$$\Gamma(X \to \chi_{c0}\pi^0) = \frac{G^2_{X\chi_{c0}\pi^0}}{60\pi m_X^6}\lambda^2(m_X^2, m^2_{\chi_{c0}}, m^2_{\pi^0})P^*_{\chi_{c0}},$$

$$\Gamma(X \to \chi_{c1}\pi^0) = \frac{G^2_{X\chi_{c1}\pi^0}}{160\pi m_X^4}\lambda^2(m_X^2, m^2_{\chi_{c1}}, m_{\pi^0})P^*_{\chi_{c1}},$$

$$\Gamma(X \to \chi_{c2}\pi^0) = \frac{1}{40\pi m_X^2}\Big[\sum_{i\le j=1}^{3} G^{(i)}_{X\chi_{c2}\pi^0}G^{(j)}_{X\chi_{c2}\pi^0}C^{ij}_{X\chi_{c2}\pi^0}\Big]P^*_{\chi_{c2}},$$